Off-shell effects in top production at NLO: the $e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ case

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Heavy Particles at the LHC

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Outline

Introduction

- precise predictions for $t\bar{t}$ production: present status
- Towards a complete $\mathcal{O}(\alpha_{EM}^4 \alpha_S^3)$ calculation

Theoretical framework

- The `HELAC-NLO` system

Phenomenological results

- a NLO study of $W^+W^-b\bar{b}$ with full leptonic decays: comparative analysis at Tevatron and LHC
- study of the narrow-width limit

Introduction

The subject of precise predictions for $t\bar{t}$ production is relevant for the physics program of both Tevatron and LHC and has been widely investigated since many years. Impressive progress has been achieved in several directions...

Inclusive production $pp(p\bar{p}) \to t\bar{t} + X$

- NLO QCD + EW — Nason *et al.* (1989); Beenakker *et al.* (1991); Frixione *et al.* (1995) ... Beenakker *et al.* (1994); Bernreuther *et al.* (2005); Kühn *et al.* (2005); ...
- NNLO QCD — Czakon *et al.* (2007); Bonciani *et al.* (2008); Körner *et al.* (2008) ...
- resummation NLL + NNLL — Kidonakis *et al.* (1997); Bonciani *et al.* (1998); ... Beneke *et al.* (2009); Czakon *et al.* (2009); ...

On-shell production × decay $pp(p\bar{p}) \to t\bar{t} \to \ell^+\nu_\ell\ell^-\bar{\nu}_\ell b\bar{b} + X$

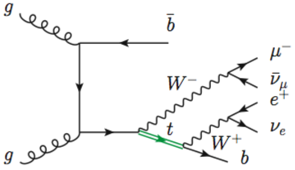
- NLO QCD and EW+QCD — Bernreuther *et al.* (2001); Melnikov *et al.* (2009) ... Bernreuther *et al.* (2010)

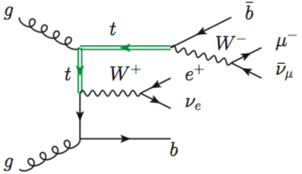
Tipically, $t\bar{t}$ production is restricted to *on-shell* states. Top quark decays, when available, are treated in the narrow-width approximation (NWA).

Full calculations just appeared: Denner *et al.* arXiv:1012.3975; Bevilacqua *et al.* arXiv:1012.4230

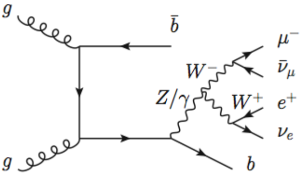
Representative examples of off-shell contributions to $t\bar{t}$ production

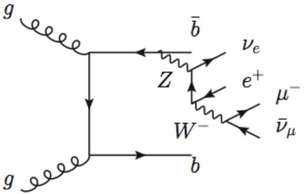
Single-resonant





Non-resonant



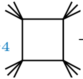
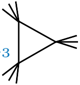
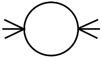
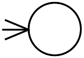


Theoretical framework

Theoretical framework - virtual corrections

We organize our one-loop calculation within the framework of the OPP method :

1. decompose amplitudes into a basis of scalar integrals:

$\mathcal{A} = \sum d_{i_1 i_2 i_3 i_4}$  $+ \sum c_{i_1 i_2 i_3}$  $+ \sum b_{i_1 i_2}$  $+ \sum a_{i_1}$  $+ R$

$a, b, c, d \rightarrow$ cut-constructible part $\qquad R \rightarrow$ rational terms

$$\mathcal{A} = \sum_{I \subset \{0,1,\cdots,m-1\}} \int \frac{\mu^{(4-d)} d^d q}{(2\pi)^d} \frac{\bar{N}_I(\bar{q})}{\prod_{i \in I} \bar{D}_i(\bar{q})}$$

Theoretical framework - virtual corrections

We organize our one-loop calculation within the framework of the OPP method :

2. evaluate the coefficients of the expansion at the *integrand* level:

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0,i_1,i_2,i_3) + \tilde{d}(q;i_0,i_1,i_2,i_3)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0,i_1,i_2) + \tilde{c}(q;i_0,i_1,i_2)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0,i_1) + \tilde{b}(q;i_0,i_1)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q;i_0)\right] \prod_{i\neq i_0}^{m-1} D_i
\end{aligned}
$$

$\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$ are ”spurious” terms (vanish upon integration). Their q-dependence is known

Ossola, Papadopoulos and Pittau, Nucl. Phys. B 763, 147 (2007)

Theoretical framework - virtual corrections

We organize our one-loop calculation within the framework of the OPP method :

3. compute the rational terms $R = R_1 + R_2$:

- R_1: originates from the ϵ dependence of *denominators*

$$D_i \to \bar{D}_i - \tilde{q}^2$$

$\hookrightarrow$ computable within the framework of OPP reduction

Ossola, Papadopoulos and Pittau, JHEP 0805 (2008) 004

- R_2: originates from the ϵ dependence of *numerators*

$$\bar{q} = q + \tilde{q} \qquad \bar{\gamma}_\mu = \gamma_\mu + \tilde{\gamma}_\mu \qquad \bar{g}^{\mu\nu} = g^{\mu\nu} + \tilde{g}^{\mu\nu}$$

$\hookrightarrow$ computable with effective tree-level Feynman rules

Draggiotis, Garzelli, Papadopoulos and Pittau, arXiv:0903:0356 [hep-ph]

Garzelli, Malamos and Pittau, arXiv:0910.3130 [hep-ph]

Theoretical framework - complex mass scheme

- Resumed corrections in two-point functions and consistency of scattering matrix elements Veltman 1962
- Naive schemes may violate badly WI and result to obviously wrong results Phys. Lett. B **349** (1995) 367
- Fixed width schemes: partially satisfying WI Argyres *et al.* Phys. Lett. B **358** (1995) 339
- Fermion-loop scheme: WI consistent Beenakker *et al.* Nucl. Phys. B **500** (1997) 255
- CMS: complex renormalized parameters Denner *et al.* Nucl. Phys. B **560** (1999) 33

  Top-quark propagator:

  $(\not{p} - m_t + i\epsilon)^{-1}$ with the resumed one $(\not{p} - \mu_t + i\epsilon)^{-1}$

  $\mu_t^2 = m_t^2 - i m_t \Gamma_t$
- Renormalization Beenakker *et al.* Nucl. Phys. B **653** (2003) 151

  $\frac{\delta m_t}{m_t} = -\frac{\alpha_s}{3\pi}\left[3\Delta + 4\right] \to \frac{\delta \mu_t}{\mu_t}$ $\Delta = \frac{\Gamma(1+\epsilon)}{\epsilon}\left(\frac{4\pi\mu^2}{m_t^2}\right)$ with $m_t^2 \to \mu_t^2$

Theoretical framework - real corrections

Subtraction terms encoding IR/collinear divergences:

$$\sigma^{NLO} = \int_m d\sigma^B + \int_{m+1} d\sigma^R - \int_{m+1} d\sigma^A + \int_{m+1} d\sigma^A + \int_m d\sigma^V$$

$$\hookrightarrow \quad \int d\sigma^B + \int_{m+1} \left[d\sigma^R - d\sigma^D \right] + \int_m \left[d\sigma^V + d\sigma^I + d\sigma^{KP} \right]$$

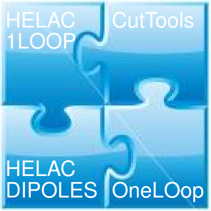
Catani-Seymour dipole formalism

Catani, Seymour, Nucl. Phys. B485, 291 (1997)
Catani, Dittmaier, Seymour, Trocsanyi, Nucl. Phys. B627, 189 (2002)

extended to arbitrary helicity eigenstates of the external partons

Czakon, Papadopoulos, Worek, arXiv:0905.0883 [hep-ph]

The HELAC−NLO system



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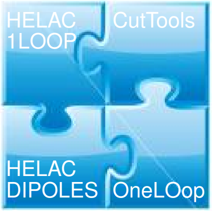
The HELAC−NLO system



HELAC−1LOOP

- evaluation of loop numerators $N(q)$ and rational terms R_2

CutTools

- reduction of tensor integrals, determination of OPP coefficient and R_1

OneLOop

- evaluation of scalar integrals

Ossola, Papadopoulos and Pittau, JHEP **0803** (2008) 042 [arXiv:0711.3596 [hep-ph]]
Van Hameren, Papadopoulos and Pittau, JHEP **0909** (2009) 106 [arXiv:0903.4665 [hep-ph]]

HELAC−DIPOLES

- Catani-Seymour dipole subtraction for massless and massive cases

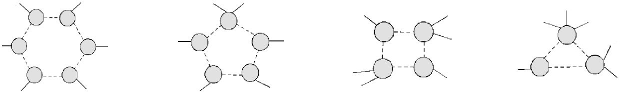
Czakon, Papadopoulos and Worek, JHEP **0908**, 085 (2009) [arXiv:0905.0883 [hep-ph]]

Integration over phase space performed with KALEU (adaptive) and cross-checked with PHEGAS (multichannel)

The one-loop calculation in a nutshell

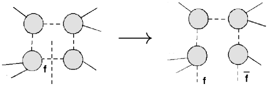
The computation of $pp(p\bar{p}) \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ involves up to six-point functions.
The most generic integrand has therefore the form

$$\mathcal{A}(q) = \sum \underbrace{\frac{N_i^{(6)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_5}}} + \underbrace{\frac{N_i^{(5)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_4}}} + \underbrace{\frac{N_i^{(4)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_3}}} + \underbrace{\frac{N_i^{(3)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}}} + \cdots$$



In order to apply the OPP reduction, HELAC evaluates numerically the numerators $N_i^6(q), N_i^5(q), ...$ with the values of the loop momentum q provided by CutTools

- generates all inequivalent partitions of 6,5,4,3... blobs attached to the loop, and check all possible flavours (and colours) that can be consistently running inside
- hard-cuts the loop (q is fixed) to get a $n+2$ *tree-like* process



The R_2 contributions (rational terms) are calculated in the same way as the tree-order amplitude, taking into account *extra vertices*

Other features

Finite width

- complex-mass scheme adopted for intermediate top quarks

Colour/helicity Monte Carlo

- sum over colours and helicities performed with Monte Carlo sampling

One-loop reweighting

- LO+V result obtained by *re-weighting* a sample of *tree-level unweighted* events

Stability checks

- check of Ward identity for virtual corrections
- check of independence on α_{max} cutoff for real corrections

Phenomenological results at Tevatron and LHC

Total cross sections

LO and NLO cross sections for different jet algorithms using *inclusive* cuts:

$$p_T(b) > 20 \text{ GeV} \quad p_T(\ell^\pm) > 20 \text{ GeV} \quad \not{p}_T > 30 \text{ GeV}$$
$$|y(b)| < 4.5 \quad |y(\ell^\pm)| < 2.5 \quad \Delta R(jj) > 0.4 \quad \Delta R(j\ell^\pm) > 0.4$$

Tevatron run II	σ^{LO} [fb]	$\sigma^{\mathrm{NLO}}_{\alpha_{\max}=1}$ [fb]	$\sigma^{\mathrm{NLO}}_{\alpha_{\max}=0.01}$ [fb]	K-factor
anti$-k_T$	34.922 ± 0.014	35.705 ± 0.047	35.697 ± 0.049	1.02
k_T	34.922 ± 0.014	35.727 ± 0.047	35.723 ± 0.049	1.02
C/A	34.922 ± 0.014	35.724 ± 0.047	35.746 ± 0.050	1.02

LHC 7 TeV	σ^{LO} [fb]	$\sigma^{\mathrm{NLO}}_{\alpha_{\max}=1}$ [fb]	$\sigma^{\mathrm{NLO}}_{\alpha_{\max}=0.01}$ [fb]	K-factor
anti$-k_T$	550.538 ± 0.175	808.463 ± 0.967	808.291 ± 1.040	1.47
k_T	550.538 ± 0.175	808.665 ± 0.966	808.855 ± 1.025	1.47
C/A	550.538 ± 0.175	808.741 ± 0.968	808.279 ± 1.027	1.47

Total cross sections

LO and NLO cross sections for different jet algorithms using *inclusive* cuts:

$$p_T(b) > 20 \text{ GeV} \quad p_T(\ell^\pm) > 20 \text{ GeV} \quad \not{p}_T > 30 \text{ GeV}$$
$$|y(b)| < 4.5 \quad |y(\ell^\pm)| < 2.5 \quad \Delta R(jj) > 0.4 \quad \Delta R(j\ell^\pm) > 0.4$$

LHC 10 TeV	$\sigma^{\rm LO}$ [fb]	$\sigma^{\rm NLO}_{\alpha_{\rm max}=1}$ [fb]	$\sigma^{\rm NLO}_{\alpha_{\rm max}=0.01}$ [fb]
anti-k_T	1394.72 ± 0.75	1993.3 ± 2.5	1993.9 ± 2.7
k_T	1394.72 ± 0.75	1995.2 ± 2.5	1994.3 ± 2.7
C/A	1394.72 ± 0.75	1995.0 ± 2.5	1994.3 ± 2.7

The narrow-width limit

We compare full results with NWA by studying the narrow-width limit (NWL) of our calculation:

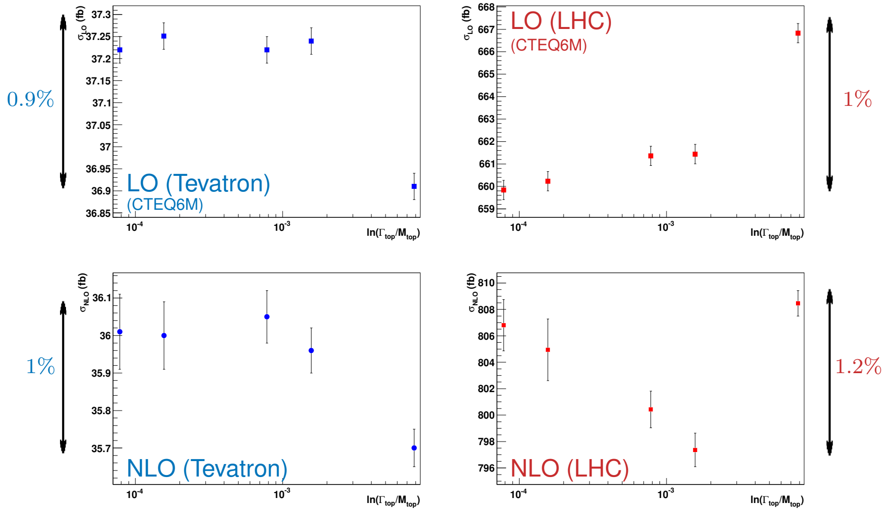
- top quark width (Γ_t) is rescaled by a large factor
- the matrix element is multiplied by the same factor squared in order to preserve proper normalization

The limit $\Gamma_t \to 0$ corresponds to the on-shell $t\bar{t}$ *production* $\times$ *decay* setup

- double-resonant contributions enhanced
- single-resonant, non-resonant contributions suppressed

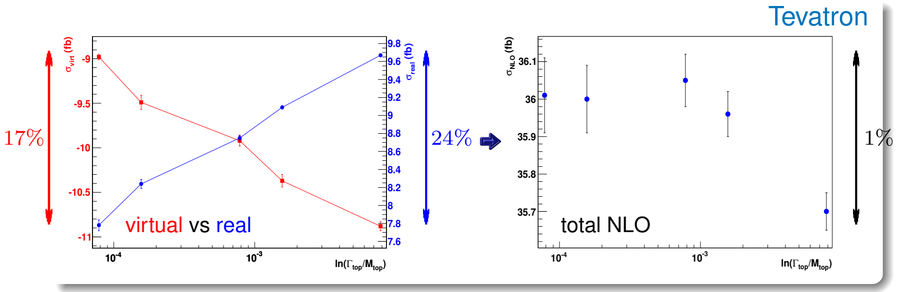
The narrow-width limit

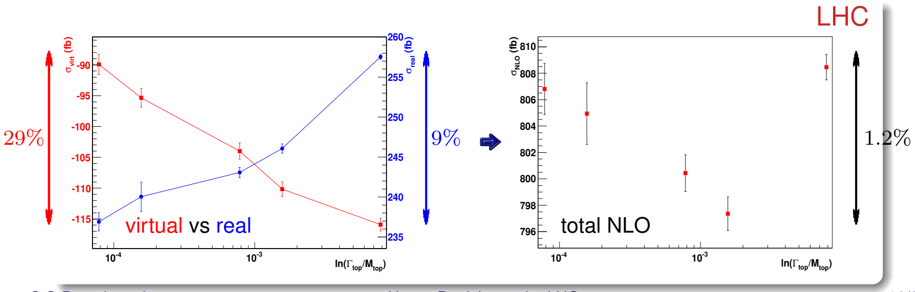
Total cross sections as a function of $\log(\Gamma_{top}/M_{top})$ at LO and NLO



Going from the full result to NWL changes cross sections at the 1% level

Note: taken separately, the virtual and real contributions to the full NLO cross section show a larger variation upon Γ_t rescaling, as expected due to logarithmic enhancements $\sim \log(\Gamma_t/M_t)$

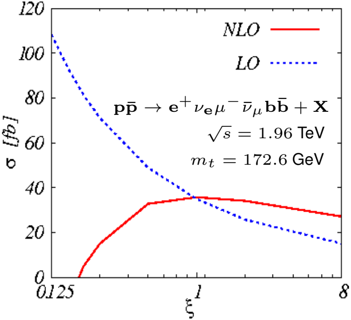




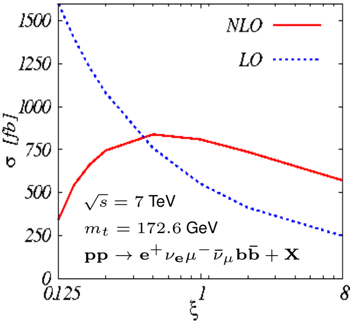
Scale dependence

Total cross section as a function of $\mu_R = \mu_F = \xi m_t$:

Tevatron



LHC



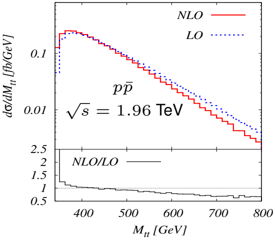
Varying the scale up and down by a factor 2:

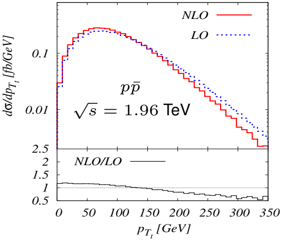
Tevatron	**LO**: $\sigma = 34.922\,^{+40\%}_{-26\%}$ fb	$\to$	**NLO**: $\sigma = 35.727\,^{-8\%}_{-4\%}$ fb
LHC	**LO**: $\sigma = 550.538\,^{+37\%}_{-25\%}$ fb	$\to$	**NLO**: $\sigma = 808.665\,^{+4\%}_{-9\%}$ fb

Differential cross sections

Kinematics of reconstructed top quarks

Tevatron





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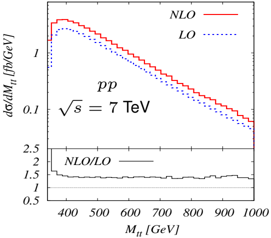
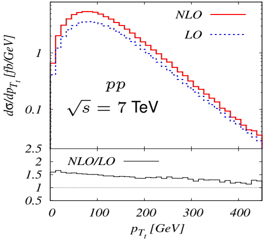
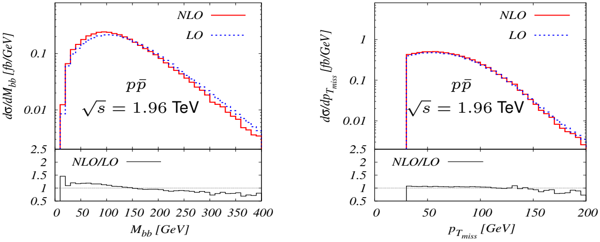




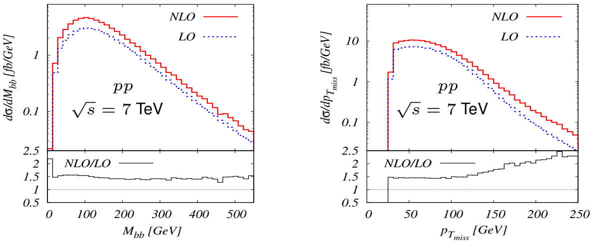
Differential cross sections

Invariant mass of $b\bar{b}$ system, missing p_T

Tevatron



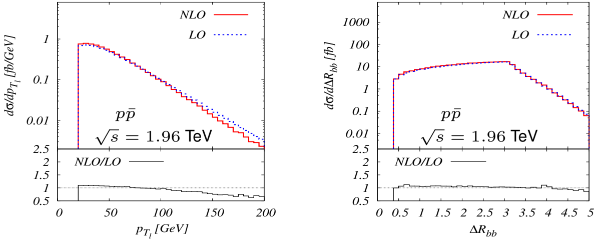
LHC



Differential cross sections

p_{Tl} and ΔR_{bb}

Tevatron



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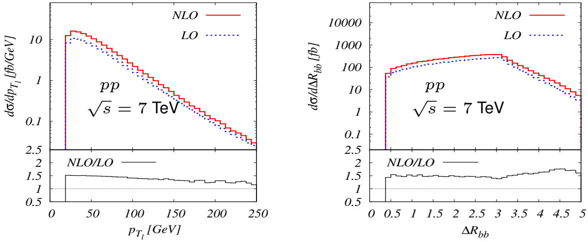
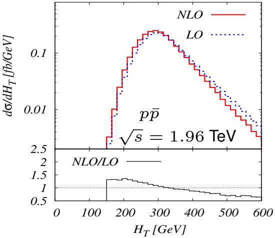
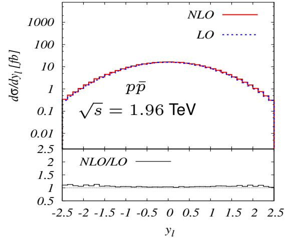


Differential cross sections

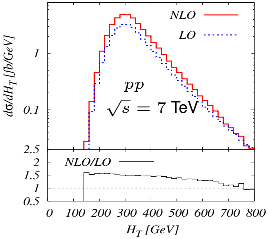
H_T and y_l

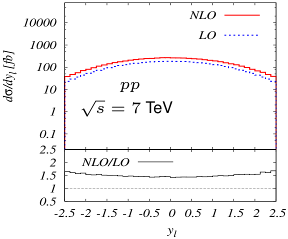
Tevatron





LHC





Top quark asymmetry at the Tevatron

QCD is invariant under charge (C), parity (P) and CP conjugation

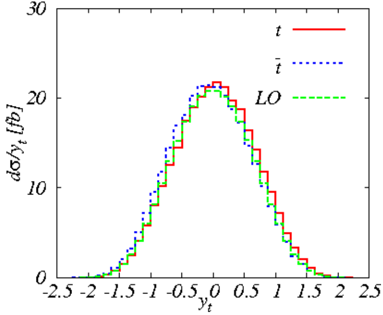
$\hookrightarrow$ $C(P)$ symmetric initial states lead to $C(P)$ symmetric final states

$\therefore$ $gg \to t\bar{t} + X$: t production is symmetric w.r.t. beam direction

$q\bar{q} \to t\bar{t} + X$: t production can show forward/backward asymmetry but $A^t_{FB} = -A^{\bar{t}}_{FB}$ due to CP invariance

- In the Standard Model, QCD predicts that the top quark production angle is forward/backward symmetric at LO
- A small charge asymmetry originates at NLO as a consequence of interference effects at the level of both real-emission and one-loop contributions

Top quark asymmetry at the Tevatron



$$A_{FB}^{t} = \frac{\int_{y_t>0} N_t(y) - \int_{y_t<0} N_t(y)}{\int_{y_t>0} N_t(y) + \int_{y_t<0} N_t(y)}$$

$$A_{FB}^{t} = -A_{FB}^{\bar{t}}$$

Tevatron (CDF): $A_{FB}^{t} = \mathbf{0.193 \pm 0.065^{stat} \pm 0.01^{syst}}$

CDF Collaboration, CDF conference note 9724 (2009)

Tevatron (D0): $A_{FB}^{t} = \mathbf{0.12 \pm 0.08^{stat} \pm 0.01^{syst}}$

D0 collaboration, hep-ph/0712.0851

NLO $t\bar{t}$ production $\times$ decay: $A_{FB}^{t} = \mathbf{0.051 \pm 0.006}$

Antunano, Kuhn and Rodrigo (2008), hep-ph/0709.1652

Our result: $A_{FB}^{t} = \mathbf{0.051 \pm 0.0013}$

Conclusions

Many analyses at Tevatron and LHC demand the highest possible precision in the description of $t\bar{t}$, both as a signal and as a background in connection with new physics searches

Results for NLO QCD corrections to $pp(p\bar{p}) \to t\bar{t} \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b} + X$ processes including complete *off-shell* effects have been presented for the first time

With *inclusive* cut selection, the impact of non-resonant contributions is at the level of 1%

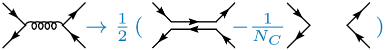
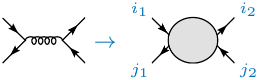
Further investigation about off-shell effects with *exclusive* cuts (*e.g.* VBF) will be addressed in a future analysis

HELAC−NLO has been tested and developed including by now all needed ingredients for NLO-QCD (+EWK): full color, massive particles, complex masses. In 2011 expected release of public code for HELAC-1LOOP

Backup slides

Structure of the one-loop calculation: Colour Sampling

Merging two complementary representations:

Colour flow decomposition	Colour assignment
	
$\|\text{Amp}\|^2 = \sum_{\sigma,\sigma'} A_\sigma A^*_{\sigma'} C_{\sigma\sigma'}$	$\|\text{Amp}\|^2 = \sum_{\{i\},\{j\}} \|\mathcal{M}^{i_1,i_2,\cdots,i_k}_{j_1,j_2,\cdots,j_k}\|^2$

✓Simple colour factors / Feynman rules for A_σ
✗Factorial growth of $A_\sigma{}'s$

✓Possiblity of MC sampling
✗More complex Feynman rules

$$\mathcal{M}^{i_1,i_2,\cdots,i_k}_{j_1,j_2,\cdots,j_k} = \sum_\sigma \delta_{i_{\sigma(1)}j_1}\delta_{i_{\sigma(2)}j_2}\cdots\delta_{i_{\sigma(k)}j_k} A_\sigma$$

$\hookrightarrow$ **Colour sampling:**

- generate random color assignment for each external particle
- find which colour connections (σ) are compatible with the given assignment
- restrict calculation to the eligible A_σ's

Exact treatment of colour sum $\leftrightarrow$ Improvement in speed

Structure of the one-loop calculation: Re-weighting

$$\sigma^{LO+V} = \int dx_1 dx_2 d\Phi_m pdf_a(x_1) pdf_b(x_2)(|\mathcal{M}|^2 + \mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L})$$

[$\mathcal{M}$ = LO amp; $\mathcal{L}$ = one-loop amp]

Factorizing $|\mathcal{M}|^2$ and dividing by σ^{LO} we recover a *tree-order* probability density:

$$\frac{\sigma^{LO+V}}{\sigma^{LO}} = \int dx_1 dx_2 d\Phi_m \, g(x_1, x_2, \Phi_m) \left(1 + \frac{\mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L}}{|\mathcal{M}|^2}\right)$$

where $g(x_1, x_2, \Phi_m) \equiv \frac{1}{\sigma^{LO}} pdf_a(x_1) pdf_b(x_2) |\mathcal{M}|^2 = \frac{1}{\sigma^{LO}} \frac{d\sigma^{LO}}{dx_1 dx_2 d\Phi_m}$

$\hookrightarrow$ We get the LO+V result by *re-weighting* a sample of *tree-level unweighted* events

- generate a sample S of tree-level unweighted events
- compute $\mathcal{L}$ event by event and get the re-weighting factor
- get the LO+V cross section: $\sigma^{LO+V} = \frac{\sigma^{LO}}{N_S} \sum_{i \in S} \left(1 + \frac{\mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L}}{|\mathcal{M}|^2}\right)$

Speed-up in the calculation of virtual corrections

Lazopoulos, Melnikov, Petriello, Phys. Rev. D76 (2007) 014001

Binoth, Ossola, Papadopoulos, R. Pittau, JHEP 0806 (2008) 082