Top-Quark Pair Production Beyond Next-To-Leading Order

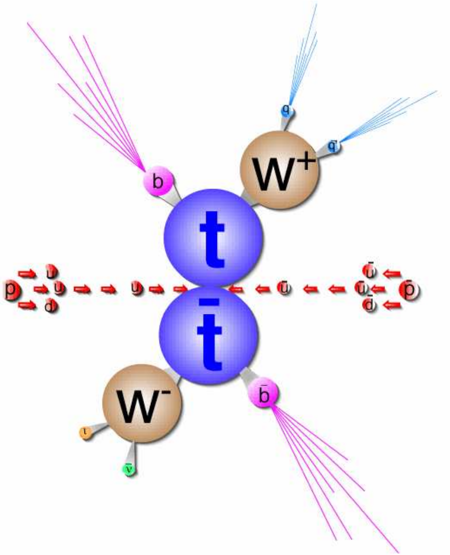
Andrea Ferroglia

New York City College of Technology

ETH Zürich, January 6, 2011

Outline

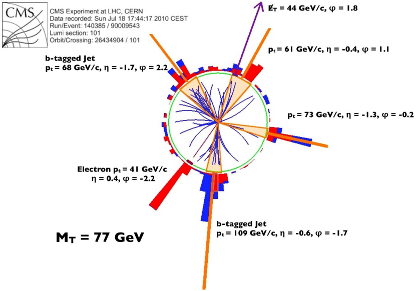
Top-Quark Pair Hadroproduction

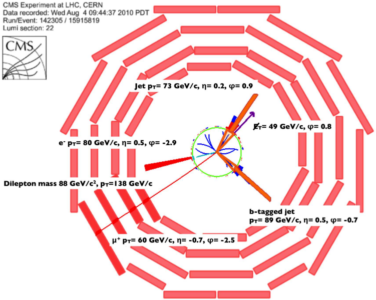


Several observables measured at Tevatron (a few thousand observed events)

- Total Cross Section
- Invariant Mass Distribution
- p_T distribution
- Charge / Forward-Backward Asymmetry
- ...

Top-Pair Production and the LHC





- Next 2 years at the LHC
  $\sim$ few 10^4 top pair events
- First measurement of the total CS at CMS

$$\sigma^{t\bar{t}} = 194 \pm 72 \pm 24 \pm 21\,\text{pb}$$

arXiv:1010.5994

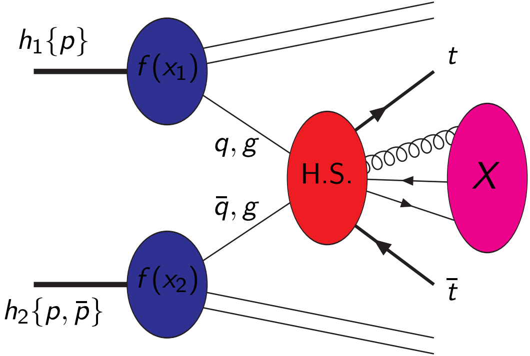
- First measurement of the total CS at ATLAS

$$\sigma^{t\bar{t}} = 145 \pm 31^{+42}_{-27}\,\text{pb}$$

arXiv:1012.594

- at $\sqrt{s} = 14\,\text{TeV}$, $10\,\text{fb}^{-1}$
  $\Longrightarrow$ millions of top-quarks

Top-quark pair production is a hard scattering process which can be computed in perturbative QCD

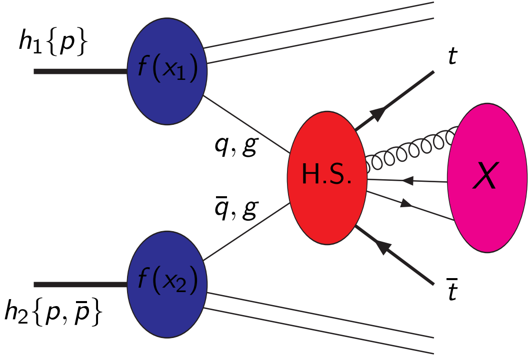


$$\sigma^{t\bar{t}}_{h_1,h_2} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_i^{h_1}(x_1, \mu_F) f_j^{h_2}(x_2, \mu_F) \hat{\sigma}_{ij}(s, m_t, \alpha_s(\mu_R), \mu_F, \mu_R)$$

$$s_{\text{had}} = (p_{h_1} + p_{h_2})^2 \, , \; s = x_1 x_2 s_{\text{had}}$$

Top Quark Pair Hadroproduction & QCD

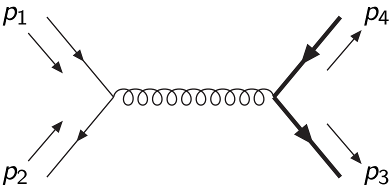
Top-quark pair production is a hard scattering process which can be computed in perturbative QCD



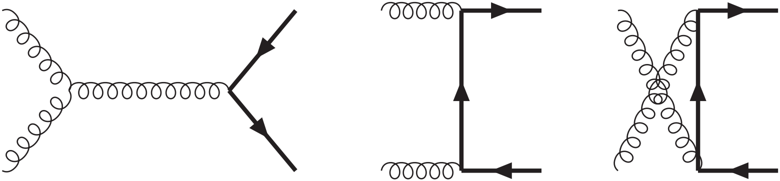
$$\sigma_{h_1,h_2}^{t\bar{t}}(s_{\rm had}, m_t^2) = \sum_{ij} \int_{4m_t^2}^{s_{\rm had}} ds \; \underbrace{L_{ij}\left(s, s_{\rm had}, \mu_f^2\right)}_{\text{partonic luminosity}} \; \overbrace{\hat{\sigma}_{ij}(s, m_t^2, \mu_f^2, \mu_r^2)}^{\text{partonic cross section}}$$

Tree Level QCD Partonic Processes

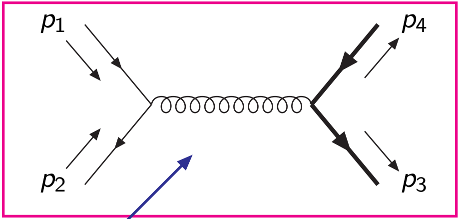
$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



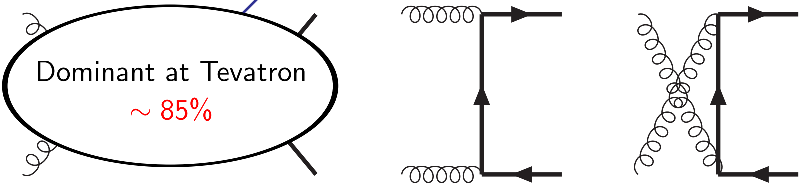
$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$

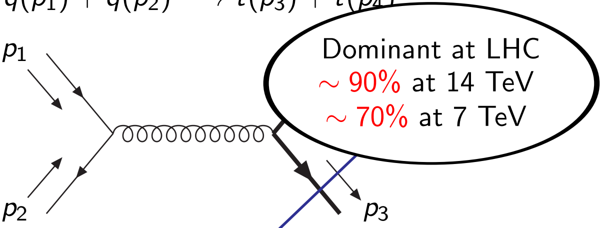


$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$

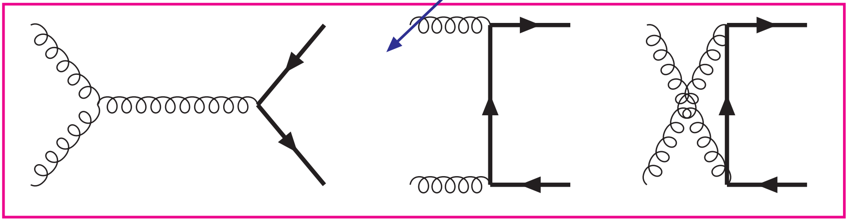


Tree Level QCD Partonic Processes

$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



NLO corrections

The NLO corrections to top-quark pair production have been a subject of active research for more than 20 years
(too many authors to list here!)

- NLO QCD corrections to the total cross section
- NLO QCD corrections to the distributions (p_T, rapidity, invariant mass, ...)
- NLL resummation of threshold effects
- Mixed QCD-EW corrections
- NLO with top decays in narrow width approximation (Melnikov's talk)
- NLO with top decays with off-shell top quarks (Pozzorini's and Papadopoulos' talks)

NLO corrections

The NLO corrections to top-quark pair production have been a subject of active research for more than 20 years
(too many authors to list here!)

- NLO QCD corrections to the total cross section
- NLO QCD corrections to the distributions (p_T, rapidity, invariant mass, ...)
- NLL resummation of threshold effects
- Mixed QCD-EW corrections
- NLO with top decays in narrow width approximation (Melnikov's talk)
- NLO with top decays with off-shell top quarks (Pozzorini's and Papadopoulos' talks)

At the LHC some observables (ex. total cross section) will be affected by experimental errors which are smaller than the current NLO + NLL theoretical uncertainties

Beyond NLO+NLL

In order to take full advantage of the LHC potential, one needs to obtain theoretical predictions that go
beyond the current NLO+NLL calculations

Beyond NLO+NLL

In order to take full advantage of the LHC potential, one needs to obtain theoretical predictions that go beyond the current NLO+NLL calculations

NNLL Resummation
Approximate NNLO

Recent developments based upon EFT methods for

- total cross section
- invariant mass distribution
- p_T distributions, etc

See talks by M. Neubert and C. Schwinn

Beyond NLO+NLL

In order to take full advantage of the LHC potential, one needs to obtain theoretical predictions that go beyond the current NLO+NLL calculations

NNLL Resummation
Approximate NNLO

Recent developments based upon EFT methods for

- total cross section
- invariant mass distribution
- p_T distributions, etc

See talks by M. Neubert and C. Schwinn

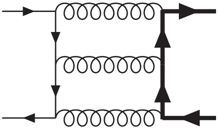
Full NNLO
Calculations

Long and technically challenging: where do we stand?

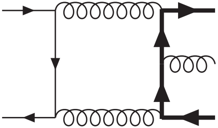
$\Longrightarrow$ This talk

NNLO Laundry List

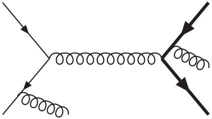
- Two-loop diagrams with a $t\bar{t}$ in the final state



- One-loop diagrams with a $t\bar{t}g(q, \bar{q})$ in the final state

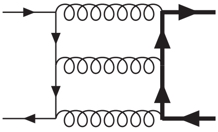


- Tree-level diagrams with a $t\bar{t}gg(gq, g\bar{q}, q\bar{q})$ in the final state



NNLO Laundry List

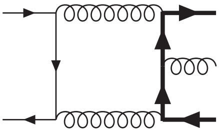
- Two-loop diagrams with a $t\bar{t}$ in the final state



  - 218 diagrams in $q\bar{q} \to t\bar{t}$
  - 789 diagrams in $gg \to t\bar{t}$

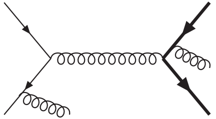
- One-loop diagrams with a $t\bar{t}g(q,\bar{q})$ in the final state

Dittmaier, Uwer, Wenzierl ('07,'08)
Bevilacqua *et al.* ('10)
Melnikov, Schulze ('10)



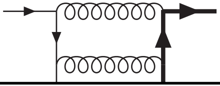
- Tree-level diagrams with a $t\bar{t}gg(gq, g\bar{q}, q\bar{q})$ in the final state

Dittmaier, Uwer, Wenzierl ('07,'08)
Bevilacqua *et al.* ('10)
Melnikov, Schulze ('10)



NNLO Laundry List

- Two-loop diagrams with a $t\bar{t}$ in the final state



- ► 218 diagrams in $q\bar{q} \to t\bar{t}$
- ► 789 diagrams in $gg \to t\bar{t}$

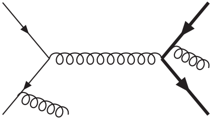
Further Problem: NNLO subtraction with massive fermions

Recent developments:

Czakon ('10-'11); Anastasiou, Herzog, Lazopoulos ('10)

(See talks by Czakon and Abelof)

- Tree-level diagrams with a $t\bar{t}gg(gq, g\bar{q}, q\bar{q})$ in the final state



Dittmaier, Uwer, Wenzierl ('07,'08)
Bevilacqua *et al.* ('10)
Melnikov, Schulze ('10)

Quark-Annihilation Channel

Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

$$|\mathcal{M}|^2 (s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c} \left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

$$|\mathcal{M}|^2 (s, t, m, \varepsilon) = \frac{4\pi^2 \alpha_s^2}{N_c} \left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi} \right) \mathcal{A}_1 + \left(\frac{\alpha_s}{\pi} \right)^2 \mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right) \right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

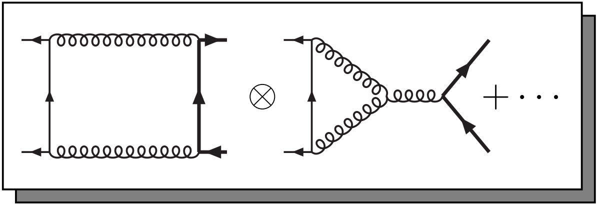
Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

$$\left|\mathcal{M}\right|^2 (s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right) \ldots\right.$$

One-Loop × One-Loop

Körner, Merebashvili, Rogal ('05,'08)

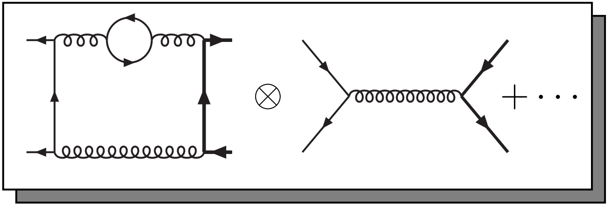
$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times0)} + \mathcal{A}_2^{(1\times1)}$$



Two-Loop × Tree

$$\ldots \alpha_s^2 \left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$



Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times 0)} \quad = \quad & N_c C_F\Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l\left(N_c D_l + \frac{E_l}{N_c}\right) \\ & + N_h\left(N_c D_h + \frac{E_h}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\Big]\end{aligned}$$

10 different color coefficients, functions of s, t, and m_t

Small Mass Approximation and Numerical Evaluation

The two-loop corrections to $q\bar{q} \to t\bar{t}$ were first evaluated in the limit in which $s, |t|, |u| \gg m_t^2$

M. Czakon, A. Mitov, and S. Moch ('07)

Small Mass Approximation and Numerical Evaluation

The two-loop corrections to $q\bar{q} \to t\bar{t}$ were first evaluated in the limit in which $s, |t|, |u| \gg m_t^2$

M. Czakon, A. Mitov, and S. Moch ('07)

Adding more terms in the expansion in powers of $m^2/s, m^2/|t|, m^2/|u|$ is not sufficient for phenomenological studies (particularly near the production threshold)

Small Mass Approximation and Numerical Evaluation

The two-loop corrections to $q\bar{q} \to t\bar{t}$ were first evaluated in the limit in which $s, |t|, |u| \gg m_t^2$

M. Czakon, A. Mitov, and S. Moch ('07)

Adding more terms in the expansion in powers of $m^2/s, m^2/|t|, m^2/|u|$ is not sufficient for phenomenological studies (particularly near the production threshold)

An exact numerical evaluation of these correction is available

M. Czakon ('08)

Method employed: Reduction to Master Integrals and + numerical variant of the Differential Equation Method

7 color structures receive contributions from the diagrams involving closed fermion loops

- N_l number of light (massless) quarks
- N_h number of heavy (massive) quarks

$$\mathcal{A}_2^{(2\times0)} = N_c C_F \Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l \left(N_c D_l + \frac{E_l}{N_c} \right) + N_h \left(N_c D_h + \frac{E_h}{N_c} \right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h \Big]$$

7 co... ...sed
ferm...

All the diagrams with a closed quark loop (massive or massless) were evaluated analytically

Bonciani, AF, Gehrmann, Maître, Studerus ('08)

Agreement with the numerical results by Czakon

$$\mathcal{A}_2^{(2\times 0)} = N_c C_F \Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l \left(N_c D_l + \frac{E_l}{N_c} \right) + N_h \left(N_c D_h + \frac{E_h}{N_c} \right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h \Big]$$

The leading color coefficient receives contributions only from planar diagrams

$$\mathcal{A}_2^{(2\times 0)} = N_c C_F \Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l \left(N_c D_l + \frac{E_l}{N_c} \right) + N_h \left(N_c D_h + \frac{E_h}{N_c} \right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h \Big]$$

The leading color coefficient receives contributions only from planar diagrams

$$\mathcal{A}_2^{(2\times 0)} = N_c C_F \Bigg[N_c^2 A + B + \frac{C}{N_c^2} + N_l \left(N_c D_l + \frac{E_l}{N_c} \right)$$

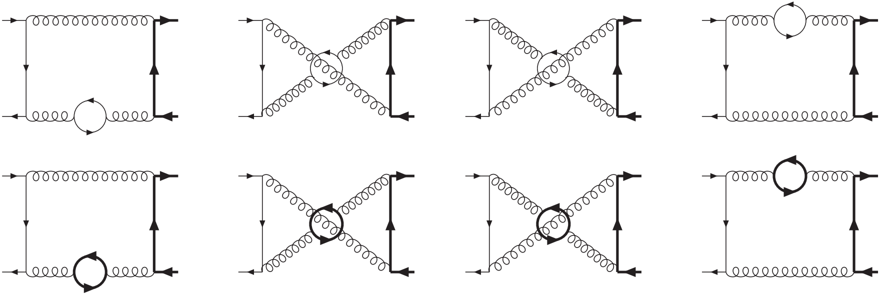
All the diagrams contributing to the leading color coefficient were evaluated analytically

Bonciani, AF, Gehrmann, Studerus ('09)

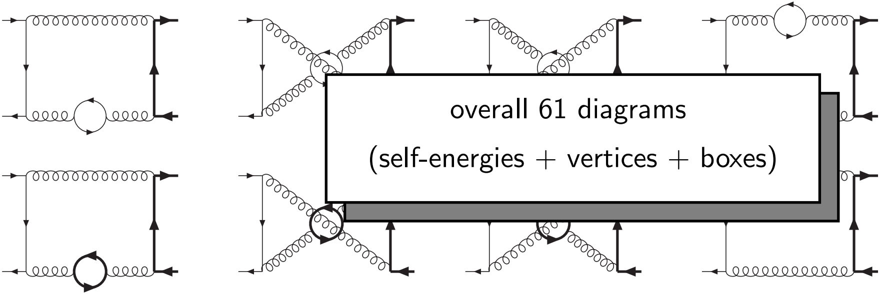
Agreement with the numerical results by Czakon

Box Diagrams Involved

Quark-loop diagrams: only two different box topologies (and all crossings)



Quark-loop diagrams: only two different box topologies (and all crossings)

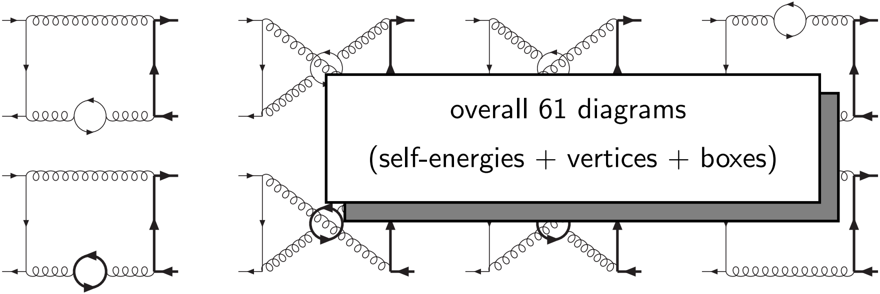


overall 61 diagrams

(self-energies + vertices + boxes)

Box Diagrams Involved

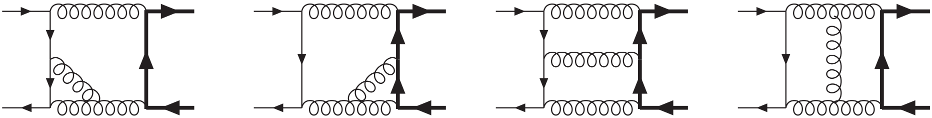
Quark-loop diagrams: only two different box topologies (and all crossings)



overall 61 diagrams

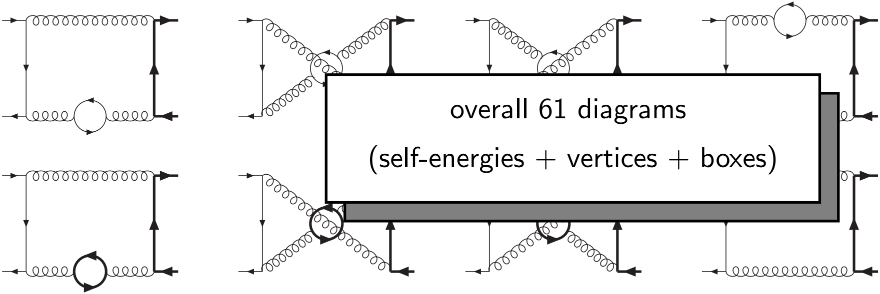
(self-energies + vertices + boxes)

Planar diagrams: four box topologies (including double boxes)



Box Diagrams Involved

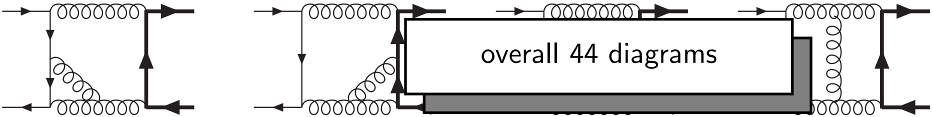
Quark-loop diagrams: only two different box topologies (and all crossings)



overall 61 diagrams

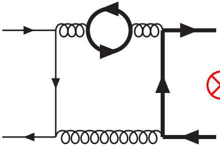
(self-energies + vertices + boxes)

Planar diagrams: four box topologies (including double boxes)



overall 44 diagrams

After interfering a two-loop graph with the Born amplitude, one obtains linear combinations of scalar integrals



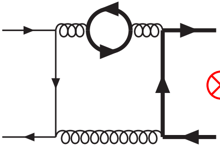
$\otimes$ Born Amplitude

$$\int \mathfrak{D}^d k_1 \mathfrak{D}^d k_2 \frac{S_1^{n_1} \cdots S_q^{n_q}}{\mathcal{D}_1^{m_1} \cdots \mathcal{D}_t^{m_t}}$$

- $k_i \rightarrow$ integration momenta
- $p_i \rightarrow$ external momenta
- $S \rightarrow$ scalar products $k_i \cdot k_j$ or $k_i \cdot p_k$
- $\mathcal{D} \rightarrow$ propagators $\left[\sum c_i k_i + \sum d_j p_j\right]^2 \; (+m_t^2)$

Method: The General Strategy

After interfering a two-loop graph with the Born amplitude, one obtains linear combinations of scalar integrals



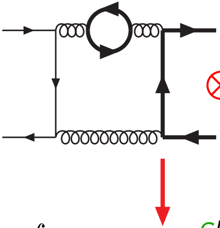
$\otimes$ Born Amplitude

$$\int \mathfrak{D}^d k_1 \mathfrak{D}^d k_2 \frac{S_1^{n_1} \cdots S_q^{n_q}}{\mathcal{D}_1^{m_1} \cdots \mathcal{D}_t^{m_t}}$$

k_i	$\rightarrow$	integration momenta
p_i	$\rightarrow$	external momenta
S	$\rightarrow$	scalar products $k_i \cdot k_j$ or $k_i \cdot p_k$
$\mathcal{D}$	$\rightarrow$	propagators $\left[\sum c_i k_i + \sum d_j p_j\right]^2 \ (+m_t^2)$

Luckily, just a “small” number of these integrals are independent: the MIs

After interfering a two-loop graph with the Born amplitude, one obtains linear combinations of scalar integrals



$\otimes$ Born Amplitude

$$\int \mathfrak{D}^d k_1 \mathfrak{D}^d k_2 \frac{S_1^{n_1} \cdots S_q^{n_q}}{\mathcal{D}_1^{m_1} \cdots \mathcal{D}_t^{m_t}}$$

$k_i \rightarrow$ integration momenta

$p_i \rightarrow$ external momenta

$S \rightarrow$ scalar products $k_i \cdot k_j$ or $k_i \cdot p_k$

$\mathcal{D} \rightarrow$ propagators $\left[\sum c_i k_i + \sum d_j p_j\right]^2 \ (+m_t^2)$

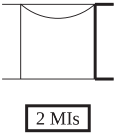
Luckily, just a "small" number of these integrals are independent: the MIs

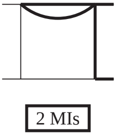
It is necessary to

- identify the MIs $\Longrightarrow$ Reduction through the Laporta Algorithm
- calculate the MIs $\Longrightarrow$ Differential Equation Method

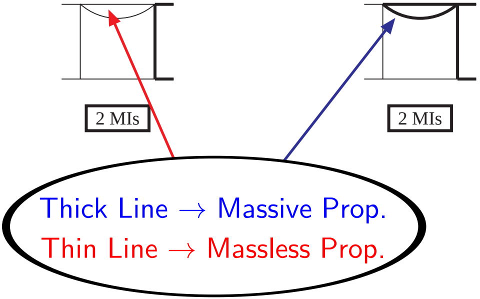
Master Integrals

Irreducible four-point topologies for the quark-loop corrections



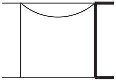


Irreducible four-point topologies for the quark-loop corrections

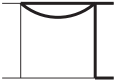


Master Integrals

Irreducible four-point topologies for the quark-loop corrections

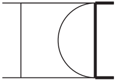


2 MIs

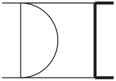


2 MIs

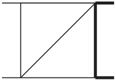
Leading Color Coefficient: 6 new irreducible four point topologies



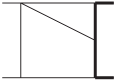
2 MIs



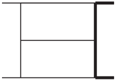
2 MIs



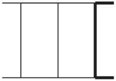
2 MIs



2 MIs



2 MIs

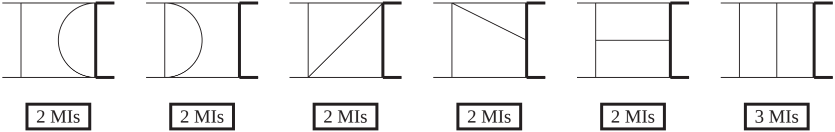


3 MIs

Master Integrals

Irreducible four-point topologies for the quark-loop corrections

Leading Color Coefficient: 6 new irreducible four point topologies



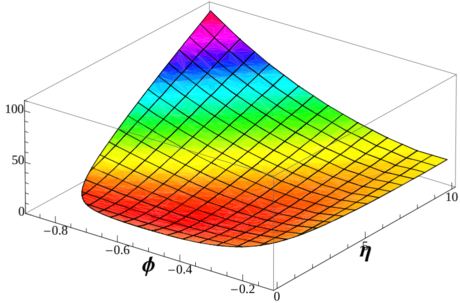
A new `GiNaC/C++` implementation of the Laporta Algorithm was developed for this project:

`Reduze` (`Studerus ('09)`), and `Reduze 2` (`Studerus, von Manteuffel ('10)`)

General Features of the Analytical Results

- All of the results are written in terms of HPLs and 2dHPLs of maximum weight 3 (fermion-loops) and weight 4 (leading color coefficient)
- It was not possible to find closed expressions for all of the the 2dHPL in term of Li_n; the numerical evaluation was carried out with the `GiNaC` package by Vollinga and Weinzierl ('04)
- The calculation of the integration constants requires the direct evaluation of the integral for special values of s and t with techniques based on Mellin-Barnes representations
- The threshold expansions $m_t \to 0$ and $\beta = \sqrt{1 - 4m_t^2/s} \to 0$ required tricks to extract the expansion of the 2dHPL from the expansion of the integrands in the integral representation

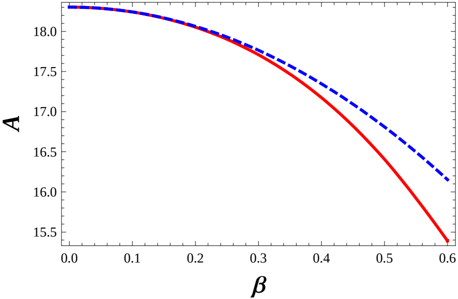
Finite part of A



$$\eta = \frac{s}{4m_t^2} - 1$$

$$\phi = -\frac{t - m_t^2}{s}$$

Threshold expansion versus exact result



$$\beta = \sqrt{1 - \frac{4m_t^2}{s}}$$

partonic c.m. scattering angle $\theta = \frac{\pi}{2}$

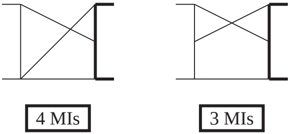
The $\beta \to 0$ expansion depends only polynomially on β and $\xi = (1 - \cos\theta)/2$

Color Coefficients B and C

Only the poles of the subleading color coefficients B and C are known analytically (more details later)

$$
\begin{aligned}
\mathcal{A}_2^{(2\times 0)} &= N_c C_F \Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l \left(N_c D_l + \frac{E_l}{N_c} \right) \\
&\quad + N_h \left(N_c D_h + \frac{E_h}{N_c} \right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h \Big]
\end{aligned}
$$

The calculation of the coefficient B and C involves topologies with a large number of MIs



Czakon, Mitov, Moch ('07)

Gluon-Fusion Channel

Two-Loop Corrections to $gg \to t\bar{t}$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

One-Loop × One-Loop

Anastasiou, Aybat ('08)
Körner, Kniehl, Merebashvili, Rogal ('08)

Two-Loop Corrections to $gg \to t\bar{t}$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times0)} + \mathcal{A}_2^{(1\times1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times0)} = (N_c^2-1)\Bigg(& N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h \\ & + N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h \\ & + N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh} \Bigg)\end{aligned}$$

16 color structures

$\mathcal{A}_2^{(2\times0)}$ is known only in the limit $s, |t|, |u| \gg m_t^2$

Czakon, Mitov, Moch ('07)

Interlude: IR Poles in QCD Amplitudes

After UV renormalization, QCD amplitudes still include soft and collinear (IR) poles

$$\mathcal{A}_1 = \frac{\mathcal{A}_1^{(-2)}}{\epsilon^2} + \frac{\mathcal{A}_1^{(-1)}}{\epsilon} + \mathcal{A}_1^{(0)}$$

$$\mathcal{A}_2 = \frac{\mathcal{A}_2^{(-4)}}{\epsilon^4} + \frac{\mathcal{A}_2^{(-3)}}{\epsilon^3} + \frac{\mathcal{A}_2^{(-2)}}{\epsilon^2} + \frac{\mathcal{A}_2^{(-1)}}{\epsilon} + \mathcal{A}_2^{(0)}$$

Interlude: IR Poles in QCD Amplitudes

After UV renormalization, QCD amplitudes still include soft and collinear (IR) poles

$$
\begin{aligned}
\mathcal{A}_1 &= \frac{\mathcal{A}_1^{(-2)}}{\epsilon^2} + \frac{\mathcal{A}_1^{(-1)}}{\epsilon} + \mathcal{A}_1^{(0)} \\
\mathcal{A}_2 &= \frac{\mathcal{A}_2^{(-4)}}{\epsilon^4} + \frac{\mathcal{A}_2^{(-3)}}{\epsilon^3} + \frac{\mathcal{A}_2^{(-2)}}{\epsilon^2} + \frac{\mathcal{A}_2^{(-1)}}{\epsilon} + \mathcal{A}_2^{(0)}
\end{aligned}
$$

It is possible to predict the analytic expression of the coefficients of the IR poles in any QCD amplitude at two-loops, also in the presence of massive partons: IR poles in QCD amplitudes can be removed by multiplicative renormalization

Becher, Neubert ('09)

$$
\mathbf{Z}^{-1}(\epsilon, \{p\}, \{m\}) \, |\mathcal{M}_n(\epsilon, \{p\}, \{m\})\rangle_{\alpha_s^{QCD} \to \xi \alpha_s} = \text{FINITE}
$$

Two-Loop Corrections to $gg \to t\bar{t}$: IR Poles

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
A	10.749	18.694	−156.82	262.15	?
B	−21.286	−55.990	−235.04	1459.8	?
C		−6.1991	−68.703	−268.11	?
D			94.087	−130.96	?
E_l					?
E_h					?
F_l					?
F_h					?
G_l					?
G_h					?
H_l			2.3888	−5.4520	?
H_{lh}				−0.0043025	?
H_h					?
I_l			−4.7302	10.810	?
I_{lh}				0.0	?
I_h					?

It was possible to recalculate the known poles in $q\bar{q} \to t\bar{t}$ and to obtain exact analytical expressions for all the IR poles in $gg \to t\bar{t}$ at two loops

AF, Neubert, Pecjak, Yang ('09)

$$t_1 = -0.45s\,, \qquad s = 5m_t^2\,, \qquad \mu = m_t$$

Two-Loop Corrections to $gg \to t\bar{t}$: IR Poles

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
A	10.749	18.694	−156.82	262.15	?
B	−21.286	−55.990	−235.04	1459.8	?
C		−6.1991	−68.703	−268.11	?
D			94.087	−130.96	?
E_l		−12.541	18.207	27.957	?
E_h			0.012908	11.793	?
F_l		24.834	−26.609	−50.754	?
F_h			0.0	−23.329	?
G_l			3.0995	67.043	?
G_h				0.0	?
H_l			2.3888	−5.4520	?
H_{lh}				−0.0043025	?
H_h					?
I_l			−4.7302	10.810	?
I_{lh}				0.0	?
I_h					?

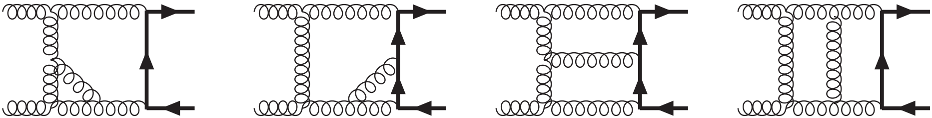
$$t_1 = -0.45s\,, \qquad s = 5m_t^2\,, \qquad \mu = m_t$$

Leading Color Coefficient

Bonciani, AF, Gehrmann, von Manteuffel, Studerus ('10)

The calculation of the leading color coefficient in the gluon fusion channel involves the same MIs which appear in the calculation of the leading color coefficient in the quark annihilation diagram (plus MIs obtained by $t \leftrightarrow u$)
$\Longrightarrow$ Analytical result available

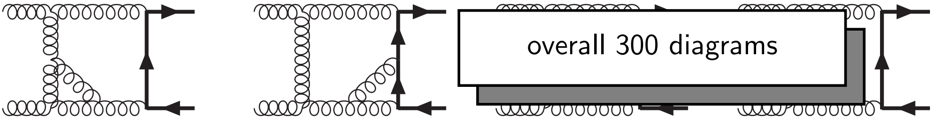
$$
\begin{aligned}
\mathcal{A}_2^{(2\times 0)} &= (N_c^2-1)\Bigg(N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h \\
&\quad + N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h \\
&\quad + N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh} \Bigg)
\end{aligned}
$$



Bonciani, AF, Gehrmann, von Manteuffel, Studerus ('10)

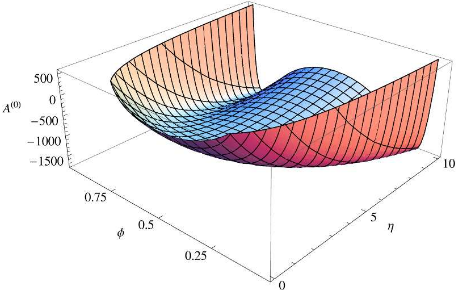
The calculation of the leading color coefficient in the gluon fusion channel involves the same MIs which appear in the calculation of the leading color coefficient in the quark annihilation diagram (plus MIs obtained by $t \leftrightarrow u$)
$\Longrightarrow$ Analytical result available

$$\mathcal{A}_2^{(2\times 0)} = (N_c^2-1)\Bigg(N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h$$
$$+N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h$$
$$+N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh}\Bigg)$$



Leading Color Coefficient: Results

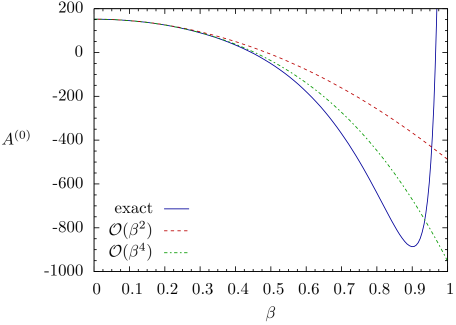
Finite part of A



$$\eta = \frac{s}{4m_t^2} - 1 \qquad \phi = -\frac{t - m_t^2}{s}$$

The symmetry $t \leftrightarrow u$, due to the two identical particles in the initial state, is manifested through $A(\phi) = A(1-\phi)$

Threshold expansion versus exact result



$$\beta = \sqrt{1 - \frac{4m_t^2}{s}}$$

partonic c.m. scattering angle $\theta = \frac{\pi}{2}$

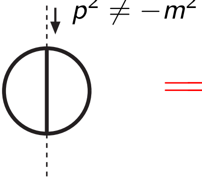
The $\beta \to 0$ expansion depends only polynomially on β and $\xi = (1 - \cos\theta)/2$

What about the finite parts of the other color coefficients?

- The diagrams involving massless quark loops can be calculated analytically in the usual way

  Bonciani, AF, Gehrmann, von Manteuffel, Studerus (in progress)

- The remaining part of the virtual corrections involve many MIs which cannot be expressed in terms of HPLs only $\Longrightarrow$ Numerical approach? New ideas?



$\Longrightarrow$

Elliptic Functions

$$K(z) = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-zx^2)}}$$

Laporta Remiddi ('04)

Summary & Conclusions

- The measurements related to the production of top-quark pairs play a crucial role at the Tevatron and at the LHC: desirable to reach NNLO accuracy in theoretical predictions

Summary & Conclusions

- The measurements related to the production of top-quark pairs play a crucial role at the Tevatron and at the LHC: desirable to reach NNLO accuracy in theoretical predictions
- Real radiation matrix elements calculated by several groups

Summary & Conclusions

- The measurements related to the production of top-quark pairs play a crucial role at the Tevatron and at the LHC: desirable to reach NNLO accuracy in theoretical predictions
- Real radiation matrix elements calculated by several groups
- Projects aiming to the calculation of all of the two-loop matrix elements are well under way:
  - $q\bar{q} \to t\bar{t}$ Full numerical result available
  - $q\bar{q} \to t\bar{t}$ Fermion loop diagrams known analytically
  - $q\bar{q} \to t\bar{t}$ Leading color coefficient known analytically
  - $gg \to t\bar{t}$ All IR poles known analytically
  - $gg \to t\bar{t}$ Leading color coefficient known analytically **NEW!**
  - $gg \to t\bar{t}$ Calculation of light quark diagrams in progress

Summary & Conclusions

- The measurements related to the production of top-quark pairs play a crucial role at the Tevatron and at the LHC: desirable to reach NNLO accuracy in theoretical predictions
- Real radiation matrix elements calculated by several groups
- Projects aiming to the calculation of all of the two-loop matrix elements are well under way:
  - $q\bar{q} \to t\bar{t}$ Full numerical result available
  - $q\bar{q} \to t\bar{t}$ Fermion loop diagrams known analytically
  - $q\bar{q} \to t\bar{t}$ Leading color coefficient known analytically
  - $gg \to t\bar{t}$ All IR poles known analytically
  - $gg \to t\bar{t}$ Leading color coefficient known analytically **NEW!**
  - $gg \to t\bar{t}$ Calculation of light quark diagrams in progress

Still a lot of work to do
but a full NNLO calculation is possible

Backup Slides

HPLs and two-dimensional HPLs

In the non physical region ($s < 0$) one needs 7 weight functions for the HPLs of argument x

$$f_w(x) = \frac{1}{x - w}, \quad \text{with} \quad w \in \left\{0, 1, -1, -y, -\frac{1}{y}, \frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right\}$$

and 6 weight functions for HPLs of argument y

$$f_w(y) = \frac{1}{y - w}, \quad \text{with} \quad w \in \left\{0, 1, -1, -x, -\frac{1}{x}, 1 - \frac{1}{x} - x\right\}$$

Weight one HPLs are defined as

$$G(0; x) = \ln x\,, \qquad G(w; x) = \int_0^x dt\, f_w(t)$$

Higher weights are defined through iterated integrations

$$G(w, \cdots; x) = \int_0^x dt\, f_w(t)\, G(\cdots; t) \qquad \left(G(\underbrace{0, \cdots, 0}_{n}; x) = \frac{1}{n!} \ln^n x \right)$$

IR Poles in QCD Amplitudes

IR poles in QCD amplitudes can be removed by a multiplicative renormalization

Becher and Neubert ('09)

$$\mathbf{Z}^{-1}(\epsilon,\{p\},\{m\})\,|\mathcal{M}_n(\epsilon,\{p\},\{m\})\rangle_{\alpha_s^{QCD}\to\xi\alpha_s} = \text{FINITE}$$

IR Poles in QCD Amplitudes

IR poles in QCD amplitudes can be removed by a multiplicative renormalization

Becher and Neubert ('09)

$$\mathbf{Z}^{-1}(\epsilon, \{p\}, \{m\}) \mathcal{M}_n(\epsilon, \{p\}, \{m\})|_{\alpha_s^{QCD} \to \xi \alpha_s} = \text{FINITE}$$

$$\mathbf{Z}^{-1} = \mathbf{1} - \alpha_s \mathbf{Z}_{(1)} + \alpha_s^2 (\mathbf{Z}_{(1)}^2 - \mathbf{Z}_{(2)}) + \mathcal{O}(\alpha_s^3)$$

$$\mathcal{M} = \alpha_s \mathcal{M}^{(0)} + \alpha_s^2 \mathcal{M}^{(1)} + \alpha_s^3 \mathcal{M}^{(2)} + \mathcal{O}(\alpha_s^4)$$

IR Poles in QCD Amplitudes

IR poles in QCD amplitudes can be removed by a multiplicative renormalization

Becher and Neubert ('09)

$$\mathbf{Z}^{-1}(\epsilon, \{p\}, \{m\}) \, |\mathcal{M}_n(\epsilon, \{p\}, \{m\})\rangle_{\alpha_s^{QCD} \to \xi \alpha_s} = \text{FINITE}$$

therefore

$$\begin{aligned} |\mathcal{M}_n^{(1),\,\mathrm{sing}}\rangle &= \mathbf{Z}^{(1)} \, |\mathcal{M}_n^{(0)}\rangle \\ |\mathcal{M}_n^{(2),\,\mathrm{sing}}\rangle &= \left[\mathbf{Z}^{(2)} - \left(\mathbf{Z}^{(1)}\right)^2\right] |\mathcal{M}_n^{(0)}\rangle + \left(\mathbf{Z}^{(1)} \, |\mathcal{M}_n^{(1)}\rangle\right)_{\mathrm{poles}} \end{aligned}$$

But what is **Z**?

Evolution Matrix

Z satisfies the evolution equation

$$\mathbf{Z}^{-1}(\epsilon, \{\underline{p}\}, \{\underline{m}\}, \mu) \frac{d}{d \ln \mu} \mathbf{Z}(\epsilon, \{\underline{p}\}, \{\underline{m}\}, \mu) = -\boldsymbol{\Gamma}(\{\underline{p}\}, \{\underline{m}\}, \mu)$$

where, in the color space formalism

$$\begin{aligned}
\boldsymbol{\Gamma} \;=\;& \sum_{(i,j)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) \\
&- \sum_{(I,J)} \frac{\mathbf{T}_I \cdot \mathbf{T}_J}{2} \gamma_{\text{cusp}}(\beta_{IJ}, \alpha_s) + \sum_I \gamma^I(\alpha_s) + \sum_{I,j} \mathbf{T}_I \cdot \mathbf{T}_j \gamma_{\text{cusp}}(\alpha_s) \ln \frac{m_I \mu}{-s_{Ij}} \\
&+ \sum_{(I,J,K)} if^{abc}\, \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_K^c F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI}) \\
&+ \sum_{(I,J)} \sum_k if^{abc}\, \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_k^c f_2\Big(\beta_{IJ}, \ln \frac{-\sigma_{Jk}\, v_J \cdot p_k}{-\sigma_{Ik}\, v_I \cdot p_k}\Big) + \mathcal{O}(\alpha_s^3)
\end{aligned}$$

Becher and Neubert ('09)

Evolution Matrix

$\mathbf{Z}$ satisfies the evolution equation

$$\mathbf{Z}^{-1}(\epsilon, \{\underline{p}\}, \{\underline{m}\}, \mu) \frac{d}{d \ln \mu} \mathbf{Z}(\epsilon, \{\underline{p}\}, \{\underline{m}\}, \mu) = -\mathbf{\Gamma}(\{\underline{p}\}, \{\underline{m}\}, \mu)$$

where, in the color space formalism

$$\mathbf{\Gamma} = \sum_{(i,j)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{\ldots} \ldots \frac{\mu^2}{\ldots} + \sum_i \ldots$$

$$- \sum_{(I,J)} \ldots \sum_{I} \ldots \sum_{I,j} \ldots \alpha_s) \ln \frac{m_I \mu}{-s_{Ij}}$$

$$+ \sum_{(I,J,K)} if^{abc}\, \mathbf{T}_I^a\, \mathbf{T}_J^b\, \mathbf{T}_K^c\, F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI})$$

$$+ \sum_{(I,J)} \sum_k if^{abc}\, \mathbf{T}_I^a\, \mathbf{T}_J^b\, \mathbf{T}_k^c\, f_2\left(\beta_{IJ}, \ln \frac{-\sigma_{Jk}\, v_J \cdot p_k}{-\sigma_{Ik}\, v_I \cdot p_k}\right) + \mathcal{O}(\alpha_s^3)$$

$\mathbf{T}_i$ ($\mathbf{T}_I$) are color generators associated to massless (massive) particles

Becher and Neubert ('09)

Evolution Matrix

Z satisfies the evolution equation

$$\mathbf{Z}^{-1}(\epsilon,\{\underline{p}\},\{\underline{m}\},\mu)\,\frac{d}{d\ln\mu}\,\mathbf{Z}(\epsilon,\{\underline{p}\},\{\underline{m}\},\mu) = -\boldsymbol{\Gamma}(\{\underline{p}\},\{\underline{m}\},\mu)$$

where, in the c

Staring at two-loop order one finds
three particle correlators
The explicit expression for the coefficient functions F_1 and f_2 was recently obtained
AF, Neubert, Pecjak, Yang ('09)

$$\boldsymbol{\Gamma} = \sum_{(i,j)} \cdots$$

$$- \sum_{(I,J)} \cdots \alpha_s) \ln \frac{m_I \mu}{-s_{Ij}}$$

$$+ \sum_{(I,J,K)} i f^{abc}\, \mathbf{T}_I^a\, \mathbf{T}_J^b\, \mathbf{T}_K^c\, F_1(\beta_{IJ},\beta_{JK},\beta_{KI})$$

$$+ \sum_{(I,J)} \sum_k i f^{abc}\, \mathbf{T}_I^a\, \mathbf{T}_J^b\, \mathbf{T}_k^c\, f_2\left(\beta_{IJ}, \ln \frac{-\sigma_{Jk}\, v_J \cdot p_k}{-\sigma_{Ik}\, v_I \cdot p_k}\right) + \mathcal{O}(\alpha_s^3)$$

Becher and Neubert ('09)