



Workshop on Heavy Particles at the LHC

Resonant Single Top Production
at Hadron Colliders

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IN COLLABORATION WITH

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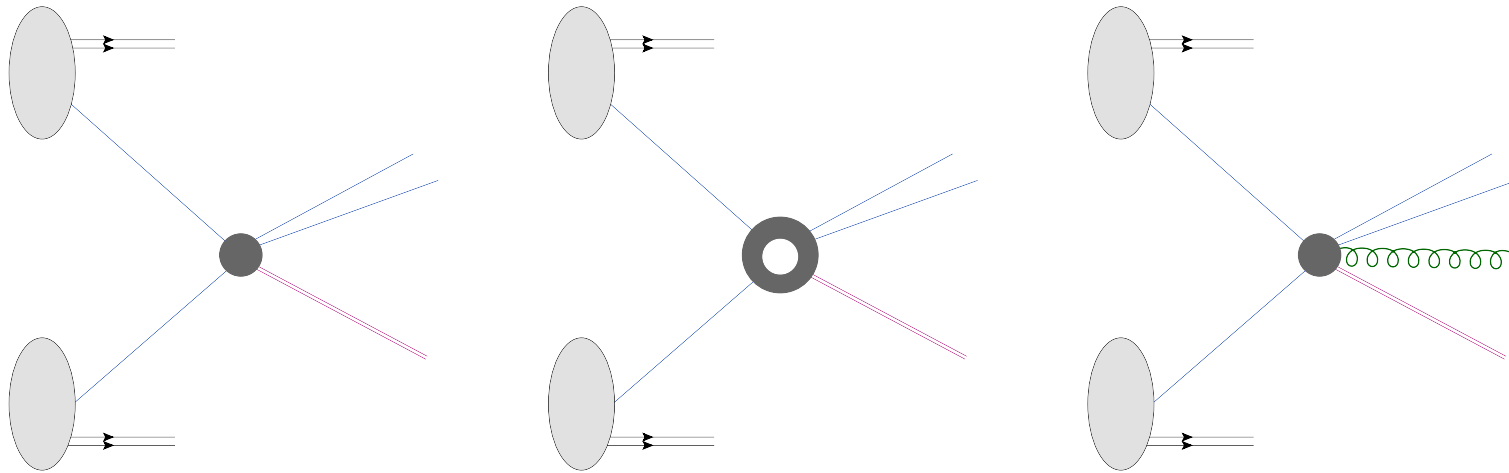
5.-7. JANUARY 2011, ZURICH, SWITZERLAND



- introduction
 - background
 - non-factorizable corrections
- method
 - effective theory approach
 - virtual corrections
 - real corrections
- application
 - overview single-top 't' and 's'-channel
 - tree level, virtual corrections, real corrections
- results
 - 't'-channel LHC
 - 's'-channel LHC
 - 't' and 's'-channel Tevatron
- conclusions
 - outlook



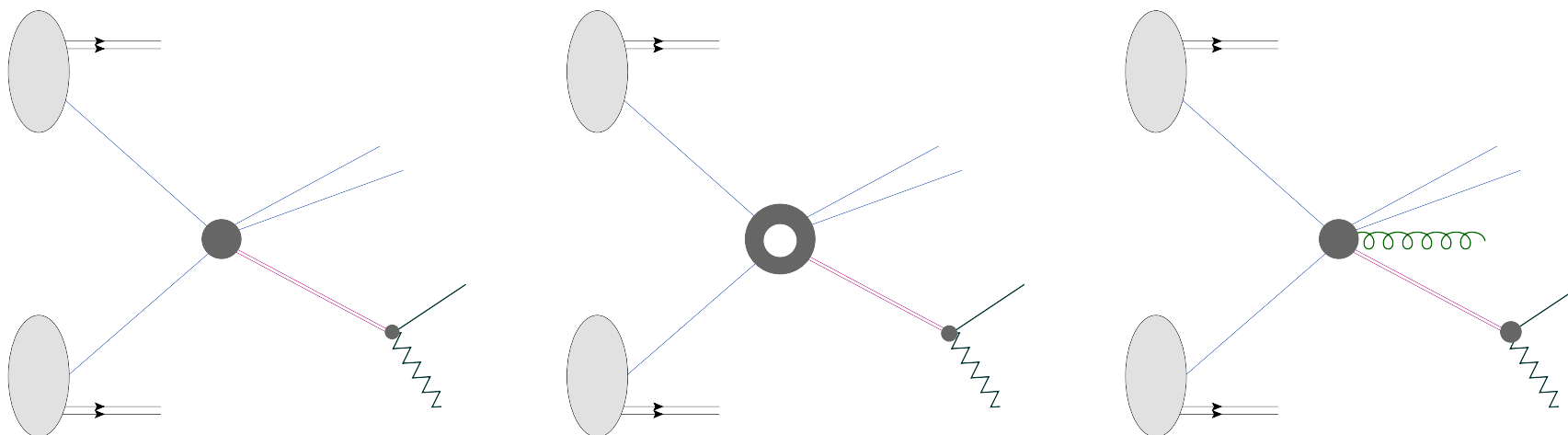
Production of an **on-shell heavy** (unstable) particle X : $p_X^2 = m_X^2$



- often this is a reasonable approximation **but**
- cuts on decay products not possible
- off-shell effects of X not taken into account



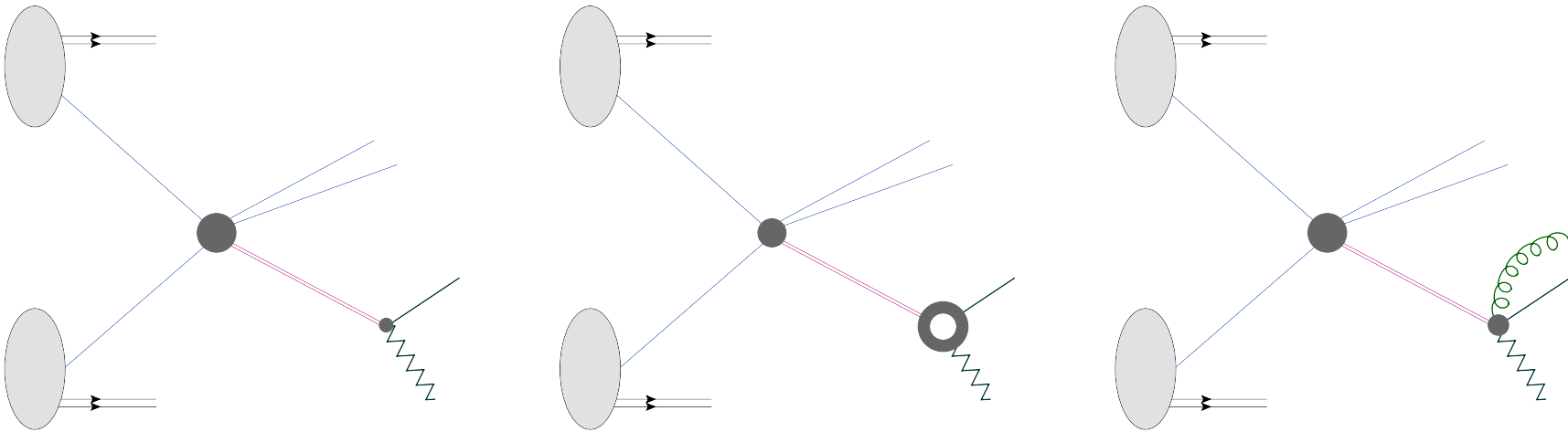
Production of an **on-shell heavy** (unstable) particle X , including decay: $p_X^2 = m_X^2$



- (improved) narrow width approximation, $M_{\text{decay}}^2 = m_X^2$
- NLO correction of production and decay included
- cuts on decay products possible but off-shell effects of X not taken into account



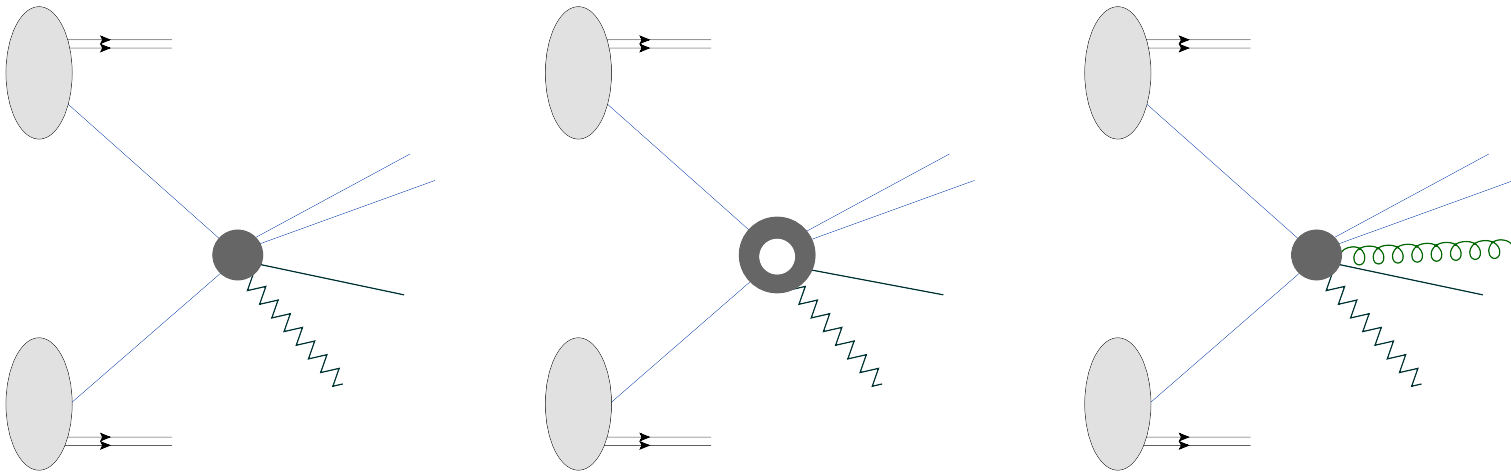
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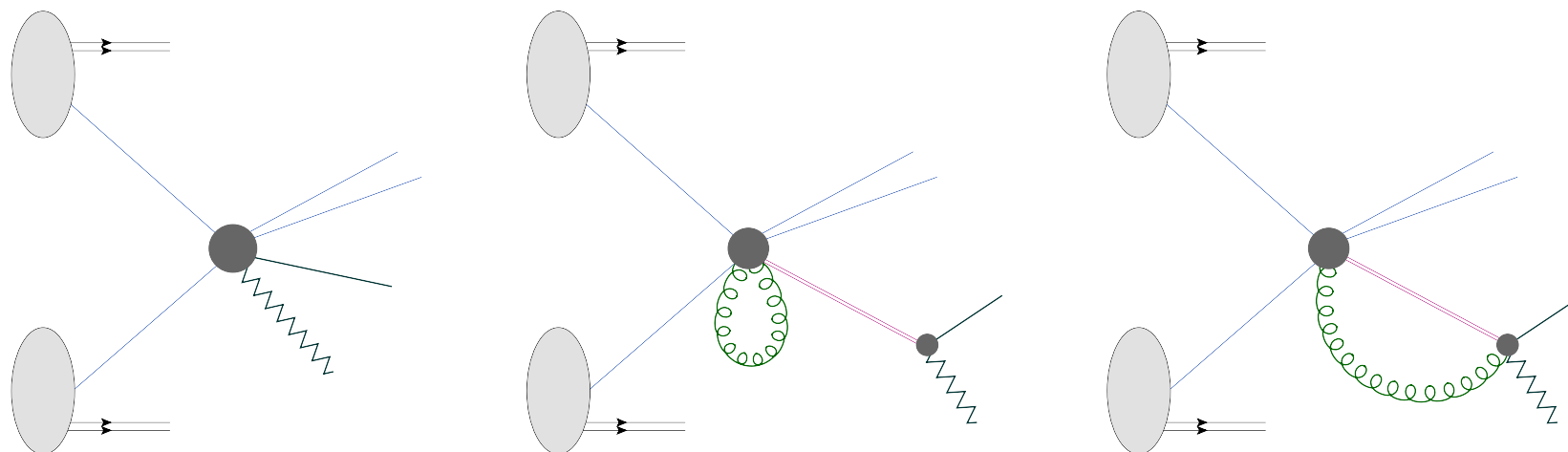
Production of an **off-shell heavy** (unstable) particle X , including decay: $p_X^2 \neq m_X^2$



- tree-level background diagrams (no particle X , but same final state)
- virtual and real background diagrams
- valid for any p_X^2 , (off-shell effects taken into account) but calculation complicated



Production of a **resonant heavy** (unstable) particle X , including decay: $p_X^2 \sim m_X^2$



- tree-level background diagrams (no particle X , but same final state)
- use pole approximation (at one loop)
 - factorizable corrections
 - non-factorizable corrections
 } gauge invariant separation
- real background diagrams
- off-shell effects of X are taken into account, calculation simplified



non-factorizable corrections have been extensively studied [Fadin et.al; Melnikov et.al; Beenakker et.al; Denner et.al.; Jadach et.al; . . .] but are usually neglected at hadron colliders, because:

- they seem to be more difficult to compute (not really)
- they are generally small [Beenakker et.al; Pittau]
 - resonant \rightarrow non-resonant propagator unless $E \lesssim \Gamma$ is small (soft)
 - cancellations for “inclusive” observables [Fadin, Khoze, Martin]

purpose of this work:

- do not neglect non-factorizable corrections
- try to obtain an efficient way to identify and compute minimal amount required

in this talk I will not consider many other (sometimes related) issues such as

- (soft) connection of unstable particle to beam remnant
- issues related to using pole mass for unstable particle $\delta m_t \simeq \Lambda_{\text{QCD}}$??



- small scale $(p_X^2 - m_X^2)/m_X^2 \sim \delta \ll 1 \rightarrow$ effective theory (ET) approach
- expand in all small parameters α and $(p_X^2 - m_X^2)/m_X^2 \rightarrow$ power counting:

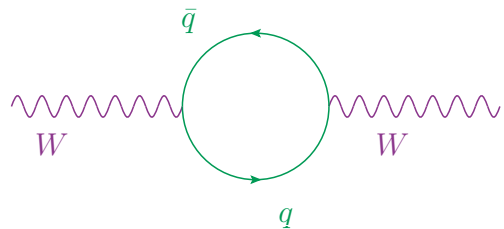
$$\alpha \sim \frac{p_X^2 - m_X^2}{m_X^2} \equiv \frac{\Delta}{m_X^2} \sim \frac{\Gamma_X}{m_X} \sim \delta \ll 1$$

- integrate out hard modes: $\mathcal{L}(\phi, A, \psi) \rightarrow \mathcal{L}_{\text{eff}}(c_i, \phi_s, A_s, A_c, \psi_s, \psi_c)$
UPET is nothing but a mix between SCET and H'Q'ET
- virtual corrections and total cross section
 - expand integrand, method of regions [Beneke, Smirnov]
 - new identification [Chapovsky, Khoze, AS, Stirling]
factorizable corrections = hard corrections (ET)
non-factorizable corrections = soft corrections (ET)
 - applications for total cross section:
 $e^+e^- \rightarrow t\bar{t}$ near threshold [Hoang et.al; Beneke et.al; Melnikov et.al; ...]
 $e^+e^- \rightarrow W^+W^-$ near threshold [Beneke et.al.]
- arbitrary real corrections problematic (new scales from definition of observable)
- follow fixed-order approach, but expand $d\sigma^{\text{real}}$ to match IR singularities of $d\sigma^{\text{virt}}$



use method of regions [Beneke, Smirnov] and expand integrand (in principle to any order):

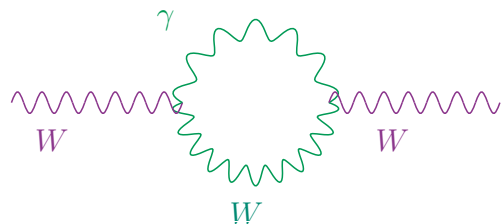
- hard corrections $\ell \sim m_X$ (= factorizable corrections)
- soft corrections $\ell \sim m_X \delta$ (= non-factorizable corrections)



$$\int \frac{d^d \ell}{(p + \ell)^2 \ell^2}$$

hard: full

$$\text{soft: } \int \frac{d^d \ell (2p \cdot \ell)}{p^2 \ell^2} = 0$$



$$\int \frac{d^d \ell}{(\ell^2 + 2p \cdot \ell + \Delta) \ell^2}$$

$$\text{hard: } \int \frac{d^d \ell}{\ell^2 (\ell^2 + 2p \cdot \ell)} \neq 0$$

$$\text{soft: } \int \frac{d^d \ell}{\ell^2 (2p \cdot \ell + \Delta)} \neq 0$$

- leads to resummation of **hard part** (= leading part in Δ) of self-energy insertions

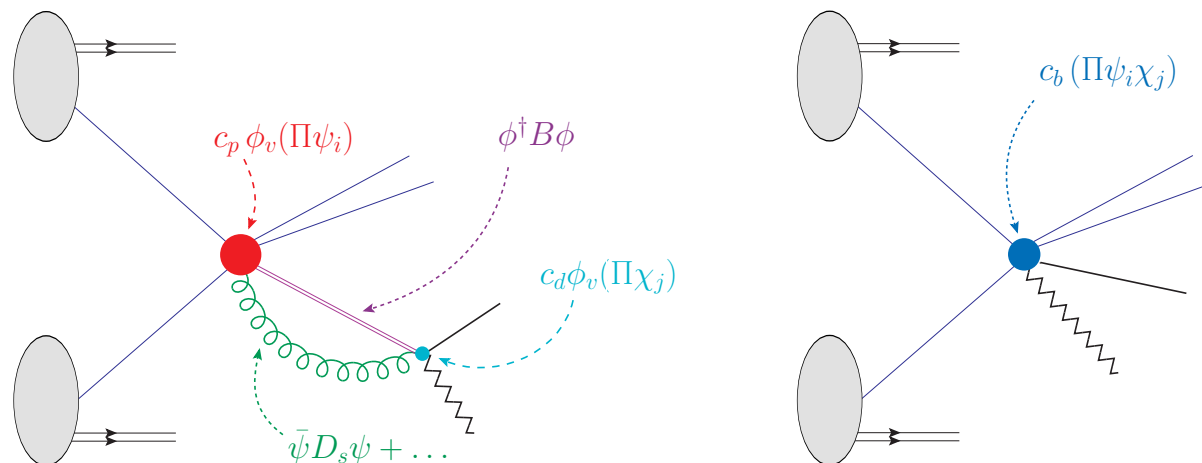
$$\mathcal{L}_{\text{eff}} = 2m_X \phi_s^\dagger \left(i v \cdot \partial - \frac{c_{\phi\phi}}{2} \right) \phi_s + \dots$$

- matching coefficients are gauge invariant ($c_{\phi\phi} = -i\Gamma$ in pole scheme)
full result is gauge invariant at each order in δ , but gauge invariance is **not** an input



integrate out hard modes \rightarrow effective Lagrangian

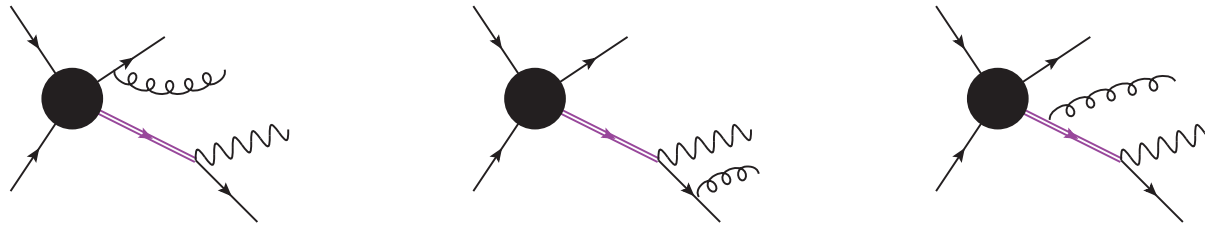
$$\mathcal{L} = \phi^\dagger B \phi + c_p \phi(\Pi\psi_i) + c_d \phi(\Pi\chi_j) + c_b (\Pi\psi_i\chi_j) + \bar{\psi} D_s \psi + \dots$$



- matching coefficients c_i contain effects of hard modes
- matching done on shell, $p_X^2 = \bar{s} = m_X^2 + \mathcal{O}(\Delta)$, with \bar{s} the complex position of pole
- soft (and collinear . . .) d.o.f. still dynamical
- can be combined with further resummations (e.g. non-relativistic \rightarrow ET has more complicated structure)
- direct link to anomalous couplings via \mathcal{L}_{eff}



- for arbitrary observable it is not clear what expansion parameter is



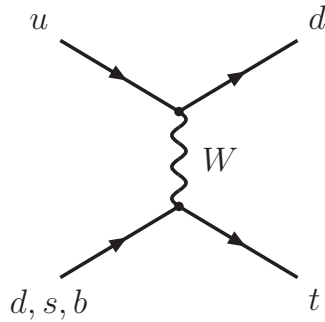
- observable can introduce new scales \rightarrow change in structure of ET
- take full real matrix element and apply (say) subtraction method

$$\begin{aligned} \int d\Phi_{n+1} |M_{n+1}|^2 &= \int d\Phi_{n+1} \left(|M_{n+1}|^2 - |M_{n(+1)}^{\text{sing}}|^2 \right) + \int d\Phi_{n+1} |M_{n(+1)}^{\text{sing}}|^2 \\ &\simeq \int d\Phi_{n+1} \left(|M_{n+1}|^2 - |M_{n(+1)}^{\text{sing}}|^2 \right) + \int d\Phi_{n+1} |M_{n(+1)}^{\text{sing exp}}|^2 \end{aligned}$$

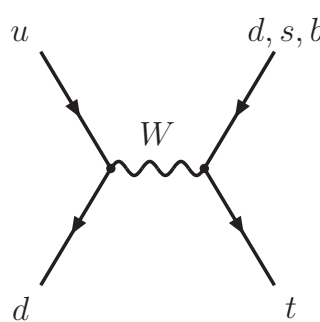
- $\int d\Phi_{n+1} |M_{n(+1)}^{\text{sing}}|^2$ matches singularity structure of full virtual correction $\int d\Phi_n |M_n^{\text{v}}|^2$
- $\int d\Phi_{n+1} |M_{n(+1)}^{\text{sing exp}}|^2$ matches singularity structure of virtual term $\int d\Phi_n |M_n^{\text{v exp}}|^2$
- we subtract something and add back something different, but difference is higher order in δ
- expansion only required for n parton kinematics



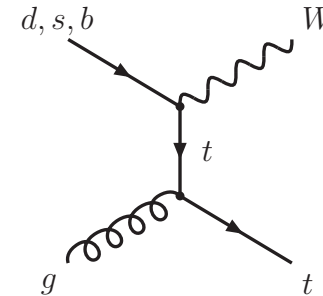
single top: t-channel



s-channel



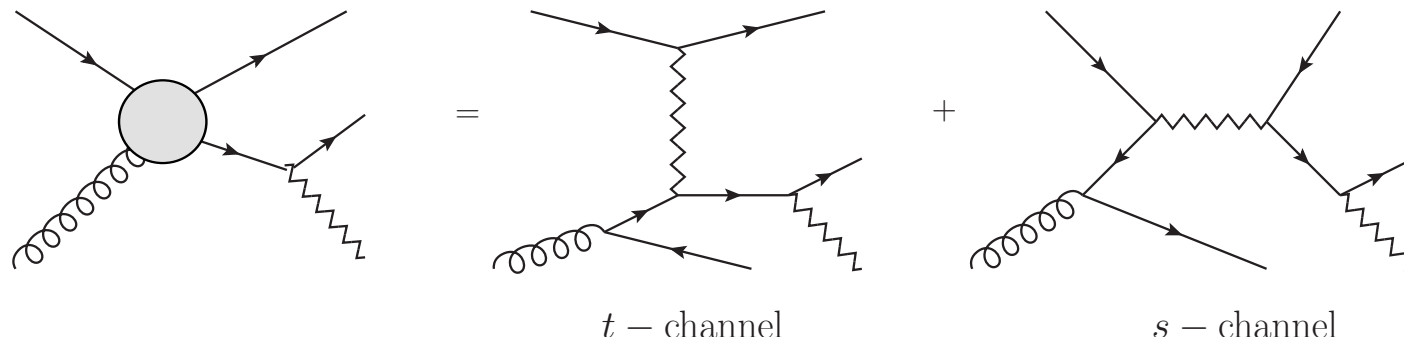
($W t$ not considered here)



- total rate and distributions for on-shell top quarks at NLO known [[Bordes et.al](#); [Stelzer et.al](#); [Harris et.al](#); [Campbell et.al](#); [Cao et.al](#); ...]
- implemented in MC@NLO [[Frixione et.al.](#)] and POWHEG [[Alioli et.al.](#)]
- comparison 5-flavour scheme vs. 4-flavour scheme [[Campbell et.al.](#)]
- EW corrections [[Beccaria et.al.](#)]
- effects of BSM operators [[Willenbrock et.al.](#)]
- resummation of threshold logs [[Kidonakis, Wang et.al.](#)]



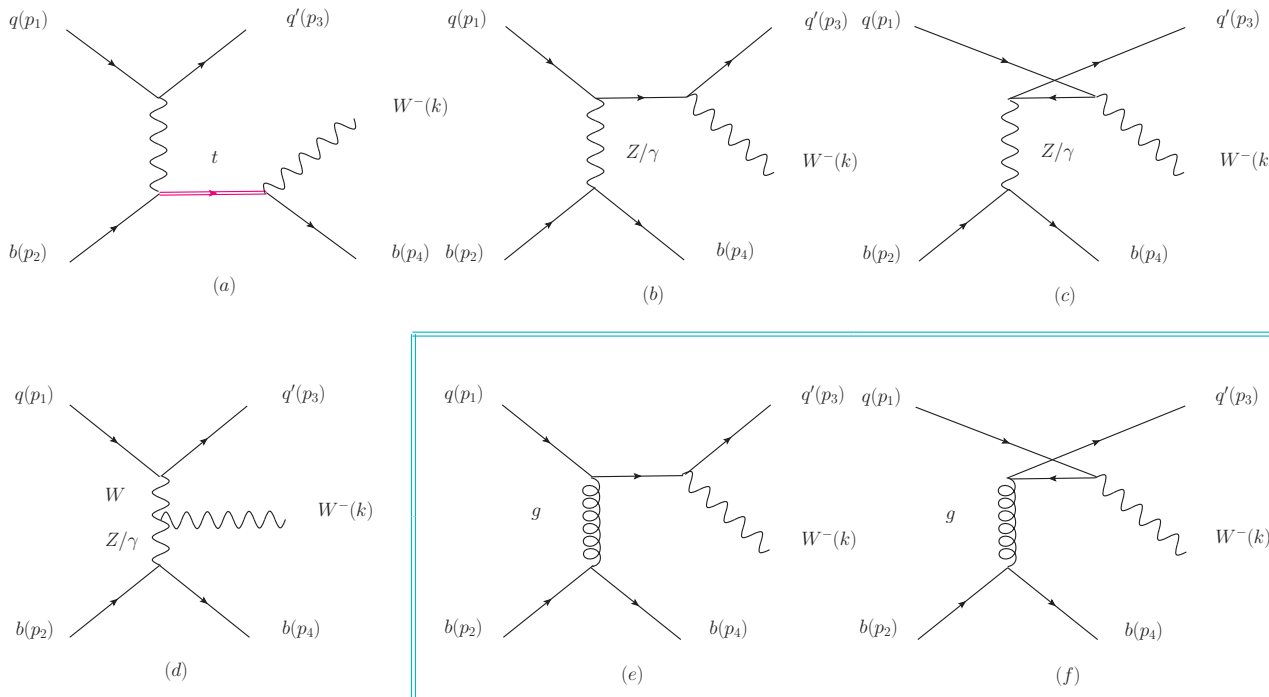
- s and t channel mix (beyond LO)
 → more appropriate to talk about (tJ) , (tb) (and (tW)) cross sections



- final state $W(= \ell\nu), J_b, J$ for 't'-channel and $W(= \ell\nu), J_b, J_{\bar{b}}$ for 's'-channel
- signal for top is WJ_b pair with invariant mass $(p_W + p_{J_b})^2 \equiv s_{Wb} \sim m_t^2$
- small parameter: $\delta \sim \frac{s_{Wb} - m_t^2}{m_t^2} \equiv \frac{\Delta}{m_t^2}$; **counting:** $\alpha_s^2 \sim \alpha_{ew} \sim \frac{\Gamma_t}{m_t} \sim \delta \ll 1$
- use (improved) narrow width for W decay
- use 5 flavour scheme, $m_b = 0$, and “fixed” order, i.e. no parton shower etc.



$$\text{amplitude: } \mathcal{A}^{\text{tree}} = \delta_{31}\delta_{42} \left(\underbrace{g_{ew}^3 A_{(-1)}^{(3,0)}}_{\delta^{1/2}} + \underbrace{g_{ew}^3 A_{(0)}^{(3,0)}}_{\delta^{3/2}} + \dots \right) + \underbrace{T_{31}^a T_{42}^a g_{ew} g_s^2 A^{(1,2)}}_{\delta \text{ signal!}}$$



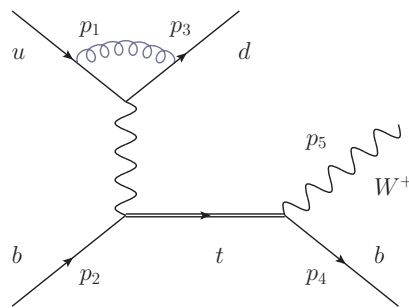
amplitude squared: (no interference due to colour \rightarrow no $\delta^{3/2}$ term)

$$|M|^2 = \underbrace{g_{ew}^6 N_c^2 \left| A_{(-1)}^{(3,0)} \right|^2}_{\delta} + \underbrace{g_{ew}^6 N_c^2 2 \text{Re} \left(A_{(-1)}^{(3,0)} [A_{(0)}^{(3,0)}]^* \right)}_{\delta^2} + \underbrace{g_{ew}^2 g_s^4 N_c C_F / 2 \left| A^{(1,2)} \right|^2}_{\delta^2} + \dots$$

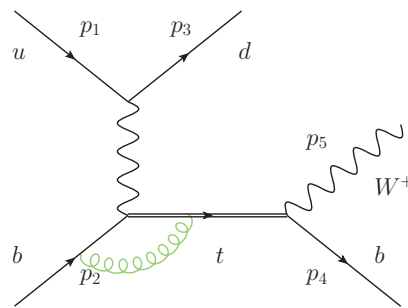


tree-level (squared) $\sim \delta$, compute all $\sim \delta^{3/2}$ contributions to $|M|^2$ ($\sim \mathcal{O}(\alpha_s)$ corrections)

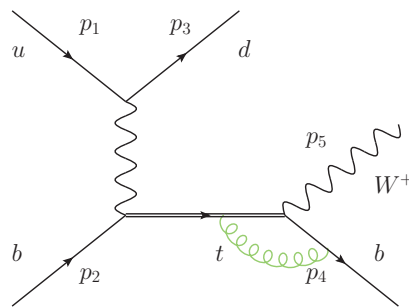
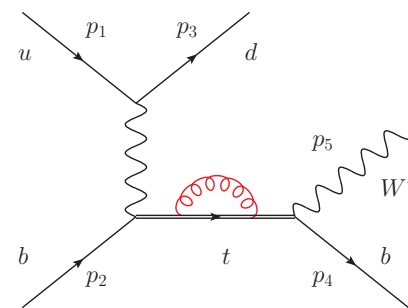
consider subset of resonant virtual diagrams (before expansion in δ this is gauge dependent)



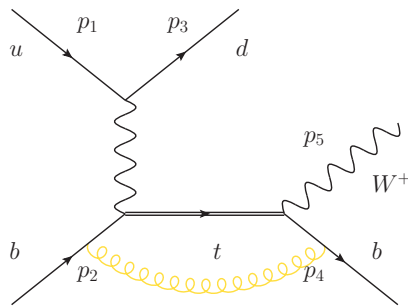
hard $\rightarrow c_{\text{prod}}$ soft = 0



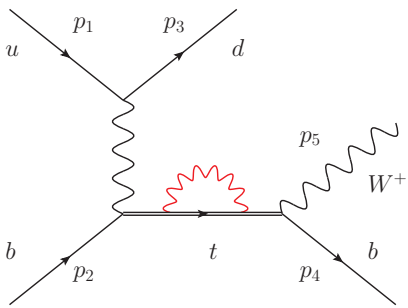
hard $\rightarrow c_{\text{prod}}$ soft \rightarrow non-fact



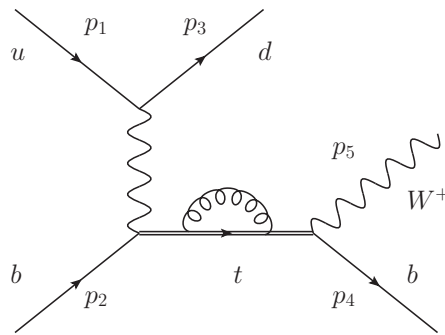
hard $\rightarrow c_{\text{dec}}$ soft \rightarrow non-fact



hard $\sim \delta^2$ soft \rightarrow non-fact



hard $\rightarrow c_{\phi\phi}$ soft \rightarrow non-fact



denominator $\Delta^2 \ell^2 [(p_t - \ell)^2 - m_t^2]$

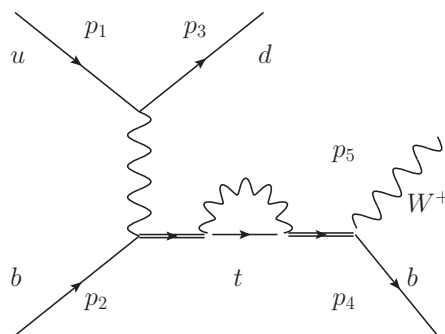
hard $\Delta^2 \ell^2 [\ell^2 - 2\ell \cdot p_t]$

soft $\Delta^2 \ell^2 [-2\ell \cdot p_t + \Delta]$

$$\sim \frac{g_{ew}^3 \cdot \alpha_s \cdot 1}{\delta^2 \cdot 1 \cdot 1} \sim 1$$

$$\sim \frac{g_{ew}^3 \cdot \alpha_s \cdot \delta^4}{\delta^2 \cdot \delta^2 \cdot \delta} \sim \delta$$

hard part of QCD self-energy is superleading, i.e. $\mathcal{O}(1)$ with LO amplitude $\sim \delta^{1/2}$
 but in pole scheme the leading hard part is precisely cancelled by counter term
 soft and subleading hard part of QCD self-energy is NLO, i.e. $\mathcal{O}(\delta^{3/2})$ for $|M|^2$



denominator $\Delta^2 \ell^2 [(p_t - \ell)^2 - m_t^2]$

hard $\Delta^2 \ell^2 [\ell^2 - 2\ell \cdot p_t]$

soft $\Delta^2 \ell^2 [-2\ell \cdot p_t + \Delta]$

$$\sim \frac{g_{ew}^3 \cdot \alpha_{ew} \cdot 1}{\delta^2 \cdot 1 \cdot 1} \sim \delta^{1/2}$$

$$\sim \frac{g_{ew}^3 \cdot \alpha_{ew} \cdot \delta^4}{\delta^2 \cdot \delta^2 \cdot \delta} \sim \delta^{3/2}$$

hard part of EW self-energy is leading, i.e. $\mathcal{O}(\delta^{1/2}) \rightarrow$ resum
 soft part of EW self-energy is beyond NLO, i.e. $\mathcal{O}(\delta^2)$ for $|M|^2$



explicit calculations and results are very simple!

$$\mathcal{A}^{(1),\text{soft}} = \mathcal{A}^{(0)} \delta V^{\text{soft}}$$

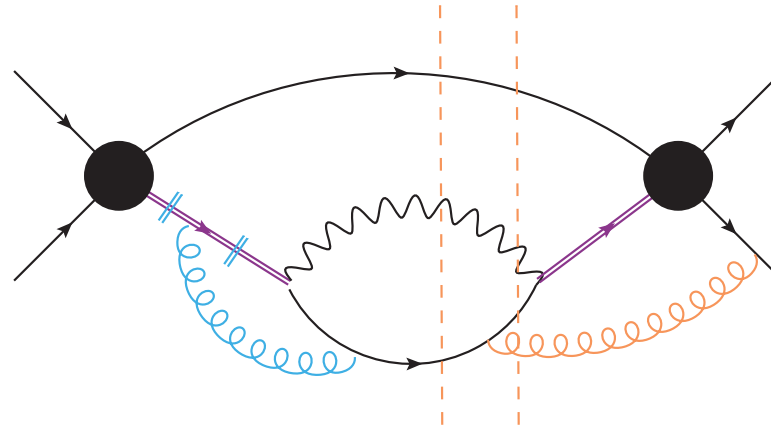
$$\delta V^{\text{soft}} = \frac{\alpha_s C_F}{2\pi} \left(-\frac{\Delta}{\mu m_t} \right)^{-2\epsilon} \left[\frac{1}{\epsilon} \left(1 - \ln \frac{s_{2t} s_{4t}}{m_t^2 s_{24}} \right) + 2 + \text{Li}_2 \left(1 - \frac{s_{2t} s_{4t}}{m_t^2 s_{24}} \right) \right]$$

$$\mathcal{A}^{\text{hard},(b)} = \mathcal{A}^{(0)} \delta V^{\text{hard},(b)} + \frac{\alpha_s C_F}{2\pi} \frac{-ig_{ew}^4 \langle 46 \rangle \langle 3|2|1 \rangle [25]}{(s_{13} + M_W^2) \Delta} \frac{m_t^2}{m_t^2 - s_{2t}} \ln \frac{s_{2t}}{m_t^2}$$

$$\delta V^{\text{hard},(b)} = \frac{\alpha_s C_F}{2\pi} \left[-\frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \left(\ln \frac{s_{2t}}{m_t \mu} - \frac{1}{2} \right) + \text{Li}_2 \left(1 - \frac{m_t^2}{s_{2t}} \right) - 2 - \frac{\pi^2}{24} \right. \\ \left. - \frac{1}{2} \ln^2 \frac{s_{2t}}{m_t \mu} + \frac{1}{8} \ln^2 \frac{m_t^2}{\mu^2} + \frac{s_{2t}}{4(m_t^2 - s_{2t})} \ln \frac{m_t^2}{\mu^2} \right. \\ \left. + \frac{1}{2} \ln \frac{s_{2t}}{m_t \mu} \left(2 - \frac{s_{2t}}{m_t^2 - s_{2t}} - \ln \frac{m_t^2}{\mu^2} \right) \right]$$



Comparison between (ET) and earlier (non-ET) NLO calculations [Campbell et.al, Yuan et.al.]



- ET virtual: hard part vanishes (at this order), soft part contributes and is included
- ET real: interference between production and decay radiation included
after expansion this cancels corresponding virtual IR singularities
- non-ET real and virtual: not included
- ET: both top quarks can be off-shell, hard and soft part contribute
- non-ET: one top is always on-shell



7 TeV LHC 't'-channel: $m_t = 171.3$ GeV, MSTW 2008 NLO pdf, $m_t/4 \leq \mu \leq m_t$

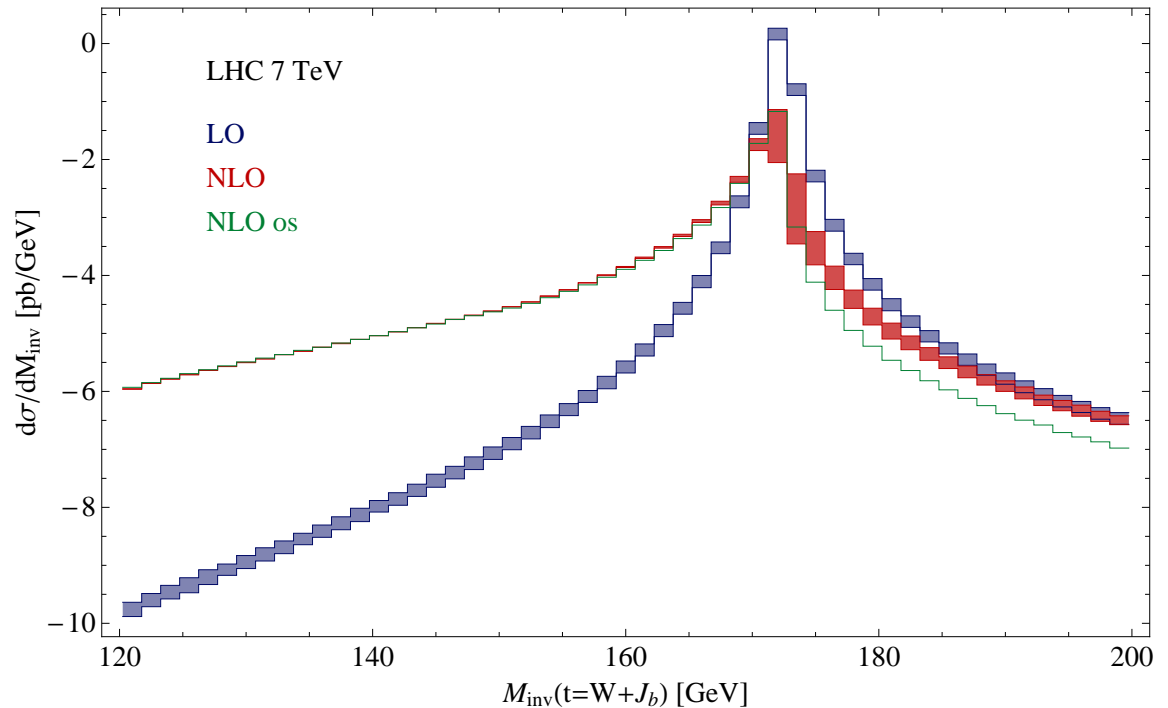
define jets: k_{\perp} cluster algorithm \Rightarrow

- J_b with $p_{\perp}(J_b) > 20$ GeV
- J_q with $p_{\perp}(J_q) > 20$ GeV
- no further $J_{\bar{b}}$ with $p_{\perp}(J_{\bar{b}}) > 15$ GeV
- $E_{\perp} + p_{\perp}(\ell) > 60$ GeV,

top window: $150 \text{ GeV} < \sqrt{(p(J_b) + p(\ell) + p(\nu))^2} < 200 \text{ GeV}$

invariant mass of 'top'

$$M_{\text{inv}}^2 \equiv (p(J_b) + p(W))^2$$





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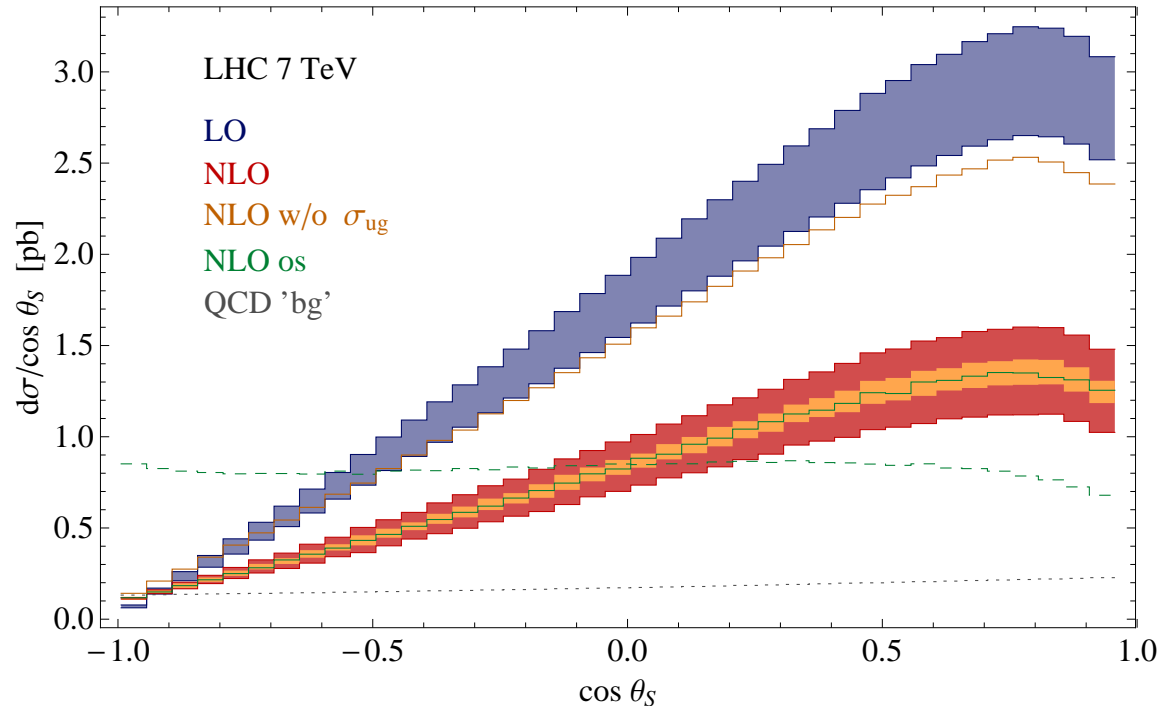
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spin correlation

$$\cos \theta_{\text{spec}} \equiv \frac{\vec{p}_q^* \cdot \vec{p}_\ell^*}{|\vec{p}_q^*| |\vec{p}_\ell^*|}$$





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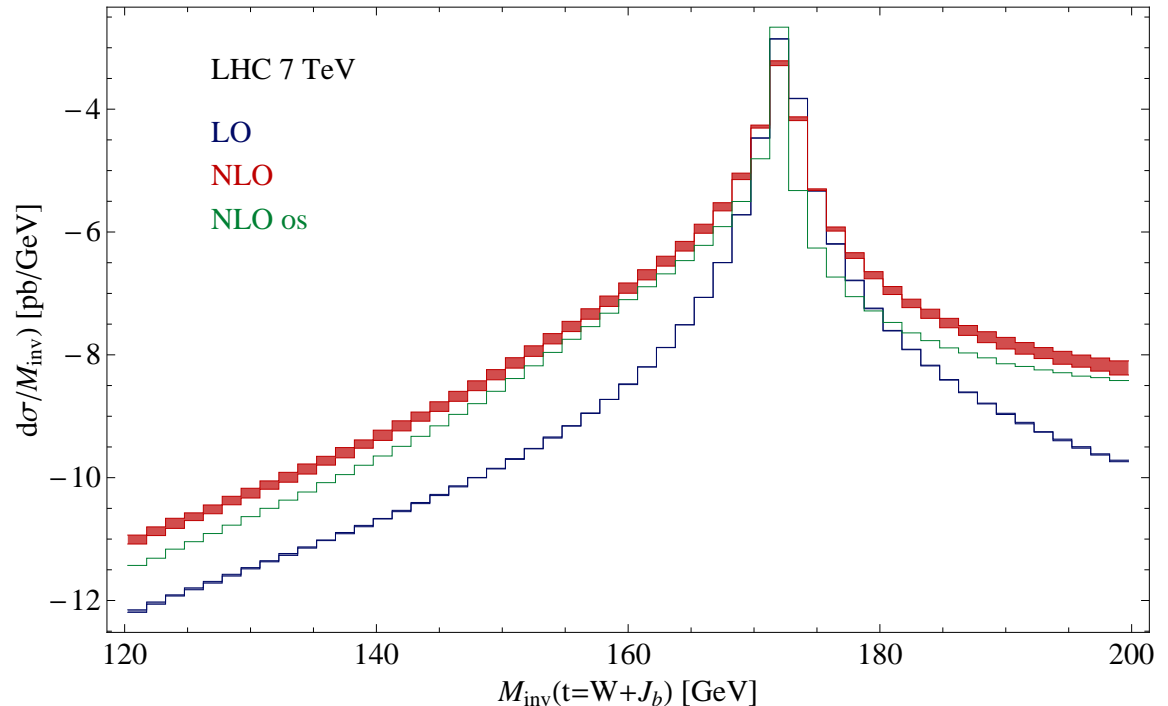
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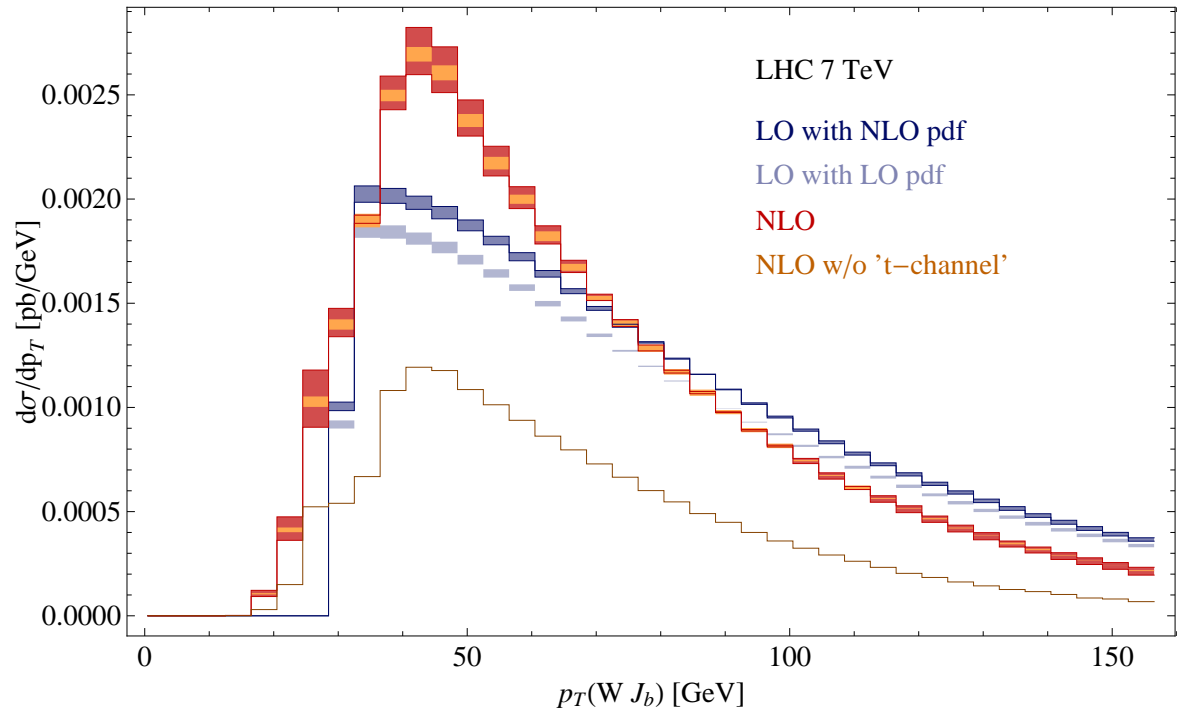
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top window: $150 \text{ GeV} < \sqrt{(p(J_b) + p(\ell) + p(\nu))^2} < 200 \text{ GeV}$

transverse momentum 'top'

$$p_T(t = J_b + W)$$





Tevatron 't'-channel: $m_t = 171.3 \text{ GeV}$, MSTW 2008 NLO pdf, $m_t/4 \leq \mu \leq m_t$

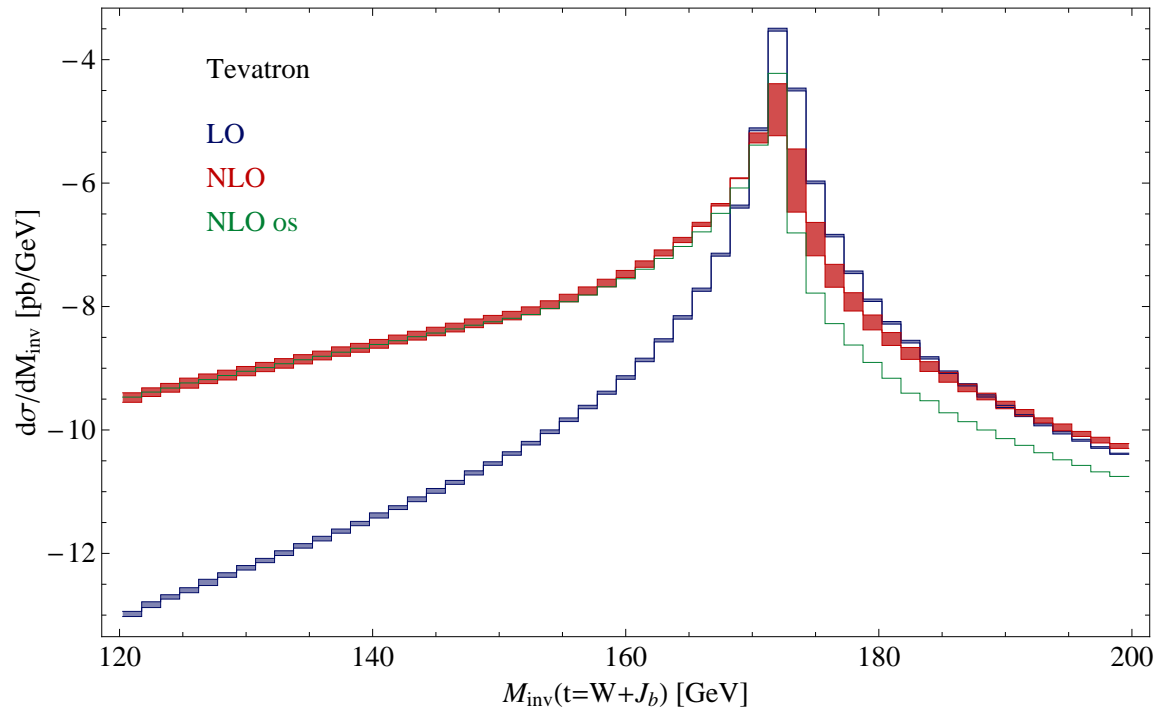
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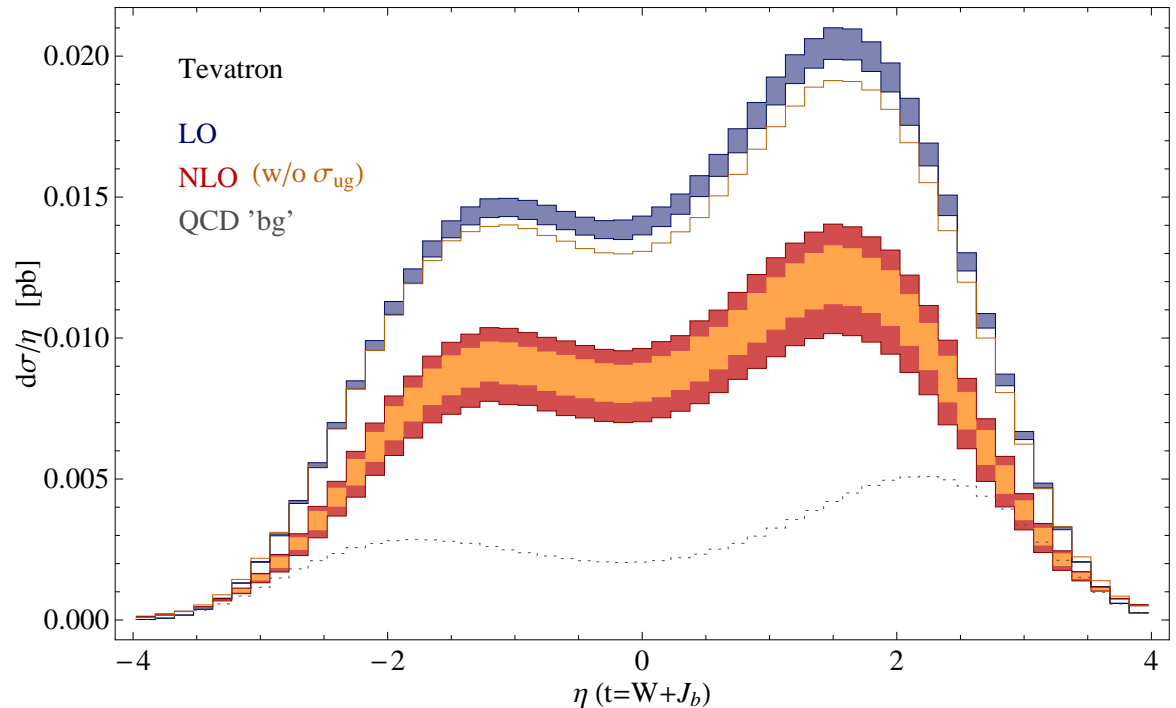
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rapidity of top

$$\eta(t = W + J_b)$$





Tevatron 's'-channel: $m_t = 171.3 \text{ GeV}$, MSTW 2008 NLO pdf, $m_t/4 \leq \mu \leq m_t$

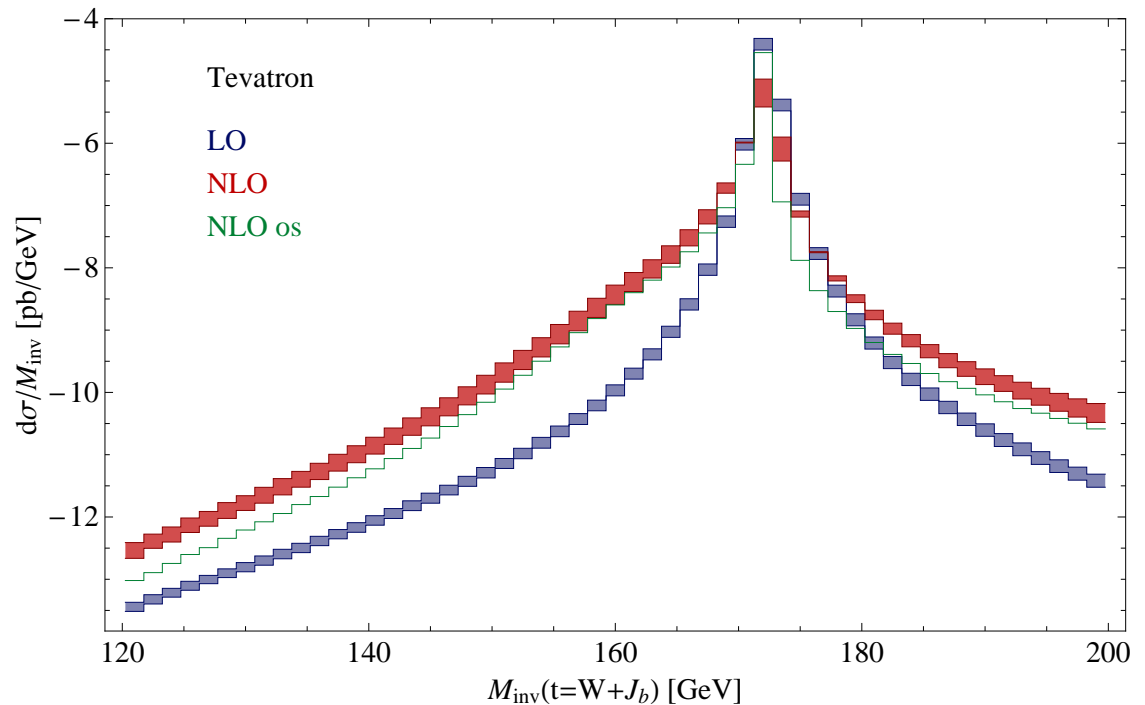
define jets: k_{\perp} cluster algorithm \Rightarrow

- J_b with $p_{\perp}(J_b) > 20 \text{ GeV}$
- J_q with $p_{\perp}(J_q) > 20 \text{ GeV}$
- no further $J_{\bar{b}}$ with $p_{\perp}(J_{\bar{b}}) > 15 \text{ GeV}$
- $E_{\perp} + p_{\perp}(\ell) > 30 \text{ GeV}$,

top window: $150 \text{ GeV} < \sqrt{(p(J_b) + p(\ell) + p(\nu))^2} < 200 \text{ GeV}$

invariant mass of 'top'

$$M_{\text{inv}}^2 \equiv (p(J_b) + p(W))^2$$





Tevatron 's'-channel: $m_t = 171.3 \text{ GeV}$, MSTW 2008 NLO pdf, $m_t/4 \leq \mu \leq m_t$

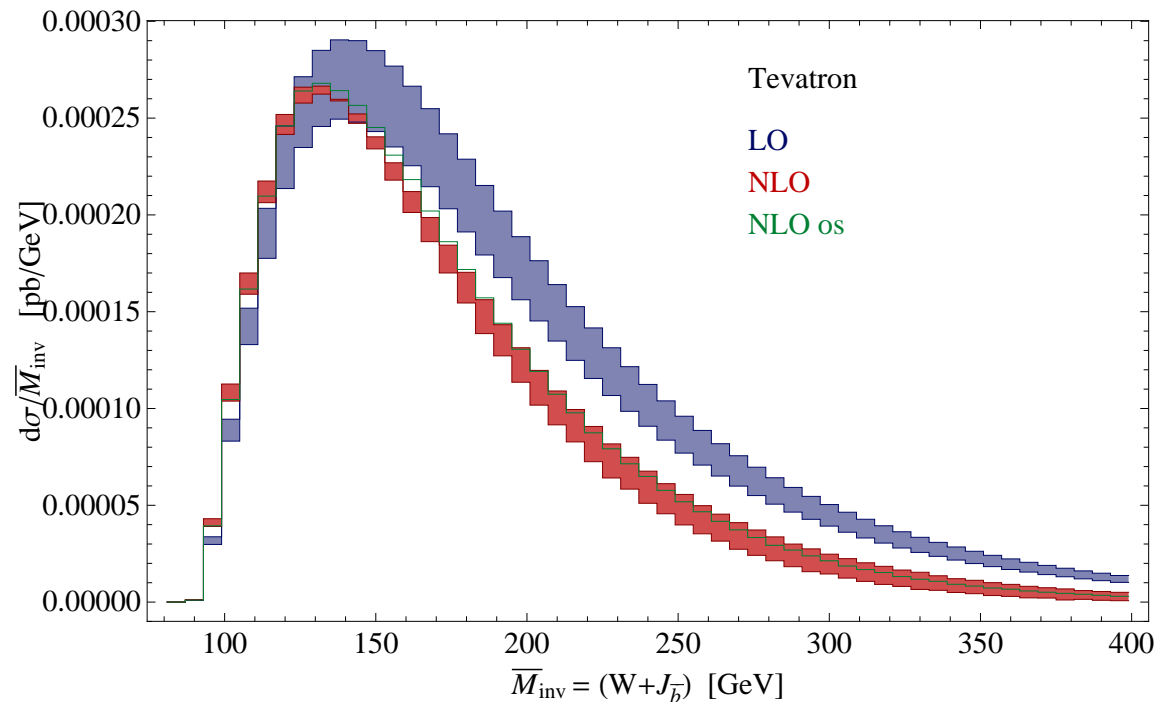
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'wrong' invariant mass

$$\bar{M}_{\text{inv}}^2 \equiv (p(J_{\bar{b}}) + p(W))^2$$





- using ET inspired approach, the computational effort to include off-shell effects at NLO for unstable particles is very modest
 - amounts to inclusion of non-factorizable (= soft) corrections (and all spin correlation effects)
 - combined with “standard” (= hard) corrections for production and decay
- can be extended beyond NLO, well defined power counting to identify minimal amount of computation required for a certain precision
- applicable to unstable fermions, gauge bosons and scalars
- example single top:
 - off-shell effects $\mathcal{O}(\alpha_s \delta)$ are small for total cross section and most observables
off-shell effects seem to be smeared by ‘reconstruction effects’
 - higher order contributions in Δ are not too difficult to compute and can be numerically important (e.g. QCD “background”)
 - full calculation beyond $\mathcal{O}(\delta^{3/2})$ for $|M|^2$ would require two-loop matching coefficient