Antenna subtraction for the production of massive final state fermions at hadron colliders

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Motivations

Why are we interested in top quarks?

- Very large cross section at the LHC:
  $\sigma_{t\bar{t}}(14\mathrm{TeV}, \mathrm{p_T^{top}} > 700\mathrm{GeV}) \approx 700\mathrm{fb}$,
- Large Yukawa coupling. Sensitivity to the mechanism of electroweak symmetry breaking,
- Background to various New Physics searches,
- Preferred channel for the decay of potential new heavy resonances,

Motivations

A few facts about $t\bar{t}$ production

- At the LHC an experimental error of $\sim$ 5% is expected for $\sigma_{t\bar{t}}$
- Theoretically, NLO[1]+NLL[2] calculations for the LHC give an uncertainty of $\sim$ 10%

- [1] Nason, Dawson, Ellis '88-'90; Beenakker, Kuijf, van Neerven, Smith '89-'91
- [2] Kidonakis, Sterman '97; Bonciani et al. '98, Cacciari et al. '08; Moch, Uwer '08; Kidonakis '08
- Most recently completed NNLL resummation: Ahrens et al. '09

To match the theoretical and experimental accuracies, a full NNLO calculation is needed

- 2-Loop corrections: Czakon '08; Bonciani et al. '08-'10
- 1$\times$1-loop corrections: Korner et al. '05, Anastasiou, Aybat '08; Kniehl et al. '08
- Real corrections:
  - Subtraction methods at NNLO (massless): Daleo et al. '09; Boughezal, Gehrmann-De Ridder, Ritzmann '10; Glover, Pires '10
  - New NNLO methods (massive): Czakon '11; Anastasiou, Herzog, Lazopoulos '10

Motivations

Towards an NNLO calculation for $t\bar{t}$ we

- Fully extended the NLO antenna subtraction method for hadronic collisions to incorporate massive particles in the final state
- Computed NLO subtraction terms for $\sigma_{t\bar{t}}$ and $\sigma_{t\bar{t}+jet}$

The NLO subtraction term for $t\bar{t} + jet$ is needed for $t\bar{t}$ at NNLO (same matrix elements, same single unresolved limits)

Subtraction at NLO for hadronic processes

Symbolically, we can write

$$\mathrm{d}\hat{\sigma}_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\hat{\sigma}^{R}_{NLO} - \mathrm{d}\hat{\sigma}^{S}_{NLO}\right) J_m^{(m+1)} + \int_{\mathrm{d}\Phi_m} \left(\mathrm{d}\hat{\sigma}^{V}_{NLO} + \mathrm{d}\hat{\sigma}^{MF}_{NLO} + \int_1 \mathrm{d}\hat{\sigma}^{S}_{NLO}\right) J_m^{(m)},$$

Subtraction term:

- Approximation to the $(m+1)$-particle matrix element in its soft and collinear limits
- Can be integrated over a factorized form of the $(m+1)$-particle phase space and added to the 1-loop m-particle contribution

$\Rightarrow$ $\mathrm{d}\hat{\sigma}^{R}_{NLO} - \mathrm{d}\hat{\sigma}^{S}_{NLO}$ is numerically finite
$\mathrm{d}\hat{\sigma}^{V}_{NLO} + \mathrm{d}\hat{\sigma}^{MF}_{NLO} + \int_1 \mathrm{d}\hat{\sigma}^{S}_{NLO}$ is free of divergencies

Real radiation contributions

For $p_1 + p_2 \to k_Q + k_{\bar{Q}} + (m-2) jets$

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}^R_{NLO}(p_1, p_2) \quad = \quad & \mathcal{N} \sum \mathrm{d}\Phi_{m+1}(k_Q, k_{\bar{Q}}, k_1, \ldots, k_{m-1}; p_1, p_2) \\
& \times \frac{1}{S_{m+1}} \left|\mathcal{M}_{m+1}(k_Q, k_{\bar{Q}}, k_1, \ldots, k_{m-1}; p_1, p_2)\right|^2 \\
& \times J_m^{(m+1)}(k_Q, k_{\bar{Q}}, k_1, \ldots, k_{m-1}).
\end{aligned}
$$

Knowing the factorization properties of $\mathcal{M}$ in its infrared limits, we can construct $\mathrm{d}\hat{\sigma}^S_{NLO}$ that reproduces all the configurations in which a parton j becomes unresolved between the hard radiators i and k.

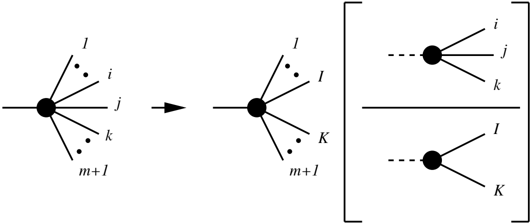
Unresolved limits of $\mathcal{M}$:

- Soft gluon limits $\to$ ϵ poles in $\mathrm{d}\hat{\sigma}$
- Collinear limits (massless partons) $\to$ ϵ poles in $\mathrm{d}\hat{\sigma}$
- Quasi-collinear limits (with at least one massive final state fermion) $\to$ $\ln(Q^2/M_Q^2)$ in $\mathrm{d}\hat{\sigma}$ [More on this later]

NLO Antenna subtraction

Subtraction terms for *m*-jet production:

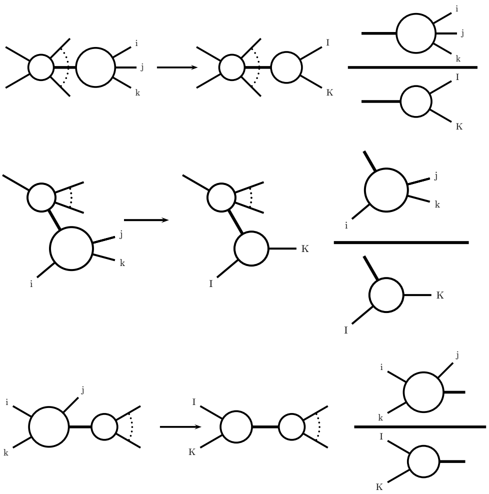
- Product of reduced matrix elements with *m* particles and antenna functions
- Antenna functions X_{ijk}: normalized three-particle matrix element
  - Two hard particles (hard radiators)
  - One particle soft, or collinear to either of the radiators



$$|\mathcal{M}_{m+1}|^2 \to |\mathcal{M}_m|^2 \cdot X_{ijk}$$

NLO Antenna subtraction for LHC processes

Require three types of antenna functions. Hard radiators can be in the initial or final state [Daleo, Gehrmann, Maître '07]



Final-final

Initial-final

Initial-initial

For $t\bar{t}$ and $t\bar{t} + jet$ at NLO...

We need

- Three types of subtraction terms (final-final, initial-final, initial-initial)
- Massive final-final and initial-final antennae as well as massless initial-final and initial-initial antennae

Example: To account for the unresolved limits of a gluon between a massless $q\bar{q}$ pair we need

$A_3^0(q, g, \bar{q})$, which is generated from $(\gamma^* \to qg\bar{q})/(\gamma^* \to q\bar{q})$

If we have a massive $Q\bar{Q}$ pair instead, we need

$A_3^0(Q, g, \bar{Q})$, which is generated from $(\gamma^* \to Qg\bar{Q})/(\gamma^* \to Q\bar{Q})$

- Massive initial-final antennae are obtained by suitable crossings on the final-final
- In addition, we also need massive flavour violating A-type antennae

Massive flavour-violating antennae

For example, in the $\bar{q}g \to Q\bar{Q}\bar{q}g$ process ($t\bar{t} + jet$)

$$\Rightarrow |M_6^0(3_{\bar{q}}6_g \to 1_Q 2_{\bar{Q}} 4_{\bar{q}} 5_g)|^2 \sim |\mathcal{M}_6^0(1_Q, 5_g, 4_{\bar{q}};; \hat{3}_{\bar{q}}, \hat{6}_g, 2_{\bar{Q}})|^2 + \ldots$$

5_g is emitted between 1_Q and $4_{\bar{q}}$ (different flavours).

To subtract the limits in which a gluon becomes unresolved between two fermions with different flavours we need $A_3^0(Q, g, \bar{q})$ (quark-antiquark of different flavours).

- It is an *A*-type antenna: it has the same spin properties as $A_3^0(q, g, \bar{q})$ except that $q \to Q$ and the same unresolved limits
- It can be generated from the physical process ratio $(W^+ \to t\bar{b}g)/(W^+ \to t\bar{b})$.

For $t\bar{t} + jet$ we also need $A_3^0(Q, g, q)$ (2 quarks of different flavours)

- Can be generated by an MSSM process initiated by a *W*-boson

Unresolved limits

In its unresolved limits a massless final-final antenna function gives

- $X_3^0(a,s,b) \xrightarrow{k_s \to 0} \frac{2s_{ab}}{s_{as}s_{bs}}$
- $X_3^0(a,s,b) \xrightarrow{a||s} \frac{P_{as}(z)}{s_{as}}$

$s_{ij} = 2p_i \cdot p_j$

In the massive case

- $X_3^0(a,s,b) \xrightarrow{k_s \to 0} \frac{2s_{ab}}{s_{as}s_{bs}} - \frac{2m_a^2}{s_{as}^2} - \frac{2m_b^2}{s_{bs}^2}$
- No strict collinear limit
- Quasi-collinear limit: Massive parton decays quasi-collinearly into two massive partons:
  - $p_j^\mu \to zp^\mu \qquad p_k^\mu \to (1-z)p^\mu \qquad p^2 = m_{(jk)}^2$
  - Constraints: $p_j \cdot p_k,\ m_j,\ m_k,\ m_{jk} \to 0$
  - Fixed ratios: $\frac{m_j^2}{p_j \cdot p_k}, \frac{m_k^2}{p_j \cdot p_k}, \frac{m_{jk}^2}{p_j \cdot p_k}$
  - Mass dependence in quasi-collinear splitting functions. E.g.:

$$P_{qg\to Q}(z,\mu_{qg}^2) = \frac{1+(1-z)^2-\epsilon z^2}{z} - \frac{2m_Q^2}{s_{qg}}$$

Initial-initial configurations

These configurations are unchanged with respect to the massless case [Daleo, Gehrmann, Maitre '07].

$$
\begin{aligned}
\mathrm{d}\sigma_{NLO}^{S,(ii)} &= \mathcal{N} \sum \mathrm{d}\Phi_{m+1}(k_Q, k_{\bar{Q}}, k_1, \ldots, k_j, \ldots, k_{m-1}; p_i, p_k) \frac{1}{S_{m+1}} \\
&\quad \times \sum_j X_{ik,j}^0 \, |\mathcal{M}_m(\tilde{k}_Q, \tilde{k}_{\bar{Q}}, \tilde{k}_1, \ldots, \tilde{k}_{j-1}, \tilde{k}_{j+1} \ldots, \tilde{k}_{m-1}; x_i p_i, x_k p_k)|^2 \\
&\quad \times J_m^{(m)}(\tilde{k}_Q, \tilde{k}_{\bar{Q}}, \tilde{k}_1, \ldots, \tilde{k}_{j-1}, \tilde{k}_{j+1} \ldots, \ldots, \tilde{k}_{m-1}).
\end{aligned}
$$

- ALL momenta need to be remapped to fulfill overall momentum conservation. The boost needed for this remapping for the massive particles Q,$\bar{Q}$ is the same as in the massless case [1]
- Phase space mapping and factorization formulae are unchanged in the massive case
- No initial-initial massive antennae

[1][Catani, Seymour '96]

Final-final configurations

For configurations with an unresolved parton *j* emitted between the final state hard radiators *i* and *k* (one, or both of them being massive)

$$
\begin{aligned}
\mathrm{d}\sigma_{NLO}^{S,(ff)} \quad = \quad & \mathcal{N} \sum \mathrm{d}\Phi_{m+1}(k_1, \ldots, k_i, k_j, k_k, k_{m+1}; p_1, p_2) \frac{1}{S_{m+1}} \\
& \times \sum_j X_{ijk}^0 \, |\mathcal{M}_m(k_1, \ldots, K_I, K_K, \ldots, k_{m+1}; p_1, p_2)|^2 \\
& \times J_m^{(m)}(k_1, \ldots, K_I, K_K, \ldots, k_{m+1})
\end{aligned}
$$

- Massive final-final case derived in [Gehrmann-De Ridder, Ritzmann '09]
- New: Massive flavour violating $A_3^0(Q, g, \bar{q})$,$A_3^0(Q, g, q)$
- $(m+1)$-particle phase space factorizes $\mathrm{d}\Phi_{m+1} \to \mathrm{d}\Phi_m \cdot \mathrm{d}\Phi_{X_{ijk}}$
- $\mathrm{d}\Phi_{X_{ijk}}$: $1 \to 3$ particle phase space with a massless parton (*j*) and two hard radiators (*i* and *k*, being one or both of them massive)
- Integrated antennae defined as in massless case

Initial-final configurations

To take into account configurations with an unresolved parton j (which can be massive), an initial state radiatior i (massless) and a final state radiator k (massive or massless)

$$
\begin{aligned}
\mathrm{d}\sigma_{NLO}^{S,(if)} &= \mathcal{N} \sum \mathrm{d}\Phi_{m+1}(k_1, \ldots, k_j, k_k, \ldots, k_{m+1}; p_i, p_2) \frac{1}{S_{m+1}} \\
&\times \sum_j X_{i,jk}^0 \, |\mathcal{M}_m(k_1, \ldots, K_K, \ldots, k_m; p_i, p_2)|^2 \\
&\times J_m^{(m)}(k_1, \ldots, K_K, \ldots, k_m)
\end{aligned}
$$

- $X_{i;jk}$ obtained by crossing i in X_{ijk}
- Unresolved parton j can be massive [e.g. $A_3^0(g; Q, \bar{Q})$] or massless [e.g. $E_3^0(q; Q, q)$ derived from $(q \to \tilde{\chi} Qq)/(g \to \tilde{\chi} Q)$]
- For $t\bar{t}$ and $t\bar{t} + jet$ we computed and integrated all relevant antennae (including flavour violating)

Initial-final configurations

- We generalized the phase space factorization to the massive case

$$\mathrm{d}\Phi_{m+1}(k_1,...,k_{m+1};p_i,p_2) = \mathrm{d}\Phi_m(k_1,...,K_k,...,k_{m+1};x_i p_i,p_2) \times \frac{(Q^2+m_j^2+m_k^2)}{2\pi}\mathrm{d}\Phi_2(k_j,k_k;p_i,q)\frac{\mathrm{d}x_i}{x_i}$$

$\mathrm{d}\Phi_2(k_j,k_k;p_i,q)$: $2 \to 2$ phase space with one or two massive final state particles

- Generalized phase space mapping

$$p_i + q \to k_j + k_k \quad \Rightarrow \quad x_i p_i + q \to K_k$$

$$x_i = \frac{Q^2+m_j^2+m_k^2}{2p_i \cdot q}$$

Integrated antennae

We did the ϵ expansion of all integrated massive antennae (final-final and initial-final) and found that all pole parts are related to

- x-dependent splitting kernels associated to initial state collinearities
- Massive $I^{(1)}$ operators [1] that contain the infrared structure of one-loop amplitudes squared

⇒Constructing our subtraction terms with antennae we are in a good shape to cancel poles in:

- Virtual contributions
- Mass factorization counterterms

[1][Catani, Dittmaier, Seymour, Trócsányi '02]

Application to heavy quark pair production at the LHC

For $t\bar{t}$ production at NLO

$$\mathrm{d}\sigma^R = \int \frac{\mathrm{d}\xi_1}{\xi_1}\frac{\mathrm{d}\xi_2}{\xi_2}\left\{ \sum_q \left[f_q(\xi_1) f_{\bar{q}}(\xi_2)\mathrm{d}\hat{\sigma}_{q\bar{q}\to Q\bar{Q}g} + f_q(\xi_1) f_g(\xi_2)\mathrm{d}\hat{\sigma}_{qg\to Q\bar{Q}q} \right.\right.$$
$$\left.\left. + f_{\bar{q}}(\xi_1) f_g(\xi_2)\mathrm{d}\hat{\sigma}_{\bar{q}g\to Q\bar{Q}\bar{q}} \right] + f_g(\xi_1) f_g(\xi_2)\mathrm{d}\hat{\sigma}_{g\bar{g}\to Q\bar{Q}g} \right\}$$

For the color decomposition of the amplitudes needed for the partonic cross-sections we consider the fictitious processes

- $0 \to Q\bar{Q}q\bar{q}g$
- $0 \to Q\bar{Q}ggg$

Application to heavy quark pair production at the LHC

Consider, for example, $q\bar{q} \to Q\bar{Q}g$:

- Take the colour decomposition of $0 \to Q\bar{Q}q\bar{q}g$
- Square the colour decomposed amplitude
- Use decoupling identities to eliminate interferences between different partial amplitudes. In this case

$$\mathcal{M}_5^0(1_Q, 2_{\bar{Q}}, 3_q, 4_{\bar{q}}, 5_\gamma) = \mathcal{M}_5^0(1_Q, 5_g, 4_{\bar{q}};; 3_q, 2_{\bar{Q}}) + \mathcal{M}_5^0(1_Q, 4_{\bar{q}};; 3_q, 5_g, 2_{\bar{Q}})$$
$$= \mathcal{M}_5^0(1_Q, 5_g, 2_{\bar{Q}};; 3_q, 4_{\bar{q}}) + \mathcal{M}_5^0(1_Q, 2_{\bar{Q}};; 3_q, 5_g, 4_{\bar{q}})$$

$$\Rightarrow |M_5^0(0 \to 1_Q, 2_{\bar{Q}}, 3_q, 4_{\bar{q}}, 5_g)|^2 = \frac{g^6(N_c^2-1)}{8}$$
$$\times \Bigg[N_c \left(|\mathcal{M}_5^0(1_Q, 5_g, 4_{\bar{q}};; 3_q, 2_{\bar{Q}})|^2 + |\mathcal{M}_5^0(1_Q, 4_{\bar{q}};; 3_q, 5_g, 2_{\bar{Q}})|^2 \right)$$
$$+ \frac{1}{N_c} \Big(|\mathcal{M}_5^0(1_Q, 5_g, 2_{\bar{Q}};; 3_q, 4_{\bar{q}})|^2 + |\mathcal{M}_5^0(1_Q, 2_{\bar{Q}};; 3_q, 5_g, 4_{\bar{q}})|^2$$
$$-2|\mathcal{M}_5^0(1_Q, 2_{\bar{Q}}, 3_q, 4_{\bar{q}}, 5_\gamma)|^2 \Big) \Bigg].$$

Application to heavy quark pair production at the LHC

- Cross the $q\bar{q}$ pair to the initial state

$$|M_5^0(3_{\bar{q}}4_q \to 1_Q, 2_{\bar{Q}}, 5_g)|^2 = \frac{g^6(N_c^2-1)}{8}$$
$$\times\Bigg[N_c\left(|\mathcal{M}_5^0(1_Q, 5_g, \hat{4}_q;;\hat{3}_{\bar{q}}, 2_{\bar{Q}})|^2 + |\mathcal{M}_5^0(1_Q, \hat{4}_q;;\hat{3}_{\bar{q}}, 5_g, 2_{\bar{Q}})|^2\right)$$
$$+\frac{1}{N_c}\left(|\mathcal{M}_5^0(1_Q, 5_g, 2_{\bar{Q}};;\hat{3}_{\bar{q}}, \hat{4}_q)|^2 + |\mathcal{M}_5^0(1_Q, 2_{\bar{Q}};;\hat{3}_{\bar{q}}, 5_g, \hat{4}_q)|^2\right.$$
$$\left.-2|\mathcal{M}_5^0(1_Q, 2_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{4}_q, 5_\gamma)|^2\right)\Bigg].$$

- The subtraction term for this partonic process is

$$\mathrm{d}\hat{\sigma}^S_{q\bar{q}\to Q\bar{Q}g} = \frac{g^6(N_c^2-1)}{8}\mathrm{d}\phi_3(k_{1Q}, k_{2\bar{Q}}, k_{5g}; p_{4q}, p_{3\bar{q}})$$
$$\times\Bigg\{N_c\Big[A_3^0(4_q; 1_Q, 5_g)|\mathcal{M}_4^0((\widetilde{15})_Q, 2_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{\hat{4}}_{\bar{q}})|^2 J_2^{(2)}(K_{\widetilde{15}}, k_2)$$
$$+A_3^0(3_{\bar{q}}; 2_{\bar{Q}}, 5_g)|\mathcal{M}_4^0(1_Q, (\widetilde{25})_{\bar{Q}}, \hat{\hat{3}}_{\bar{q}}, \hat{4}_q)|^2 J_2^{(2)}(k_1, K_{\widetilde{25}})\Big]$$
$$-\frac{1}{N_c}\Big[A_3^0(1_Q, 5_g, 2_{\bar{Q}})|\mathcal{M}_4^0((\widetilde{15})_Q, (\widetilde{25})_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{4}_q)|^2 J_2^{(2)}(k_{\widetilde{15}}, k_{\widetilde{25}})$$
$$+A_3^0(4_q, 3_{\bar{q}}; 5_g)|\mathcal{M}_4^0(\tilde{1}_Q, \tilde{2}_{\bar{Q}}, \hat{\hat{3}}_{\bar{q}}, \hat{\hat{4}}_q)|^2 J_2^{(2)}(\tilde{k}_1, \tilde{k}_2)\Big]\Bigg\}.$$

Application to heavy quark pair production at the LHC

For $t\bar{t} + jet$ production the unphysical processes are:

- $0 \rightarrow Q\bar{Q}q\bar{q}q'\bar{q}'$
- $0 \rightarrow Q\bar{Q}q\bar{q}gg$
- $0 \rightarrow Q\bar{Q}gggg$

Calculations are more involved because

- More partial amplitudes
- More unresolved limits to subtract
- Identical flavour contributions
- Decoupling identities do not always eliminate all interference terms

Colour interferences

For $t\bar{t} + jet$ we find squared amplitudes whose interferences between partial amplitudes cannot be removed with decoupling identities. For example (ommiting quark labels)

$$|M_6^0(0 \to 1_Q, 2_{\bar{Q}}, 3_g, 4_g, 5_g, 6_g)|^2 = \frac{g^8(N_c-1)}{16N_c^3}$$

$$\times \Bigg\{ \sum_{(i,j,k,l)\in P(3,4,5,6)} \Bigg[N_c^6|\mathcal{M}(i,j,k,l)|^2 - N_c^4|\mathcal{M}(i,j,k;l)|^2 + \frac{N_c^2}{2!}|\mathcal{M}(i,j;k,l)|^2$$

$$-N_c^4\mathrm{Re}\Bigg[\Big(\mathcal{M}(j,i,l,k) + \mathcal{M}(j,l,i,k) + \mathcal{M}(j,l,k,i)$$

$$+\mathcal{M}(k,i,l,j) + \mathcal{M}(k,j,l,i) + \mathcal{M}(l,i,k,j)$$

$$+\mathcal{M}(l,j,i,k) + \mathcal{M}(l,k,j,i)\Big) \times \mathcal{M}(i,j,k,l)^\dagger\Bigg]\Bigg]$$

$$+(N_c^4 - 3N_c^2 - 1)|\bar{\mathcal{M}}(3,4,5,6)|^2\Bigg\},$$

(Checked with [Mangano, Parke '90])

Colour interferences

- Collinear singularities: an interference term develops a collinear singularity only when a collinear limit is shared by both partial amplitudes
- Problem: Soft singularities?

At the amplitude level (massless case)

$$\mathcal{M}^0_{n+1}(..., a, s^+, b, ...) \xrightarrow{k_s \to 0} \frac{\langle ab \rangle}{\langle as \rangle \langle sb \rangle} \mathcal{M}^0_n(..., a, b, ...)$$

$$\mathcal{M}^0_{n+1}(..., a, s^-, b, ...) \xrightarrow{k_s \to 0} \frac{[ab]}{[as][sb]} \mathcal{M}^0_n(..., a, b, ...)$$

Colour interferences

With some algebra and after spin averaging,

$$\mathcal{M}^0_{n+1}(...,a,s,b,...)\mathcal{M}^0_{n+1}(...,c,s,d,...)^\dagger \xrightarrow{k_s\to 0} \left(\frac{s_{ad}}{s_{as}s_{ds}}+\frac{s_{bc}}{s_{bs}s_{cs}}-\frac{s_{ac}}{s_{as}s_{cs}}-\frac{s_{bd}}{s_{bs}s_{ds}}\right) \times\mathcal{M}^0_n(...,a,b,...)\mathcal{M}^0_n(...,c,d,...).$$

$\Rightarrow$ We can subtract the soft singularities of the interference terms with

$$\frac{1}{2}X^0_3(a,s,d)\mathcal{M}^0_{n,1}(...,\tilde{as},\tilde{bs},...)\mathcal{M}^0_{n,1}(...,\tilde{cs},\tilde{ds},...)$$
$$+\frac{1}{2}X^0_3(b,s,c)\mathcal{M}^0_{n,2}(...,\tilde{as},\tilde{bs},...)\mathcal{M}^0_{n,2}(...,\tilde{cs},\tilde{ds},...)$$
$$-\frac{1}{2}X^0_3(a,s,c)\mathcal{M}^0_{n,3}(...,\tilde{as},\tilde{bs},...)\mathcal{M}^0_{n,3}(...,\tilde{cs},\tilde{ds},...)$$
$$-\frac{1}{2}X^0_3(b,s,d)\mathcal{M}^0_{n,4}(...,\tilde{as},\tilde{bs},...)\mathcal{M}^0_{n,4}(...,\tilde{cs},\tilde{ds},...).$$

without introducing any extra collinear singularities!

Colour interferences

This way of treating soft singularities in interference terms also gives all the correct limits in the massive case if we replace

- Massless antenae $\rightarrow$ massive antennae
- Massless eikonal factors $\rightarrow$ massive eikonal factors

Subtraction terms grow in size:

- Only one antenna function is needed to subtract the soft limits of a gluon in $|\mathcal{M}|^2$
- Subtraction of one soft limit in an interference term requires four antenna functions

Checks performed on all subtraction terms

As a consistency check we have verified that, for a given process, the sum of all colour-ordered subtraction terms reproduces the collinear limits of the full $|M_m^0|^2$

$$\mathrm{d}\sigma^S \xrightarrow{a||b} g^2\, C\, \frac{P_{ab}(z)}{s_{ab}} \times |M_m^0|^2 \times \mathrm{d}\Phi_m\, J_m^{(m)}$$

- M_m^0 is the full Born amplitude
- $C = C_A, C_F, T_R$ is the corresponding Casimir

For example

$$\mathrm{d}\hat{\sigma}^S_{q\bar{q}\to Q\bar{Q}g} \xrightarrow{1_Q||5_g} g^6 \mathrm{d}\Phi_2(k_{(1+5)Q}, k_{2\bar{Q}}; p_{4q} p_{3\bar{q}}) J_2^{(2)}(k_{(1+5)}, k_2)$$
$$\times C_F \frac{P_{qg\to Q}(z, \mu^2_{qg})}{s_{15}} |M_4^0(3_{\bar{q}} 4_q \to (1+5)_Q, 2_{\bar{Q}})|^2$$

Summary and conclusions

- We extended the antenna subtraction method at NLO for initial-final configurations with massive final state fermions:
  - Computed and integrated massive initial-final antenna functions relevant for $t\bar{t}$ and $t\bar{t} + jet$,
  - Generalized phase space mapping and factorization formulae for the massive case,
  - Computed and integrated flavour violating antenna functions.
- We developed a way of subtracting soft singularities from interferences between different partial amplitudes at NLO.
- We constructed subtraction terms for all partonic processes involved in $t\bar{t}$ and $t\bar{t} + jet$.
- NEXT: We shall start working on a NNLO extension of the method for the inclusion of massive final state fermions.

Summary and conclusions

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  - Computed and integrated massive initial-final antenna functions relevant for $t\bar{t}$ and $t\bar{t} + jet$,
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- We constructed subtraction terms for all partonic processes involved in $t\bar{t}$ and $t\bar{t} + jet$.
- NEXT: We shall start working on a NNLO extension of the method for the inclusion of massive final state fermions.

THANK YOU!

Backup Slide: Check of collinear limits

Take the subtraction term we discussed

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}^{S}_{q\bar{q}\to Q\bar{Q}g} &= \frac{g^6(N_c^2-1)}{8}\mathrm{d}\phi_3(k_{1Q},k_{2\bar{Q}},k_{5g};p_{4q},p_{3\bar{q}}) \\
&\times\Bigg\{N_c\Big[A_3^0(4_q;1_Q,5_g)|\mathcal{M}_4^0((\widetilde{15})_Q,2_{\bar{Q}},\hat{3}_{\bar{q}},\hat{\hat{4}}_{\bar{q}})|^2 J_2^{(2)}(K_{\widetilde{15}},k_2) \\
&\qquad +A_3^0(3_{\bar{q}};2_{\bar{Q}},5_g)|\mathcal{M}_4^0(1_Q,(\widetilde{25})_{\bar{Q}},\hat{\hat{3}}_{\bar{q}},\hat{4}_q)|^2 J_2^{(2)}(k_1,K_{\widetilde{25}})\Big] \\
&\quad -\frac{1}{N_c}\Big[A_3^0(1_Q,5_g,2_{\bar{Q}})|\mathcal{M}_4^0((\widetilde{15})_Q,(\widetilde{25})_{\bar{Q}},\hat{3}_{\bar{q}},\hat{4}_q)|^2 J_2^{(2)}(k_{\widetilde{15}},k_{\widetilde{25}}) \\
&\qquad +A_3^0(4_q,3_{\bar{q}};5_g)|\mathcal{M}_4^0(\tilde{1}_Q,\tilde{2}_{\bar{Q}},\hat{\hat{3}}_{\bar{q}},\hat{\hat{4}}_q)|^2 J_2^{(2)}(\tilde{k}_1,\tilde{k}_2)\Big]\Bigg\}. \\
&\xrightarrow{1_Q||5_g} g^6\frac{N_c^2-1}{8}\mathrm{d}\Phi_2(k_{(1+5)Q},k_{2\bar{Q}};p_{4q}p_{3\bar{q}})J_2^{(2)}(k_{(1+5)},k_2)\frac{P_{qg\to Q}(z,\mu_{qg}^2)}{s_{15}} \\
&\quad\times\Big[N_c|\mathcal{M}_4^0((1+5)_Q,2_{\bar{Q}},\hat{3}_{\bar{q}},\hat{4}_q)|^2-\frac{1}{N_c}|\mathcal{M}_4^0((1+5)_Q,2_{\bar{Q}},\hat{3}_{\bar{q}},\hat{4}_q)|^2\Big]
\end{aligned}
$$

Backup Slide: Check of collinear limits

$$= g^6 \mathrm{d}\Phi_2(k_{(1+5)Q}, k_{2\bar{Q}}; p_{4q} p_{3\bar{q}}) J_2^{(2)}(k_{(1+5)}, k_2)$$

$$\times \frac{P_{qg\to Q}(z, \mu_{qg}^2)}{s_{15}} \frac{N_c^2 - 1}{2N_c} \frac{N_c^2 - 1}{4} |\mathcal{M}_4^0((1+5)_Q, 2_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{4}_q)|^2$$

$$= g^6 \mathrm{d}\Phi_2(k_{(1+5)Q}, k_{2\bar{Q}}; p_{4q} p_{3\bar{q}}) J_2^{(2)}(k_{(1+5)}, k_2)$$

$$\times C_F \frac{P_{qg\to Q}(z, \mu_{qg}^2)}{s_{15}} |M_4^0(3_{\bar{q}} 4_q \to (1+5)_Q, 2_{\bar{Q}})|^2$$

Massive flavour violating antenna functions

Processes determined according to spin properties: (omitting couplings):

$A_3^0(Q,g,\bar{q})$ can be generated in the SM from
$(W^+ \to t\bar{b}g)/(W^+ \to t\bar{b})$

- t (spin $1/2$, massive) plays the role of Q
- $\bar{b}$ (spin $1/2$ massless) plays the role of $\bar{q}$

$A_3^0(Q,g,q)$ is generated from MSSM process [1] ratio
$(W^+ \to \chi_i^0\chi_j^+ Z^0)/(W^+ \to \chi_i^0\chi_j^+)$

- χ_i^0: Neutralino (Majorana fermion, spin $1/2$, massless). Plays the role of q (or $\bar{q}$)
- χ_j^+: Chargino (Massive fermion, spin $1/2$). Plays the role of Q
- Z^0: Vector boson (taken massless, spin 1). Plays the role of the gluon. Can be radiated from χ_i^0, χ_j^+

[1][Rosiek '95]