

"Two-loop numerical results for Bhabha process at small and large angles"

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Outline

- 1 Introduction
- 2 Virtual corrections
- 3 Hadronic part: more insight
- 4 Adding real radiation
 - Low energy
 - High energy

The Luminosity Monitor

Luminosity of a collider depends on the **machine** and the **beam**:
all is complicated (not mentioning errors estimation)

$$\frac{dN}{dt} = \mathcal{L}(t), \quad N = \sigma \int dt \mathcal{L}(t) = \sigma \mathcal{L} \quad (1)$$

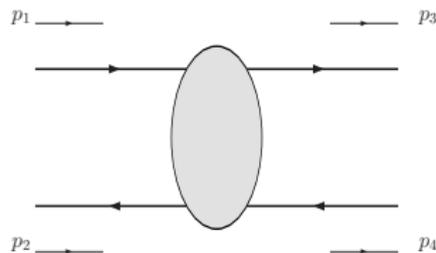
better is to choose a well known process:

$$\mathcal{L} = \frac{1}{\sigma_{Bhabha}} N_{Bhabha}$$

then we may determine any cross section in (1)

Kinematical Regions for Bhabha

Two regions where the Bhabha-scattering cross section is **large** and **QED** dominated



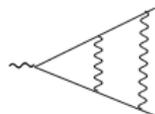
$$s = (p_1 + p_2)^2 = 4E^2 > 4m_e^2, \quad t = (p_1 - p_3)^2 = -4(E^2 - m_e^2) \sin^2 \frac{\theta}{2} < 0$$

- $\sqrt{s} \sim 10^2 \text{ GeV} \Rightarrow$ **small θ**
- SABS $\Rightarrow \mathcal{L}$ at LEP, ILC
 \sim a few degrees

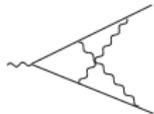
- $\sqrt{s} \sim 1\text{-}10 \text{ GeV} \Rightarrow$ **large θ**
- LABS $\Rightarrow \mathcal{L}$ at KLOE
 $\theta \sim 55^\circ - 125^\circ$

ILC: 31-63 mrad: 1.78-3.61 deg

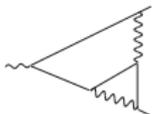
NNLO photonic and fermionic $N_f = 1, 2$ topologies



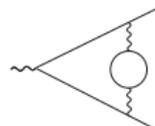
V1



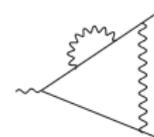
V2



V3



V4



V5



B1



B2



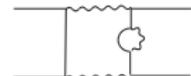
B3



B4



B5

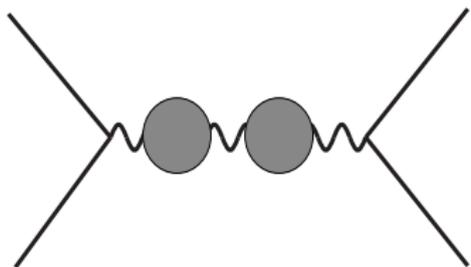
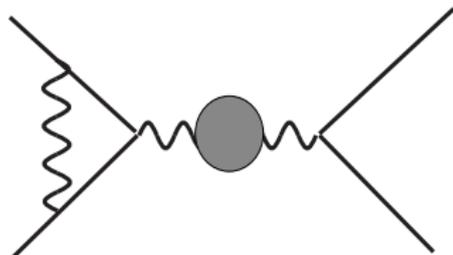
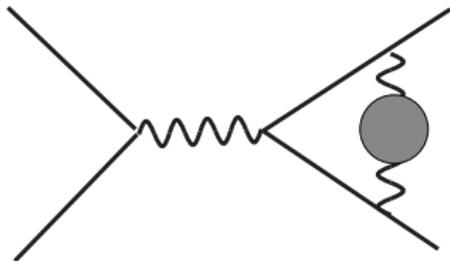
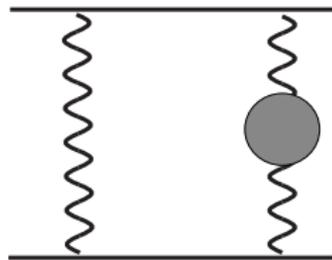


B6

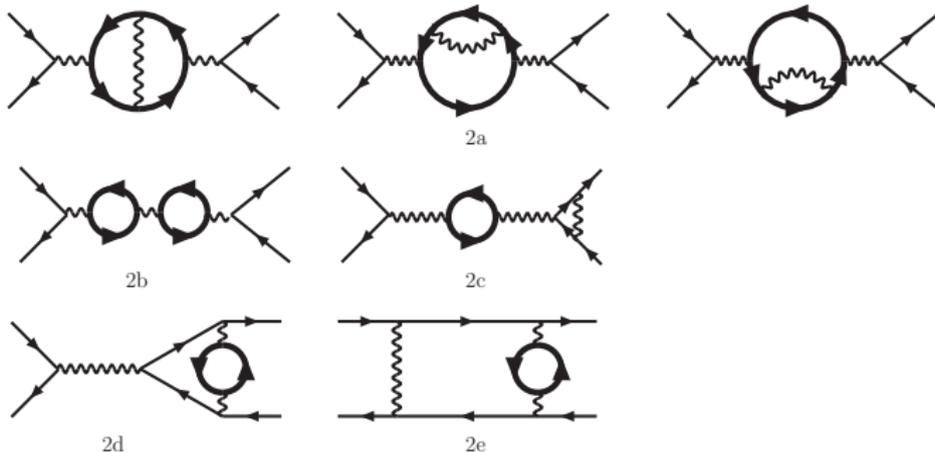
2-loop boxes for Bhabha process

- ▶ **SE** loop insertions (without photonic line) are so called **fermionic** diagrams, rest represents **photonic**.
- ▶ Closed fermionic loop can be muon, tau, top or light quarks.
- ▶ In general, box **B5** is a 4-scale problem: $m_e, m_f, s, t(u)$.

Basic topologies

*(a)**(b)**(c)**(d)*

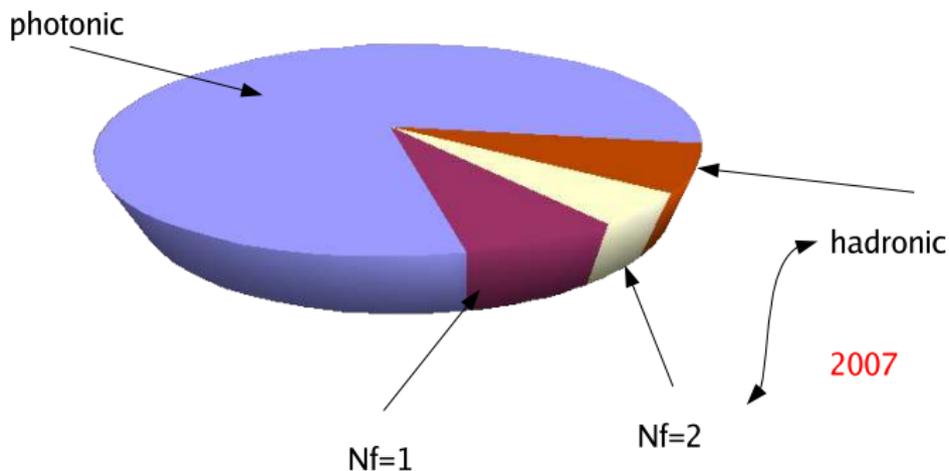
more precisely



Classes of Bhabha-scattering **2-loop diagrams** containing at least one fermion loop. After combining the **2-loop** terms with the **loop-by-loop** terms and with **soft real** corrections:

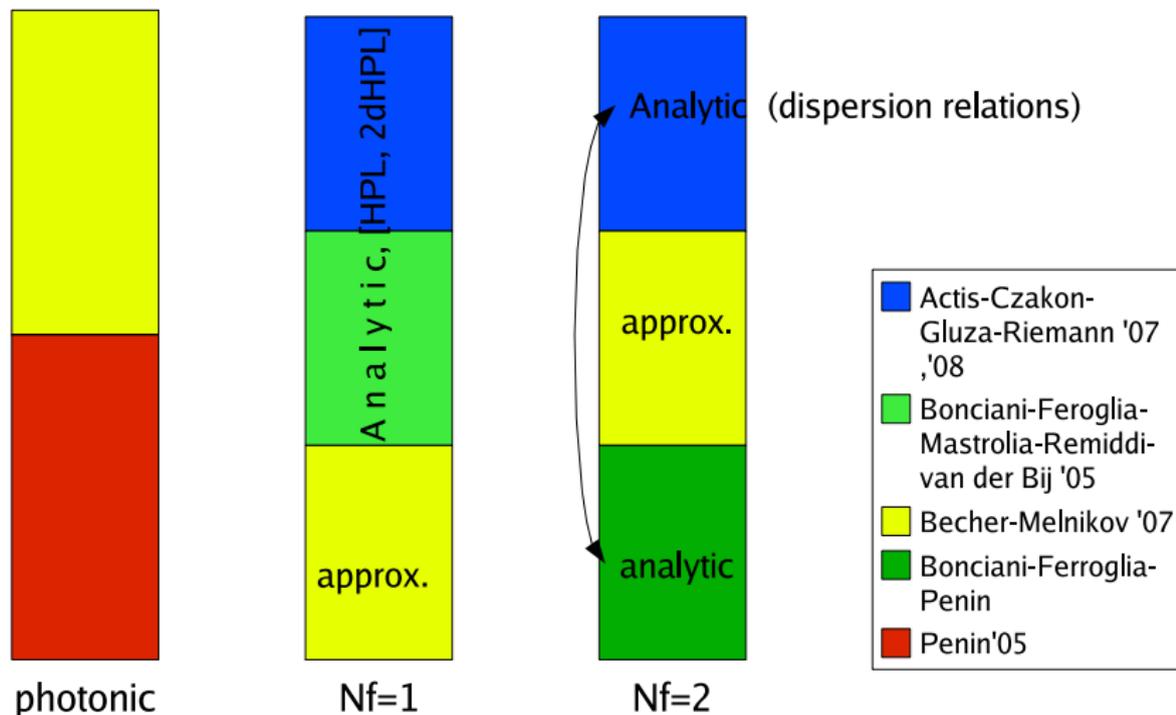
$$\frac{d\sigma^{\text{NNLO}}}{d\Omega} + \frac{d\sigma_{\gamma}^{\text{NLO}}}{d\Omega} = \frac{d\sigma^{\text{NNLO},e}}{d\Omega} + \sum_{f \neq e} Q_f^2 \frac{d\sigma^{\text{NNLO},f^2}}{d\Omega} + \sum_{f \neq e} Q_f^4 \frac{d\sigma^{\text{NNLO},f^4}}{d\Omega} + \sum_{f_1, f_2 \neq e} Q_{f_1}^2 Q_{f_2}^2 \frac{d\sigma^{\text{NNLO},2f}}{d\Omega}.$$

Status



Remarkable: photonic, $N_f = 1$, $N_f = 2$ NNLO corrections doubly (triply) cross-checked

Present situation, virtual NNLO QED



How to evaluate the $N_f = 2$ diagrams?

We did it in 2 ways

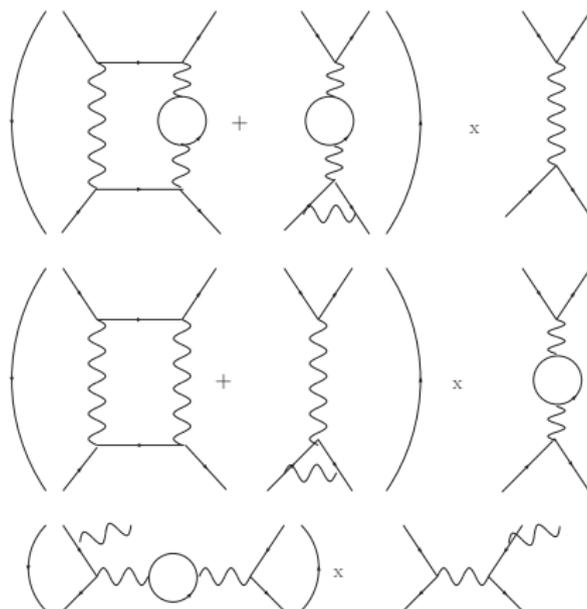
- Decompose the 2-loop integrals to master integrals, solve them.
Here: In the limit $m_e^2 \ll m_f^2 \ll s, t, u$
Actis, Czakon, JG, Riemann, NPB 786 (2007)
- Alternatively, rewrite the 2-loop integrals as dispersion integrals.
Actis, Czakon, JG, Riemann, PRD (2008)
Decompose the loop integrals afterwards into master integrals
The master integrals are simpler, of one-loop type, but the numerical dispersion integration remains then.

Advantages of the dispersion integrals:

- get easily the range $m_e^2 \ll m_f^2, s, t, u$
- method applies also to hadronic insertions

Cut dependent results, Actis, Czakon, JG, Riemann, PRD2008

but boxes IR finite in the following minimal set “finite remainder” in the following tables:



Cut dependent results, Actis, Czakon, JG, Riemann, PRD2008

θ [°] \sqrt{s} [GeV]	$\theta = 20$ 1	$\theta = 20$ 10	$\theta = 3$ M_Z	$\theta = 3$ 500
QED Born	214.903	2.14903	53.0348	1.76398
weak Born	214.903	2.14930	53.0376	1.76390
vertices [$\mu+\tau$ +hadr.]	-0.001086	-0.00022513	-0.007982	-0.00129296
vertices [e]	-0.102787	-0.00325449	-0.092546	-0.00574577
rest: e	0.235562	0.00497834	0.135650	0.00672652
μ	0.009518	0.00135040	0.040792	0.00287809
	-0.017214	0.00134282	0.040688	0.00287795
τ	0.000074	0.00005385	0.002706	0.00087639
	×	×	-0.009610	0.00083969
hadr.	0.008642	0.00269490	0.087618	0.00810781

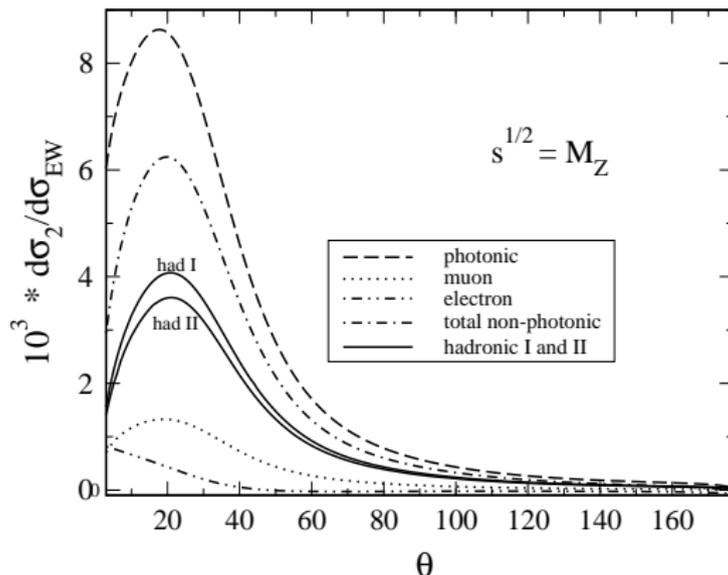
Numerical values for the differential cross section in nanobarns, in units of 10^2 . For the finite remainder with irreducible box diagrams we show for each fermion flavor the result obtained through the dispersion-based approach (first line) and the one coming from the analytical expansion (second line), neglecting $\mathcal{O}(m_f^2/x)$, where $x = s, |t|, |u|$. When $m_f^2 > x$, the entry is suppressed.

Cut dependent results, Actis, Czakon, JG, Riemann, PRD2008

\sqrt{s} [GeV]	1	10	M_Z	500
QED Born	466537	4665.37	56.1067	1.86615
weak Born	466526	4654.16	1238.7500	0.92890
vertices [$\mu+\tau$ +hadr.]	-16.351	-2.0437	-0.125208	-0.0104275
vertices [e]	-477.620	-12.3010	-0.298589	-0.0155751
rest: e	807.476	14.5277	0.270575	0.0119285
μ	160.197	6.0819	0.147046	0.0072579
	152.890	6.0809	0.147046	0.0072579
τ	2.383	1.3335	0.075268	0.0045713
	×	1.0739	0.075214	0.0045712
hadr.	232.674	16.0670	0.469944	0.0246035

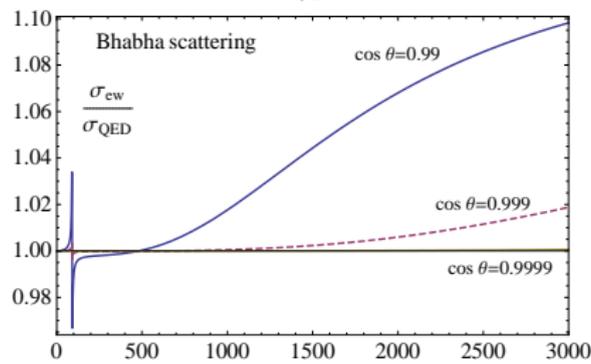
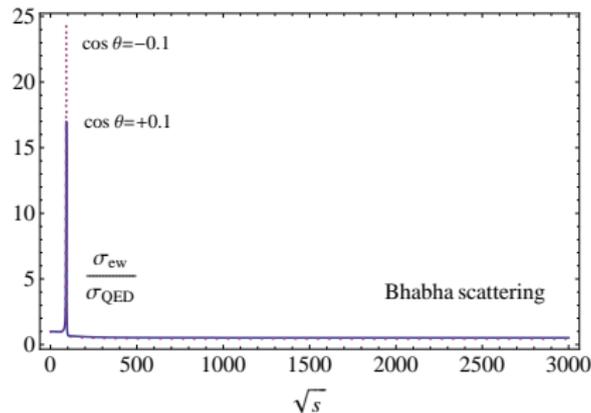
Numerical values for the differential cross section in nanobarns at a scattering angle $\theta = 90^\circ$, in units of 10^{-4} .

Cut dependent results, Actis, Czakon, JG, Riemann, PRD2008

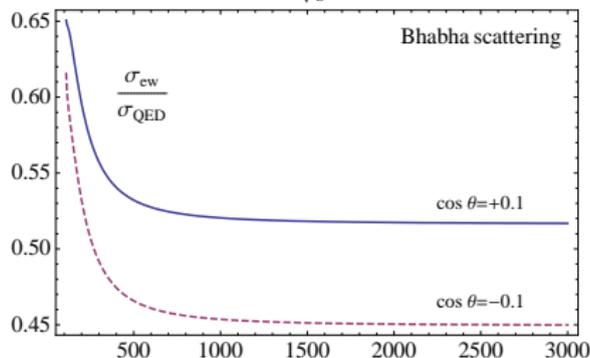
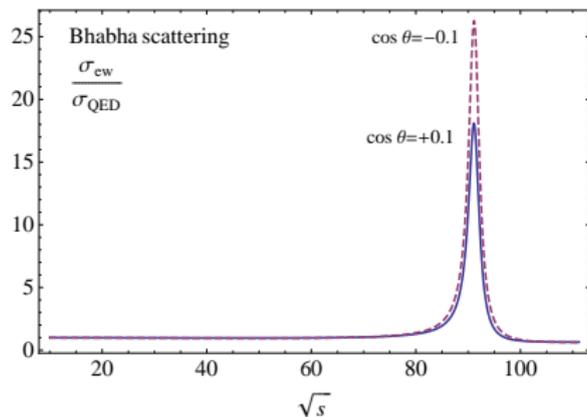


General conclusions on separate contributions not justified for at least two reasons: (i) real radiation not included, (ii) e-weak contributions are substantial for large angles

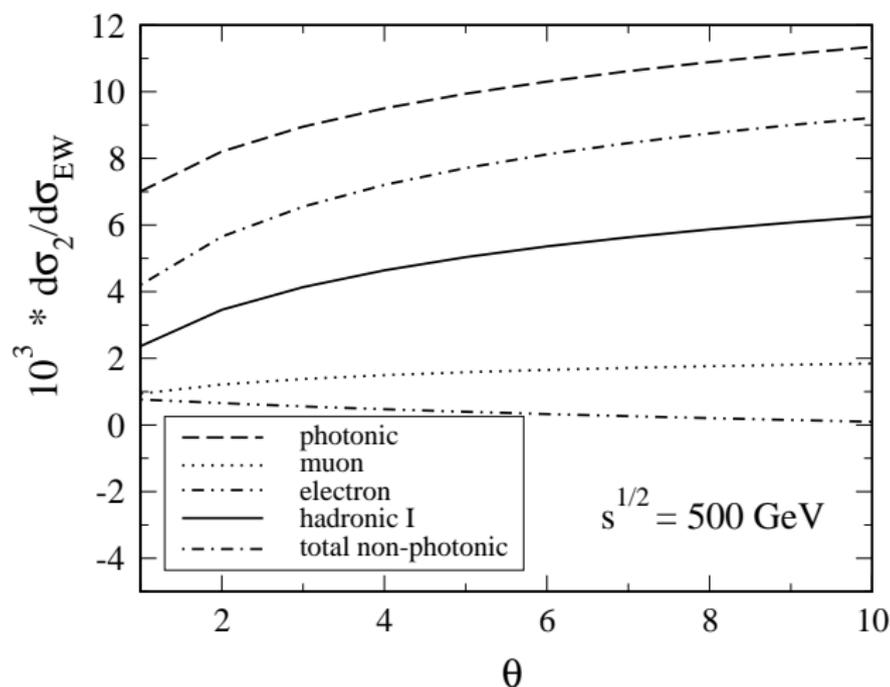
Ratio of electroweak to QED Bhabha scattering cross-section at large and small angles as a function of CoM



Ratio of electroweak to QED Bhabha scattering cross-section at large angles in the energy ranges of LEP1/GigaZ (up) and ILC (down).



Cut dependent results, Actis, Czakon, JG, Riemann, PRD2008



Dispersion integrals

$$\frac{g_{\mu\nu}}{q^2 + i\delta} \rightarrow \frac{g_{\mu\alpha}}{q^2 + i\delta} \left(q^2 g^{\alpha\beta} - q^\alpha q^\beta \right) \Pi_{\text{had}}(q^2) \frac{g_{\beta\nu}}{q^2 + i\delta},$$

the once-subtracted dispersion integral

$$\Pi_{\text{had}}(q^2) = -\frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dz}{z} \frac{\text{Im} \Pi_{\text{had}}(z)}{q^2 - z + i\delta}.$$



Dispersion Relations

- ▶ Obtain fermionic corrections inserting Π_R in $\Delta_\gamma^{\mu\nu}$

$$\frac{g_{\mu\nu}}{q^2 + i\delta} \rightarrow \frac{g_{\mu\alpha}}{q^2 + i\delta} (q^2 g^{\alpha\beta} - q^\alpha q^\beta) \Pi_R(q^2) \frac{g_{\beta\nu}}{q^2 + i\delta}$$

- ▶ Represent Π_R through a dispersion integral

$$\Pi_R(q^2) = -\frac{q^2}{\pi} \int_{4M^2}^{\infty} \frac{dz}{z} \frac{\text{Im } \Pi(z)}{q^2 - z + i\delta}$$

Leptons, top (perturbative)

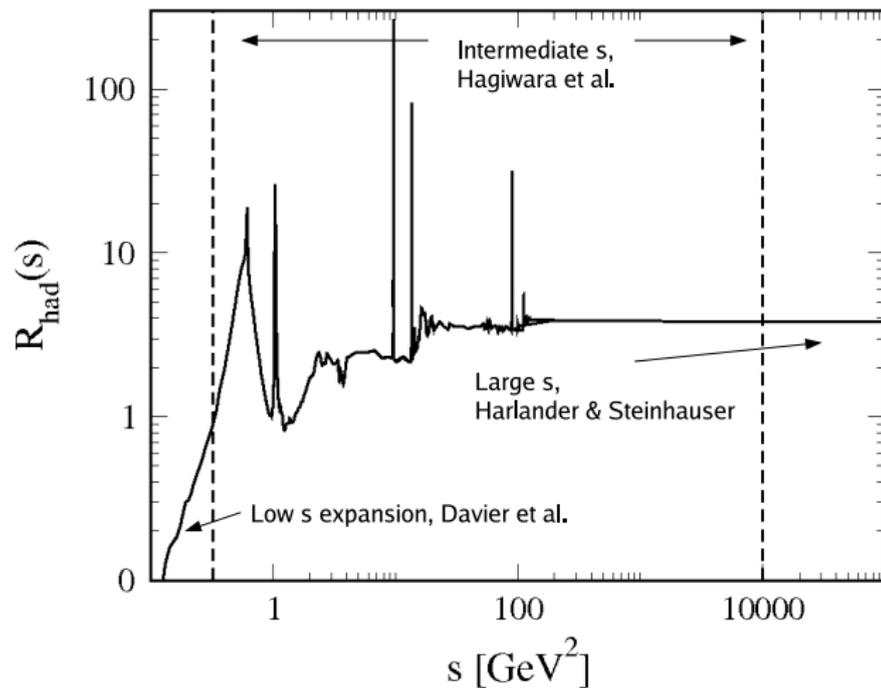
$$\text{Im } \Pi_f(z) = -\left(\frac{\alpha}{\pi}\right) F_\epsilon \left(\frac{m_e^2}{m_f^2}\right)^\epsilon Q_f^2 C_f \times \\ \times \theta(z - 4m_f^2) \frac{\pi}{3} \left\{ \frac{\beta_f(z)}{2} \left[3 - \beta_f^2(z) \right] \right\}$$

Light quarks (non pert.)

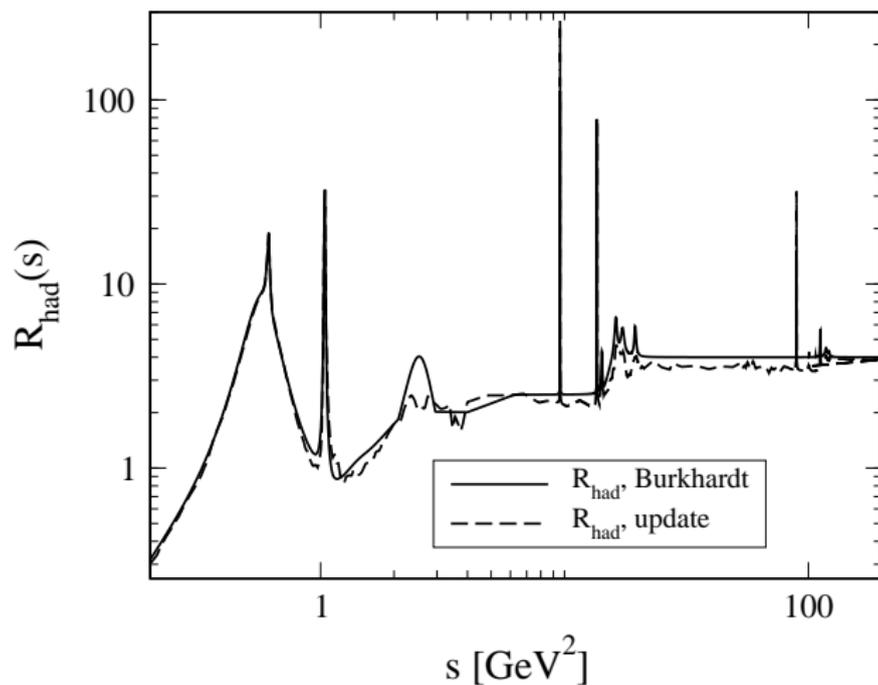
$$\text{Unitarity} \Rightarrow \text{Im } \Pi_{\text{had}}(z) = -\frac{\alpha}{3} R_{\text{had}}(z)$$

$$R_{\text{had}}(z) = \frac{\sigma(\{e^+ e^- \rightarrow \gamma^* \rightarrow \text{hadrons}\}; z)}{(4\pi\alpha^2)/(3z)}$$

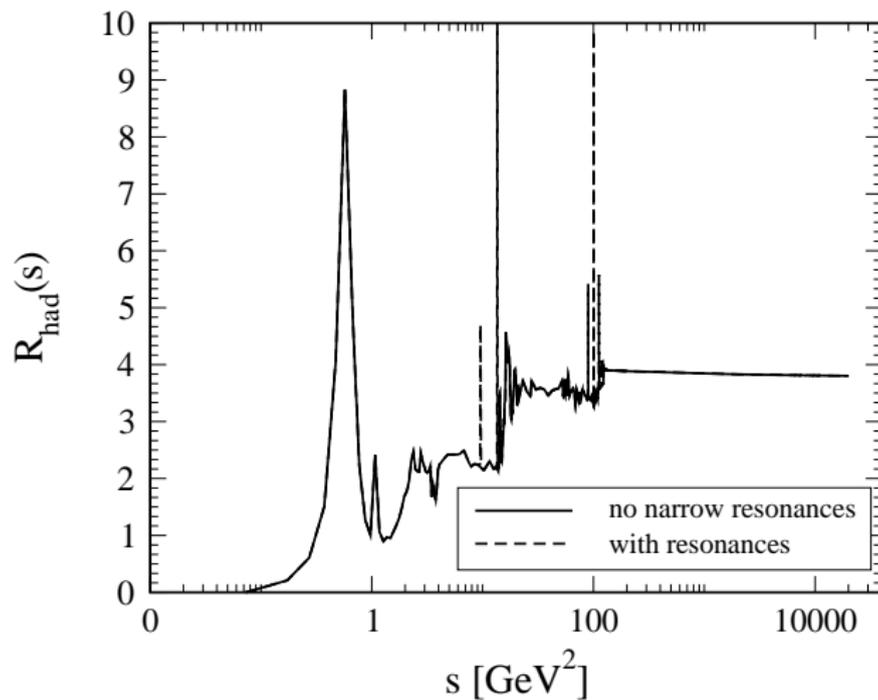
Rhad, Actis, Czakon, JG, Riemann, PRD2008



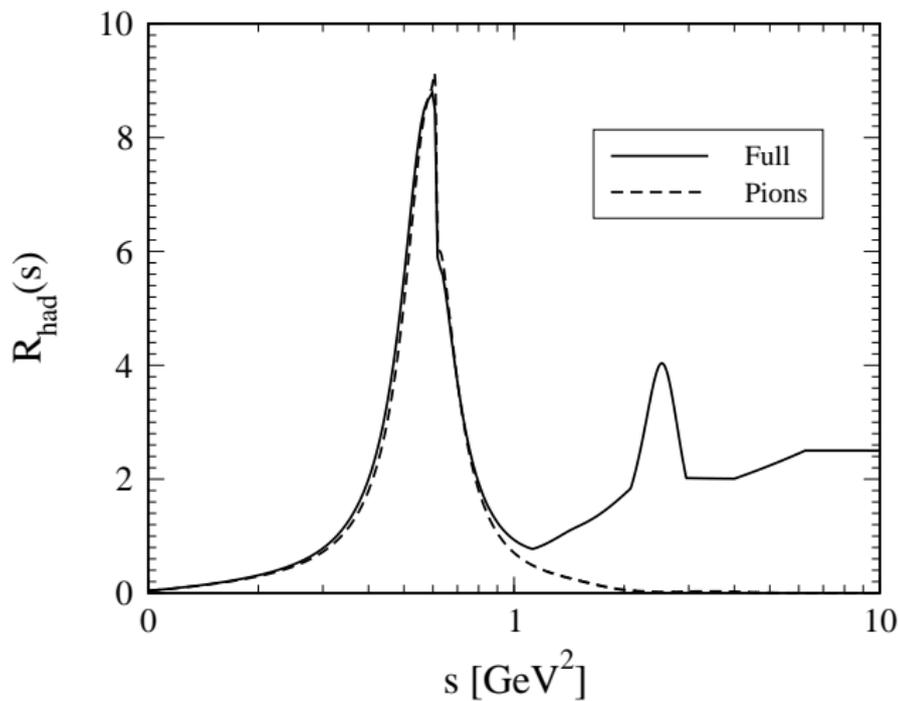
Rhad, Actis, Czakon, JG, Riemann, PRD2008

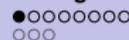


Rhad update 2010 (from Thomas Teubner)



Pion form factor only: does it give reliable description?

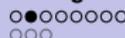




Realistic low energy results, in nanobarnes

Electron pair corrections						
	σ_B	σ_h	σ_{v+s}	σ_{v+s+h}	σ_{pairs}	$ \sigma_{v+s+h}/\sigma_B $
KLOE	529.469	9.502	-11.567	-2.065	0.271	0.0039
BaBar	6.744	0.246	-0.271	-0.025	0.017	0.0037
Muon pair corrections						
	σ_B	σ_h	σ_{v+s}	σ_{v+s+h}	σ_{pairs}	$ \sigma_{v+s+h}/\sigma_B $
KLOE	529.469	1.494	-1.736	-0.241	-	0.00045
BaBar	6.744	0.091	-0.095	-0.004	0.0005	0.00059
Tau pair corrections						
	σ_B	σ_h	σ_{v+s}	σ_{v+s+h}	σ_{pairs}	$ \sigma_{v+s+h}/\sigma_B $
KLOE	529.469	0.020	-0.023	-0.003	-	neglig.
BaBar	6.744	0.016	-0.017	-0.0007	$< 10^{-7}$	neglig.
Pion pair corrections						
	σ_B	σ_h	σ_{v+s}	σ_{v+s+h}	σ_{pairs}	$ \sigma_{v+s+h}/\sigma_B $
KLOE	529.469	1.174	-1.360	-0.186	-	0.00035
BaBar	6.744	0.062	-0.065	-0.003	0.00003	0.00044

Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data, S. Actis et al, EPJ 2009



Work in progress, C. Carloni Calame, H. Czyż, JG, M. Gunia, G. Montagna, O. Nicrosini, F. Piccinini, T. Riemann, M. Worek

B-factories BABAR/PEP-II (SLAC) & BELLE/KEKB (KEK)

(a) $\sqrt{s} = 10.56$ GeV

(b) $|\vec{p}_+|/E_{beam} > 0.75$ and $|\vec{p}_-|/E_{beam} > 0.50$

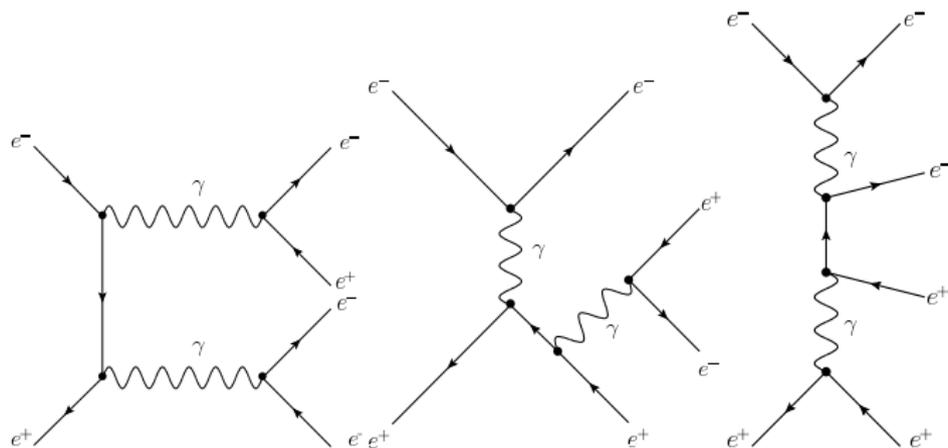
or $|\vec{p}_-|/E_{beam} > 0.75$ and $|\vec{p}_+|/E_{beam} > 0.50$

(c) For $|\cos(\theta_{\pm})|$ the following selections have to be checked

i. $|\cos(\theta_{\pm})| < 0.70$ and $|\cos(\theta_+)| < 0.65$ or $|\cos(\theta_-)| < 0.65$

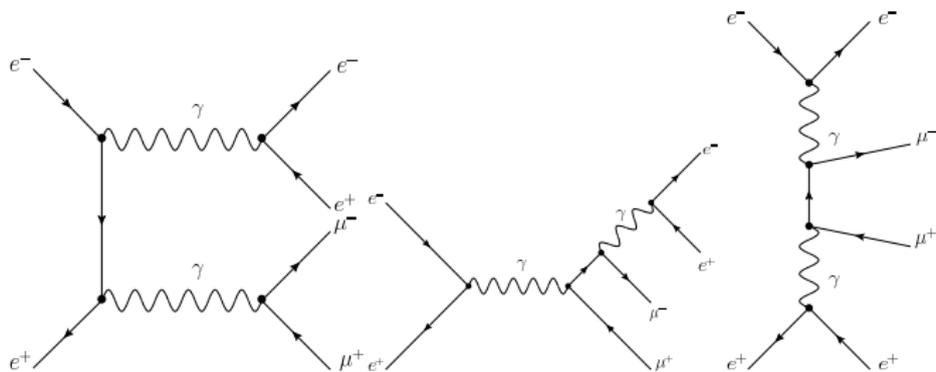
ξ_{max}^{3d}	σ_h	σ_{v+s}	$\sigma_{tot} [= \sigma_{v+s} + \sigma_h]$	σ_{pairs}	σ_{pairs}/σ_B	$(\sigma_{tot} + \sigma_{pairs})/\sigma_B$
electron, BaBar, $E_{cut}/E_{beam} = 10^{-3}$ [GeV]						
20	0.173979(6)	-0.202669	-0.028691(6)			
22	0.174956(6)	-0.202669	-0.027713(6)			
24	0.175799(6)	-0.202669	-0.026870(6)			
26	0.176531(6)	-0.202669	-0.026138(6)			
28	0.177172(6)	-0.202669	-0.025498(6)			
30	0.177736(6)	-0.202669	-0.024933(6)			
...
40	0.179766(6)	-0.202669	-0.022903(6)			
$E_{cut}/E_{beam} = 10^{-4}$ [GeV]						
20	0.242736(8)	-0.271429	-0.028694(8)			
22	0.243714(8)	-0.271429	-0.027716(8)			
24	0.244556(8)	-0.271429	-0.026873(8)			

Real electron pairs



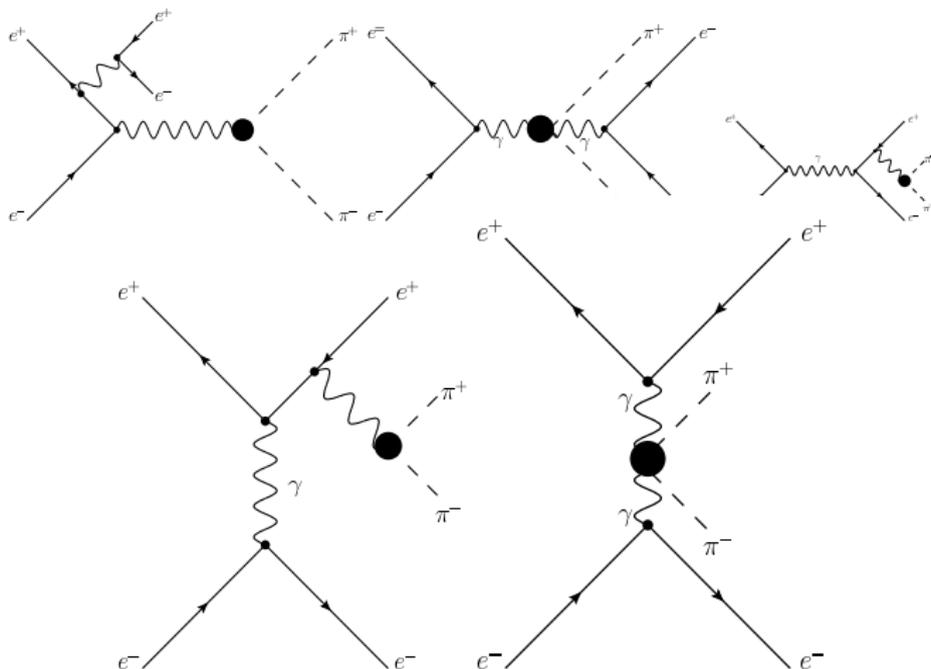
Samples of the 36 diagrams contributing to $e^+e^- \rightarrow e^+e^-(e^+e^-)$, as calculated by M. Worek using Helac-Phegas code.

Real muon pairs



Samples of the 12 diagrams contributing to
 $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$.

Real pion pairs



Sample diagrams with real pion pair emission.

Cuts

The cuts applied for the KLOE experiment are

- $\sqrt{s} = 1.02 \text{ GeV}$,
- $E_{\min} = 0.4 \text{ GeV}$,
- $55^\circ < \theta_{\pm} < 125^\circ$,
- $\xi_{\max} = 9^\circ$,

and for the BaBar experiment

- $\sqrt{s} = 10.56 \text{ GeV}$,
- $|\cos(\theta_{\pm})| < 0.7$ and $|\cos(\theta_+)| < 0.65$ or $|\cos(\theta_-)| < 0.65$,
- $|\vec{p}_+|/E_{\text{beam}} > 0.75$ and $|\vec{p}_-|/E_{\text{beam}} > 0.5$ or
 $|\vec{p}_-|/E_{\text{beam}} > 0.75$ and $|\vec{p}_+|/E_{\text{beam}} > 0.5$,
- $\xi_{\max}^{3d} = 30^\circ$.

Cuts

E_{\min} - the energy threshold for the final-state electron/positron, θ_{\pm} - the electron/positron polar angles, ξ_{\max} - the maximum allowed polar angle acollinearity:

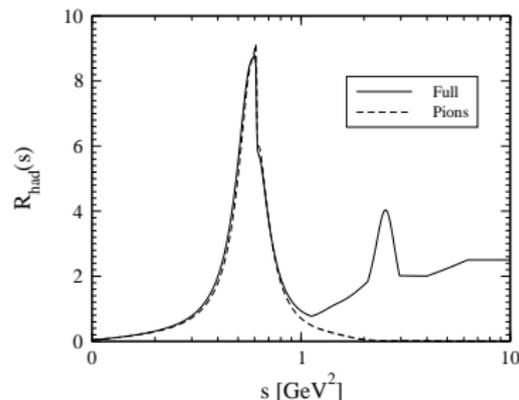
$$\xi = |\theta_+ + \theta_- - 180^\circ|,$$

ξ_{\max}^{3d} is the maximum allowed three dimensional acollinearity:

$$\xi^{3d} = \left| \arccos \left(\frac{\vec{p}_+ \cdot \vec{p}_-}{|\vec{p}_-| |\vec{p}_+|} \right) \times \frac{180^\circ}{\pi} - 180^\circ \right|.$$

For $e^+ e^- \rightarrow e^+ e^- \mu^+ \mu^-$, cuts are applied only to the $e^+ e^-$ pair. In the case of $e^+ e^- \rightarrow e^+ e^- e^+ e^-$, all possible $e^\pm e^\mp$ combinations are checked and if at least one pair fulfils the cuts the event is accepted.

Modelling with pion form-factor only, preliminary



	KLOE	Babar
σ_{S+V}, R_{pions}	-2.58	-0.15
$\sigma_{S+V}, R_{had}, \text{sign}(z,0)$	-0.915	-0.167
$\sigma_{S+V+H}, R_{pions}$	0.48	-0.011
$\sigma_{S+V+H}, R_{had}, \text{sign}(z,0)$	-0.281	-0.0081

- For Babar it looks not bad, for KLOE definitely not (the sign), higher energies?
- adding narrow resonances will not improve

Electron 2-loop virtual+soft+real

$\sqrt{s} = 500$ GeV, cuts for acollinearity: 5 mrad

Dear Wolfgang,

I think it would be very dangerous to cut on the acollinearity. The angular distribution is very steep and even with a good resolution it may introduce distortion difficult to correct for. The energy cut is bad enough. We probably have a natural cut due to the requirement of shower containment, but that's very large angles. ... Best regards, HA

$\omega [\times \sqrt{s}/2]$	10^{-3}	10^{-4}	10^{-5}
virtual and soft	-0.109318	-0.146432	-0.183546
hard	0.084631567	0.012172138	0.015879122
pairs	?	?	?
sum (v+s+h)	-0.024686	-0.0247106	-0.0247551
Born	3.28486	3.28486	3.28486
$ \sigma_{(v+s+h)}/\text{Born} $	0.00751509	0.00752258	0.0075361

Total cross sections in nanobarns, scattering angle 31-63 mrad.

Muon 2-loop virtual+soft+real

$\sqrt{s} = 500 \text{ GeV}$, no cuts on acollinearity

$\omega [\times \sqrt{s}/2]$	10^{-3}	10^{-4}	10^{-5}
virtual and soft	-0.0414923012	-0.0567141686	-0.0719360419
hard	0.037836979	0.053128351	0.06841173
pairs	?	?	?
sum (v+s+h)	-0.003655	-0.0035858	-0.003524
Born	3.28486	3.28486	3.28486
$ \sigma_{(v+s+h)}/\text{Born} $	0.0011	0.0011	0.0011

Total cross sections in nanobarns, scattering angle 31-63 mrad.

hadronic contributions? to be done

Conclusions

- due to planned kinematical cuts, numerical predictions for NNLO Bhabha scattering at small angles are possible
- numerics for real pair emission needs clarification (negligible?)
- large angles needs much more work - electroweak contributions enters