

# $J/\psi$ Production at NLO in NRQCD

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LHC  
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In collaboration with Mathias Butenschön  
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# Production and Decay Rates of Heavy Quarkonia

**Heavy quarkonia:** Bound states of heavy quark and its antiquark.

- Charmonia ( $c\bar{c}$ ) and Bottomonia ( $b\bar{b}$ )
- Top decays too fast for bound state

**The classic approach: Color-singlet model**

- Calculate cross section for heavy quark pair in physical **color-singlet** (= color neutral) state. In case of  $J/\psi$ :  $c\bar{c}[{}^3S_1^{[1]}]$
- Multiply by quarkonium wave function (or its derivative) at origin
- Strong disagreement with Tevatron data

**Nonrelativistic QCD (NRQCD):**

- 1995: Rigorous effective field theory by Bodwin, Braaten, Lepage
- Based on **factorization of soft and hard scales**  
(Scale hierarchy:  $Mv^2, Mv \ll \Lambda_{\text{QCD}} \ll M$ )
- Could explain hadroproduction at Tevatron

# $J/\psi$ Production with NRQCD

**Factorization theorem:**  $\sigma_{J/\psi} = \sum_n \sigma_{c\bar{c}[n]} \cdot \langle O^{J/\psi}[n] \rangle$

- $n$ : Every possible Fock state, including **color-octet** states.
- $\sigma_{c\bar{c}[n]}$ : Production rate of  $c\bar{c}[n]$ , calculated in perturbative QCD.
- $\langle O^{J/\psi}[n] \rangle$ : Long distance matrix elements (LDMEs): describe  $c\bar{c}[n] \rightarrow J/\psi$ , universal, extracted from experiment.

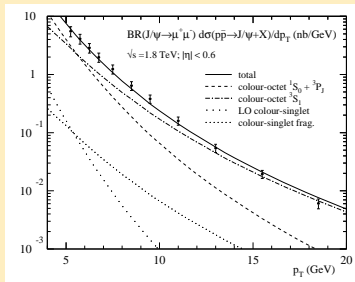
**Scaling rules:** MEs scale with relative velocity  $v$  ( $v^2 \approx 0.2$ ):

scaling	$v^3$	$v^7$	$v^{11}$
$n$	$^3S_1^{[1]}$	$^1S_0^{[8]}, ^3S_1^{[8]}, ^3P_{0/1/2}^{[8]}$	...

- **Double expansion** in  $v$  and  $\alpha_s$ .
- Leading term in  $v$  ( $n = ^3S_1^{[1]}$ ) equals **color-singlet model**.

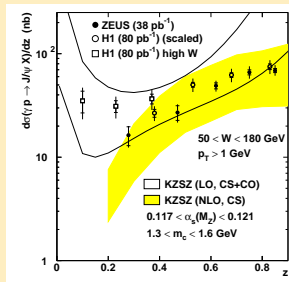
# Production of $J/\psi$ : NRQCD vs. Experiment (History)

## Hadroproduction at Tevatron:



- Color octet states important  
 $\Rightarrow$  **Great success** for NRQCD

## Photoproduction at HERA:



- MEs from fits to Tevatron data.
- Importance of color octet **unclear**

**This work:** **NLO** NRQCD calculation for photo- and hadroproduction  
 $\Rightarrow$  Aim: Establish universality of long distance matrix elements.

# Production of $J/\psi$ : Summary of Calculations

## Hadroproduction:

	$^3S_1^{[1]}$	$^1S_0^{[8]}, ^3S_1^{[8]}, ^3P_{0/1/2}^{[8]}$
<b>Born</b>	Baier, Rückl (1980)	Cacciari, Krämer (1996)
<b>NLO</b>	Campbell et al. (2007)	<b>Butenschön, BK (2010)</b> Ma et al. (2010)

## Photoproduction:

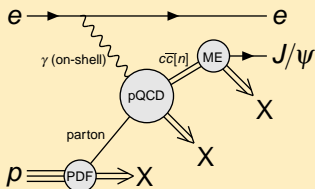
	$^3S_1^{[1]}$	$^1S_0^{[8]}, ^3S_1^{[8]}, ^3P_{0/1/2}^{[8]}$
<b>Born</b>	Berger, Jones (1981)	Ko, Lee, Song (1996)
<b>NLO</b>	Krämer (1995)	<b>Butenschön, BK (2009)</b>

## Open question of ME universality:

- NLO NRQCD calculation: after **14 years!**
- Difficulty: virtual corrections to  **$P$  states**

# Direct $J/\psi$ Production

**Factorization formulas:** (e.g. photoproduction)



- Convolute partonic cross sections with **proton PDFs**:

$$\sigma_{\text{hadr}} = \sum_i \int dx f_{i/p}(x) \cdot \sigma_{\text{part},i}$$

- NRQCD factorization:**

$$\sigma_{\text{part},i} = \sum_n \sigma(\gamma i \rightarrow c\bar{c}[n] + X) \cdot \langle O^{J/\psi}[n] \rangle$$

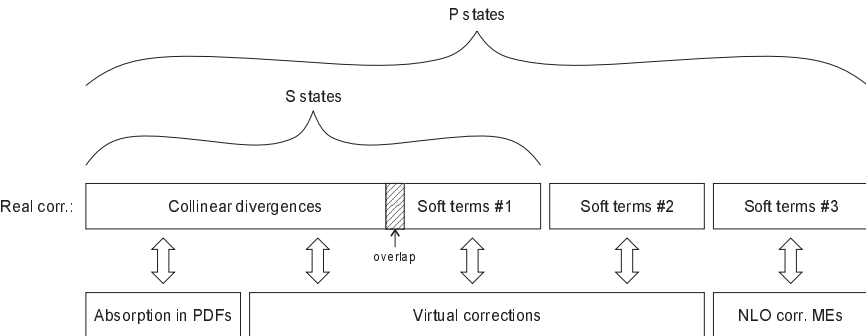
**Amplitudes for  $c\bar{c}[n]$  production by projector application, e.g.:**

$$A_{c\bar{c}[3S_1^{[1/8]}]} = \varepsilon_\alpha \text{Tr} [C \Pi^\alpha A_{c\bar{c}}] |_{q=0}$$

$$A_{c\bar{c}[3P_J^{[8]}]} = \varepsilon_{\alpha\beta} \frac{d}{dq_\beta} \text{Tr} [C \Pi^\alpha A_{c\bar{c}}] |_{q=0}$$

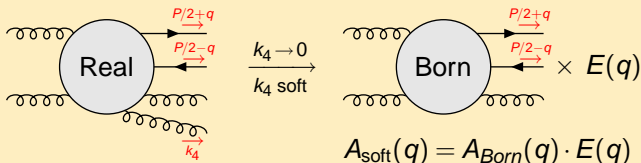
- $A_{c\bar{c}}$ : Amputated pQCD amplitude for open  $c\bar{c}$  production.
- $q$ : Relative momentum between  $c$  and  $\bar{c}$ .

# Overview of IR Singularity Structure



# Structure of Soft Singularities

## Soft limits of the real corrections:



## S and P states: Soft #1 + Soft #2 + Soft #3 terms:

$$A_{\text{soft},s} = A_{\text{soft}}(0) = A_{\text{Born},s} \cdot E(0)$$

$$A_{\text{soft},p} = A'_{\text{soft}}(0) = A_{\text{Born},p} \cdot E(0) + A_{\text{Born},s} \cdot E'(0)$$

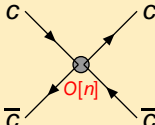
$$|A_{\text{soft},s}|^2 = |A_{\text{Born},s}|^2 \cdot E(0)^2$$

$$|A_{\text{soft},p}|^2 = |A_{\text{Born},p}|^2 \cdot E(0)^2 + 2 \operatorname{Re} A_{\text{Born},s}^* A_{\text{Born},p} \cdot E(0)E'(0) + |A_{\text{Born},s}|^2 \cdot E'(0)^2$$



# Radiative Corrections to Long Distance MEs

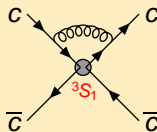
In NRQCD: Long distance MEs =  $c\bar{c}$  scattering amplitudes:

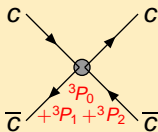
$$\langle O^{J/\psi}[n] \rangle =$$


$$O[n] = \text{4-fermion operators}$$

$$(n = {}^3S_1^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_{0/1/2}^{[8]}, \dots)$$

Corrections to  $\langle O^{J/\psi}[{}^3S_1^{[1/8]}] \rangle$  with NRQCD Feynman rules:

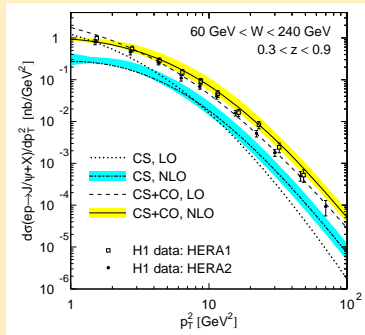
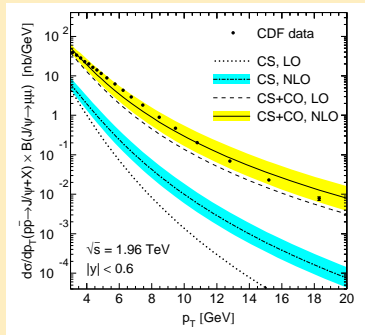


$$+ \text{similar diagrams} \propto \frac{4\alpha_s}{3\pi m_c^2} \left( \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) \cdot$$


- **UV singularity** cancelled by renormalization of 4-fermion operat.
- **IR singularity** cancels soft #3 terms of  $P$  states.

# Combined Fit to Tevatron and HERA (1)

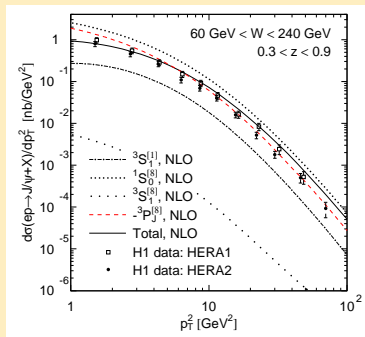
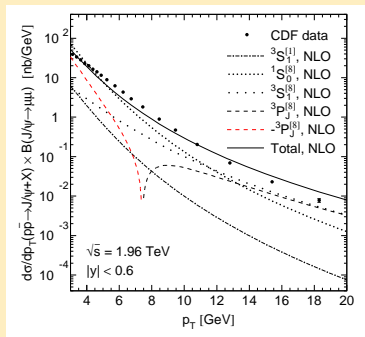
## Fit CO LDMEs to $p_T$ distributions from Tevatron II and HERA II:



- Best fit values:  $\langle \theta(1S_0^{[8]}) \rangle = (0.0476 \pm 0.0071) \text{ GeV}^3$ ,  
 $\langle \theta(3S_1^{[8]}) \rangle = (0.00265 \pm 0.00091) \text{ GeV}^3$ ,  $\langle \theta(3P_0^{[8]}) \rangle = -(0.0132 \pm 0.0035) \text{ GeV}^5$
- $\propto v^4 \langle O_1(3S_1) \rangle \rightsquigarrow$  NRQCD velocity scaling rules  $\sqrt{\phantom{x}}$

# Combined Fit to Tevatron and HERA (2)

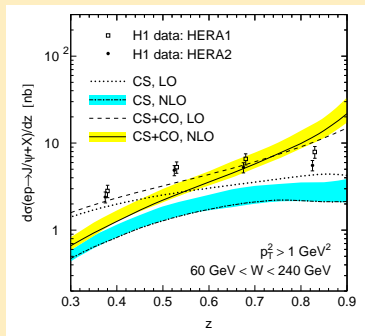
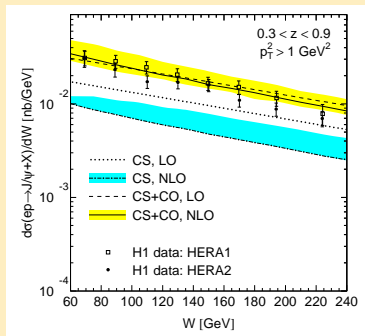
## Contribution of individual states:



- Hadroprod.: Short-distance  $\sigma(c\bar{c}[{}^3P_J^{[8]}])$  **negative** for  $p_T \gtrsim 7$  GeV
- But: Short-distance cross sections and LDMEs **unphysical** (NRQCD scale and scheme dependence)  $\implies$  No problem!

# Further Predictions for HERA

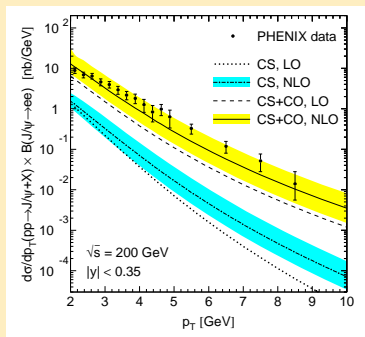
## Use LDMEs to make $W$ and $z$ distribution predictions:



- Proton rest frame:  $z =$  fraction of photon energy going to  $J/\psi$ .
- $z \lesssim 0.45$ : **Resolved** photoproduction important (not yet included).
- $W$  distribution also well described. Data not part of the fit.

# Predictions for RHIC

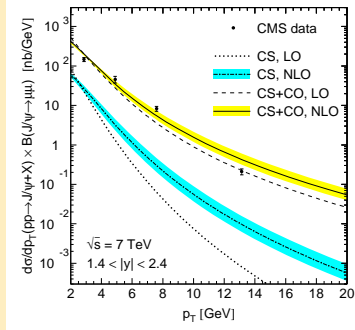
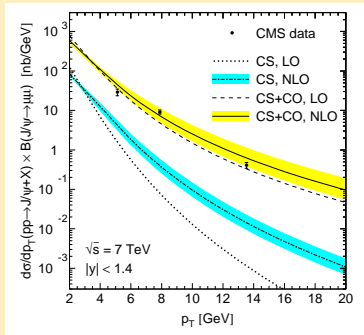
## Use LDMEs to make prediction for RHIC:



- Also RHIC data **well described** by CS+CO.
- Like at Tevatron: **CS** orders of magnitudes **below** the data.
- These data **not part** of the fit, outcome not trivial.

# Predictions for LHC

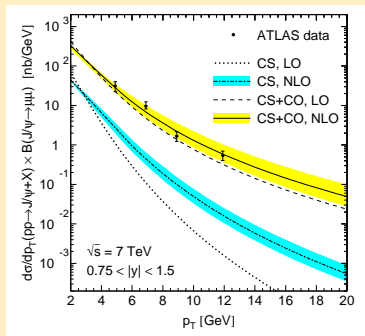
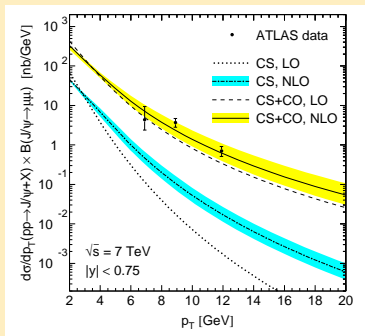
## Use LDMEs to make predictions for CMS:



- Also CMS data **well described** by CS+CO.
- Like at Tevatron: **CS** orders of magnitudes **below** the data.
- These data **not part** of the fit, outcome not trivial.

# Predictions for LHC (2)

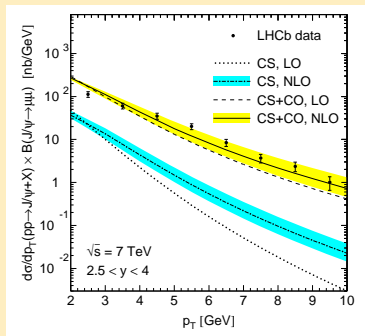
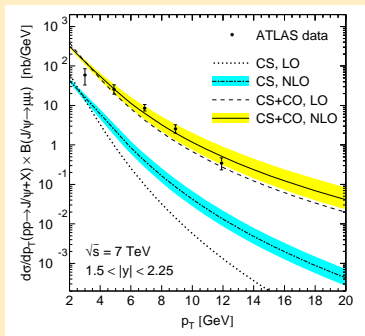
## Use LDMEs to make predictions for ATLAS:



- Also ATLAS data **well described** by CS+CO.
- Like at Tevatron: **CS** orders of magnitudes **below** the data.
- These data **not part** of the fit, outcome not trivial.

# Predictions for LHC (3)

## Use LDMEs to make predictions for ATLAS and LHCb:

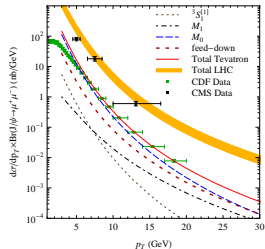
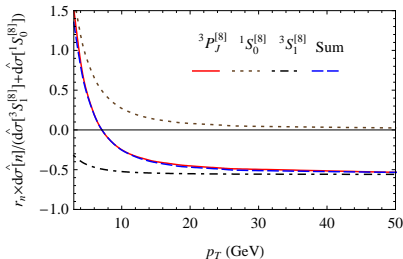


- Also ATLAS and LHCb data **well described** by CS+CO.
- Like at Tevatron: **CS** orders of magnitudes **below** the data.
- These data **not part** of the fit, outcome not trivial.



# Comparison with Ma, Wang, Chao arXiv:1009.3655

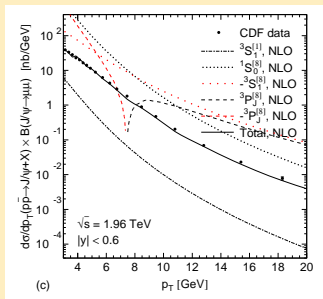
## Fit only to Tevatron data with $p_T > 7$ GeV, including feed-down



- Observe that  $d\hat{\sigma}(3P_J^{[8]}) \approx r_0 d\hat{\sigma}(1S_0^{[8]}) + r_1 d\hat{\sigma}(3S_1^{[8]})$  with  $r_0 = 3.9$  and  $r_1 = -0.56$
- Define  $M_0 = \langle \mathcal{O}(1S_0^{[8]}) \rangle + \frac{r_0}{m_c} \langle \mathcal{O}(3P_0^{[8]}) \rangle$  and  $M_1 = \langle \mathcal{O}(3S_1^{[8]}) \rangle + \frac{r_1}{m_c} \langle \mathcal{O}(3P_0^{[8]}) \rangle$
- Substitute  $\langle \mathcal{O}(1S_0^{[8]}) \rangle \rightarrow M_0$  and  $\langle \mathcal{O}(3S_1^{[8]}) \rangle \rightarrow M_1$  and discard  $d\hat{\sigma}(3P_J^{[8]})$ .
- Fit yields  $M_0 = (7.4 \pm 1.9) \times 10^{-2} \text{ GeV}^3$  and  $M_1 = (0.05 \pm 0.02) \times 10^{-2} \text{ GeV}^3$  with  $\chi^2/\text{d.o.f.} = 0.33$
- Cf.  $M_0 = (2.47 \pm 0.93) \times 10^{-2} \text{ GeV}^3$  and  $M_1 = (0.59 \pm 0.13) \times 10^{-2} \text{ GeV}^3$  from BK

# Significance of HERA data

Fit only to Tevatron data with  $p_T > 3$  GeV, excluding feed-down


 $10^{-2} \text{ GeV}^{3+2L}$ 

BK default

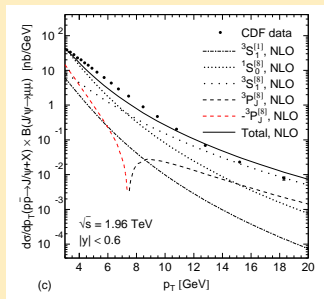
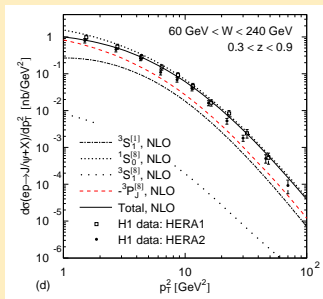
Tevatron only

● $\langle \sigma(1S_0^{[8]}) \rangle$	$4.76 \pm 0.71$	59
$\langle \sigma(3S_1^{[8]}) \rangle$	$0.265 \pm 0.091$	-6.6
$\langle \sigma(3P_0^{[8]}) \rangle$	$-1.32 \pm 0.35$	-29

- CO MEs too large in magnitude  $\rightsquigarrow$  NRQCD velocity scaling rules violated
- Substantial fine tuning  $\rightsquigarrow$  unnatural

# Significance of $p_T$ cut on Tevatron data

## Fit to Tevatron $p_T > 7$ GeV and HERA data, excluding feed-down

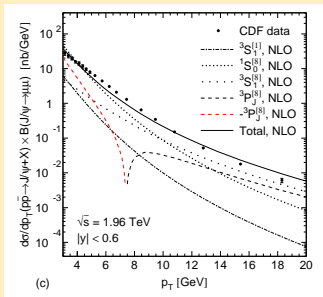
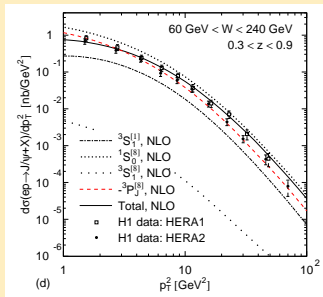


$10^{-2} \text{ GeV}^{3+2L}$	BK default	Tevatron $p_T > 7$ GeV
$\langle \sigma(1S_0^{[8]}) \rangle$	$4.76 \pm 0.71$	2.9
$\langle \sigma(3S_1^{[8]}) \rangle$	$0.265 \pm 0.091$	0.4
$\langle \sigma(3P_0^{[8]}) \rangle$	$-1.32 \pm 0.35$	-0.58

- 
- Shift of  $p_T$  cut on Tevatron data from 3 GeV to 7 GeV less dramatic
- Thanks to stabilizing influence of HERA data

# Significance of feed-down

## Fit to HERA and Tevatron data w/ feed-down subtracted



- feed-down/prompt  $\approx (32 - 0.62 \frac{p_T}{\text{GeV}})\%$  from Tevatron I,  $\approx 15\%$  at HERA from MC

$10^{-2} \text{ GeV}^{3+2L}$

BK default

feed-down subtracted

- |  |                   |       |
|--|-------------------|-------|
| ● $\langle \sigma(1S_0^{[8]}) \rangle$ | $4.76 \pm 0.71$   | 3.1   |
| $\langle \sigma(3S_1^{[8]}) \rangle$   | $0.265 \pm 0.091$ | 0.23  |
| $\langle \sigma(3P_0^{[8]}) \rangle$   | $-1.32 \pm 0.35$  | -0.82 |

- Feed-down corrections  $\ll$  theoretical uncertainties

# Comparison with MWC arXiv:1009.3655 (2)

## Summary of $M_0$ and $M_1$ in ( $10^{-2} \text{ GeV}^3$ ) from BK and MWC

authors	data	feed-down	$M_0$	$M_1$
BK	default	—	2.5	0.59
BK	HERA $p_T > \sqrt{5} \text{ GeV}$	—	2.5	0.60
BK	Tevatron $p_T > 7 \text{ GeV}$	—	1.9	0.54
BK	default	✓	1.7	0.43
BK	only Tevatron	—	8.7	0.62
BK	only Tevatron $p_T > 7 \text{ GeV}$	—	9.3	0.30
MWC	only Tevatron $p_T > 5 \text{ GeV}$	✓	5.2	0.16
MWC	only Tevatron $p_T > 7 \text{ GeV}$	✓	7.4	0.05

- Lower  $p_T$  cuts on HERA or Tevatron data marginal in joint fit
- Feed-down corrections moderate  $\rightsquigarrow$  no qualitative change
- Exclusion of HERA data  $\rightsquigarrow$  fit greatly underconstrained

# Comparison with MWC arXiv:1009.3655 (3)

## Errors on $M_0$ and $M_1$ in 3-parameter fit to only Tevatron $p_T > 7$ GeV

Covariance matrix  $V_{ij}$  defined through

$$(V^{-1})_{ij} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial x_i \partial x_j}, \quad (x_1, x_2, x_3) = (\langle \theta(1S_0^{[8]}) \rangle, \langle \theta(3S_1^{[8]}) \rangle, \langle \theta(3P_0^{[8]}) \rangle)$$

has eigenvalues and eigenvectors:

$\lambda_1 = 0.127:$	$\vec{v}_1 = (0.864, -0.114, -0.491)$
$\lambda_2 = 1.78 \times 10^{-6}:$	$\vec{v}_2 = (0.502, 0.0968, 0.860)$
$\lambda_3 = 4.69 \times 10^{-8}:$	$\vec{v}_3 = (0.0507, 0.989, -0.141)$

Cf.  $\vec{v}_{M_0} = (0.500, 0, 0.866)$  and  $\vec{v}_{M_1} = (0, 0.970, -0.242)$  from 2-parameter fit of MWC

$$\Delta M_0 = (\vec{v}_{M_0}^T V \vec{v}_{M_0})^{1/2} / (\vec{v}_{M_0})_1 = 0.95 \times 10^{-2}$$

$$\Delta M_1 = (\vec{v}_{M_1}^T V \vec{v}_{M_1})^{1/2} / (\vec{v}_{M_1})_2 = 0.29 \times 10^{-2}$$

I.e. 3-parameter fit yields:  $M_0 = (9.3 \pm 0.95)$  and  $M_1 = (0.30 \pm 0.29)$  in  $10^{-2}$  GeV<sup>3</sup>

↪ **almost 100% error on  $M_1$  in 3-parameter fit**

Cf.  $M_0 = (7.4 \pm 1.9)$  and  $M_1 = (0.05 \pm 0.02)$  from 2-parameter fit by MWC

- $M_1$  corresponds mostly to  $\vec{v}_3$ , but contains small admixtures of  $\vec{v}_1$  and  $\vec{v}_2$
- $\vec{v}_1$  very badly constrained ↪ large error on  $M_1$
- Not exhibited in 2-parameter fit, where variations orthogonal to  $M_0$  and  $M_1$  are forbidden

# Comparison with MWC arXiv:1009.3655 (4)

## Errors on $\langle \mathcal{O}(^1S_0^{[8]}) \rangle$ , $\langle \mathcal{O}(^3S_1^{[8]}) \rangle$ and $\langle \mathcal{O}(^3P_0^{[8]}) \rangle$ in 3-parameter fits

$10^{-2} \text{ GeV}^{3+2L}$	BK default	Tevatron $p_T > 7 \text{ GeV}$ only
$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$	$4.76 \pm 0.71$	$25.1 \pm 30.7$
$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$	$0.265 \pm 0.091$	$-1.96 \pm 4.07$
$\langle \mathcal{O}(^3P_0^{[8]}) \rangle$	$-1.32 \pm 0.35$	$-9.09 \pm 17.48$

Correlation matrix  $V_{ij}$  of default fit has eigenvalues and eigenvectors:

$$\lambda_1 = 6.39 \times 10^{-5}: \quad \vec{v}_1 = (0.893, -0.111, -0.435)$$

$$\lambda_2 = 1.92 \times 10^{-7}: \quad \vec{v}_2 = (0.440, 0.0193, 0.898)$$

$$\lambda_3 = 4.00 \times 10^{-8}: \quad \vec{v}_3 = (0.0910, 0.994, -0.0659)$$

- No such strong hierarchy among eigenvalues
- Fit results meaningful

# Summary (1)

## Our project: Test NRQCD

- NRQCD provides rigorous **factorization theorem** for production and decay of heavy quarkonia: Include **color-octet** (CO) states.
- But: Need to proof **universality** of CO LDMEs.
- **This work**: Technological breakthrough: After **14 years** finally NLO NRQCD photo- and hadroproduction calculation.

## Our Results:

- **CSM predictions**: Could verify all previous results: CS contributions **far below data** in all considered experiments.
- **Fitted** CO LDMEs to  $p_T$  distributions at Tevatron and HERA.
- Used LDMEs for RHIC, LHC and HERA  $W$  and  $z$  distributions  $\implies$  CS+CO: Good **agreement** with data.



# Summary (2)

## Discussion of Fit:

- **NLO** hadroproduction  $P$  states: Shape changes, even negative.  
⇒ Fit **all three** CO LDMEs (not linear combination like at LO).
- Negative unphysical quantities no problem.
- Photo- and hadroproduction **consistently described** by NRQCD.

## Still to be done:

- Include **resolved** photoproduction.
- Extend analysis to  $e^+e^-$  collisions (LEP,  $B$  factories).
- Include **feed-down** processes.
- Do **polarization** analysis.