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Status of Higher Order QCD Calculations

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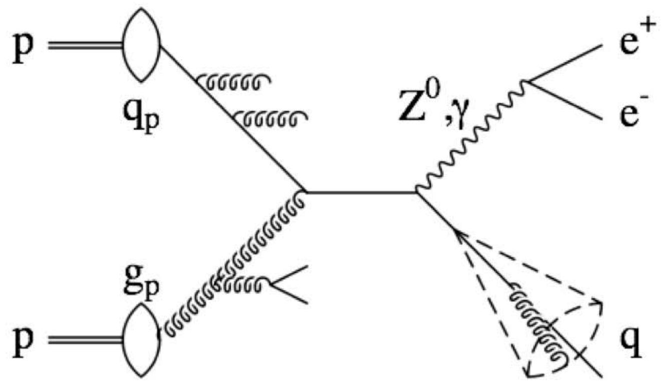
QCD at High Energy Colliders

- QCD: successful theory of strong interactions
- QCD is omnipresent in high energy collisions



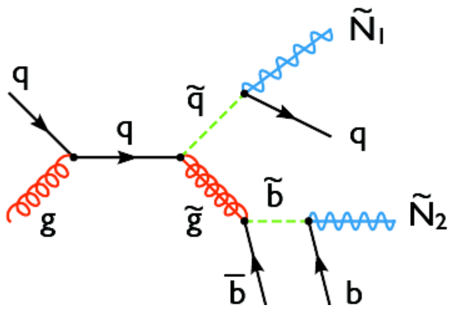
QCD effects
- initial state: parton distributions
- final state: jets
- hard scattering matrix elements with multiple radiation

- Detailed understanding of QCD mandatory for
  - Interpretation of collider data
  - Precision studies
  - Searches for new physics

Expectations at LHC

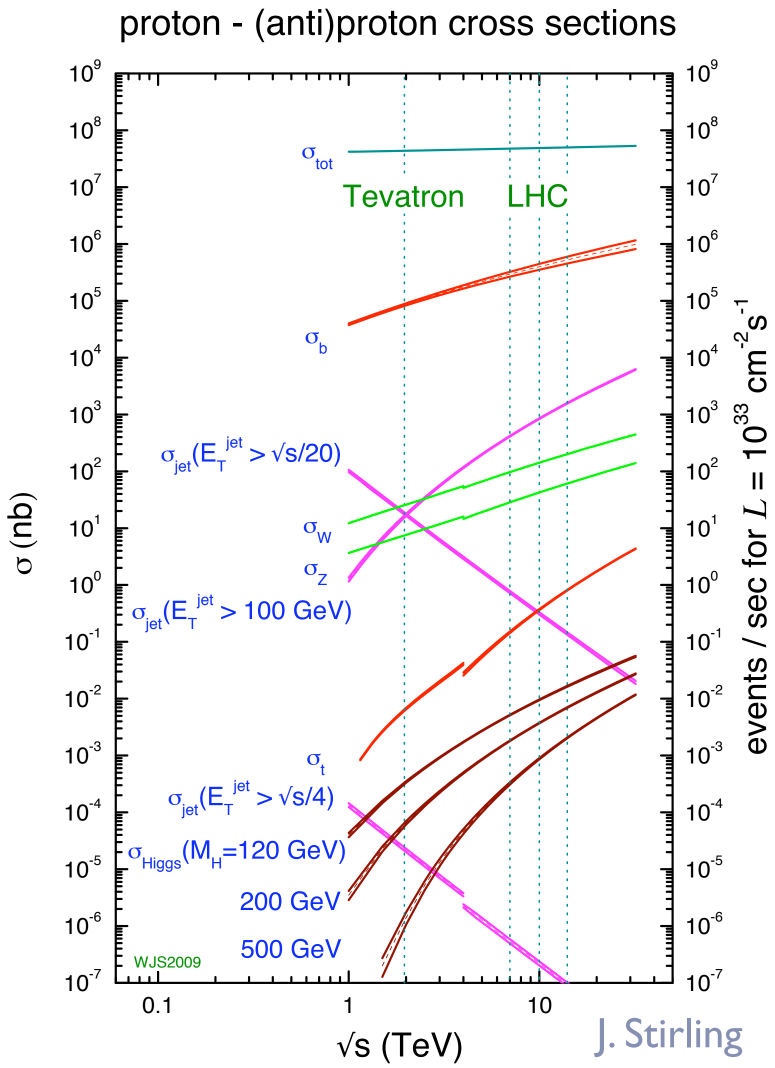
- LHC brings new frontiers in energy and luminosity
- Production of short-lived heavy states (Higgs, SUSY,…)
  - detected through their decay products
  - yield multi-particle final states involving jets, leptons, γ , $\not{E}_T$
- Search for new effects in multi-particle final states
  - typically involving jets
  - need to understand signal and background processes
- Require precise predictions: NLO

Example: SUSY signature:
3j + $\not{E}_T$



Expectations at LHC

- Large production rates for Standard Model processes
  - jets
  - top quark pairs
  - vector bosons
- Allow precision measurements
  - masses
  - couplings
  - parton distributions
- Require precise theory: NNLO



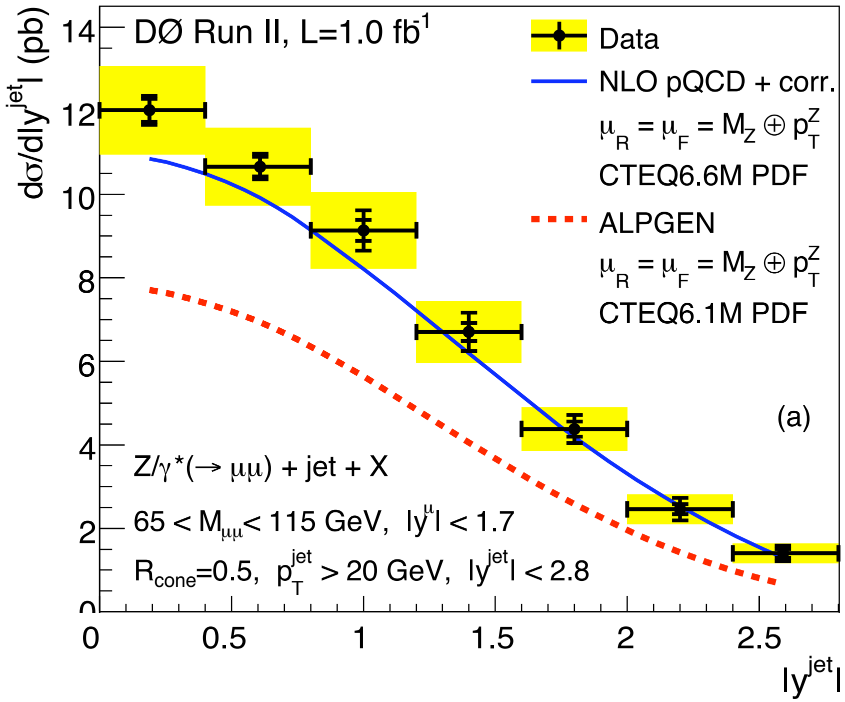
J. Stirling

Outline

- Multiparticle production at NLO
- Precision observables at NNLO

NLO Multiparticle Production

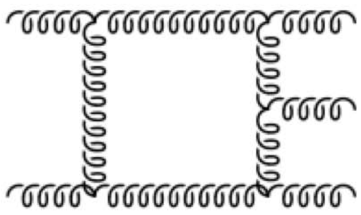
- Why NLO?
  - reduce uncertainty of theory prediction
  - reliable normalization and shape
  - accounts for effects of extra radiation
  - jet algorithm dependence

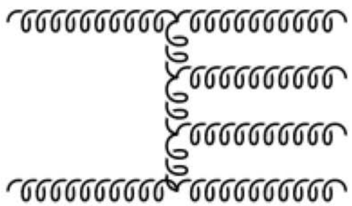


- Example: Z+j at Tevatron
  - NLO error: ~15%
  - substantial NLO effect
  - correction not constant

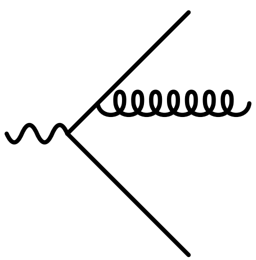
NLO Multiparticle Production

- Require two principal ingredients (here: pp → 3j)
  - one-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all 2 → 2 processes
      - known for many 2 → 3 processes
      - current frontier 2 → 4: major challenge
  - tree-level matrix elements
    - implicit poles from soft/collinear emission





- Implicit poles appear only upon phase space integration



$$\sim \frac{1}{s_{qg}} = \frac{1}{2E_q E_g (1-\cos\Theta_{qg})}$$

Soft gluon: $E_g \to 0$

Collinear qg: $\cos\Theta_{qg} \to 1$

NLO Multiparticle Production

- Structure of NLO m-jet cross section

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{R}_{NLO} - \mathrm{d}\sigma^{S}_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{S}_{NLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V}_{NLO}\right]$$

- General methods for combining virtual and real emission:
  - extract process-independent implicit poles from real emission
    - residue subtraction (S. Frixione, Z. Kunszt, A. Signer)
    - dipole subtraction (S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi)
    - antenna subtraction
      (D. Kosower; J. Campbell, M. Cullen, E.W.N. Glover; A. Daleo, T. Gehrmann, D. Maitre, M. Ritzmann, AG)

NLO Multiparticle Production

- Automated subtraction tools
  - dipole method
    - SHERPA (T. Gleisberg, F. Krauss)
    - MadDipole (R. Frederix, T. Gehrmann, N. Greiner)
    - TeVJet (M. Seymour, C. Tevlin)
    - Helac/Phegas (M. Czakon, C. Papadopoulos, M. Worek)
  - residue method
    - MadFKS (R. Frederix, S. Frixione, F. Maltoni, T. Stelzer)

Bottleneck up to now: one-loop multileg matrix elements

NLO: One-loop multi-leg amplitudes

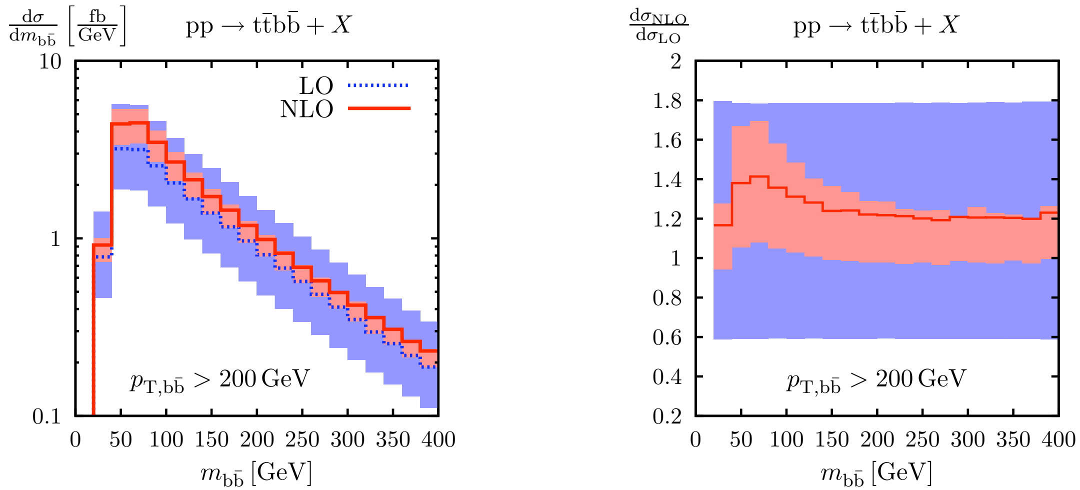
- General structure

$$\mathcal{A} = \sum_i d_i \text{ Box}_i + \sum_i c_i \text{ Triangle}_i + \sum_i b_i \text{ Bubble}_i + \sum_i a_i \text{ Tadpole}_i + R$$

- One-loop scalar integrals known analytically
  (G. Passarino, M.Veltman; G.J. van Oldenbourgh; K. Ellis, G. Zanderighi; A. Denner, S. Dittmaier)
- Task: compute integral coefficients
- Challenges
  - complexity: number of diagrams, number of scales
  - stability: linear dependence among external momenta
- Enormous progress using two approaches
  - traditional: Feynman diagram based
  - unitarity based: reconstruct integral coefficients from cuts

NLO multi-leg: traditional approach

- Based on one-loop Feynman diagrams
  - contain high-rank tensor integrals
  - reduced to basis integrals: with analytical (A. Denner, S. Dittmaier) or semi-numerical (GOLEM: T. Binoth, J.P. Guillet, G. Heinrich, E. Pilon, C. Schubert) approach
- Successfully applied in first complete 2 → 4 calculation: $pp \to t\bar{t}b\bar{b}$ (A. Bredenstein, A. Denner, S. Dittmaier, S. Pozzorini)



NLO multi-leg: unitarity-based method

- Generalized unitarity
  - apply multi-particle cuts: one or more loop propagators on-shell (Z. Bern, L. Dixon, D. Dunbar, D. Kosower, R. Britto, F. Cachazo, B. Feng; P. Mastrolia; D. Forde)
  - result: integral coefficients are products of tree-level amplitudes evaluated at complex momenta
- Reduction at integrand level (OPP: G. Ossola, C. Papadopoulos, R. Pittau)
- Rational terms not determined by unitarity
  - Special recursion relations (C. Berger et al.)
  - Feynman diagram approach (OPP)
  - D-dimensional unitarity (R. Ellis, W. Giele, Z. Kunszt, K. Melnikov)
- Algorithmic procedure: can be automated

Automating NLO calculations

- Virtual corrections: implementations
  - semi-numerical form factor decomposition: GOLEM
    (T. Binoth, J.P. Guillet, G. Heinrich, E. Pilon, T. Reiter)
  - unitarity and multi-particle cuts: BlackHat
    (C.F. Berger, Z. Bern, L.J. Dixon, F. Febres Cordero, D. Forde, H. Ita, D.A. Kosower, D. Maitre)
  - reduction at integrand level: CutTools (G. Ossola, C. Papadopoulos, R. Pittau)
  - generalized D-dimensional unitarity: Rocket (W. Giele, G. Zanderighi)
  - generalized D-dimensional unitarity: Samurai
    (P. Mastrolia, G. Ossola, T. Reiter, F. Tranmontano)
  - several more packages in progress
    (A. Lazopoulos; W. Giele, Z. Kunszt, J. Winter; K. Melnikov, M. Schulze)
- NLO multileg results available

The Les Houches Wish List (2010)

2010

process wanted at NLO	background to
1. $pp \to VV + $ jet	$t\bar{t}H$, new physics Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi
2. $pp \to H + 2$ jets	H in VBF Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$ Bredenstein, Denner Dittmaier, Pozzorini; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$ Bevilacqua, Czakon, Papadopoulos, Worek
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$ VBF: Bozzi, Jäger, Oleari, Zeppenfeld
7. $pp \to V + 3$ jets	new physics Berger Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi
8. $pp \to VVV$	SUSY trilepton Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld; Binoth, Ossola, Papadopoulos, Pittau
9. $pp \to b\bar{b}b\bar{b}$	Higgs, new physics GOLEM

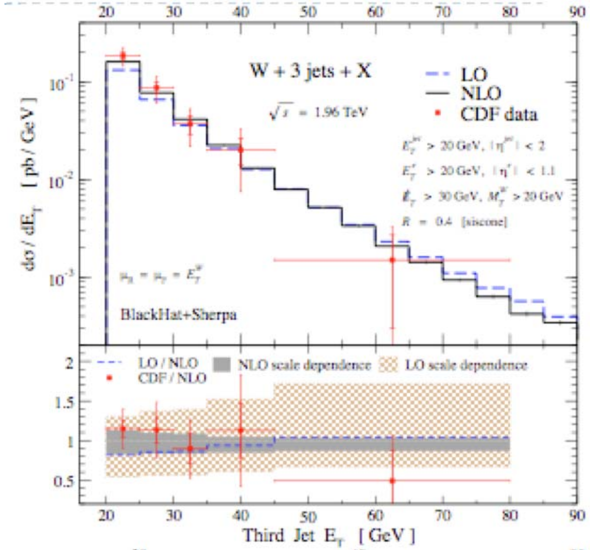
Feynman diagram methods

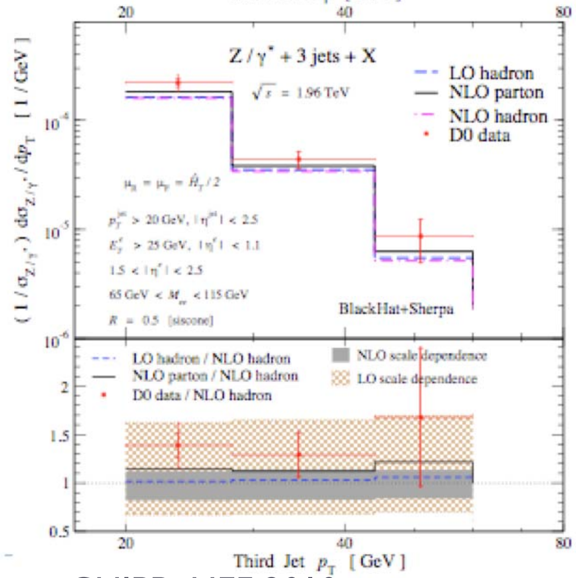
now joined by

unitarity based methods

NLO multileg: $W^{\pm}$ + 3j, Z^{0} + 3j

- Calculations of $W^{\pm}$ + 3j
  - Blackhat + Sherpa (C.F. Berger, Z. Bern, L. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D.A. Kosower, D. Maitre)
  - Rocket (R.K. Ellis, K. Melnikov, G. Zanderighi)
- excellent description of Tevatron data
  - moderate corrections
  - precise predictions
  - rich phenomenology
- Calculation of Z^{0} + 3j (Blackhat + Sherpa)
- Ongoing: $W^{\pm}$ + 4j (Blackhat + Sherpa)





Outline

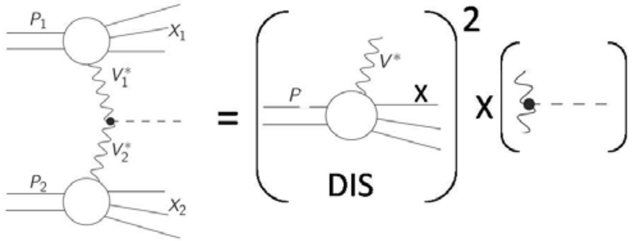
- Multiparticle production at NLO
- Precision observables at NNLO

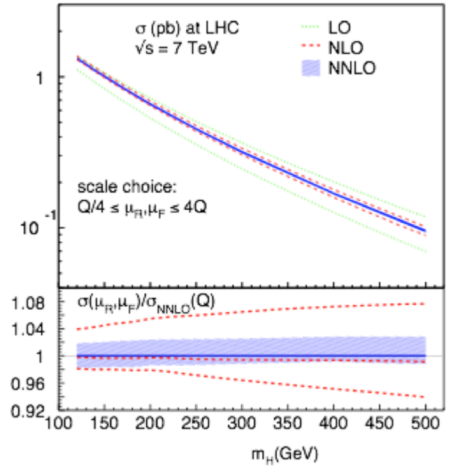
Where are NNLO corrections needed?

- Processes measured to few per cent accuracy
  - $e^+e^- \to 3$ jets
  - 2+1 jet production in deep inelastic scattering
  - hadron collider processes:
    - jet production
    - vector boson (+jet) production
    - top quark pair production
- Processes with potentially large perturbative corrections
  - Higgs or vector boson pair production
- Require NNLO corrections for
  - meaningful interpretation of experimental data
  - precise determination of fundamental parameters

What is known to NNLO?

- fully inclusive observables
  - total cross sections: R-ratio, Drell-Yan and Higgs production
  - structure functions in deep inelastic scattering
  - evolution of parton distributions
  - Higgs production in vector boson fusion (P. Bolzoni, F. Maltoni, S. Moch, M. Zaro)

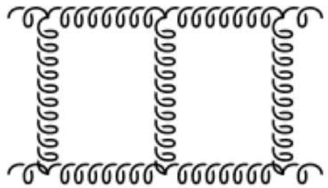


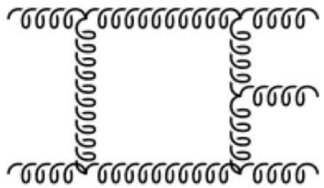


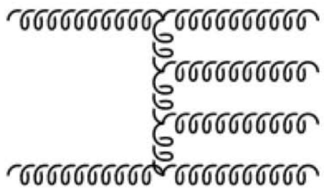
- single differential observables
  - rapidity distribution in Drell-Yan process (C. Anastasiou, L. Dixon, K. Melnikov, F. Petriello)
- fully differential observables
  - colourless high mass system including decays
  - jet production

NNLO calculations

- Require three principal ingredients (here: pp → 2j)
  - two-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all massless 2 → 2 processes
  - one-loop matrix elements
    - explicit infrared poles from loop integral
    - and implicit poles from soft/collinear emission
      - usually known from NLO calculations
  - tree-level matrix elements
    - implicit poles from two partons unresolved
      - known from LO calculations



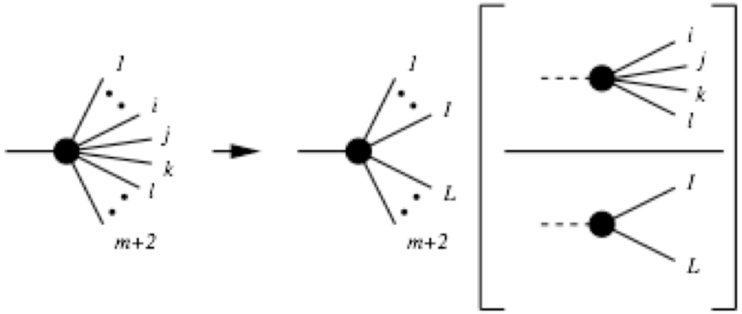




- Challenge: combine contributions into parton-level generator
- need method to extract implicit infrared poles

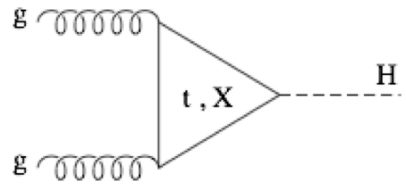
NNLO calculations

- Solutions
  - sector decomposition: expansion in distributions, numerical integration (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello; M. Czakon)
  - subtraction: add and subtract counter-terms: process-independent approximations in all unresolved limits, analytical integration
    - several well-established methods at NLO
    - NNLO for specific hadron collider processes: q_T subtraction
      (S. Catani, M. Grazzini)
    - NNLO for jet observables in e^+e^- processes: antenna subtraction
      (T. Gehrmann, E.W.N. Glover, AG)



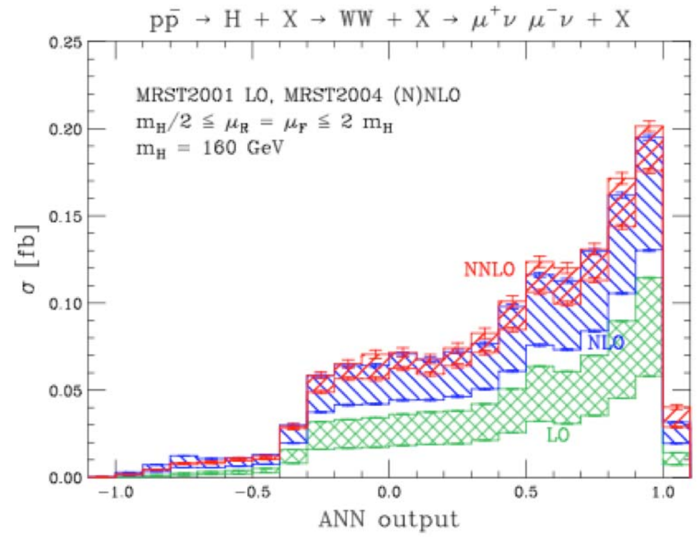
Higgs boson production at NNLO

- Dominant production process: gluon fusion



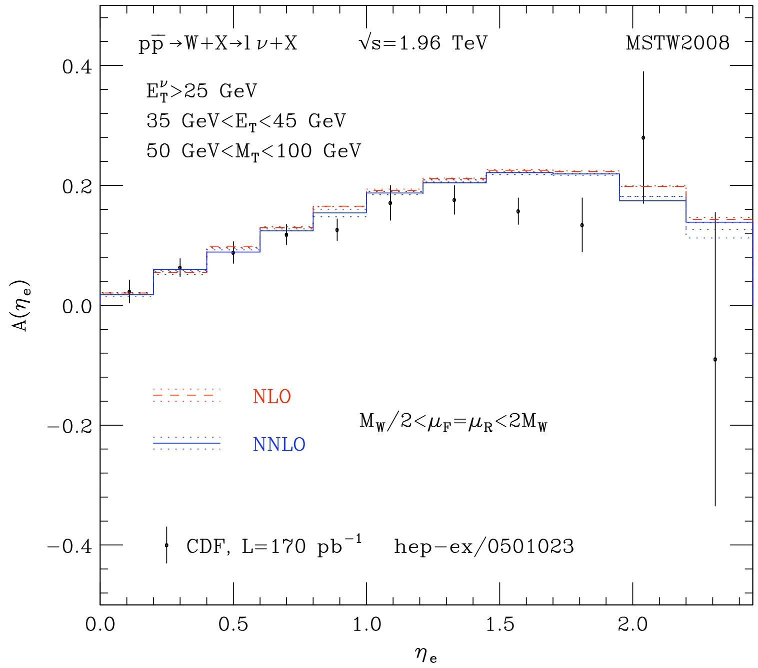
- exclusive calculations to NNLO, including H decay
  - using sector decompostion (C. Anastasiou, K. Melnikov, F. Petriello)
  - using q_T-subtraction (S. Catani, M. Grazzini)

- Application: Higgs at Tevatron $H \to WW \to l\nu\ l\nu$
  - all distributions to NNLO (C. Anastasiou, G. Dissertori, M. Grazzini, F. Stöckli, B. Webber)
  - cuts on jet activity
  - neural-network output to NNLO



Vector boson production at NNLO

- Fully exclusive calculations
  - parton-level event generator
    - using sector decomposition (K. Melnikov, F. Pertriello)
    - using q_T subtraction (S. Catani, L. Cieri, G. Ferrera, D. de Florian, M. Grazzini)
  - including vector boson decay
  - allowing arbitrary final-state cuts
- Application: lepton charge asymmetry (S. Catani, G. Ferrera, M. Grazzini)
  - small NNLO corrections
  - determine quark distributions

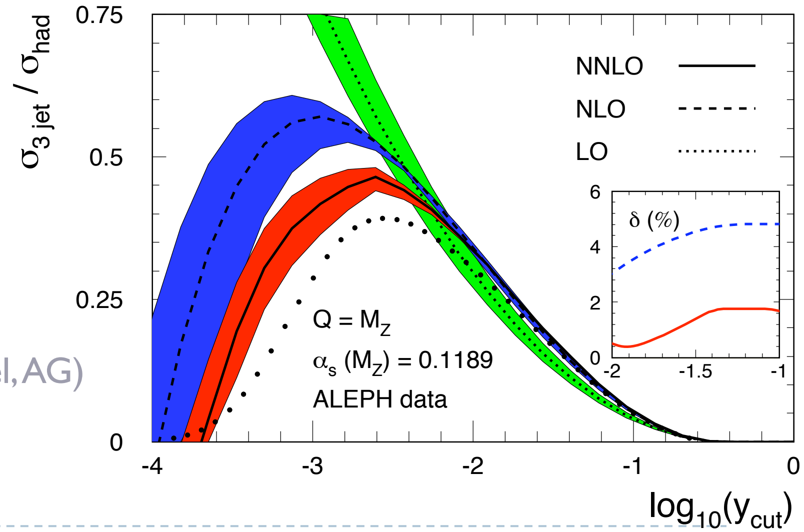


Jet production at NNLO: e^+e^- collisions

- Two calculations of NNLO corrections to $e^+e^- \to$ 3 jets
  - using antenna subtraction (T. Gehrmann, E.W.N. Glover, G. Heinrich, AG; S. Weinzierl)
  - as parton-level event generator
  - allow evaluation of event shapes and jet rates
- improved description of data with reduced scale uncertainty
  - one per cent for three-jet rate
- use to extract α_s from LEP data:

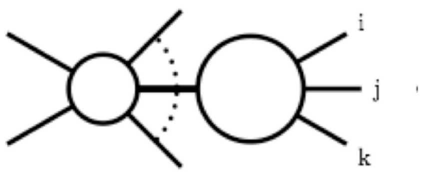
$\alpha_s(M_Z) = 0.1175 \pm 0.0020(\mathrm{exp}) \pm 0.0015(\mathrm{th})$

(G. Dissertori, T. Gehrmann, E.W.N. Glover, G. Heinrich, H. Stenzel, AG)

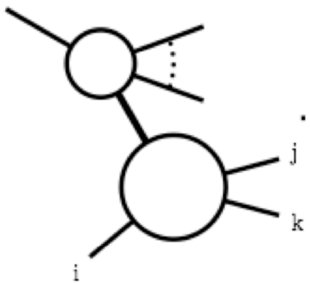


NNLO jet cross sections at hadron colliders

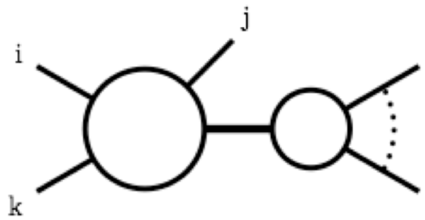
- two-loop matrix elements known for
  - two-jet production
    (C. Anastasiou, E.W.N. Glover, C. Oleari, M.E. Tejeda-Yeomans; Z. Bern, A. De Freitas, L. Dixon)
  - vector-boson-plus-jet production (T. Gehrmann, E. Remiddi)
  - (2+1) jet production in DIS (T. Gehrmann, E.W.N. Glover)
- antenna subtraction formalism at NNLO: with radiators in initial state



final-final



initial-final



initial-initial

NNLO jet cross sections at hadron colliders

- First implementation of antenna subtraction
  - $gg \to 4g$ subtraction constructed and tested (E.W.N. Glover, J. Pires)
- Integration of antenna functions
  - final-final antennae known
  - initial-final antennae derived recently:
    sufficient for (2+1) jets in DIS (A. Daleo, T. Gehrmann, G. Luisoni, AG)
  - initial-initial in progress (R. Boughezal, M. Ritzmann, AG)

Conclusions and Outlook

- QCD is crucial for the success of LHC physics
  - interpretation of collider data
  - searches for new physics
  - precision studies
- Particle theory is getting ready
  - impressive progress in automated multiparticle NLO cross sections
  - high precision NNLO calculations for fully differential observables in benchmark processes are in progress