EW Symmetry Breaking & Higgs Searches

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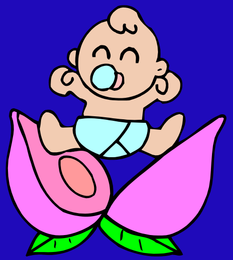
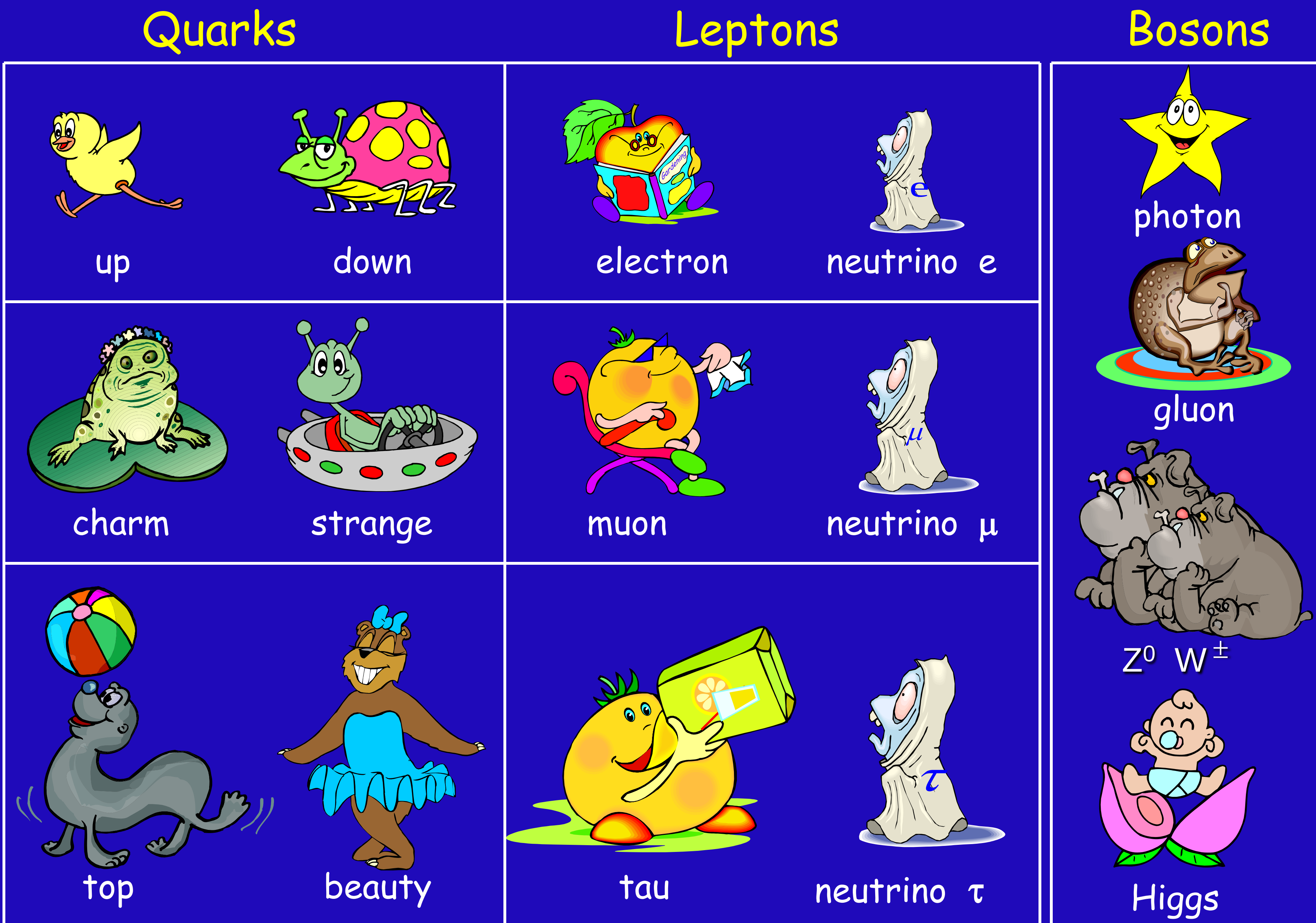
- The $SU(2)_L \otimes U(1)_L$ Electroweak Theory
- The Higgs Boson
- Goldstone Dynamics
- Fermion Flavour



The 2011 Hadron Collider Physics Summer School
CERN, Geneva, June 8-17, 2011


Quarks
Leptons
Bosons
up
down
electron
neutrino e
photon
gluon
charm
strange
muon
neutrino μ
Z^0 $W^{\pm}$
top
beauty
tau
neutrino τ
Higgs

EXPERIMENTAL FACTS

Three Families

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix} , \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix} , \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

Family Structure

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \left\{ \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , \; (\nu_l)_R \; , \; l^-_R \right\} \; ; \; \left\{ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L , \; (q_u)_R \; , \; (q_d)_R \right\}$$

Charged Currents $\quad W^\pm$

- Left–handed Fermions only
- Flavour Changing: $\nu_l \Leftrightarrow l \; , \; q_u \Leftrightarrow q_d$

Neutral currents $\quad \gamma \, , \, Z \quad$ Flavour Conserving

Universality $\quad$ (Family – Independent Couplings)

$$(\nu_l)_R \quad ?$$

$SU(2)_L \otimes U(1)_Y$

GAUGE THEORY

Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

Free Lagrangian for Massless Fermions:

$$\mathcal{L}_0 = \sum_j i\, \bar{\psi}_j\, \gamma^\mu \partial_\mu \psi_j$$

$SU(2)_L \otimes U(1)_Y$ **Flavour Symmetry:**

$$U_L \equiv \exp\left\{ i\, \frac{\vec{\sigma}}{2}\, \vec{\alpha} \right\}$$

$$\psi_1 \to e^{i y_1 \beta}\, U_L\, \psi_1 \quad ; \quad \psi_2 \to e^{i y_2 \beta}\, \psi_2 \quad ; \quad \psi_3 \to e^{i y_3 \beta}\, \psi_3$$

$$\bar{\psi}_1 \to \bar{\psi}_1\, U_L^\dagger\, e^{-i y_1 \beta} \quad ; \quad \bar{\psi}_2 \to \bar{\psi}_2\, e^{-i y_2 \beta} \quad ; \quad \bar{\psi}_3 \to \bar{\psi}_3\, e^{-i y_3 \beta}$$

Gauge Principle: $\vec{\alpha} = \vec{\alpha}(x) \quad , \quad \beta = \beta(x)$

$$\mathbf{D}_\mu \psi_1 \equiv \left[\partial_\mu + i\, g\, \mathbf{W}_\mu(x) + i\, g'\, y_1\, B_\mu(x)\right] \psi_1 \quad \to \quad \mathrm{e}^{i\, y_1 \beta(x)}\, \mathbf{U}_\mathrm{L}(x)\; \mathbf{D}_\mu \psi_1$$

$$\mathbf{D}_\mu \psi_k \equiv \left[\partial_\mu + i\, g'\, y_k\, B_\mu(x)\right] \psi_k \quad \to \quad \mathrm{e}^{i\, y_k \beta(x)}\, \mathbf{D}_\mu \psi_k \qquad (k = 2, 3)$$

$$B_\mu(x) \quad \to \quad B_\mu(x) - \frac{1}{g'}\, \partial_\mu \beta(x)$$

$$\mathbf{W}_\mu(x) \quad \to \quad \mathbf{U}_\mathrm{L}(x)\; \mathbf{W}_\mu(x)\; \mathbf{U}_\mathrm{L}^\dagger(x) + \frac{i}{g}\, \partial_\mu \mathbf{U}_\mathrm{L}(x)\; \mathbf{U}_\mathrm{L}^\dagger(x)$$

$$\mathrm{U}(x) \equiv \exp\left\{ i\, \frac{\vec{\sigma}}{2}\, \vec{\alpha}(x) \right\} \quad ; \quad \mathbf{W}_\mu(x) \equiv \frac{\vec{\sigma}}{2}\, \vec{W}_\mu(x) \quad ; \quad \delta W_\mu^i = -\frac{1}{g}\, \partial_\mu(\delta\alpha^i) - \varepsilon^{ijk}\, \delta\alpha^j\, W_\mu^k$$

4 Massless Gauge Bosons $\quad W_\mu^\pm \; , \; W_\mu^3 \; , \; B_\mu^0$

CHARGED CURRENTS

$$\mathcal{L} = \sum_j i\,\bar{\psi}_j \gamma^\mu D_\mu \psi_j \quad \longrightarrow \quad -g\;\bar{\psi}_1 \gamma^\mu \mathbf{W}_\mu \psi_1 \;-\; g'\, B_\mu \sum_j y_j\; \bar{\psi}_j \gamma^\mu \psi_j$$

$$\mathbf{W}_\mu \equiv \frac{\vec{\sigma}}{2}\cdot \vec{W}_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}\,W_\mu^\dagger \\ \sqrt{2}\,W_\mu & -W_\mu^3 \end{pmatrix} \quad ; \quad W_\mu \equiv \left(W_\mu^1 + i\,W_\mu^2\right)/\sqrt{2}$$

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\; W_\mu^\dagger \left[\, \bar{q}_u \gamma^\mu (1-\gamma_5)\, q_d \;+\; \bar{\nu}_l \gamma^\mu (1-\gamma_5)\, l \,\right] \;+\; \text{h.c.}$$

Quark / Lepton Universality

Left – Handed Interaction

NEUTRAL CURRENTS

$$\mathcal{L}_{\mathrm{NC}} = -g\, W_\mu^3\; \bar\psi_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - g'\, B_\mu \sum_j y_j\, \bar\psi_j \gamma^\mu \psi_j$$

Neutral Fields ➡ Arbitrary Combination

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

(broken symmetry, since $M_Z \neq 0$)

A_μ has the QED Interaction IF $\quad g\sin\theta_W = g'\cos\theta_W = e$

$$y_1 = Q_u - \frac{1}{2} = Q_d + \frac{1}{2} \quad ; \quad y_2 = Q_u \quad ; \quad y_3 = Q_d$$

Electroweak Unification

$$\mathcal{L}_{\mathrm{NC}} = -e\, A_\mu \sum_j \bar\psi_j\, \gamma^\mu Q_j\, \psi_j + \mathcal{L}_{\mathrm{NC}}^Z$$

$$Q_1 = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix} \quad ; \quad Q_2 = Q_u \quad ; \quad Q_3 = Q_d$$

$$\mathcal{L}_{NC}^{Z} = -\frac{e}{\sin\theta_W \cos\theta_W} Z_\mu \left\{ \bar\psi_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - \sin^2\theta_W \sum_j \bar\psi_j \gamma^\mu Q_j \psi_j \right\}$$

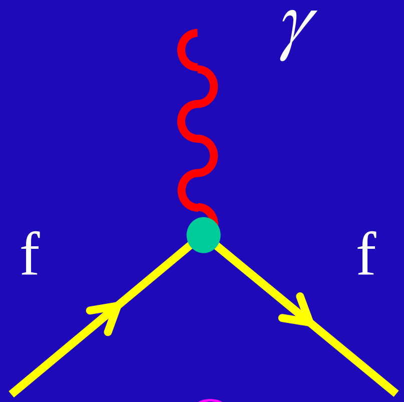
$$= -\frac{e}{2\sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu [v_f - a_f \gamma_5] f$$

	q_u	q_d	ν_l	l^-
$2\,v_f$	$1-\frac{8}{3}\sin^2\theta_W$	$-1+\frac{4}{3}\sin^2\theta_W$	1	$-1+4\sin^2\theta_W$
$2\,a_f$	1	-1	1	-1

IF ν_R do exist: $y(\nu_R) = Q_\nu = 0$ ➡ No ν_R Interactions

Sterile Neutrinos

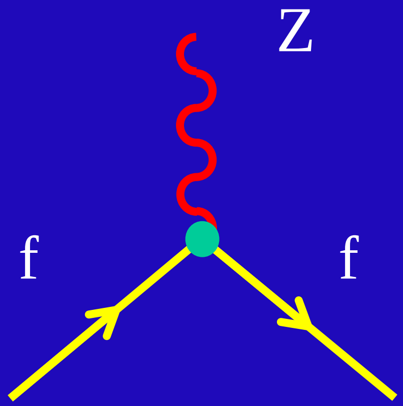
NEUTRAL CURRENTS



$e\, Q_f$

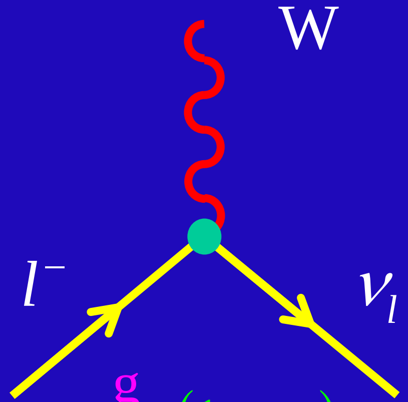
$$a_f = T_3^f = \pm\frac{1}{2}$$

$$v_f = T_3^f\left(1 - 4\,|Q_f|\,\sin^2\theta_W\right)$$

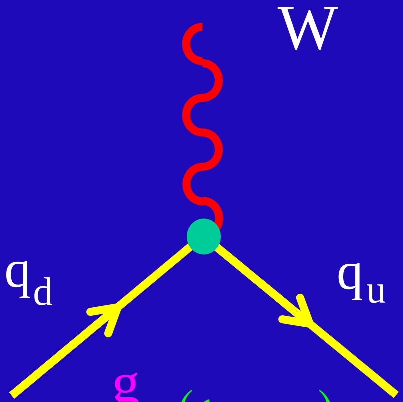


$$\frac{e}{2\, s_\theta c_\theta}\left(v_f - a_f\,\gamma_5\right)$$

CHARGED CURRENTS



$$\frac{g}{2^{3/2}}\left(1-\gamma_5\right)$$



$$\frac{g}{2^{3/2}}\left(1-\gamma_5\right)$$

$y(\nu_R) = Q_\nu = 0$ ➡ No ν_R Interactions **Sterile Neutrinos**

$$\mathbf{W}_{\mu\nu} \equiv -\frac{i}{g}\left[\mathbf{D}_\mu , \mathbf{D}_\nu\right] \equiv \frac{\vec{\sigma}}{2}\cdot\vec{W}_{\mu\nu} \;\rightarrow\; \mathbf{U}_{\mathrm{L}}\, \mathbf{W}_{\mu\nu}\, \mathbf{U}_{\mathrm{L}}^\dagger \qquad ; \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \;\rightarrow\; B_{\mu\nu}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\varepsilon^{ijk}\, W^j_\mu W^k_\nu$$

$$\mathcal{L}_{\mathrm{K}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2}\mathrm{Tr}\left(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right) = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}_{\mu\nu} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_3 + \mathcal{L}_4$$

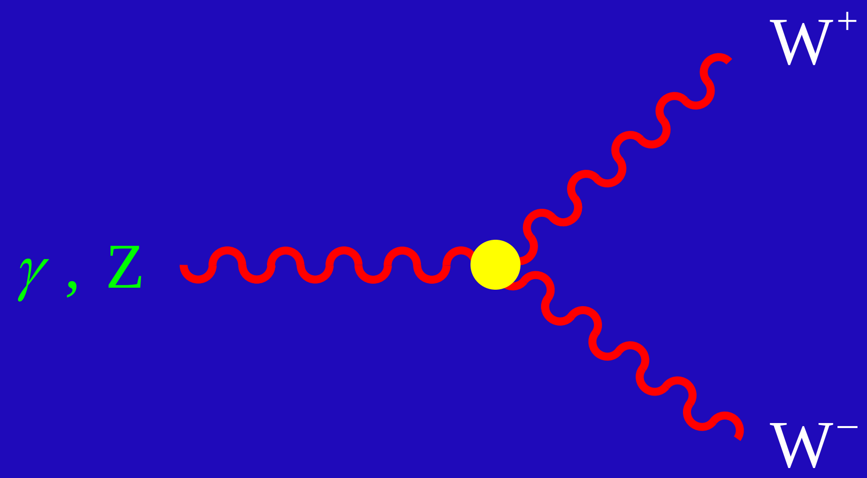
$$\mathcal{L}_3 = i\, e \cot\theta_W \left\{(\partial^\mu W^\nu - \partial^\nu W^\mu) W^\dagger_\mu Z_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}) W_\mu Z_\nu + W_\mu W^\dagger_\nu (\partial^\mu Z^\nu - \partial^\nu Z^\mu)\right\}$$

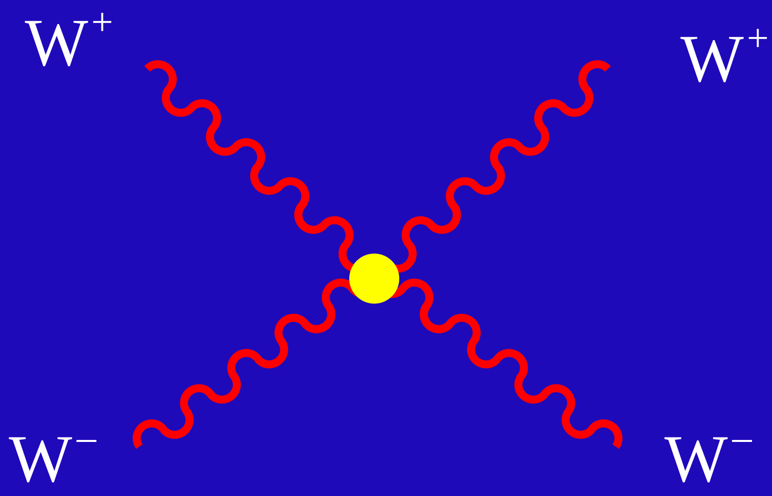
$$+ i\, e \left\{\left(\partial^\mu W^\nu - \partial^\nu W^\mu\right) W^\dagger_\mu A_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right) W_\mu A_\nu + W_\mu W^\dagger_\nu \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\right\}$$

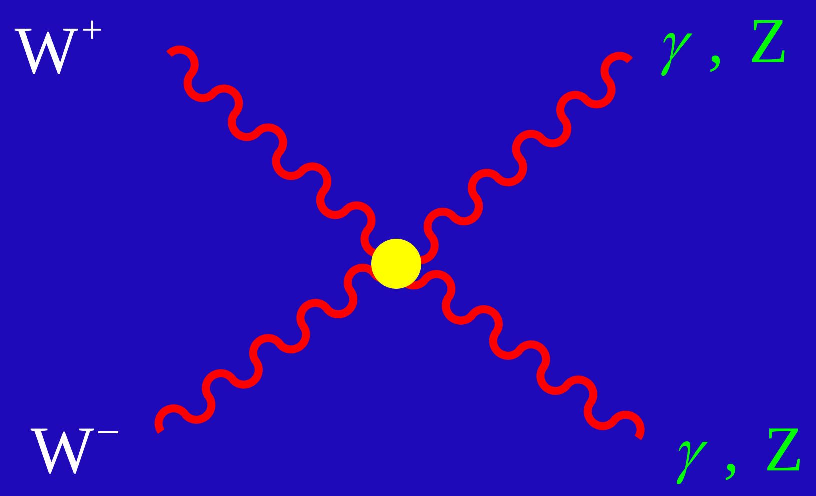
$$\mathcal{L}_4 = -\frac{e^2}{2\sin^2\theta_W}\left\{\left(W^\dagger_\mu W^\mu\right)^2 - W^\dagger_\mu W^{\mu\dagger} W_\nu W^\nu\right\} - e^2\cot^2\theta_W \left\{W^\dagger_\mu W^\mu Z_\nu Z^\nu - W^\dagger_\mu Z^\mu W_\nu Z^\nu\right\}$$

$$-e^2\cot\theta_W \left\{2 W^\dagger_\mu W^\mu Z_\nu A^\nu - W^\dagger_\mu Z^\mu W_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu Z^\nu\right\} - e^2\left\{W^\dagger_\mu W^\mu A_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu A^\nu\right\}$$

GAUGE SELF-INTERACTIONS







PROBLEM WITH MASS SCALES

$$\mathcal{L}_M = \frac{1}{2}\, m_B^2\, B^\mu B_\mu + \frac{1}{2}\, m_W^2\, W^{i\mu} W_\mu^i$$

Not Gauge Invariant

Gauge Symmetry → $m_\gamma = 0$ **Good**; $M_W = M_Z = 0$ **Bad!**

$M_W = 80.40\ \text{GeV}$

$M_Z = 91.19\ \text{GeV}$

Moreover $\quad \mathcal{L}_{m_f} \equiv -m_f\, \bar{f} f = -m_f\, (\bar{f}_L f_R + \bar{f}_R f_L)$

Also Forbidden by Gauge Symmetry → $m_f = 0 \quad \forall f$

All Particles Massless

Bosonic Degrees of Freedom

Massless $W^{\pm}\,,\; Z$

3 x 2 polarizations **= 6**

Massive $W^{\pm}\,,\; Z$

3 x 3 polarizations **= 9**

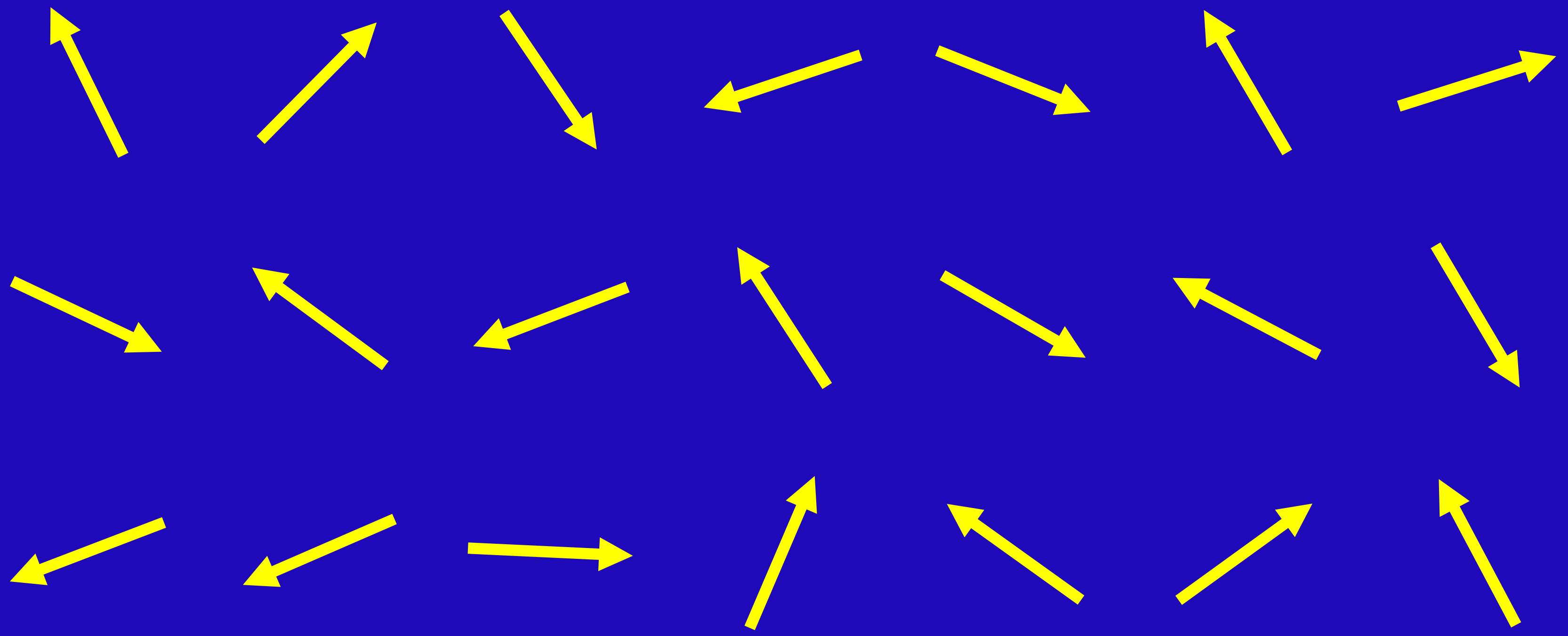
3 d.o.f. Missing

Spontaneous Symmetry Breaking

Spontaneous Symmetry Breaking

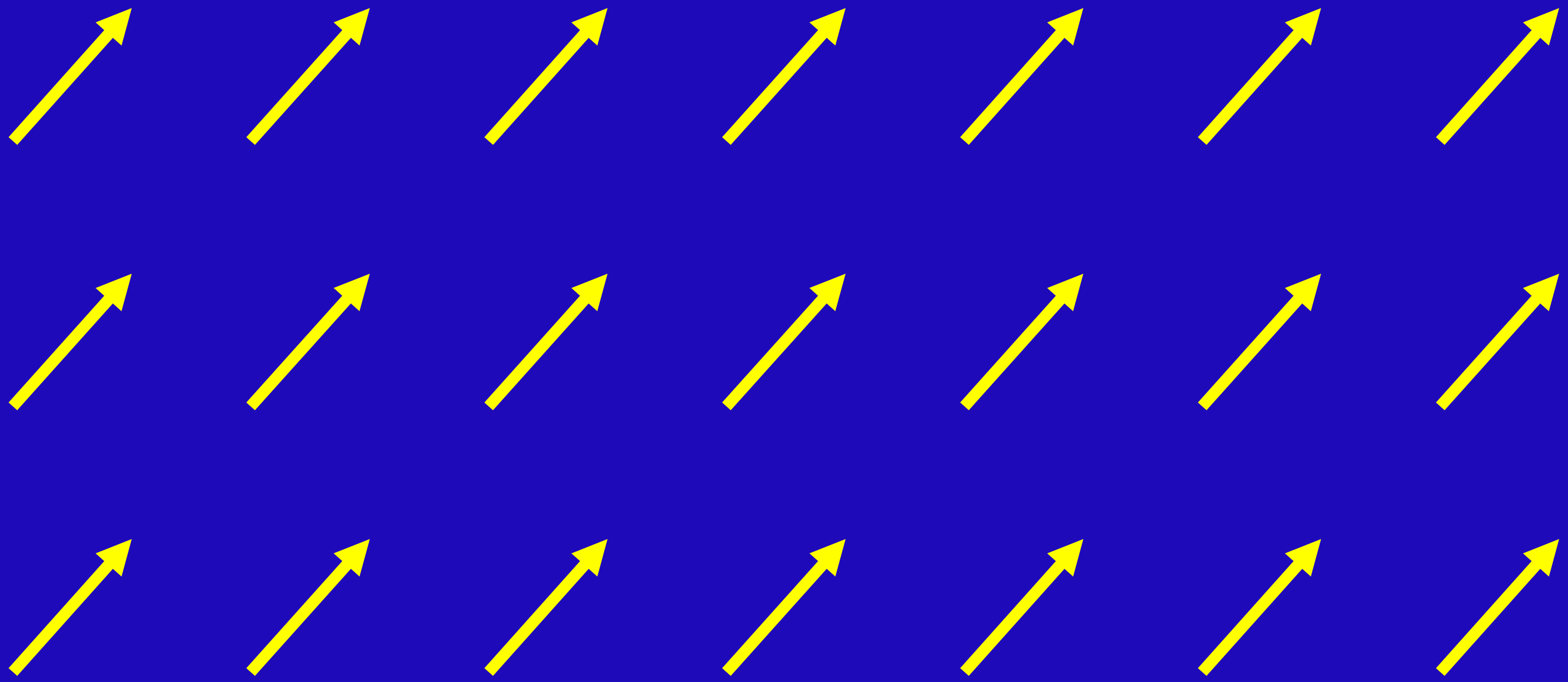
FERROMAGNET

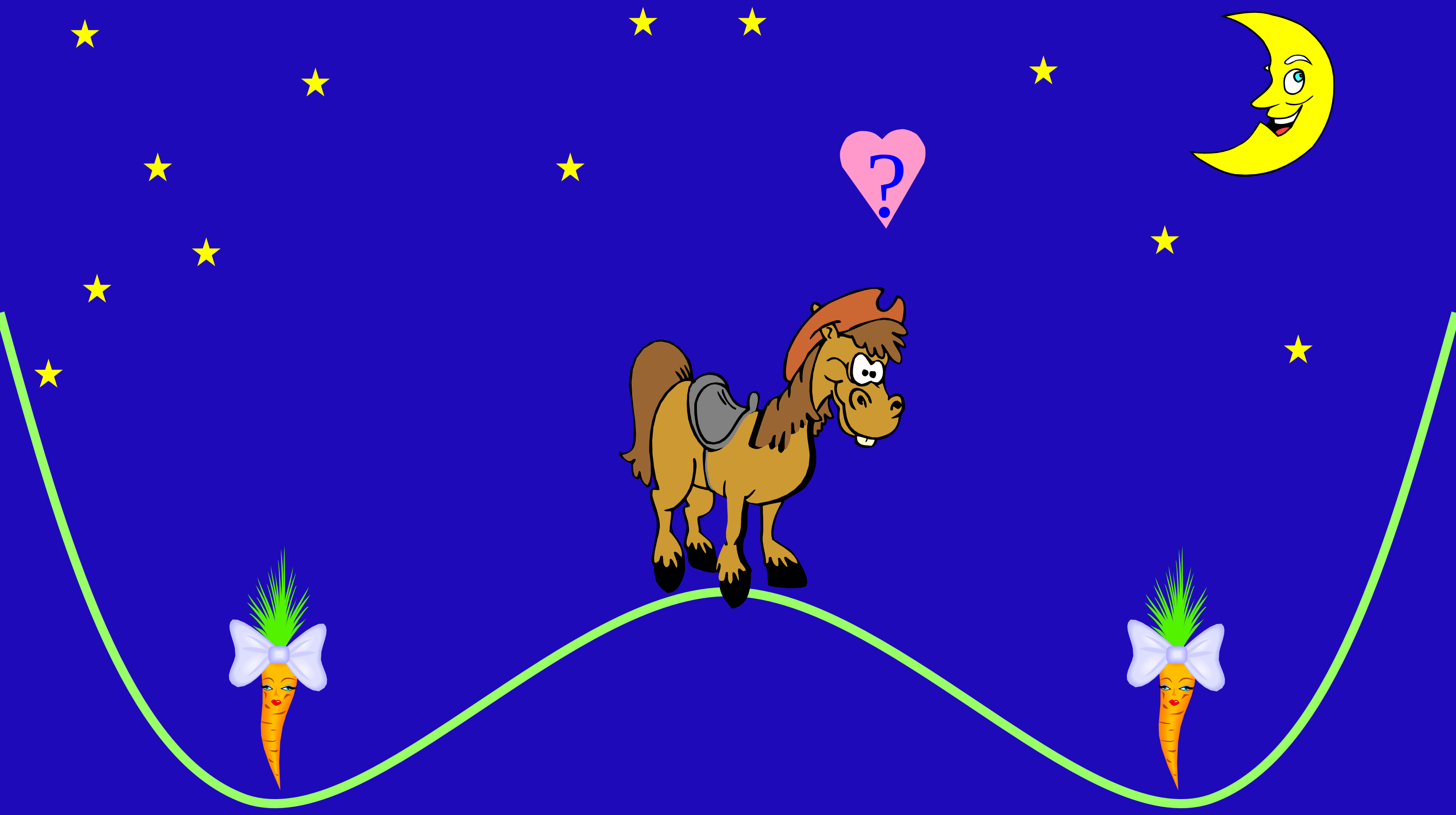
$T > T_C$

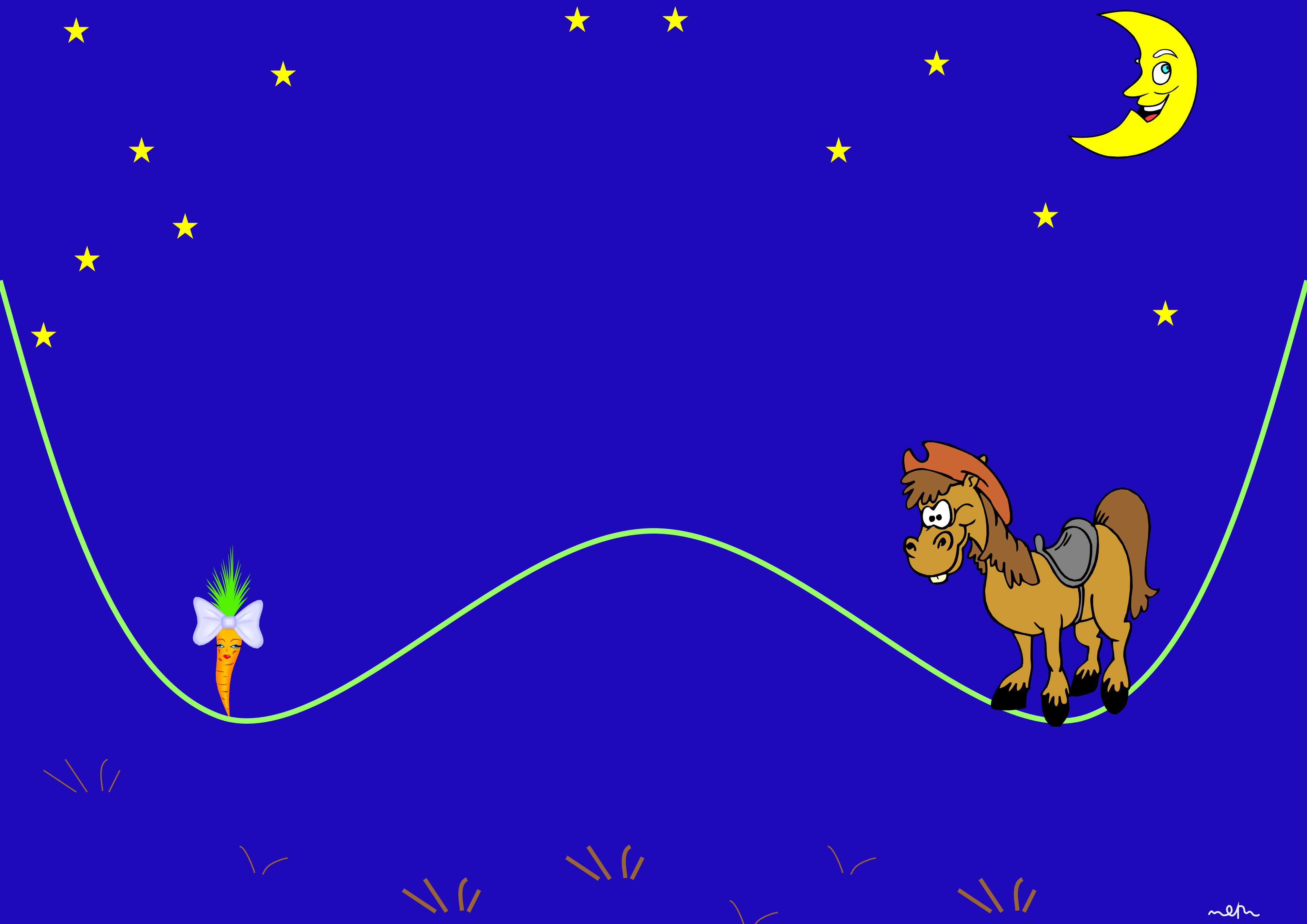


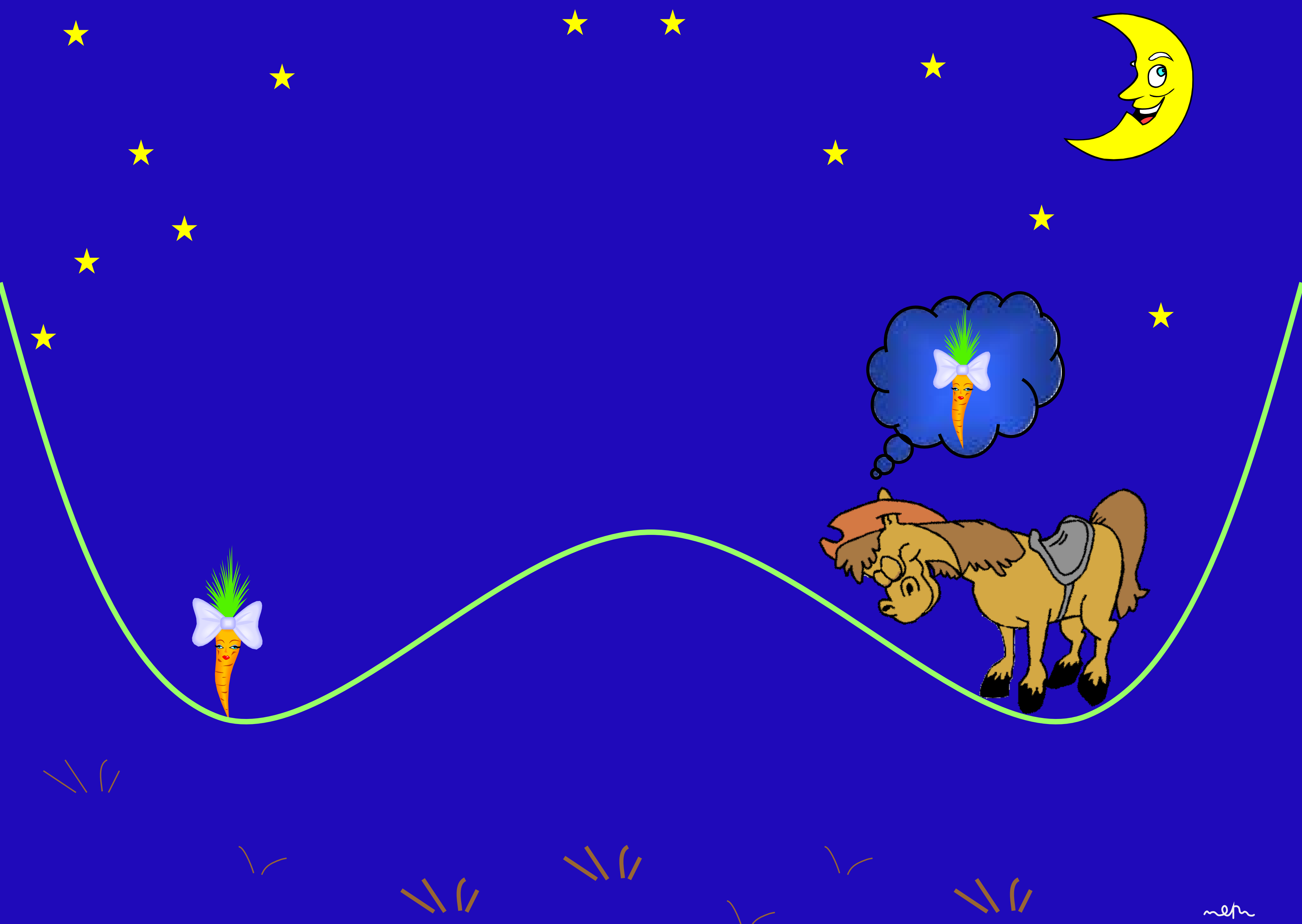
FERROMAGNET

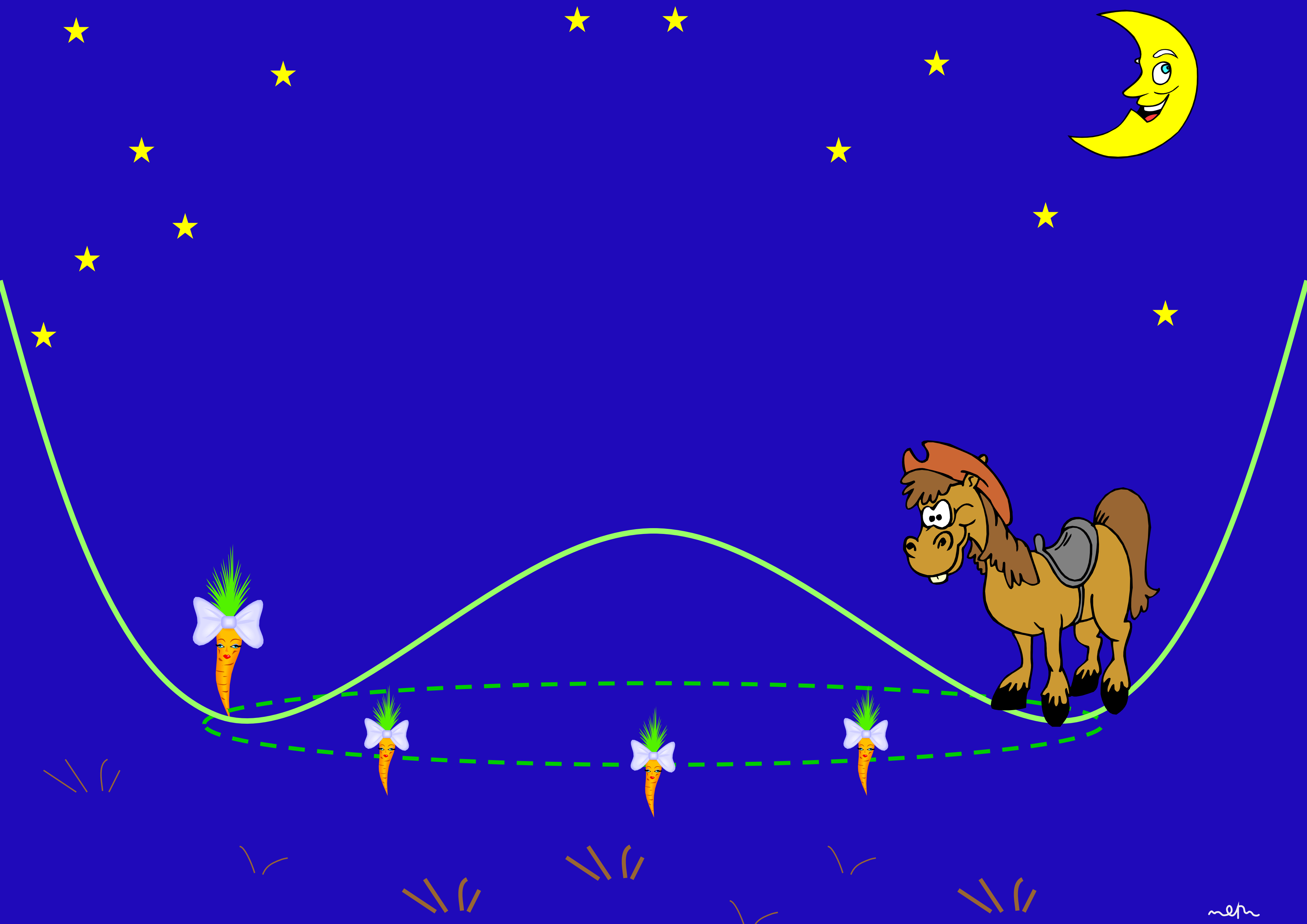
$T < T_C$











SCALAR POTENCIAL

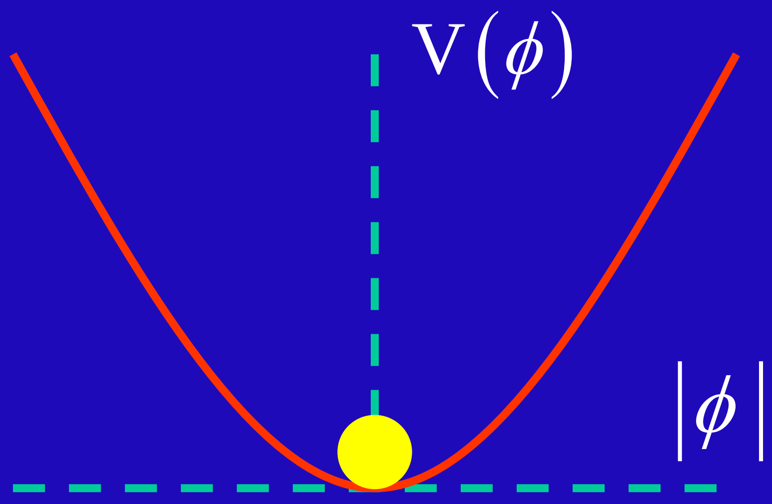
$$\mathcal{L}(\phi) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)$$

$$\mathrm{V}(\phi) = \mu^2 \ \phi^\dagger \phi + h \ (\phi^\dagger \phi)^2$$

Phase Symmetry: $$\phi(x) \to \mathrm{e}^{i\theta} \phi(x)$$

$$h > 0 \quad ; \quad \mu^2 > 0$$

$$M_\phi = \mu$$



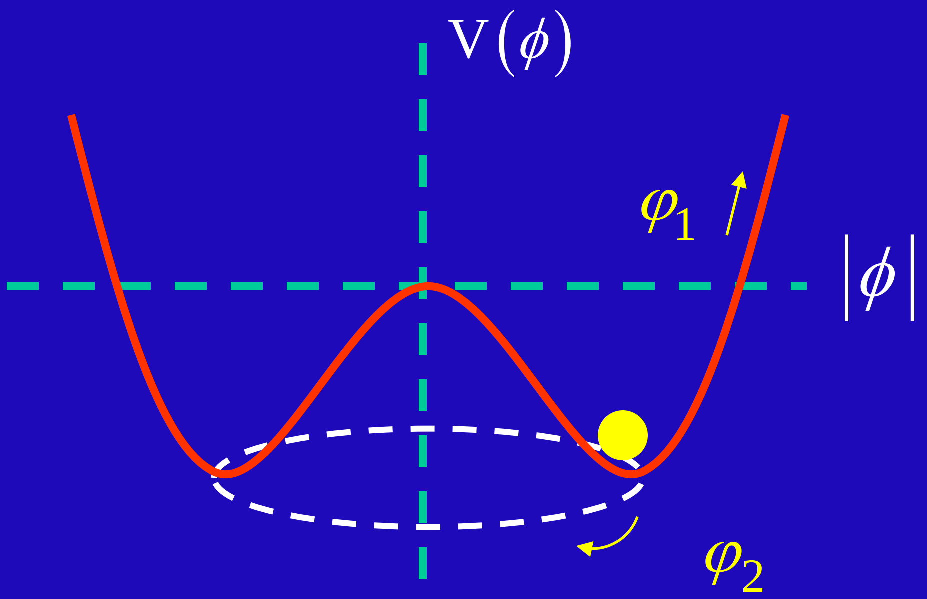
Trivial Minimum (Ground State / Vacuum): $$\phi = \phi_0 = 0$$

$$V(\phi) = \mu^2 \, \phi^\dagger \phi + h \, (\phi^\dagger \phi)^2$$

Phase Symmetry:

$$\phi(x) \to e^{i\theta} \, \phi(x)$$

$$\mu^2 < 0$$



Degenerate Minima

(Ground State / Vacuum)

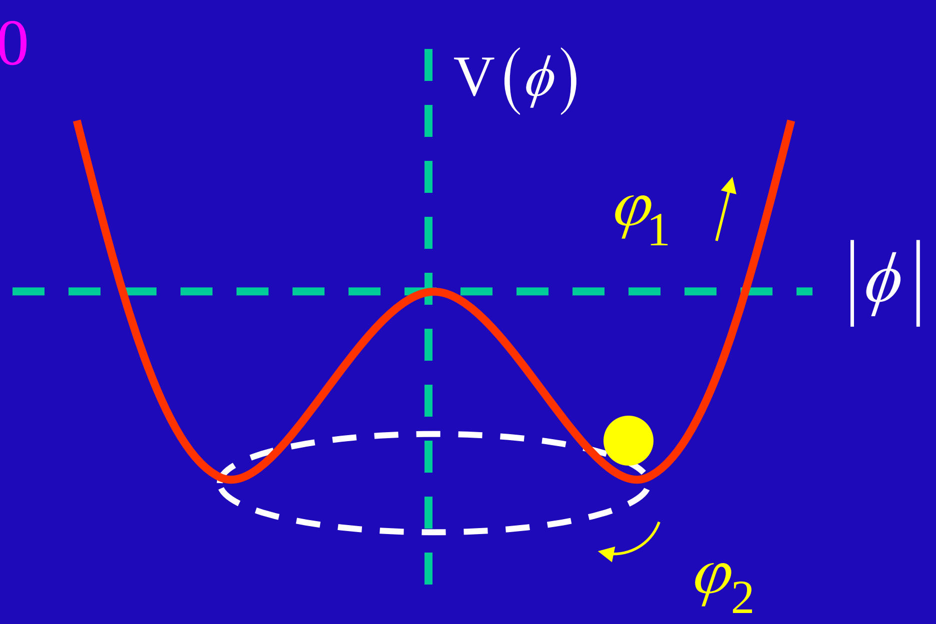
$$|\phi_0| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}} > 0 \quad ; \quad V(\phi_0) = -\frac{1}{4} h v^4$$

Spontaneous Symmetry Breaking:

$$\phi \equiv \frac{1}{\sqrt{2}} \left[v + \varphi_1(x) \right] \, e^{i\varphi_2(x)/v}$$

Vacuum Choice

Spontaneous Symmetry Breaking



$$\phi \equiv \frac{1}{\sqrt{2}}\left[\mathrm{v} + \varphi_1(x)\right]\, \mathrm{e}^{i\varphi_2(x)/\mathrm{v}}$$

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\varphi_1\,\partial^\mu\varphi_1 + \frac{1}{2}\left(1+\frac{\varphi_1}{\mathrm{v}}\right)^2 \partial_\mu\varphi_2\,\partial^\mu\varphi_2 - V(\phi)$$

$$V(\phi) = V(\phi_0) + \frac{1}{2} M_{\varphi_1}^2\,\varphi_1^2 + h\,\mathrm{v}\,\varphi_1^3 + \frac{1}{4} h\,\varphi_1^4$$

$$M_{\varphi_1}^2 = -2\mu^2 > 0 \quad ; \quad M_{\varphi_2}^2 = 0$$

1 Massless Goldstone Boson

ELECTROWEAK SSB

New Scalar Doublet $\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix}$; $y_\phi = Q_\phi - T_3 = \frac{1}{2}$

$$\mathcal{L}(\phi) = (\mathbf{D}_\mu \phi)^\dagger \, \mathbf{D}^\mu \phi - \mu^2 \, \phi^\dagger \phi - h \, (\phi^\dagger \phi)^2$$

$$\mathbf{D}^\mu \phi = \left[\partial^\mu + i\, g\, \mathbf{W}^\mu + i\, g'\, y_\phi \, B^\mu \right] \phi \qquad ; \qquad \mathbf{W}^\mu = \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu$$

$SU(2)_L \otimes U(1)_Y$ Symmetry

Degenerate Vacuum States:

$(\mu^2 < 0 \; , \; h > 0)$

$$\left| \langle 0 | \, \phi^{(0)} \, | 0 \rangle \right| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}}$$

Spontaneous Symmetry Breaking: $\phi(x) = \exp\left\{ i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$

HIGGS MECHANISM

$$\phi(x) = \exp\left\{ i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathrm{v} + H(x) \end{pmatrix}$$

$SU(2)_L$ Invariance ➡ $\vec{\theta}(x)$ can be gauged away

Unitary Gauge: $\vec{\theta}(x) = 0$

$$(D_\mu \phi)^\dagger D^\mu \phi \quad \to \quad \frac{1}{2} \partial_\mu H \, \partial^\mu H + \frac{g^2}{4} (\mathrm{v} + H)^2 \left\{ W_\mu^\dagger W^\mu + \frac{1}{2 \cos^2\theta_W} Z_\mu Z^\mu \right\}$$

➡ $$M_Z \cos\theta_W = M_W = \frac{1}{2} \mathrm{v}\, g$$

Massive Gauge Bosons

Bosonic Degrees of Freedom

Massless $W^{\pm}\, ,\; Z$

3 x 2 polarizations **= 6**

$\pm$

3 Goldstones $\vec{\theta}$

SSB ↓

Massive $W^{\pm}\, ,\; Z$

3 x 3 polarizations **= 9**

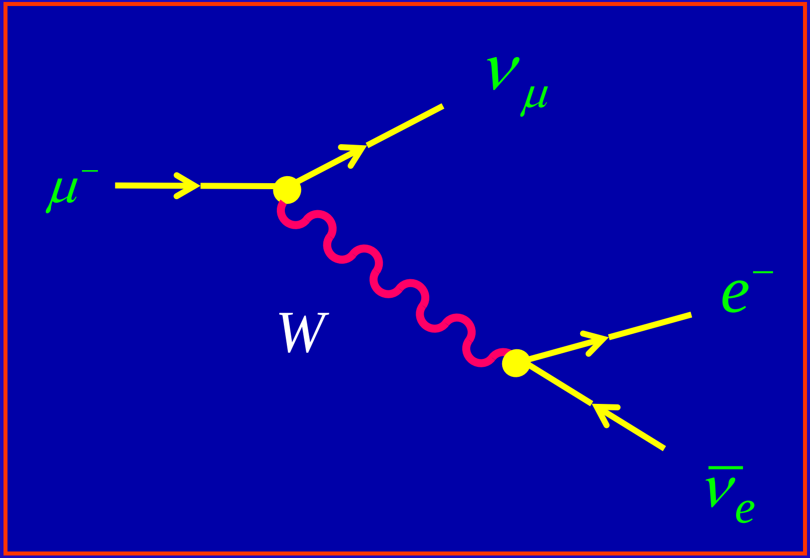
SAME PHYSICS

$$M_Z \cos\theta_W = M_W = \frac{1}{2}\, v\, g$$

$$M_Z = 91.1875 \text{ GeV} > M_W = 80.399 \text{ GeV} \;\Rightarrow\; \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223$$

$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} \equiv 4\sqrt{2}\, G_F$$

$$\frac{1}{\tau_\mu} \equiv \Gamma = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}$$



$$\left.\begin{array}{l} G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2} \\ g = \dfrac{e}{\sin\theta_W} \quad , \quad M_W \end{array}\right\} \Rightarrow \begin{array}{l} \sin^2\theta_W = 0.215 \\ v = \left(\sqrt{2}\, G_F\right)^{-1/2} = 246 \text{ GeV} \end{array}$$

FERMION MASSES

Scalar – Fermion Couplings allowed by Gauge Symmetry

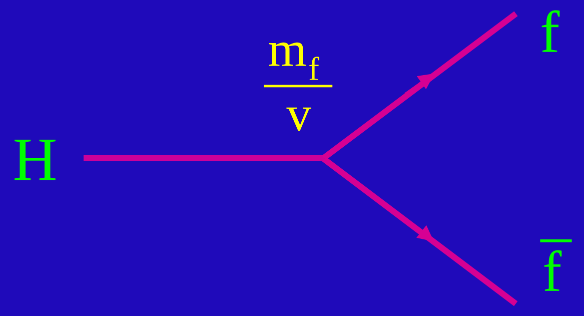
$$\mathcal{L}_Y = (\bar{q}_u, \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] + (\bar{\nu}_l, \bar{l})_L \; c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

⬇ **SSB**

$$\mathcal{L}_Y = -\left(1 + \frac{H}{\text{v}}\right) \left\{ m_{q_d} \, \bar{q}_d \, q_d + m_{q_u} \, \bar{q}_u \, q_u + m_l \, \bar{l} \, l \right\}$$

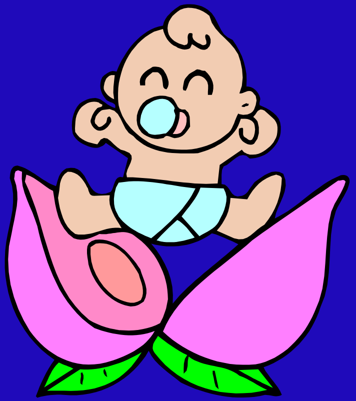
Fermion Masses are New Free Parameters

$$\left[m_{q_d} \,,\; m_{q_u} \,,\; m_l \right] = -\left[c^{(d)} \,,\; c^{(u)} \,,\; c^{(l)} \right] \frac{\text{v}}{\sqrt{2}}$$



Couplings Fixed: $$g_{Hf\bar{f}} = \frac{m_f}{v}$$

THE HIGGS BOSON



$$\mathcal{L}_S = \frac{h\,\mathrm{v}^4}{4} + \mathcal{L}_H + \mathcal{L}_{HG^2}$$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H\,\partial^\mu H - \frac{1}{2}M_H^2\,H^2 - \frac{M_H^2}{2\,\mathrm{v}}\,H^3 - \frac{M_H^2}{8\,\mathrm{v}^2}\,H^4$$

$$\mathcal{L}_{HG^2} = \left[M_W^2\,W_\mu^\dagger W^\mu + \frac{1}{2}M_Z^2\,Z_\mu Z^\mu\right]\left\{1 + \frac{2}{\mathrm{v}}\,H + \frac{H^2}{\mathrm{v}^2}\right\}$$

1 Scalar Particle H^0 to be Discovered

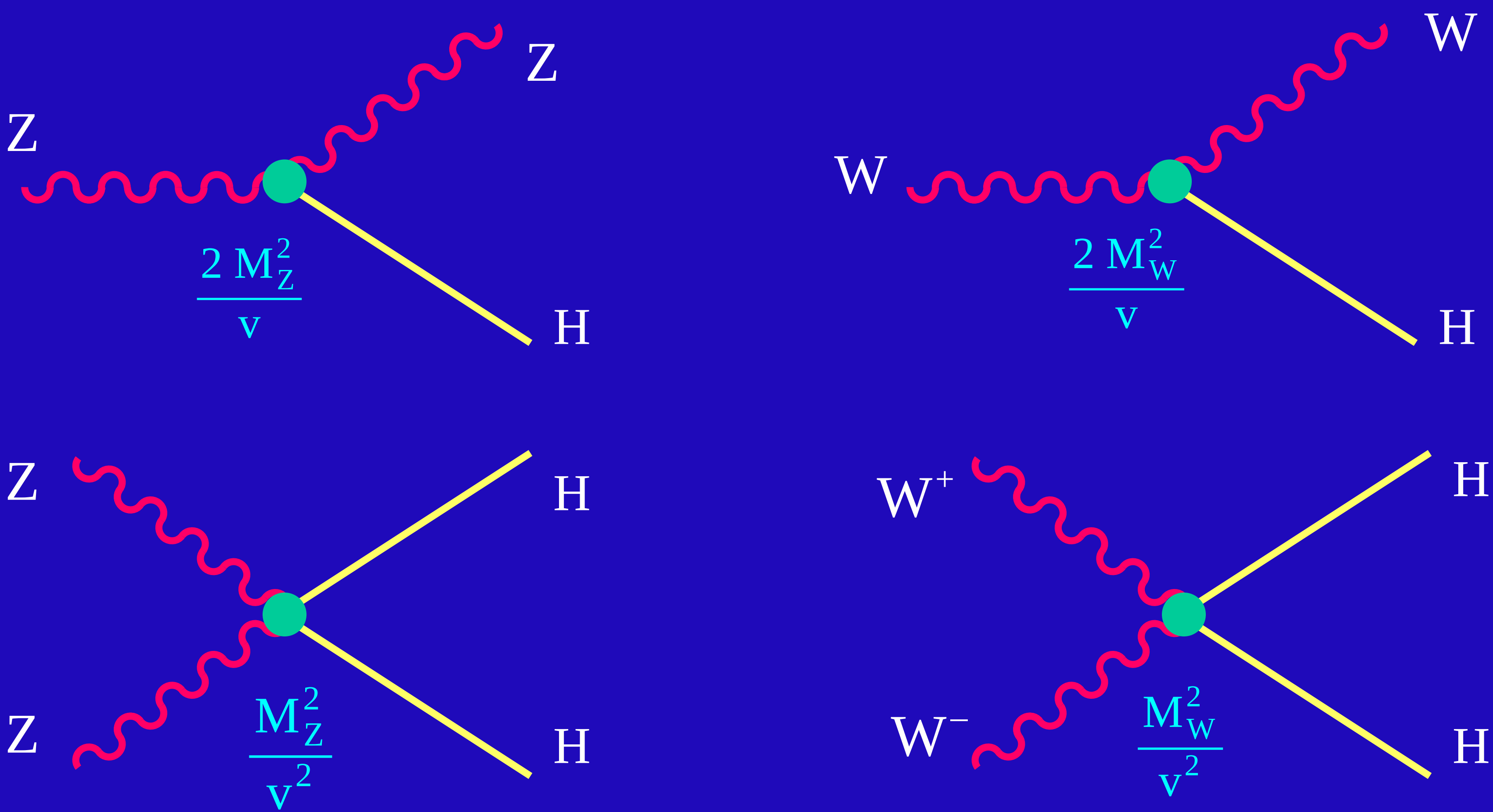
$$M_H = \sqrt{-2\,\mu^2} = \sqrt{2\,h}\;\mathrm{v}$$

Free Parameter

LEP: $114.4\ \mathrm{GeV} < M_H < 158\ \mathrm{GeV}$ (95% CL)

(Direct) (Indirect)

Higgs Couplings $\propto$ Masses



$$v = \left(\sqrt{2}\,G_F\right)^{-1/2} = 246 \quad \text{GeV}$$

Backup Slides

FREE Dirac fermion:

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi$$

Phase Invariance: $\psi \to \psi' = e^{iQ\theta}\,\psi \quad ; \quad \bar{\psi} \to \bar{\psi}' = e^{-iQ\theta}\,\bar{\psi}$

Absolute phases are not observable in Quantum Mechanics

GAUGE PRINCIPLE: $\theta = \theta(x)$

Phase Invariance should hold LOCALLY

BUT $\partial_\mu\psi \to e^{iQ\theta}\,(\partial_\mu + i\,Q\,\partial_\mu\theta)\,\psi$

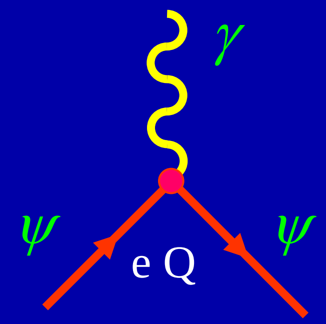
SOLUTION: Covariant Derivative

$$D_\mu\psi \equiv (\partial_\mu + i\,e\,Q\,A_\mu)\,\psi \quad \to \quad e^{iQ\theta}\,D_\mu\psi$$

One needs a spin-1 field A_μ satisfying $A_\mu \to A_\mu - \frac{1}{e}\,\partial_\mu\theta$

QUANTUM ELECTRODYNAMICS

$$\mathcal{L} = \bar{\psi}\,(i\,\gamma^\mu D_\mu - m)\,\psi$$
$$= \bar{\psi}\,(i\,\gamma^\mu \partial_\mu - m)\,\psi \;-\; e\,Q\;A_\mu\;(\bar{\psi}\,\gamma^\mu \psi\,)$$



Kinetic term: $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

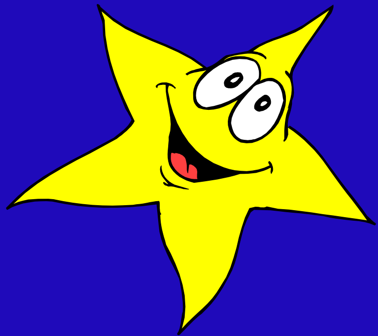
$$\mathcal{L}_K = -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} \;\Rightarrow\; \partial_\mu F^{\mu\nu} = eQ\;(\bar{\psi}\,\gamma^\nu\,\psi\,)$$ Maxwell

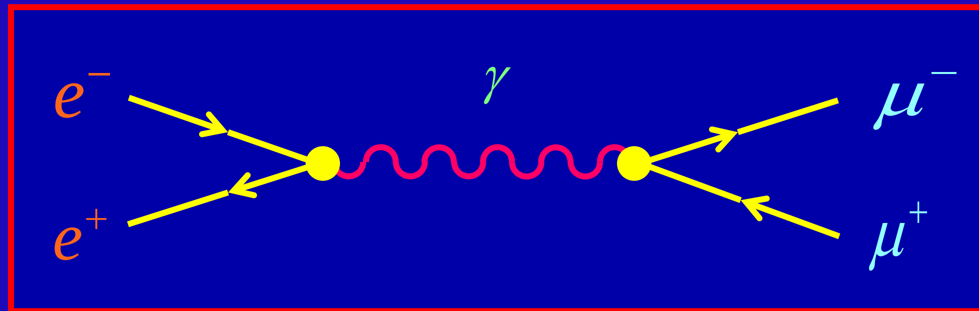
Mass term: $[\,\text{exp:}\quad m_\gamma < 1\cdot 10^{-18}\ \text{eV}\,]$

$$\mathcal{L}_M = \frac{1}{2}\,m_\gamma^2\;A^\mu A_\mu$$ Not Gauge Invariant $\Rightarrow$ $m_\gamma = 0$

Gauge Symmetry $\Rightarrow$ QED Dynamics

Successful Theory

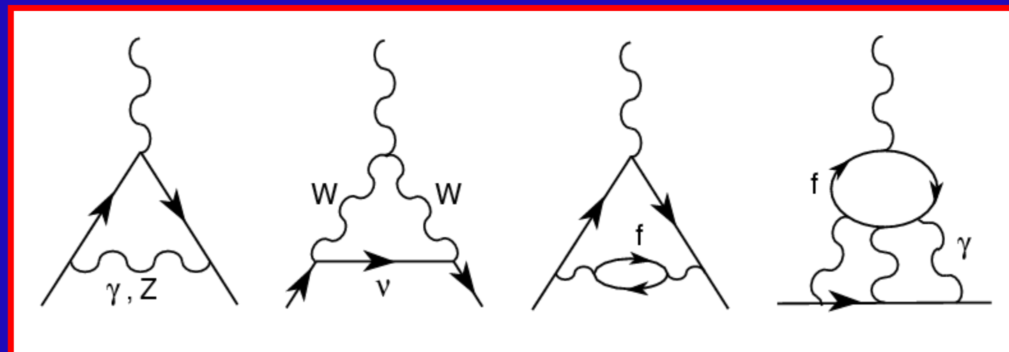




Anomalous Magnetic Moment

$$\mu_l \equiv g_l \, \frac{e}{2\, m_l}$$

$$a_l \equiv \frac{1}{2}\,(g_l - 2)$$



$$a_e = (1\,159\,652\,180.73 \pm 0.28) \times 10^{-12} \quad \Rightarrow \quad \alpha^{-1} = 137.035\,999\,084 \,\pm\, 0.000\,000\,051$$

$$\Rightarrow \quad a_\mu^{\mathrm{th}} = (11\,659\,184 \pm 5) \times 10^{-10} \qquad [\,\mathrm{Exp:}\quad (11\,659\,208.9 \pm 6.3) \times 10^{-10}\,]$$

QUANTUM CHROMODYNAMICS

$$\mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

FREE QUARKS: $\mathcal{L} = \bar{\mathbf{q}}\,[\,i\gamma^\mu\partial_\mu - m\,]\,\mathbf{q}$ $\qquad N_C = 3$

SU(3) Colour Symmetry: $\mathbf{q} \to \mathbf{U}\,\mathbf{q} \quad ; \quad \bar{\mathbf{q}} \to \bar{\mathbf{q}}\,\mathbf{U}^\dagger$

$$\mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\mathbf{U} = \mathbf{1} \quad ; \quad \det\mathbf{U} = 1 \quad ; \quad \mathbf{U} = \exp\left\{ i\,\frac{\lambda^a}{2}\,\theta_a \right\}$$

Gauge Principle: Local Symmetry $\qquad \theta_a = \theta_a(x)$

$$\mathbf{D}^\mu\mathbf{q} \equiv (\mathbf{I}_3\,\partial^\mu + i\,g_s\,\mathbf{G}^\mu)\,\mathbf{q} \quad\to\quad \mathbf{U}\;\mathbf{D}^\mu\mathbf{q}$$

$$\mathbf{D}^\mu \to \mathbf{U}\;\mathbf{D}^\mu\,\mathbf{U}^\dagger \quad ; \quad \mathbf{G}^\mu \to \mathbf{U}\,\mathbf{G}^\mu\,\mathbf{U}^\dagger + \frac{i}{g_s}(\partial^\mu\mathbf{U})\;\mathbf{U}^\dagger$$

$$[\mathbf{G}^\mu]_{\alpha\beta} \equiv \frac{1}{2}\,(\lambda^a)_{\alpha\beta}\;G^\mu_a(x)$$

8 Gluon Fields

Infinitesimal SU(3) Transformation:

$$\delta \mathrm{q}^{\alpha} = i\ \delta\theta_a \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} \mathrm{q}^{\beta} \qquad ; \qquad \delta G_a^{\mu} = -\frac{1}{g_s}\, \partial^{\mu}(\delta\theta_a) - f^{abc}\, \delta\theta_b\, G_c^{\mu}$$

Non Abelian Group:

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = i\ f^{abc}\, \frac{\lambda^c}{2}$$

- δG_a^{μ} depends on G_a^{μ}
- Universal g_s
- Same colour charge for all quarks

Kinetic Term:

$$\mathbf{G}^{\mu\nu} \equiv -\frac{i}{g_s}\,[\mathbf{D}^{\mu},\mathbf{D}^{\nu}] = \partial^{\mu}\mathbf{G}^{\nu} - \partial^{\nu}\mathbf{G}^{\mu} + i\,g_s\,[\mathbf{G}^{\mu},\mathbf{G}^{\nu}] \quad\to\quad \mathbf{U}\;\mathbf{G}^{\mu\nu}\;\mathbf{U}^{\dagger}$$

$$\mathbf{G}^{\mu\nu} \equiv \frac{\lambda^a}{2}\,G_a^{\mu\nu} \quad ; \quad G_a^{\mu\nu} = \partial^{\mu}G_a^{\nu} - \partial^{\nu}G_a^{\mu} - g_s\, f^{abc}\, G_b^{\mu}\, G_c^{\nu}$$

$$\mathcal{L}_K = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\mathbf{G}_{\mu\nu}) = -\frac{1}{4}\,G_a^{\mu\nu}\,G^a_{\mu\nu}$$

Mass Term:

$$\mathcal{L}_M = \frac{1}{2}\,m_G^2\;G_a^{\mu}\,G^a_{\mu}$$

Not Gauge Invariant ➡ $m_G = 0$

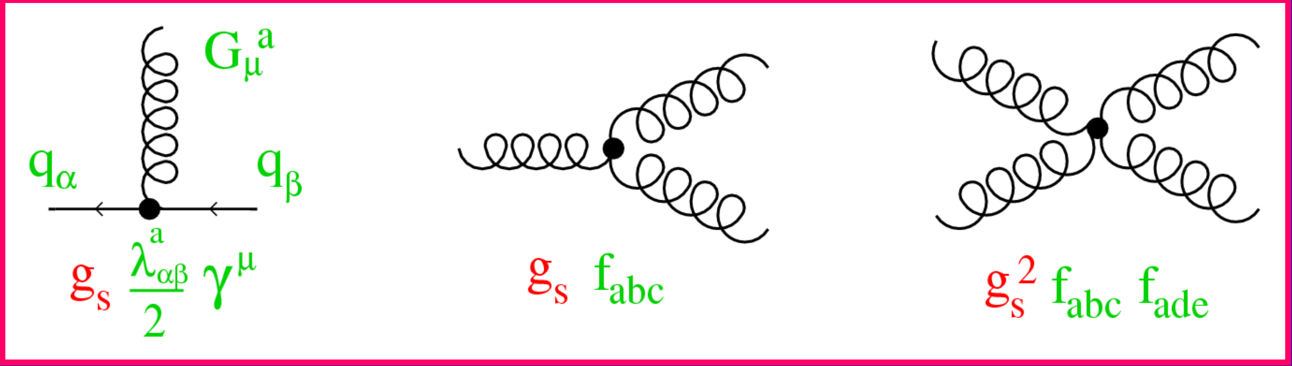
Massless Gluons

$$\mathcal{L}_{QCD} = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\,\mathbf{G}_{\mu\nu}) + \bar{\mathbf{q}}\,[\,i\,\gamma^{\mu}\,\mathbf{D}_{\mu} - m_q\,]\,\mathbf{q}$$

$$= -\frac{1}{4}\left(\partial^{\mu}G^{\nu}_{a} - \partial^{\nu}G^{\mu}_{a}\right)\left(\partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu}\right) + \sum_{q}\bar{q}_{\alpha}\,[\,i\,\gamma^{\mu}\partial_{\mu} - m_q\,]\,q_{\alpha}$$

$$-\frac{1}{2}\sum_{q} g_s\,[\bar{q}_{\alpha}\,(\lambda^{a})_{\alpha\beta}\,\gamma^{\mu}q_{\beta}\,]\,G^{a}_{\mu}$$

$$+\frac{1}{2}\,g_s\,f_{abc}\,(\partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu})\,G^{\mu}_{b}\,G^{\nu}_{c} - \frac{1}{4}\,g_s^2\,f_{abc}\,f_{ade}\,G^{\mu}_{b}\,G^{\nu}_{c}\,G^{d}_{\mu}\,G^{e}_{\nu}$$



- **Gluon Self – interactions** $\quad G^3\ ,\ G^4$
- **Universal Coupling** $\mathbf{g_s}$ **(No Colour Charges)**