SUSY-Yukawa sum rule

A road towards testing weak scale naturalness

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Workshop on heavy particles at the LHC

Zürich – January 6, 2011

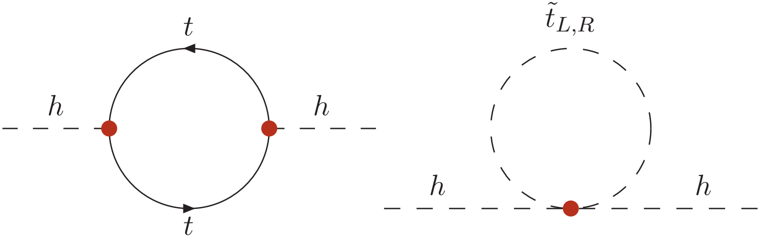
Goals for the next 25 minutes

1. Introducing the SUSY-Yukawa sum rule
2. Mass measurements – overview
3. LHC prospects for the SUSY-Yukawa sum rule
   - Parton level analysis
   - Full simulation - *preliminary!*
   - Results
4. Conclusions

MB, D. Curtin, M. Perelstein, 1004.5350, 110x.xxxx

SUSY cancellation of quadratic divergences

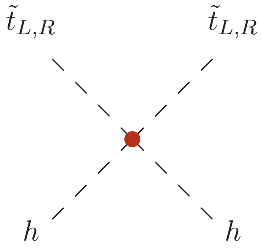
- **hierarchy problem:** loop contributions of SM particles (e. g. tops) let the Higgs potential depend quadratically on the cut-off scale



- new particles (stops) with sub-TeV masses required to cancel these contributions
- couplings to the Higgs boson have to be equal

How to access the stop-Higgs coupling at the LHC?

We want to measure the coupling $hh\tilde{t}_{L,R}\tilde{t}^c_{L,R}$:



- direct measurement not feasible at the LHC

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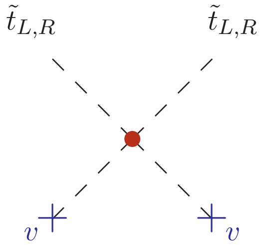


- direct measurement not feasible at the LHC
- EWSB ➢ contribution to stop mass matrix

The stop mass matrix

- stop mass matrix: $\mathcal{L} = (\tilde{t}_L^c, \tilde{t}_R^c)\mathcal{M}_t^2(\tilde{t}_L, \tilde{t}_R)$

$$\mathcal{M}_t^2 = \begin{pmatrix} m_L^2 + {m_t}^2 + \Delta_u & m_t(A_t + \mu\cot\beta) \\ m_t(A_t + \mu\cot\beta) & m_R^2 + {m_t}^2 + \Delta_{\bar{u}} \end{pmatrix}$$

- rotation to mass eigenstates via

$$\begin{aligned} \tilde{t}_1 &= \cos\theta_t\,\tilde{t}_L + \sin\theta_t\,\tilde{t}_R \\ \tilde{t}_2 &= -\sin\theta_t\,\tilde{t}_L + \cos\theta_t\,\tilde{t}_R \end{aligned}$$

- then re-express $\mathcal{M}_{11}^2$

$$m_L^2 + {m_t}^2 + \Delta_u = m_{\tilde{t}_1}^2 \cos^2\theta_t + m_{\tilde{t}_2}^2 \sin^2\theta_t$$

- analogously for sbottom system

The SUSY-Yukawa sum rule

eliminating m_L^2 yields the SUSY-Yukawa sum rule ($m_b \to 0$)

$$m_t{}^2 + \Delta_{ud} = m_{\tilde{t}_1}^2 \cos^2\theta_t + m_{\tilde{t}_2}^2 \sin^2\theta_t - m_{\tilde{b}_1}^2 \cos^2\theta_b - m_{\tilde{b}_2}^2 \sin^2\theta_b$$

where $\Delta_{ud} = \Delta_u - \Delta_d = m_Z^2 \cos^2\theta_W \cos 2\beta \approx -m_W^2$

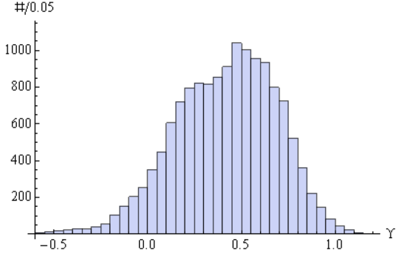
- sum rule expresses stop-Higgs coupling in terms of measurable quantities (masses, mixing angles)
- SUSY weak scale stabilization (in principle) testable at the LHC!

. . . and what about radiative corrections?

- above derivation valid at tree level
- to quantify effect of radiative corrections, define

$$\Upsilon = \frac{1}{v^2}(m_{\tilde{t}_1}^2 \cos^2\theta_t + m_{\tilde{t}_2}^2 \sin^2\theta_t - m_{\tilde{b}_1}^2 \cos^2\theta_b - m_{\tilde{b}_2}^2 \sin^2\theta_b)$$

- SUSY tree level prediction: $\Upsilon_{\text{tree}} = 0.28 \quad (\tan\beta >$ a few)



SuSpect scan over pMSSM parameter space yields

$|\Upsilon| < 1$

'generic' prediction: $|\Upsilon| < 16\pi^2$

Parameters to be determined

- masses

$$m_{\tilde{t}_1}, m_{\tilde{t}_2}$$
$$m_{\tilde{b}_1}, m_{\tilde{b}_2}$$

- mixing angles

$$\sin\theta_t$$
$$\sin\theta_b$$
$\tan\beta$ (minor impact)

- additional information helpful (to pin down radiative corrections)

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➢ most promising at the LHC

Mass measurements in missing energy events

see e. g. Burns, Kong, Matchev, Park, 0810.5576

SUSY mass measurements complicated, as event cannot fully be reconstructed (LSP escapes detection)

- **endpoint method**
  measure kinematic endpoints of invariant mass distributions of SM decay products
- **polynomial method**
  attempt exact event reconstruction
- $\boldsymbol{M_{T2}}$ **method**
  reconstruct endpoint of transverse invariant mass

for short decay chains ($n \leq 2$), we need to rely on M_{T2}!

Recall: how to measure the W mass

- consider decay $W \to \ell\nu$: invariant mass

$$m_W^2 = m_\ell^2 + m_\nu^2 + 2p_\ell \cdot p_\nu$$

- but we know only $\boldsymbol{p}_T^\nu = \not{\boldsymbol{p}}_T$
- consider transverse mass instead

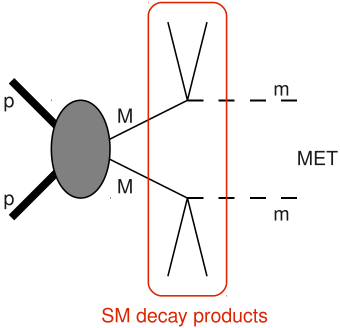
$$m_T^2 = m_\ell^2 + m_\nu^2 + 2(E_T^\ell E_T^\nu - \boldsymbol{p}_T^\ell \cdot \boldsymbol{p}_T^\nu)$$

- m_W is then determined by the endpoint

$$m_W = \max_{\text{all events}} \{m_T\}$$

- Note: m_ν and $\boldsymbol{p}_T^\nu$ are known

The stransverse mass M_{T2}



Barr, Lester, Stephens, hep-ph/0304226

SUSY events complicated by

- two missing particles
- LSP mass unknown

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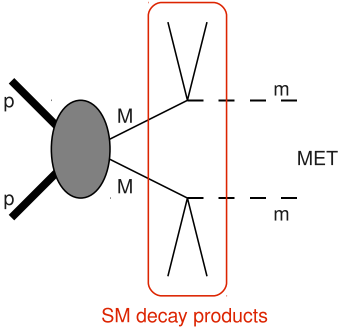

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SUSY events complicated by

- two missing particles
- LSP mass unknown

➢ **the best we can do**

- use trial LSP mass χ
- minimize over all possible LSP momentum configurations

➢ define the stransverse mass M_{T2} by

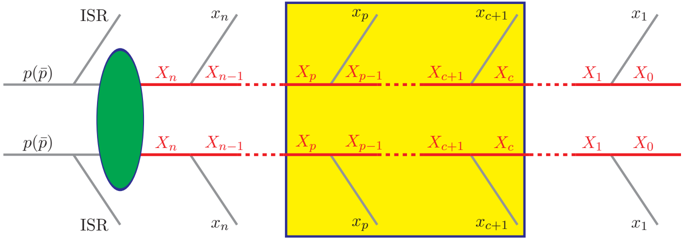
$$M_{T2}(\chi) = \min_{\boldsymbol{p}_T^{(1)} + \boldsymbol{p}_T^{(2)} = \not{\boldsymbol{p}}_T} \left\{ \max\{m_T^{(1)}, m_T^{(2)}\} \right\}$$

edge of distribution: $M_{T2}(\chi)_{\text{max}} = \frac{M^2-m^2}{2M} + \sqrt{(\frac{M^2-m^2}{2M})^2 + \chi^2}$

Extension: the subsystem M_{T2}

Burns, Kong, Matchev, Park, 0810.5576

for $n > 1$ step decay chains:



generalize M_{T2} concept to subsystem $M_{T2}^{(n,p,c)}(\chi)$
(n: grandparent index, p: parent index, c: child index)

➢ $M_{T2}^{(n,p,c)}(\chi)$ endpoint yields relation between m_n, m_p and m_c

Our benchmark scenario – main virtues

$m_{\tilde{t}_1}$	$=$	371 GeV	$\tan\beta$	$=$	10
$m_{\tilde{t}_2}$	$=$	800 GeV	$\sigma(pp \to \tilde{t}_1 \tilde{t}_1^c)$	$=$	2 pb
$\sin\theta_t$	$=$	-0.09	$\sigma(pp \to \tilde{g}\tilde{g})$	$=$	11 pb
$m_{\tilde{b}_1}$	$=$	341 GeV	$Br(\tilde{g} \to b\tilde{b}_1)$	$=$	100%
$m_{\tilde{g}}$	$=$	525 GeV	$Br(\tilde{b}_1 \to b\tilde{\chi}_1^0)$	$=$	100%
$m_{\tilde{\chi}_1^0}$	$=$	98 GeV	$Br(\tilde{t}_1 \to t\tilde{\chi}_1^0)$	$=$	100%

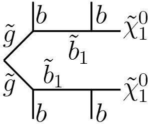
- study $pp \to \tilde{g}\tilde{g} \to bb\tilde{b}_1\tilde{b}_1 \to 4b + \not{E}_T$ and $pp \to \tilde{t}_1\tilde{t}_1^c \to t\bar{t} + \not{E}_T$
- *for now:* parton level analysis using `MG/ME` and `BRIDGE`
- more realistic analysis (`Pythia, PGS`) in progress

MB, D. Curtin, M. Perelstein, 1004.5350, 110x.xxxx

Gluino pair production – $4b + \not{E}_T$

It's all about finding edges!

- consider $pp \to 2\tilde{g} \to 2b + 2\tilde{b}_1 \to 4b + \not{E}_T$
- $\sigma(pp \to 2\tilde{g}) \simeq 11.6\,\text{pb}$ for $\sqrt{s} = 14\,\text{TeV}$
- basic E_T, $\not{E}_T$ cuts & require 4 b-tags
- with $\mathcal{L} = 10\,\text{fb}^{-1}$: ~ 4800 signal events, SM background negligible!



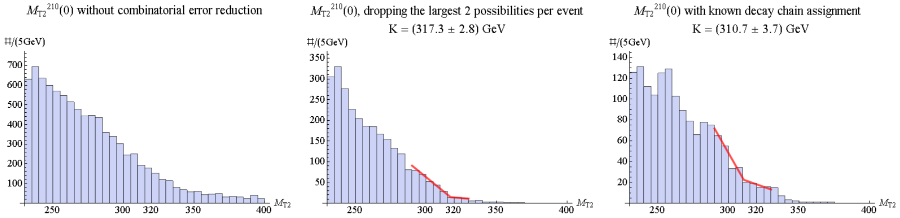
- however: combinatorial background – which b-jet is which?

several possible ways to get rid of 'wrong' pairings (e. g. ΔR)
➢ we always require **two independent methods** to yield **consistent results** (otherwise measurement is rejected)

Mass determination for $\tilde{g}$, $\tilde{b}_1$ and $\tilde{\chi}_1^0$

- M_{bb} invariant mass endpoint can easily be recovered
- need two more edges to pin down all masses ➢ $M_{T2}^{(2,2,0)}$, $M_{T2}^{(2,1,0)}$

Barr et al, hep-ph/0304226; Burns et al, 0810.5576



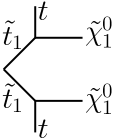
➢ combining those, we obtain the mass measurements

	68% C.L.	theory
$m_{\tilde{b}_1}$	(316,356)	341 GeV
$m_{\tilde{g}}$	(508,552)	525 GeV
$m_{\tilde{\chi}_1^0}$	$(45^{(*)}$,115)	98 GeV

$^{(*)}$ LEP lower bound

Stop pair production – extracting $m_{\tilde{t}_1}$

- analyze $pp \to 2\tilde{t}_1 \to 2t + \not{E}_T$
- $\sigma(pp \to 2\tilde{t}_1) \simeq 2\,\text{pb}$ for $\sqrt{s} = 14\,\text{TeV}$
- impose standard cuts & use hadronic tops
  (following Meade, Reece, hep-ph/0601124)
- with $\mathcal{L} = 100\,\text{fb}^{-1}$: $S/B \simeq 14$, $S/\sqrt{B} = 140$
- straightforward to extract



$$(M_{T2})_{\text{max}}(\chi = 0) = (340 \pm 4)\,\text{GeV}$$

theory: $336.7\,\text{GeV}$

- using our previous $m_{\tilde{\chi}_1^0}$ measurement we find (68% C.L.)

$$356\,\text{GeV} \leq m_{\tilde{t}_1} \leq 414\,\text{GeV}$$

theory: $371\,\text{GeV}$

Effects of initial state radiation etc.

- ISR introduces "unbalance" in transverse momenta
  ➢ M_{T2} edges are ISR-dependent

Alwall et al., 0905.1201
Konar et al., 0910.3679
Nojiri, Sakurai, 1008.1813

2 ways out:

1. perform ISR-binned analysis ➢ $M_{T2}(p_{T,\text{ISR}})$ ➢ true M_{T2}
2. introduce $M_{T2\perp}$: analogous to M_{T2}, but consider only $p_T \perp p_{T,\text{ISR}}$
   ➢ ISR-independent

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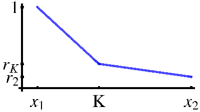
- FSR, hadronization, detector effects:
  minor impact, mostly taken care of by jet clustering

➢ loss in statistics, but combining both methods **reliable edge determinations are still possible**!

Improved kink fitting

so far: simple linear kink fit, three fit parameters
➢ good results for sharp edges



but: edges are smeared out by jet energy smearing

➢ convolute linear kink fit function with Gaussian distribution

- semi-realistic fit function
- reliably determines edges with significant smearing
- fit uncertainties better under control

So what did we learn?

- 4 MSSM masses determined – only 120 parameters left ☺

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- 4 MSSM masses determined – only 120 parameters left ☺
- rewrite Υ as

$$\Upsilon = \underbrace{\frac{1}{v^2}(m^2_{\tilde{t}_1} - m^2_{\tilde{b}_1})}_{\Upsilon'} + \underbrace{\frac{\sin^2\theta_t}{v^2}(m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1})}_{\Delta\Upsilon_t} - \underbrace{\frac{\sin^2\theta_b}{v^2}(m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1})}_{\Delta\Upsilon_b}$$

- our measurements yield $\Upsilon' = 0.53^{+0.20}_{-0.15}$ theory: 0.35

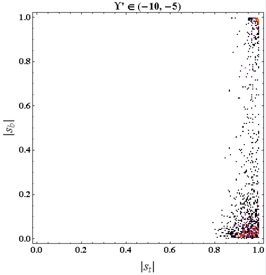
- however no information on $\Delta\Upsilon_t, \Delta\Upsilon_b$
  ➢ sum rule test has to wait for lepton collider

Information from Υ' measurement

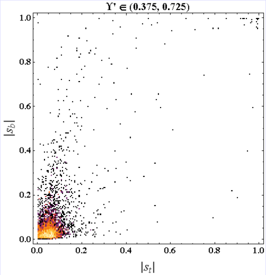
- Υ' alone does not test the sume rule
- but assuming the MSSM it can be used to restrict θ_t and θ_b

$$\Upsilon' = \underbrace{\Upsilon}_{|\Upsilon|<1} - \underbrace{\frac{\sin^2\theta_t}{v^2}(m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1})}_{\Delta\Upsilon_t>0} + \underbrace{\frac{\sin^2\theta_b}{v^2}(m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1})}_{\Delta\Upsilon_b>0}$$

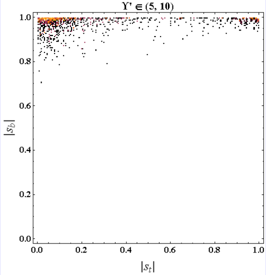
$\Upsilon' \ll 0 \succ t_1 \sim t_R$



$\Upsilon' \sim 0 \succ t_1 \sim t_L, b_1 \sim b_L$



$\Upsilon' \gg 0 \succ b_1 \sim b_R$



for direct θ_t measurement, see e. g. Hisano et al, hep-ph/0304214; Perelstein, Weiler, 0811.1042; Rolbiecki et al, 0909.3196

Summary

1. SUSY-Yukawa sum rule relates stop-Higgs coupling to stop and sbottom masses and mixing angles ➢ measurable quantities
2. verifying (or falsifying) the sum rule means testing SUSY as the origin of the weak scale stabilization
3. full measurement will have to wait for lepton collider
4. we can make significant progress at the LHC in some regions of parameter space:
   masses of $\tilde{t}_1$, $\tilde{b}_1$, $\tilde{g}$ and $\tilde{\chi}_1^0$ can be determined
   ➢ prediction for θ_t, θ_b
5. we also developed new techniques to reduce combinatorial backgrounds in M_{T2} analyses

Back-up slides

Definition of benchmark scenario

parameter	EWSB scale value
M_1	100 GeV
$M_{2,3}$	450 GeV
A_t	390 GeV
μ	400 GeV
$\tan\beta$	10
M_A	600 GeV
$m_{\tilde{e}_{L,R},\tilde{\tau}_{L,R},\tilde{q}_L\tilde{u}_R,\tilde{d}_R}$	1000 GeV
$m_{\tilde{Q}_L}$	310 GeV
$m_{\tilde{t}_R}$	780 GeV

SM backgrounds to $4b + \not{p}_T$

Background	Generator	$\epsilon_{\rm b}\sigma$	$\epsilon_{\rm b}\epsilon_{\rm kin}\sigma$
$4j + (Z \to \nu\nu)$	MGME, ALPGEN	$10\,\mathrm{fb}$	
diboson + jets	—	$< 10\,\mathrm{fb}$	
$tt \to n\tau + X$	MGME, BRIDGE	$21.6\,\mathrm{pb}$	$25\,\mathrm{fb}$
t	—		$\ll 30\,\mathrm{fb}$

assumed b-tagging efficiencies: 0.6 (b), 0.1 (c, τ), 0.01 (light jet)

Some technical details – parton level

- SUSY spectrum and decays calculated using `SUSY-HIT`
- parton-level analysis for $\sqrt{s} = 14\,\text{TeV}$ pp collisions
- Monte Carlo event samples generated by `MadGraph/MadEvent`
- fully decayed final state obtained with `BRIDGE`
- leading order analysis, using CTEQ6l1 pdf sets
- Gaussian smearing of jet energies

$$\frac{\Delta E}{E} = \frac{50\%}{\sqrt{E[\text{GeV}]}} \oplus 3\%$$

to simulate detector response