

# Quarkonium Physics in CASCADE

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## P L A N   O F   T H E   T A L K

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## 1. PREFACE ON $k_t$ -FACTORIZATION

**CASCADE** is a Monte-Carlo generator using the **CCFM** equation for the evolution of parton densities (in the backward evolution scheme). On the technical side, similar to other generators.

Point of importance: using the  $k_t$ -, not the collinear factorization.

In the **collinear scheme**, the evolution is only used to calculate the parton densities and has no effect on the hard interaction subprocess.

In the  **$k_t$ -factorization**, the parton evolution changes the character of the hard interaction: both the kinematics (due to the initial parton transverse momentum) and polarization properties (longitudinal component for the off-shell gluons).

The evolution cascade is part of the hard interaction. By means of the evolution equation we resum a subset of Feynman diagrams (up to infinitely high order) representing higher-order contributions: i.e., the ladder diagrams enhanced with  $\alpha_s^n [\ln(1/x)]^n$ .

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## THE BENEFIT:

With the LO matrix elements for the hard subprocess we get access to effects requiring complicated next-to-leading order calculations in the collinear scheme. Many important results have been obtained in the  $k_t$ -factorization much earlier than in the collinear case.

Upon including more NLO, NNLO,.. corrections, the collinear results become closer to the  $k_t$ -factorization predictions.

## EXAMPLES:

- Azimuthal correlations in open Heavy Flavor production;
- $p_t$  dependence of the  $J/\psi$  and  $\Upsilon$  cross sections ( $1/p_t^8$  versus  $1/p_t^4$ )
- $J/\psi$  and  $\Upsilon$  spin alignment (transverse versus longitudinal)

Now concentrate on the Quarkonium physics, see below

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## 2. THEORETICAL FRAMEWORK

### 2.1 COLOR-SINGLET GLUON-GLUON FUSION

Perturbative production of a heavy quark pair within QCD;

Gluon polarization vectors:  $\epsilon_g^\mu = k_T^\mu / |k_T|$

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, *Sov. Phys. JETP* 45, 199 (1977);  
Ya. Balitsky, L.N. Lipatov, *Sov. J. Nucl. Phys.* 28, 822 (1978);  
L.V. Gribov, E.M. Levin, M. G. Ryskin, *Phys. Rep.* 100, 1 (1983).

Spin projection operators to guarantee the proper quantum numbers:

for Spin-triplet states  $\mathcal{P}(^3S_1) = \not{\epsilon}_V(\not{p}_Q + m_Q)/(2m_Q)$

for Spin-singlet states  $\mathcal{P}(^1S_0) = \gamma_5(\not{p}_Q + m_Q)/(2m_Q)$

Probability to form a bound state is determined by the wave function:

for  $S$ -wave states  $|R_S(0)|^2$  is known from leptonic decay widths;

for  $P$ -wave states  $|R'_P(0)|^2$  is taken from potential models.

E. J. Eichten, C. Quigg, *Phys. Rev. D* 52, 1726 (1995)

If  $L \neq 0$  and  $S \neq 0$  we use the Clebsch-Gordan coefficients to reexpress the  $|L, S\rangle$  states in terms of  $|J, J_z\rangle$  states, namely, the  $\chi_0, \chi_1, \chi_2$  mesons.

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## 2.2 ACCESSING THE POLARIZATION OF VECTOR MESONS

Polarization is measured via the angular distributions of the decay products. The most general form for  $V \rightarrow \mu^+ \mu^-$ :

$$\frac{d\sigma}{d\cos\theta d\phi} \propto 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi$$

Four conventional frame definitions:

- Recoil  $\vec{z} = -\vec{p}_1 - \vec{p}_2$
  - Gottfried-Jackson  $\vec{z} = \vec{p}_1$
  - Target  $\vec{z} = -\vec{p}_2$
  - Collins-Soper  $\vec{z} = \vec{p}_1/|p_1| - \vec{p}_2/|p_2|$
- and always  $\vec{y} = [\vec{p}_1 \times \vec{p}_2]$ ,  $\vec{x} = [\vec{y} \times \vec{z}]$

Vector meson ( $V = J/\psi, \psi', \Upsilon, \Upsilon', \Upsilon''$ ) spin density matrix:

$$\epsilon_V^\mu \epsilon_V^{*\nu} = 3(l_1^\mu l_2^\nu + l_2^\mu l_1^\nu - m_V^2 g^{\mu\nu} / 2) / m_V^2$$

Equivalent to  $-g^{\mu\nu} + p_V^\mu p_V^\nu / m_V^2$  but gives access to the decay variables.  
Mode d'emploi: generate MC events including decays and apply a three-parametric fit.

## 2.3 FEED-DOWN FROM P-WAVE STATES

Assuming the dominance of electric dipole transitions, we have:

Angular distributions in the polarized  $\chi_J$  decays

$$d\Gamma(\chi_1 \rightarrow V\gamma)/d\cos\theta \propto \left[ \left(1 + \frac{1}{2}\rho\right) + \left(1 - \frac{3}{2}\rho\right) \cos^2\theta \right]$$

$$d\Gamma(\chi_2 \rightarrow V\gamma)/d\cos\theta \propto \left[ \left(\frac{5}{6} - \frac{1}{12}\xi - \frac{1}{3}\tau\right) - \left(\frac{1}{2} - \frac{1}{4}\xi - \tau\right) \cos^2\theta \right]$$

where  $\rho = d\sigma_{\chi_1(|h|=1)}/d\sigma_{\chi_1}$ ,  $\xi = d\sigma_{\chi_2(|h|=1)}/d\sigma_{\chi_2}$ ,  $\tau = d\sigma_{\chi_2(|h|=2)}/d\sigma_{\chi_2}$   
(all known from the  $\chi_J$  production matrix elements)

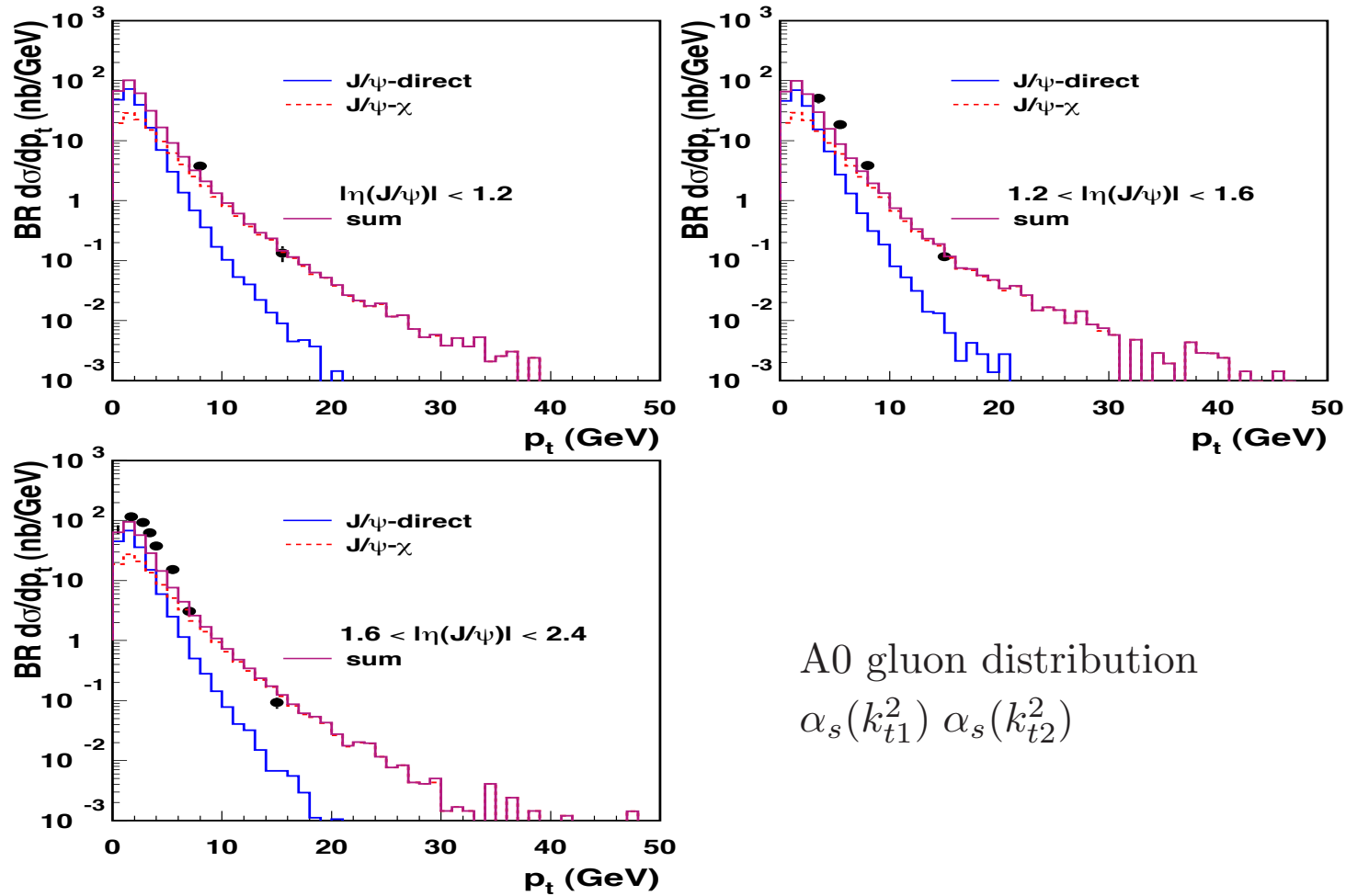
Polarization of the decay products

$$\begin{aligned} \sigma_{V(h=0)} &= B(\chi_1 \rightarrow V\gamma) \left[ (1/2) \sigma_{\chi_1(|h|=1)} \right] \\ &+ B(\chi_2 \rightarrow V\gamma) \left[ (2/3) \sigma_{\chi_2(h=0)} + (1/2) \sigma_{\chi_2(|h|=1)} \right] \\ \sigma_{V(|h|=1)} &= B(\chi_1 \rightarrow V\gamma) \left[ \sigma_{\chi_1(h=0)} + (1/2) \sigma_{\chi_1(|h|=1)} \right] \\ &+ B(\chi_2 \rightarrow V\gamma) \left[ (1/3) \sigma_{\chi_2(h=0)} + (1/2) \sigma_{\chi_2(|h|=1)} + \sigma_{\chi_2(|h|=2)} \right]. \end{aligned}$$

P.Cho, M.Wise, S.Trivedi, *Phys. Rev. D* **51**, R2039 (1995)

### 3. NUMERICAL RESULTS

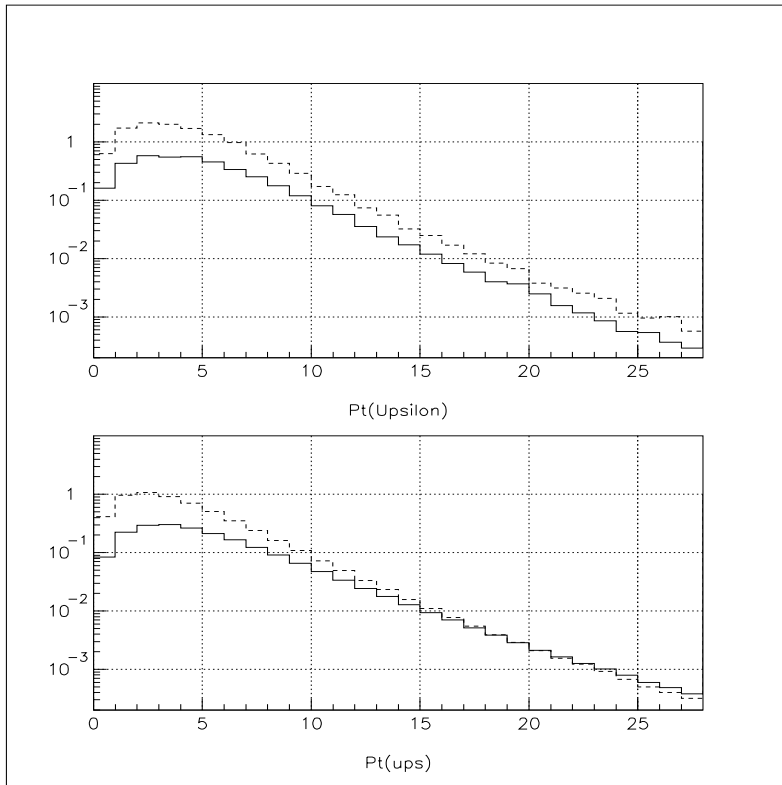
#### COMPARISON WITH LHC DATA ON THE $J/\psi$ PRODUCTION



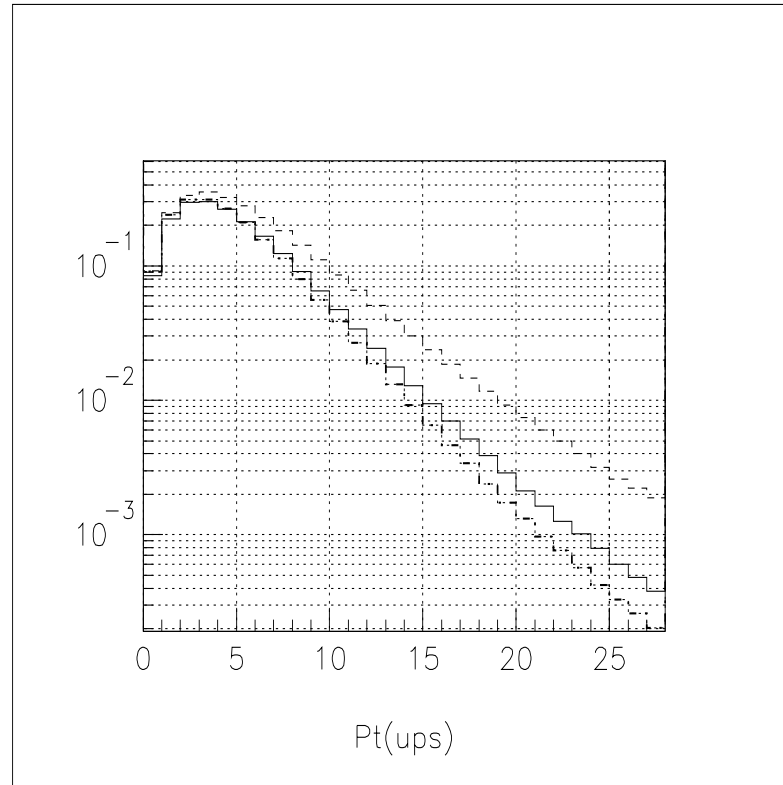
A0 gluon distribution

$$\alpha_s(k_{t1}^2) \alpha_s(k_{t2}^2)$$

## MORE ON THEORETICAL UNCERTAINTIES



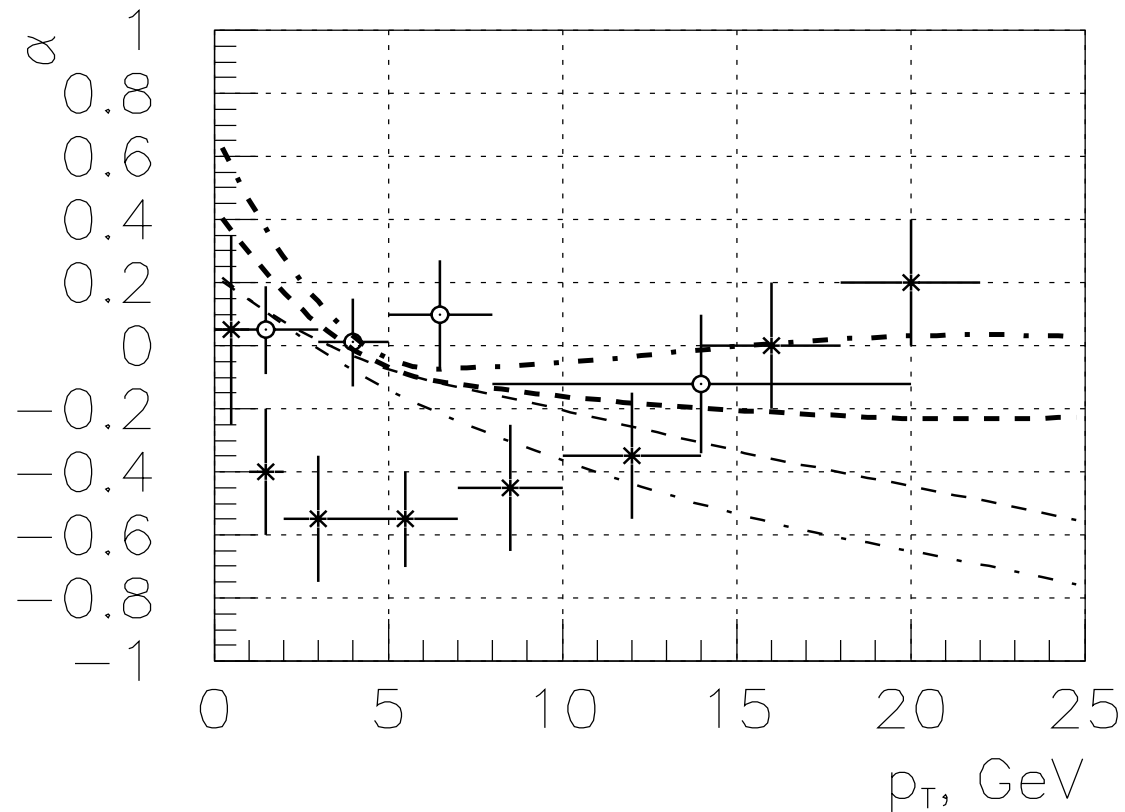
Effect of the scale in the  $\alpha_s(\mu^2)$ :  
 Upper (dashed) lines –  $\mu^2 = k_t^2$ ;  
 lower (solid) lines –  $\mu^2 = p_t^2 + m^2$   
 Upper panel –  $\Upsilon$ , lower panel –  $\chi_b$



Effect of the flux definition:  
 Solid lines –  $1/\lambda^{1/2}(\hat{s}, k_{t1}^2, k_{t2}^2)$   
 dashed lines –  $1/\hat{s}$   
 thick dash-dotted –  $1/(p_t^2 + m^2)$



## $\Upsilon(1S)$ SPIN ALIGNMENT AT THE TEVATRON



Dash-dotted lines – JB gluons; dashed – dGRV gluons;  
Thin lines – direct  $\Upsilon$  only; thick lines – with  $\chi_b$  decays added.  
○ D.Acosta et al.(CDF), Phys. Rev. Lett. **88**, 161802 (2002);  
× V.M.Abazov et al.(DO), Phys. Rev. Lett. **101**, 182004 (2008)

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## 4. CONCLUSIONS

For the Quarkonium Physics tasks CASCADE is equipped with:

- Off-shell  $g^* + g^* \rightarrow V + g$  matrix elements  
for  $V = J/\psi, \psi(2S), \Upsilon(1S), \Upsilon(2S), \Upsilon(3S)$
- Full spin density matrix for leptonic decays  $V \rightarrow l^+ + l^-$
- Off-shell  $g^* + g^* \rightarrow \chi_J$  matrix elements  
for  $\chi_J = \chi_{cJ}(1S), \chi_{bJ}(1S), \chi_{bJ}(2S)$  with  $J = 0, 1, 2$
- Full information on the  $\chi_J$  spin alignment parameters  
to generate the decays  $\chi_J \rightarrow V + \gamma$  followed by  $V \rightarrow l^+ + l^-$

ALL IS READY FOR USE

YOU ARE WELCOME!