

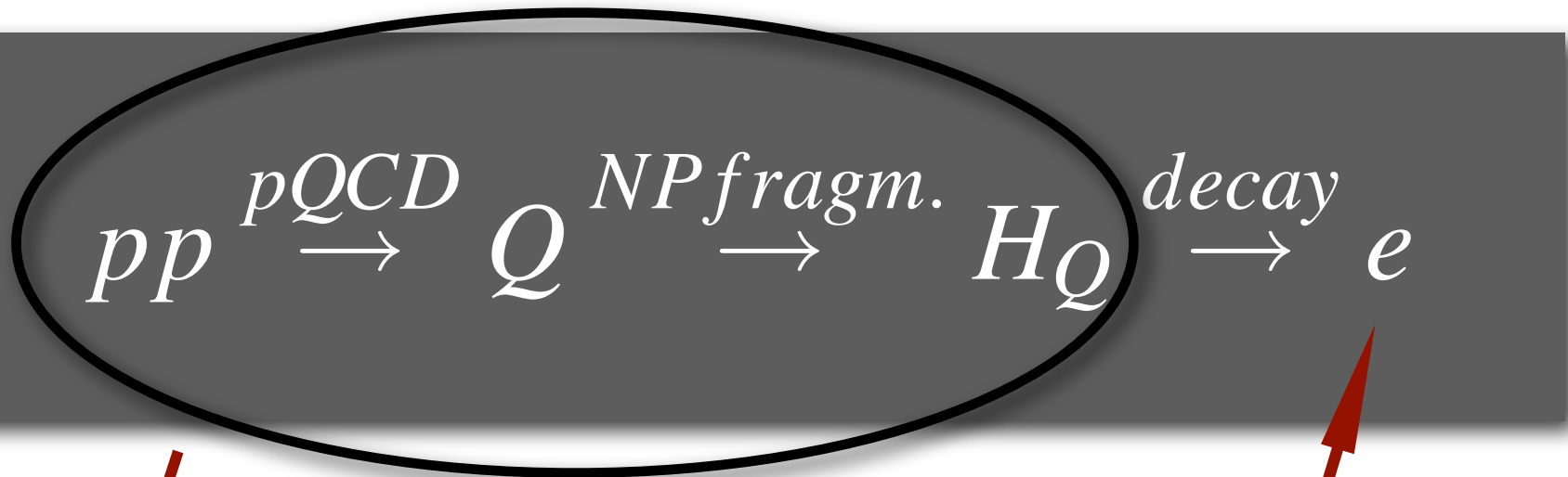
# Status of QCD predictions for heavy quark production

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(+ paper in preparation with Frixione, Mangano, Nason, Ridolfi)

# Heavy Quark production



This part is QCD.  
How accurately can we predict it?  
What ingredients do we need?



A generic final state  
observable



Compare at this level, if possible.  
A quark is not a physical object

The starting point is the 20+ years old NLO calculation

[Nason, Dawson, Ellis, '88  
Beenakker et al., '91]

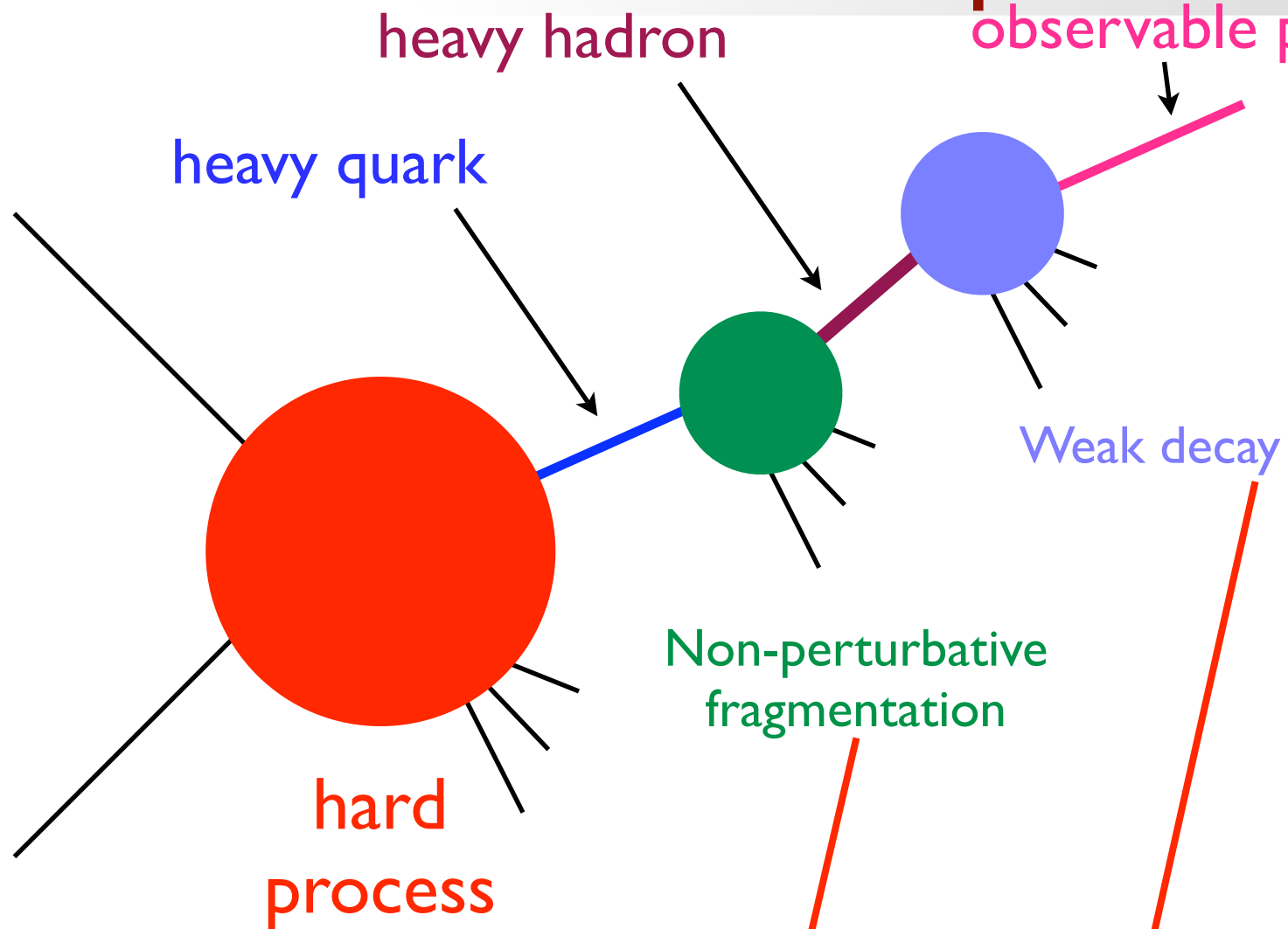
Due to the presence of a *parametrically large mass*  $m$ :

- ▶ It allows one to *predict the total cross section*
- ▶ For  $(p_t \gg m)$  differential distributions are also reliable (modulo the inclusion of a smallish non-perturbative contribution)

**Any modern implementation/prediction should tend to the NLO result<sup>(\*)</sup> in the small/moderate  $p_t$  limit or for total cross sections**

(\*) within the expected residual (i.e. NNLO) uncertainty

# Sketch of inclusive production



$$\frac{d\sigma(b \rightarrow B \rightarrow J/\psi)}{dp_T} = \frac{d\sigma(b)}{d\hat{p}_T} \otimes f(b \rightarrow B) \otimes g(B \rightarrow J/\psi)$$

# How is it done?

- ▶ Calculate perturbative corrections **as well as you can**  
(usually NLO + resummation of large logs)
- ▶ Fit remaining **(small) non-perturbative** contribution to data  
(usually  $e^+e^-$ , CLEO/BELLE, LEP,...)
- ▶ Set up code to calculate as **realistic** as possible cross sections  
(cuts, weak decays to observed particles -- **the latter typically taken from measured decay spectra**)

(Residual uncertainty usually dominated by perturbative one)

# Perturbative corrections

## NLO + Logs (without double-counting)

### ▶ FONLL

MC, Greco '94, MC, Greco, Nason '98

(An example of a GM-VFNS calculation. See Kniehl's talk for a different one)

### ▶ MC@NLO

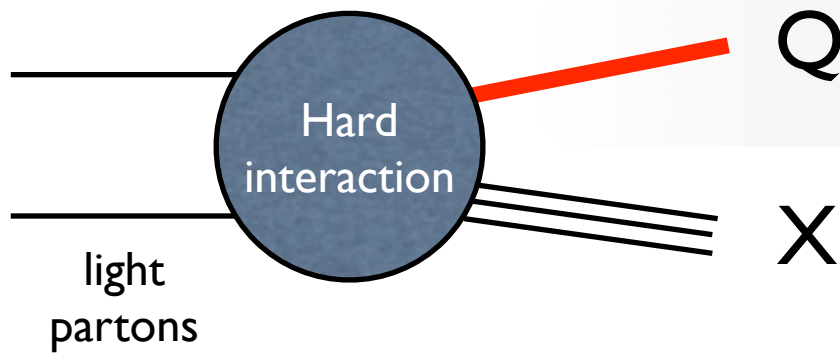
Frixione, Webber '02

### ▶ POWHEG

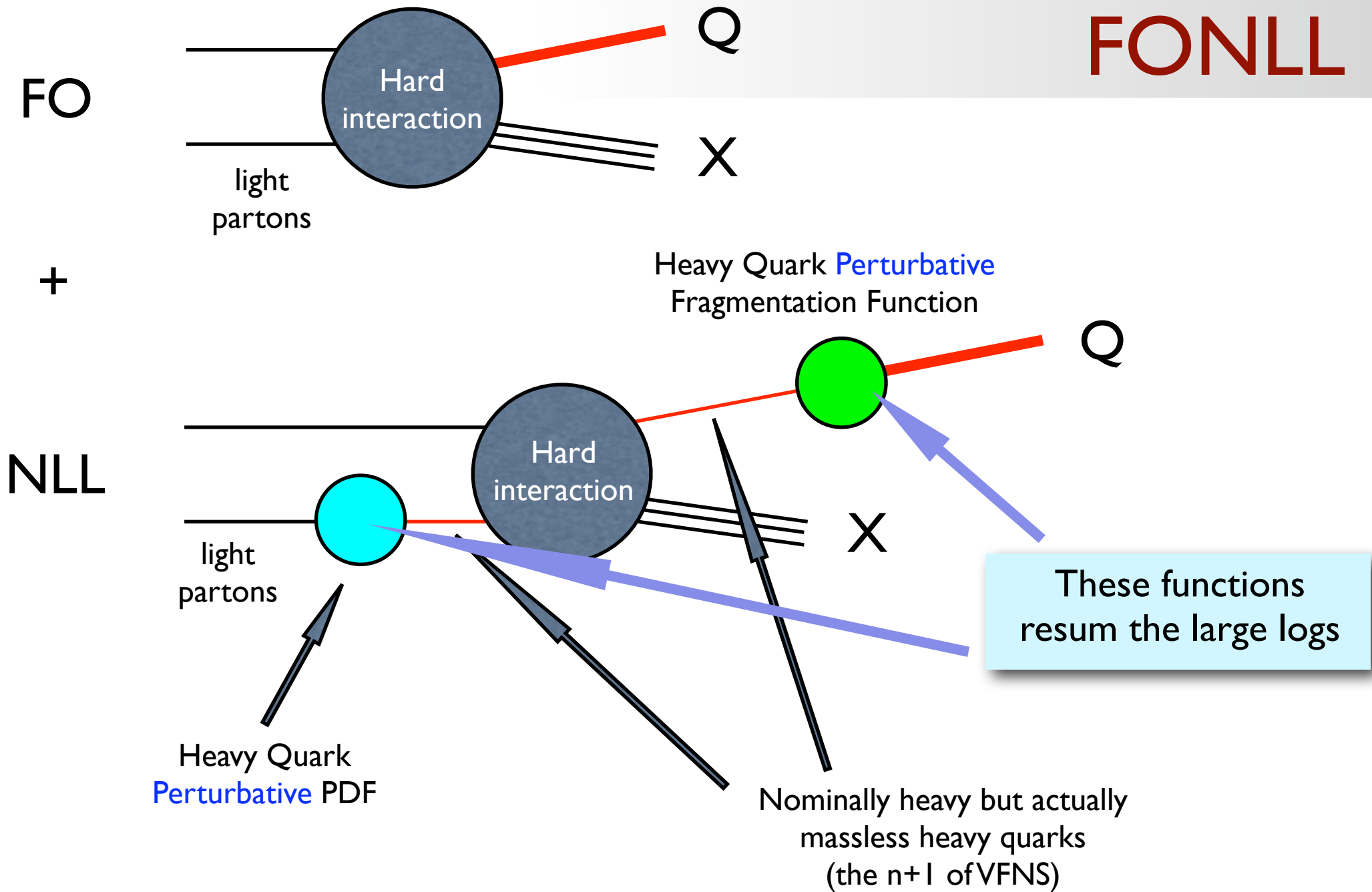
Nason, '04

In all cases, matching between a NLO fixed order calculation (Nason, Dawson, Ellis, '88) and the resummation of large logs, either semi-numerically (FONLL, NLL accuracy) or via a parton shower Montecarlo (LL accuracy)

FO

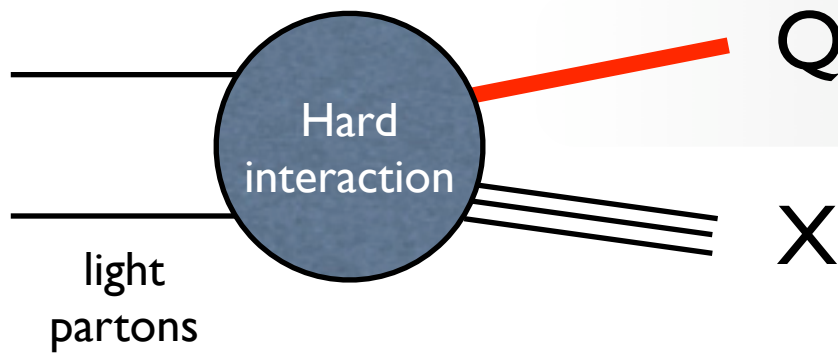


+





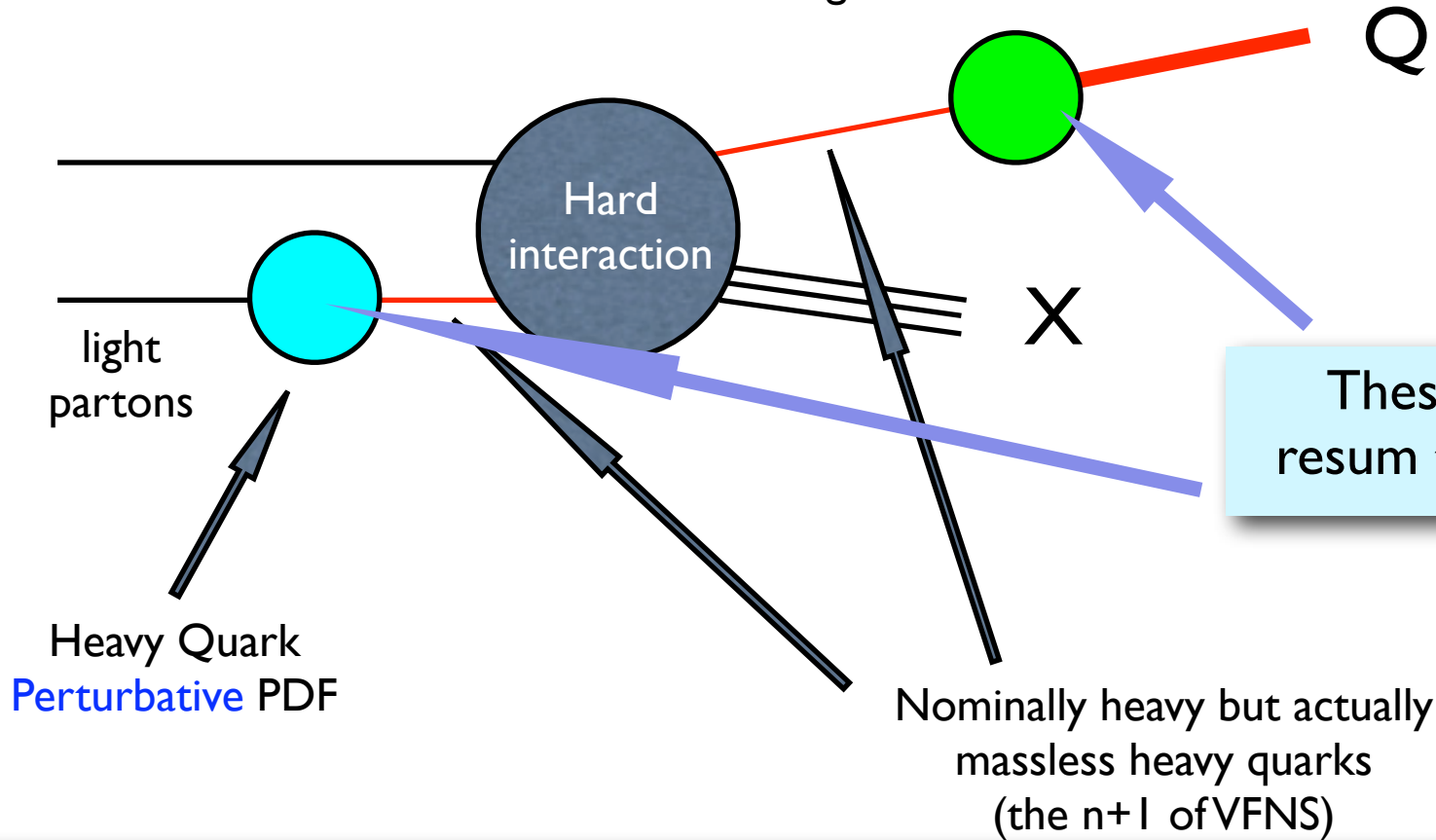
FO



+

Heavy Quark **Perturbative**  
Fragmentation Function

NLL



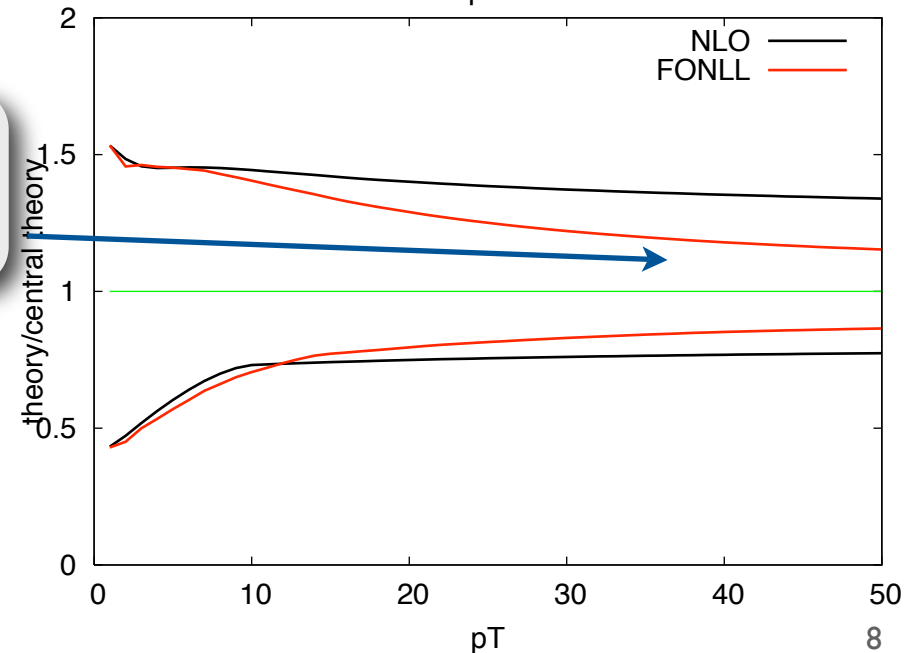
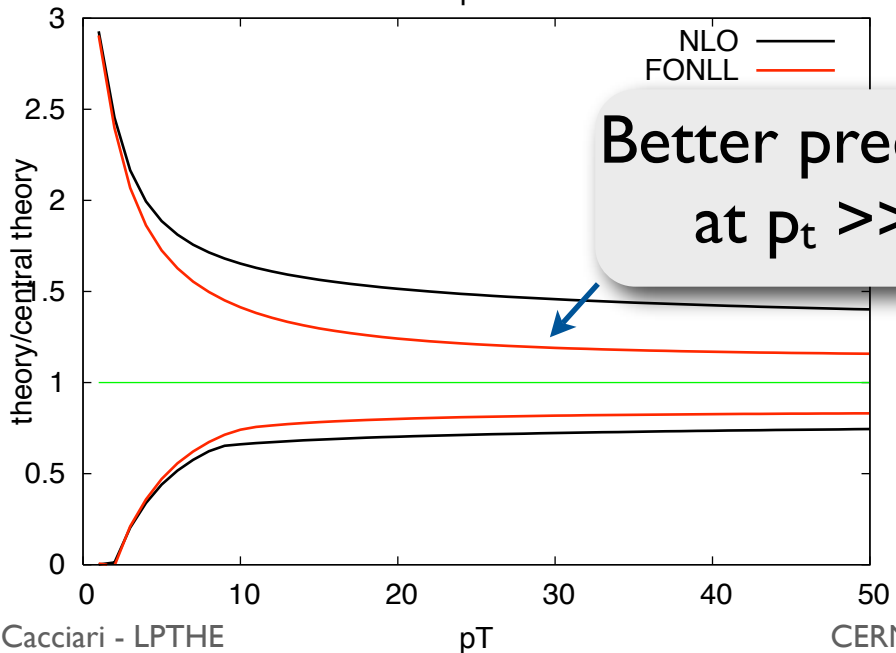
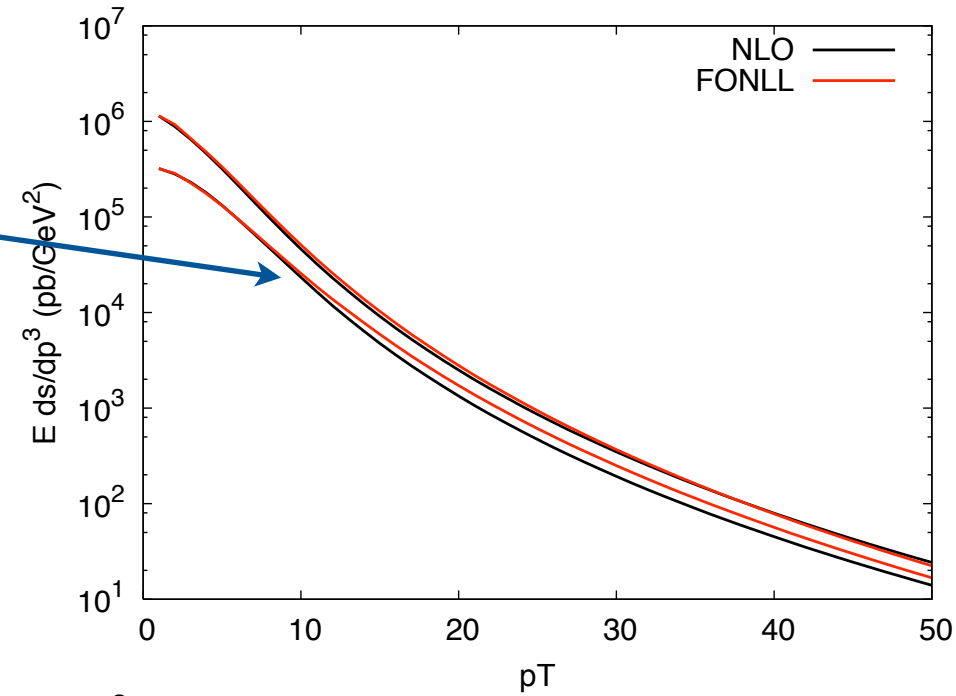
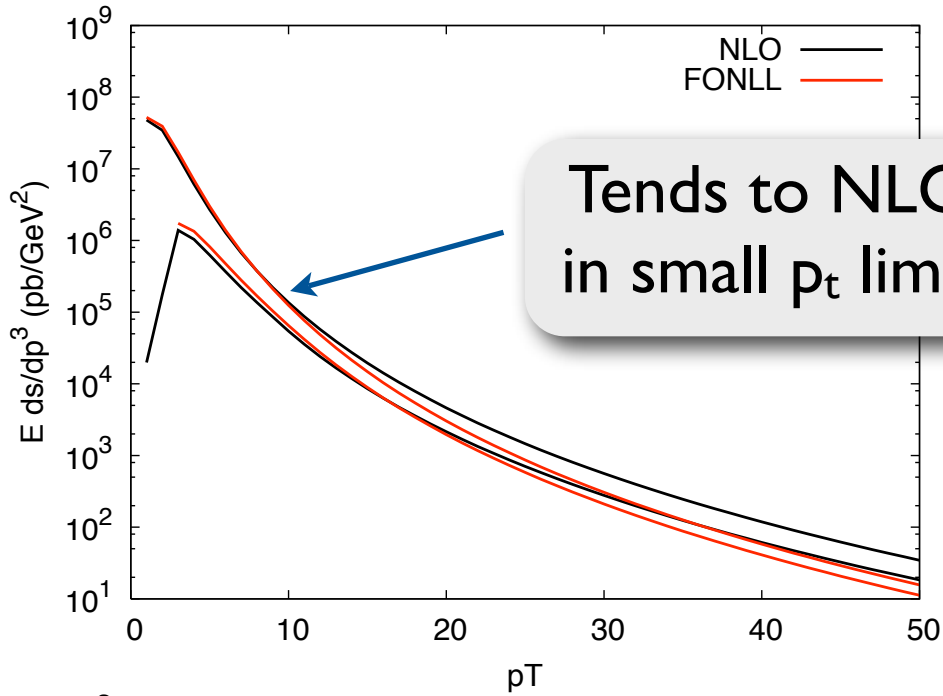
These functions  
resum the large logs

**- double-counting, properly merged**  
(accurate at NLO+NLL)

# Charm & bottom quarks @ LHC14

## Charm

## Bottom

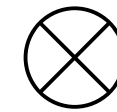
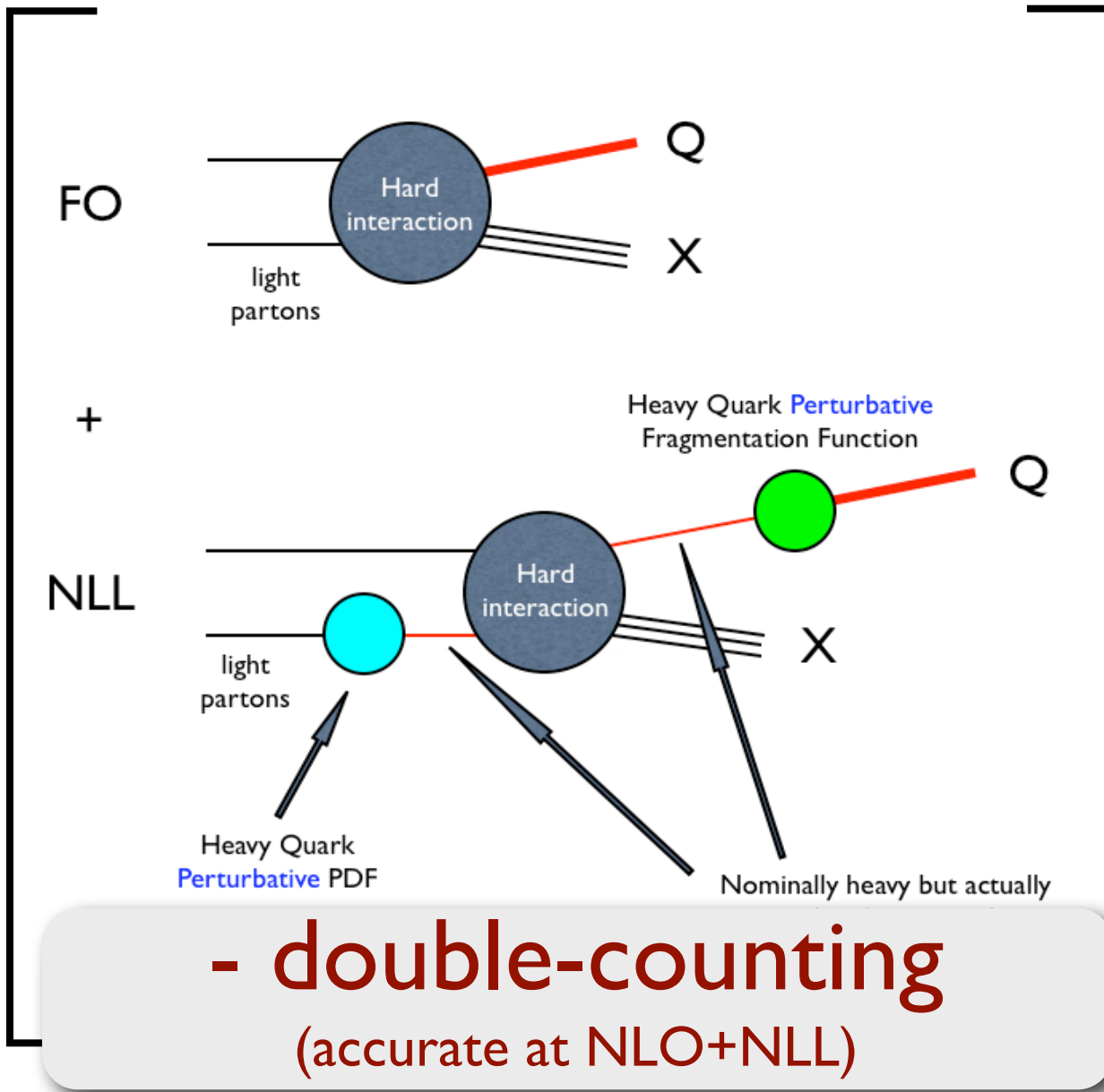


$$\frac{d\sigma_H}{dp_T} = \frac{d\sigma_Q}{dp_T} \otimes D^{np}$$

measured  
cross section

NLO (+NLL)  
calculation

non-perturbative  
fragmentation  
(usually extracted  
from e<sup>+</sup>e<sup>-</sup> data)



$$D_{Q \rightarrow H}$$

Fitted to  $e^+e^-$  data  
in the **same scheme**

# Non-perturbative fragmentation

## What do we know about it?

If the quark is **light**, not much. It's a process-independent artificial object (factorisation theorem) which we must extract from data (e.g. pion fragmentation functions)

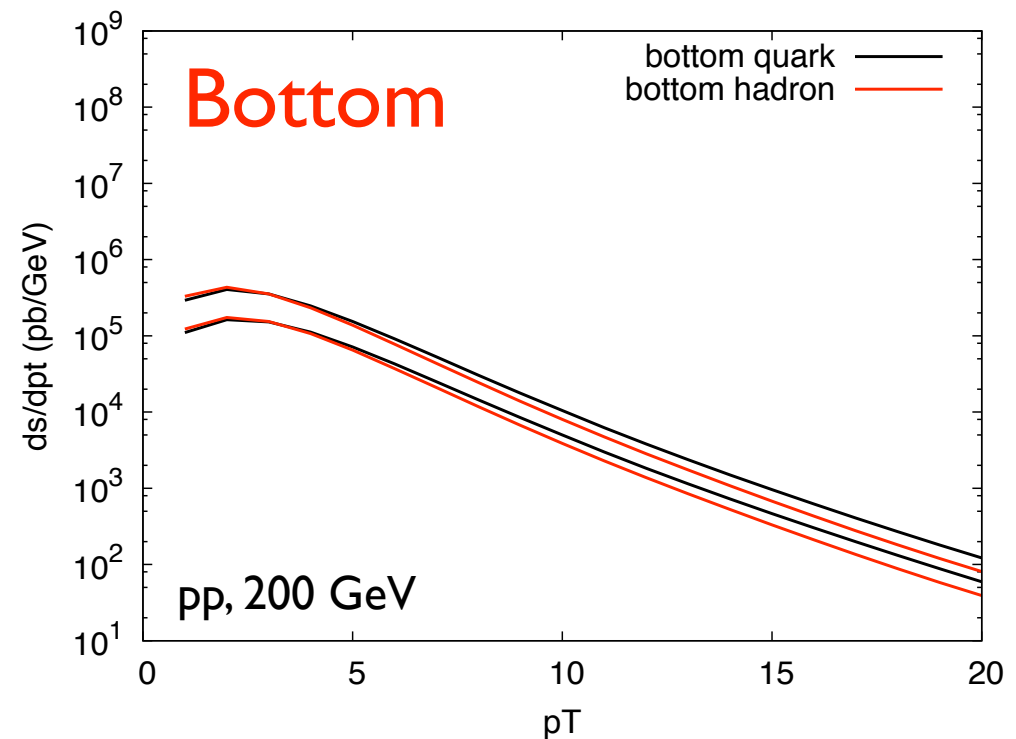
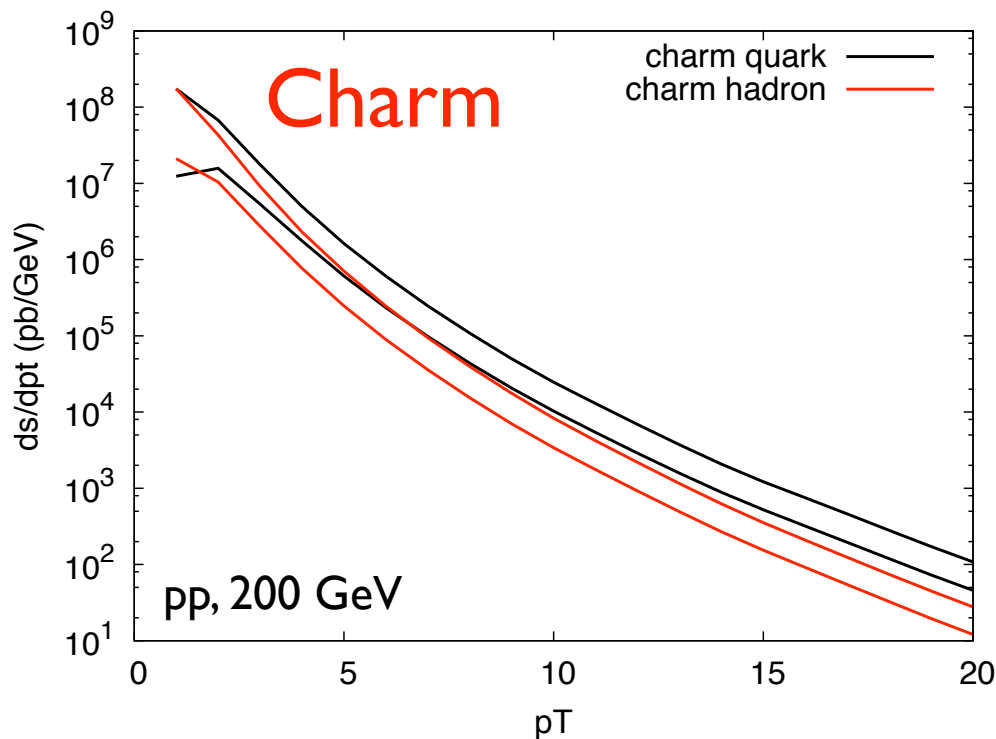
If the quark is **heavy**, its fragmentation function is still ambiguous, but we can tell something more about it:

- ▶ we know it's a (parametrically) small effect,  $O(\Lambda/m)$
- ▶ we can relate it to the hadronisation scale and to the heavy quark mass
- ▶ we can test this on D and B data

# Effect of NP fragmentation

The **total number** (and of **heavy hadrons**) of heavy quarks is a **genuine prediction of pQCD**

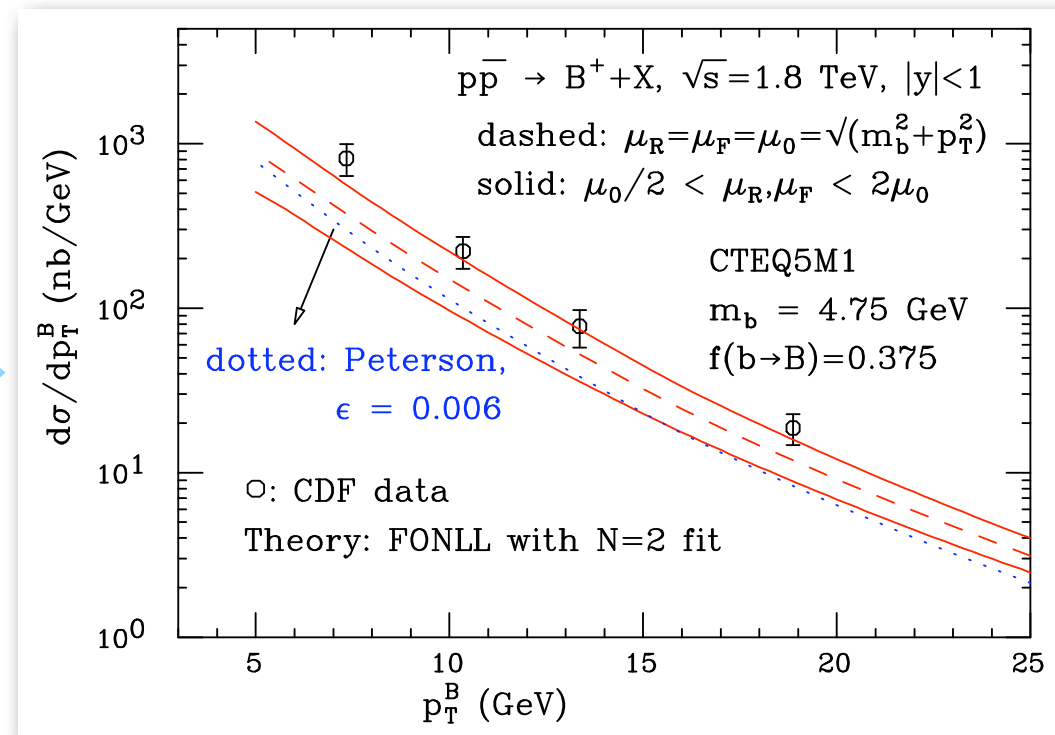
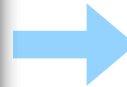
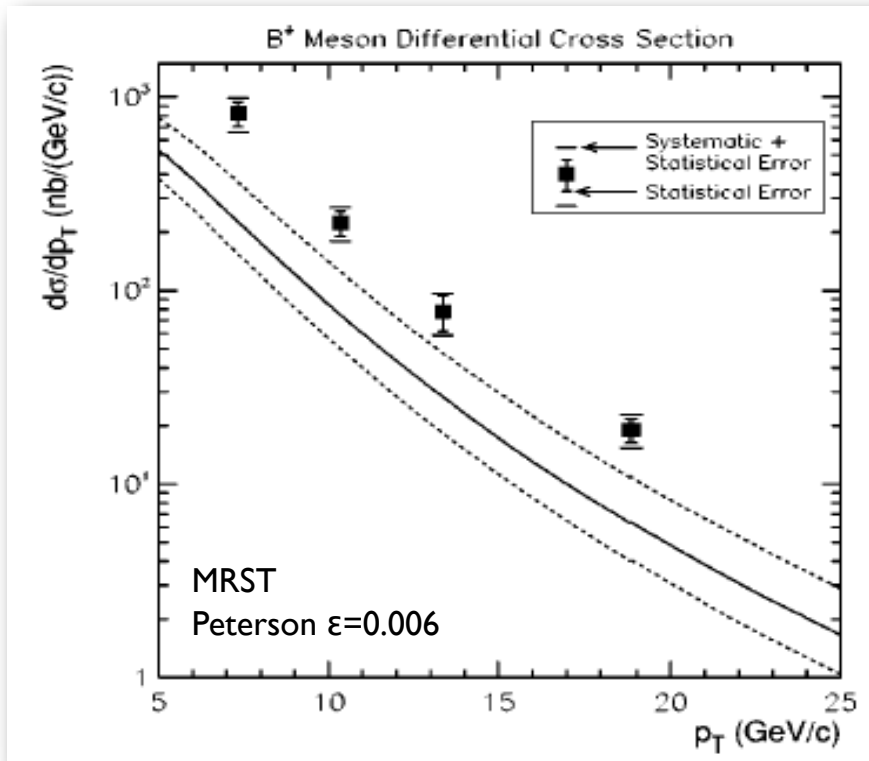
At the **differential** level instead, hadrons and quarks differ



However, the non-perturbative correction is expected (and observed) to be **parametrically small,  $O(\Lambda/m)$**

(Still, at large  $p_T$  the effect can be large)

# Lessons from Tevatron



Using proper inputs (PDFs,  $\alpha_s$ ) and correct fragmentation leads to an acceptable description of data

# It's the moment that matters

$$\frac{d\sigma_Q}{dp_T} \sim \frac{A}{p_T^n}$$

heavy **quark**  
cross section



heavy **hadron** cross section

$$\frac{d\sigma_H}{dp_T} = \frac{d\sigma_Q}{dp_T} \otimes D^{np} = \int \frac{d\sigma_Q}{dp_T}(p_T/z) D^{np}(z) \frac{dz}{z} = \int \frac{A}{(p_T/z)^n} D^{np}(z) \frac{dz}{z} = \frac{d\sigma_Q}{dp_T}(p_T) D_n^{np}$$

It's the  **$n^{\text{th}}$  moment** of the non-perturbative fragmentation function that controls the effect of hadronisation at large transverse momentum

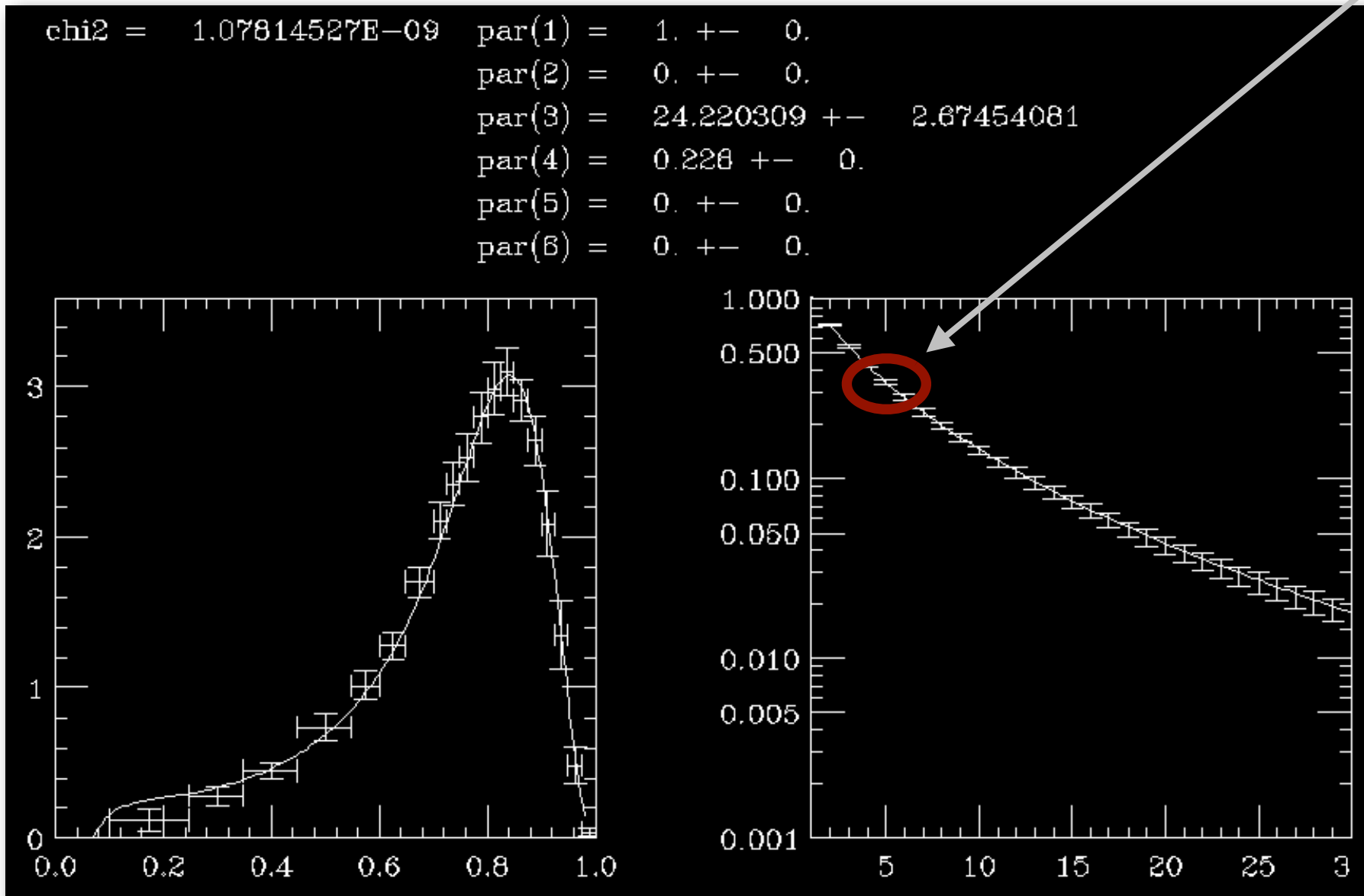
**NB. In hadronic collisions,  $n$  is typically  $\sim 5$**



# Quality of moment-space fits

ALEPH B hadrons

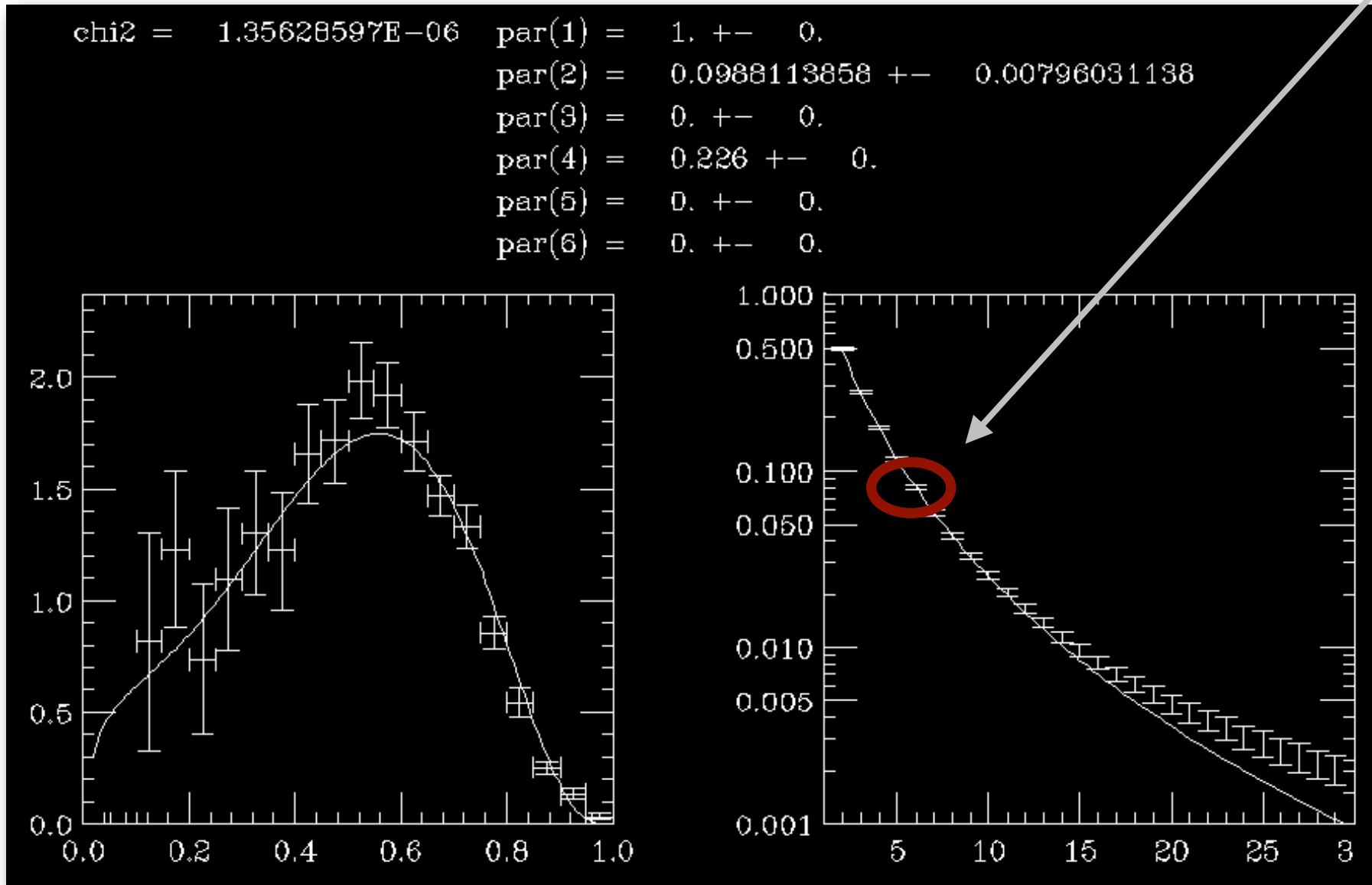
Fit to N=5



# Quality of moment-space fits

ALEPH  $D^{*+}$  hadrons

Fit to N=5



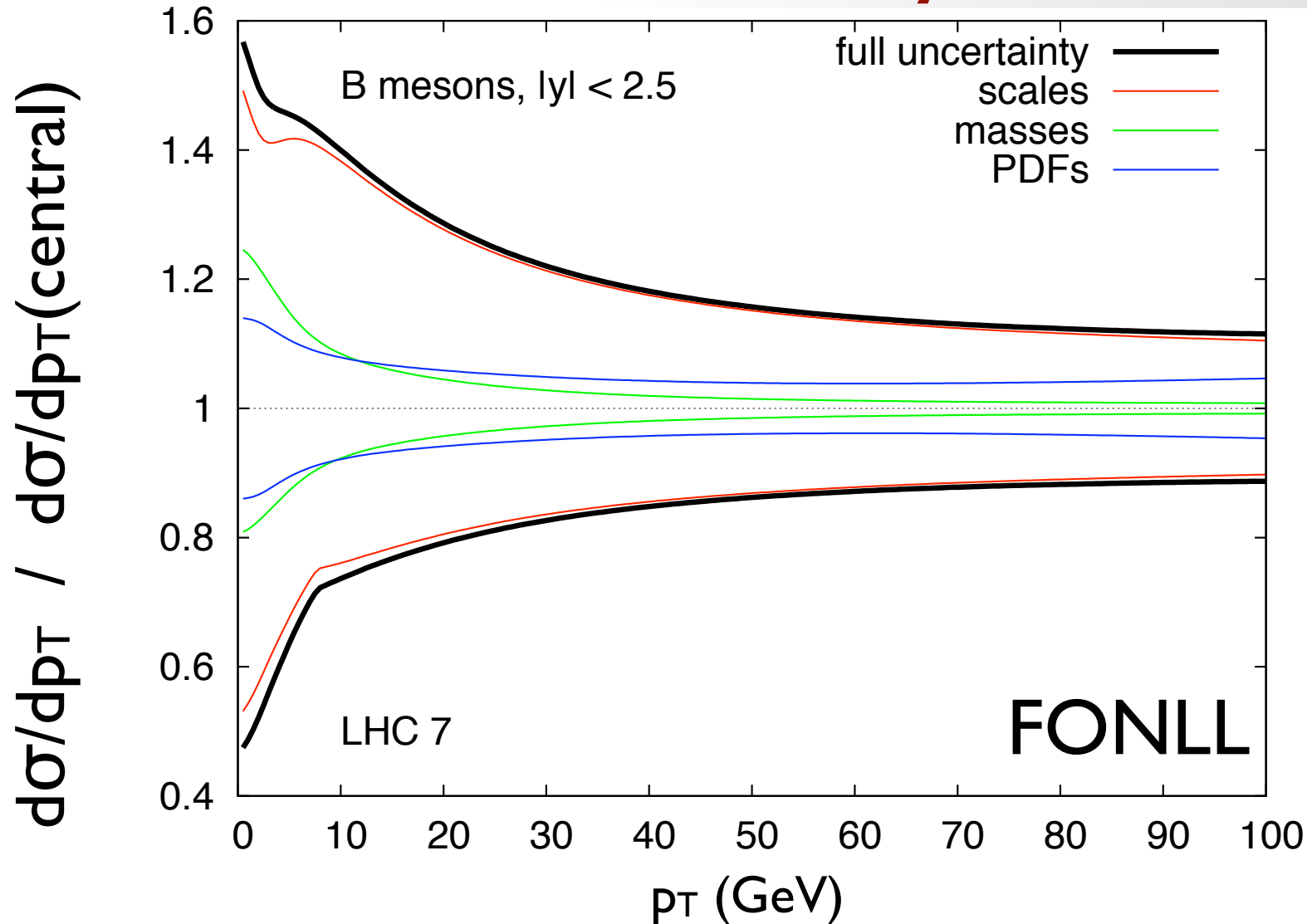
# Theoretical uncertainties

# Scales, masses and PDFs

- ▶ Renormalisation and factorization scales varied ***independently*** in the range  $1/2 < \mu/m_T < 2$ , with the *additional constraint*  $1/2 < \mu_R/\mu_F < 2$
- ▶ masses varied as (4.5, 4.75, 5) GeV for bottom, (1.3, 1.5, 1.7) GeV for charm. Non perturbative parameters fitted accordingly.
- ▶ PDF uncertainties calculated from the usual N 'plus' and 'minus' eigenvectors of the PDF set (CTEQ6.6 in this case)

These three uncertainties summed in quadrature

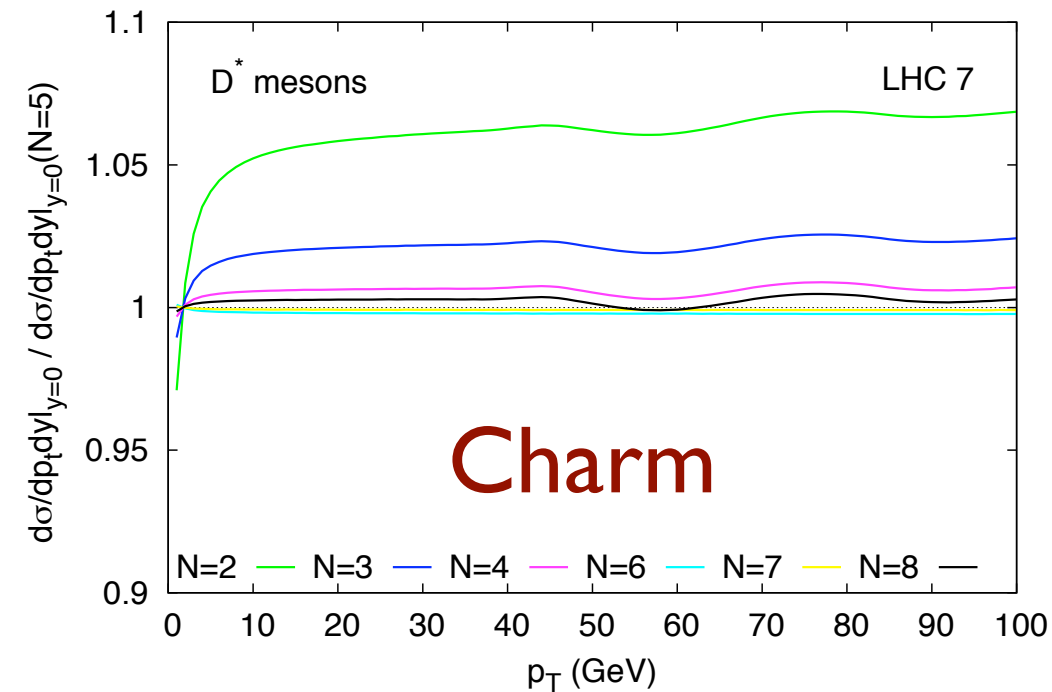
# Anatomy of uncertainty



Typically dominated by scales uncertainty

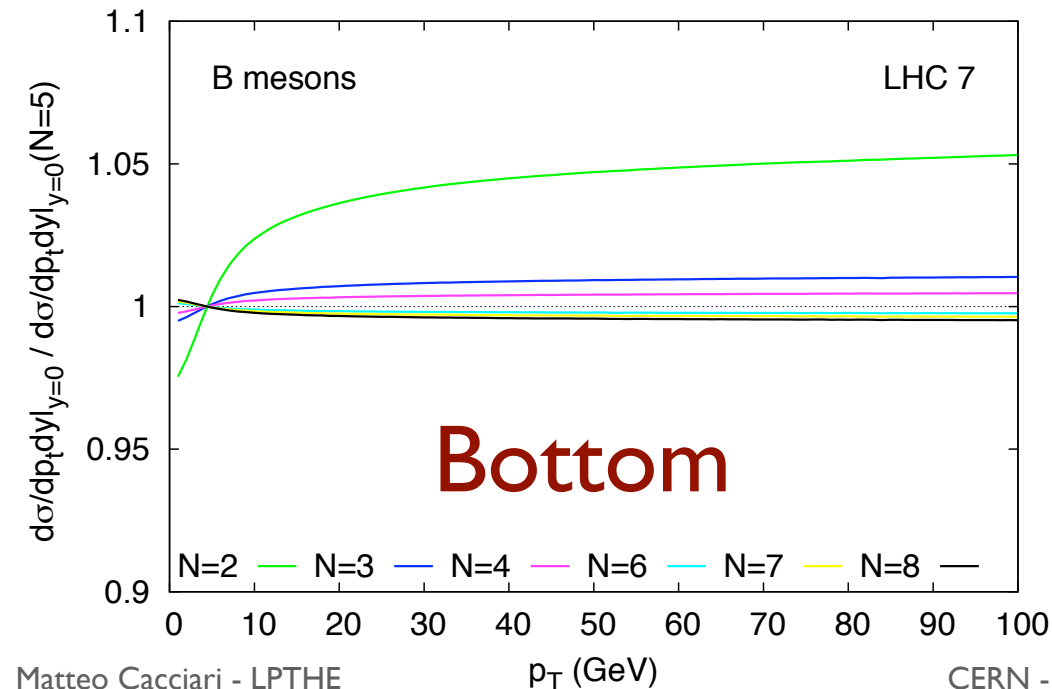
What about the uncertainty from NP fragmentation?

# Choice of non-perturbative parameter



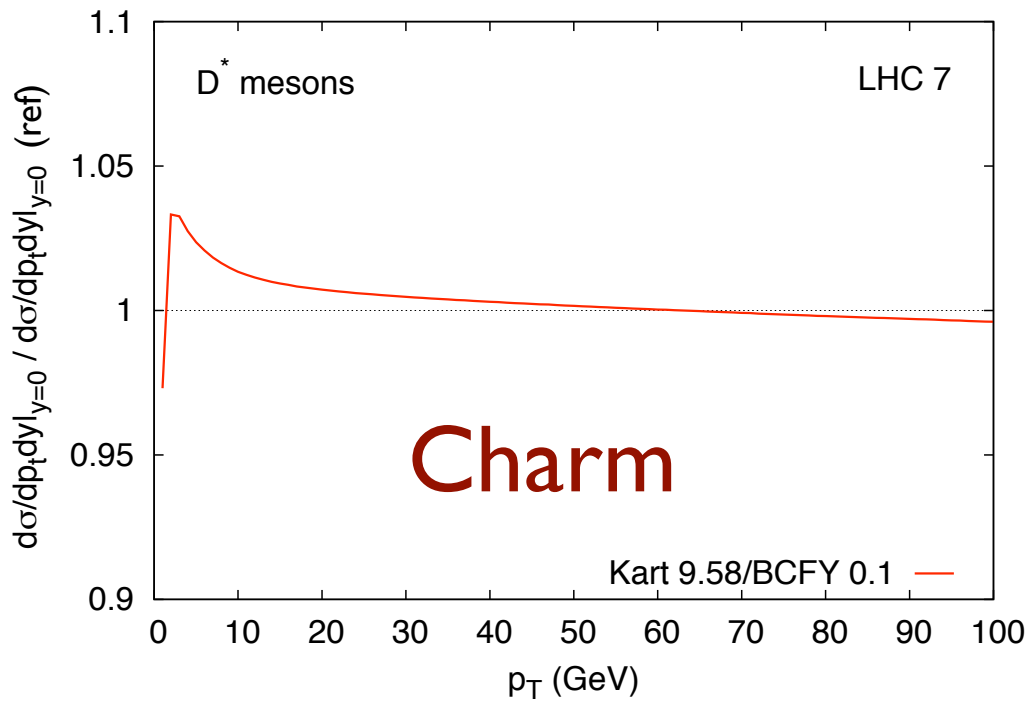
Using the **N=2** fit in place of the **N=5** one gives at most a **~5%** difference

Other choices only result in **O(1%)** differences



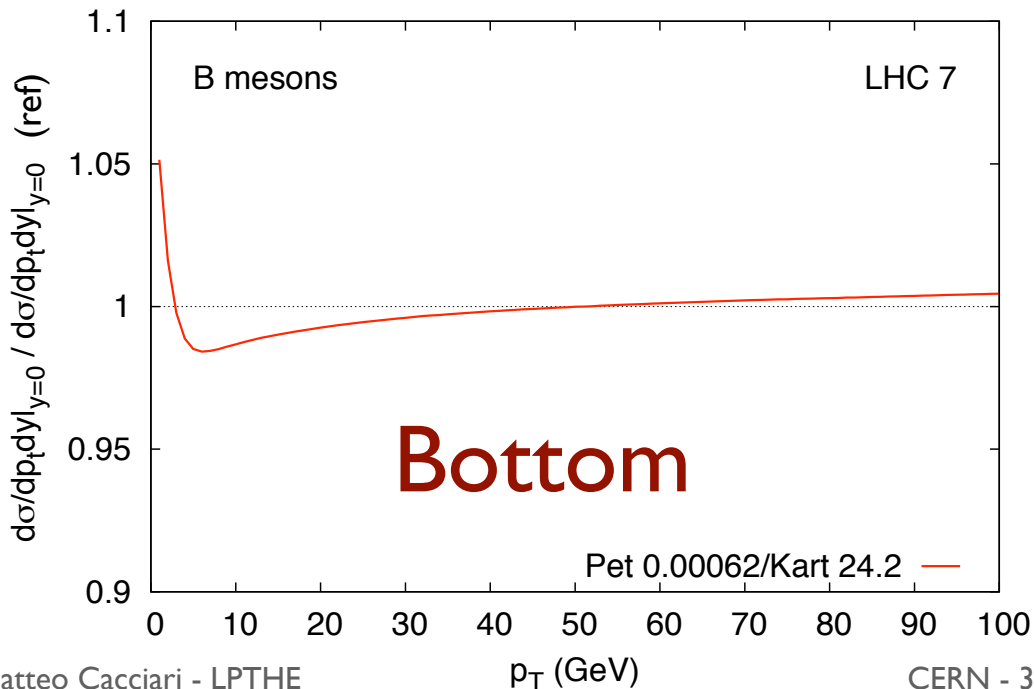
The shape of the  $p_t$  distribution is largely unaffected by the choice

# Choice of non-perturbative form



Kartvelishvili  $\alpha=9.58$

BCFY  $r=0.1$



Peterson  $\varepsilon=0.00062$

Kartvelishvili  $\alpha=24.2$

# Did experimentalists and theorists converge?

Talk in BNL - 2005  
Still actual today

Not much room to wiggle around:

+5  
↑

The NLO calculation has been around for 15 years. With the addition of the NLL resummation, its perturbative uncertainty at large transverse momentum is not larger than a few 10%

The uncertainty from the PDFs should be fairly constrained. Say 10-15% Probably quite smaller today

The non-perturbative fragmentation contribution is tightly constrained by  $e^+e^-$  data. It is definitely known to better than 10% Make it a few %

So, at large transverse momenta, where the theoretical framework is better under control, the overall uncertainty of the theoretical prediction should be smaller than 40-50%. → Quite conservative. Make it less than 20%

==> No room for factors of three discrepancies

(BTW: the expt. accuracy is actually often better than the theoretical one!)

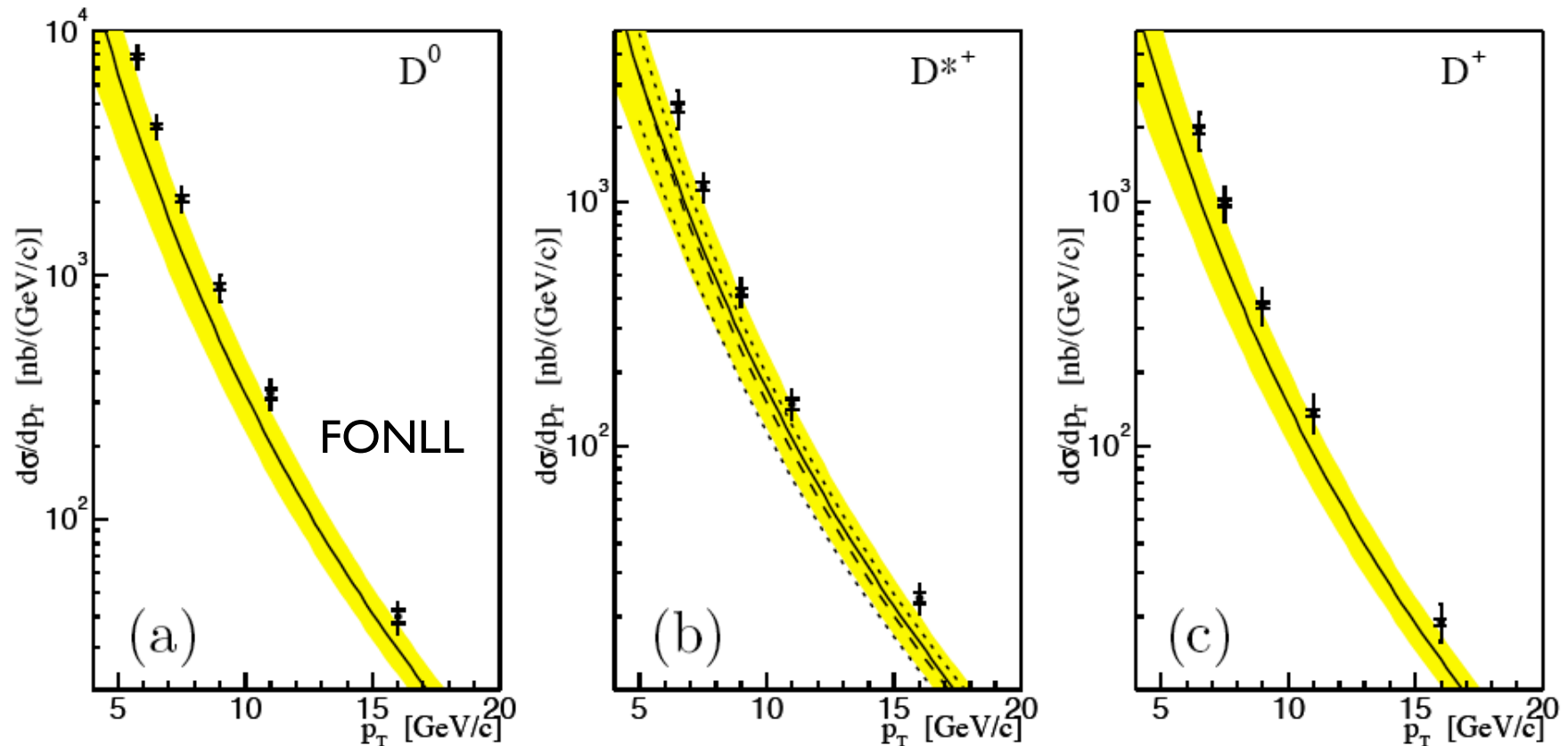


# Results

- ▶ The FONLL framework has not significantly changed since 1998:
  - ▶ Successful description of Tevatron bottom data in 2002
  - ▶ Most recent non-perturbative fits in 2005, consistent with what had been used before

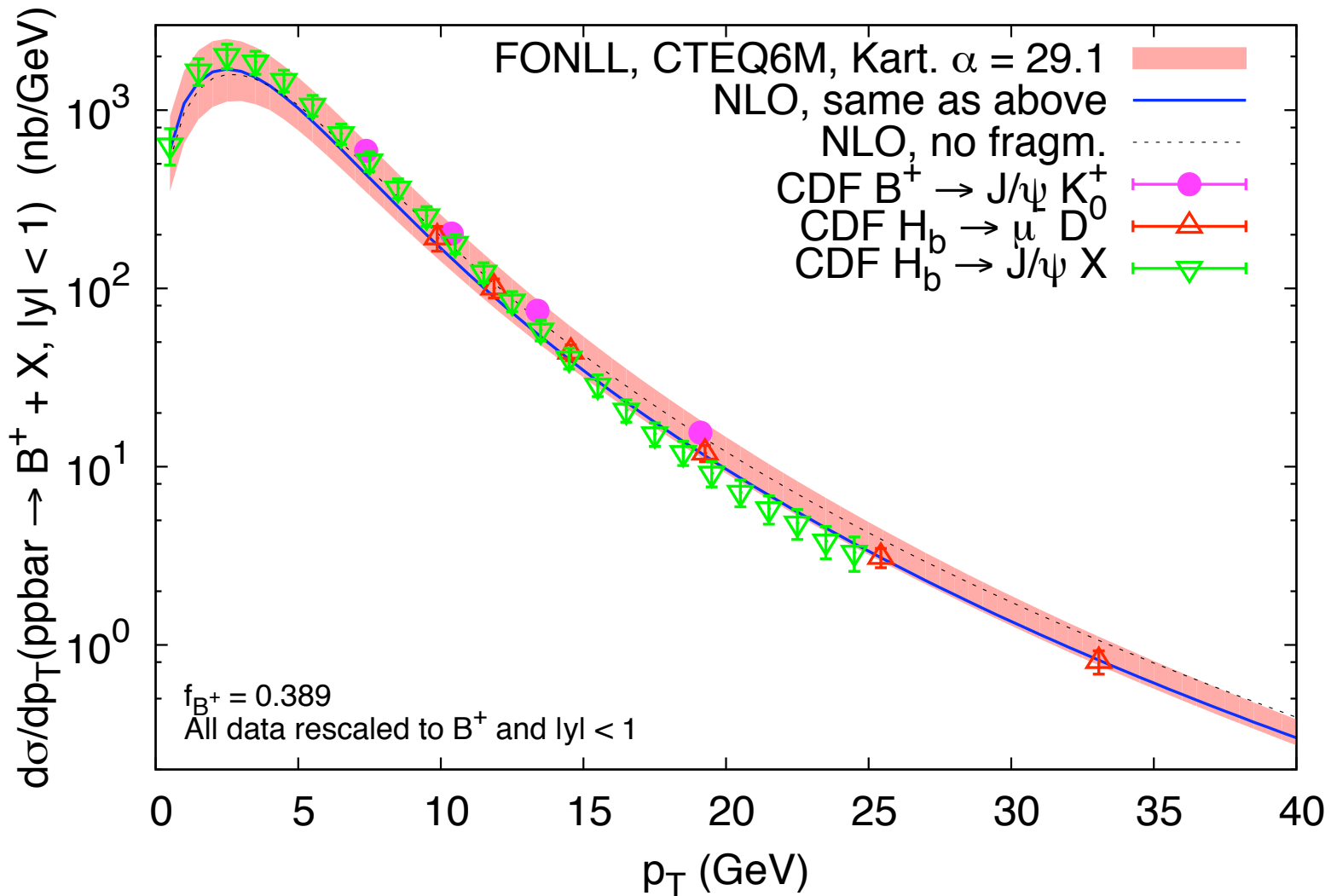
Everything still actual today:  
will use framework unchanged at the LHC

CDF Run II  $c \rightarrow D$  data [PRL 91:241804,2003]



Non-perturbative charm fragmentation needed to describe the  $c \rightarrow D$  hadronization extracted from moments of ALEPH data at LEP.

# bottom @ Tevatron

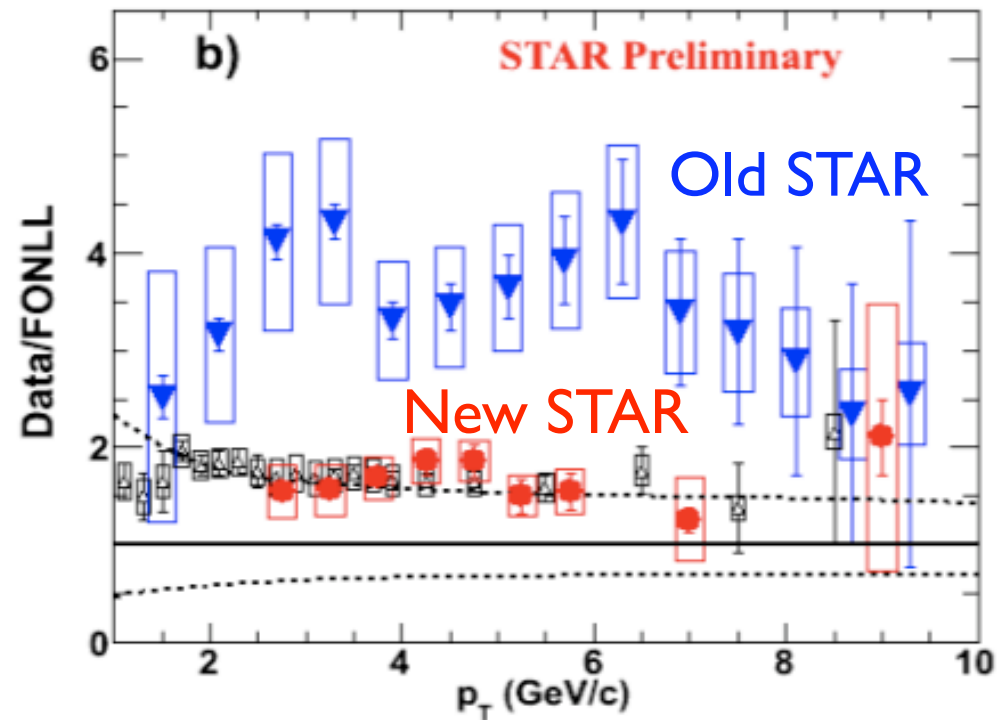
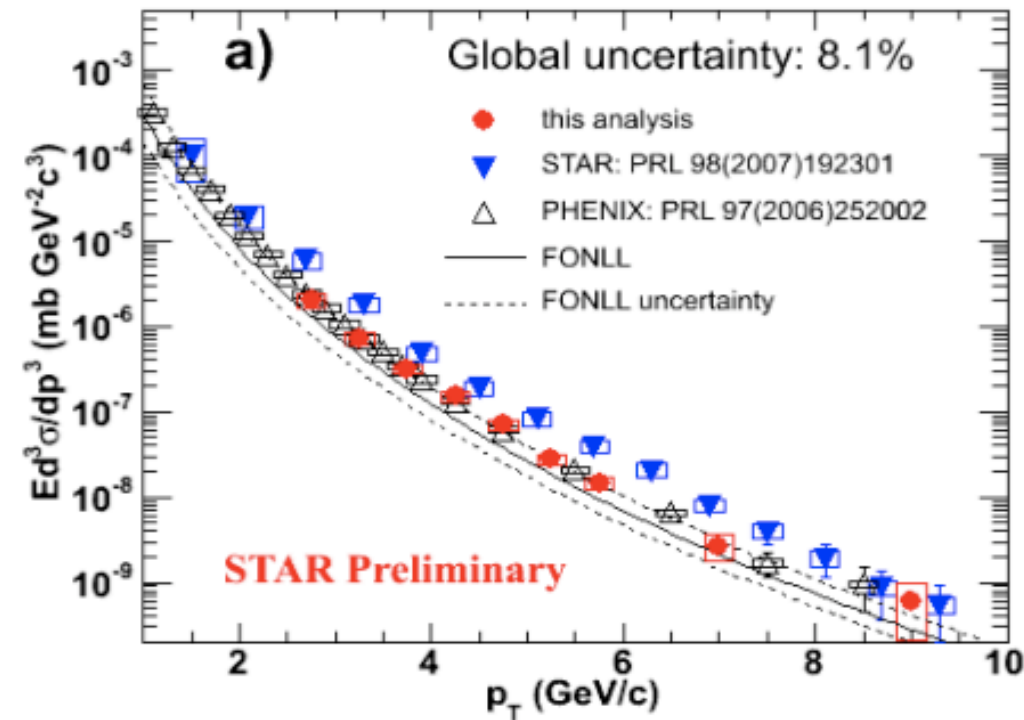


Good agreement, with minimal non-perturbative correction

NLO is sufficient for correct total rate prediction

# charm and bottom @ RHIC

‘non-photonic’ electrons:  $pp \rightarrow c,b \rightarrow e$



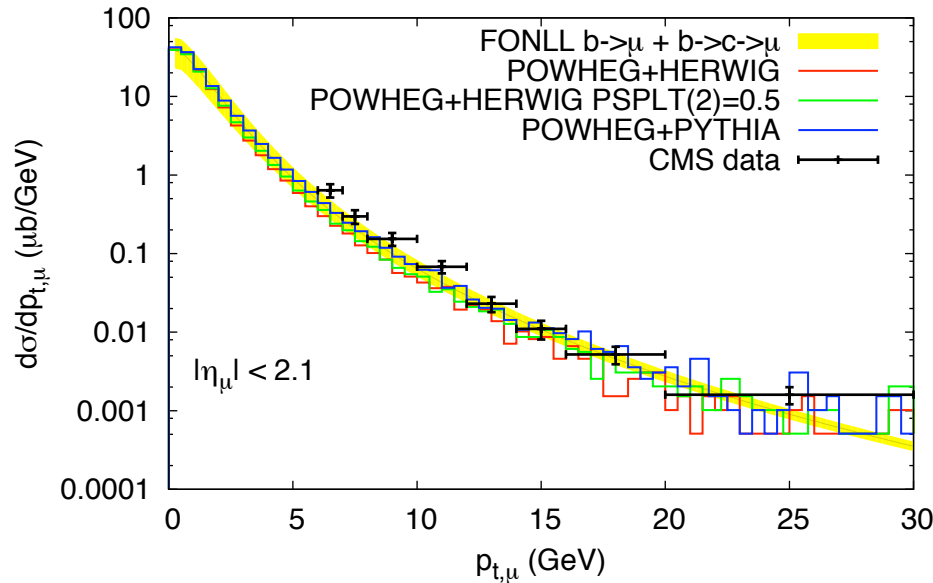
Theory holding firm,  
STAR data initially showing an excess have come down

# Lessons from Tevatron and RHIC

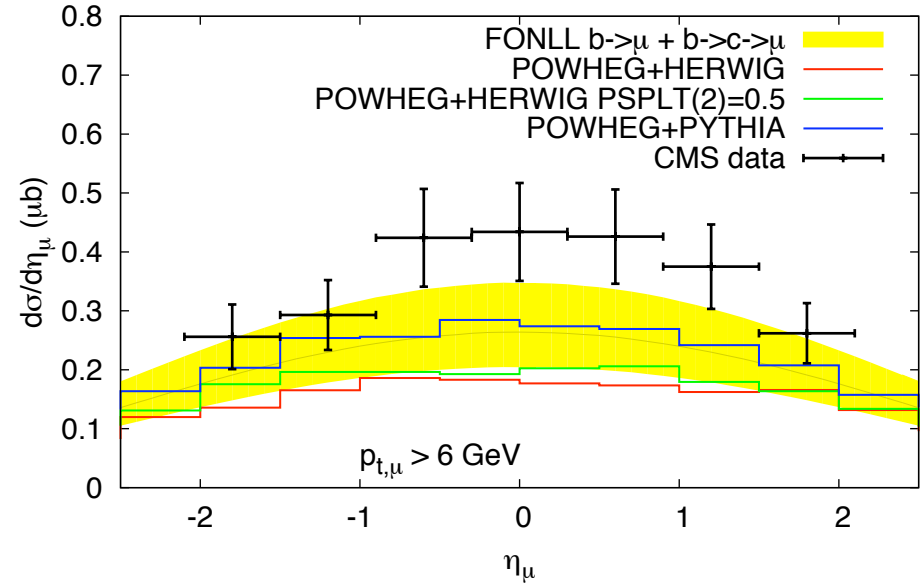
- ▶ NLO QCD predicts correctly the 'total' heavy quark bottom cross sections
- ▶ FONLL with non-perturbative fragmentation extracted from CLEO/BELLE and LEP describes correctly the differential distributions

# First results from LHC

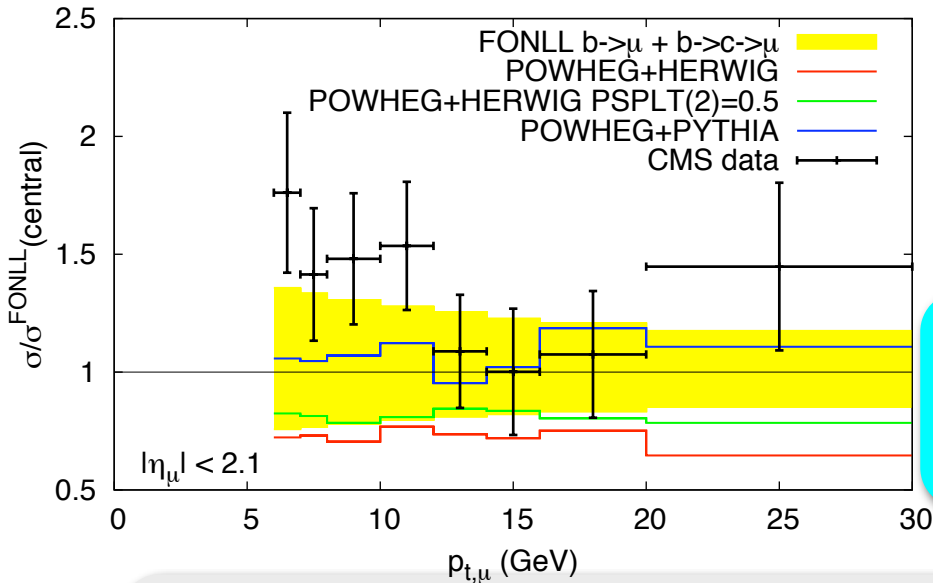
$\mu$  from b,bbar



$\mu$  from b,bbar



$\mu$  from b,bbar



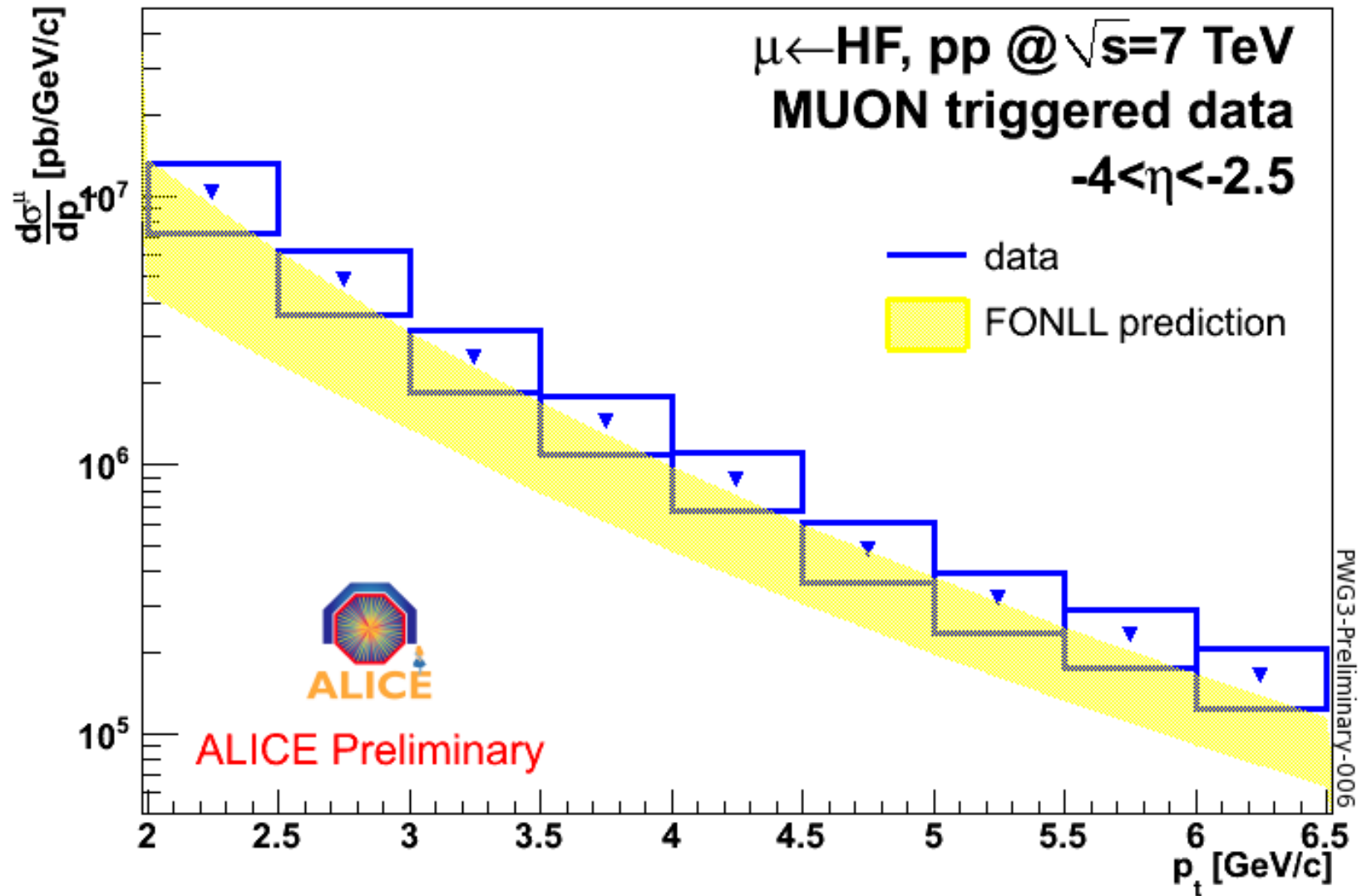
- ▶ FONLL and POWHEG+PYTHIA perfectly compatible
- ▶ CMS slightly high, but compatible within uncertainties

**FONLL**  
 $0.97^{+0.31}_{-0.22}$  (scales/masses)  $\pm 0.06$  (PDFs)  $\mu\text{b}$

$$\sigma(pp \rightarrow b + X \rightarrow \mu + X', p_{\perp}^{\mu} > 6 \text{ GeV}, |\eta^{\mu}| < 2.1) = (1.48 \pm 0.04_{\text{stat}} \pm 0.22_{\text{syst}} \pm 0.16_{\text{lumi}}) \mu\text{b}.$$



# $\mu$ from HF @ ALICE

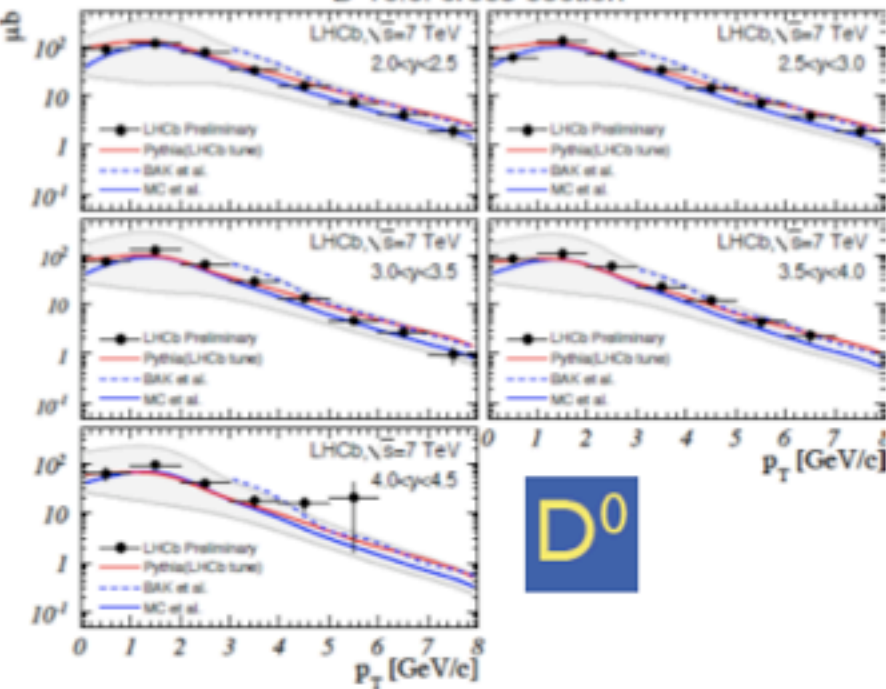


# Open charm cross-sections

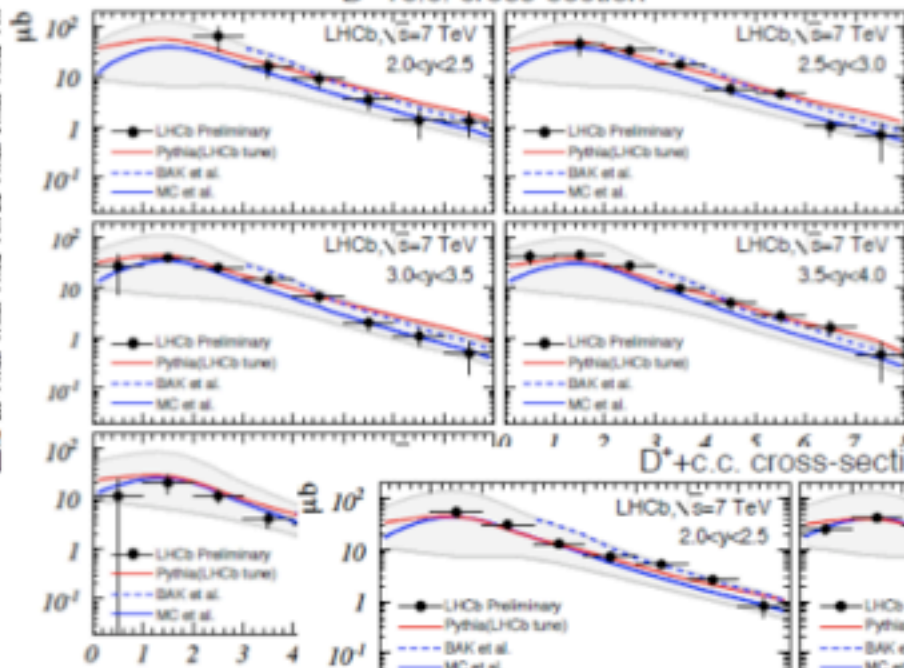
$D^{*+}$

$D^0+c.c.$  cross-section

$D^{*+}+c.c.$  cross-section



$D^0$

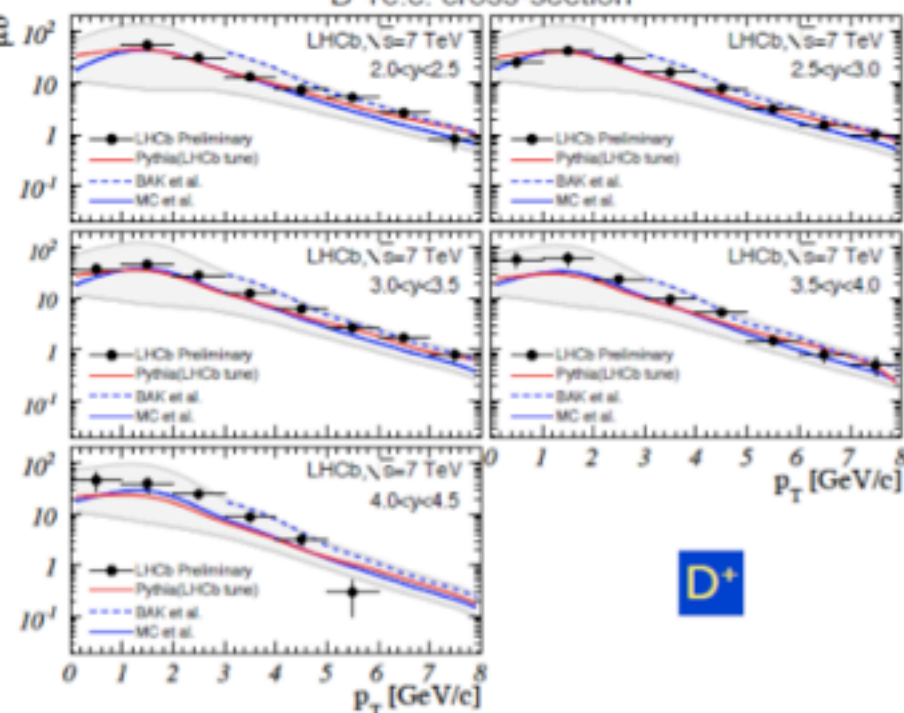


$D^{*+}+c.c.$  cross-section

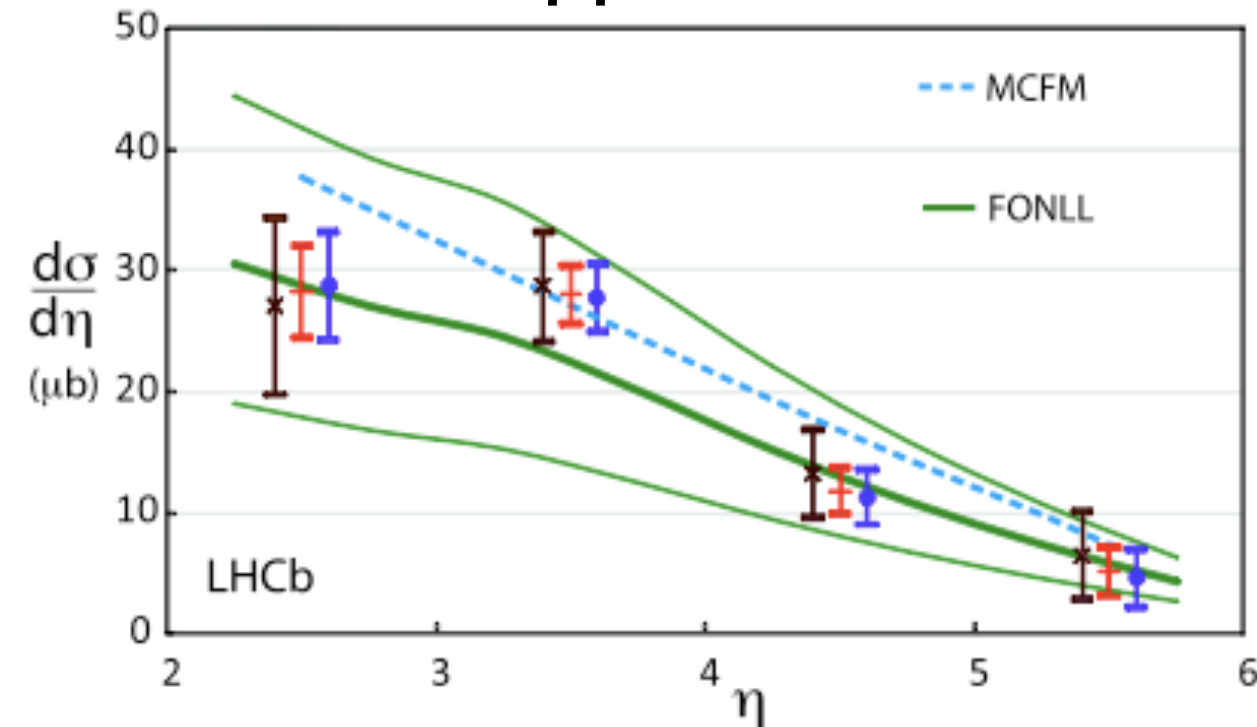
$D^+$

Extrapolating to all  $p_T$  and 4 can also confirm the expectation on ratio at  $\sqrt{s}=7\text{TeV}$   
 $(pp \rightarrow c\bar{c}X) \approx 20 \times (pp \rightarrow b\bar{b}X)$

→ Good news for LHCb charm program



$pp \rightarrow H_b X$



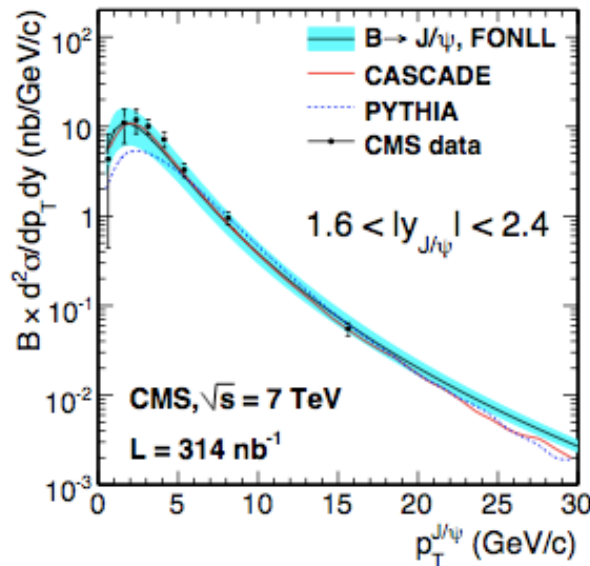
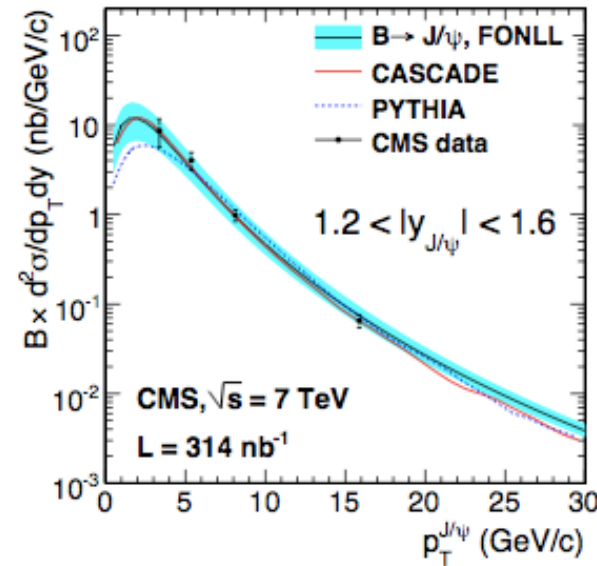
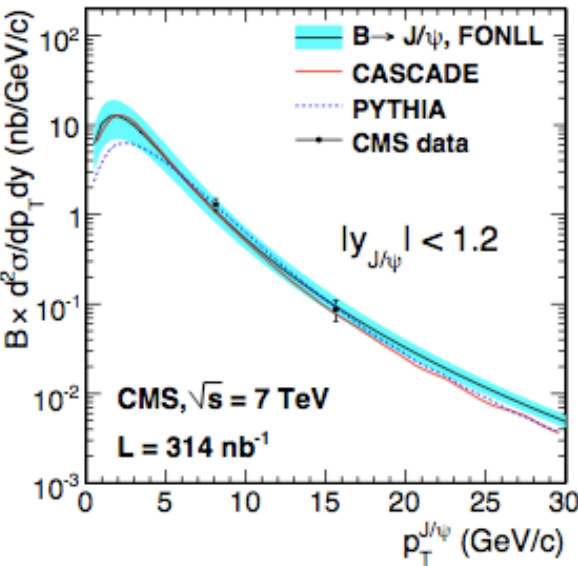
LHCb Collab. arXiv:1009.2731

$$\sigma(pp \rightarrow H_b X) = (75.3 \pm 5.4 \pm 13.0) \mu\text{b}$$

$$[2 < \eta < 6]$$

**FONLL:**  $71^{+33}_{-26}$  (scales)  $^{+10}_{-12}$  (mass)  $\pm 7$  (PDFs)  $\mu\text{b}$

# J/ψ from B @ CMS



CMS Collab. arXiv:1011.4193

**FONLL**

$23.5^{+8.3}_{-5.7}$  (scales/masses)  $\pm 1.7$  (PDFs) nb

$$\sigma(pp \rightarrow bX \rightarrow J/\psi X) \cdot \text{BR}(J/\psi \rightarrow \mu^+ \mu^-) = 26.0 \pm 1.4 \text{ (stat)} \pm 1.6 \text{ (syst)} \pm 2.9 \text{ (luminosity) nb}$$

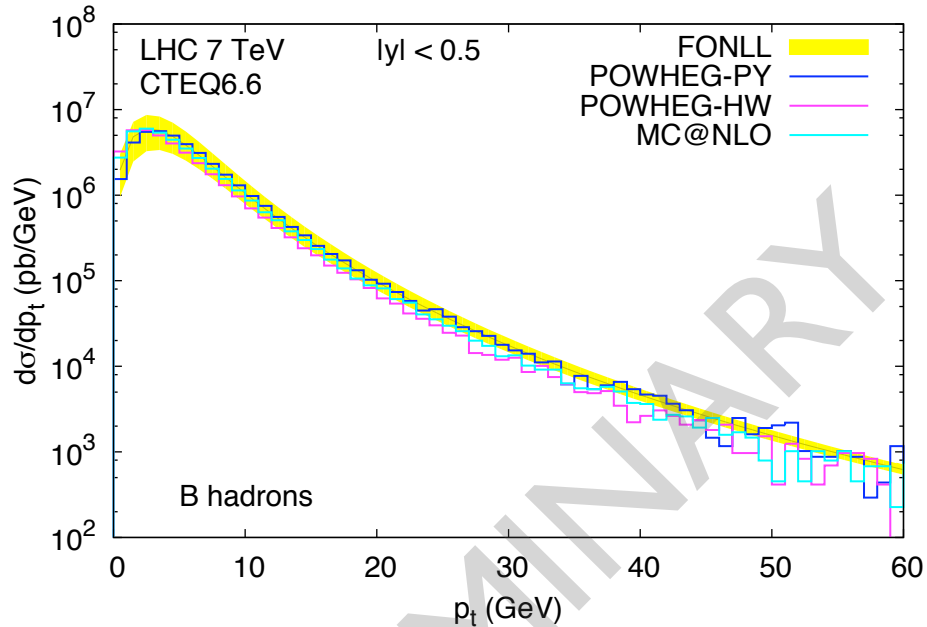
# First lessons from LHC

- ▶ Picture successful at Tevatron and RHIC still working very well
- ▶ No critical threshold apparently crossed going from Tevatron to LHC (no large small-x effects visible so far)

# FONLL, MC@NLO, POWHEG

- ▶ Detailed comparisons of predictions of FONLL, MC@NLO and POWHEG is in progress
- ▶ A paper (MC, Frixione, Mangano, Nason, Ridolfi) presenting this work, and specific predictions for the four LHC experiments, is expected to appear (soon?)

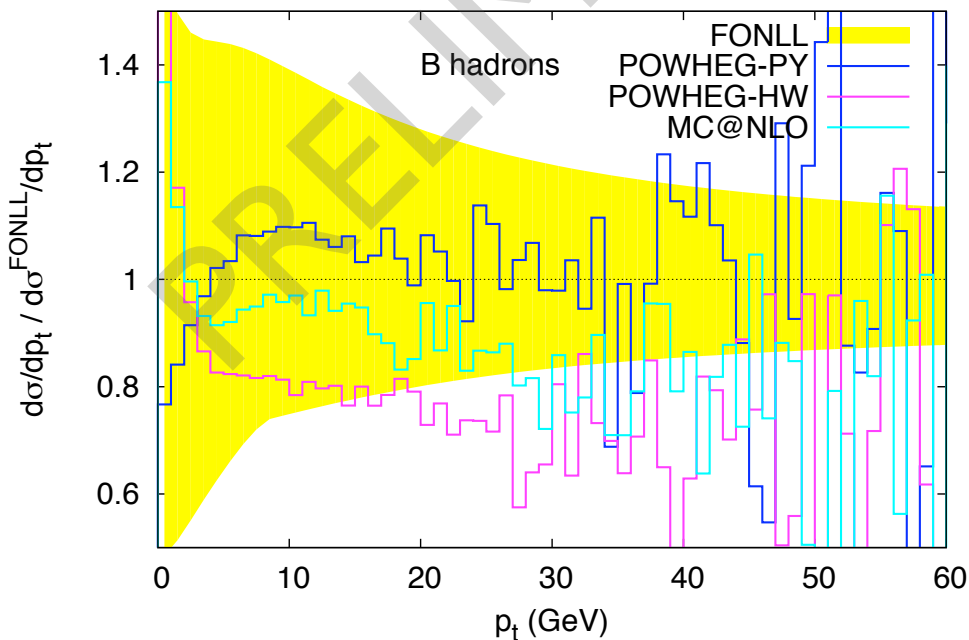
# FONLL, MC@NLO, POWHEG



► POWHEG and MC@NLO in good agreement with FONLL

► POWHEG+PYTHIA in *excellent* agreement

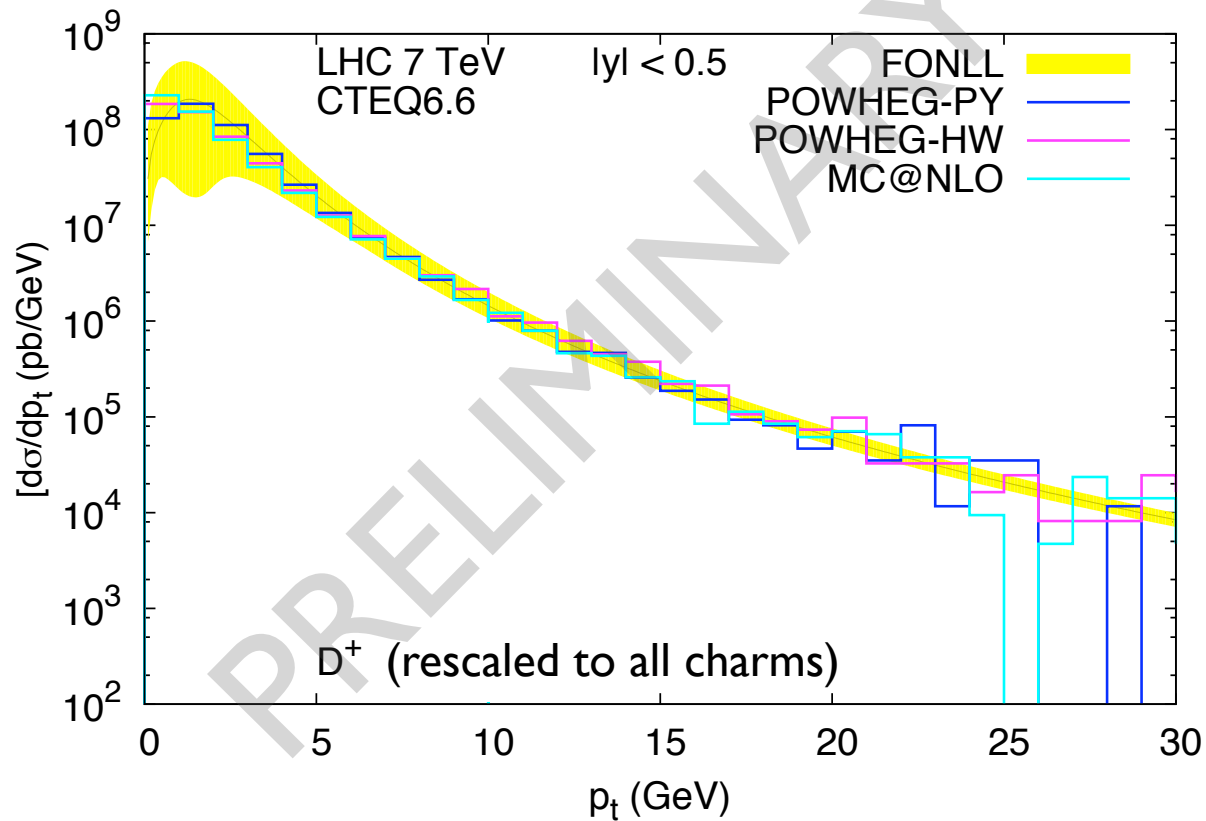
(Similar results hold at non-central rapidities)



Not that fragmentation in the Montecarlos has NOT been tuned to data as precisely as in FONLL. This may hold the key to the residual differences

# FONLL, MC@NLO, POWHEG

Situation for charmed hadrons production largely similar



Agreement within  $O(10-20\%)$



# Conclusions

- ▶ NLO and resummations successfully matched in various frameworks: normalization is a genuine prediction.

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- ▶ Non-perturbative contributions under control. Residual uncertainties mainly of perturbative origin.
- ▶ Predictions successful in early data.  
POWHEG/MC@NLO appear reliable, but NP fragmentation tuning in Montecarlo may need more work
- ▶ Phenomenological inputs and theory quite **constraining**:  
**All predictions (these and others) should probably agree within residual uncertainties (i.e. very few 10% at  $p_t \gg m$ )**



# Non-perturbative forms

Bottom: B

$$D_{\text{NP}}(x) = (\alpha + 1)(\alpha + 2)x^\alpha(1 - x)$$

Kartvelishvili et al., PLB78 (1978)

Charm: D\*

$$D_{Q \rightarrow V}(z) = 3N \frac{rz(1-z)^2}{(1-(1-r)z)^6} \left[ 2 - 2(3-2r)z + 3(3-2r+4r^2)z^2 - 2(1-r)(4-r+2r^2)z^3 + (1-r)^2(3-2r+2r^2)z^4 \right].$$

Braaten et al, hep-ph/9409316

Other functional forms are possible.

No significant differences in predictions if fitted properly

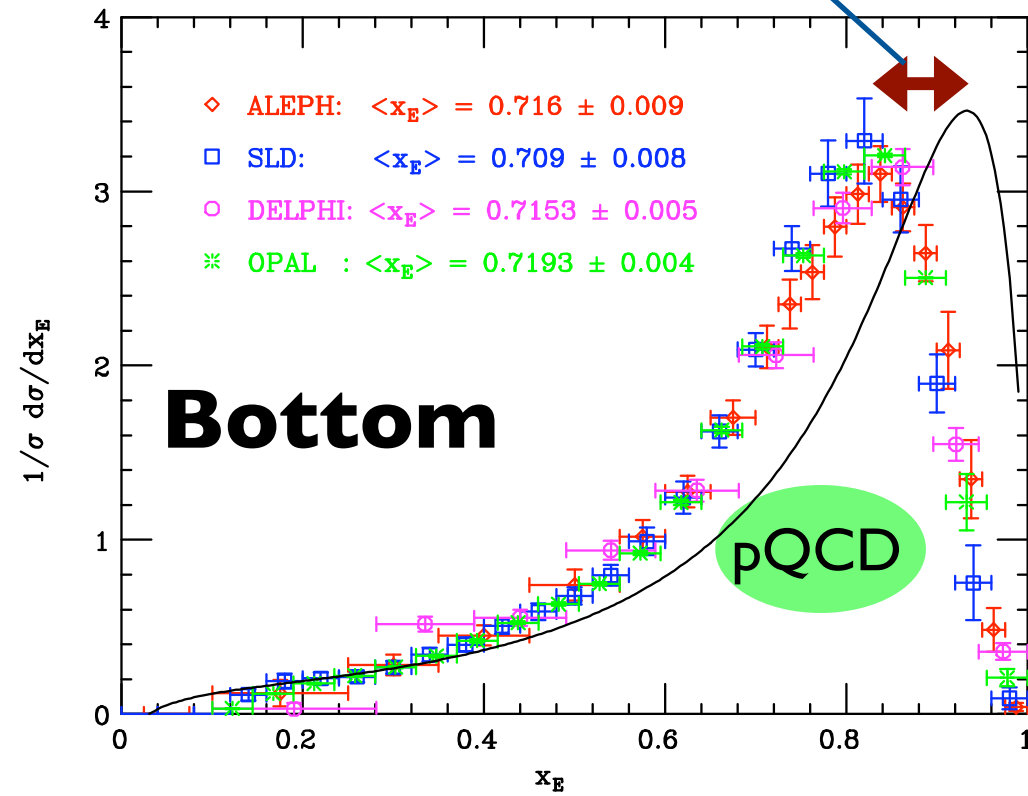
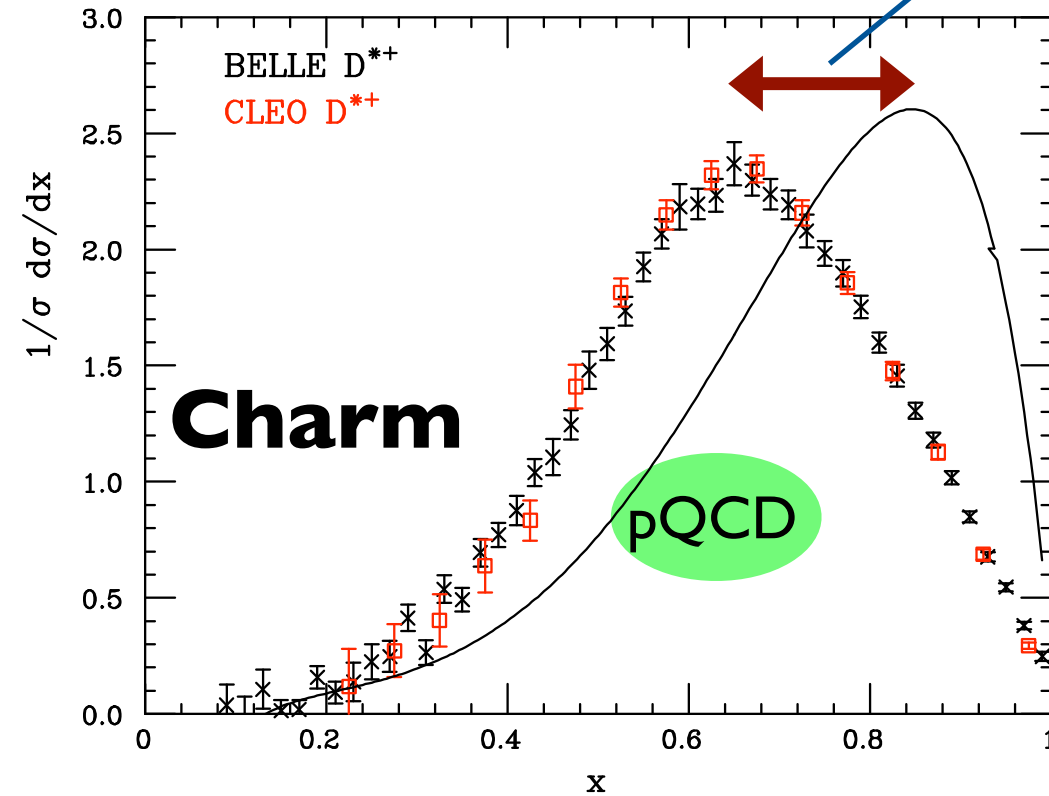
# Non perturbative fragmentation

$$e^+e^- \rightarrow QX \rightarrow H_Q X$$

non-perturbative contribution

$O(\Lambda/m_{\text{charm}})$

$O(\Lambda/m_{\text{bottom}})$

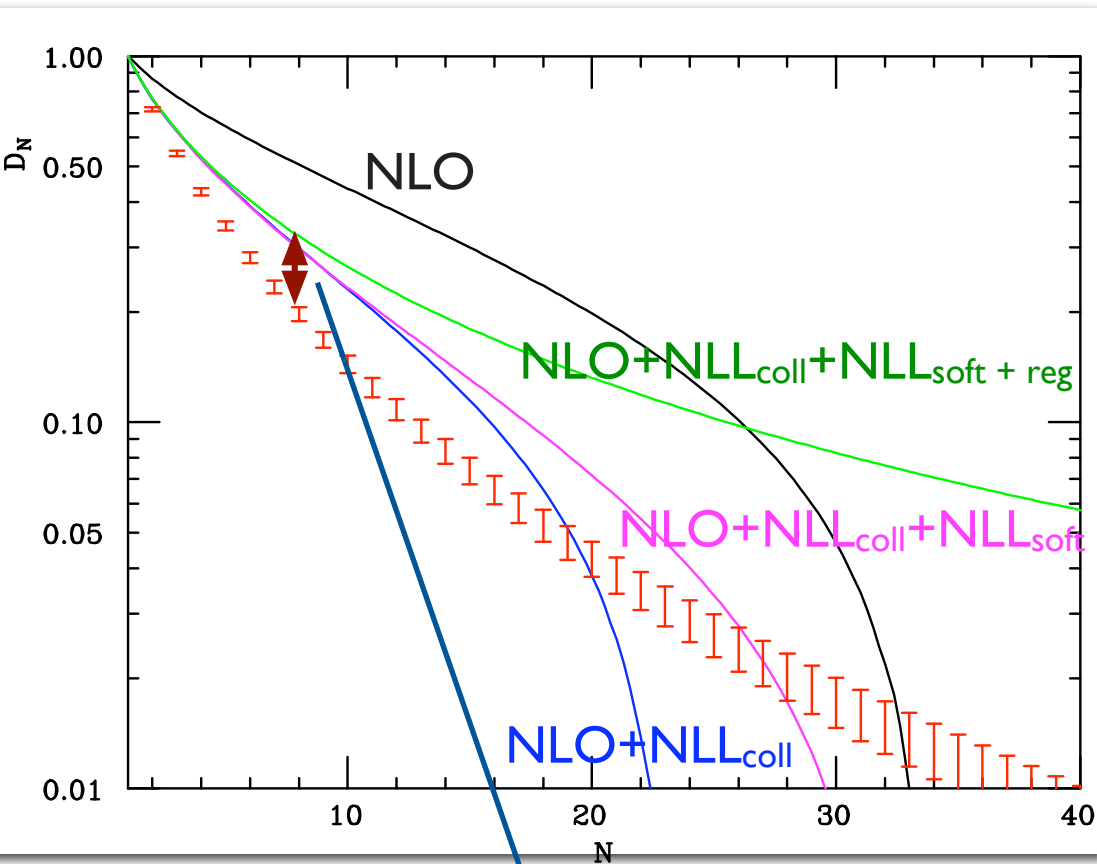


- ▶ non-perturbative contribution **limited in size** and compatible with expectations
- ▶ high-accuracy expt. data allow it to be precisely determined



# Non perturbative fragmentation

LEP B meson data translated to Mellin space:



This gap:  
non-perturbative QCD

$$f_N \equiv \int_0^1 x^{N-1} f(x) dx = \langle x^{N-1} \rangle$$

In this space  
**convolutions become products**

$$\langle x \rangle_{expt} = \langle x \rangle_{pQCD} \langle x \rangle_{np}$$

# NP fragmentation: quantitative picture

N=2 moments (i.e.  $\langle x \rangle$ )

$N$	2
$c$ @ 10.58 GeV	0.7359
$c$ @ 91.2 GeV (NS)	0.5858
$c$ @ 91.2 GeV (full)	0.5954
$b$ @ 91.2 GeV	0.7634
BELLE $D^{*+} \rightarrow D^0$ (ISR corr.)	$0.6418 \pm 0.0042$
ALEPH $D^{*+}$ (ISR corr.)	$0.4920 \pm 0.0152$
ALEPH $B$	$0.7163 \pm 0.0085$
CLEO $D^{*+}$	$0.877^{+0.009}_{-0.010}$
BELLE $D^{*+} \rightarrow D^0$	$0.872^{+0.005}_{-0.006}$
ALEPH $D^{*+}$	$0.840^{+0.022}_{-0.031}$
Tab. 2 and eq. (4.2)	0.868
ALEPH $B$	$0.938^{+0.009}_{-0.014}$
SLD $B$	$0.931^{+0.016}_{-0.030}$

→ pQCD (NLL)

→ data  
(very precise!)

→  $D^{np} = \frac{\text{data}}{\text{pQCD}}$

charm  $\sim 1 - 0.16$   
bottom  $\sim 1 - 0.06$

Compatible with  $D_N^{np} = 1 - \frac{(N-1)\Lambda}{m} + \dots$  and  $\Lambda \simeq 0.25 \text{ GeV}$