

Heavy-quark production and running masses

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DESY, Zeuthen

in collaboration with **S. Alekhin** on arXiv:1011.5790

with **S. Alekhin , J. Blümlein , P. Jimenez-Delgado , E. Reya** on arXiv:1011.6259

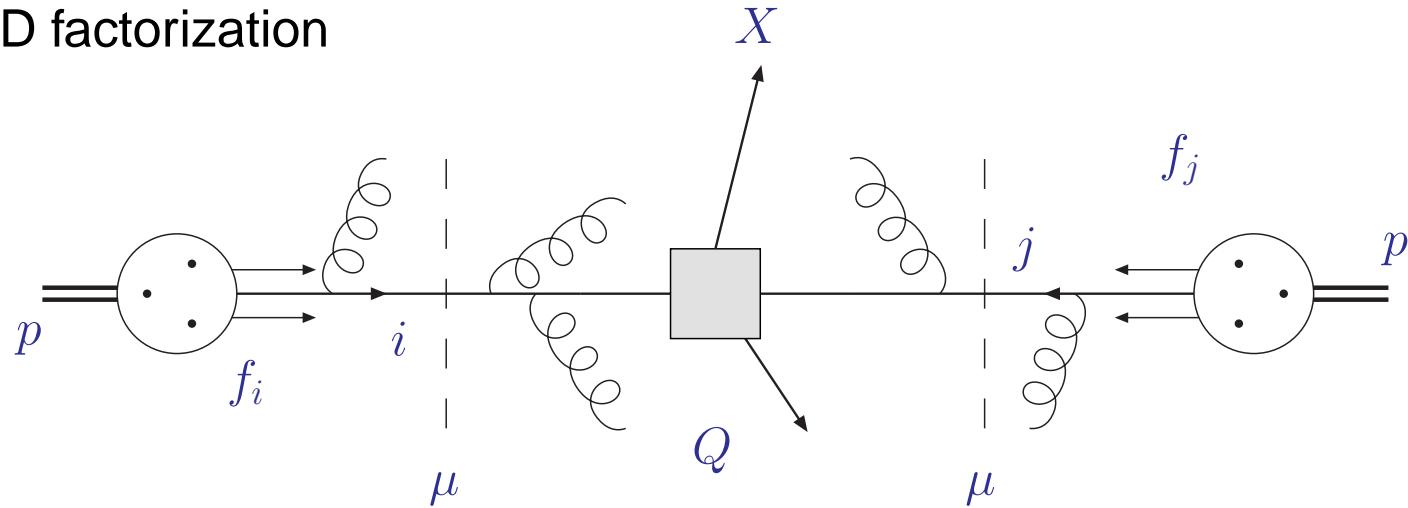
with **H. Kawamura A. Lo Presti A. Vogt** on arXiv:1008.0951

with **U. Langenfeld** and **P. Uwer** on arXiv:0907.5273

– *Workshop on Heavy Particles at the LHC, Zurich, Jan 06, 2011* –

Introduction

- QCD factorization



$$\sigma_{pp \rightarrow X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \rightarrow X} (\alpha_s(\mu^2), Q^2, \mu^2, m_X^2)$$

- Hard parton cross section $\hat{\sigma}_{ij \rightarrow X}$ calculable in perturbation theory
 - known to NLO, NNLO, ... ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i , strong coupling α_s , particle masses m_X
 - known from global fits to exp. data, lattice computations, ...

Heavy-quark masses

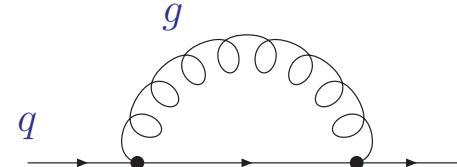
QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\text{flavors}} \bar{q} (\mathrm{i} \not{D} - m_q) q$$

- Covariant derivative $\not{D}_\mu = \partial_\mu + \mathrm{i} g_s A_\mu$
- Formal parameters of the theory (no observables)
 - strong coupling $\alpha_s = g_s^2/(4\pi)$
 - quark masses m_q
- Quantum corrections (loop integrals) require UV renormalization; (scheme dependence):
 - α_s → asymptotic freedom, running coupling (\overline{MS} scheme)
 - m_q → pole mass or running mass (\overline{MS} scheme)

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q) \Big|_{p^2 = m_q^2}$$


- heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$

Running quark masses

- \overline{MS} mass definition $m(\mu_R)$ realizes running mass (scale dependence)
 - renormalization group equation (mass anomalous dimension γ)
$$\left(\mu_R^2 \frac{\partial}{\partial \mu_R^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) m(\mu_R) = \gamma(\alpha_s) m(\mu_R)$$
 - short distance mass probes at scale of hard scattering
 $m_{\text{pole}} = m_{\text{short distance}} + \delta m$
 - conversion between pole mass and \overline{MS} mass definition in perturbation theory: $m = m(\mu_R) \left(1 + a_s(\mu_R) d^{(1)} + a_s(\mu_R)^2 d^{(2)} \right)$

Scale dependence

- Renormalization group equation for scale dependence

- strong coupling α_s and mass m

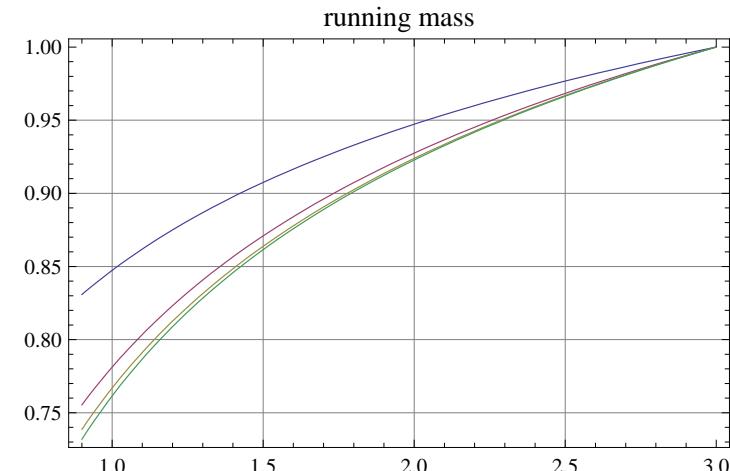
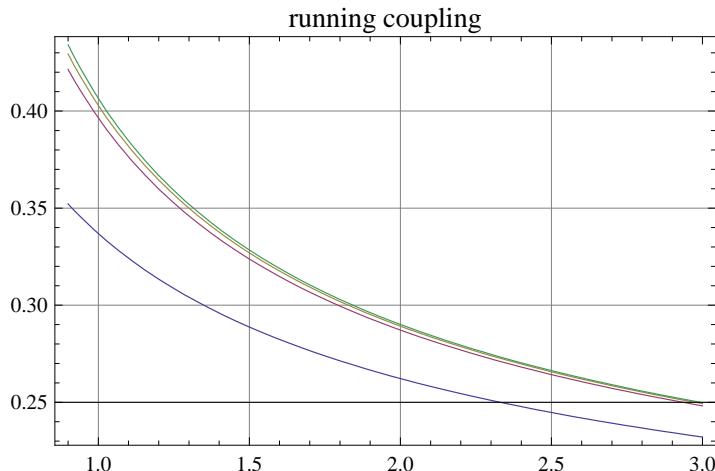
$$\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) = \beta(\alpha_s) \quad \mu^2 \frac{d}{d\mu^2} m(\mu) = \gamma(\alpha_s)m(\mu)$$

- Perturbative expansion known to four loops

- β -function van Ritbergen, Vermaseren, Larin '97 and mass anomalous dimension γ Chetyrkin '97; Larin, van Ritbergen, Vermaseren '97
 - very good convergence of perturbative series even at low scales

- Plot at low scales $\mu = 1.0 \dots 3.0 \text{ GeV}$

α_s (left) and mass ratio $m(3\text{GeV})/m(\mu)$ (right)

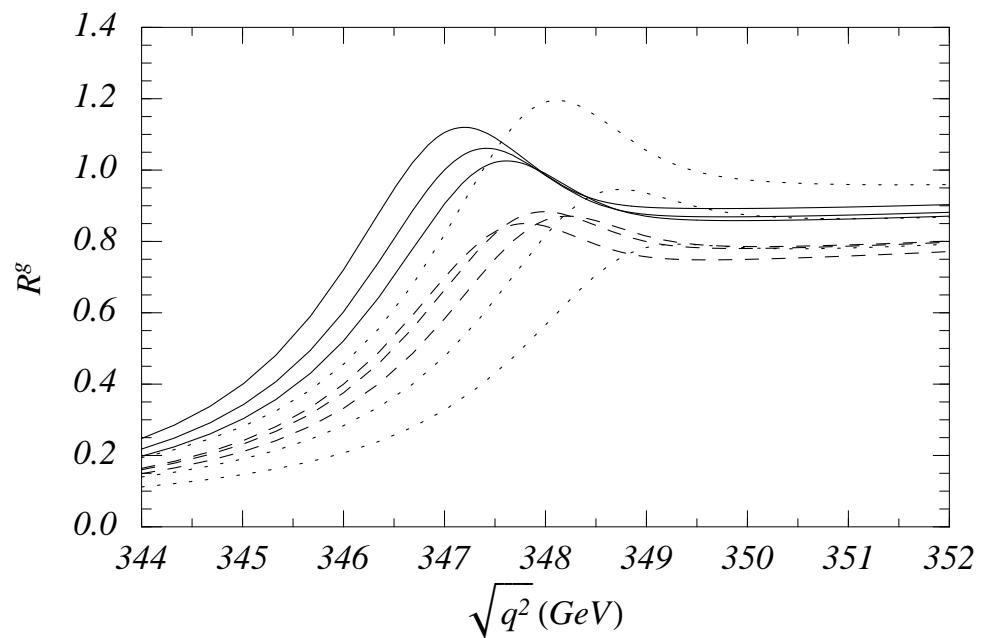


- Use of charm-quark mass $m_c(m_c)$ is well justified

The top-quark mass

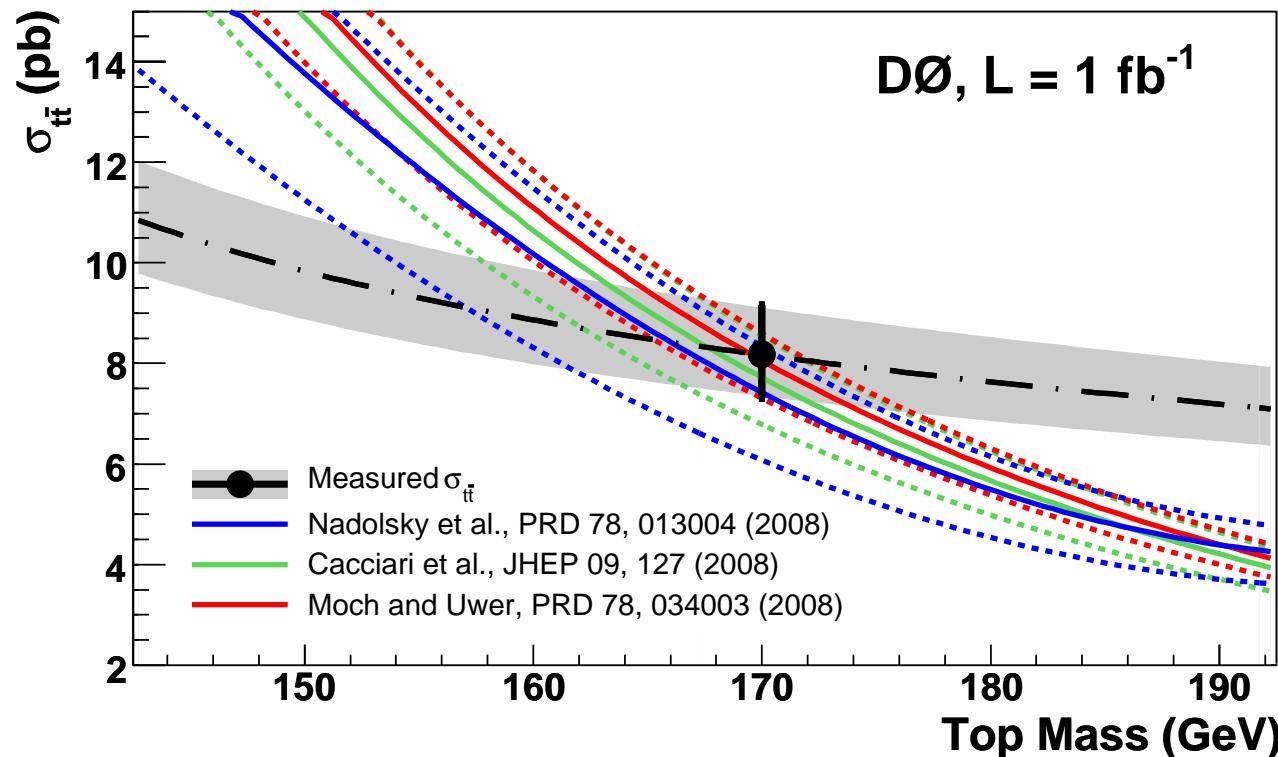
Illustration for linear collider

- Idea: top-quark mass measurement from threshold scan of cross section in e^+e^- collision
- Pole mass measurements are strongly order-dependent
 - ILC study
 - Beneke, Signer, Smirnov '99;
 - Hoang, Teubner '99;
 - Melnikov, Yelkhovsky '98;
 - Penin, Pivovarov '99;
 - Yakovlev '99
 - LO (dotted), NLO (dashed), NNLO (solid)



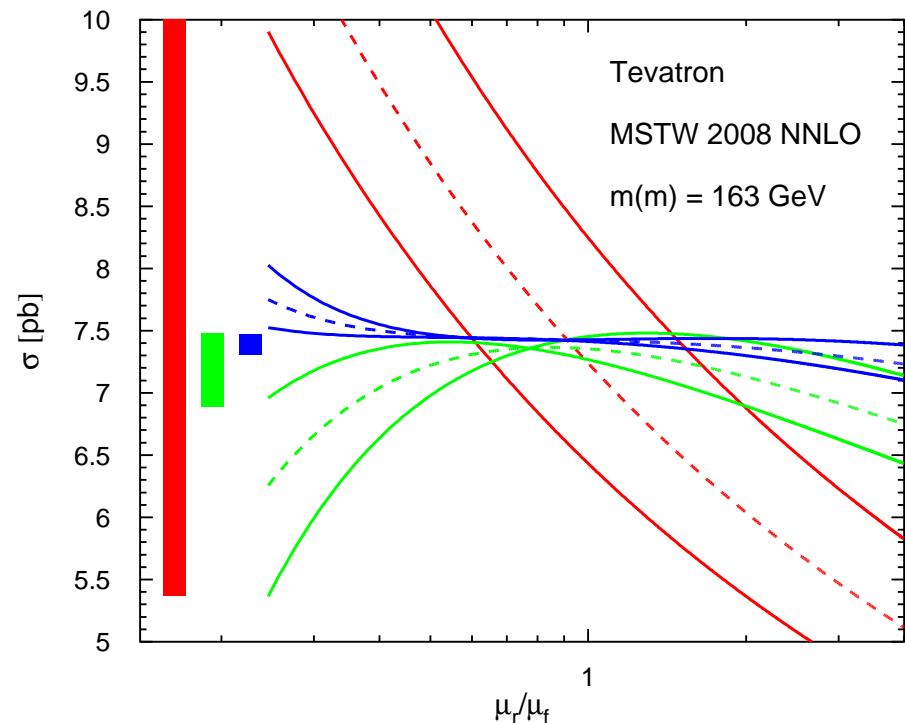
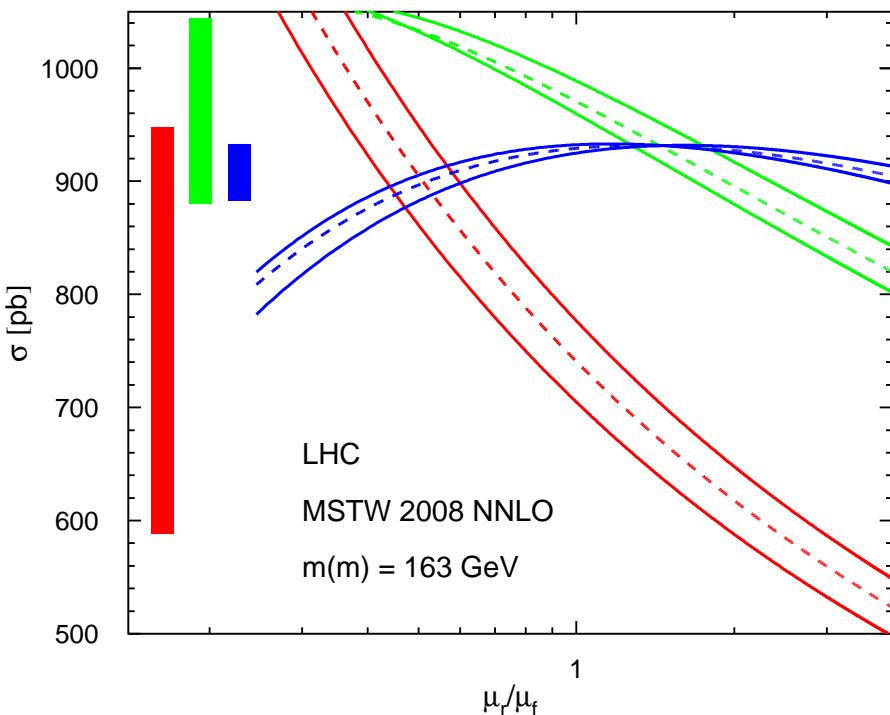
Tevatron analyses

- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- Determination of m_t from total cross section (slope $d\sigma/dm_t$)
 - e.g. D0 '09: NLO $m_t = 165.5^{+6.1}_{-5.9}$; NNLO $m_t = 169.1^{+5.9}_{-5.2}$, ...



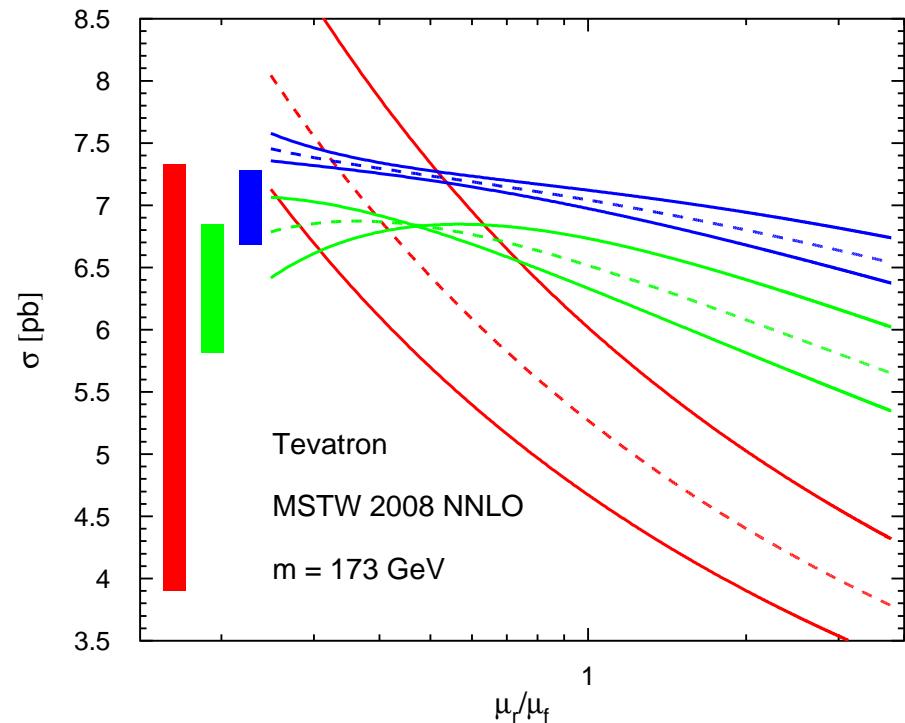
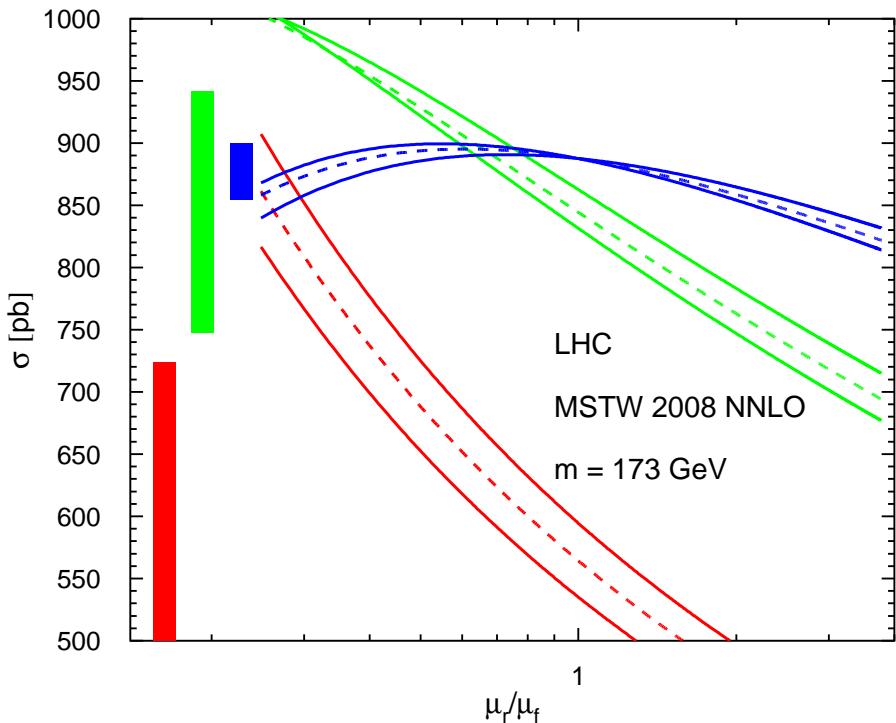
The running top-quark mass

- \overline{MS} mass definition $m(\mu_R)$ realizes running mass (scale dependence)
 - short distance mass probes at scale of hard scattering
 - conversion between pole mass and \overline{MS} mass definition in perturbation theory: $m_t = m(\mu_R) \left(1 + a_s(\mu_R)d^{(1)} + a_s(\mu_R)^2 d^{(2)}\right)$
- Scale dependence greatly reduced



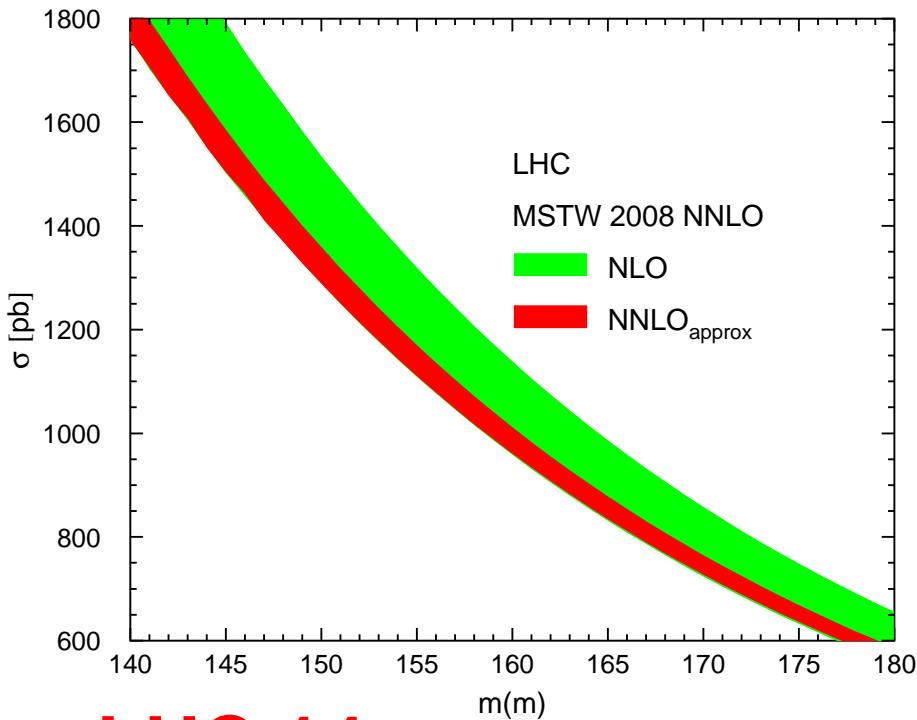
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- Pole mass scheme for comparison

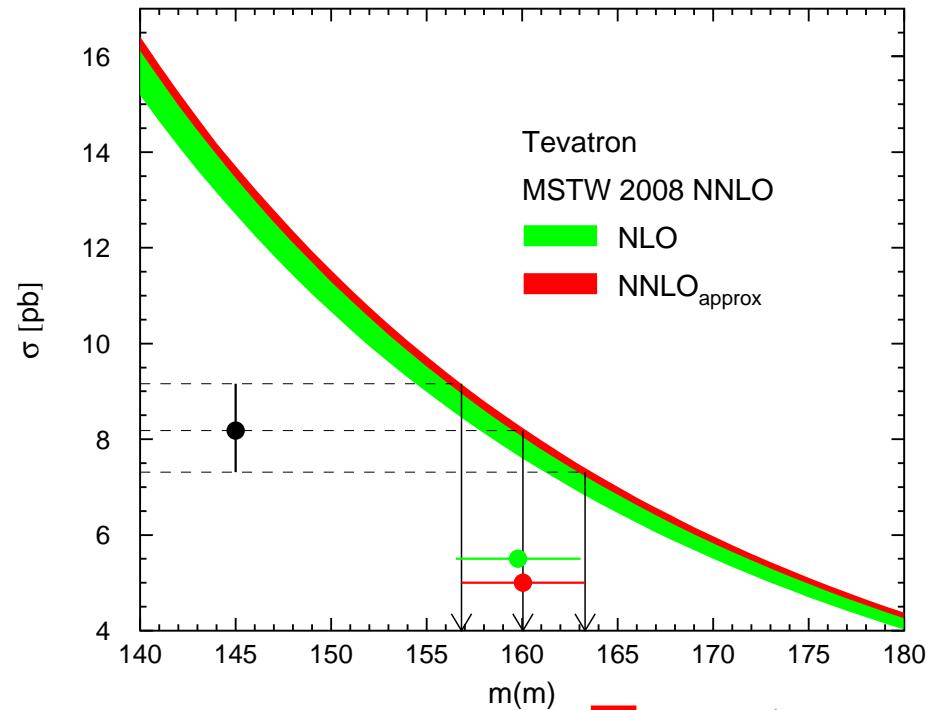


Mass dependence in \overline{MS} mass scheme

- Total top-quark cross section as function of \overline{m}
 - theoretical uncertainty (band) due to variation of $\mu_R \in [\overline{m}/2, 2\overline{m}]$ for fixed set $\mu_F \in \overline{m}/2, \overline{m}, 2\overline{m}$



LHC 14

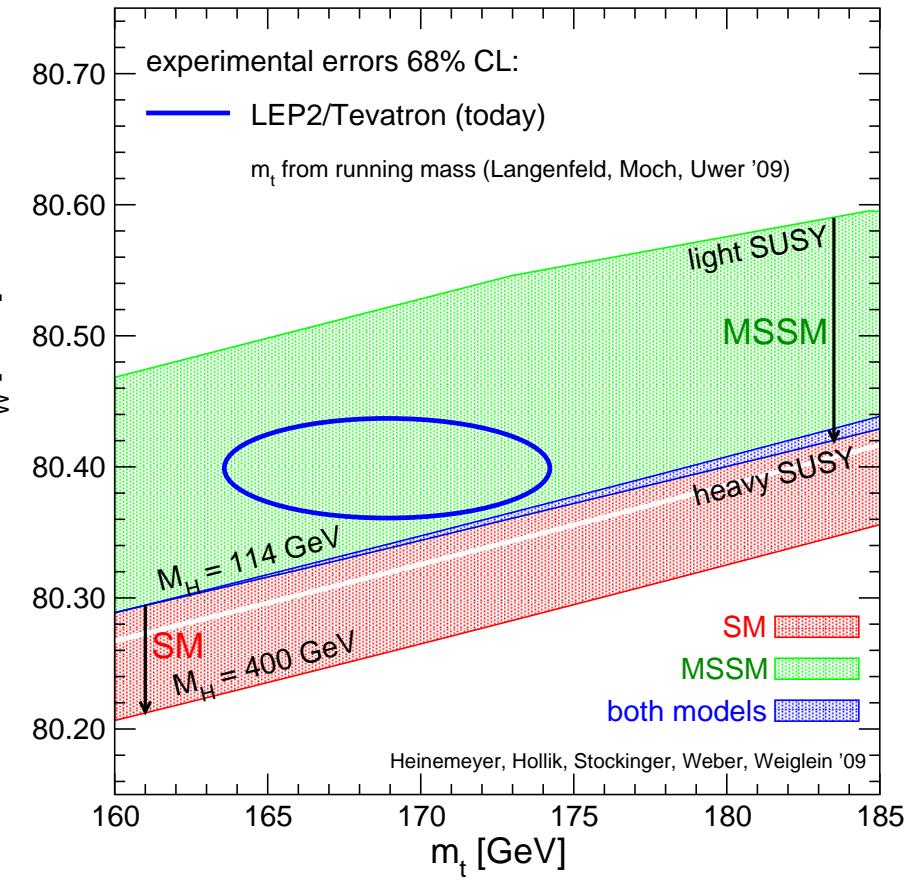
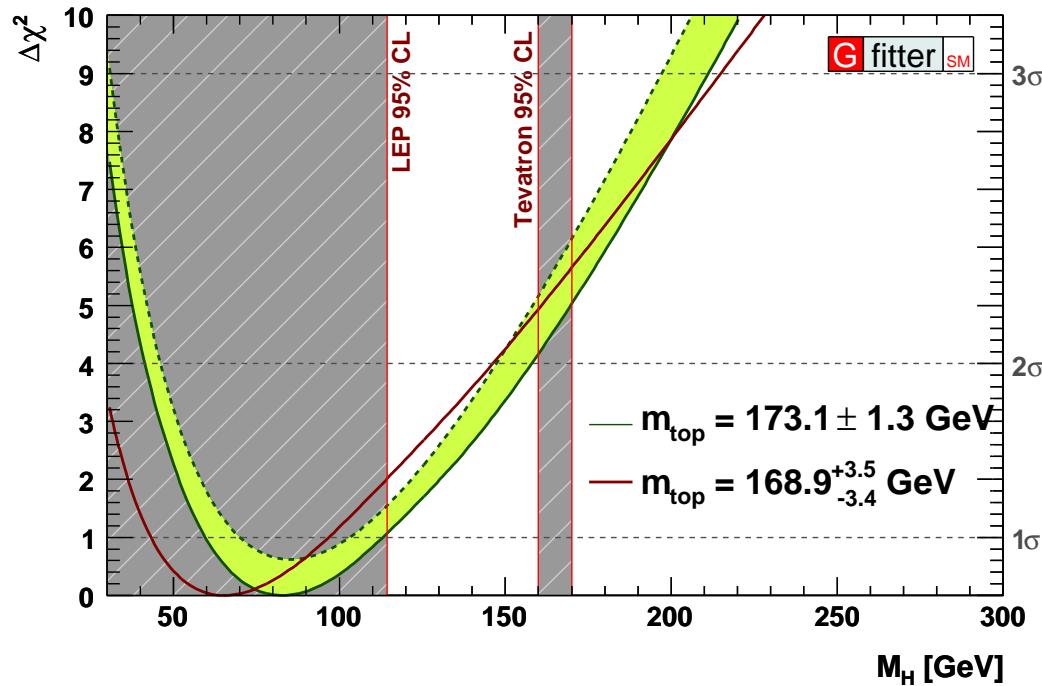


Tevatron

- First direct determination of $m(m)$ Langenfeld, S.M., Uwer '09
 - \overline{MS} mass $m(m) = 160.0^{+3.3}_{-3.2} \text{ GeV}$
 - conversion to pole mass $m_t = 168.9^{+3.5}_{-3.4} \text{ GeV}$

Implications for indirect Higgs searches

- Electroweak precision data constrains M_H



- pole mass $m_t = 168.9^{+3.5}_{-3.4}$ GeV
(lighter top-quark masses disfavor SM Higgs sector)

HATHOR

C++ package

Aliev, Lacker, Langenfeld, S.M., Uwer, Wiedermann '10

- Cross section evaluation done entirely in Hathor class

```
unsigned int scheme = Hathor::LO | Hathor::NLO | Hathor::NNLO;  
double mt = 171., muf=171., mur=171.;  
double val,err,chi2a,up,down;  
  
Lhapdf pdf( "MSTW2008nnlo68cl" ); ← PDF choice  
Hathor XS(pdf)
```



HATHOR

C++ package

Aliev, Lacker, Langenfeld, S.M., Uwer, Wiedermann '10

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```
unsigned int scheme = Hathor::LO | Hathor::NLO | Hathor::NNLO;  
double mt = 171., muf=171., mur=171.;  
double val,err,chi2a,up,down;  
  
Lhapdf pdf( "MSTW2008nnlo68cl" );    ← PDF choice  
Hathor XS(pdf)  
  
XS.setPrecision(Hathor::MEDIUM);  
XS.getXsection(mt,mur,muf);    →  $\sigma = 164.3^{+4.6}_{-9.2}$  pb (scale unc.)  
XS.getResult(0,val,err,chi2a);
```



HATHOR

C++ package

Aliev, Lacker, Langenfeld, S.M., Uwer, Wiedermann '10

- Cross section evaluation done entirely in Hathor class

```
unsigned int scheme = Hathor::LO | Hathor::NLO | Hathor::NNLO;  
double mt = 171., muf=171., mur=171.;  
double val,err,chi2a,up,down;  
  
Lhapdf pdf( "MSTW2008nnlo68cl" );    ← PDF choice  
Hathor XS(pdf)  
  
XS.setPrecision(Hathor::MEDIUM);  
XS.getXsection(mt,mur,muf);   →  $\sigma = 164.3^{+4.6}_{-9.2}$  pb (scale unc.)  
XS.getResult(0,val,err,chi2a);  
  
XS.setScheme(scheme | Hathor::PDF_SCAN); ← with PDF error scan  
XS.setPrecision(Hathor::LOW);  
XS.getXsection(mt,mur,muf); } →  $\sigma = 164.3^{+4.6}_{-9.2}$  pb (sc.)  $^{+4.4}_{-4.4}$  pb (PDF unc.)  
XS.getPdfErr(up,down);
```



Parton distributions

Quark masses in PDF fits

- Choice of value for heavy-quark masses part of uncertainty
- PDF fits assume pole mass scheme for heavy-quarks
 - numerical values systematically lower than those from PDG (2-loop conversion to pole mass)

[GeV]	PDG	ABKM	GJR	HERAPDF	MSTW	CTEQ	NNPDF
m_c	$1.66^{+0.09}_{-0.15}$	$1.5^{+0.25}_{-0.25}$	1.3	1.4	1.4	1.3	$\sqrt{2}$
m_b	$4.79^{+0.19}_{-0.08}$	$4.5^{+0.5}_{-0.5}$	4.2	4.75	4.75	4.5	4.3

PDG

- PDG quotes running masses:
charm: $m_c(m_c) = 1.27^{+0.07}_{-0.11} \text{ GeV}$, bottom: $m_b(m_b) = 4.20^{+0.17}_{-0.07} \text{ GeV}$

Heavy-quark production in DIS

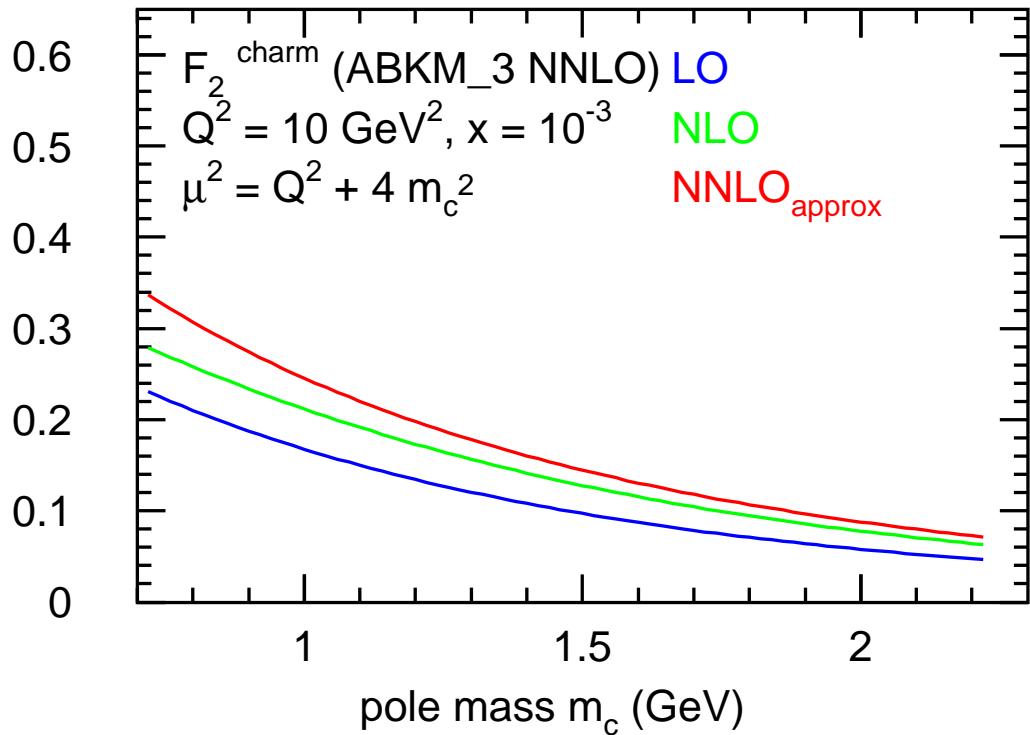
- NLO for charm structure function
 - neutral current Laenen, Riemersma, Smith, van Neerven '93
 - charged current Gottschalk '81; Glück, Kretzer, Reya '96

Theory improvements beyond NLO

- Parton cross section close to threshold $s \simeq 4m^2$
 - Sudakov logarithms $\ln(\beta)$ with velocity of heavy quark
 $\beta = \sqrt{1 - 4m^2/s}$ at n^{th} -order: $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$
- Resummation in Mellin space (renormalization group equation)
predicts fixed orders in perturbation theory
 - approximate expressions to NNLO
Laenen, S.M. '98; Alekhin, S.M. '08; Lo Presti, Kawamura, S.M., Vogt '10
- Asymptotics beyond NLO
 - NNLO corrections at large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein '09

Running quark masses in DIS

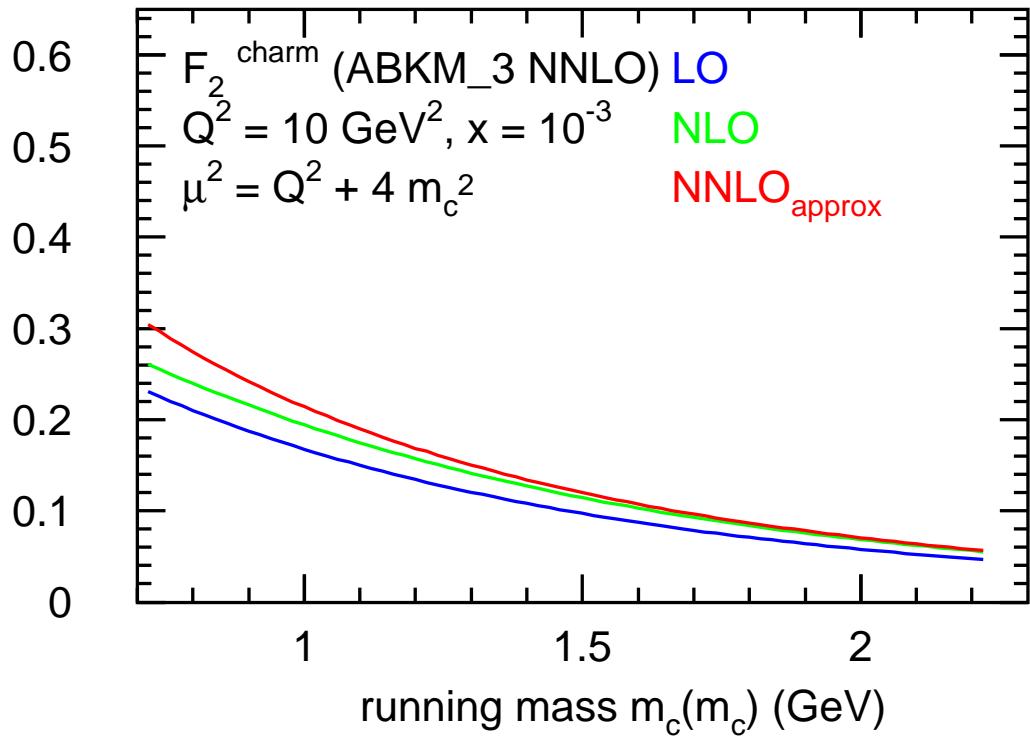
- Charm structure function Alekhin, S.M. '10
 - improved convergence
 - scale dependence reduced



- pole mass scheme for comparison

Running quark masses in DIS

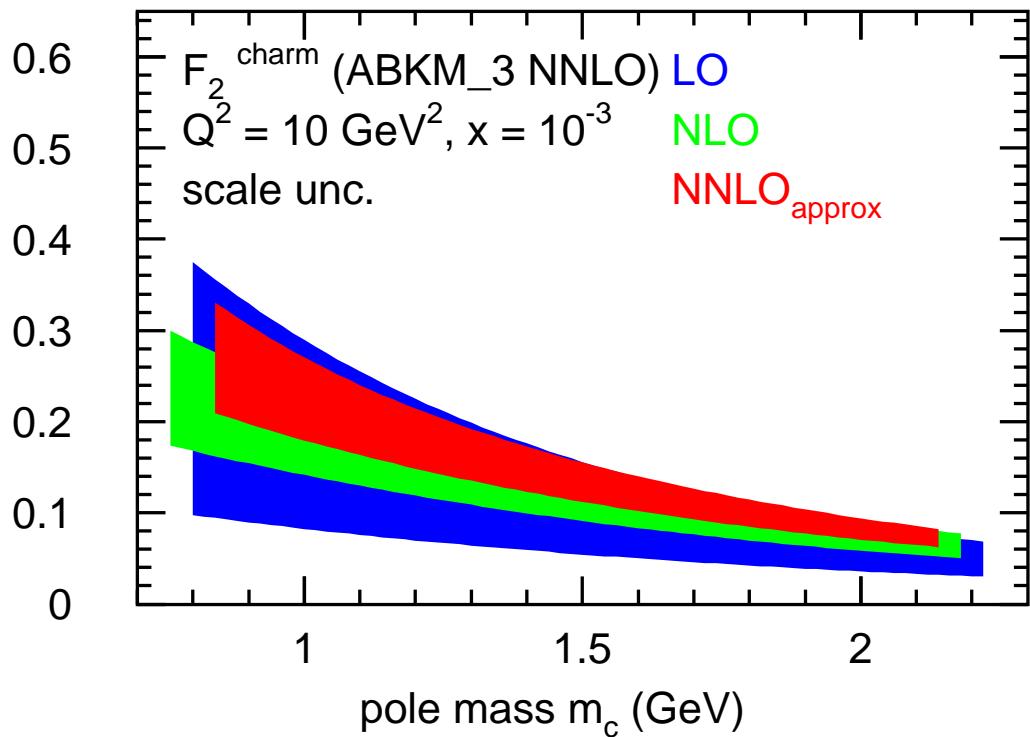
- Charm structure function Alekhin, S.M. '10
 - improved convergence
 - scale dependence reduced



- running mass
 - direct determination of $m_c(m_c)$
- NLO
 $1.26 \pm 0.09 \text{ (exp)} \pm 0.11 \text{ (th)} \text{ GeV}$
- NNLO_{approx}
 $1.01 \pm 0.09 \text{ (exp)} \pm 0.03 \text{ (th)} \text{ GeV}$

Running quark masses in DIS

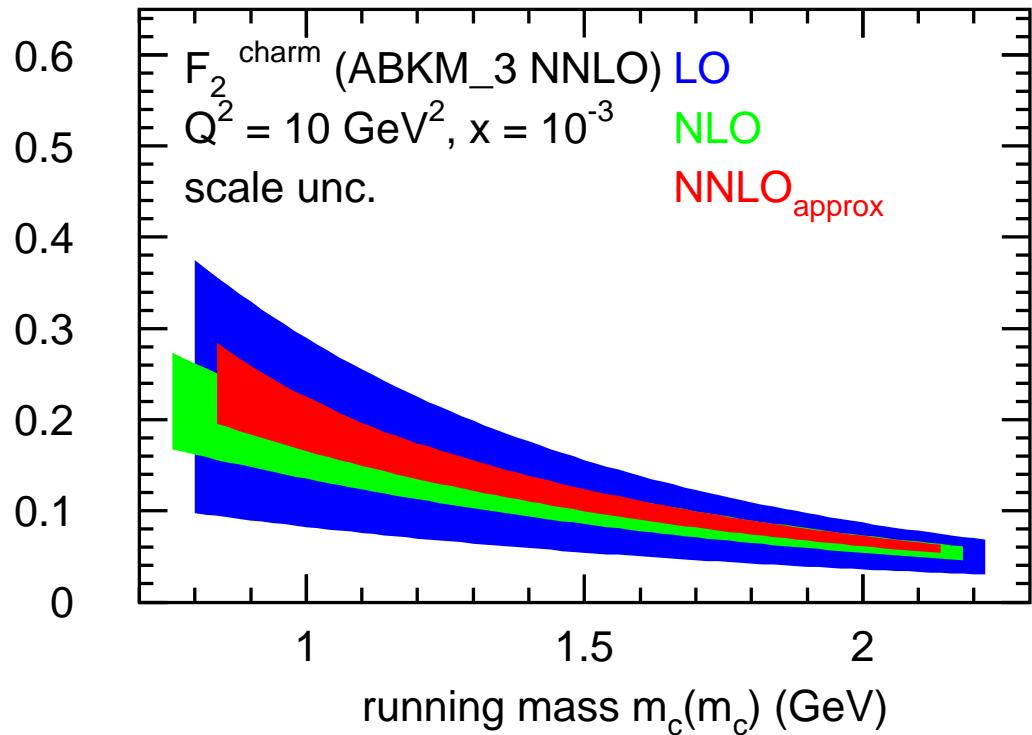
- Charm structure function Alekhin, S.M. '10
 - improved convergence
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- pole mass scheme for comparison
- scale variation $0.5 \leq \kappa \leq 2$ around $\mu_R, \mu_F = \kappa \sqrt{Q^2 + 4m_c^2}$

Running quark masses in DIS

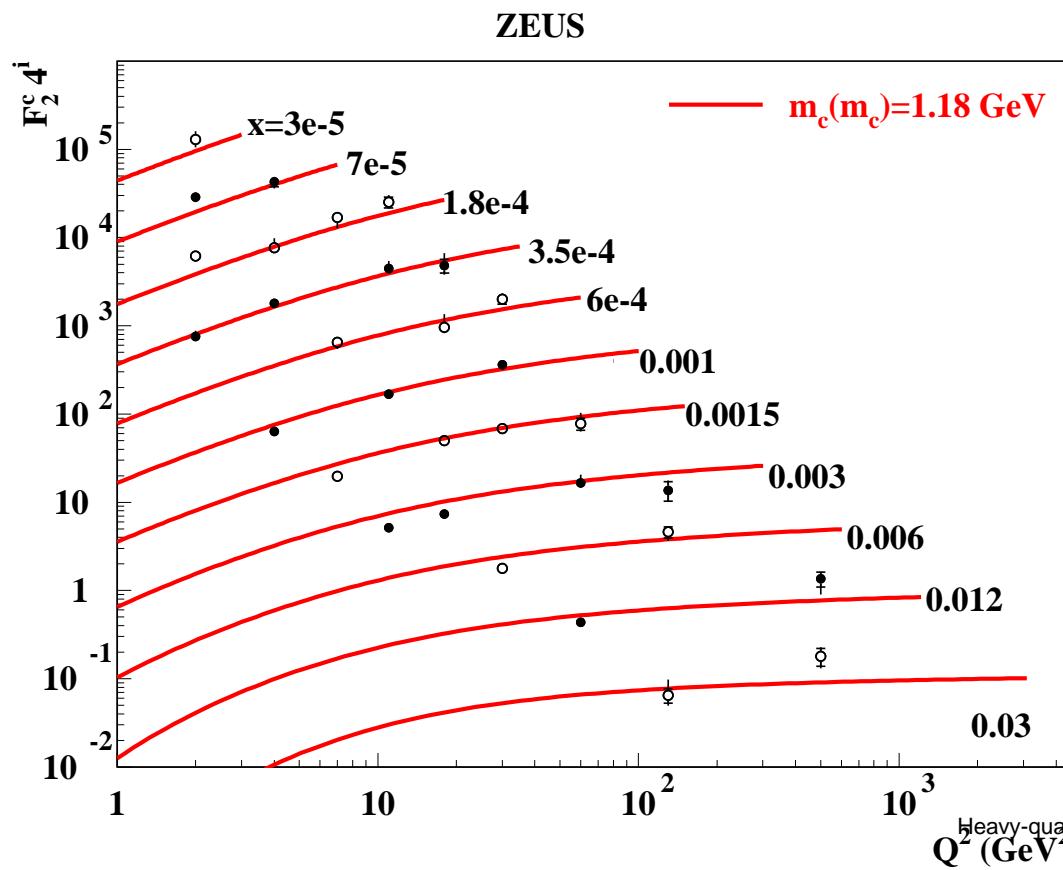
- Charm structure function Alekhin, S.M. '10
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- running mass
- scale variation $0.5 \leq \kappa \leq 2$ around $\mu_R, \mu_F = \kappa \sqrt{Q^2 + 4m_c^2}$

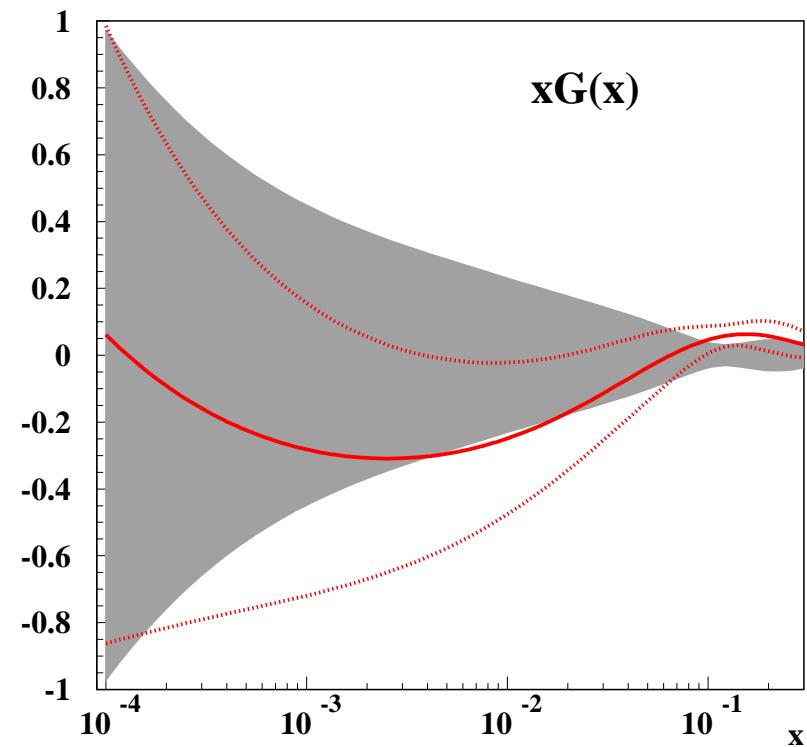
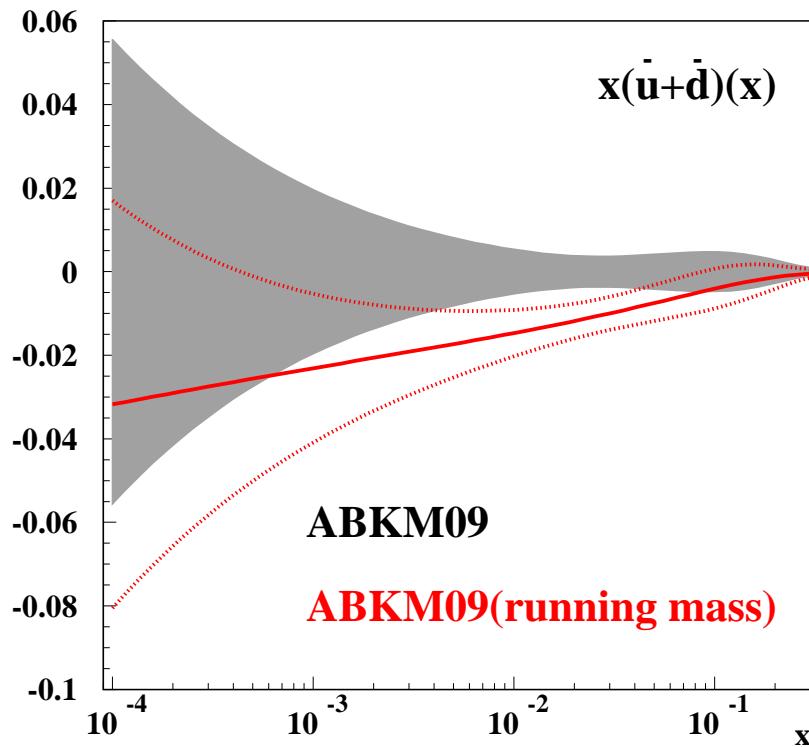
Global fit

- NNLO fit of PDFs (variant of ABKM fit Alekhin, Blümlein, Klein, S.M. '09)
 - add PDG value for $m_c(m_c)$ as additional constraint
 - fit gives $m_c(m_c) = 1.18 \pm 0.06 \text{ (exp)} \pm 0.03 \text{ (th)} \text{ GeV}$
- Comparison with ZEUS data shows good agreement
(theory somewhat low in Q^2 -region at small values of x)



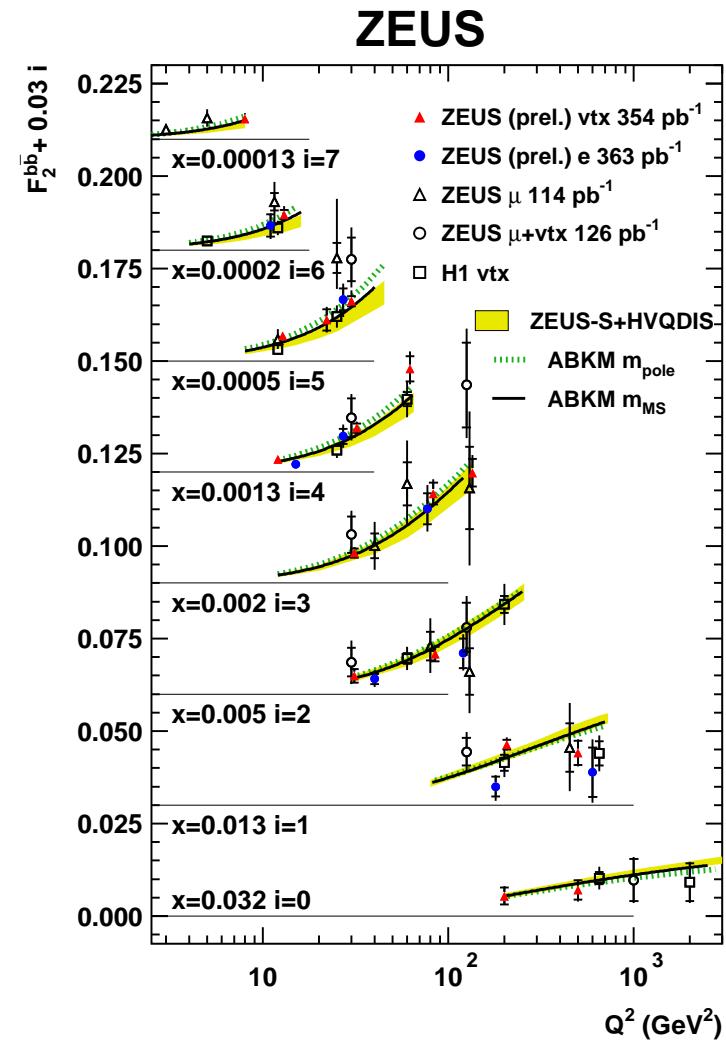
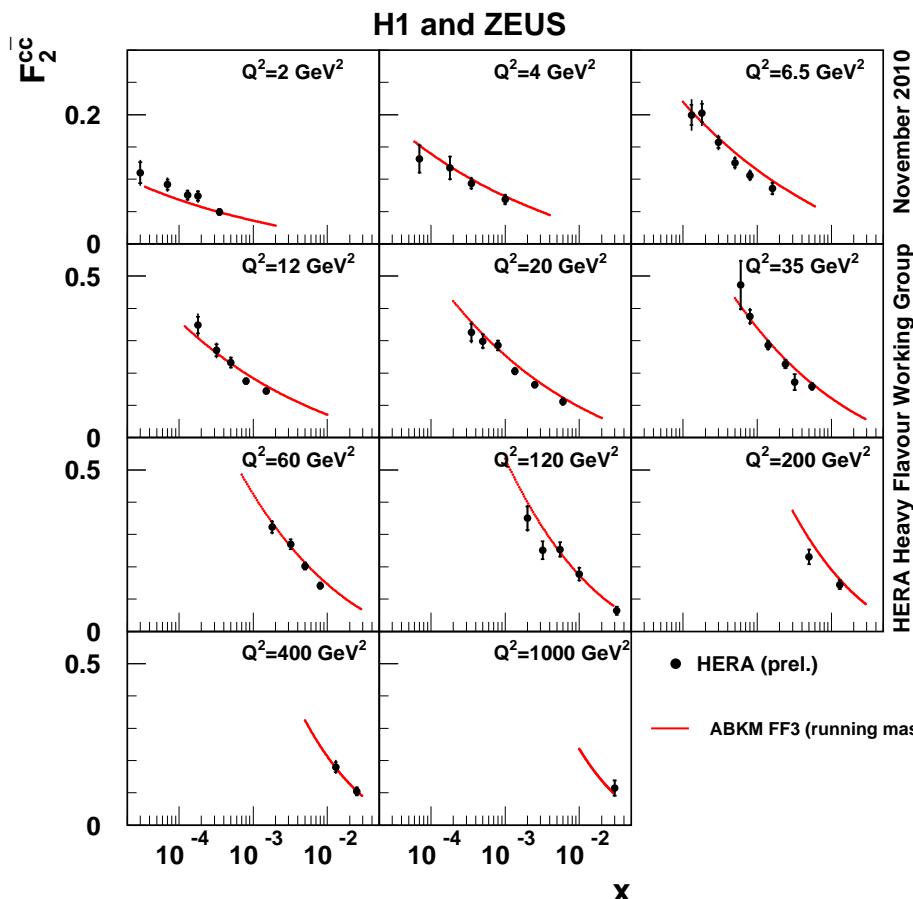
Impact on PDFs

$\mu=3 \text{ GeV}$



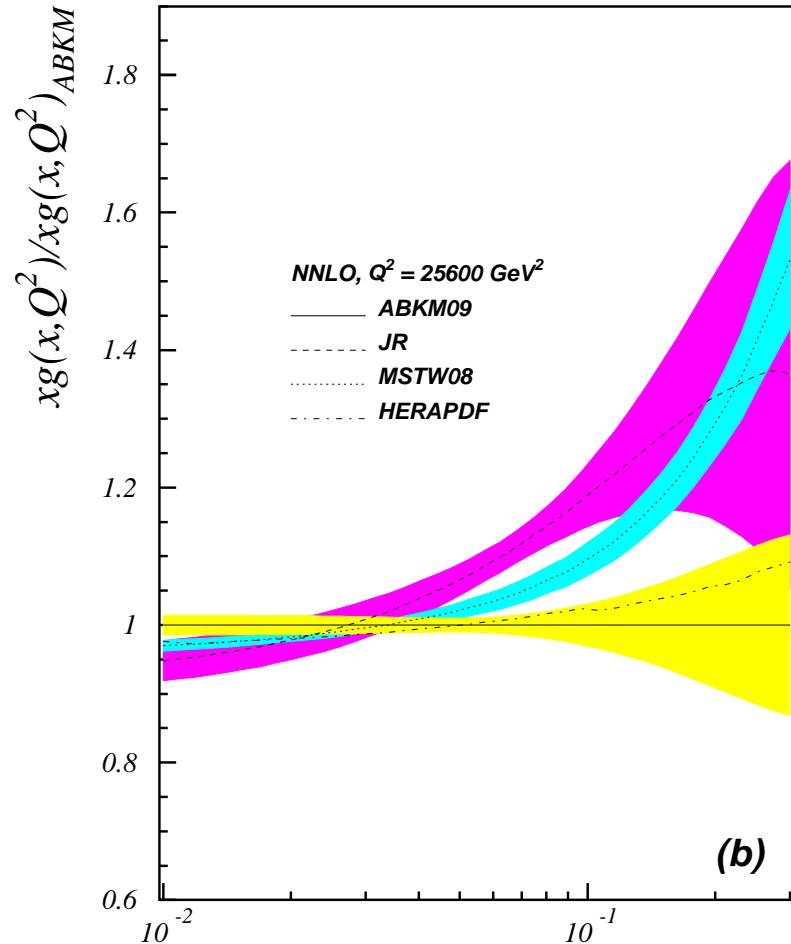
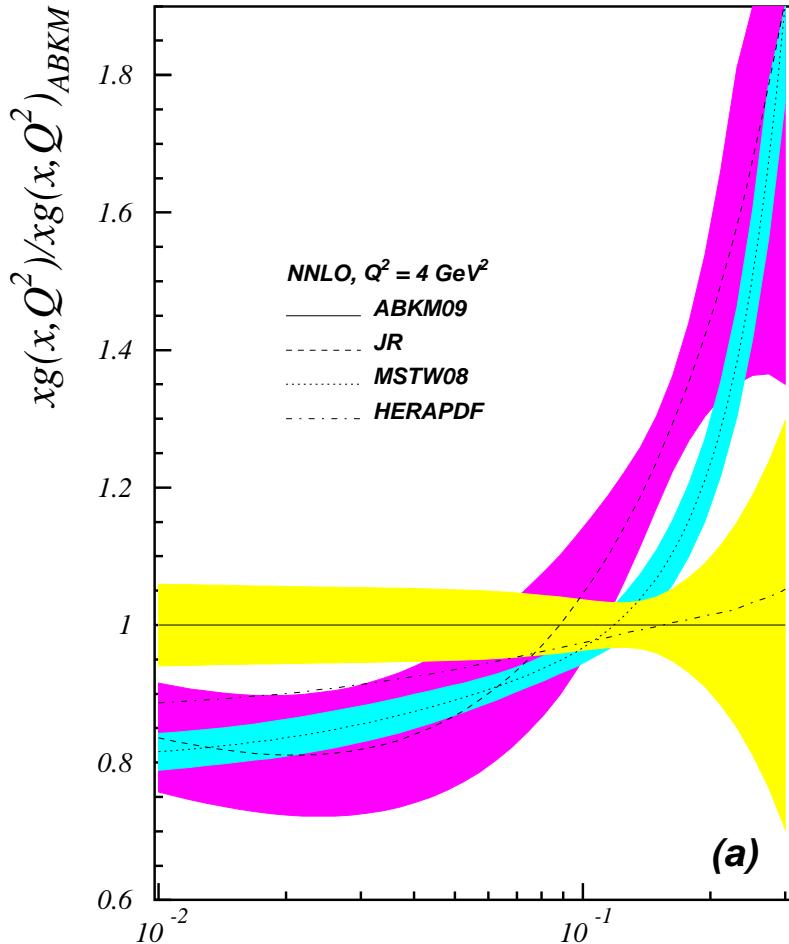
- Light-quark and gluon PDFs with absolute uncertainties at $\pm 1\sigma$
Alekhin, S.M. '10
 - red lines from fit with running masses
1.18 GeV (charm) and 4.19 GeV (bottom)

Phenomenology



- Comparison to new data for F_2^{charm} and F_2^{bottom} from HERA

Gluon distribution

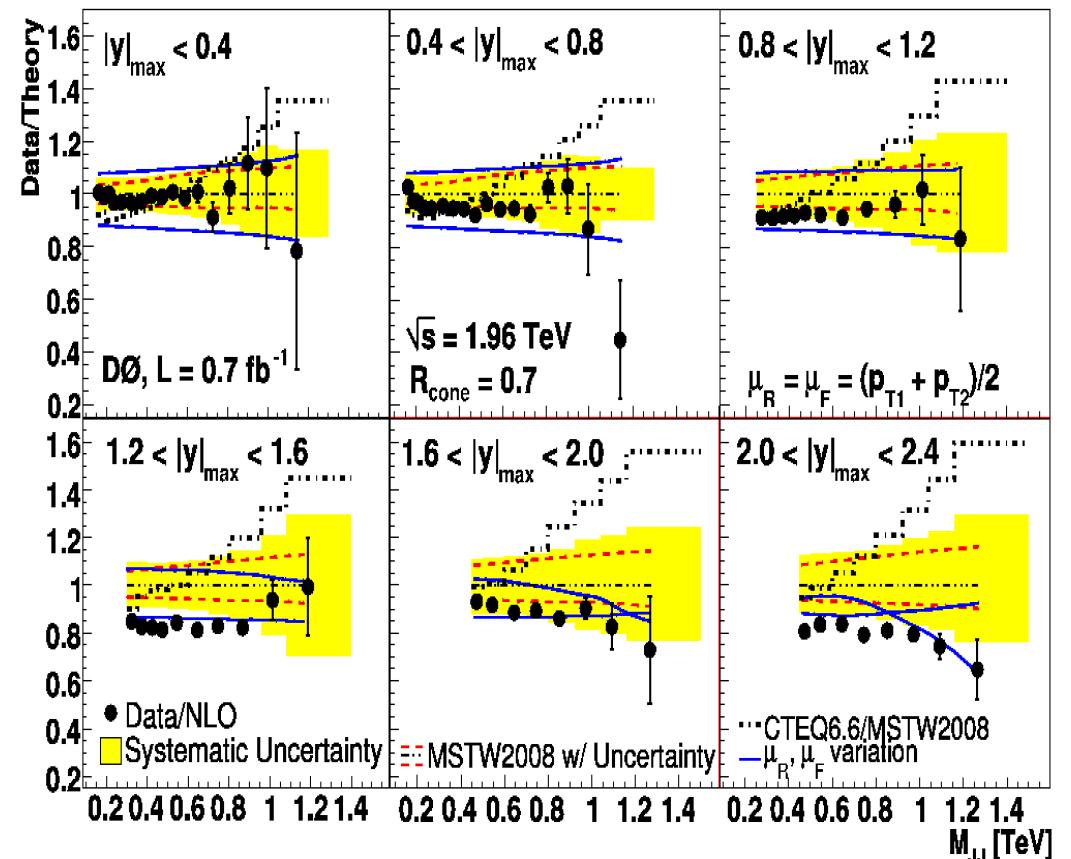
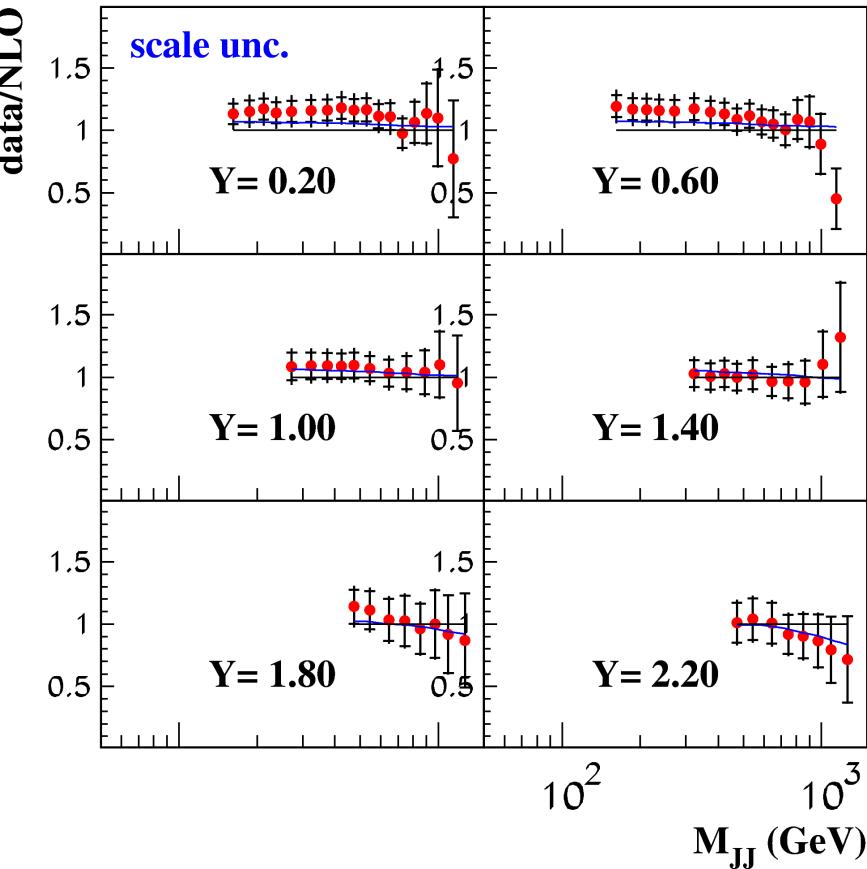


- Comparison of gluon PDFs from NNLO fits with PDF
Alekhin, Blümlein, Jimenez-Delgado, S.M., E. Reya '10
 - uncertainties bands at 1σ

Run II D0 dijet data in the ABKM fit

D0 Collaboration PRL 101, 062001 (2008)

ABKM09 (no re-fit)



The NLO ABKM09 predictions compared with the D0 Run II dijet data:

5-flavor PDFs generated from the 3-flavor ones,

$$\mu_r = \mu_F = M_{JJ}$$

Impact of the data on ABKM PDFs is marginal

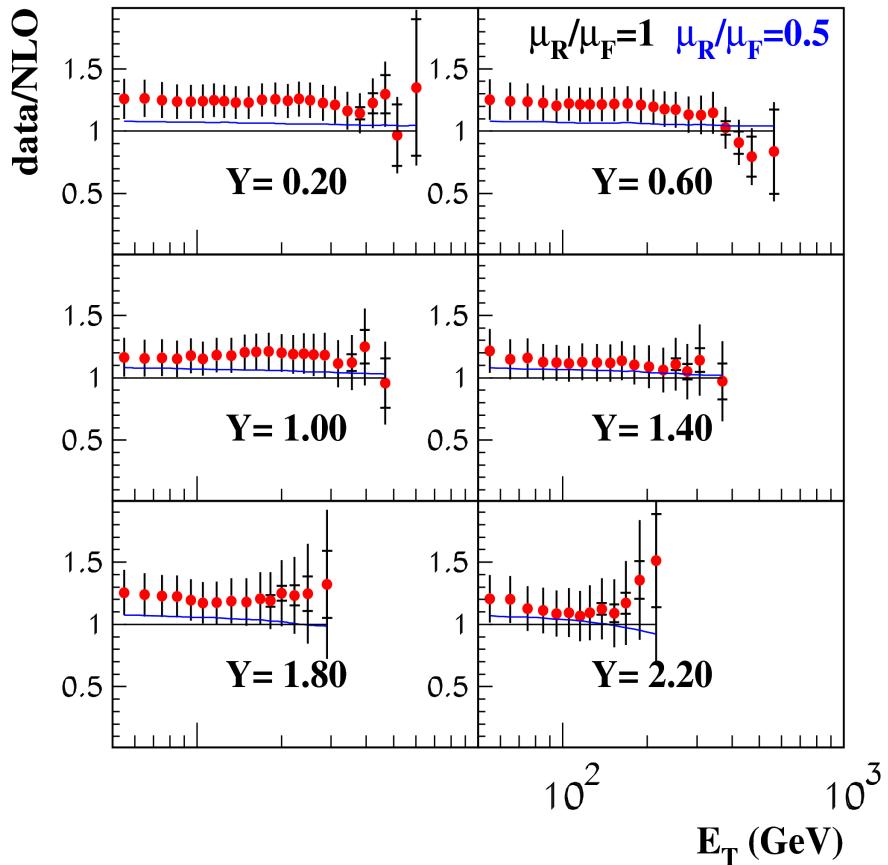
ABKM describes jet data better than the “truly global fits” based on the Run II data??

Alekhin
INT Seattle Oct 2010

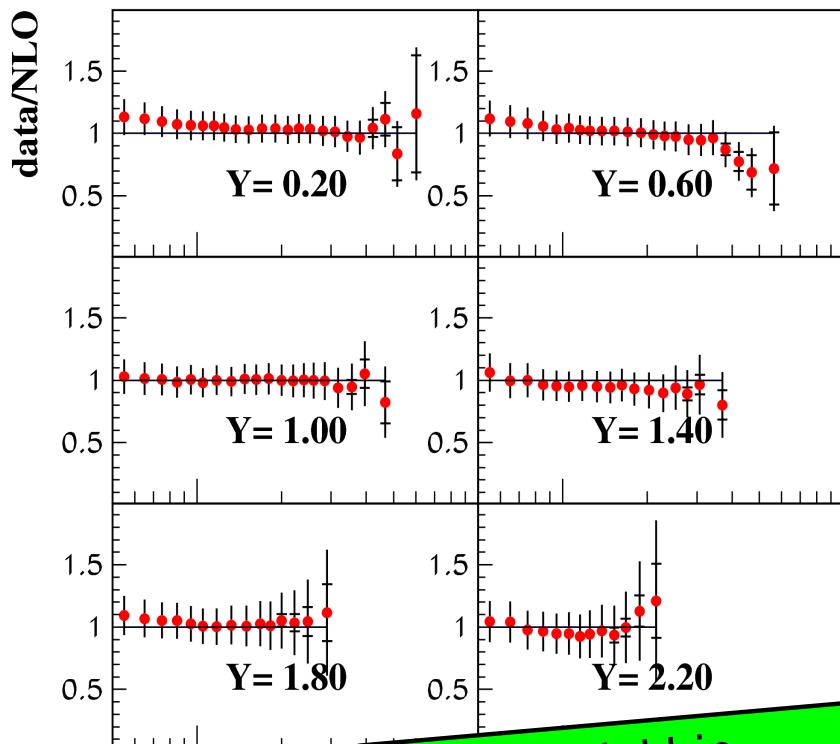
Run II D0 inclusive data in the ABKM fit

D0 Collaboration PRL 101, 062001 (2008)

Before the fit



After the fit

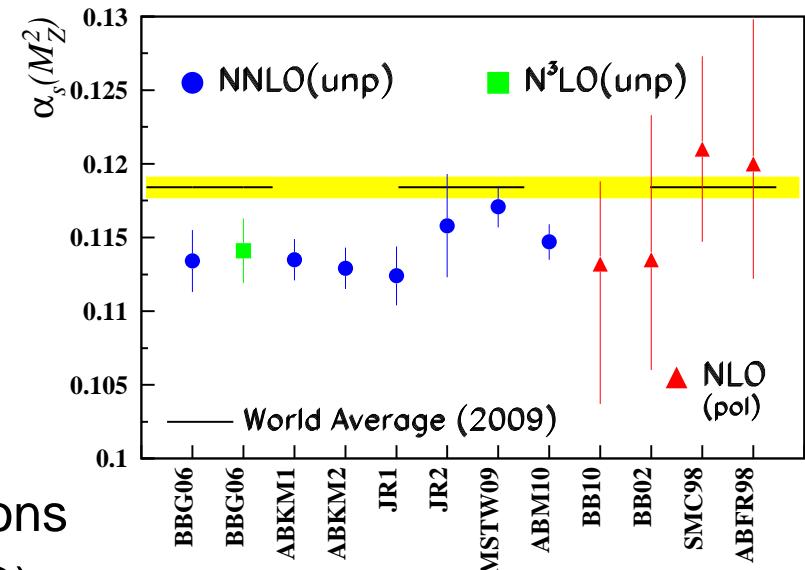


Alekhin
INT Seattle Oct 2010

- The NLO variant of the ABKM09 fit with the D0 Run II inclusive midpoint data included (*uncertainty due to missing NNLO corrections*)
- Mixed scheme: 3-flavor PDFs for the DIS and 5-flavor PDFs for jets, $\mu_F = E_T$
- The value of χ^2 for D0 data is 104/110 → jet data can be easily combined with others

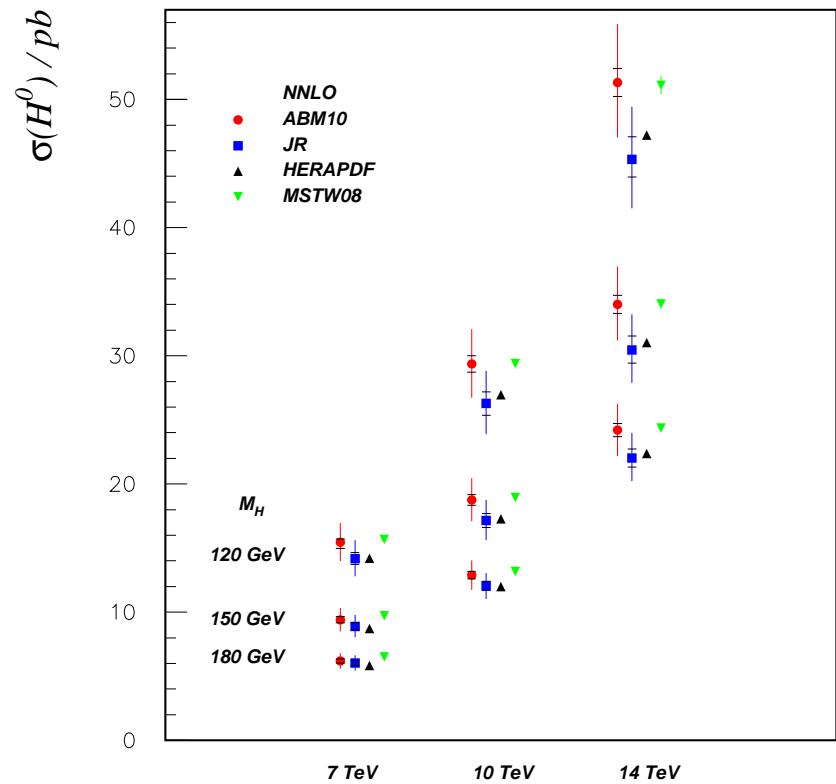
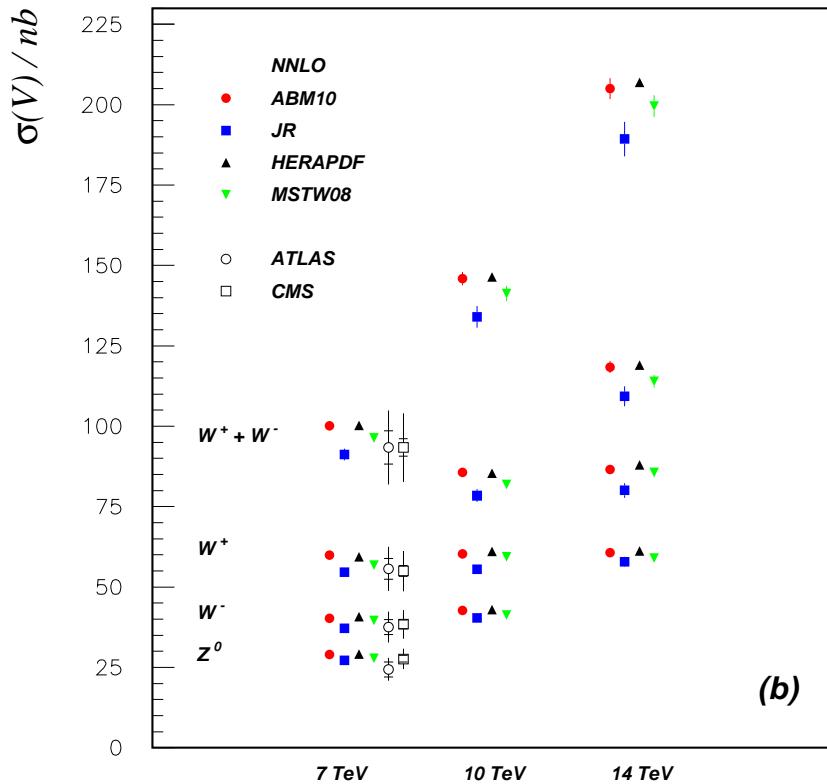
Strong coupling constant

- $\alpha_s(M_Z) = 0.1179 \pm 0.0016$ in ABKM 09 at NLO
- $\alpha_s(M_Z) = 0.1135 \pm 0.0014$ in ABKM 09 at NNLO
- $\alpha_s(M_Z) = 0.1147 \pm 0.0012$ in ABKM 10 (including comb. HERA data)
- Compilation of $\alpha_s(M_Z)$ values
Blümlein, Böttcher '10



- Consistency with other determinations
 - $\alpha_s(M_Z) = 0.1161 \pm 0.0045$ (NLO)
from the Tevatron jet data D0 Coll. '10
 - $\alpha_s(M_Z) = 0.1135 \pm 0.0002(\text{exp}) \pm 0.0005(\text{had}) \pm 0.0009(\text{pert})$
from $e^+e^- \rightarrow 3 \text{ jets}$ data (NNLO + NNLL res.)
Abbate, Fickinger, Hoang, Mateu, Steward '10

Impact on LHC cross sections



Alekhin, Blümlein, Jimenez-Delgado, S.M., E. Reya '10

- W^\pm, Z and Higgs boson production cross sections at LHC
- PDF uncertainties (inner error bars) at 1σ
- Scale uncertainty (outer error bars) in range $M/2 \leq \mu_F = \mu_R \leq 2M$

Summary

Heavy quark masses

- Running mass definition for heavy quark $m(\mu_R)$
 - greatly reduced scale dependence
 - much improved convergence of perturbation theory
- Top-quark mass: $m_t(m_t) = 160.0^{+3.3}_{-3.2}(\text{exp})\text{GeV}$
- Charm-quark mass: $m_c(m_c) = 1.01 \pm 0.09(\text{exp}) \pm 0.03(\text{th})\text{GeV}$

Parton distributions

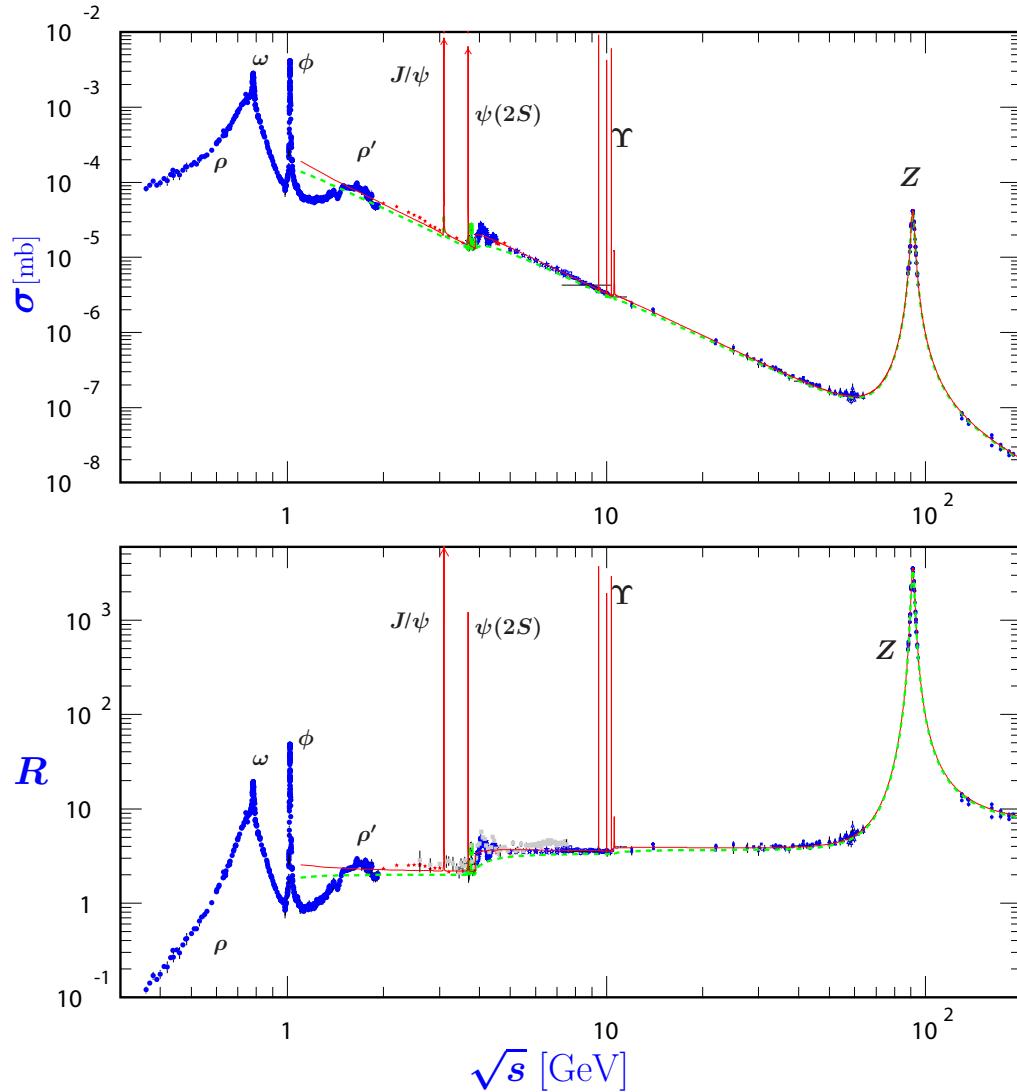
- Consequences for heavy particle production at LHC and Tevatron
 - sizable differences in PDFs at NNLO
 - impact of Tevatron jet data likely to be small
 - other sources under investigation

Phenomenology

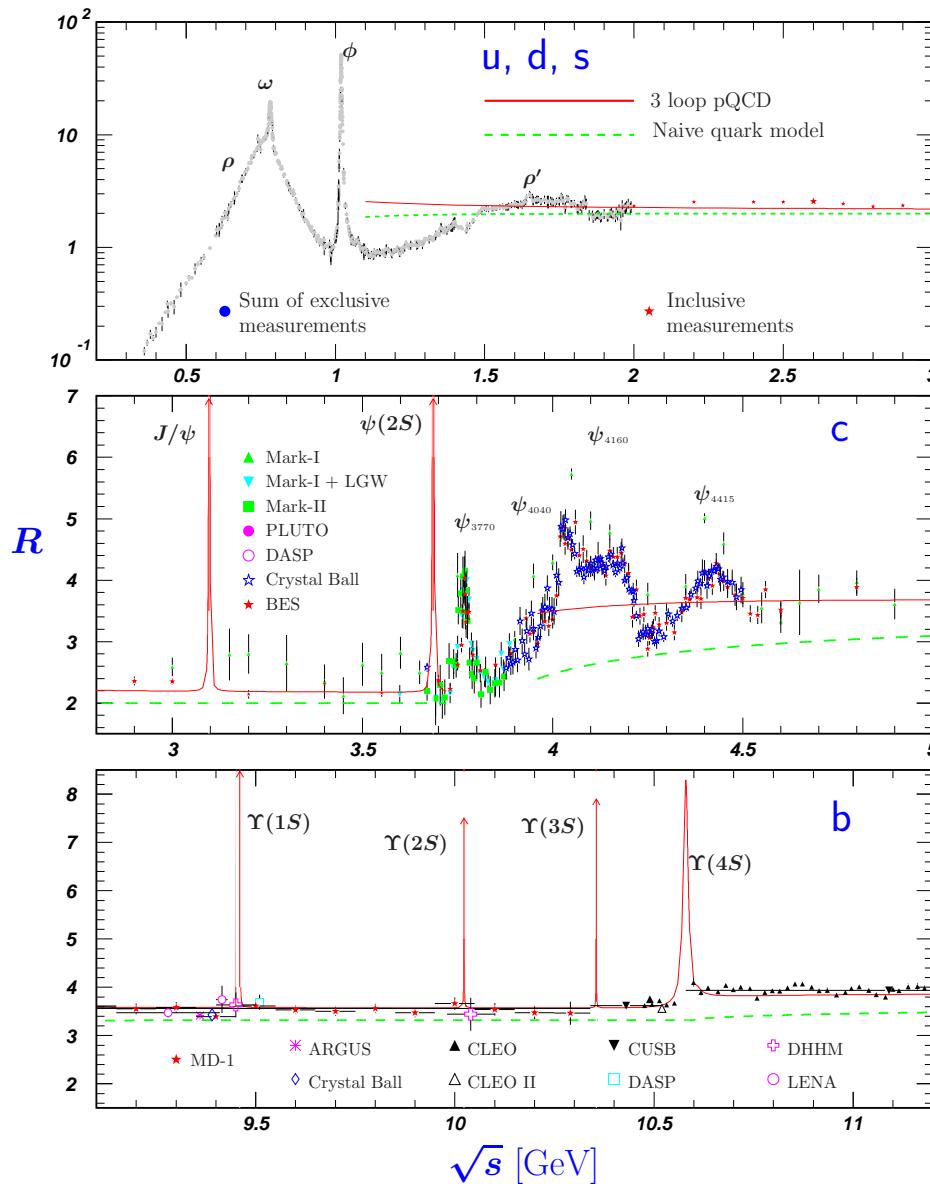
- Precision predictions need best possible non-perturbative input

Extra Slides

R -ratio in e^+e^- -annihilation



R-ratio in e^+e^- -annihilation (zoom)



- Relativistic sum rules

$$M_n = \int \frac{ds}{s^{n+1}} R(s)$$

- Most advance theory:

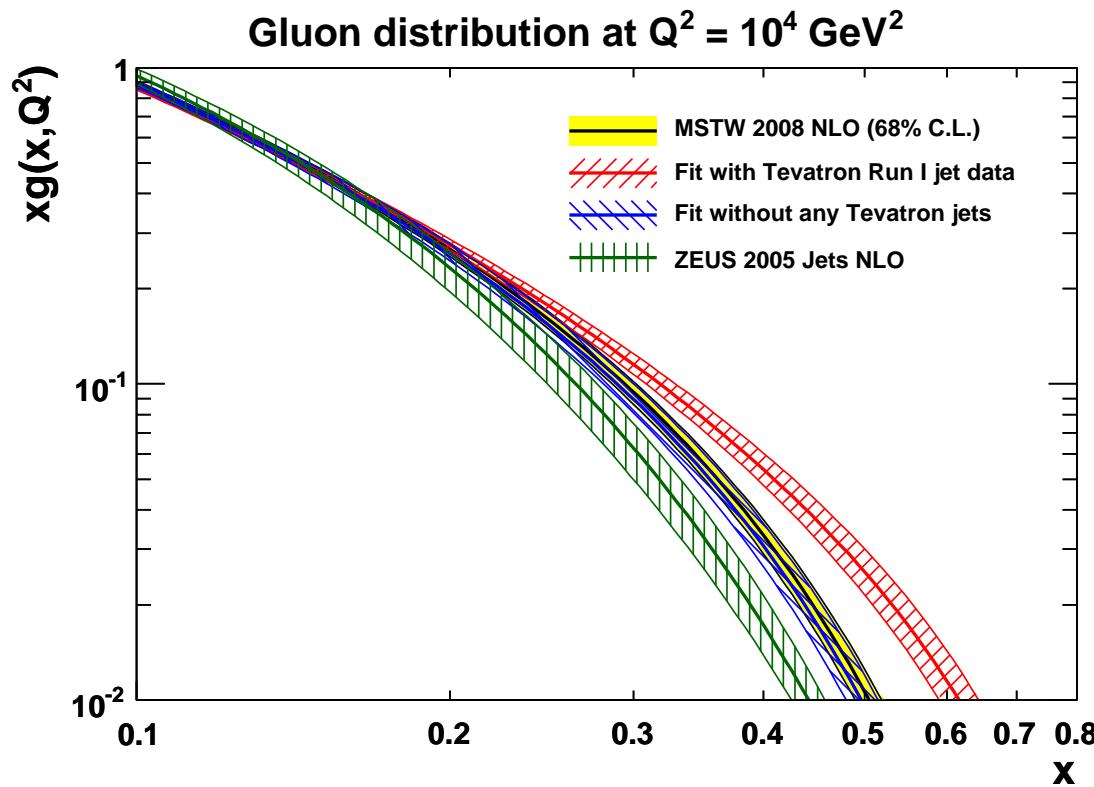
$$M_n^{\text{theory}} = \frac{12\pi^2}{n!} \left(\frac{d}{q^2} \right)^n \Pi(q^2) \Big|_{q^2=0}$$

- $\Pi(q^2)$ known at $\mathcal{O}(\alpha_s^3)$
- explicit mass dependence for heavy quarks, e.g. charm

$$\Pi_c(q^2) = e_c^2 \frac{3}{16\pi^2} \sum_{n \geq 0} C_n \left(\frac{q^2}{4m_c^2} \right)^n$$

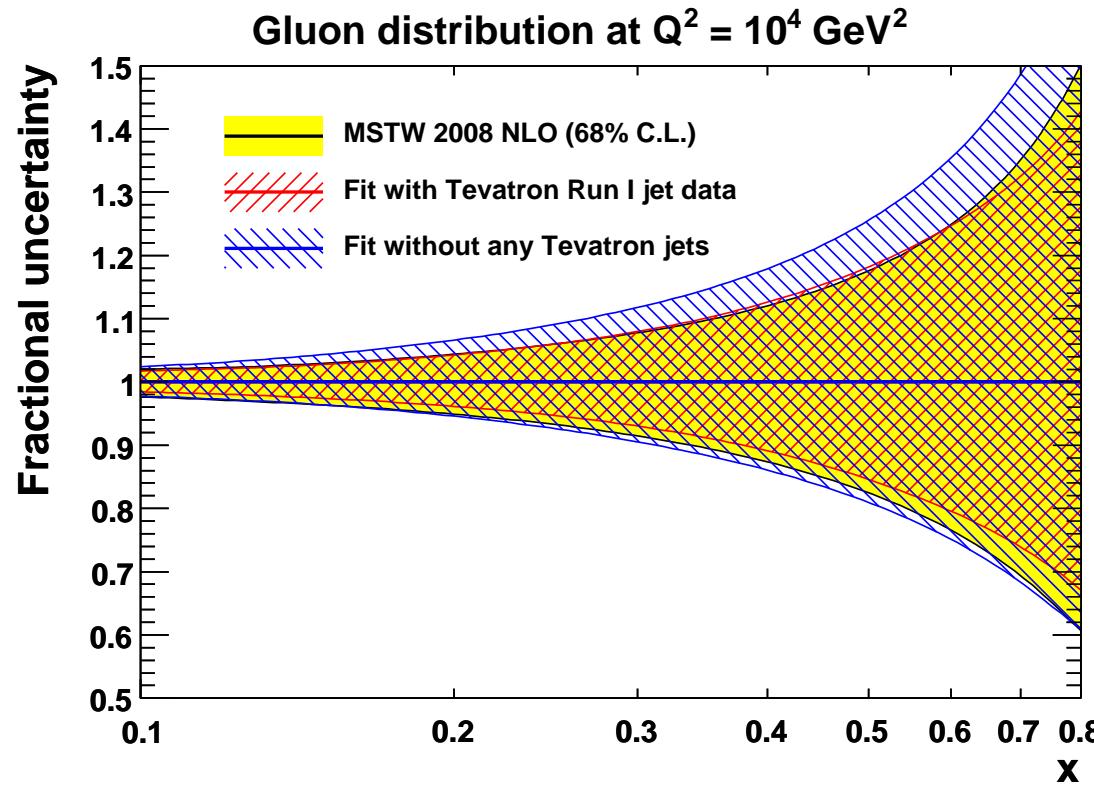
- precision determinations
Chetyrkin, Kühn, Meier, Meierhofer, Marquard, Steinhauser '10
 $m_c(3 \text{ GeV}) = 986 \text{ MeV} \pm 13 \text{ MeV}$
 $m_c(m_c) = 1279 \text{ MeV} \pm 13 \text{ MeV}$
also: Bodenstein, Bordes, Dominguez, Penarrocha, Schilcher '10

Gluon PDF at large- x (absolute)



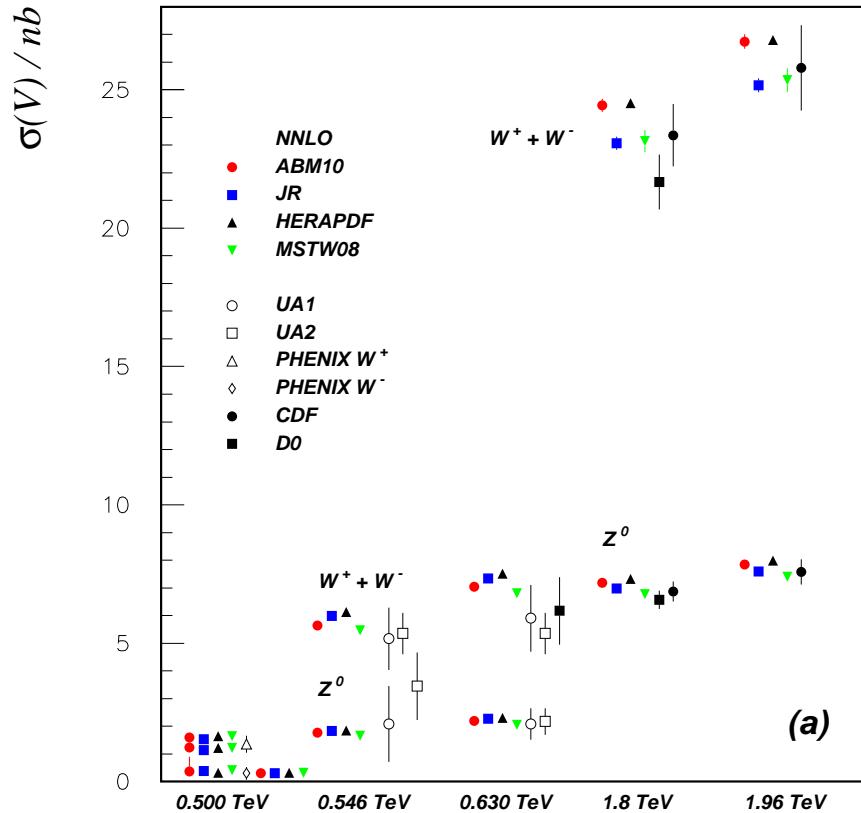
- Impact of Tevatron jet data on high- x gluon PDF at NLO
Martin, Roberts, Stirling, Watt '09
- Additional constraint $\mathcal{O}(\text{few})\%$ at highest x
 - e.g. at $\sqrt{S} = 7 \text{ TeV}$ LHC
 $\langle x \rangle \sim \mathcal{O}(0.3)$ corresponds to mass scales $M = \mathcal{O}(1 - 2) \text{ TeV}$

Gluon PDF at large- x (relative)



- Impact of Tevatron jet data on high- x gluon PDF at NLO
Martin, Roberts, Stirling, Watt '09
- Additional constraint $\mathcal{O}(\text{few})\%$ at highest x
 - e.g. at $\sqrt{S} = 7 \text{ TeV}$ LHC
 $\langle x \rangle \sim \mathcal{O}(0.3)$ corresponds to mass scales $M = \mathcal{O}(1 - 2) \text{ TeV}$

W^\pm and Z cross sections



Alekhin, Blümlein, Jimenez-Delgado, S.M., E. Reya '10

- Energy dependence of W^\pm and Z production cross sections