Cosmic String Loop Distribution with a Gravitational Wave Cutoff

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Outline

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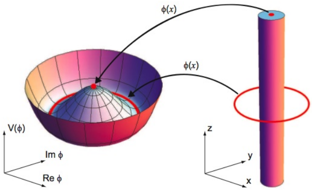
What are cosmic strings?

- Cosmic strings are line-like defects produced in GUT phase transitions in the early Universe (Kibble 1976).
- After formation, the cosmic string network undergoes
  - stretching,
  - intercommutation events and
  - energy loss by gravitational radiation.

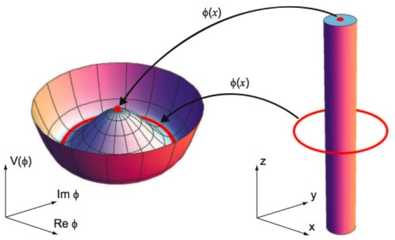


Source: J. Rocha (2008)

Under these effects, the network rapidly reaches a scaling regime.

What are cosmic strings?

- Energy density $\rho_\infty \propto 1/d_\mathrm{h}^2$: strings never dominate the Universe at late times.
- Long cosmic strings are characterized by their tension U (or dimensionless GU, $G \equiv m_{\mathrm{P}l}^{-2}$).
- Once a candidate for structure formation, now constrained by WMAP to contribute $< 10\%$.

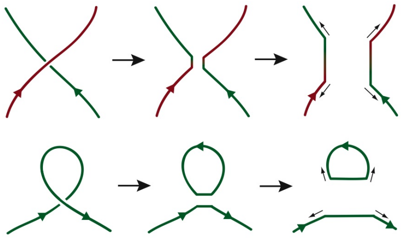


Source: J. Rocha (2008)

for $a \propto t^\nu$: $d_\mathrm{h}(t) = \dfrac{t}{1-\nu}$

from CMB: $GU < 7 \times 10^{-7}$

Cosmic string loops



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- Loops are formed at intersection or auto-commutation events ($P = 1$ for Nambu Goto).
- Loops evacuate energy density from the network so that scaling is reached.
- Cosmic string loops are characterized by their length ℓ.

We study the number density distribution $\mathbf{n(\ell, t)}$ of cosmic string loops of size ℓ at time t.

Outline

Distribution of loops

At which size are cosmic string loops typically formed?

- "one scale model" (Kibble 1976):
  length ℓ given by typical distance between long strings, *i.e.* (fraction of) the horizon size d_{h}
- but simulations (Ringeval et al. 2005) indicate that there is also a loop population at smaller ℓ!

Distribution of loops

At which size are cosmic string loops typically formed?

- "one scale model" (Kibble 1976): length ℓ given by typical distance between long strings, *i.e.* (fraction of) the horizon size d_h
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using $\alpha = \ell / d_\mathrm{h}$: $$\frac{\mathrm{d}n}{\mathrm{d}\alpha} = \frac{\mathcal{S}(\alpha)}{\alpha d_\mathrm{h}^3}\,, \qquad \mathcal{S}(\alpha) = C_\circ\, \alpha^{-p}$$

$\mathcal{S}(\alpha)$ is the "scaling function", where $C_\circ$ and p are fixed from simulations

But:
Simulations do not include the gravitational wave emission (GW) of loops which dominantes for scales $\alpha < \alpha_\mathrm{d} \propto \Gamma G U$.

The Polchinski Rocha model of loop formation

Polchinski & Rocha (2006, 2007):

- consider stretching the dominant process for long cosmic strings and add loop formation as a perturbation
- loop production function obtained from tangent vector correlators along long strings in scaling
- with $\mathrm{d}\langle N\rangle$ the average loop number and $\mathrm{d}\sigma$ the unit distance along the long string, it is found that

$$\frac{\mathrm{d}\langle\mathcal{N}\rangle}{\mathrm{d}\sigma\,\mathrm{d}\ell\,\mathrm{d}t} \propto \frac{1}{\ell^3}\left(\frac{\ell}{t}\right)^{2\chi}$$

loop production (LP)

power law shape in good agreement with simulations

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Rocha (2007):

- study evolution of loop number density with LP and GW

Extending the Polchinski Rocha model

The emission of gravitational waves (for $\alpha < \alpha_{\rm d} = \Gamma GU$) renders the cosmic string loops *smoother and smoother* on short scales.

No (or much less) loops should be formed below the scale of gravitational backreaction (BR) given by $\alpha_{\rm c} = \Upsilon (GU)^{1+2\chi}$.

here: Γ, Υ coefficients with $\Gamma \simeq 10^2$, $\Upsilon \simeq 10$

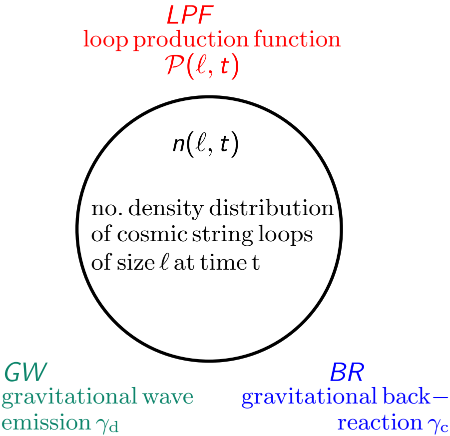
Our approach:

- Use Polchinski Rocha loop production function with coefficients adjusted to numerical simulations in the region $\alpha > \alpha_{\rm d}$.
- Account for gravitational wave emission.
- Change loop production function below $\alpha_{\rm c}$ to phenomenologically include gravitational backreaction.
- Do not neglect transient solutions.

What is the "full scaling regime"?

Outline

Loop number density distribution $n(\ell, t)$



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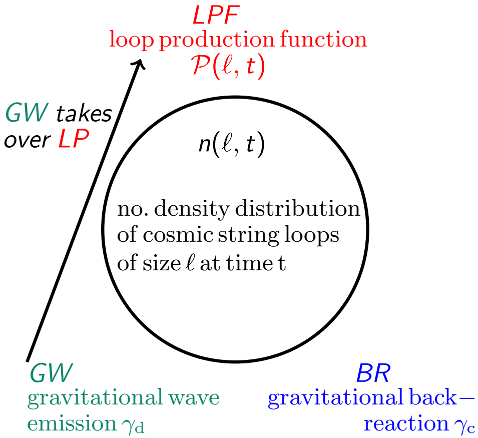
Loop number density distribution $n(\ell, t)$

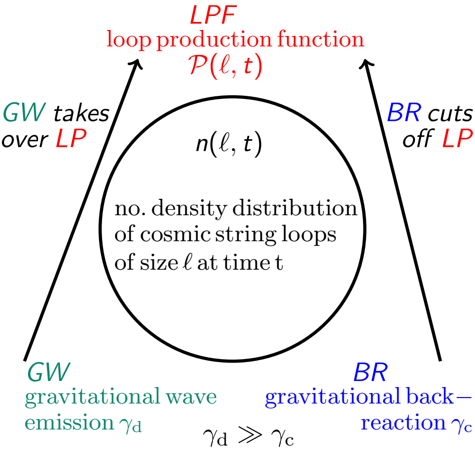


$$\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) = a^3 \mathcal{P}(\ell, t)$$

shrinking by GW emission

$$\frac{\partial}{\partial t}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) - \gamma_{\mathrm{d}} \frac{\partial}{\partial \ell}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) = a^3 \mathcal{P}(\ell, t)$$

Loop number density distribution $n(\ell, t)$



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shrinking by GW emission:

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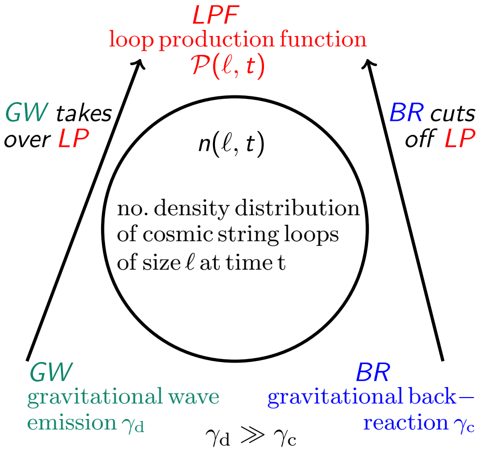

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change of variables:

- $(\ell, t) \to (\gamma, t)$ with $\gamma \equiv \frac{\ell}{t}$
- $\mathcal{F}(\gamma, t) \equiv \mathrm{d}n/\mathrm{d}\ell$

Evolution of $\mathcal{F}(\gamma, t)$

$$t\frac{\partial(a^3\mathcal{F})}{\partial t} - (\gamma + \gamma_\mathrm{d})\frac{\partial(a^3\mathcal{F})}{\partial\gamma} = a^3 t\mathcal{P}(\gamma, t)$$

- $\gamma_\mathrm{d} = \Gamma GU$: GW emission scale, $\gamma_\mathrm{c} = \Upsilon(GU)^{1+2\chi}$: gravitational BR scale
- $\mathcal{P}(\gamma, t)$ piecewise, but continuous for $\gamma > \gamma_\mathrm{c}$ and $\gamma < \gamma_\mathrm{c}$
- variable ranges for γ and t:
  - $0 < \gamma < \gamma_\mathrm{max}$ (horizon-sized loops)
  - $t_\mathrm{ini} < t < t_0$

What is a phenomenological form for $\mathcal{P}$?

Outline

Shape of the loop production function $\mathcal{P}(\gamma, t)$

- Polchinski Rocha: $t^5 \mathcal{P}(\gamma, t) = c\,\gamma^{2\chi - 3}$
- for $\gamma > \gamma_{\rm d}$, fix parameters c and χ from simulations *(this includes "loops from loops" etc.)*
- assumption for $\gamma < \gamma_{\rm c}$: $t^5 \mathcal{P}(\gamma, t) = c_{\rm c}\,\gamma_{\rm c}^{2\chi_{\rm c} - 3}$
- impose continuity at $\gamma = \gamma_{\rm c}$, *i.e.* $c_{\rm c} = c\gamma_{\rm c}^{2(\chi - \chi_{\rm c})}$

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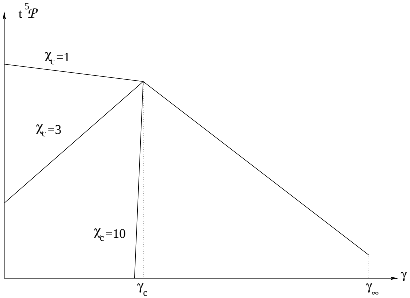

$1 \le \chi_{\rm c} < \infty$: details of smoothing

solve evolution equation in the two domains and choose initial conditions

Solving the evolution equation

$$t\frac{\partial(a^3\mathcal{F})}{\partial t} - (\gamma + \gamma_\mathrm{d})\frac{\partial(a^3\mathcal{F})}{\partial \gamma} = a^3 t \begin{cases} c\gamma^{2\chi-3}, & \gamma > \gamma_\mathrm{c} \\ c_\mathrm{c}\gamma^{2\chi_\mathrm{c}-3}, & \gamma < \gamma_\mathrm{c} \end{cases}$$

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The solutions to this equation with $a \propto t^\nu$ are:

$$\mathcal{F}(\gamma, t) = \frac{c_{\mathrm{c}}}{\mu_{\mathrm{c}}}\frac{(\gamma + \gamma_{\mathrm{d}})^{2\chi_{\mathrm{c}}-3}}{t^4}\,{}_2\mathrm{F}_1\left(3 - 2\chi_{\mathrm{c}}, \mu_{\mathrm{c}}; \mu_{\mathrm{c}} + 1; \frac{\gamma_{\mathrm{d}}}{\gamma + \gamma_{\mathrm{d}}}\right) + \frac{\mathcal{I}_{\mathrm{c}}(\gamma t + \gamma_{\mathrm{d}} t)}{a^3}$$

- $\mu_{\mathrm{c}} \equiv 3\nu - 2\chi_{\mathrm{c}} - 1$
- $\mathcal{I}_{\mathrm{c}}(x)$ are two unknown functions related by continuity:

$$\mathcal{I}_{\mathrm{c}}(x) = \mathcal{I}(x) + K\frac{(\gamma_{\mathrm{d}} + \gamma_{\mathrm{c}})^4}{x^4}\left[a\left(\frac{x}{\gamma_{\mathrm{c}} + \gamma_{\mathrm{d}}}\right)\right]^3,$$

where K is a constant

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  where K is a constant
- The unknown functions can be fixed from the initial loop number density distribution $\mathcal{N}_\mathrm{ini}(\ell)$ at t_ini.

With $\mathcal{I}(x)$ and $\mathcal{I}_{\rm c}(x)$ fixed, the solution has **three** parts:

$$t^4\mathcal{F}(\gamma \geq \gamma_{\rm c}, t) = \left(\frac{t}{t_{\rm ini}}\right)^4 \left(\frac{a_{\rm ini}}{a}\right)^3 t_{\rm ini}^4\, \mathcal{N}_{\rm ini}\left\{\left[\gamma + \gamma_{\rm d}\left(1 - \frac{t_{\rm ini}}{t}\right)\right] t\right\} + C(\gamma+\gamma_{\rm d})^{2\chi-3} f\left(\frac{\gamma_{\rm d}}{\gamma+\gamma_{\rm d}}\right)$$
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$$t^4\mathcal{F}(\gamma_\tau \leq \gamma < \gamma_\mathrm{c}, t) = \left(\frac{t}{t_\mathrm{ini}}\right)^4 \left(\frac{a_\mathrm{ini}}{a}\right)^3 t_\mathrm{ini}^4 \mathcal{N}_\mathrm{ini}\left\{\left[\gamma + \gamma_\mathrm{d}\left(1 - \frac{t_\mathrm{ini}}{t}\right)\right] t\right\} + C_\mathrm{c}(\gamma+\gamma_\mathrm{d})^{2\chi_\mathrm{c}-3} f_\mathrm{c}\left(\frac{\gamma_\mathrm{d}}{\gamma+\gamma_\mathrm{d}}\right)$$
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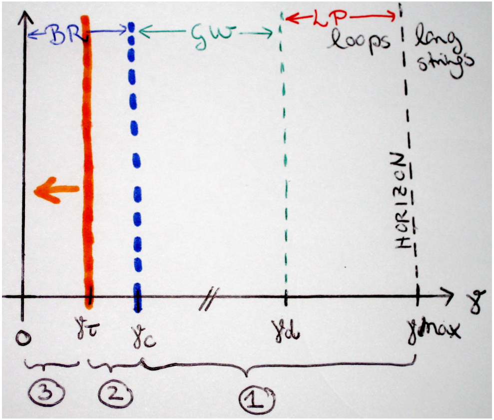
where $C \equiv c/\mu$, $f(x) \equiv {}_2\mathrm{F}_1(3-2\chi, \mu; \mu+1; x)$ to simplify notations

$$\gamma_\tau(t) \equiv (\gamma_\mathrm{c}+\gamma_\mathrm{d})\frac{t_\mathrm{ini}}{t} - \gamma_\mathrm{d}$$

Why three?



1. $\gamma \geq \gamma_c$
2. $\gamma_\tau \leq \gamma < \gamma$:
   "on-site production" vs. "produced and shrunk"
3. $\gamma < \gamma_\tau$:
   "non-contaminated"

Solution 3 dies out when $\gamma_\tau(t_\tau) = 0$, that is at

$$\frac{t_\tau - t_{\rm ini}}{t_{\rm ini}} = \frac{\gamma_c}{\gamma_d} \, .$$

Asymptotic expansions

With $\mathcal{I}(x)$ and $\mathcal{I}_c(x)$ fixed, the solution **soon** has **two** parts:

$$t^4\mathcal{F}(\gamma \geq \gamma_c, t) = \left(\frac{t}{t_{ini}}\right)^4 \left(\frac{a_{ini}}{a}\right)^3 t_{ini}^4 \mathcal{N}_{ini}\left\{\left[\gamma + \gamma_d\left(1 - \frac{t_{ini}}{t}\right)\right] t\right\} + C(\gamma+\gamma_d)^{2\chi-3} f\left(\frac{\gamma_d}{\gamma+\gamma_d}\right)$$
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After traces of $\mathcal{N}_{ini}$ have vanished:

$\gamma \ll \gamma_c \ll \gamma_d$

$$t^4\mathcal{F} \simeq \frac{C\mu}{2-2\chi}\frac{\gamma_c^{2\chi-2}}{\gamma_d}$$

independent of χ_c!

$\gamma_c < \gamma \ll \gamma_d$

$$t^4\mathcal{F} \simeq \frac{C\mu}{2-2\chi}\frac{\gamma^{2\chi-2}}{\gamma_d}$$

smooth matching

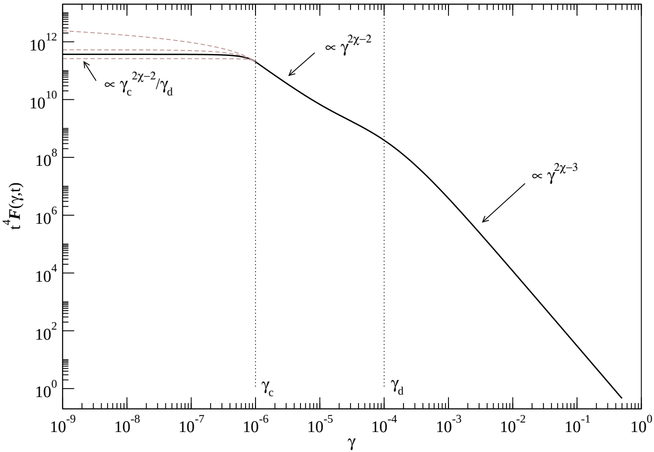
$\gamma \gg \gamma_d$

$$t^4\mathcal{F} \simeq C\,\gamma^{2\chi-3}$$

C, χ from simulations

Outline

Scaling solution in the radiation era



$$\nu = 1/2$$
$$\chi_c = 2$$
$$\gamma_d = 10^{-4}$$
$$\gamma_c = 10^{-6}$$

C, χ from Ringeval et al. (2005)

Outline

Relaxations and processes

The loop number density distribution $\mathcal{F}(\gamma, t)$ undergoes two relaxations:

1. from initial distribution $\mathcal{N}_{\mathrm{ini}}(\ell)$ to radiation scaling solution $\mathcal{F}_*(\gamma, t_*)$
2. from radiation scaling solution $\mathcal{F}_*(\gamma, t_*)$ to matter scaling solution

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Model cosmological background in terms of z using (H_0, Ω_{r_0},Ω_{m_0}):

$$t_{\mathrm{rad}}(z) \simeq \frac{1}{2H_0\sqrt{\Omega_{\mathrm{r}_0}}}\frac{1}{(1+z)^2}$$

$$t_{\mathrm{mat}}(z) \simeq \frac{2}{3H_0\sqrt{\Omega_{\mathrm{m}_0}}}\frac{1}{(1+z)^{3/2}}$$

- good approximation apart from transition and for $z > 2$
- instantaneous transition from radiation to matter at

$$z_* = \frac{9}{16}\frac{\Omega_{\mathrm{m}_0}}{\Omega_{\mathrm{r}_0}} - 1 = \frac{9}{16}(1+z_{\mathrm{eq}}) - 1$$

- $h = 0.72$, $\Omega_{\mathrm{m}_0} h^2 = 0.13$, $\Omega_{\mathrm{r}_0} h^2 = 2.471 \times 10^{-5}$

Relaxations and processes

The loop number density distribution $\mathcal{F}(\gamma, t)$ undergoes two relaxations:

1. from initial distribution $\mathcal{N}_{\mathrm{ini}}(\ell)$ to radiation scaling solution $\mathcal{F}_*(\gamma, t_*)$
2. from radiation scaling solution $\mathcal{F}_*(\gamma, t_*)$ to matter scaling solution

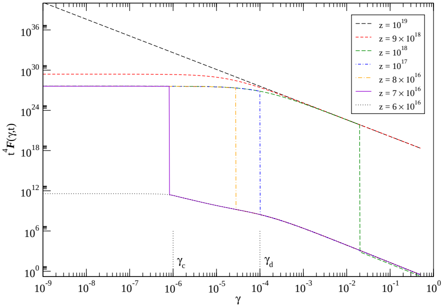
There are two relaxation processes:

1. "sweeping leftwards": $\mathcal{N}_{\mathrm{ini}}[\ell > d_{\mathrm{h}}(t_{\mathrm{ini}})] = 0$
2. damping of terms with $\mathcal{N}_{\mathrm{ini}}$

$$t^4 \mathcal{F}(\gamma \geq \gamma_{\mathrm{c}}, t) = \left(\frac{t}{t_{\mathrm{ini}}}\right)^4 \left(\frac{a_{\mathrm{ini}}}{a}\right)^3 t_{\mathrm{ini}}^4 \mathcal{N}_{\mathrm{ini}}\left\{\left[\gamma + \gamma_{\mathrm{d}}\left(1 - \frac{t_{\mathrm{ini}}}{t}\right)\right] t\right\} + C(\gamma + \gamma_{\mathrm{d}})^{2\chi - 3} f\left(\frac{\gamma_{\mathrm{d}}}{\gamma + \gamma_{\mathrm{d}}}\right) - C(\gamma + \gamma_{\mathrm{d}})^{2\chi - 3}\left(\frac{t}{t_{\mathrm{ini}}}\right)^{2\chi + 1}\left(\frac{a_{\mathrm{ini}}}{a}\right)^3 f\left(\frac{\gamma_{\mathrm{d}}}{\gamma + \gamma_{\mathrm{d}}} \frac{t_{\mathrm{ini}}}{t}\right)$$

$$t^4 \mathcal{F}(0 \leq \gamma < \gamma_{\mathrm{c}}, t) = \left(\frac{t}{t_{\mathrm{ini}}}\right)^4 \left(\frac{a_{\mathrm{ini}}}{a}\right)^3 t_{\mathrm{ini}}^4 \mathcal{N}_{\mathrm{ini}}\left\{\left[\gamma + \gamma_{\mathrm{d}}\left(1 - \frac{t_{\mathrm{ini}}}{t}\right)\right] t\right\} + C_{\mathrm{c}}(\gamma + \gamma_{\mathrm{d}})^{2\chi_{\mathrm{c}} - 3} f_{\mathrm{c}}\left(\frac{\gamma_{\mathrm{d}}}{\gamma + \gamma_{\mathrm{d}}}\right) - C(\gamma + \gamma_{\mathrm{d}})^{2\chi - 3}\left(\frac{t}{t_{\mathrm{ini}}}\right)^{2\chi + 1}\left(\frac{a_{\mathrm{ini}}}{a}\right)^3 f\left(\frac{\gamma_{\mathrm{d}}}{\gamma + \gamma_{\mathrm{d}}} \frac{t_{\mathrm{ini}}}{t}\right) + K\left(\frac{\gamma_{\mathrm{c}} + \gamma_{\mathrm{d}}}{\gamma + \gamma_{\mathrm{d}}}\right)^4 \left[\frac{a\left(\frac{\gamma + \gamma_{\mathrm{d}}}{\gamma_{\mathrm{c}} + \gamma_{\mathrm{d}}} t\right)}{a(t)}\right]^3$$

Initial distribution and relaxation



Vachaspati & Vilenkin (1984): random walk model

$$t_{\rm ini}^4 \mathcal{N}_{\rm ini}(\ell) = C_{\rm i} \left(\frac{t_{\rm ini}}{\ell} \right)^{5/2} .$$

where $1 \leq C_{\rm i} \leq (GU)^{-3/4}$

- no damping (cancellations)
- "sweeping" complete at $\dfrac{1+z_{\rm h}(\gamma)}{1+z_{\rm ini}} = \sqrt{\dfrac{\gamma+\gamma_{\rm d}}{2+\gamma_{\rm d}}}$

Relaxation from radiation to matter

For $\nu = 2/3$, the ${}_2F_1$ function becomes a polynomial expression:

$$f(x) = \frac{1}{(1-x)^{2-2\chi}} \left(1 - \frac{x}{2-2\chi} \right)$$

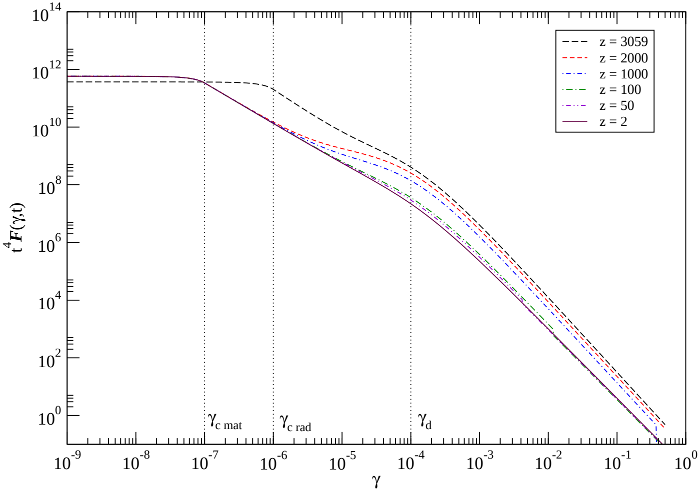
- damping of $\mathcal{N}_*$ complete by $(\gamma \gg \gamma_\mathrm{d})$

$$\frac{1+z_\mathrm{d}(\gamma)}{1+z_*} = \left(\frac{C_\mathrm{M}}{C_\mathrm{R}} \right)^{1/(3/2-3\chi_\mathrm{R})} \gamma^{2(\chi_\mathrm{M}-\chi_\mathrm{R})/(3/2-3\chi_\mathrm{R})}$$

- "sweeping" less efficient

for large γ, $z_\mathrm{d} \simeq 300$, but intermediate scales $\gamma_\mathrm{c} < \gamma \ll \gamma_\mathrm{d}$ relax last

Relaxation from radiation to matter



$\gamma_\mathrm{d} = 10^{-4}$, $\gamma_\mathrm{c}(rad) = 10^{-6}$, $\gamma_\mathrm{d}(mat) = 10^{-7}$ “production resumed”

Conclusions

- We studied the evolution of the cosmic string loop number density distribution $\mathrm{d}n/\mathrm{d}\ell(\ell, t)$
  - using the Polchinski Rocha model for loop production with numerically adjusted coefficients,
  - including emission of gravitational waves and
  - including smoothing by gravitational backreaction.
- $\mathrm{d}n/\mathrm{d}\ell(\ell, t)$ rapidly assumes a a universal form on all length scales, insensitive to the details of the backreaction.
- Our study may be extended by
  - recalculating the GW constraints using the new loop distribution,
  - using the parameters to describe other energy loss mechanisms or
  - considering the case $P \neq 1$ (cosmic superstrings).

Can cosmic string loops be Dark Matter?

in the matter era:

$$t^4 \mathcal{F}(\gamma \geq \gamma_c, t) = \frac{C}{\gamma^{2-2\chi}(\gamma + \gamma_d)} \left(1 - \frac{1}{2 - 2\chi} \frac{\gamma_d}{\gamma + \gamma_d}\right)$$

$$t^4 \mathcal{F}(\gamma < \gamma_c, t) = \frac{C_c \gamma^{2\chi_c - 2}}{\gamma + \gamma_d} \left(1 + \frac{1}{2\chi_c - 2} \frac{\gamma_d}{\gamma + \gamma_d}\right) + K \left(\frac{\gamma_c + \gamma_d}{\gamma + \gamma_d}\right)^2$$

use these calculate loop energy density:

$$\rho_\circ = \frac{U}{t^2} \int_0^{\gamma_{\max}} t^4 \mathcal{F}(\gamma, t) \gamma \, \mathrm{d}\gamma,$$

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then one finds $\Omega_\circ \equiv \rho_\circ/\rho_{\rm crit}$:

$$\Omega_\circ = \frac{3\pi^2 C}{(1-\chi)\sin(2\pi\chi)}\frac{GU}{\gamma_d^{1-2\chi}}$$

which for our parameters is

$$\Omega_\circ \simeq 0.10 \times (GU)^{0.59} < 10^{-5}$$

for current GU values

Cosmic string loops are not a suitable DM candidate.

How far to the nearest loop?

$$t^3 n_L = \int_0^{L/t} t^4 \mathcal{F}(\gamma, t)\,\mathrm{d}\gamma$$

n_L: number density of loops with length $\ell \leq L$ at t

for $\gamma_\mathrm{c} < \gamma_\mathrm{d} \ll L/t$:

$$t^3 n_L = \frac{2\chi_\mathrm{c} - 2\chi}{2\chi_\mathrm{c} - 1} \frac{C}{\gamma_\mathrm{d}\gamma_\mathrm{c}^{1-2\chi}}\,.$$

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with our parameters:

$$n_L \simeq \frac{6.1 \times 10^{-5}}{t^3} (GU)^{-1.65}$$

today $t = t_0$ for $GU \simeq 7 \times 10^{-7}$:

$$n_L \simeq 5.5 \times 10^{-6}\,\mathrm{Mpc}^{-3}$$

loops from very light cosmic strings more numerous