Higher order threshold effects for top and squark pair production

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(Based on M.Beneke, P.Falgari, CS, arXiv:0907.1443 [hep-ph], arXiv:1007.5414 [hep-ph]

M.Beneke, M.Czakon, P.Falgari, A.Mitov, CS arXiv:0911.5166 [hep-ph]

M.Beneke, P.Falgari, S. Klein, CS, in progress)

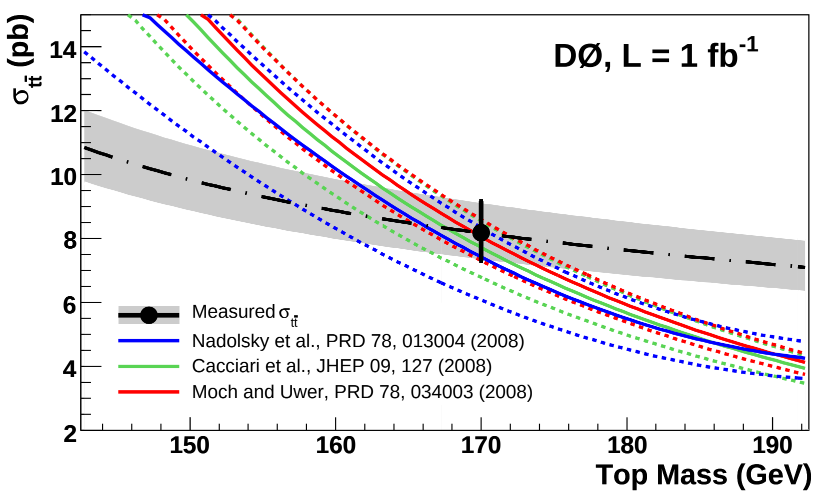
Pair production of heavy coloured particles at Tevatron/LHC

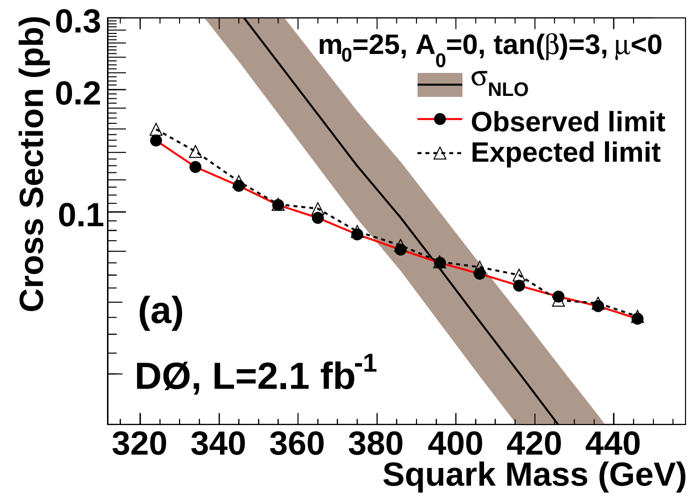
$$NN' \to HH' + X$$

- N, N': pp, $p\bar{p}$; HH': top-quark, squark, gluino... pairs

Precise knowledge of total cross sections:

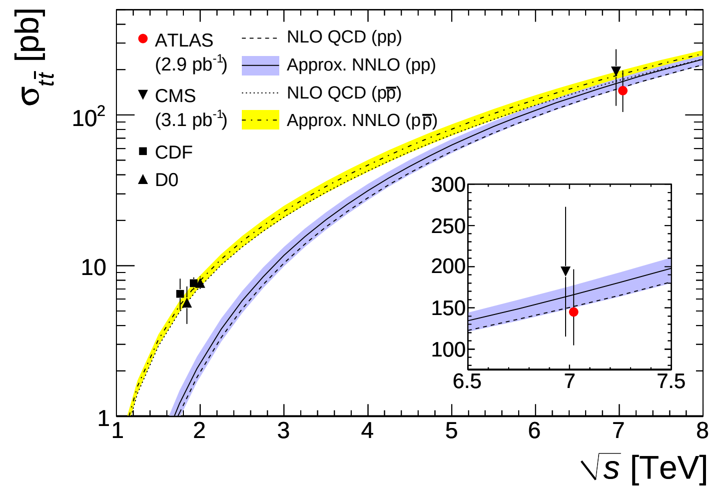
- top-quarks: sensitivity on mass, constraining gluon PDFs
- new particles: Exclusion bounds, model discrimination,...





Total $t\bar{t}$ cross section:

Tevatron: $\Delta\sigma_{t\bar{t}} = 6.8\%$;
LHC Goal: $\Delta\sigma_{t\bar{t}} \approx 5\%$



Theory status:

NLO+NLL: $\Delta\sigma_{t\bar{t}} \approx 10-20\%$

(Cacciari et.al., Moch/Uwer; Kidonakis/Vogt,...)

NNLO: in progress ($\Rightarrow$ talks by Czakon, Ferroglia)

Estimate of dominant higher order corrections: NNLO$_{\text{approx}}$, NNLL

($\Rightarrow$ this talk, also Moch/Uwer(+Langenfeld), Ahrens et.al., Kidonakis)

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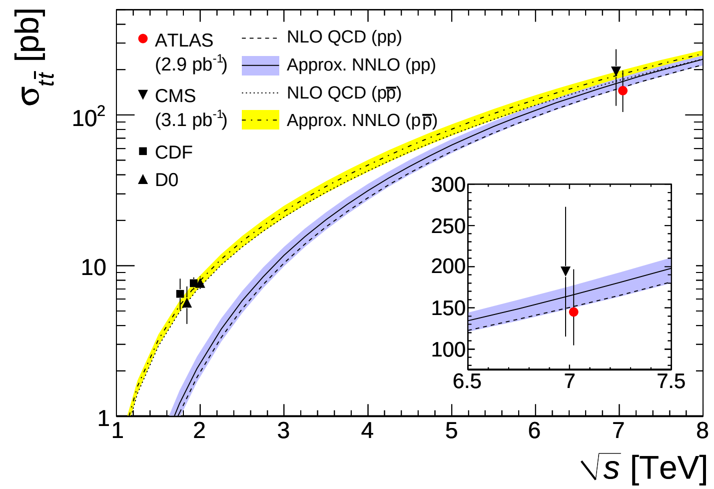

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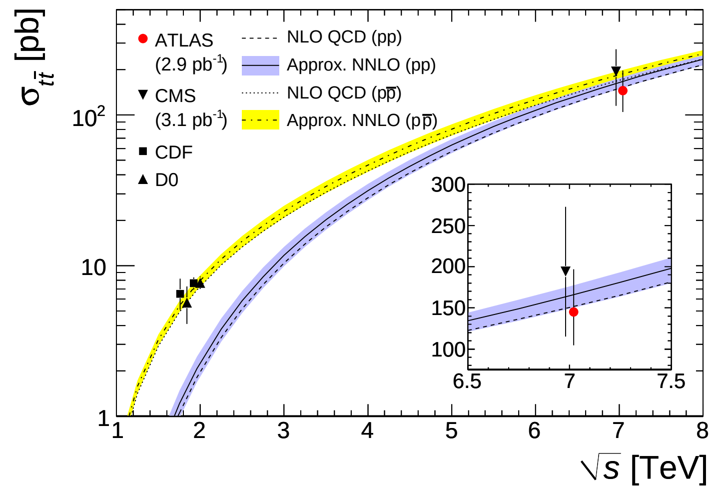

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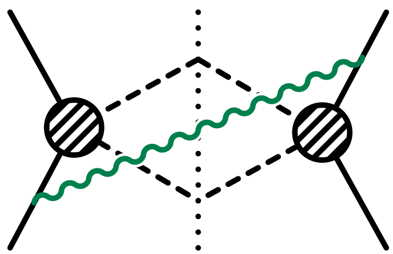
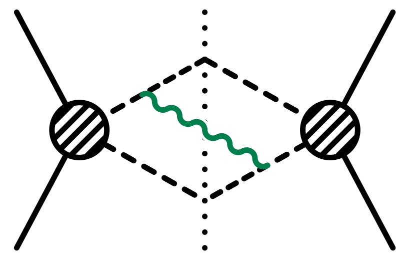
Squark and gluino production processes:

- NLO SUSY-QCD (Beenakker et.al. 96, implemented in PROSPINO)
- EW corrections (Bornhauser et.al. 07, Hollik et.al. , 07-10)
- NLL, NLLO$_{\text{approx}}$ (Kulesza/Motyka; Beenakker et.al.; Langenfeld/Moch 09/10)

Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, . . .)



$\Rightarrow \alpha_s \;\log^2(8\beta^2)$



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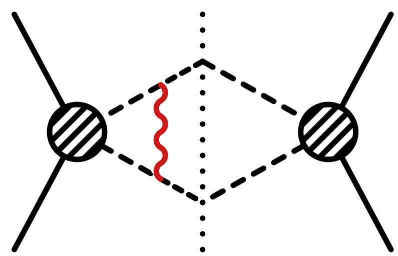
Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,. . .)

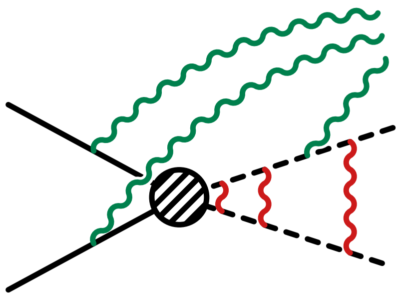


$\Rightarrow \alpha_s \;\frac{1}{\beta}$

Counting of threshold corrections:

$$\hat\sigma_{pp'} \quad\propto\quad \sigma^{(0)}\,\exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s\ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s\ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s\ln\beta)}_{\text{(NNLL)}} + \ldots\Big]$$
$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \left\{1\,\text{(LL,NLL)};\alpha_s,\beta\,\text{(NNLL)};\ldots\right\}\,:$$

Combination of Coulomb- and soft effects?



Heavy particles nonrelativistic near threshold:

$$E \sim m\beta^2\,, \qquad |\vec{p}| \sim m\beta$$

soft gluon momenta of same order: $q_s \sim m\beta^2 \sim E$

$\Rightarrow$ heavy particles "feel" soft radiation

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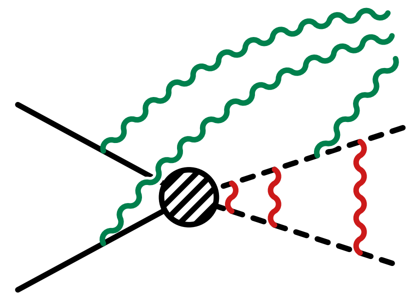

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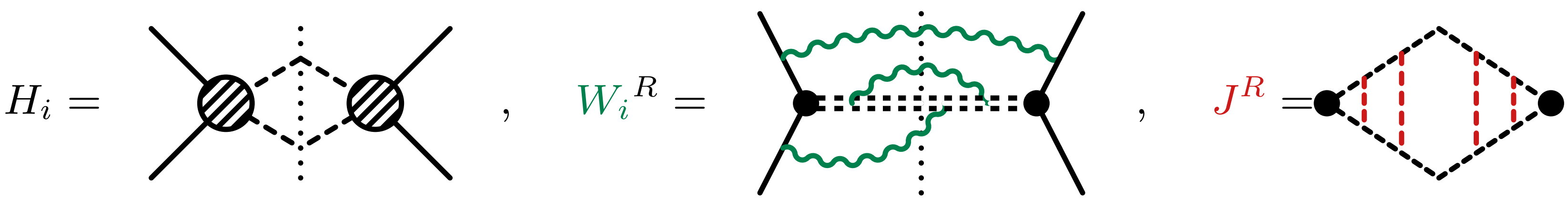
Factorization of cross section (Beneke, Falgari, CS 09/10)

$$\hat{\sigma}_{pp'\to HH'}|_{\hat{s}\to 4M^2} = \sum_{R,i} H_i \int d\omega\, W_i^R(\omega) J^R(E-\omega)$$

Hard, soft and Coulomb functions:



Soft radiation "sees" only total colour charge R of heavy particles

(Singlet, octet,... Extends one-loop results by Sterman/Kidonakis 97, Bonciani et.al. 98, Kulesza/Moytka 08, Beenakker et.al. 09)

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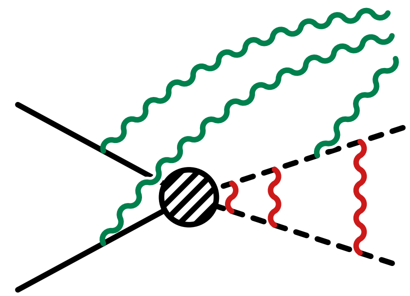

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- disentangles hard, soft and Coulomb contribution for total cross section for S-wave production and up to NNLL

  (more complicated colour structure for other observables: Ferroglia e.a., Ahrens e.a. 09)

- can perform simultaneous summation of threshold Logs and Coulomb corrections (also Hagiwara, Sumino, Yokoya; Kiyo et.al. 08)

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu}$ $\Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu}$ $\Rightarrow$ related to IR singlarities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

$$\frac{d}{d\log\mu} W_i^{R_\alpha}(z^0,\mu) = \left(2\gamma_{\text{cusp}}(C_r + C_{r'})\log\left(\frac{iz_0\bar{\mu}}{2}\right) - 2(\gamma_{H.s}^{R_\alpha} + \underbrace{\gamma_s^r + \gamma_s^{r'}}_{\text{as for Drell-Yan/Higgs}})\right) W_i^{R_\alpha}(z^0,\mu)$$

Solution in Mellin space (Korchemsky/Marchesini 92);

momentum space (Becher/Neubert 06)

Soft anomalous dimension (Beneke, Falgari, CS 09; Czakon, Mitov, Sterman 09)

$$\gamma_{H,s}^{R_\alpha} = \frac{\alpha_s}{4\pi}(-2C_{R_\alpha}) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{R_\alpha}\left[-C_A\left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{40}{18}n_f\right] + \mathcal{O}(\alpha_s^3).$$

(extracted from Becher/Neubert 09, Korchemsky/Radyushkin 92, Kidonakis 09)

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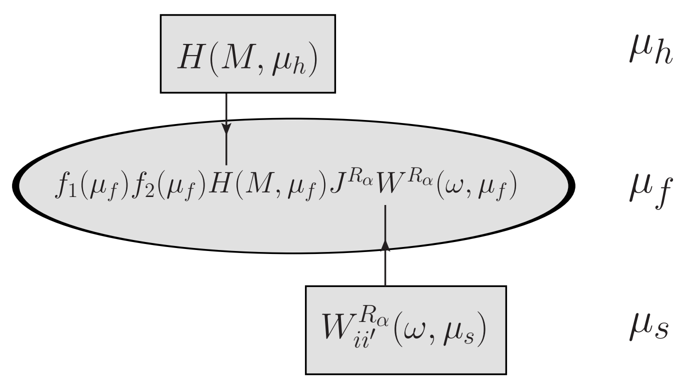
Resummation:

- evolve hard function from $\mu_h \sim 2m_t$ to μ_f
- evolve soft function from μ_s to μ_f
- (N)LO Coulomb-Green function (as in $e^-e^+ \to t\bar{t}$: Fadin/Khoze 87; Beneke/Signer/Smirnov 99,...)



Squark -antisquarks at LHC

- Two production channels:

$$q_i\bar{q}_j \longrightarrow \tilde{q}_k\overline{\tilde{q}_l} \quad , \qquad gg \longrightarrow \tilde{q}_k\overline{\tilde{q}_l}$$

- Simplified setup: equal squark masses, no stop
- Matching to NLO result (Beenakker et.al. 96, PROSPINO)

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Choice of scales for resummation in momentum space

Soft scale: $\tilde{\mu}_s/m_{\tilde{q}} \approx 0.5\ldots0.2$

for $m_{\tilde{q}} = 0.5,\ldots 2$ TeV

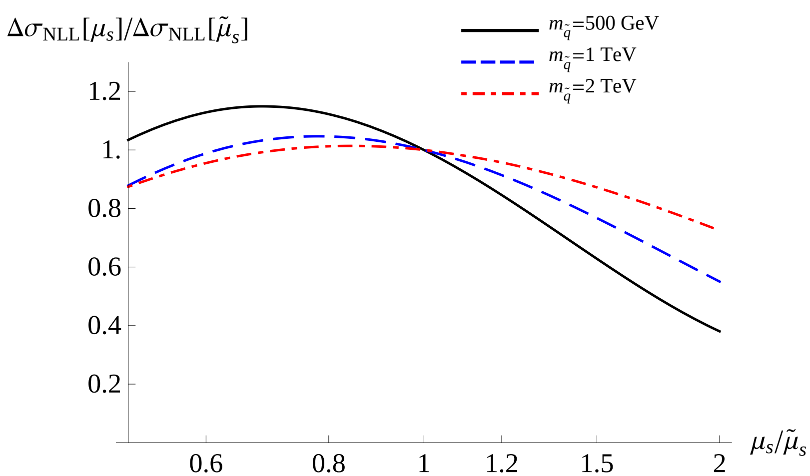
from minimizing $\Delta\sigma^{\text{NLO}}_{\text{soft}}$

(c.f. Becher e.a. 07)

Hard scale: $\tilde{\mu}_h = 2m_{\tilde{q}}$

Coulomb scale:

$\mu_C =$ max$\{2m_{\tilde{q}}\beta, C_F m_{\tilde{q}}\alpha_s\}$



($\sqrt{s} = 14$ TeV, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$)

Resumed Results:

NLL: full Coulomb $\otimes$ res. soft

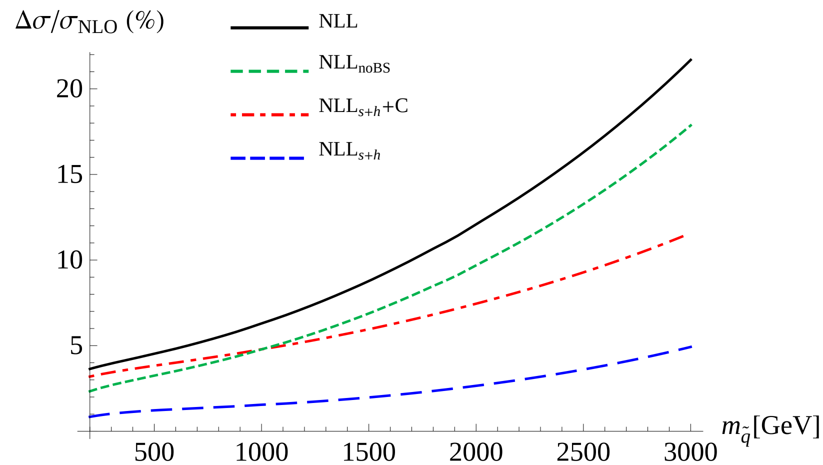
noBS:

NLL without bound states

$\mathbf{NLL_{s+h}}$:

resummation of H and W

C: Coulomb resummation



($\sqrt{s} = 14$ TeV, $m_{\tilde g}/m_{\tilde q} = 1.25$MSTW08NLO)

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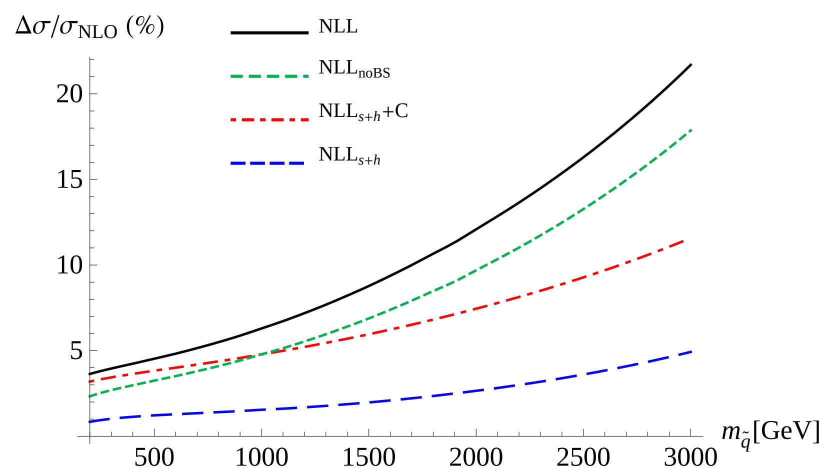


($\sqrt{s} = 14$ TeV, $m_{\tilde g}/m_{\tilde q} = 1.25$MSTW08NLO)

Comparison to Mellin-approach: (Kulesza, Motyka 08/09, Beenakker et.al. 09)

Good agreement for appropriate choice of scales ($\mu_h = \mu_f$: $\rm NLL_s$):

$m_{\tilde q}$[GeV]	NLO[pb]	$\rm NLL_{Mellin}$[pb]	$\rm NLL_s$[pb]	NLL [pb]
500	1.6×10^1	1.61×10^1 (1.2%)	1.62×10^1 (1.3%)	1.67×10^1 (4.2%)
1000	2.89×10^{-1}	2.93×10^{-1}(1.7%)	2.94×10^{-1}(1.7%)	3.06×10^{-1}(5.8%)
2000	1.11×10^{-3}	1.14×10^{-3}(3.4%)	1.14×10^{-3}(3.1%)	1.24×10^{-3} (11%)

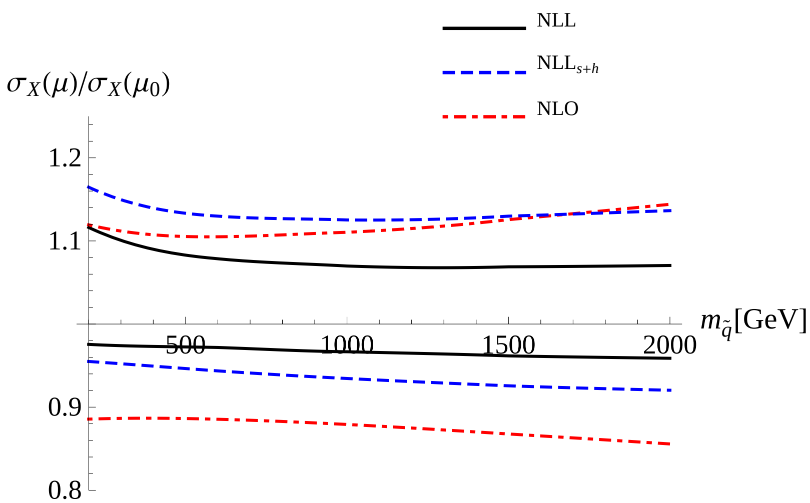
(LHC 14 TeV, $m_{\tilde g} = m_{\tilde q}$)

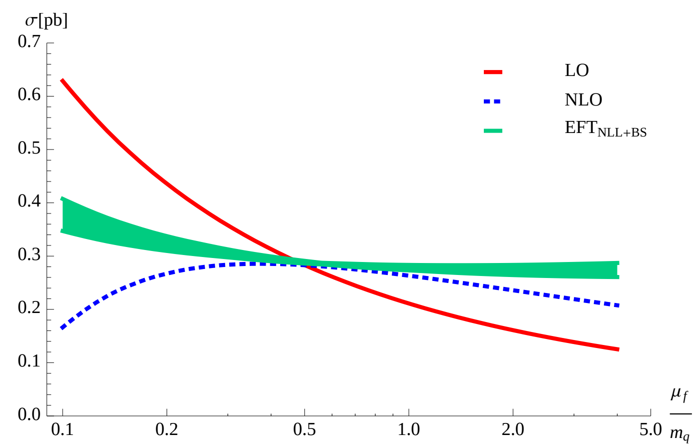
Scale uncertainty reduced by combined resummation

NLO $\frac{m_{\tilde{q}}}{2} < \mu_f < 2m_{\tilde{q}}$

NLL: vary all scales $\frac{\tilde{\mu}_i}{2} < \mu_i < 2\tilde{\mu}_i$, add in quadrature

$\Rightarrow$ significant reduction for combined resummation!





($\sqrt{s} = 14$ TeV, MSTW08NLO, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$)

($m_{\tilde{q}} = 1$ TeV, $\mu_s^0/2 < \mu_s < 2\mu_s^0$)

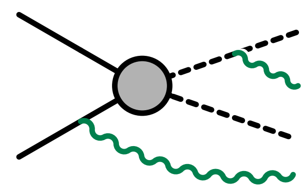
Threshold expansion at $\mathcal{O}(\alpha_s^2)$

All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms (Beneke, Czakon, Falgari, Mitov, CS 09

Implemented in HATHOR, Aliev et.al. 10)

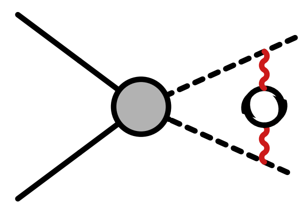
Pure soft corrections: (also Moch/Uwer+Langenfeld (08/09))

$$\Delta\sigma_{\mathsf{s}}^{(2)} \sim \alpha_s^2 (c_{\mathsf{LL}}^{(2)} \ln^4\beta + c_{\mathsf{NLL}}^{(2)} \ln^3\beta + c_{\mathsf{NNLL},2}^{(2)} \ln^2\beta + \underbrace{c_{\mathsf{NNLL},1}^{(2)} \ln\beta}_{\text{2-loop } \gamma_{H,s}})$$



Potential corrections: 2nd Coulomb, NLO potentials

$$\Delta\sigma_{\mathsf{p}}^{(2)} \sim \alpha_s^2 \left(\frac{c_{C2}}{\beta^2} + \frac{1}{\beta}(c_{\mathsf{C},0}^{(2)} + c_{\mathsf{C},1}^{(2)} \log\beta) + \underbrace{c_{\mathsf{n\text{-}C}}^{(2)} \ln\beta}_{\text{spin-dependent}} \right)$$

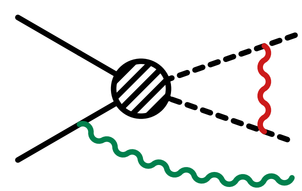


(using Beneke, Signer, Smirnov 99, Czarnecki/Melnikov 97/01)

mixed Coulomb/soft/hard corrections:

$$\Delta\sigma_{\mathsf{p}\otimes\,\mathsf{sh}}^{(2)} \sim \frac{\alpha_s}{\beta} \alpha_s (c_{\mathsf{LL}}^{(1)} \ln\beta^2 + c_{\mathsf{NLL}}^{(1)} \ln\beta + c + H^{(1)})$$

$$\Delta\sigma_{\mathsf{s}\otimes\mathsf{h}}^{(2)} \sim \alpha_s^2 H^{(1)} (c_{\mathsf{LL}}^{(1)} \ln\beta^2 + c_{\mathsf{NLL}}^{(1)} \ln\beta)$$



(H_1: process and colour-channel dependent, $t\bar{t}$: Czakon/Mitov 09)

Total top-pair production cross-section

$\sigma_{t\bar{t}}$(pb)	Tevatron	LHC7	LHC10	LHC14
NLO	$6.50^{+0.32+0.33}_{-0.70-0.24}$	150^{+18+8}_{-19-8}	380^{+44+17}_{-46-17}	842^{+97+30}_{-97-32}
NLO+NLL	$6.57^{+0.52+0.33}_{-0.30-0.24}$	151^{+23+8}_{-12-9}	382^{+60+17}_{-32-18}	$848^{+136+30}_{-75-32}$
NLO+NNLL	$6.77^{+0.27+0.35}_{-0.48-0.25}$	155^{+4+8}_{-9-9}	390^{+14+17}_{-26-18}	858^{+35+31}_{-64-33}
NNLO$_{\rm app}(\beta)$	$7.10^{+0.0+0.36}_{-0.26,-0.26}$	162^{+2+9}_{-3-9}	407^{+9+17}_{-5-18}	895^{+24+31}_{-6-33}
NNLO$_{\rm app}(\beta)$ + NNLL	$7.13^{+0.22+0.36}_{-0.24-0.26}$	162^{+4+9}_{-1-9}	405^{+14+17}_{-2-18}	892^{+38+31}_{-3-33}
NNLO$_{\rm app}(\beta)$ + NNLL+BS	$7.14^{+0.14+0.36}_{-0.22-0.26}$	162^{+4+9}_{-1-9}	407^{+14+17}_{-2-18}	896^{+38+31}_{-3-33}

($m_t = 173.1$ GeV, $\tilde{\mu}_f = mt$, MSTW08NNLO) (Beneke, Falgari, Klein, CS preliminary)

- Resummation in momentum space using fixed μ_s from minimising $\Delta\sigma^{\rm NLO}_{\rm soft}(\mu_s)$
  $\Rightarrow \tilde{\mu}_s = 85/146$ GeV for Tevatron/LHC7: no big scale hierarchy
- vary μ_s, μ_h, μ_f from $0.5\tilde{\mu} < \mu < 2\tilde{\mu}$, add uncertainties in quadrature
- (N)NLL includes (N)LO Coulomb resummation
- BS: include bound-state contributions below threshold
- Preliminary estimate of uncertainty from $\alpha_s^2 C^{(2)}$ terms: $\sim 3\%$

Alternative threshold expansions

Pair invariant mass cross sections (Kidonakis,Sterman 97, Ahrens et.al. 10)

$$\frac{d\sigma(t\bar{t})}{dM_{t\bar{t}}} \Rightarrow \left[\frac{\log^n(1-z)}{1-z}\right]_+ , \; z = \frac{M_{t\bar{t}}^2}{\hat{s}}$$

One particle inclusive cross sections: (Laenen, Oderda, Sterman 98)

$$\frac{d\sigma(t+X)}{ds_4} \Rightarrow \left[\frac{\log^n(s_4/m^2)}{s_4}\right] , \; s_4 = p_X^2 - m_t^2$$

$\sigma_{t\bar{t}}$(pb)	Tevatron	LHC7	LHC10	LHC14
NLO	$6.50^{+0.32+0.33}_{-0.70-0.24}$	150^{+18+8}_{-19-8}	380^{+44+17}_{-46-17}	842^{+97+30}_{-97-32}
NNLO$_{\rm app}(\beta)$	$7.10^{+0.0+0.36}_{-0.26,-0.26}$	162^{+2+9}_{-3-9}	407^{+9+17}_{-5-18}	895^{+24+31}_{-6-33}
NLO + NNLL ($M_{t\bar{t}}$) (Ahrens et.al. 10)	$6.48^{+0.17+0.32}_{-0.21-0.25}$	146^{+7+8}_{-7-8}	368^{+20+19}_{-14-15}	813^{+50+30}_{-36-35}
NNLO$_{\rm app}(s_4)$ (mt=173; Kidonakis 10)	$7.08^{+0.00+0.36}_{-0.24-0.27}$	163^{+7+9}_{-5-9}	415^{+17+18}_{-21-19}	920^{+50+33}_{-39-35}

($m_t = 173.1$ GeV, $\mu_f = mt$, MSTW08NNLO)

Threshold corrections $\sim \log^n \beta$, $\frac{1}{\beta^n}$

- Factorization of soft and Coulomb corrections
- $\log \beta$ resummation from momentum space solution to RGEs
- combined Soft and Coulomb resummation possible
- theoretical progress: now NNLL resummation feasible

Squark-antisquark production

- total corrections $4 - 10\%$ for $m_{\tilde{q}} = 300$ GeV-2 TeV
- reduced μ_f-dependence for combined soft/gluon resummation

Threshold expansion to $\mathcal{O}(\alpha_s^2)$ of $t\bar{t}$ cross section

NNLL resummation for $t\bar{t}$

- dominant higher-order corrections included in NNLO$_{\text{approx}}$
- discrepancy to NNLL from integrated $\frac{d\sigma}{dM_{tt}^2}$? (Ahrens et.al. 10)

Bonus slides

Matching of scattering amplitude (for S-wave production)

$$\mathcal{A}_{pp'\to HH'X} = \sum_i C^{(i)}_{\{\alpha\}}(M,\mu)\, c^{(i)}_{\{a\}}\, \langle HH'X|\phi_{c;a_1\alpha_1}\phi_{\bar{c};a_2\alpha_2}\psi^\dagger_{a_3\alpha_3}\psi'^\dagger_{a_4\alpha_4}|pp'\rangle_{\mathrm{EFT}}$$

- $\psi^\dagger$, $\psi'^\dagger$: non-relativistic fields that create H and H'
  $\Rightarrow$ (P)NRQCD
- ϕ_c ($\phi_{\bar{c}}$): collinear (anti-collinear) fields that destroy p and p'
  $\Rightarrow$ SCET
- α_i: spin, a_i: colour indices, $c^{(i)}_{\{a\}}$: colour basis
- only (u)soft hadronic final states X for threshold kinematics

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Collinear and nonrelativistic fields only connected by (u)soft gluons
$\Rightarrow$ Soft-gluon decoupling field redefinition (Bauer, Pirjol, Stewart 01)

$$\phi_c(x) = S_n(x_-)\phi_c^{(0)}(x) \qquad S_n(x) = \mathrm{P}\exp\left[ig_s\int_{-\infty}^0 dt\, n\cdot A_s^a(x+nt)T^a\right]$$

LO NRQCD Lagrangian for particles H, H' in representations R, R':

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi'$$
$$+ \int d^3\vec{r} \left[\psi^\dagger \mathbf{T}^{(R)a} \psi \right](\vec{r}) \left(\frac{\alpha_s}{r} \right) \left[\psi'^\dagger \mathbf{T}^{(R')a} \psi' \right](0) \,,$$

with $D_s^0 = \partial^0 - ig_s A_s^0(x_0, \vec{0})$.

EFT approach to heavy pair production

LO NRQCD Lagrangian for particles H, H' in representations R, R':

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$$+ \int d^3\vec{r} \, \left[\psi^\dagger \mathbf{T}^{(R)a} \psi\right](\vec{r}) \left(\frac{\alpha_s}{r}\right) \left[\psi'^\dagger \mathbf{T}^{(R')a} \psi'\right](0) \,,$$

with $D_s^0 = \partial^0 - i g_s A_s^0(x_0, \vec{0})$.

Decoupling for heavy particle fields:

$$\psi(x) = S_v^{(R)}(x_0) \psi^{(0)\dagger}(x), \quad S_v^{(R)}(x) = \overline{\mathrm{P}} \exp\left[-i g_s \int_0^\infty ds\, v \cdot A^a(x + vs) \mathbf{T}^{(R)a} \right]$$
$$\Rightarrow D_s^0 \psi = S_v \partial^0 \psi^0$$

same $v = (1, \vec{0})$ for both heavy particles at threshold

Works at leading order in PNRQCD

(sufficient at NNLL: Beneke, Czakon, Falgari, Mitov, CS 09; Beneke, Falgari, CS 10)

Apply soft-gluon decoupling to amplitude:

$$\mathcal{A}_{pp'\to HH'X} \Rightarrow \sum_i C^{(i)} \langle HH'|\psi^{(0)\dagger}\psi'^{(0)\,\dagger}|0\rangle\ \langle 0|\phi_c^{(0)}|p\rangle\langle 0|\phi_{\bar{c}}^{(0)}|p'\rangle\ \langle X|S_n S_{\bar{n}} \boldsymbol{c}^{(i)} S_v^\dagger S_v^\dagger|0\rangle$$

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Inserting into formula for σ, summing over complete set of $|X\rangle$...

$$\hat{\sigma}_{pp'}(\hat{s}, \mu) = \sum_{i,i'} H_{ii'}(M, \mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(\sqrt{\hat{s}} - 2M - \frac{\omega}{2}) W_{ii'}^{R_\alpha}(\omega, \mu)$$

Irreducible representations $R \otimes R' = \sum_{R_\alpha} R_\alpha$ e.g. $3 \otimes \overline{3} = 1 \oplus 8$.

EFT approach to heavy pair production

Apply soft-gluon decoupling to amplitude:

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Irreducible representations $R \otimes R' = \sum_{R_\alpha} R_\alpha$ e.g. $3 \otimes \overline{3} = 1 \oplus 8$.

Soft function:

$$W_{ii'}^{R_\alpha}(\omega) = \int \frac{dz_0}{4\pi}\, e^{i\omega z_0/2}\, \langle 0|\overline{\mathsf{T}}[S_v S_v c^{(i')*} S_{\bar{n}}^\dagger S_n^\dagger](0) P^{R_\alpha}\, \mathsf{T}[S_n S_{\bar{n}} c^{(i)} S_v^\dagger S_v^\dagger](x_0)|0\rangle$$

Potential function:

$$J_{R_\alpha}(E) = \int d^4 z e^{iEz^0}\, \langle 0|[\psi^{(0)}\psi'^{(0)}](z^0) P^{R_\alpha} [\psi^{(0)\dagger}\psi'^{(0)\dagger}](0)|0\rangle \quad = 2\,\mathrm{Im} G_C^{R_\alpha}(0,0,E)$$

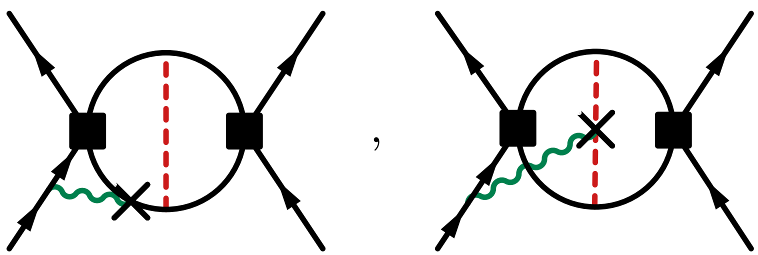
(Same as for $e^- e^+ \to t\bar{t}$: Fadin, Khoze 87; Beneke, Signer, Smirnov; Hoang, Teubner 99,...)

Subleading PNRQCD and SCET interactions:

$$\psi^\dagger \vec{x}\cdot \vec{E}_{us}(x_0,0)\psi'^\dagger\,, \quad \bar{\xi}\left(x_\perp^\mu n_-^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger\right)\frac{\not n_+}{2}\,\xi\,\ldots$$

Soft gluons not decoupled by field redefinitions.

Possibly relevant at NNLL in soft $\otimes$ potential corrections :



$$\sim \frac{\alpha_s}{\beta}\times \alpha_s\beta\log\beta \sim \alpha_s^2\log\beta$$

Related to three-parton colour correlations in IR singularities of amplitudes (Ferroglia et.al. 09)

$\sigma_{\mathbf{tot}}$: effects vanish at NNLL! (Beneke, Czakon,Falgari, Mitov, CS 09)

- no collinear/potential correction $\sim\beta$ for $k_\perp = 0$
- no potential/soft corrections due to rotational invariance

  (no heavy particle three-momentum available)

Potential corrections:

- 2nd Coulomb correction
- NLO Coulomb potentials:

$$\tilde{V}_{\mathrm{C}}^{(1)}(\boldsymbol{p},\boldsymbol{q}) = \frac{D_{R_\alpha}\alpha_s^2}{\boldsymbol{q}^2}\left(a_1 - \beta_0 \ln\frac{\boldsymbol{q}^2}{\mu^2}\right)$$

- Non-Coulomb potential:

$$\tilde{V}_{\mathrm{nC}}^{(1)}(\boldsymbol{p},\boldsymbol{q}) = \frac{4\pi D_{R_\alpha}\alpha_s}{\boldsymbol{q}^2}\left[\frac{\pi\alpha_s|\boldsymbol{q}|}{4m}\left(\frac{D_{R_\alpha}}{2}+C_A\right)+\frac{\boldsymbol{p}^2}{m^2}+\frac{\boldsymbol{q}^2}{m^2}\,v_{\mathrm{spin}}\right],$$

($v_{\mathrm{spin}} = 0$ (singlet); $-2/3$ (triplet))

Corrections to cross section:

$$\Delta\hat{\sigma}_{\mathrm{nC}} = \hat{\sigma}^{(0)}\,\alpha_s^2\,\ln\beta\,\left[-2D_{R_\alpha}^2\,(1+v_{\mathrm{spin}})+D_{R_\alpha}C_A\right]$$

(extracted from Beneke, Signer, Smirnov 99, Pineda, Signer 06)