NLO $t\bar{t} + 1$ jet production merged with shower in POWHEG

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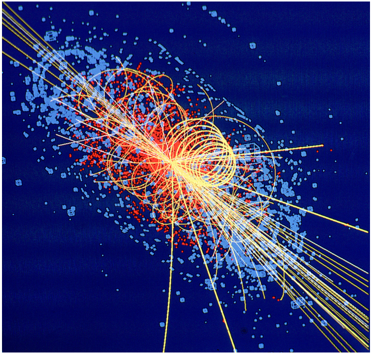
Workshop on Heavy Particles at the LHC

ETH - Zurich

5 January 2011

OUTLINE



- The POWHEG method and the POWHEG-BOX
- $t\bar{t} + 1jet$ production in the POWHEG-BOX
- Conclusions & future outlooks

INTRODUCTION

SMC (LO+SHOWER)

- ✗ LO accuracy. Large dependence on $\mu_{\rm R}$ and $\mu_{\rm F}$
- ✗ Extra emissions accurate only in soft/collinear approx.
- ✓ Sudakov suppression of soft/collinear emissions
- ✓ Realistic events in the output

NLO

- ✓ Accuracy up to a further order in $\alpha_{\rm S}$
- ✓ Reduced dependence on $\mu_{\rm R}$ and $\mu_{\rm F}$
- ✗ Parton level output only. Low final-state multiplicity.
- ✗ Numerical instability due to large cancellations

Try to merge benefits (and avoid drawbacks) of both approaches!

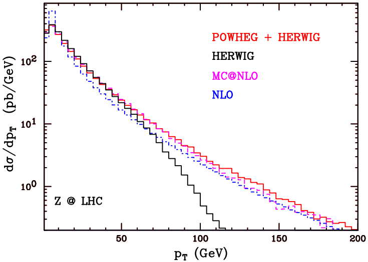
- ✗ A K factor = $\frac{\sigma_{NLO}}{\sigma_{LO}}$ correction may improve inclusive quantities
- ✗ Matrix element corrections added to obtain better shape predictions (*e.g.* CKKW, MLM)
  - ⇒ Only add further real contributions (maintaining LO normalization)
  - ⇒ A matching prescription to avoid double-counting of radiation must be defined
  - ⇒ Large uncertainty under scale variations due to the lack of virtual corrections
    $\alpha_{\rm S}^n(f\mu) \approx \alpha_{\rm S}^n(\mu)(1 - b_0\alpha_{\rm S}(\mu)\log(f^2))^n \approx \alpha_{\rm S}^n(\mu)(1 \pm n\alpha_{\rm S}(\mu))$

INTRODUCTION

✓ Use full NLO calculation as "hard subprocess" for the SMC ⇒ NLO+PS

Many ideas to avoid double-counting, but two general method perform this merging for hadronic collisions fully tested

- MC@NLO [Frixione & Webber,JHEP 0206:029,2002]
- POWHEG [Nason,JHEP 0411:040,2004] [Frixione, Nason & Oleari, JHEP 0711:070,2007]



MC@NLO can now also be interfaced with PYTHIA and HERWIG++ showers.
POWHEG method adopted also in HERWIG++ and SHERPA programs.

✓ Merging of NLO+PS with ME corrections. NLO accuracy can be reached reweighting ME+PS by a Φ_B-dependent $K-$factor. [Nason& Hamilton, arXiv:1004.1764]
Unaffordable evaluation! Approximate solution MENLOPS tested for W and $t\bar{t}$. Similar approach also implemented in SHERPA [Hoche,Krauss,Schonherr&Siegert,arXiv:1009.1127]

NLO AND SMC FORMULAS

- NLO calculation (subtraction method): $d\Phi_{n+1} = d\Phi_n \, d\Phi_{\rm rad} \qquad d\Phi_{\rm rad} \div dt\, dz\, \frac{d\varphi}{2\pi}$

$$d\sigma_{\tt NLO} = \left\{ B(\Phi_n) + V(\Phi_n) + \Big[\underbrace{\overbrace{R(\Phi_n,\Phi_{\rm rad})}^{divergent} - \overbrace{C(\Phi_n,\Phi_{\rm rad})}^{divergent}}_{finite} \Big] d\Phi_{\rm rad} \right\} d\Phi_n$$

$$\overbrace{\int d\sigma_{\tt NLO}\, d\Phi_{\rm rad} = \bar{B}(\Phi_n)}^{\text{Inclusive NLO cross section at fixed underlying Born}} \;, \qquad V(\Phi_n) = \underbrace{\overbrace{V_b(\Phi_n)}^{divergent} + \overbrace{\int C(\Phi_n,\Phi_{\rm rad})\, d\Phi_{\rm rad}}^{divergent}}_{finite}$$

- Standard SMC's first emission:

$$d\sigma_{\tt SMC} = \overbrace{B(\Phi_n)}^{Born} d\Phi_n \left\{ \Delta_{\tt SMC}(t_0) + \Delta_{\tt SMC}(t) \overbrace{\frac{\alpha_S(t)}{2\pi}\frac{1}{t}P(z)}^{\lim_{k_T\to 0} R(\boldsymbol{\Phi}_{n+1})/B(\boldsymbol{\Phi}_n)} d\Phi_{\rm rad}^{\tt SMC} \right\}$$

$$\Delta_{\tt SMC}(t) = \underbrace{\exp\left[- \int d\Phi'_{\rm rad} \frac{\alpha_S(t')}{2\pi}\frac{1}{t'} P(z')\,\theta(t'-t) \right]}_{\tt SMC\; Sudakov}$$

$$d\sigma_{\tt POWHEG} = \bar{B}(\Phi_n)\, d\Phi_n \left\{ \Delta_{\tt POWHEG}(\Phi_n, p_{\rm T}^{\rm min}) + \Delta_{\tt POWHEG}(\Phi_n, k_{\rm T}) \frac{R(\Phi_n, \Phi_{\rm rad})}{B(\Phi_n)}\, \theta\left(k_{\rm T} - p_{\rm T}\right)\, d\Phi_{\rm rad} \right\}$$

- ✓ NLO cross section for inclusive quantities.

- ✓ $\bar{B} = B(\Phi_n) + V(\Phi_n) + \int \left[R(\Phi_n, \Phi_{\rm rad}) - C(\Phi_n, \Phi_{\rm rad})\right]\, d\Phi_{\rm rad} < 0$

  Negative weights where NLO $>$ LO, i.e. where perturbation expansion breaks down!

- ✓ Probability of not emitting with transverse momentum harder than $p_{\rm T}$:

$$\Delta_{\tt POWHEG}(\Phi_n, p_{\rm T}) = \overbrace{\exp\left[- \int d\Phi'_{\rm rad} \frac{R(\Phi_n, \Phi'_{\rm rad})}{B(\Phi_n)} \theta\left(k_{\rm T}(\Phi_n, \Phi'_{\rm rad}) - p_{\rm T}\right) \right]}^{\tt POWHEG\ \mathit{Sudakov}}$$

  It has the same LL accuracy of a SMC since for small $k_{\rm T}$'s

$$\frac{R(\Phi_n, \Phi_{\rm rad})}{B(\Phi_n)} d\Phi_{\rm rad} \approx \frac{\alpha_{\rm S}(t)}{2\pi} \frac{1}{t}\, P(z)\, dt\, dz\, \frac{d\varphi}{2\pi} \qquad \text{and} \qquad \bar{B} \approx B\left(1 + \mathcal{O}(\alpha_{\rm S})\right)$$

- ✓ The large $k_{\rm T}$'s accuracy is preserved since $\Delta_{\tt POWHEG}(\Phi_n, p_{\rm T}) \approx 1$ and

$$d\sigma_{\tt POWHEG} \approx \frac{\bar{B}(\Phi_n)}{B(\Phi_n)}\, R(\Phi_n, \Phi_{\rm rad})\, d\Phi_n\, d\Phi_{\rm rad} \approx R(\Phi_n, \Phi_{\rm rad})\left(1 + \mathcal{O}(\alpha_{\rm S})\right)\, d\Phi_n\, d\Phi_{\rm rad}$$

THE POWHEG-BOX [S.A., NASON, OLEARI, RE, JHEP 1006:043,2010]

- Framework for the implementation of a POWHEG generator for a generic NLO process
- Practical implementation of the theoretical construction of the POWHEG general formulation presented in [Frixione, Nason, Oleari, JHEP 0711:070,2007]
- FKS subtraction approach automatically implemented, hiding all technicalities to the user
- Publicly available code at the webpage

http://powhegbox.mib.infn.it

The user should only communicate to the POWHEG-BOX :

- The list of flavour of Borns and Reals, *e.g.* `[5,2,23,6,3,0]` for $b\,u \rightarrow Z\,t\;s\,g$
- The Born phase space `Born_phsp(xborn)` for `xborn(1...ndims)` randoms
- The inizialization of the couplings `init_couplings` and the setting of the scales `set_fac_ren_scales(muf,mur)`
- `setborn(p(0:3,1:nlegborn),bflav(1:nlegborn),born,bjk,bmunu)` The Born squared amplitudes $\mathcal{B} = |\mathcal{M}|^2$, the color-ordered Born squared amplitudes $\mathcal{B}_{jk}$ and the helicity correlated Born squared amplitudes $\mathcal{B}_{k,\mu\nu}$
- `setreal(p(0:3,1:nlegreal),rflav(1:nlegreal),amp2)` The Real squared amplitudes $\mathcal{R}$
- `setvirtual(p(0:3,1:nlegborn),vflav(1:legborn),virtual)` The finite part of the interference of Born and virtual amplitude contributions $\mathcal{V}_{\rm b} = 2{\rm Re}\{\mathcal{B}\times\mathcal{V}\}$, after factorizing out $\mathcal{N} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\frac{\mu_R^2}{Q^2}\right)^\epsilon$

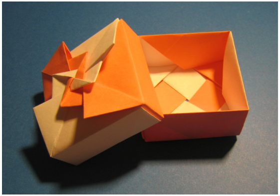
Common ingredients of any NLO calculation in a subtraction method

The POWHEG−BOX program automatically does

- ✓ The combinatorics and the projection of real contributions over the singular regions
- ✓ The counterterms, built up from soft and collinear approximations of real emissions, and the ISR and FSR phase space.
- ✓ The NLO differential cross section. BYPRODUCT: NLO distributions in FKS subtraction
- ✓ The calculation of upper bounds for an efficient generation of Sudakov-suppressed events
- ✓ The generation of hardest radiation, according to the POWHEG Sudakov
- ✓ The communication with a SMC program, either passing the generated events on-the-fly or storing them on a LesHouches events file.

Available processes

- Single vector-boson production with decay
- Vector boson plus one jet production with decay NEW
- Single-top production in the $s-$, $t-$ and $Wt-$ channel
- Higgs boson production in gluon and vector boson fusion
- Jet pair production NEW
- Heavy-quark pair production



$t\bar{t} + 1jet$ PRODUCTION

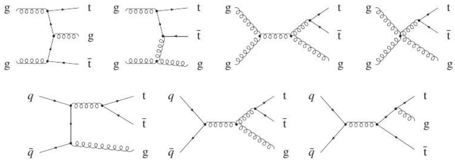
MOTIVATIONS:

- Top quark physics allows to study the EWSB mechanism, due to the larger mass
- Large fraction of inclusive $t\bar{t}$ events contains additional jet(s). Increasing relative importance of the $t\bar{t} + jet(s)$ sample at the LHC with respect to the TeVatron.
- Dominant background to Higgs production in VBF, for configurations that avoid the large rapidity gap between jets veto. Also important background for many SUSY signals.
- Fully exclusive NLO calculations have been performed: Dittmaier,Uwer and Weinzierl [Phys.Rev.Lett.98:262002,2007] for stable top-quarks and Melnikov and Shulze [Nucl.Phys.B840:129-159,2010] with unitarity methods and LO top-quark decay correlations included.
- Up to $t\bar{t} + 2$ jets at NLO in HELAC-NLO

STRATEGY AND IMPROVEMENTS:

- Initial goal is to have a fully exclusive NLO calculations, merged with shower, including spin correlations for top decays products at the LO.
- Extension to include the factorizable NLO corrections to the top-decay
- Inclusion of $b\bar{b} + 1$ jet and $c\bar{c} + 1$ jet.

Outline of the calculation – Born and Reals

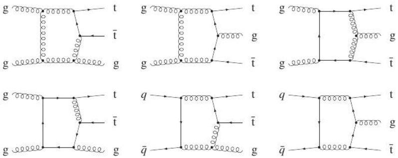


- Born amplitudes evaluated with `MadGraph` and compared with Mangano,Nason, Ridolfi routines for real emission in $Q\bar{Q}$ production [Nucl.Phys.B373:295-345,1992].
- Color-correlated Born amplitudes obtained from `MadGraph` routines, modifying `MadGraph` calls to include appropriate color insertion operators (Eikonal Approx.).
- Helicity-correlated Born amplitudes obtained from `MadGraph` routines, modifying `MadGraph` calls to keep track of amplitudes values for different helicities and making use of the completeness relation

$$\mathcal{B}_j^{\mu\nu} = N \sum_{\{i\},s_j,s_j'} \mathcal{M}\left(\{i\},s_j\right) \mathcal{M}^\dagger\left(\{i\},s_j'\right) \left(\epsilon^\mu_{s_j}\right)^* \epsilon^\nu_{s_j'}$$

- Real emissions matrix elements obtained with `MadGraph`, factorization in soft and collinear limits explicitly checked in double and quadruple precision.
- All results also checked against Dittmaier-Uwer-Weinzierl paper for single phase space point values published on [Phys.Rev.Lett.98:262002,2007]. Accuracy up to ten digits.

OUTLINE OF THE CALCULATION – VIRTUALS



- Library of virtual amplitudes built out of Dittmaier-Uwer-Weinzierl code.
- Decompositon according to helicity and color structure times scalar functions depending on the external momenta only.
- Amplitudes evaluated analytically, then further manipulated with computer algebra programs and translated in C++ code
- Reduction of up to 4 points tensor integrals performed with Passarino-Veltman reduction
- 5-points tensor integrals reduced *à la* Denner-Dittmaier, avoiding inverse Gram determinants
- Scalar integrals evaluated with FF package.
- Light flavours renormalized in the $\overline{\mathrm{MS}}$ scheme, top quark loop in gluon self-energy subtracted at zero momentum. On-shell scheme for top mass renormalization.
- C++ interface returns the finite part of virtual amplitudes, as required by the `POWHEG-BOX`. Soft-virtuals automatically constructed by the `POWHEG-BOX` using color-correlated Born amplitudes. Collinear remnants (finite leftover of factorization of initial-state collinear singularities in $\overline{\mathrm{MS}}$ scheme) also automated.

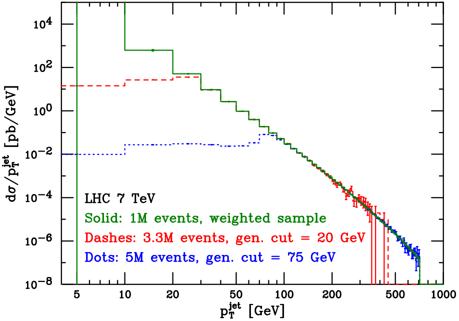
THE POWHEG−BOX: $t\bar{t} + 1jet$ PRODUCTION

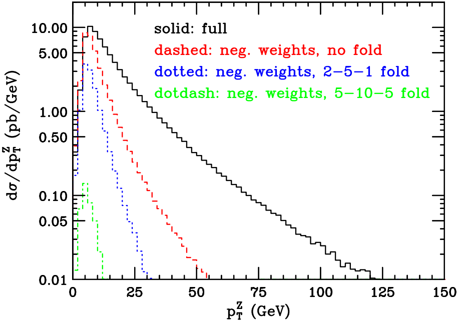
- Non trivial process definition because Born contributions are IR divergent. Need to introduce a process-defining cutoff.
- In a NLO computation is sufficient to ask that the observable $\mathcal{O}_n$ is infrared safe and that $\mathcal{O}_{n+1}$ vanish fast enough if two singular region are approached at the same time.
- POWHEG generates the Born process first, then it attaches radiation. Need to introduce a process-defining cutoff, but still not possible to generate an unweighted set of underlying Born configurations covering the whole phase space. Same problem as in $V + 1\ jet$ and Di-jets productions.
- Using an analysis cut larger than the process defining cut is not enough because the shower can raise or lower the jet and recoiling momenta ($p_{\rm T}^{V}$ or $p_{\rm T}^{t\bar{t}}$) independently
- Two possible solutions implemented in POWHEG−BOX:
  - Use a generation cut much smaller than the analysis cut and consider its variations to asses the independence of results. Then combine different samples to get full phase space coverage, avoiding overlaps.
  - Generate weighted events, suppressing the divergence

$$\bar{B}_{\rm supp} = \bar{B} \times F(p_{\rm T}), \qquad F(p_{\rm T}) = \left(\frac{p_{\rm T}^2}{p_{\rm T}^2 + {p_{\rm T,\,supp}}^2} \right)^n$$

$n = 1$ for $t\bar{t} + 1$ jet or $V + 1$ jet, $n = 3$ for Dijets. Event weight $= F^{-1}$.

Generation cut and negative-weighted events





- Negative values of $\tilde{B}(\Phi_B, X) = B(\Phi_B) + V(\Phi_B) + \left|\frac{\partial \Phi_{\rm rad}}{\partial X}\right| [R(\Phi_B, \Phi_{\rm rad}) - C(\Phi_B, \Phi_{\rm rad})]$ are expected in extreme regions of the phase-space. Only after integration over $d\Phi_{\rm rad}$ negative weights should disappear.
- Folding the radiative phase-space reduces the occurrence of negative weights, *e.g.*

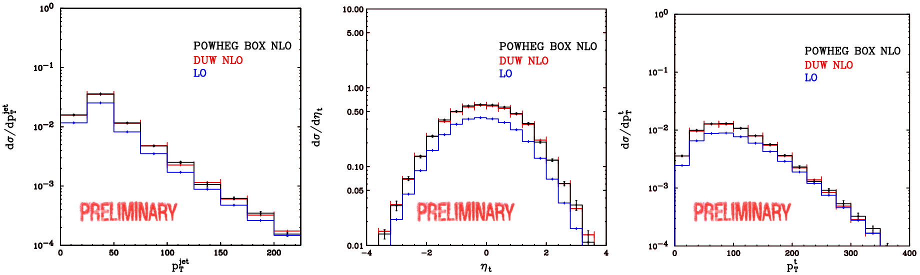
$$\tilde{B}_{\rm folded}(\Phi_B, x_1, X_2, X_3) = \tilde{B}(\Phi_B, x_1, X_2, X_3) + \tilde{B}(\Phi_B, 1/2 + x_1, X_2, X_3)$$

- Fully analogous to the negative weights in the $\mathcal{S}$ events in MC@NLO, but negative weights in the $\mathcal{H}$ event sample of MC@NLO cannot be reduced (due to shower approx. subtraction).
- Using signed events, weighted events or positive-weights only does not change the final results. Performance costs for obtaining positive-weighted events may be balanced if the analysys includes detector simulations or requires positive weights only.

Fixed order comparisons

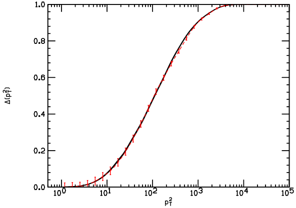
- Non trivial check due to the different subtraction methods:
  - (Massive) Dipole subtraction in Dittmaier-Uwer-Weinzierl
  - FKS subtraction, extendend to deal with soft emissions out of massive colored particles, in `POWHEG-BOX`

  Regularized (subtracted) reals and virtuals are different, but results independent from method chosen.
- Fixed renormalization and factorization scales at $m_t = 174$ GeV, CTEQ6M pdf .
- Inclusive$-k_{\rm T}$ jet algo (Collins-Soper) $k_{\rm T} > 20$ GeV, $R = 1$ with E_t recombination scheme via FastJet. Comparisons for TeVatron $\sqrt{S} = 1.96\ \mathrm{TeV}$
- Top quarks always tagged, excluded from jet reconstruction

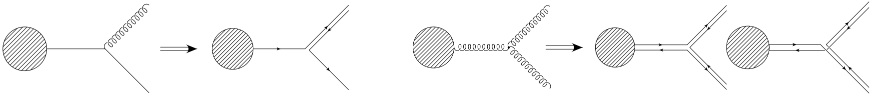


Generation of radiation and colour assignments

- Hardest radiation generated according the exact differential of Sudakov FF with veto method. Checked against independent integration of single emission probability ⇒



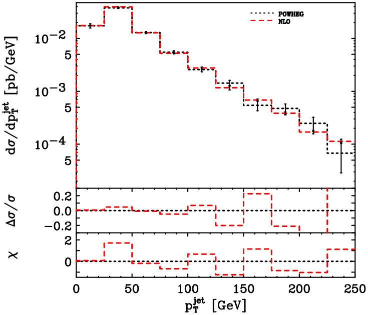
- To obtain the large$-N_c$ NLL accuracy of the Sudakov form factor the color connections in the large N_c limits must be specified for events with 4 or more colored partons at LO.
- These are assigned evaluating, at fixed underlying Born kinematics, all the planar contributions to the amplitude and choosing between them according to the respective weight. For $t\bar{t} + 1jet$, the large N_c amplitudes have been evaluated analytically.

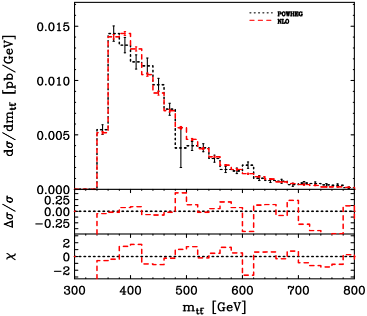


- If radiation is generated, the color connections are fully reconstructed assuming that the emitted parton is color connected to the emitter.

POWHEG FIRST EMISSION

PRELIMINARY

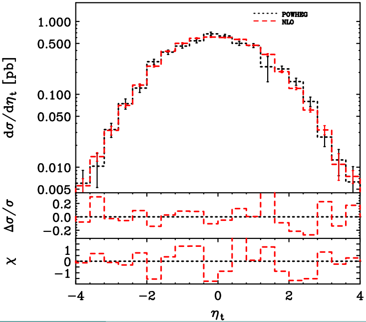




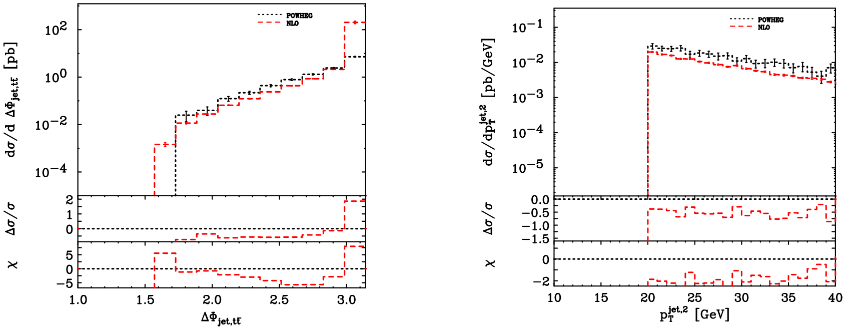
- For inclusive quantities, NLO and POWHEG hardest emission only coincide up to NNLO contributions.
- Fractional difference and difference over stat. error defined as

$$\frac{\Delta\sigma}{\sigma} = \frac{\sigma_1 - \sigma_2}{(\sigma_1+\sigma_2)/2}, \qquad \chi = \frac{\sigma_1 - \sigma_2}{\sqrt{\delta\sigma_1^2 + \delta\sigma_2^2}}$$

- Results for TeVatron, same analysis as NLO



PRELIMINARY

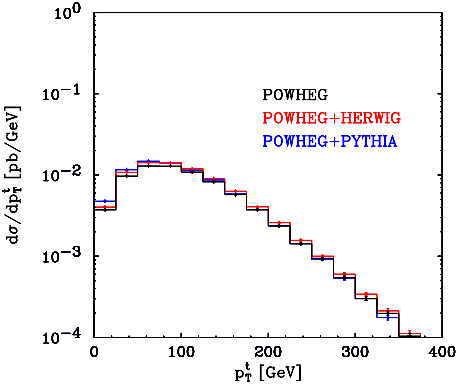


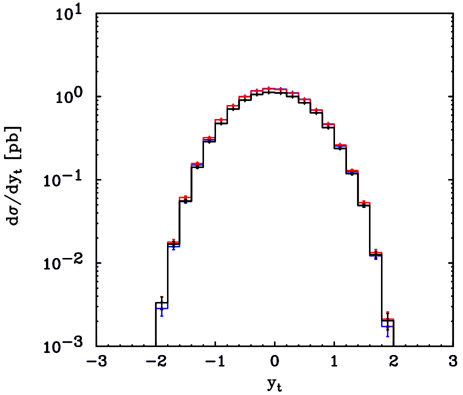
- For more exclusive quantities, more marked differences
- Resummation of soft/coll. logs partly included in the POWHEG formula
- Still "unphysical" distributions, only hardest emission is present

Matching with shower

PRELIMINARY

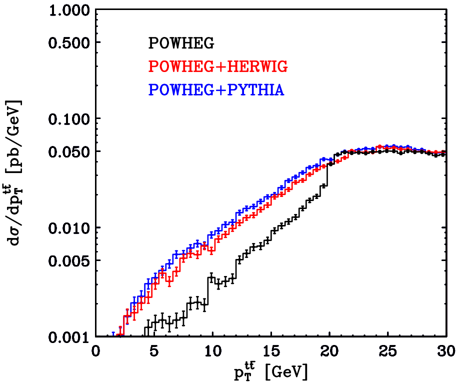
- Merging with `HERWIG` and `PYTHIA` showers
- Top quarks momenta reconstruted according to shower history
- Small differences introduced by showering for inclusive quantities

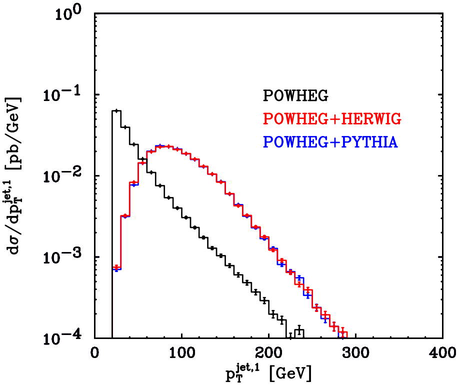




MATCHING WITH SHOWER

- Effects of the shower clearly visible in most observables.
- Dependence on jet cuts changes the normalization
- More realistic final states, kinematic constraints avoided





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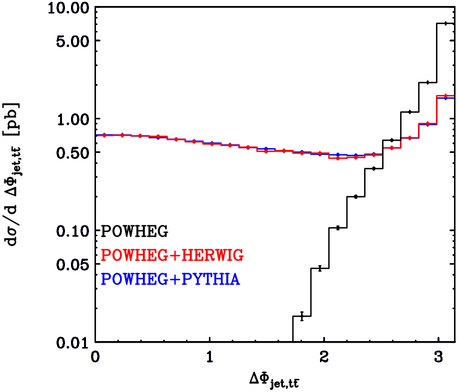

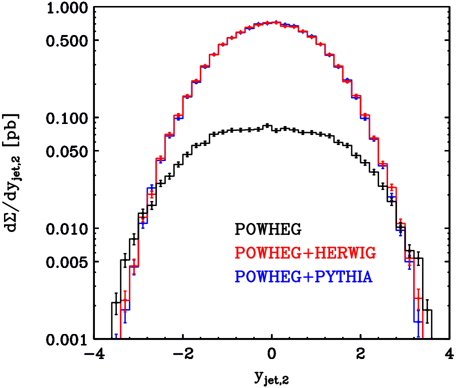

Matching with shower

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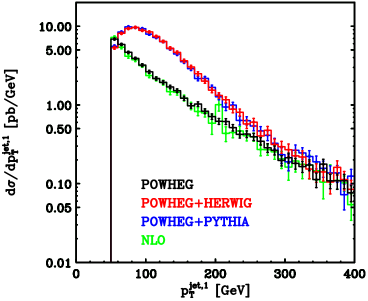
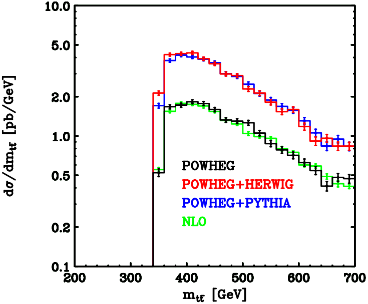
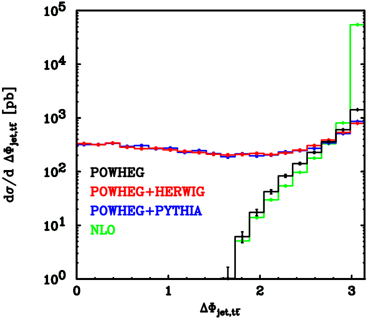




- Results for LHC show more marked differences after shower also for inclusive observables
- Different jet cut: $p_T^{\rm jet} > 50\ {\rm GeV}$
- Similar behaviour for exclusive observables sensible to shower effects



Conclusions & Outlooks

So far:

- The POWHEG-BOX proved to be a useful tool to match NLO calculations to SMC programs
- $t\bar{t} + 1\mathrm{jet}$ process implemented quite straightforwardly despite its complexity: virtuals available as external library
- NLO results in accord with Dittmaier,Uwer&Weinzierl despite different subtraction method
- Generation problems solved thanks to $V + 1\mathrm{jet}$ and Di-jet studies.

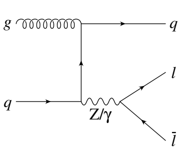
Ongoing work and prospective studies:

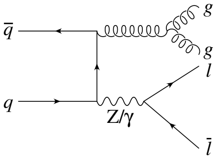
- More extensive validations and comparisons with available NLO and resummed results.
- Inclusion of top-quark correlations in decays, similarly to [Frixione,Nason and Ridolfi,JHEP 0709]
- Extension to deal with $b\bar{b} + 1\mathrm{jet}$ and $c\bar{c} + 1\mathrm{jet}$
- Phenomenological studies at the TeVatron and at the LHC (e.g. charge asimmetry, scales and PDFs dependence)
- Merging $t\bar{t}$ and $t\bar{t} + 1\mathrm{jet}$ samples

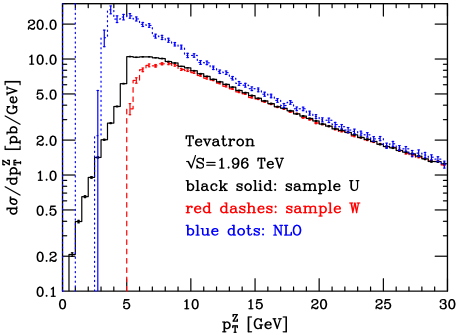
Thank you for your attention!

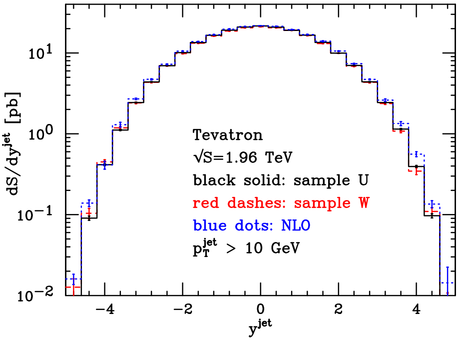
EXTRA SLIDES

VECTOR BOSON PLUS JET PRODUCTION AND DECAY

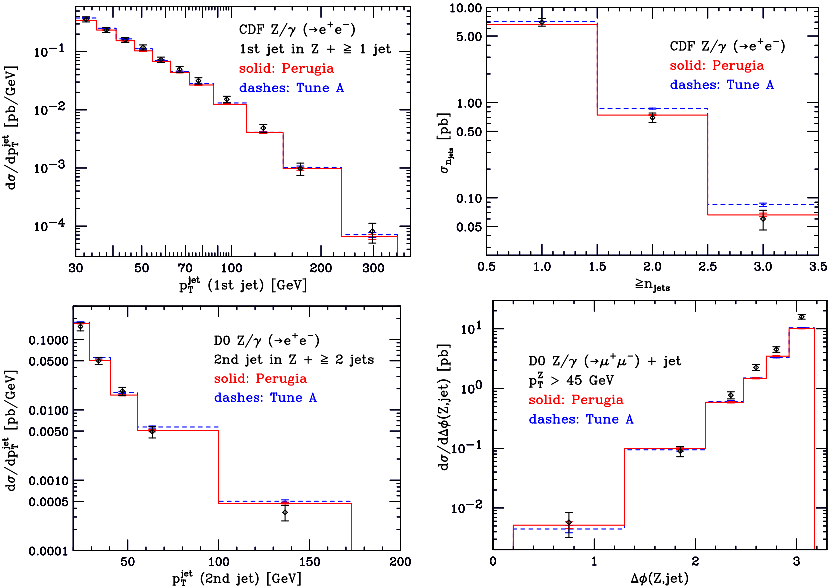




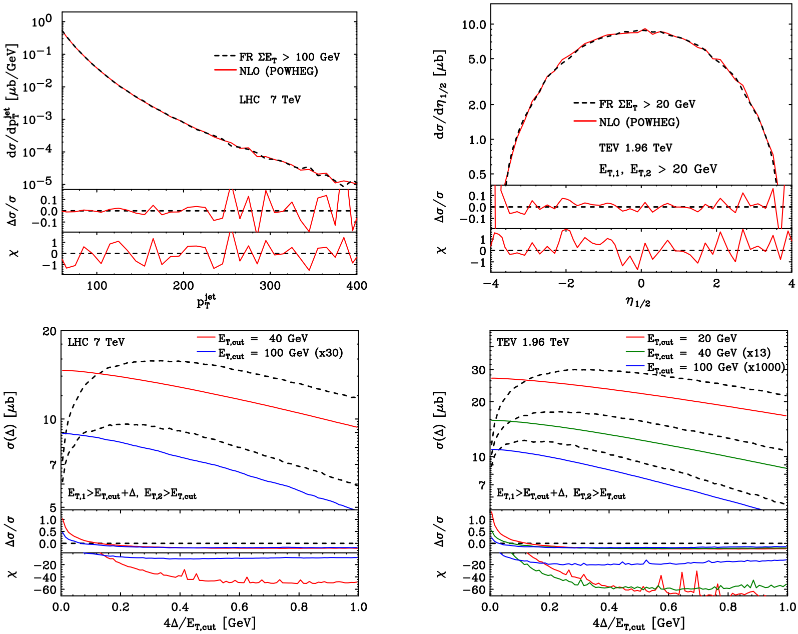




Comparison with $Z + 1j$ TeVatron data

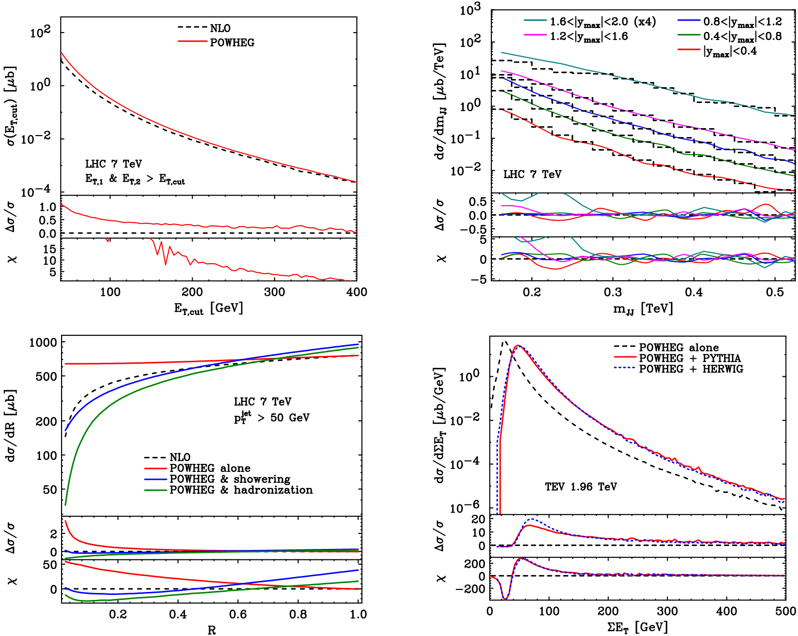


DIJETS



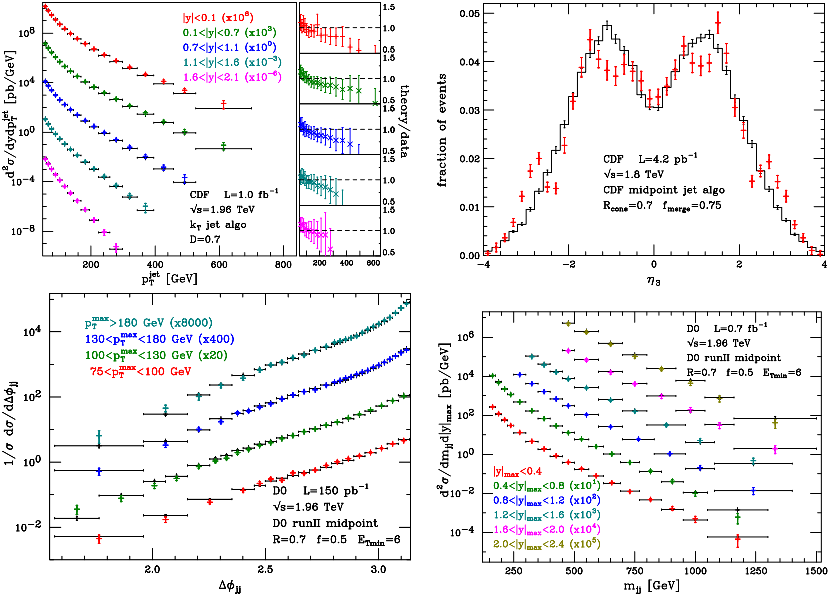
FR ΣE_T > 100 GeV
NLO (POWHEG)
LHC 7 TeV
$d\sigma/dp_T^{jet}$ [μb/GeV]
$\Delta\sigma/\sigma$
χ
p_T^{jet}
FR ΣE_T > 20 GeV
NLO (POWHEG)
TEV 1.96 TeV
$E_{T,1}$, $E_{T,2}$ > 20 GeV
$d\sigma/d\eta_{1/2}$ [μb]
$\eta_{1/2}$
LHC 7 TeV
$E_{T,cut}$ = 40 GeV
$E_{T,cut}$ = 100 GeV (x30)
$E_{T,1}$>$E_{T,cut}$+Δ, $E_{T,2}$>$E_{T,cut}$
$\sigma(\Delta)$ [μb]
$4\Delta/E_{T,cut}$ [GeV]
TEV 1.96 TeV
$E_{T,cut}$ = 20 GeV
$E_{T,cut}$ = 40 GeV (x13)
$E_{T,cut}$ = 100 GeV (x1000)
$E_{T,1}$>$E_{T,cut}$+Δ, $E_{T,2}$>$E_{T,cut}$
$\sigma(\Delta)$ [μb]
$4\Delta/E_{T,cut}$ [GeV]

DIJETS

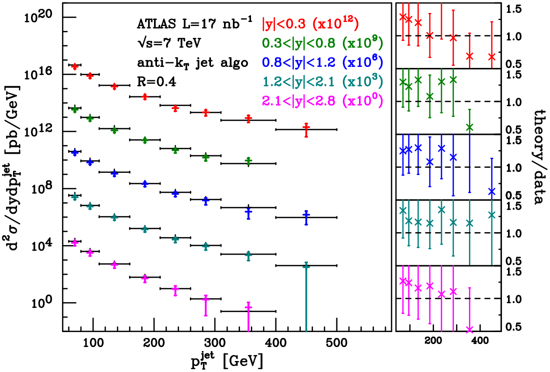


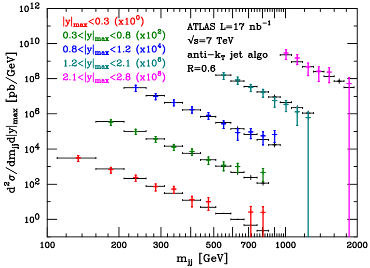
NLO
POWHEG
LHC 7 TeV
$E_{T,1}$ & $E_{T,2}$ > $E_{T,cut}$
$\sigma(E_{T,cut})$ [μb]
$\Delta\sigma/\sigma$
χ
$E_{T,cut}$ [GeV]
1.6<|y_{max}|<2.0 (x4)
1.2<|y_{max}|<1.6
0.8<|y_{max}|<1.2
0.4<|y_{max}|<0.8
|y_{max}|<0.4
LHC 7 TeV
$d\sigma/dm_{JJ}$ [μb/TeV]
m_{JJ} [TeV]
LHC 7 TeV
p_T^{jet} > 50 GeV
NLO
POWHEG alone
POWHEG & showering
POWHEG & hadronization
$d\sigma/dR$ [μb]
R
POWHEG alone
POWHEG + PYTHIA
POWHEG + HERWIG
TEV 1.96 TeV
$d\sigma/d\Sigma E_T$ [μb/GeV]
ΣE_T [GeV]

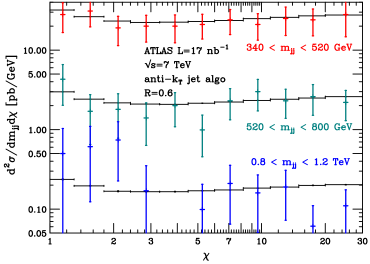
COMPARISON WITH TEVATRON DIJET DATA



Comparison with ATLAS Dijet data







NLO ACCURACY OF POWHEG FORMULA (1)

- Use the POWHEG formula

$$d\sigma = \bar{B}(\Phi_n)\, d\Phi_n \left\{ \Delta(\Phi_n, p_{\mathrm{T}}^{\min}) + \Delta(\Phi_n, k_{\mathrm{T}}) \frac{R(\Phi_{n+1})}{B(\Phi_n)}\, \theta(k_{\mathrm{T}} - p_{\mathrm{T}}^{\min})\, d\Phi_{\mathrm{rad}} \right\}$$

- to calculate the expectation value of a generic observable $< \mathcal{O} > =$

$$= \int \bar{B}(\Phi_n)\, d\Phi_n \left\{ \Delta(\Phi_n, p_{\mathrm{T}}^{\min}) O_n(\Phi_n) + \int_{p_{\mathrm{T}}^{\min}} \Delta(\Phi_n, k_{\mathrm{T}}) \frac{R(\Phi_{n+1})}{B(\Phi_n)} O_{n+1}(\Phi_{n+1})\, d\Phi_{\mathrm{rad}} \right\}$$

$$= \int \bar{B}(\Phi_n)\, d\Phi_n \left\{ \left[\Delta(\Phi_n, p_{\mathrm{T}}^{\min}) + \int_{p_{\mathrm{T}}^{\min}} \Delta(\Phi_n, k_{\mathrm{T}}) \frac{R(\Phi_{n+1})}{B(\Phi_n)}\, d\Phi_{\mathrm{rad}} \right] O_n(\Phi_n) \right.$$

$$\left. + \int_{p_{\mathrm{T}}^{\min}} \Delta(\Phi_n, k_{\mathrm{T}}) \frac{R(\Phi_{n+1})}{B(\Phi_n)} \left[O_{n+1}(\Phi_{n+1}) - O_n(\Phi_n) \right]\, d\Phi_{\mathrm{rad}} \right\}$$

- O_n, O_{n+1} are the actual forms of $\mathcal{O}$ in the $n, n+1$-body phase space.
- $\mathcal{O}$ is required to be infrared-safe and to vanish fast enough when two singular regions are approached at the same time

NLO ACCURACY OF POWHEG FORMULA (2)

- Now observe that

$$\int_{p_{\rm T}^{\rm min}} d\Phi_{\rm rad} \frac{R(\Phi_{n+1})}{B(\Phi_n)} \Delta(\Phi_n, k_{\rm T}) \;=\; \int_{p_{\rm T}^{\rm min}}^{\infty} dp'_{\rm T} \int d\Phi_{\rm rad}\, \delta(k_{\rm T} - p'_{\rm T}) \frac{R(\Phi_{n+1})}{B(\Phi_n)} \Delta(\Phi_n, p'_{\rm T})$$

$$= - \int_{p_{\rm T}^{\rm min}}^{\infty} dp'_{\rm T} \Delta(\Phi_n, p'_{\rm T}) \frac{d}{dp'_{\rm T}} \int_{p_{\rm T}^{\rm min}} d\Phi_{\rm rad}\, \theta(k_{\rm T} - p'_{\rm T}) \frac{R(\Phi_{n+1})}{B(\Phi_n)}$$

$$= \int_{p_{\rm T}^{\rm min}}^{\infty} dp'_{\rm T} \frac{d}{dp'_{\rm T}} \Delta(\Phi_n, p'_{\rm T}) \;=\; 1 - \Delta(\Phi_n, p_{\rm T}^{\rm min})$$

- Furthermore we can replace $\bar{B}(\Phi_n) \approx B(\Phi_n)\,(1 + \mathcal{O}(\alpha_{\rm S}))$
- and also $\Delta(\Phi_n, k_{\rm T}) \approx 1 + \mathcal{O}(\alpha_{\rm S})$ since $[O_{n+1} - O_n] \to 0$ at small $k_{\rm T}$'s
- The final result is (up to $p_{\rm T}^{\rm min}$ power-suppressed terms)

$$\begin{aligned} <\mathcal{O}> \;&=\; \int d\Phi_n \bar{B}(\Phi_n)\, 1\, O_n(\Phi_n) \\ &+\; \int 1 \frac{R(\Phi_{n+1})}{1} \left[O_{n+1}(\Phi_{n+1}) - O_n(\Phi_n)\right]\, d\Phi_{\rm rad} + \mathcal{O}(\alpha_{\rm S}) \end{aligned}$$

$$d\sigma_{\texttt{MC@NLO}} = \overbrace{\bar{B}_{\texttt{SMC}}(\Phi_n)}^{\texttt{MC@NLO}}\, d\Phi_n \overbrace{\left\{\Delta_{\texttt{SMC}}(t_0) + \Delta_{\texttt{SMC}}(t)\frac{R_{\texttt{SMC}}(\Phi_n,\Phi_{\rm rad}^{\texttt{SMC}})}{B(\Phi_n)}\, d\Phi_{\rm rad}^{\texttt{SMC}}\right\}}^{\texttt{SMC}}$$

$$+ \underbrace{\left[R(\Phi_n,\Phi_{\rm rad}^{\texttt{SMC}}) - R_{\texttt{SMC}}(\Phi_n,\Phi_{\rm rad}^{\texttt{SMC}})\right]}_{\texttt{MC@NLO}} d\Phi_n\, d\Phi_{\rm rad}^{\texttt{SMC}}$$

$$\bar{B}_{\texttt{SMC}}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int \left[R_{\texttt{SMC}}(\Phi_n,\Phi_{\rm rad}^{\texttt{SMC}}) - C(\Phi_n,\Phi_{\rm rad}^{\texttt{SMC}})\right]\, d\Phi_{\rm rad}^{\texttt{SMC}}$$

$$\Delta_{\texttt{SMC}}(t) = \exp\left[-\int d\Phi'_{\rm rad}\frac{R_{\texttt{SMC}}(\Phi_n,\Phi'_{\rm rad})}{B(\Phi_n)}\theta\left(t'-t\right)\right] \quad \Leftarrow \texttt{HERWIG}\text{ or }\texttt{PYTHIA}\text{ Sudakov!}$$

- ✓ NLO accuracy for IR safe observables
- ✓ Exclusive observables are described no worse than in usual (N)LL SMC's
- ✗ Dependence of PS algorithm. Need to express NLO calulation in $\Phi_{\rm rad}^{\texttt{SMC}}$ variables
- ✗ $R - R_{\texttt{SMC}}$ not singular only if $R_{\texttt{SMC}}$ reproduces exactly all the singularities of R. Issue: azimuthal dependence of collinear sing. usually neglected in $R_{\texttt{SMC}}$.
- ✗ $\bar{B}_{\texttt{SMC}}$ may be negative ! Negative weighted events

MC@NLO DIP IN HARDEST RADIATION

$$\Delta_{\mathtt{HW}}(t) = \exp\left[-\int d\Phi'_{\rm rad}\frac{R_{\mathtt{HW}}(\Phi_n,\Phi'_{\rm rad})}{B(\Phi_n)}\theta\left(t'-t\right)\right] \Leftarrow \text{HERWIG Sudakov!}$$

$$d\sigma_{\mathtt{MC@NLO}} = \bar{B}_{\mathtt{HW}}(\Phi_n)\, d\Phi_n \left\{\Delta_{\mathtt{HW}}(t_0) + \Delta_{\mathtt{HW}}(t)\frac{R_{\mathtt{HW}}(\Phi_n,\Phi_{\rm rad})}{B(\Phi_n)}\, d\Phi_{\rm rad}\right\} + \left[R(\Phi_n,\Phi_{\rm rad}) - R_{\mathtt{HW}}(\Phi_n,\Phi_{\rm rad})\right]\, d\Phi_n\, d\Phi_{\rm rad}$$

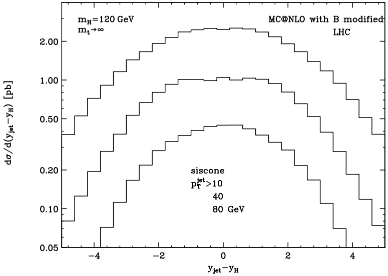
$$\bar{B}_{\mathtt{HW}}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int \left[R_{\mathtt{HW}}(\Phi_n,\Phi_{\rm rad}) - C(\Phi_n,\Phi_{\rm rad})\right]\, d\Phi_{\rm rad}$$

At high $p_{\rm T}$ the cross section goes as

$$d\sigma_{\mathtt{MC@NLO}} \approx \left(\frac{\bar{B}_{\mathtt{HW}}(\Phi_n)}{B(\Phi_n)} - 1\right) R_{\mathtt{HW}}(\Phi_n,\Phi_{\rm rad}) d\Phi_n d\Phi_{\rm rad} + R(\Phi_n,\Phi_{\rm rad}) d\Phi_n d\Phi_{\rm rad}$$

Test : Replace $\bar{B}_{\mathtt{HW}}(\Phi_n)$ with $B(\Phi_n)$ in generation of $\mathbb{S}$-type events

The dip seems to disappear



NLL ACCURACY OF THE POWHEG SUDAKOV FORM FACTOR

Substitute $\alpha_S \to A\left(\alpha_S\left(k_T^2\right)\right)$ in the Sudakov exponent, with

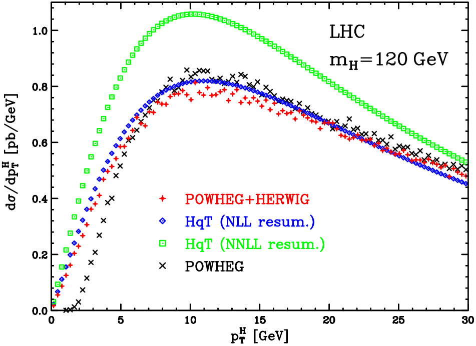
$$A(\alpha_S) = \alpha_S \left\{1 + \frac{\alpha_S}{2\pi}\left[\left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} n_f\right]\right\}$$

and one-loop expression for α_S, to get NLL resummed results for process with up to 3 coloured partons at the Born level [Catani,Marchesini and Webber Nucl.Phys.B349]

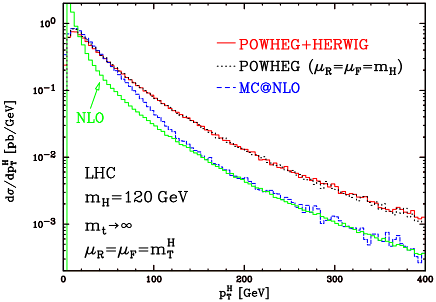
For > 3 coloured partons, soft NLL contributions exponentiates only in a matrix sense

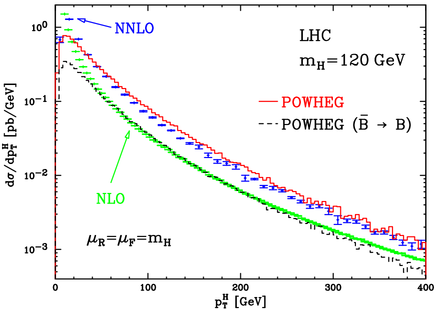
- Need to diagonalize the colour structures
- Always possible to take the large N_c limit and get NLL

Comparison with HqT program [Bozzi,Catani,de Florian and Grazzini, Nucl.Phys.B737] $\Rightarrow$



HIGH$-p_T$ BEHAVIOUR





$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int [R(\Phi_n,\Phi_{\rm rad}) - C(\Phi_n,\Phi_{\rm rad})]\, d\Phi_{\rm rad}$$

$$d\sigma = \bar{B}(\Phi_n)\, d\Phi_n \left\{ \Delta(\Phi_n, p_T^{\rm min}) + \Delta(\Phi_n, p_T)\frac{R(\Phi_n,\Phi_{\rm rad})}{B(\Phi_n)}\, d\Phi_{\rm rad} \right\}$$

if $p_T >> 1 \Rightarrow \Delta(\Phi_n, p_T) \approx 1$ and

$$d\sigma_{\rm rad} \approx \frac{\bar{B}(\Phi_n)}{B(\Phi_n)} R(\Phi_n,\Phi_{\rm rad})\, d\Phi_n\, d\Phi_{\rm rad} \approx \underbrace{\{1+O(\alpha_S)\}}_{\approx 2 \text{ for } gg\to H} R(\Phi_n,\Phi_{\rm rad})\, d\Phi_n\, d\Phi_{\rm rad}$$

Better agreement with NNLO results, but still enough flexibility to get rid of this feature!

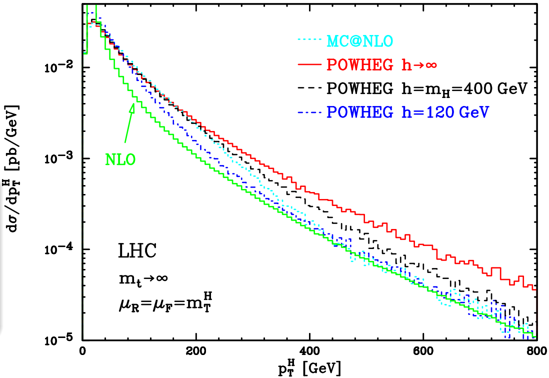
Reduction of real contribution entering the Sudakov FF

$$R = \underbrace{R \times F}_{singular} + \underbrace{R \times (1-F)}_{regular}$$
$$= R_{\bar{B}} + R_{\rm reg}$$

$F < 1, \qquad F \to 1$ when $p_{\rm T} \to 0,$

$F \to 0$ when $p_{\rm T} \to \infty$

$$\Rightarrow \; F = \frac{h^2}{p_T^2 + h^2}$$



$$\sigma = \sigma_{\bar{B}} + \sigma_{\rm reg}$$

$$\sigma_{\bar{B}} = \int d\Phi_n \Big\{ B(\Phi_n) + V(\Phi_n) + \Big[\, R_{\bar{B}}(\Phi_n, \Phi_{\rm rad}) - C(\Phi_n, \Phi_{\rm rad}) \,\Big]\, d\Phi_{\rm rad} \Big\}$$

$$\sigma_{\rm reg} = \int R_{\rm reg}(\Phi_n, \Phi_{\rm rad})\, d\Phi_n\, d\Phi_{\rm rad}$$