

# *Open heavy-flavour hadroproduction at NLO in the GM-VFN scheme*

Bernd Kniehl (Hamburg University)  
[kniehl@desy.de](mailto:kniehl@desy.de)

in collaboration with G. Kramer, I. Schienbein, and H. Spiesberger

LHC Physics Day: Charm and bottom quark production at the LHC  
CERN, 3 December 2010

# OUTLINE

- 1 OVERVIEW AND MOTIVATION
- 2 GENERAL-MASS VARIABLE-FLAVOUR-NUMBER SCHEME
- 3 FRAGMENTATION FUNCTIONS FOR  $D^0, D^+, D^{*+}$
- 4 HADROPRODUCTION OF  $D$  MESONS
- 5 PREDICTIONS FOR THE LHC
- 6 SUMMARY

# OVERVIEW

- One-particle inclusive production of heavy hadrons  $H = D, B, \Lambda_c, \dots$
- General-Mass Variable Flavour Number Scheme (**GM-VFNS**):
  - ▶ Collinear logarithms of the heavy-quark mass  $\ln \mu/m_h$  are subtracted and resummed [1]
  - ▶ Finite non-logarithmic  $m_h/Q$  terms are kept in the hard part/taken into account
  - ▶ Scheme guided by the factorization theorem of **Collins** with heavy quarks [2]

Ongoing effort to compute all relevant processes in the **GM-VFNS** at NLO:

- Available:
  - ▶  $e^+ + e^- \rightarrow (D^0, D^+, D^{*+}) + X$ : FFs [3]
  - ▶  $\gamma + \gamma \rightarrow D^{*+} + X$ : direct process [4]
  - ▶  $\gamma + \gamma \rightarrow D^{*+} + X$ : single-resolved process [5]
  - ▶  $\gamma + p \rightarrow D^{*+} + X$ : direct process [6]
  - ▶  $\gamma + p \rightarrow D^{*+} + X$ : resolved process [7]
  - ▶  $p + \bar{p} \rightarrow (D^0, D^+, D^{*+}, D_s^+, \Lambda_c^+, B^0, B^+) + X$  [1]

---

[1] B.K.,Kramer,Schienbein,Spiesberger, PRD71(2005)014018; EPJC41(2005)199; PRL96(2006)012001; PRD77(2008)014011

[2] Collins, PRD58(1998)094002

[3] Kneesch,B.K.,Kramer,Schienbein, NPB799(2008)34

[4] Kramer,Spiesberger, EPJC22(2001)289; [5] EPJC28(2003)495; [6] EPJC38(2004)309

[7] B.K.,Kramer,Schienbein,Spiesberger, arXiv:0902.3166[hep-ph], EPJC(in press)

# OVERVIEW

- One-particle inclusive production of heavy hadrons  $H = D, B, \Lambda_c, \dots$
- General-Mass Variable Flavour Number Scheme (**GM-VFNS**):
  - ▶ Collinear logarithms of the heavy-quark mass  $\ln \mu/m_h$  are subtracted and resummed [1]
  - ▶ Finite non-logarithmic  $m_h/Q$  terms are kept in the hard part/taken into account
  - ▶ Scheme guided by the factorization theorem of **Collins** with heavy quarks [2]

Ongoing effort to compute all relevant processes in the **GM-VFNS** at NLO:

- Available:
  - ▶  $e^+ + e^- \rightarrow (D^0, D^+, D^{*+}) + X$ : FFs [3]
  - ▶  $\gamma + \gamma \rightarrow D^{*+} + X$ : direct process [4]
  - ▶  $\gamma + \gamma \rightarrow D^{*+} + X$ : single-resolved process [5]
  - ▶  $\gamma + p \rightarrow D^{*+} + X$ : direct process [6]
  - ▶  $\gamma + p \rightarrow D^{*+} + X$ : resolved process [7]
  - ▶  $p + \bar{p} \rightarrow (D^0, D^+, D^{*+}, D_s^+, \Lambda_c^+, B^0, B^+) + X$  [1]

---

[1] B.K.,Kramer,Schienbein,Spiesberger, PRD71(2005)014018; EPJC41(2005)199; PRL96(2006)012001; PRD77(2008)014011

[2] Collins, PRD58(1998)094002

[3] Kneesch,B.K.,Kramer,Schienbein, NPB799(2008)34

[4] Kramer,Spiesberger, EPJC22(2001)289; [5] EPJC28(2003)495; [6] EPJC38(2004)309

[7] B.K.,Kramer,Schienbein,Spiesberger, arXiv:0902.3166[hep-ph], EPJC(in press)

# OVERVIEW -CONTINUED-

Input for the computation: Fragmentation Functions (FFs) into heavy hadrons  $H$

- FFs from fits to  $e^+e^-$  data from  $Z$  factories
- Include also  $B$  factories → Switch from ZM to GM
- Use initial scale  $\mu_0 = m$  (instead of  $\mu_0 = 2m$ ) for consistency with PDFs → important for gluon fragmentation

$H$	Data	Scheme	Reference
$D^{*+}$	ALEPH,OPAL	ZM $2m$	BKK, PRD58(1998)014014
$D^0, D^+, D_s^+, \Lambda_c^+$	OPAL	ZM $2m$	KK, PRD71(2005)094013
$D^0, D^+, D^{*+}, D_s^+, \Lambda_c^+$	OPAL	ZM $m$	KK, PRD74(2006)037502
$D^0, D^+, D_s^+$	Belle,CLEO,ALEPH,OPAL	GM $m$	KKKSc, NPB799(2008)34
$B^0, B^+$	OPAL	ZM $2m$	BKK, PRD58(1998)034016
$B^0, B^+$	ALEPH,OPAL,SLD	ZM $m$	KKScSp, PRD77(2008)014011

Goal:

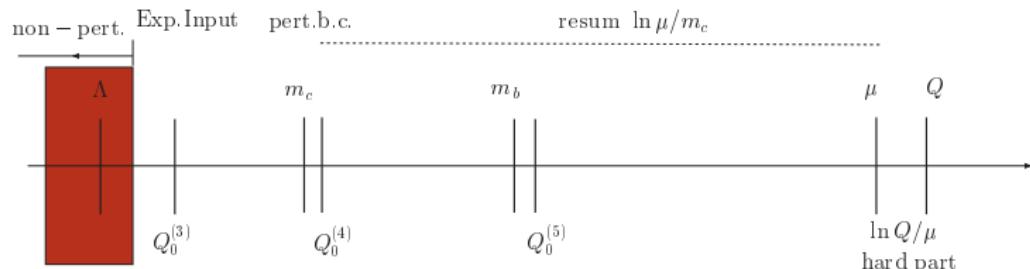
- Test pQCD formalism, scaling violations and universality of FFs in as many processes as possible

# MOTIVATION

## HEAVY QUARKS AND PQCD

### Fixed Order Perturbation Theory:

- finite collinear logs  $\ln Q/m_c$  arise → can be kept in hard part  
Of course need exp. Input for  $u, d, s, g$  PDFs at scale  $Q_0^{(3)}$



### Variable Flavour Number Scheme (VFNS):

- often large ratios of scales involved: multi-scale problems  
For  $Q \gg m_c$ : write  $\ln Q/m_c = \ln Q/\mu + \ln \mu/m_c$ , subtract  $\ln \mu/m_c$  and resum  $\ln \mu/m_c$  by introducing charm PDF at  $Q_0^{(4)} \simeq m_c$  using a perturbative boundary condition

# GENERAL-MASS VARIABLE-FLAVOUR-NUMBER SCHEME

Two basic approaches:

- Fixed Order Perturbation Theory (FFNS)
- Parton Model (ZM-VFNS)

Interpolating scheme combining the good features:

- Parton Model with quark masses (GM-VFNS, ACOT)

Glossary:

- ZM: Zero Mass
- GM: General Mass
- VFNS: Variable Flavour Number Scheme
- FFNS: Fixed Flavour Number Scheme

# OUR THEORETICAL BASIS FOR $p\bar{p} \rightarrow D^* X$

Factorization Formula:

[1]

$$d\sigma(p\bar{p} \rightarrow D^* X) = \sum_{i,j,k} \int dx_1 dx_2 dz f_i^p(x_1) f_j^{\bar{p}}(x_2) \times \\ d\hat{\sigma}(ij \rightarrow kX) D_k^{D^*}(z) + \mathcal{O}(\alpha_s^{n+1}, (\frac{\Lambda}{Q})^p)$$

Q: hard scale,  $p = 1, 2$

---

- $d\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})$ : hard scattering cross sections free of long-distance physics  $\rightarrow m_h$  kept
- PDFs  $f_i^p(x_1, \mu_F), f_j^{\bar{p}}(x_2, \mu_F)$ :  $i, j = g, q, c$  [ $q = u, d, s$ ]
- FFs  $D_k^{D^*}(z, \mu'_F)$ :  $k = g, q, c$

$\Rightarrow$  need short distance coefficients including heavy quark masses

---

[1] J. Collins, 'Hard-scattering factorization with heavy quarks: A general treatment', PRD58(1998)094002

## LIST OF SUBPROCESSES: GM-VFNS

Only light lines

- 1  $gg \rightarrow qX$
- 2  $gg \rightarrow gX$
- 3  $qg \rightarrow gX$
- 4  $qg \rightarrow qX$
- 5  $q\bar{q} \rightarrow gX$
- 6  $q\bar{q} \rightarrow qX$
- 7  $qg \rightarrow \bar{q}X$
- 8  $qg \rightarrow \bar{q}'X$
- 9  $qg \rightarrow q'X$
- 10  $qq \rightarrow gX$
- 11  $qq \rightarrow qX$
- 12  $q\bar{q} \rightarrow q'X$
- 13  $q\bar{q}' \rightarrow gX$
- 14  $q\bar{q}' \rightarrow qX$
- 15  $qq' \rightarrow gX$
- 16  $qq' \rightarrow qX$

⊕ charge conjugated processes

Heavy quark initiated ( $m_Q = 0$ )

- 1 -
- 2 -
- 3  $Qg \rightarrow gX$
- 4  $Qg \rightarrow QX$
- 5  $Q\bar{Q} \rightarrow gX$
- 6  $Q\bar{Q} \rightarrow QX$
- 7  $Qg \rightarrow \bar{Q}X$
- 8  $Qg \rightarrow \bar{q}X$
- 9  $Qg \rightarrow qX$
- 10  $QQ \rightarrow gX$
- 11  $QQ \rightarrow QX$
- 12  $Q\bar{Q} \rightarrow qX$
- 13  $Q\bar{q} \rightarrow gX, q\bar{Q} \rightarrow gX$
- 14  $Q\bar{q} \rightarrow QX, q\bar{Q} \rightarrow qX$
- 15  $Qq \rightarrow gX, qQ \rightarrow gX$
- 16  $Qq \rightarrow QX, qQ \rightarrow qX$

Mass effects:  $m_Q \neq 0$

- 1  $gg \rightarrow QX$
- 2 -
- 3 -
- 4 -
- 5 -
- 6 -
- 7 -
- 8  $qg \rightarrow \bar{Q}X$
- 9  $qg \rightarrow QX$
- 10 -
- 11 -
- 12  $q\bar{q} \rightarrow QX$
- 13 -
- 14 -
- 15 -
- 16 -

## ADOPTED PROCEDURE –CONTINUED–

- Compare  $m \rightarrow 0$  limit of massive calculation with massless  $\overline{\text{MS}}$  calculation [1]

$$\lim_{m \rightarrow 0} d\sigma(m) = d\hat{\sigma}(\overline{\text{MS}}) + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{SUB}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\sigma(m) - d\hat{\sigma}(\overline{\text{MS}})$$

- Subtract  $d\sigma_{\text{SUB}}$  from massive partonic cross section while keeping mass terms

$$d\hat{\sigma}(m) = d\sigma(m) - d\sigma_{\text{SUB}}$$

→  $d\hat{\sigma}(m)$  short distance coefficient including  $m$

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization ⊕ massive short dist. cross sections

- Treat contributions with charm in the initial state with  $m_c = 0$ ;  
~~ scheme choice of practical importance; tiny effect in DIS [2]

---

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

[2] Kretzer, Schienbein, PRD58(1998)094035

## ADOPTED PROCEDURE –CONTINUED–

- Compare  $m \rightarrow 0$  limit of massive calculation with massless  $\overline{\text{MS}}$  calculation [1]

$$\lim_{m \rightarrow 0} d\sigma(m) = d\hat{\sigma}(\overline{\text{MS}}) + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{SUB}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\sigma(m) - d\hat{\sigma}(\overline{\text{MS}})$$

- Subtract  $d\sigma_{\text{SUB}}$  from massive partonic cross section while keeping mass terms

$$d\hat{\sigma}(m) = d\sigma(m) - d\sigma_{\text{SUB}}$$

→  $d\hat{\sigma}(m)$  short distance coefficient including  $m$

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization ⊕ massive short dist. cross sections

- Treat contributions with charm in the initial state with  $m_c = 0$ ;  
~~ scheme choice of practical importance; tiny effect in DIS [2]

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

[2] Kretzer, Schienbein, PRD58(1998)094035

## ADOPTED PROCEDURE –CONTINUED–

- Compare  $m \rightarrow 0$  limit of massive calculation with massless  $\overline{\text{MS}}$  calculation [1]

$$\lim_{m \rightarrow 0} d\sigma(m) = d\hat{\sigma}(\overline{\text{MS}}) + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{SUB}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\sigma(m) - d\hat{\sigma}(\overline{\text{MS}})$$

- Subtract  $d\sigma_{\text{SUB}}$  from massive partonic cross section while keeping mass terms

$$d\hat{\sigma}(m) = d\sigma(m) - d\sigma_{\text{SUB}}$$

→  $d\hat{\sigma}(m)$  short distance coefficient including  $m$

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization ⊕ massive short dist. cross sections

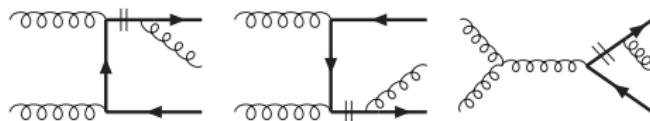
- Treat contributions with charm in the initial state with  $m_c = 0$ ; ↵ scheme choice of practical importance; tiny effect in DIS [2]

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

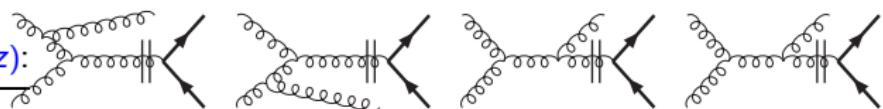
[2] Kretzer, Schienbein, PRD58(1998)094035

# GRAPHICAL REPRESENTATION OF SUBTRACTION TERMS FOR $gg \rightarrow Q\bar{Q}g$

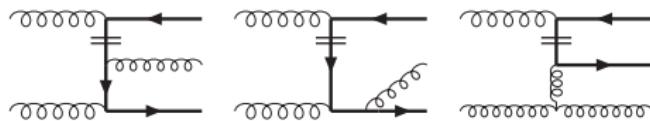
$d\hat{\sigma}^{(0)}(gg \rightarrow Q\bar{Q}) \otimes d_{Q \rightarrow Q}^{(1)}(z)$ :



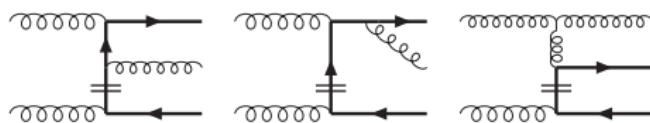
$d\hat{\sigma}^{(0)}(gg \rightarrow gg) \otimes d_{g \rightarrow Q}^{(1)}(z)$ :



$f_{g \rightarrow Q}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(Qg \rightarrow Qg)$ :

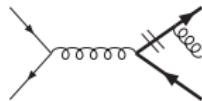


$f_{g \rightarrow Q}^{(1)}(x_2) \otimes d\hat{\sigma}^{(0)}(gQ \rightarrow Qg)$ :

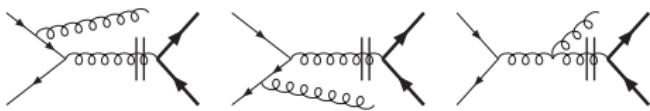


# GRAPHICAL REPRESENTATION OF SUBTRACTION TERMS FOR $q\bar{q} \rightarrow Q\bar{Q}$ AND $gq \rightarrow Q\bar{Q}q$

$d\hat{\sigma}^{(0)}(q\bar{q} \rightarrow Q\bar{Q}) \otimes d_{Q \rightarrow Q}^{(1)}(z)$ :



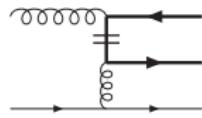
$d\hat{\sigma}^{(0)}(q\bar{q} \rightarrow gg) \otimes d_{g \rightarrow Q}^{(1)}(z)$ :



$d\hat{\sigma}^{(0)}(gq \rightarrow gq) \otimes d_{g \rightarrow Q}^{(1)}(z)$ :



$f_{g \rightarrow Q}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(Qq \rightarrow Qq)$ :



# $D^0, D^+, D^{*+}$ FFs WITH FINITE-MASS CORRECTIONS [1]

## FORMALISM

- $e^+ + e^- \rightarrow (\gamma, Z) \rightarrow H + X, \quad H = D^0, D^+, D^{*+}, \dots$
- $x = 2(p_H \cdot q)/q^2 = 2E_H/\sqrt{s} \quad \sqrt{\rho_H} \leq x \leq 1 \quad (\rho_H = 4m_H^2/s)$

$$\frac{d\sigma}{dx}(x, s) = \sum_a \int_{y_{\min}}^{y_{\max}} \frac{dy}{y} \frac{d\sigma_a}{dy}(y, \mu, \mu_f) D_a \left( \frac{x}{y}, \mu_f \right)$$

$d\sigma_a/dy$  at NLO with  $m_q = 0$  [2] and  $m_q \neq 0$  [1,3]

- $x_p = p/p_{\max} = \sqrt{(x^2 - \rho_H)/(1 - \rho_H)} \quad 0 \leq x_p \leq 1$

$$\frac{d\sigma}{dx_p}(x_p) = (1 - \rho_H) \frac{x_p}{x} \frac{d\sigma}{dx}(x)$$

---

[1] Kneesch,B.K.,Kramer,Schienbein, NPB799(2008)34

[2] Baier, Fey, ZPC2(1979)339; Altarelli et al. NPB160(1979)301

[3] Nason, Webber, NPB421(1994)473

# $D^0, D^+, D^{*+}$ FFs WITH FINITE-MASS CORRECTIONS [1]

## FORMALISM

- $e^+ + e^- \rightarrow (\gamma, Z) \rightarrow H + X, \quad H = D^0, D^+, D^{*+}, \dots$
- $x = 2(p_H \cdot q)/q^2 = 2E_H/\sqrt{s} \quad \sqrt{\rho_H} \leq x \leq 1 \quad (\rho_H = 4m_H^2/s)$

$$\frac{d\sigma}{dx}(x, s) = \sum_a \int_{y_{\min}}^{y_{\max}} \frac{dy}{y} \frac{d\sigma_a}{dy}(y, \mu, \mu_f) D_a \left( \frac{x}{y}, \mu_f \right)$$

$d\sigma_a/dy$  at NLO with  $m_q = 0$  [2] and  $m_q \neq 0$  [1,3]

- $x_p = p/p_{\max} = \sqrt{(x^2 - \rho_H)/(1 - \rho_H)} \quad 0 \leq x_p \leq 1$

$$\frac{d\sigma}{dx_p}(x_p) = (1 - \rho_H) \frac{x_p}{x} \frac{d\sigma}{dx}(x)$$

[1] Kneesch,B.K.,Kramer,Schienbein, NPB799(2008)34

[2] Baier, Fey, ZPC2(1979)339; Altarelli et al. NPB160(1979)301

[3] Nason, Webber, NPB421(1994)473

# $D^0, D^+, D^{*+}$ FFs WITH FINITE-MASS CORRECTIONS [1]

## FORMALISM

- $e^+ + e^- \rightarrow (\gamma, Z) \rightarrow H + X, \quad H = D^0, D^+, D^{*+}, \dots$
- $x = 2(p_H \cdot q)/q^2 = 2E_H/\sqrt{s} \quad \sqrt{\rho_H} \leq x \leq 1 \quad (\rho_H = 4m_H^2/s)$

$$\frac{d\sigma}{dx}(x, s) = \sum_a \int_{y_{\min}}^{y_{\max}} \frac{dy}{y} \frac{d\sigma_a}{dy}(y, \mu, \mu_f) D_a \left( \frac{x}{y}, \mu_f \right)$$

$d\sigma_a/dy$  at NLO with  $m_q = 0$  [2] and  $m_q \neq 0$  [1,3]

- $x_p = p/p_{\max} = \sqrt{(x^2 - \rho_H)/(1 - \rho_H)} \quad 0 \leq x_p \leq 1$

$$\frac{d\sigma}{dx_p}(x_p) = (1 - \rho_H) \frac{x_p}{x} \frac{d\sigma}{dx}(x)$$

---

[1] Kneesch,B.K.,Kramer,Schienbein, NPB799(2008)34

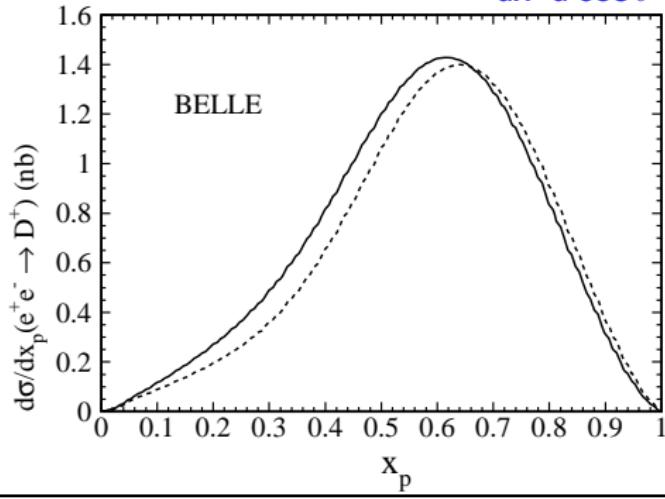
[2] Baier, Fey, ZPC2(1979)339; Altarelli et al. NPB160(1979)301

[3] Nason, Webber, NPB421(1994)473

## INITIAL-STATE RADIATION

- Use radiator  $D_{e^\pm}$  [1]

$$\frac{d\sigma_{\text{ISR}}}{dx}(x, s) = \int dx_+ dx_- dx' d\cos\theta' \delta(x - x(x_+, x_-, x', \cos\theta')) \\ \times D_{e^+}(x_+, s) D_{e^-}(x_-, s) \frac{d^2\sigma}{dx' d\cos\theta'}(x', \cos\theta', x_+ x_- s)$$



[1] Kuraev, Fadin, SJNP41(1985)466; Niclòsini, Trentadue, PLB196(1987)551

# INPUT

- Experimental data

Type	$\sqrt{s}$ [GeV]	H	Collaboration
$d\sigma/dx_p$	10.52	$D^0, D^+, D^{*+}$	Belle 06
$d\sigma/dx_p$	10.52	$D^0, D^+, D^{*+}$	CLEO 04
$(1/\sigma_{\text{tot}})d\sigma/dx$	91.2	$D^{*+}$	ALEPH 00
$(1/\sigma_{\text{tot}})d\sigma/dx$	91.2	$D^0, D^+, D^{*+}$	OPAL 96,98

- Theoretical input

- ▶  $m_c = 1.5 \text{ GeV}$ ,  $m_b = 5.0 \text{ GeV}$ ,  $\alpha(m_T) = 1/132$ ,  
 $\alpha_s(m_Z) = 0.1176 \rightsquigarrow \Lambda_{\text{QCD}}^{(5)} = 221 \text{ MeV}$
- ▶ Bowler ansatz [1]

$$D_Q^{H_c}(z, \mu_0) = Nz^{-(1+\gamma^2)}(1-z)^a e^{-\gamma^2/z}$$

---

[1] Bowler, ZPC11(1981)169

# INPUT

- Experimental data

Type	$\sqrt{s}$ [GeV]	H	Collaboration
$d\sigma/dx_p$	10.52	$D^0, D^+, D^{*+}$	Belle 06
$d\sigma/dx_p$	10.52	$D^0, D^+, D^{*+}$	CLEO 04
$(1/\sigma_{\text{tot}})d\sigma/dx$	91.2	$D^{*+}$	ALEPH 00
$(1/\sigma_{\text{tot}})d\sigma/dx$	91.2	$D^0, D^+, D^{*+}$	OPAL 96,98

- Theoretical input

- ▶  $m_c = 1.5 \text{ GeV}$ ,  $m_b = 5.0 \text{ GeV}$ ,  $\alpha(m_T) = 1/132$ ,  
 $\alpha_s(m_Z) = 0.1176 \rightsquigarrow \Lambda_{\text{QCD}}^{(5)} = 221 \text{ MeV}$
- ▶ Bowler ansatz [1]

$$D_Q^{H_c}(z, \mu_0) = Nz^{-(1+\gamma^2)}(1-z)^a e^{-\gamma^2/z}$$

---

[1] Bowler, ZPC11(1981)169

## RESULTS: GOODNESS

- $\chi^2/\text{d.o.f.}$

$H$	VFNS	Belle/CLEO	ALEPH/OPAL	Global
$D^0$	GM	3.15	0.794	4.03
	ZM	3.25	0.789	4.66
$D^+$	GM	1.30	0.509	1.99
	ZM	1.37	0.507	2.21
$D^{*+}$	GM	3.74	2.06	6.90
	ZM	3.69	2.04	7.64

- Quark mass effects improve global fits and Belle/CLEO fits for  $D^0, D^+$ , but have no impact on ALEPH/OPAL fits.
- Belle and CLEO data on  $D^0, D^{*+}$  moderately compatible.
- OPAL fits for  $D^0, D^+$  excellent; ALEPH and OPAL data on  $D^{*+}$  moderately compatible.
- Tension between Belle/CLEO and ALEPH/OPAL data.

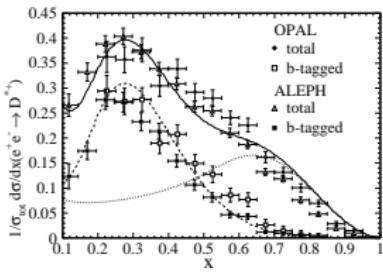
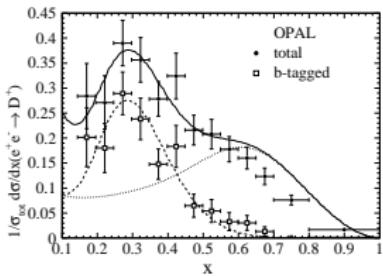
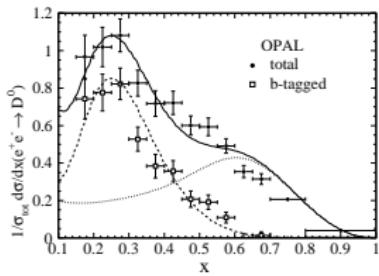
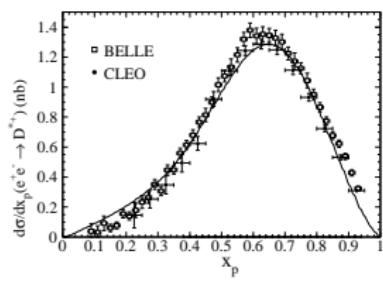
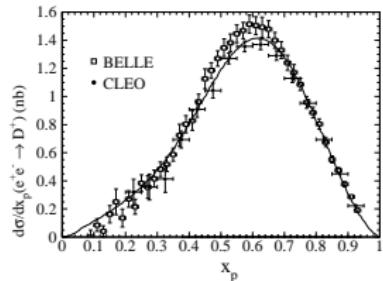
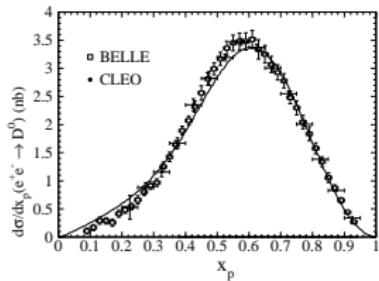
## RESULTS: GOODNESS

- $\chi^2/\text{d.o.f.}$

$H$	VFNS	Belle/CLEO	ALEPH/OPAL	Global
$D^0$	GM	3.15	0.794	4.03
	ZM	3.25	0.789	4.66
$D^+$	GM	1.30	0.509	1.99
	ZM	1.37	0.507	2.21
$D^{*+}$	GM	3.74	2.06	6.90
	ZM	3.69	2.04	7.64

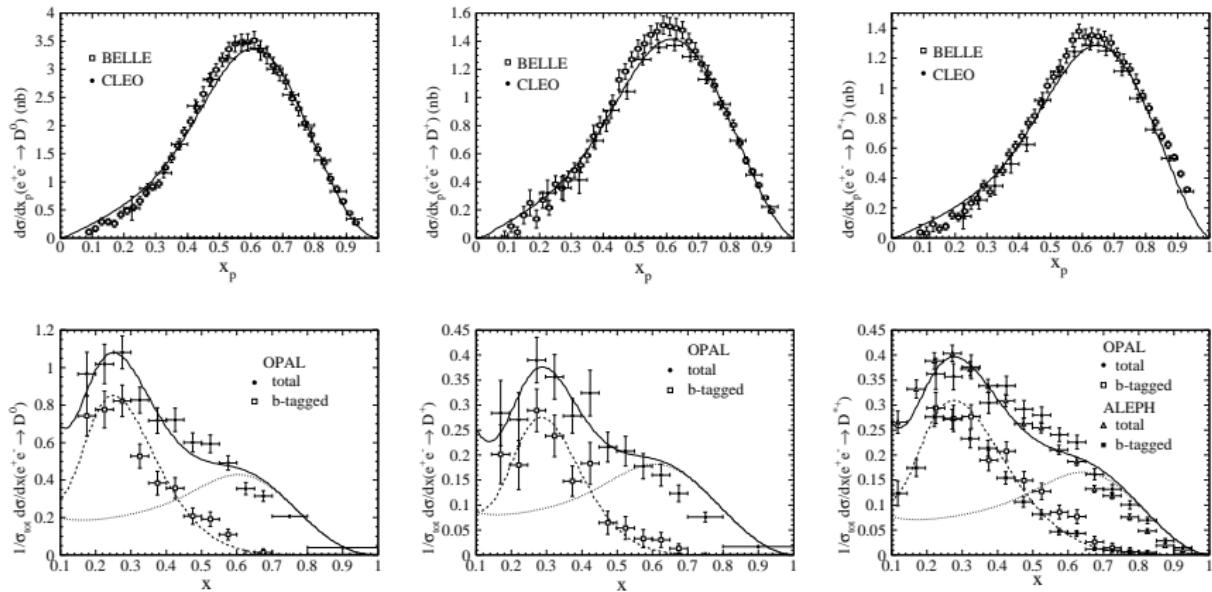
- Quark mass effects improve global fits and Belle/CLEO fits for  $D^0, D^+$ , but have no impact on ALEPH/OPAL fits.
- Belle and CLEO data on  $D^0, D^{*+}$  moderately compatible.
- OPAL fits for  $D^0, D^+$  excellent; ALEPH and OPAL data on  $D^{*+}$  moderately compatible.
- Tension between Belle/CLEO and ALEPH/OPAL data.

## RESULTS: GLOBAL FITS



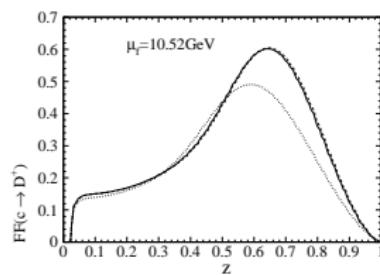
- Belle/CLEO data push  $\langle z \rangle_c(m_Z)$  up by 0.03–0.04

## RESULTS: GLOBAL FITS

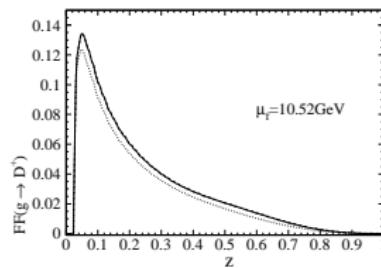


- Belle/CLEO data push  $\langle z \rangle_c(m_Z)$  up by 0.03–0.04

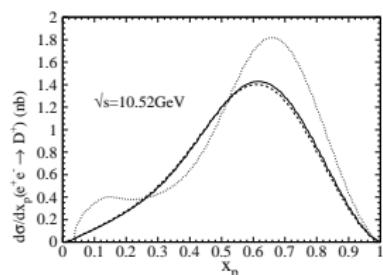
## RESULTS: QUARK AND HADRON MASS EFFECTS



$c \rightarrow D^+$  FF



$g \rightarrow D^+$  FF



$d\sigma/dx_p$  w/ Belle/CLEO-GM FFs

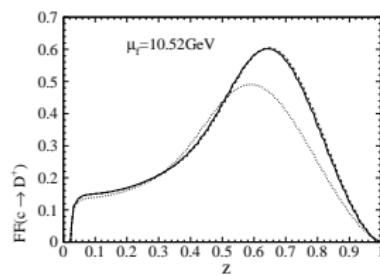
dotted:  $m_c = m_H = 0$

dashed:  $m_c = 0 \neq m_H$  (ZM-VFNS)

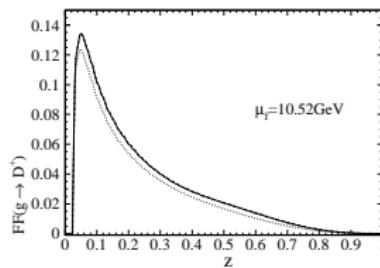
solid:  $m_c \neq 0 \neq m_H$  (GM-VFNS)

- Hadron mass effects on FFs important, quark mass effects marginal

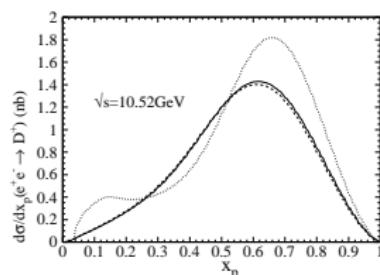
## RESULTS: QUARK AND HADRON MASS EFFECTS



$c \rightarrow D^+$  FF



$g \rightarrow D^+$  FF



$d\sigma/dx_p$  w/ Belle/CLEO-GM FFs

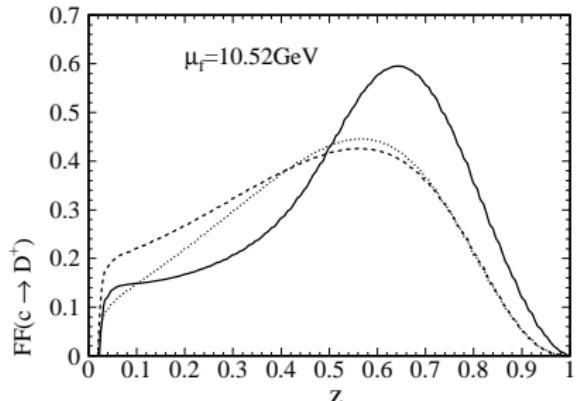
dotted:  $m_c = m_H = 0$

dashed:  $m_c = 0 \neq m_H$  (ZM-VFNS)

solid:  $m_c \neq 0 \neq m_H$  (GM-VFNS)

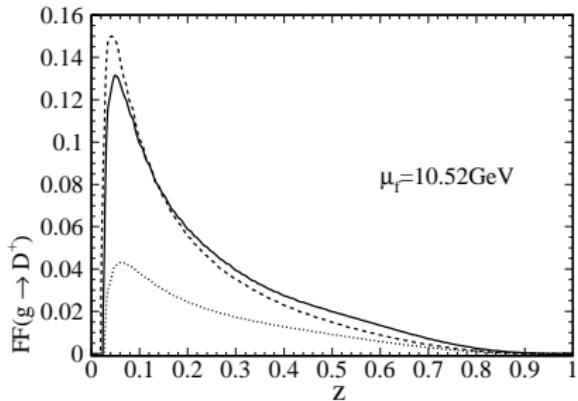
- Hadron mass effects on FFs important, quark mass effects marginal

## RESULTS: COMPARISONS W/ PREVIOUS FFs



$c \rightarrow D^+$  FF

- dotted:  $m_c = 0 = m_H$     $\mu_0 = 2m_c$
- dashed:  $m_c = 0 = m_H$     $\mu_0 = m_c$
- solid:       $m_c \neq 0 \neq m_H$     $\mu_0 = m_c$

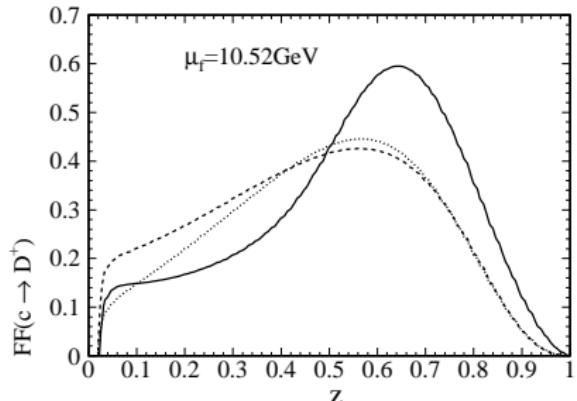


$g \rightarrow D^+$  FF

- |          |                 |          |
|----------|-----------------|----------|
| Peterson | OPAL            | KK 05    |
| Peterson | OPAL            | KK 06    |
| Bowler   | Belle,CLEO,OPAL | KKKSc 08 |

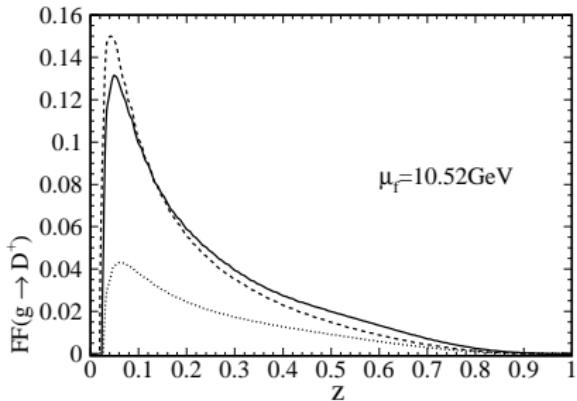
- Strong pull of Belle/CLEO data on  $c \rightarrow D^+$  FF
- Reduction in  $\mu_0$  increases  $g \rightarrow D^+$  FF

## RESULTS: COMPARISONS W/ PREVIOUS FFs



$c \rightarrow D^+$  FF

- dotted:  $m_c = 0 = m_H$     $\mu_0 = 2m_c$
- dashed:  $m_c = 0 = m_H$     $\mu_0 = m_c$
- solid:       $m_c \neq 0 \neq m_H$     $\mu_0 = m_c$



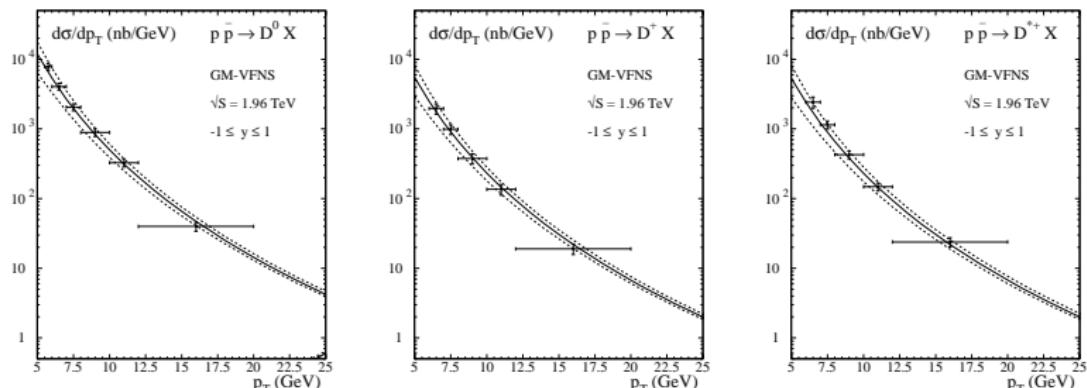
$g \rightarrow D^+$  FF

- |          |                 |          |
|----------|-----------------|----------|
| Peterson | OPAL            | KK 05    |
| Peterson | OPAL            | KK 06    |
| Bowler   | Belle,CLEO,OPAL | KKKSc 08 |

- Strong pull of Belle/CLEO data on  $c \rightarrow D^+$  FF
- Reduction in  $\mu_0$  increases  $g \rightarrow D^+$  FF

# HADROPRODUCTION OF $D^0$ , $D^+$ , $D^{*+}$ , $D_s^+$

GM-VFNS RESULTS w/ KKKSc FFs [1]

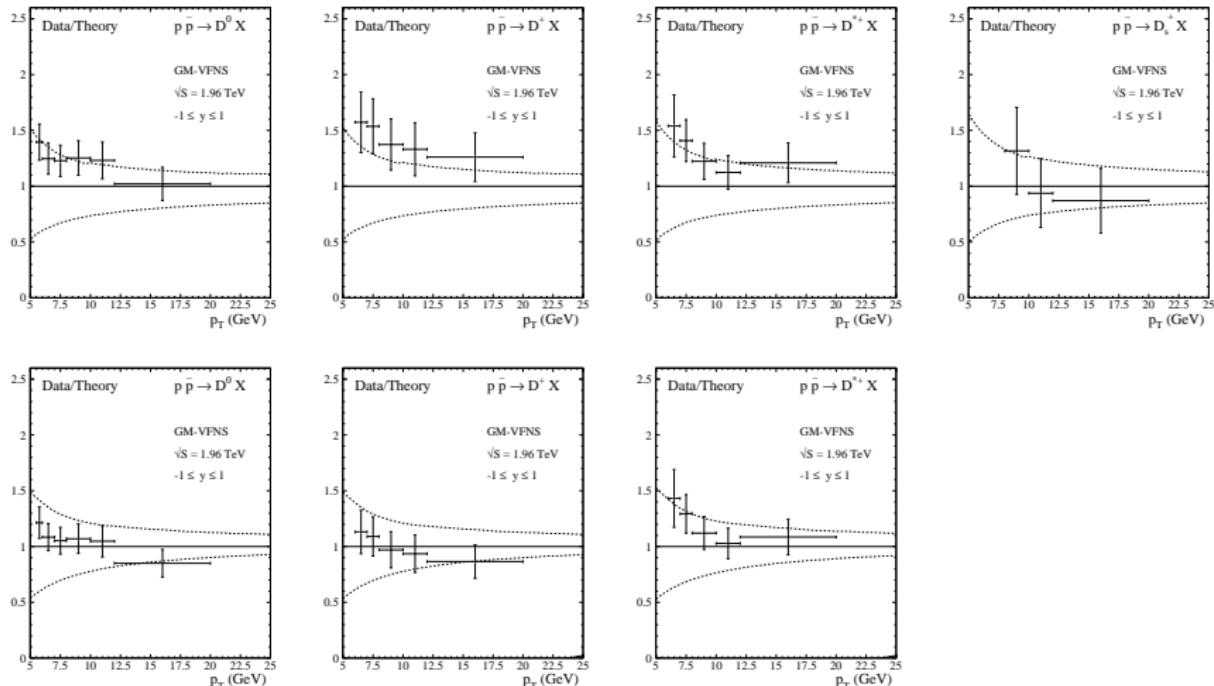


- $d\sigma/dp_T$  [nb/GeV]  $|y| \leq 1$  prompt charm
- Uncertainty band:  $1/2 \leq \mu_R/m_T, \mu_F/m_T \leq 2$  ( $m_T = \sqrt{p_T^2 + m_c^2}$ )
- CDF data from run II [2]
- GM-VFNS describes data within errors

[1] B.K., Kramer, Schienbein, Spiesberger, arXiv:0901.4130[hep-ph], PRD(to appear)

[2] Acosta et al., PRL91(2003)241804

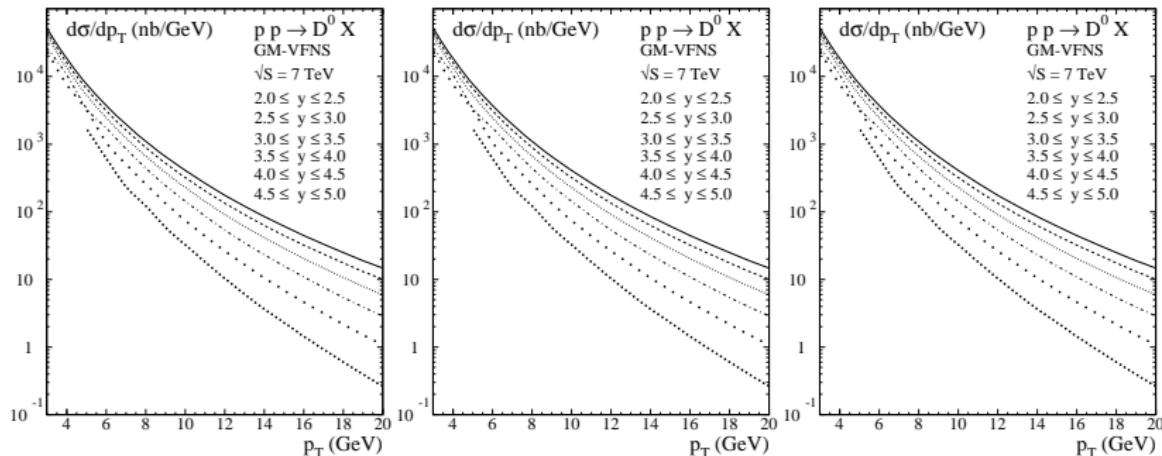
# COMPARISON W/ PREVIOUS KK FFs [1]



- New KKKSc FFs improve agreement w/ CDF data.

[1] B.K., Kramer, PRD74(2006)037502

# NLO GM-VFNS PREDICTIONS FOR THE LHC LHCb CONDITIONS



- Inputs: CTEQ6.5 PDFs, KKKSc FFs [1]
- Uncertainty band:  $1/2 \leq \mu_R/m_T, \mu_F/m_T \leq 2$  ( $m_T = \sqrt{p_T^2 + m_c^2}$ )

---

[1] Kneesch,B.K.,Kramer,Schienbein, NPB799(2008)34

# SUMMARY

- GM-VFNS (cf. ACOT scheme) with non-perturbative FFs provides rigorous theoretical framework for global analysis of inclusive heavy-hadron production:
  - ▶ full mass dependence
  - ▶ scaling violations and universality of FFs
  - ▶ no spurious  $x \rightarrow 1$  problems to be fixed
  - ▶ no ad-hoc weight functions, no hidden theoretical errors
- Processes available at NLO:
  - ▶  $e^+e^- \rightarrow H + X$
  - ▶  $\gamma\gamma \rightarrow H + X$  direct, singly and doubly resolved
  - ▶  $\gamma p \rightarrow H + X$  direct and resolved
  - ▶  $p\bar{p}, pp \rightarrow H + X$
- New  $D^0, D^+, D^{*+}$  FFs w/ mass effects from Belle, CLEO, ALEPH, and OPAL data
- $D$  (and  $B$ ) hadroproduction predictions in good agreement w/ CDF data
  - ▶ Mass effects positive and moderate

# SUMMARY

- GM-VFNS (cf. ACOT scheme) with non-perturbative FFs provides rigorous theoretical framework for global analysis of inclusive heavy-hadron production:
  - ▶ full mass dependence
  - ▶ scaling violations and universality of FFs
  - ▶ no spurious  $x \rightarrow 1$  problems to be fixed
  - ▶ no ad-hoc weight functions, no hidden theoretical errors
- Processes available at NLO:
  - ▶  $e^+e^- \rightarrow H + X$
  - ▶  $\gamma\gamma \rightarrow H + X$  direct, singly and doubly resolved
  - ▶  $\gamma p \rightarrow H + X$  direct and resolved
  - ▶  $p\bar{p}, pp \rightarrow H + X$
- New  $D^0, D^+, D^{*+}$  FFs w/ mass effects from Belle, CLEO, ALEPH, and OPAL data
- $D$  (and  $B$ ) hadroproduction predictions in good agreement w/ CDF data
  - ▶ Mass effects positive and moderate

# SUMMARY

- GM-VFNS (cf. ACOT scheme) with non-perturbative FFs provides rigorous theoretical framework for global analysis of inclusive heavy-hadron production:
  - ▶ full mass dependence
  - ▶ scaling violations and universality of FFs
  - ▶ no spurious  $x \rightarrow 1$  problems to be fixed
  - ▶ no ad-hoc weight functions, no hidden theoretical errors
- Processes available at NLO:
  - ▶  $e^+e^- \rightarrow H + X$
  - ▶  $\gamma\gamma \rightarrow H + X$  direct, singly and doubly resolved
  - ▶  $\gamma p \rightarrow H + X$  direct and resolved
  - ▶  $p\bar{p}, pp \rightarrow H + X$
- New  $D^0, D^+, D^{*+}$  FFs w/ mass effects from Belle, CLEO, ALEPH, and OPAL data
- $D$  (and  $B$ ) hadroproduction predictions in good agreement w/ CDF data
  - ▶ Mass effects positive and moderate

# SUMMARY

- GM-VFNS (cf. ACOT scheme) with non-perturbative FFs provides rigorous theoretical framework for global analysis of inclusive heavy-hadron production:
  - ▶ full mass dependence
  - ▶ scaling violations and universality of FFs
  - ▶ no spurious  $x \rightarrow 1$  problems to be fixed
  - ▶ no ad-hoc weight functions, no hidden theoretical errors
- Processes available at NLO:
  - ▶  $e^+e^- \rightarrow H + X$
  - ▶  $\gamma\gamma \rightarrow H + X$  direct, singly and doubly resolved
  - ▶  $\gamma p \rightarrow H + X$  direct and resolved
  - ▶  $p\bar{p}, pp \rightarrow H + X$
- New  $D^0, D^+, D^{*+}$  FFs w/ mass effects from Belle, CLEO, ALEPH, and OPAL data
- $D$  (and  $B$ ) hadroproduction predictions in good agreement w/ CDF data
  - ▶ Mass effects positive and moderate

# SUMMARY

- GM-VFNS (cf. ACOT scheme) with non-perturbative FFs provides rigorous theoretical framework for global analysis of inclusive heavy-hadron production:
  - ▶ full mass dependence
  - ▶ scaling violations and universality of FFs
  - ▶ no spurious  $x \rightarrow 1$  problems to be fixed
  - ▶ no ad-hoc weight functions, no hidden theoretical errors
- Processes available at NLO:
  - ▶  $e^+e^- \rightarrow H + X$
  - ▶  $\gamma\gamma \rightarrow H + X$  direct, singly and doubly resolved
  - ▶  $\gamma p \rightarrow H + X$  direct and resolved
  - ▶  $p\bar{p}, pp \rightarrow H + X$
- New  $D^0, D^+, D^{*+}$  FFs w/ mass effects from Belle, CLEO, ALEPH, and OPAL data
- $D$  (and  $B$ ) hadroproduction predictions in good agreement w/ CDF data
  - ▶ Mass effects **positive and moderate**

## SUMMARY (CONT.)

- Up-to-date NLO GM-VFNS predictions for  $D^0$ ,  $D^\pm$ ,  $D^{*\pm}$  and  $D_s^\pm$  hadroproduction at LHC available upon request
- Just specify  $D$ -meson species and  $p_T$  and  $y$  bins
- Same for  $B$  hadrons

## SUMMARY (CONT.)

- Up-to-date NLO GM-VFNS predictions for  $D^0$ ,  $D^\pm$ ,  $D^{*\pm}$  and  $D_s^\pm$  hadroproduction at LHC available upon request
- Just specify  $D$ -meson species and  $p_T$  and  $y$  bins
- Same for  $B$  hadrons

## SUMMARY (CONT.)

- Up-to-date NLO GM-VFNS predictions for  $D^0$ ,  $D^\pm$ ,  $D^{*\pm}$  and  $D_s^\pm$  hadroproduction at LHC available upon request
- Just specify  $D$ -meson species and  $p_T$  and  $y$  bins
- Same for  $B$  hadrons

## DIGRESSION: FONLL SCHEME [1]

$$\text{FONLL} = \text{FO} + (\text{RS} - \text{FOM0}) \times G(m, p_T)$$

- FO: *fixed order*, FOM0: *massless limit thereof*; RS: *resummed*
- shift  $p_T \rightarrow m_T$  in  $(\text{RS} - \text{FOM0})$
- $G(m, p_T) = \frac{p_T^2}{p_T^2 + c^2 m^2}$  with  $c = 5$  to suppress  $(\text{RS} - \text{FOM0})$  being *abnormally large* for  $m_T < 5m$
- NB:  $G(m, p_T) = 4\%, 14\%, 26\%$  for  $p_T/m = 1, 2, 3$
- convolute  $(\text{RS} - \text{FOM0})$  with perturbative  $q \rightarrow Q$  and  $g \rightarrow Q$  FFs subject to DGLAP evolution in  $\mu_F$
- convolute FONLL with non-perturbative  $\mu_F$ -independent  $Q \rightarrow X_c$  FF of form [2]  
$$D_{\text{NP}}(x) = \text{Norm.} \times \frac{1}{1+c} [\delta(1-x) + c N_{a,b}^{-1} (1-x)^a x^b]$$
 with  $N_{a,b} = \int_0^1 dx (1-x)^a x^b$

---

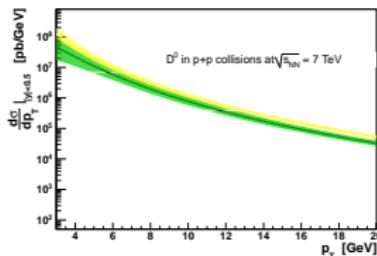
[1] Cacciari, Greco, Nason, JHEP05(1998)007; Cacciari, Nason, PRL89(2002)122003

[2] Cacciari, Nason, Oleari, JHEP04(2006)006

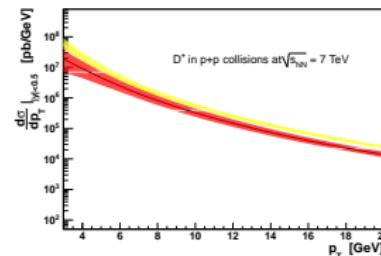
# GM-VFNS vs. FONLL

D MESONS AT ALICE

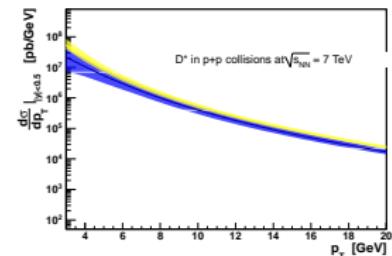
$$(D^0 + \bar{D}^0)/2$$



$$(D^+ + D^-)/2$$



$$(D^{*+} + D^{*-})/2$$



- GM-VFNS inputs: CTEQ6.5 PDFs, KKKSc FFs [1]
- Uncertainty band:  $1/2 \leq \mu_R/m_T, \mu_F/m_T \leq 2$  ( $m_T = \sqrt{p_T^2 + m_c^2}$ )
- FONLL inputs: ?

[1] Kneesch,B.K.,Kramer,Schienbein, NPB799(2008)34