NLO QCD corrections to $\mathrm{W^+W^-b\bar{b}}$ production at hadron colliders

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Workshop on Heavy Particles at the LHC, Zürich, January 5, 2011

Outline of the talk

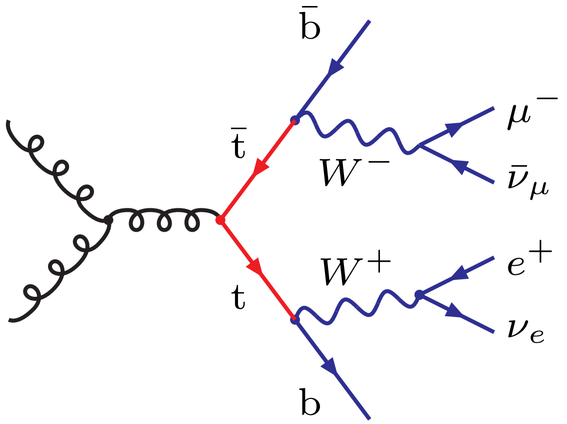
1. **Why NLO for $\mathrm{W^+W^-b\bar{b}}$ production?**
2. **Technical aspects of the calculation**
3. **NLO predictions for Tevatron and LHC**

NLO priority list (Les Houches '05): completed $2 \to 4$ calculations

- **Two calculations for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ with permille agreement**
  - arXiv:0905.0110 and arXiv:1001.4006 by Bredenstein, Denner, Dittmaier and S. P.
    Feynman diagrams and tensor integrals
  - arXiv:0907.4723 by Bevilacqua, Czakon, Papadopoulos, Pittau and Worek
    OPP reduction and HELAC
- **Two calculations for $\mathrm{pp} \to Vjjj$**
  - arXiv:0906.1445 by Ellis, Melnikov and Zanderighi
    D-dimensional unitarity (leading colour)
  - arXiv:0907.1984 ($Wjjj$) and arXiv:1004.1659 ($Zjjj$) by Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower and Maitre
    generalized unitarity (full colour)
- **$q\bar{q}$-channel contribution to $\mathrm{pp} \to \mathrm{b\bar{b}b\bar{b}}$**
  - arXiv:0910.4379 by Binoth, Greiner, Guffanti, Reuter, Guillet and Reiter
    Feynman diagrams and tensor integrals (GOLEM)
- **First result for $\mathrm{pp} \to \mathrm{t\bar{t}}jj$**
  - arXiv:1002.4009 by Bevilacqua, Czakon, Papadopoulos and Worek
    OPP reduction and HELAC

- **One calculation for** $\mathrm{pp} \to W^+W^+jj$
  - arXiv:1007.5313 by Melia, Melnikov, Rontsch and Zanderighi
    D-dimensional unitarity
- **First result for** $\mathrm{pp} \to W + 4j$
  - arXiv:1009.2338 by Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower and Maitre
    generalized unitarity (leading colour)
- **Two (very recent) calculations for** $\mathrm{pp} \to W^+W^-\mathrm{b}\bar{\mathrm{b}}$
  - arXiv:1012.3975 by Denner, Dittmaier, Kallweit and S. P.
    Feynman diagrams and tensor integrals
  - arXiv:1012.4230 by Bevilacqua, Czakon, van Hameren, Papadopoulos and Worek
    OPP reduction and HELAC

Why $\mathrm{W}^+\mathrm{W}^-\mathrm{b}\bar{\mathrm{b}}$ production at NLO?



Full description of $\mathrm{t}\bar{\mathrm{t}}$ prod×decay

- off-shell tops and non-resonant backgr.
- $\mathrm{W} \to l\nu$ decays in spin-correlated NWA

Huge $\mathrm{t}\bar{\mathrm{t}}$ samples at hadron colliders

- Tevatron: few 10^4 events $\Rightarrow \frac{\delta\sigma}{\sigma} < 10\%$
- LHC at 7(14) TeV: $1.5(9) \times 10^5$ events per fb^{-1} $\Rightarrow \frac{\delta\sigma}{\sigma} = \text{few}\,\%$

Crucial measurements and tests

- precise studies of rich variety of (differential) observables
- checks and tuning of many theoretical/experimental tools
- $\delta m_\mathrm{t}^{\mathrm{exp}} \sim 1\,\mathrm{GeV}$ measurements

Relevance for discoveries

- leptons + jets + missing E_T is a typical discovery signature (SUSY, $\mathrm{H} \to \mathrm{W}^+\mathrm{W}^-, \ldots$)
- various BSM scenarios predict heavy resonances decaying into $\mathrm{t}\bar{\mathrm{t}}$

Precise predictions for hadronic $t\bar{t}$ production (and decay)

NLO QCD corrections

Beenakker, Dawson, Ellis, Frixione, Kuijf, Meng, Nason, van Neerven, Schuler, Smith

Electroweak NLO corrections

Beenakker, Bernreuther, Denner, Fücker, Hollik, Kao, Kollar, Kühn, Ladinsky, Mertig, Moretti, Nolten, Ross, Sack, Scharf, Si, Uwer, Wackeroth, Yuan

From LL to NNLL resummations

Ahrens, Beneke, Berger, Bonciani, Catani, Contopanagos, Czakon, Falgari, Ferroglia, Frixione, Kidonakis, Kiyo, Laenen, Mangano, Mitov, Moch, Nason, Neubert, Pecjak, Ridolfi, Schwinn, Sterman, Uwer, Vogt, Yang

Towards full NNLO predictions

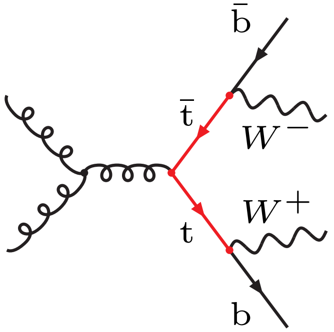
Anastasiou, Aybat, Bonciani, Czakon, Dittmaier, Ferroglia, Gehrmann, Gerhmann–De Ridder, Kniehl, Körner, Langenfeld, Maitre, Merebashvili, Mitov, Moch, Ritzmann, Rogal, Studerus, von Manteuffel, Uwer, Weinzierl

NLO $t\bar{t}$ production×decay in spin-correlated narrow-width approx.

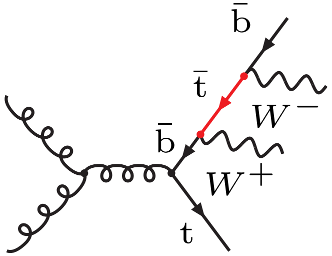
Bernreuther, Brandenburg, Melnikov, Schulze, Si, Uwer

Full $\mathrm{W^+W^-b\bar{b}}$ description vs Narrow-Width Approximation in LO

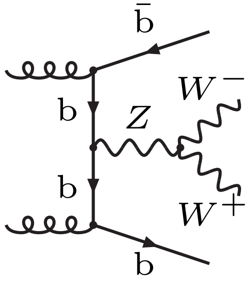
Doubly-Resonant (DR)



Singly-Resonant (SR)



Non-Resonant (NR)



Narrow-Width Approximation

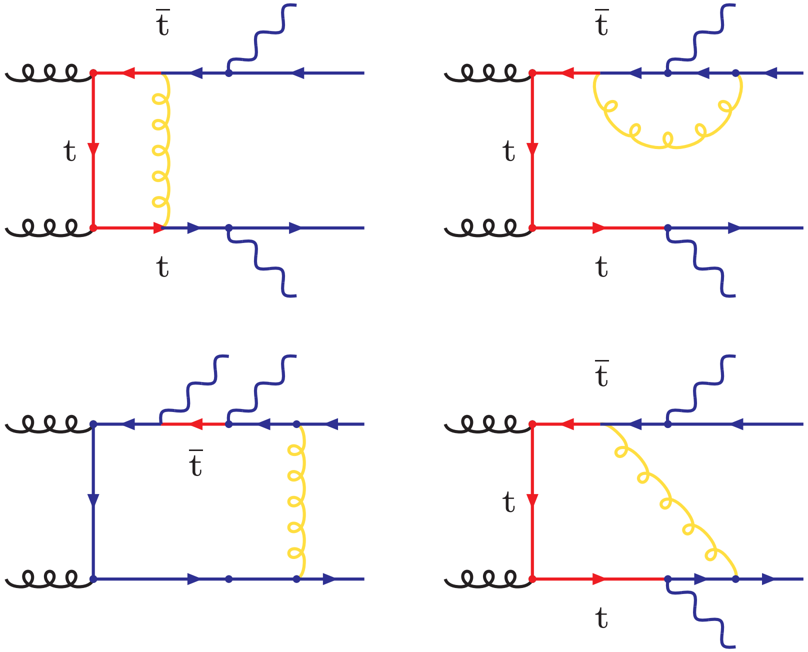
- only DR channels
- narrow-with limit of Breit-Wigner top resonances

$$\lim_{\Gamma_\mathrm{t}\to 0}\left|\frac{1}{p_\mathrm{t}^2 - m_\mathrm{t}^2 + \mathrm{i}\Gamma_\mathrm{t} m_\mathrm{t}}\right|^2 = \frac{\pi}{\Gamma_\mathrm{t} m_\mathrm{t}}\delta(p_\mathrm{t}^2 - m_\mathrm{t}^2)$$

Finite-width contributions to $\mathrm{W^+W^-b\bar{b}}$

- Off-shell corrections to DR channels
- SR+NR channels and interferences
- $\mathcal{O}(\Gamma_\mathrm{t}/m_\mathrm{t})$ corrections to inclusive observables

Full $W^+W^-b\bar{b}$ description vs Narrow-Width Approximation in NLO



Narrow-Width Approximation

- only factorisable corrections to DR channels
- huge technical simplification

Finite-width contributions to $W^+W^-b\bar{b}$

- involve pentagons and hexagons
- non-DR and non-factorisable corrections

Virtual and real NF corrections contain $\ln(\Gamma_t/m_t)$ soft-gluon enhancements which cancel in the sum. Thus finite-width corrections remain $\mathcal{O}(\Gamma_t/m_t)$ suppressed for inclusive observables [Fadin/Khoze/Martin '94].

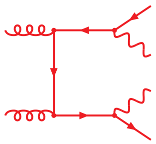
Finite-width effects can be important for

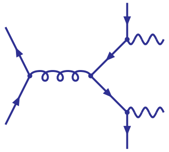
- percent-level precision in σ_{incl}
- Shape of top resonance and related observables (m_t measurement)
- off-shell regime of $W^+W^-b\bar{b}$ background

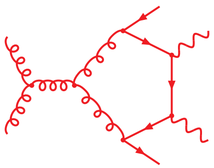
(2) Technical aspects of the calculation

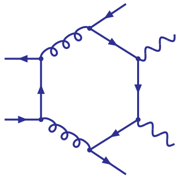
Ingredients of $pp \to W^+W^-b\bar{b}$ at NLO

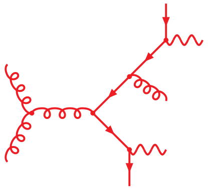
Partonic channels

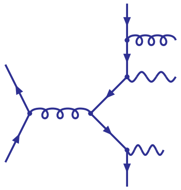

31 trees

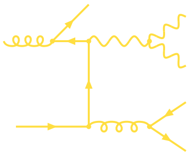

14 trees


788 loops


280 loops


222 NLO trees


90 NLO trees


90 NLO trees

Full calculation twice and independently

Generation of Feynman diagrams

- `FeynArts 1.0 / 3.2`

Algebraic reduction

- `MATHEMATICA` / `FormCalc` [Hahn]

Tensor integrals & numerics

- `Fortran77 / C++` executables: 0.25–1.2 GB

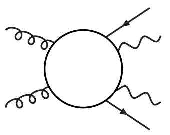
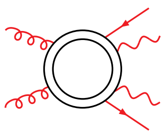
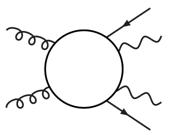
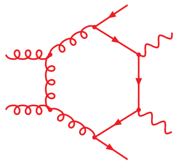
Real emission & IR Subtraction

- `Madgraph` & spinors
- Dipoles [Catani/Dittmaier/Seymour/Trócsányi ’97/’02] & `AutoDipole` [Hasegawa/Moch/Uwer ’09]

Integration over 11-dim PS

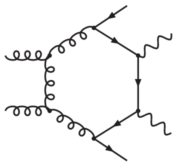
- adaptive multi-channel Monte Carlo with 250–650 mappings per partonic channel

Feynman diagrams and tensor integrals

$\sum_{\text{col,pol}}$ ( $)^*$  $=\sum_{\text{col,pol}}$ ( $)^*$  $+\ \mathcal{O}(1000)$ more diagrams

Colour sums at zero cost

thanks to *colour factorisation*

 $\times\ f^{a_1bd}f^{a_2cd}\left(T^cT^b\right)_{i_5i_6}$

$$\sum a_{i_1\ldots j_P}\ \underbrace{\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}\left[\bar{v}_5\,\gamma_{\mu_5}\ldots\gamma_{\mu_k}\,u_6\right]}\ \{g\ldots p\}^{\mu_1\ldots\nu_P}_{i_1\ldots j_P}\ \underbrace{\int \mathrm{d}^Dq\,\frac{q_{\nu_1}\ldots q_{\nu_P}}{N_0\ldots N_{N-1}}}$$

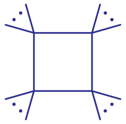
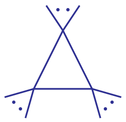
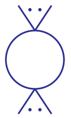
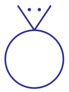
Algebraic reduction of helicity structures

$\mathcal{O}(10^3\text{–}10^4)$ *compact spinor chains* → fast helicity sums

$$\sum \underbrace{T^{(N)}_{j_1\ldots j_P}}\{g\ldots p\}^{\nu_1\ldots\nu_P}_{j_1\ldots j_P}$$

Numerical tensor-integral reduction

avoids *gigantic expressions* and *instabilities*

$\sum d_i$  $+\,c_j$  $+\,b_k$  $+\,a_l$ 

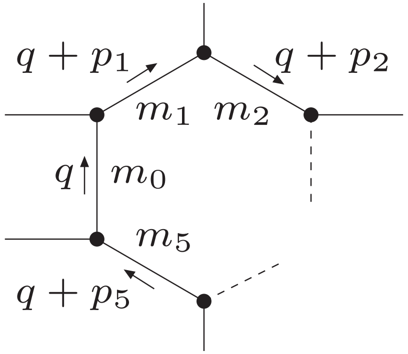
Reduction of tensor integrals – *collection of* $e^+e^- \to 4f$ *methods* [Denner/Dittmaier '05]

(A) **Space-time 4-dim ($N \geq 5$ prop.)**

simultaneous prop. & rank reduction

Melrose '65; Denner/Dittmaier '02&'05; Binoth et. al. '05

$$\begin{vmatrix} q^\mu & 2qp_1 & \dots & 2qp_5 \\ p_1^\mu & 2p_1p_1 & \dots & 2p_1p_5 \\ \vdots & \vdots & \ddots & \vdots \\ p_4^\mu & 2p_4p_1 & \dots & 2p_4p_5 \\ 0 & f_1 & \dots & f_5 \end{vmatrix} = \mathcal{O}(D-4)$$



(B) **Lorentz invariance ($N \leq 4$ prop.)**

reduction of rank (P)

Passarino/Veltman '79; Denner '93

$$2(D+P-N-1)\, T^{(P)}_{00i_3\dots i_P} = \sum_{k=1}^{N-1} f_k\, T^{(P-1)}_{ki_3\dots i_P} + 2m_0^2\, T^{(P-2)}_{i_3\dots i_P} + \text{lower-point}$$

$$\sum_{n=1}^{N-1} Z_{mn}\, T^{(P)}_{ni_2\dots i_P} = -2\sum_{r=2}^{P} \delta_{mi_r}\, T^{(P)}_{00i_2\dots\hat{i}_r\dots i_P} - f_m\, T^{(P-1)}_{i_2\dots i_P} + \text{lower-point}$$

inversion of Gram matrix $Z_{mn} = 2p_mp_n$ **unstable** when $\det(Z) \to 0$

(C) General and robust solution of instability problems

iterative $\det(Z)$-expansion (and various alternative methods)

$$\tilde{X}_{0j}T^{(P)}_{i_1\ldots i_P} = \det(Z)\,T^{(P+1)}_{ji_1\ldots i_P} + 2\sum_{n=1}^{N-1}\tilde{Z}_{jn}\sum_{r=1}^{P}\delta_{ni_r}T^{(P+1)}_{00i_1\ldots\hat{i}_r\ldots i_P} + \text{lower-point}$$

$$2\tilde{Z}_{kl}T^{(P+1)}_{00i_2\ldots i_P} = \Big\{-\det(Z)\,T^{(P+1)}_{kli_2\ldots i_P} + 2m_0\tilde{Z}_{kl}T^{(P-1)}_{i_2\ldots i_P} + \sum_{n,m=1}^{N-1}\Big[f_nf_mT^{(P-1)}_{i_2\ldots i_P} + 2\sum_{r=2}^{P}(f_n\delta_{mi_r}+f_m\delta_{ni_r})$$

$$\times T^{(P)}_{00i_2\ldots\hat{i}_r\ldots i_P} + 4\sum_{\substack{r,s=2\\ r\neq s}}^{P}\delta_{ni_r}\delta_{mi_s}T^{(P+1)}_{0000i_2\ldots\hat{i}_r\ldots\hat{i}_s\ldots i_P}\Big]\tilde{\tilde{Z}}_{(kn)(lm)} + \text{lower-point}\Big\}\Big(D+1+P-N+\sum_{r=2}^{P}\bar{\delta}_{i_r0}\Big)^{-1}$$

First physical application up to tensor rank $P = 5$

- **CPU cost of colour/helicity summed** $gg \to W^+W^-b\bar{b}$ **loop amplitudes very low** (450ms) similarly as for $gg \to t\bar{t}b\bar{b}$ (180 ms) where $P = 4$
- σ_{NLO} with **statistical accuracy of** $\mathcal{O}(10^{-3})$ requires $\mathcal{O}(10^8)$ events obtained within **5–10 days on single CPU**
- Total CPU cost at LHC dominated by real and virtual gg-channel corrections

Treatment of unstable particles

Regularisation of unstable-particle propagators via $\mathrm{Im}[\Sigma(M^2)] = M\Gamma$ resummation

$$\frac{1}{p^2 - M^2 + \mathrm{i}\epsilon} \rightarrow \frac{1}{p^2 - M^2 + \mathrm{i}M\Gamma + \mathrm{i}\epsilon}$$

can violate gauge invariance

Complex mass scheme (introduced for $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{W}^+\mathrm{W}^- \rightarrow 4f$ Denner/Dittmaier '05)

- Γ is absorbed into the renormalised pole mass $M^2 \rightarrow \mu^2 = M^2 - \mathrm{i}M\Gamma$ without modifying the bare Lagrangian
- Lagrangian symmetries require (in general) complex couplings

Technical aspects

- On-shell renormalisation with complex momenta: $\hat{\Sigma}(p^2) = 0$ at $p^2 = \mu^2$
- Scalar box integrals with complex masses (subtle analytic continuations!)
  - 't Hooft/Veltman approach: $24 \rightarrow 108$ Li_2 Nhung/Ninh '09; van Hameren '10
  - Denner/Niertse/Scharf approach: $16 \rightarrow 32$ Li_2 Denner/Dittmaier '10

(3.1) $W^+W^-b\bar{b}$ cross section at the Tevatron (1.96 TeV) and the LHC (7 TeV)

Setup and input parameters for Tevatron (LHC)

Particle masses and widths ($M_{\mathrm{H}} = \infty$, $m_{\mathrm{b}} = 0$)

$$m_{\mathrm{t}} = 172.0\,\mathrm{GeV} \qquad M_{\mathrm{W}} = 80.399\,\mathrm{GeV} \qquad M_{\mathrm{Z}} = 91.1876\,\mathrm{GeV}$$

$$\Gamma_{\mathrm{t,LO}} = 1.4655\,\mathrm{GeV} \qquad \Gamma_{\mathrm{t,NLO}} = 1.3376\,\mathrm{GeV} \qquad \Gamma_{\mathrm{W,NLO}} = 2.0997\,\mathrm{GeV}$$

G_μ-scheme couplings ($G_\mu = 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}$)

$$\sin^2\theta_{\mathrm{w}} = 1 - M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2, \qquad \alpha = \sqrt{2}G_\mu M_{\mathrm{W}}^2 \sin^2\theta_{\mathrm{w}}/\pi$$

PDFs and α_{S}: MSTW2008NLO(LO) with $1/2 \le \mu_{\mathrm{R,F}}/m_{\mathrm{t}} \le 2$ variations

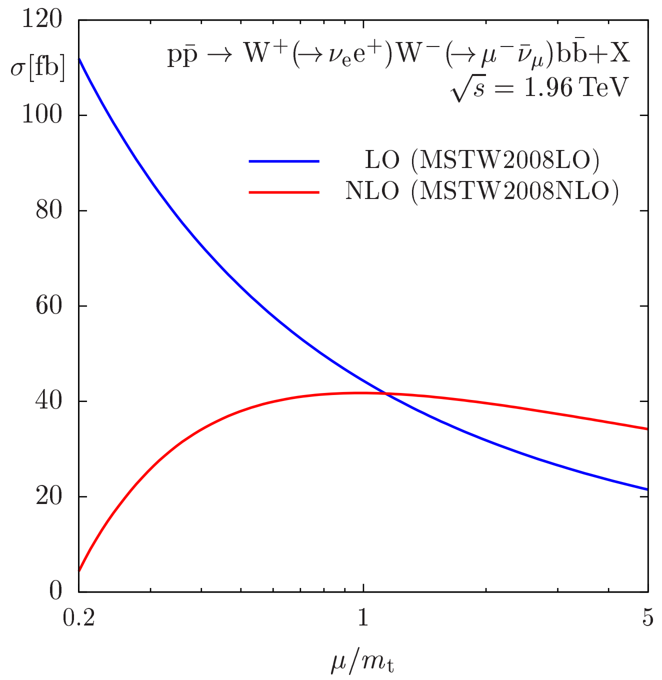
Anti-k_{T} Jet Algorithm

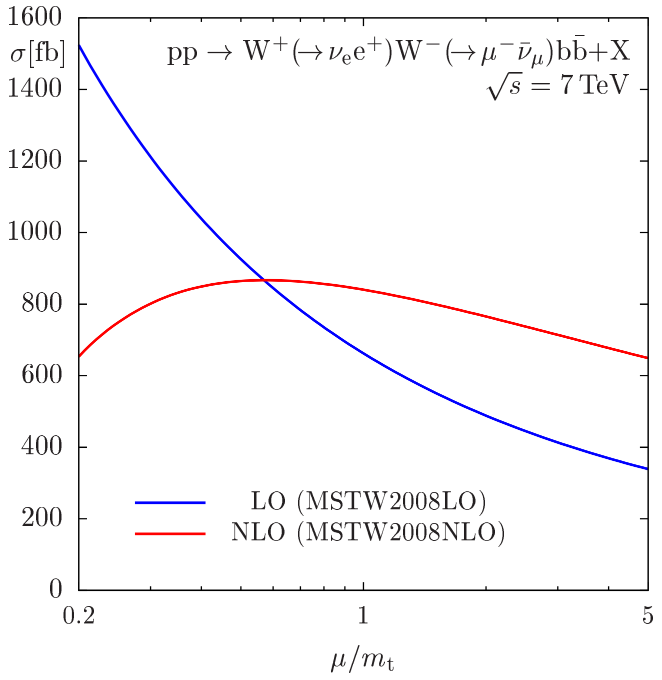
QCD partons with $|\eta| < 5 \quad \Rightarrow \quad$ jets with $\sqrt{\Delta\phi^2 + \Delta y^2} > R = 0.4\,(0.5)$

Typical Tevatron (LHC) cuts

b-jets: $p_{\mathrm{T,b}} > 20\,(30)\,\mathrm{GeV}$ $\qquad |\eta_{\mathrm{b}}| \le 2.5$

leptons: $p_{\mathrm{T},l} > 20\,\mathrm{GeV}$ $\qquad |\eta_l| \le 2.5$ $\qquad p_{\mathrm{T,miss}} > 25\,(20)\,\mathrm{GeV}$





Integrated $e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ cross section

Predictions for $\mu_{R,F} = m_t$ and $m_t/2 \leq \mu_{R,F} \leq 2m_t$

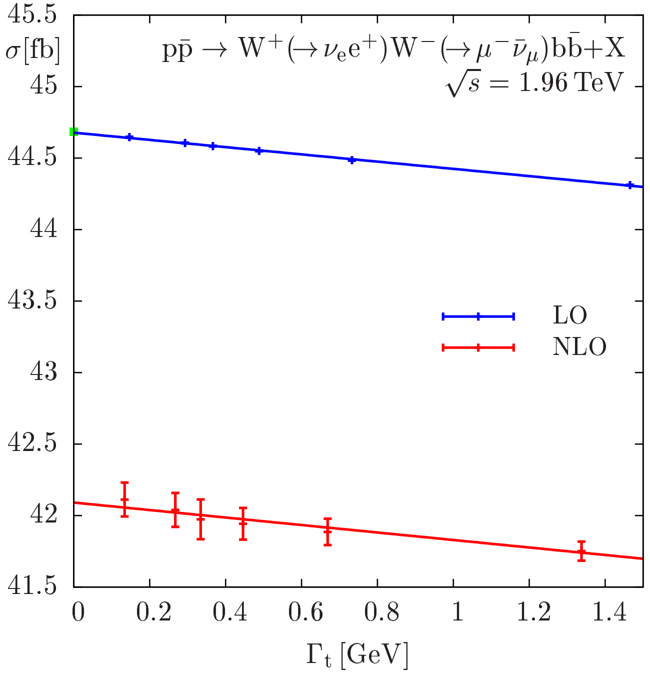
σ	LO	NLO	NLO/LO
Tevatron	$44.31^{+19.68}_{-12.49}$ fb	$41.75^{+0.00}_{-3.79}$ fb	$0.942^{+0.000}_{-0.085}$
LHC	$662.4^{+263.4}_{-174.1}$ fb	840^{+27}_{-75} fb	$1.27^{+0.04}_{-0.11}$

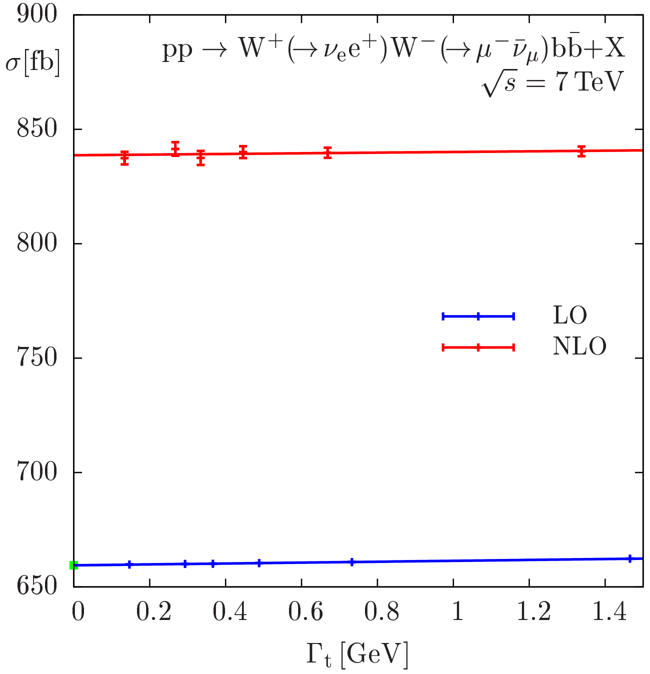
Scale uncertainty at the Tevatron (LHC)

- 44% (40%) LO uncertainty is mostly due to $\frac{\Delta\sigma_{LO}}{\sigma_{LO}} \simeq \frac{\Delta\alpha_S^2(\mu)}{\alpha_S^2(\mu)}$ and reduces to 9%(9%) at NLO

NLO corrections

- plots reflect σ_{NLO} stability and moderate corrections
- different sign and size for $q\bar{q}$-dominated σ_{Tevatron} ($K \simeq 0.94$) and gg-dominated σ_{LHC} ($K \simeq 1.27$)





Off-shell and non-resonant contributions to $\sigma_{\text{int.}}$

Assessment of finite-width effects $\sigma(\Gamma_t) - \sigma(0)$

- numerical extrapolation to $\Gamma \to 0$ using five rescaled values $\Gamma_t \to \xi\Gamma_t$ with $0.1 \lesssim \xi \leq 1$

Cancellation of soft-gluon $\ln(\Gamma_t/m_t)$ singularities

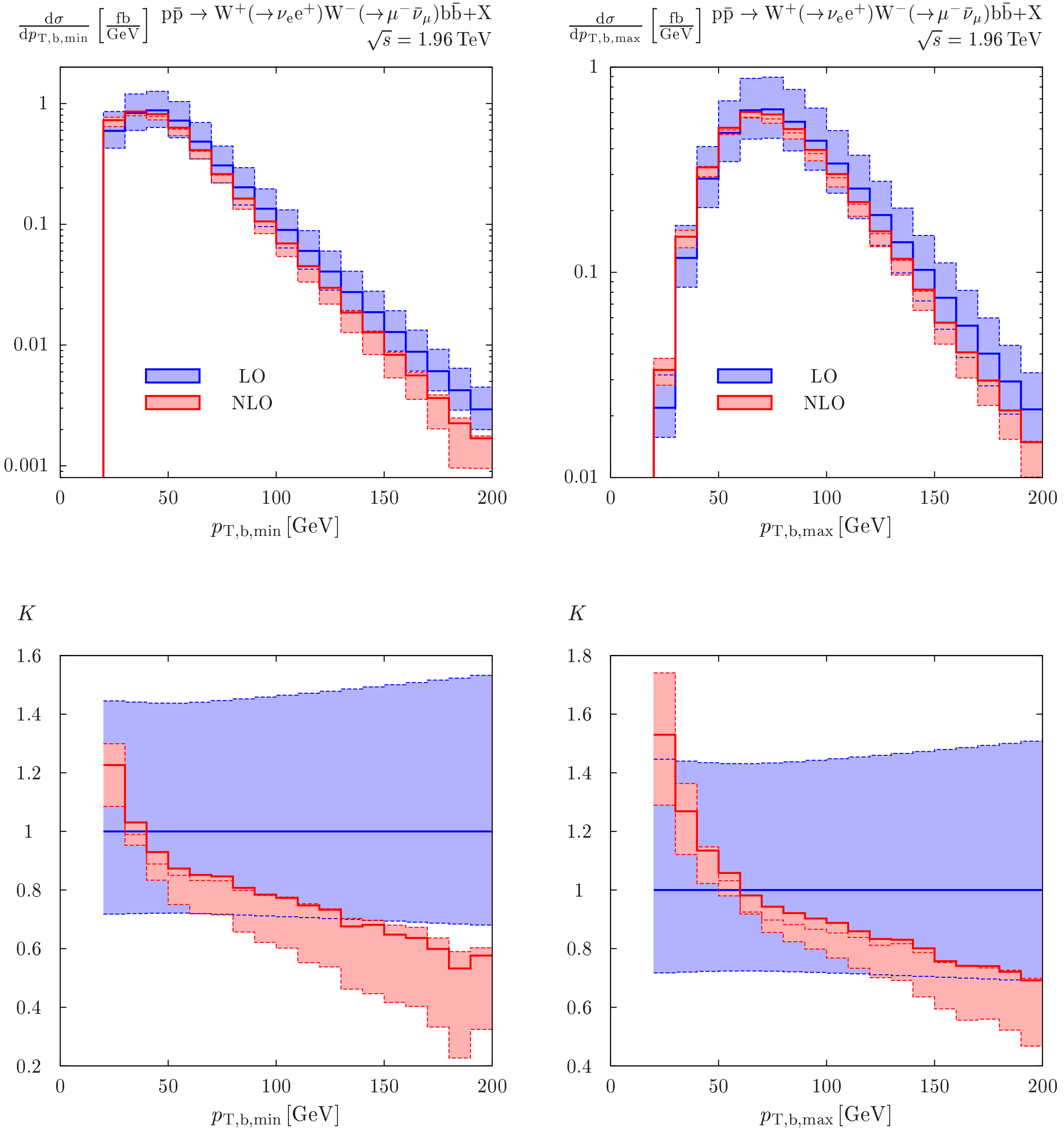
- dipole-subtracted virtual and real parts diverge logarithmically when $\Gamma \to 0$
- linear convergence of $\sigma(\Gamma_t) \to \sigma(0)$ provides non-trivial consistency and stability check

Finite-width effects comparable to $\Gamma_t/m_t \simeq 0.8\%$

	$\sigma_{\text{LO}}(\Gamma_t)/\sigma_{\text{LO}}(0) - 1$	$\sigma_{\text{NLO}}(\Gamma_t)/\sigma_{\text{NLO}}(0) - 1$
Tevatron	-0.8%	-0.9%
LHC	$+0.4\%$	$+0.2\%$

quantifies precision of NWA for σ_{incl}

(3.2) Differential $\mathrm{W^+W^-b\bar{b}}$ distributions at the Tevatron



b-jet p_T distributions

Soft b-jet (left)

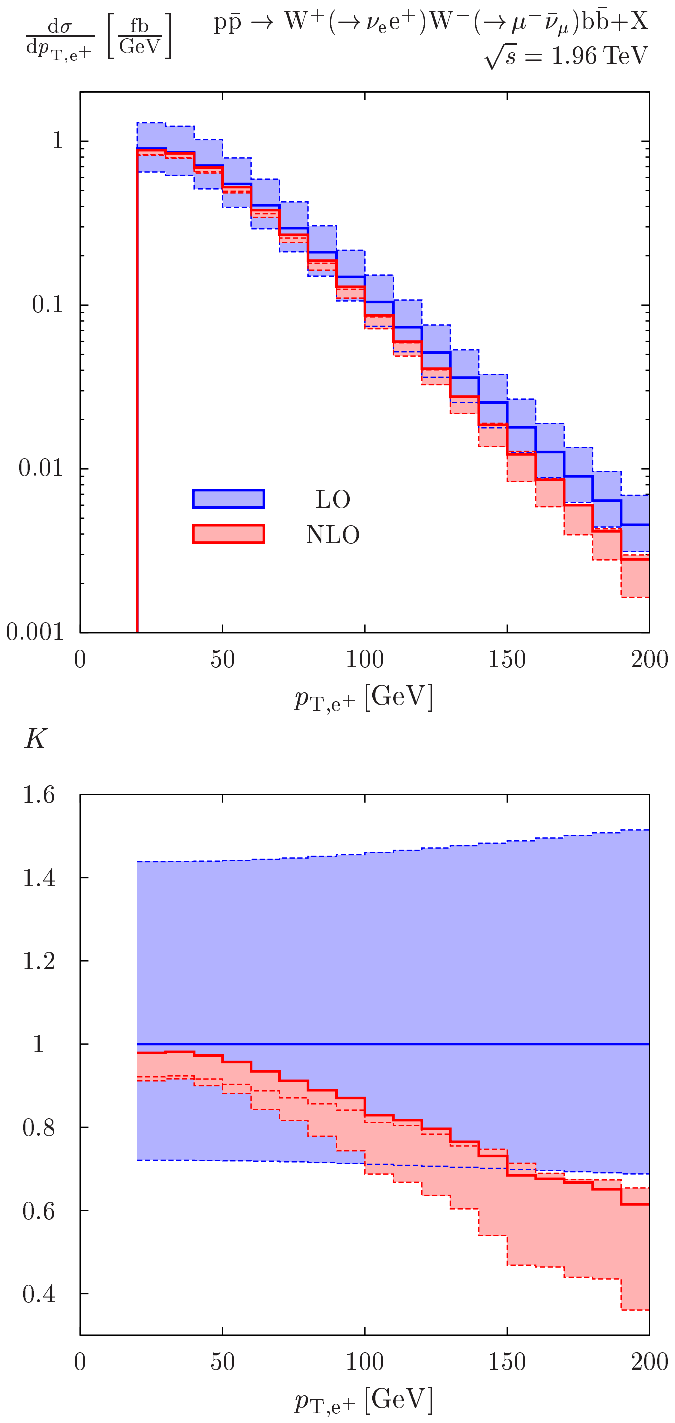
- saturates cut at 20 GeV
- +20% to −40% corrections

Hard b-jet (right)

- peaked around 80 GeV
- +50% to −30% corrections

Strong shape distortions

- especially at small p_T (impact on acceptance!)
- to be compared with parton-shower effects



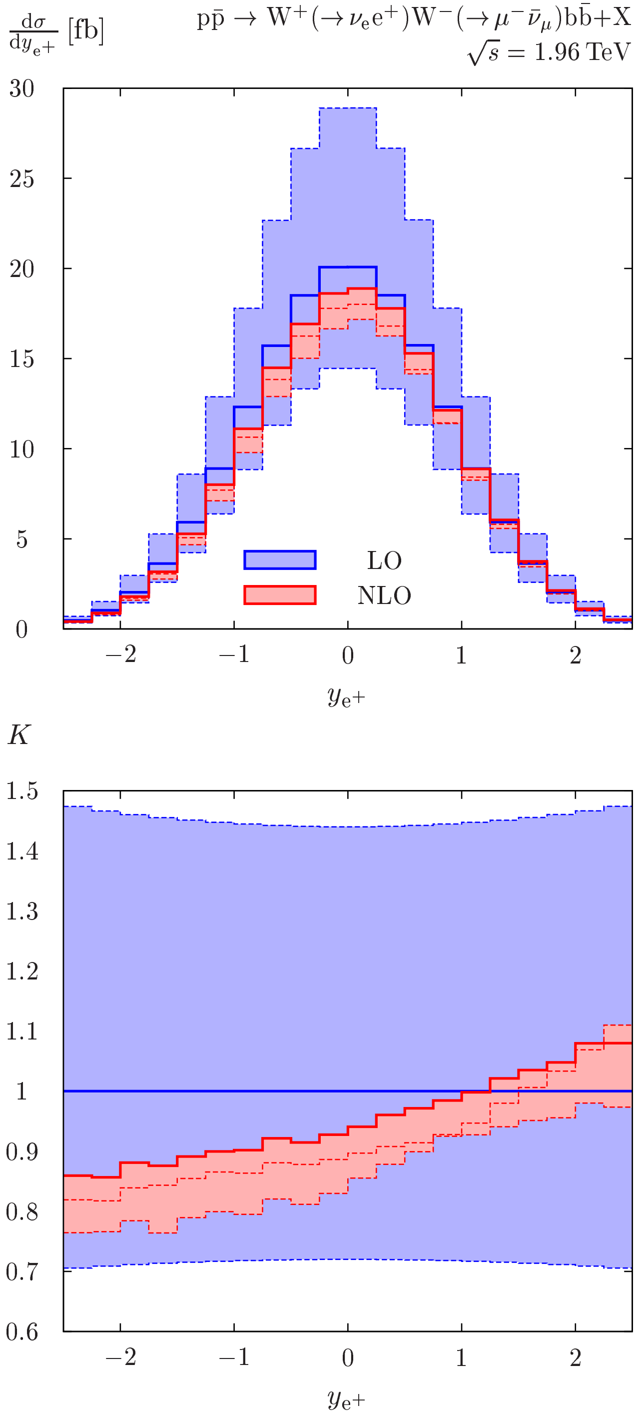
Lepton p_T distributions

e^+ from W^+ decay

- have typically $p_T \lesssim 100\,\text{GeV}$ and tend to saturate the cut at 20 GeV
- corrections range from 0% to −40%

Shape distortion

- mild in the vicinity of the cut but fairly strong at high p_T
- relevant for boosted tops and NP searches
- when $p_T \gtrsim 100\,\text{GeV}$ fixed $\mu = m_t$ should be replaced by dynamical QCD scale



Charged-lepton rapidity

LO y_{e^+} distribution

- e^+ populates central region
- almost exactly symmetric due to $t \leftrightarrow \bar{t}$ invariance of $q\bar{q}/gg \to t\bar{t}$

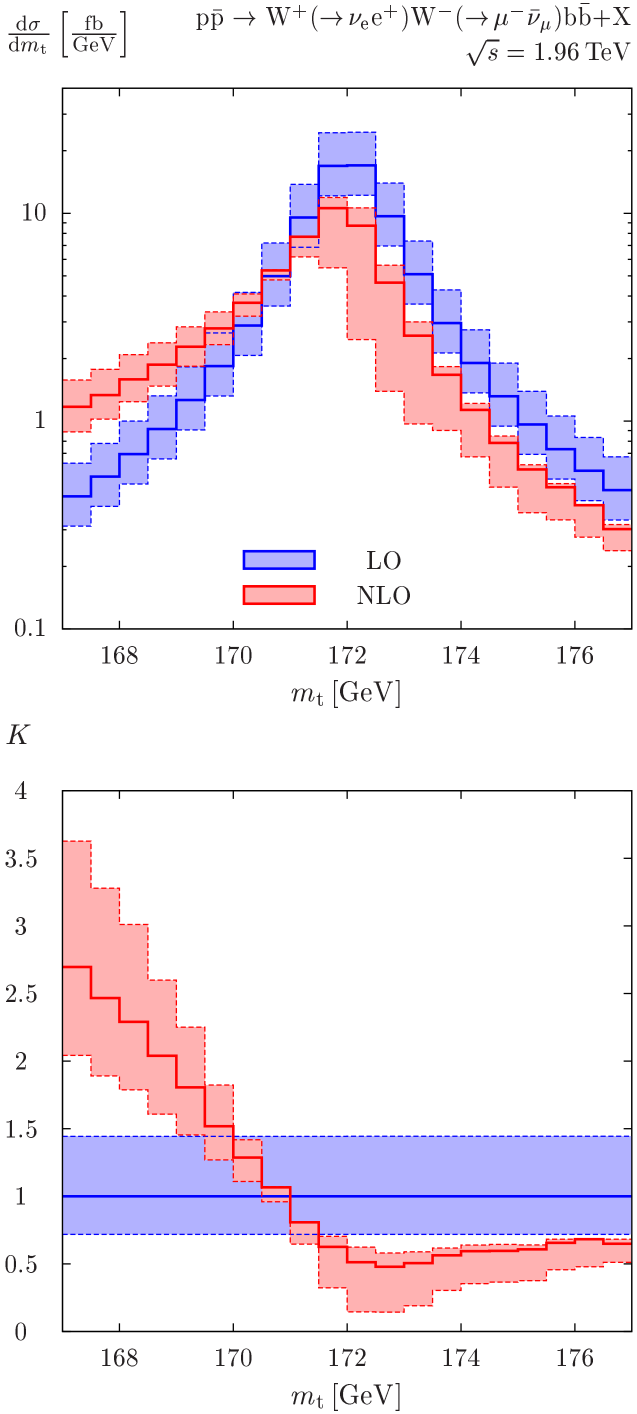
NLO charge and FB asymmetry

- IS–FS gluon exchange induces $t\bar{t}$ charge asymmetry
- reflected in y_{e^+} shape distortion (−15% to +10% corrections) and FB asymmetry

$$A_{\mathrm{FB}} = \frac{\sigma(y_{e^+} > 0) - \sigma(y_{e^+} < 0)}{\sigma(y_{e^+} > 0) + \sigma(y_{e^+} < 0)} = 0.035(2)$$

consistent with NWA [Bernreuther/Si ’10]

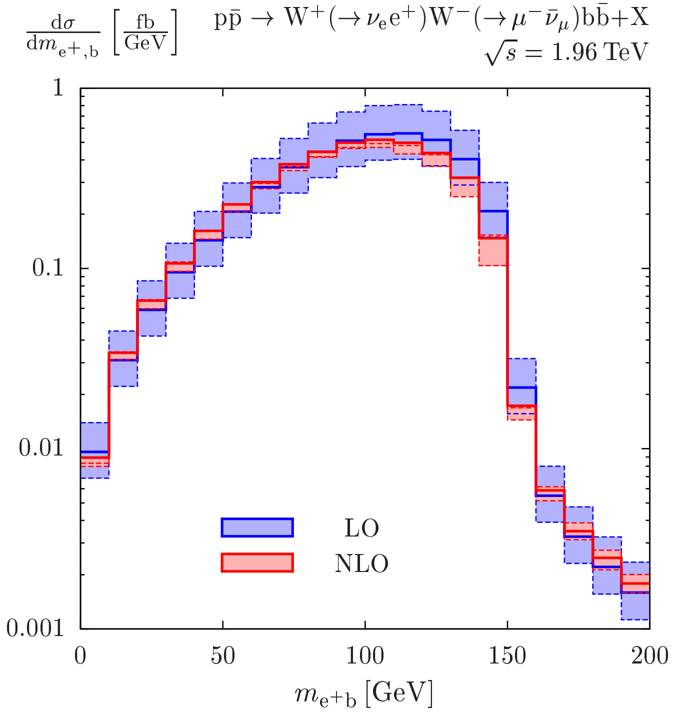
Top-quark invariant mass

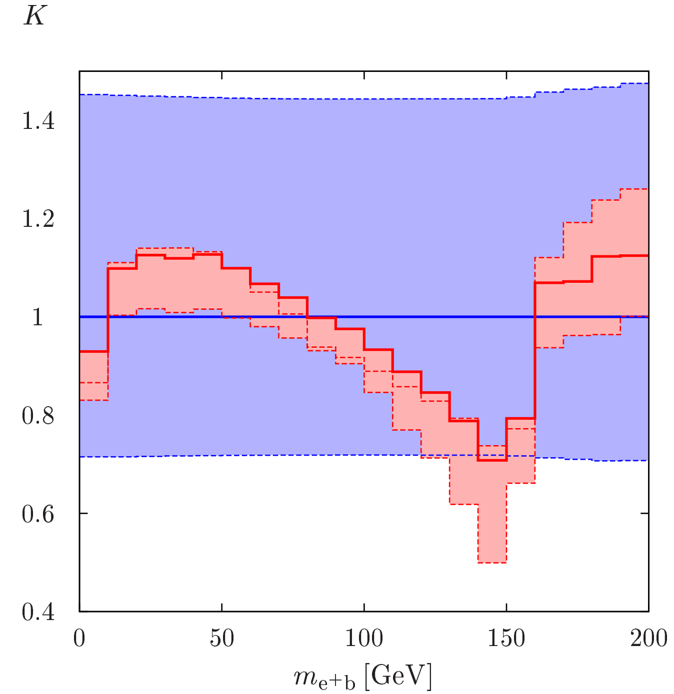


Although not observable $M_t = M_{be^+\nu_e}$ reflects off-shell nature of $2 \to 4$ calculation

- Breit–Wigner shape in the resonance region
- $\delta\Gamma_{NLO}/\Gamma_{LO} \simeq -9\%$ crucial for consistent normalisation of $\sigma_{incl.} \sim 1/\Gamma_t^2$
- Pole of top-quark progagator not shifted in on-shell scheme, but QCD radiation leads to $\mathcal{O}(1\,\text{GeV})$ invariant-mass losses
- m_t-shift depends on jet algorithm

NLO and Γ_t effects will improve description of observables used for m_t determination





Invariant mass of b-jet–e^+ pair

Observable related to m_t measurement

- visible decay products in $t \to bW^+ \to be^+\nu_e$ retain significant fraction of m_t
- good sensitivity to m_t via kinematic bound

$$M^2_{e^+b} \leq m_t^2 - M_W^2 \simeq (152\,\mathrm{GeV})^2$$

in LO and narrow-width approximation

Off-shell and NLO corrections

- M_{e^+b} bound violated by LO off-shell effects
- additional violation from NLO radiation
- strong NLO shape distortion below the bound: from +15% to −30% corrections

Conclusions

NLO QCD calculation for $\mathrm{W^+W^-b\bar{b}}$ production

- precise description of $\mathrm{t\bar{t}}$ production and decay
- including off-shell effects, non-resonant backgrounds and interferences

Inclusive cross section at the Tevatron (LHC)

- moderate corrections K=0.94 (1.27) and stable NLO predictions ($\delta\sigma/\sigma \simeq 9\%$)
- quantitative assessment of finite-width effects $\lesssim \Gamma_{\mathrm{t}}/m_{\mathrm{t}} = 0.8\%$ supports NWA

NLO corrections to differential distributions at the Tevatron

- rich and non-trivial kinematic dependence
- potentially large impact on acceptances and shape-dependent precision measurements (like m_{t})

Coming soon: LHC distributions, off-shell regime, . . .