

# Electroproduction of vector mesons and GPDs

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## Outline:

- **Handbag factorization**
- **Modeling the GPDs**
- **Results for L-L transitions**
- **Generalization to other transitions**

talk based on work with S. Goloskokov, hep-ph/0501242, hep-ph/0611290

# Handbag factorization

Radyushkin (96); Collins et al (97):

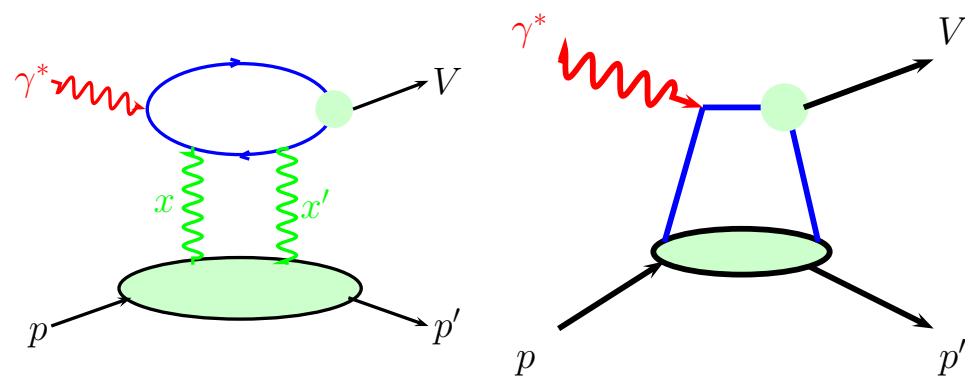
rigorous proof of factorization

for  $Q^2 \rightarrow \infty$  into

hard subprocesses

$\gamma^* g \rightarrow Vg$  and  $\gamma^* q \rightarrow Vq$

and GPDs ( $x \neq x'$ )



dominant transition  $\gamma_L^* \rightarrow V_L$  (others power suppressed)

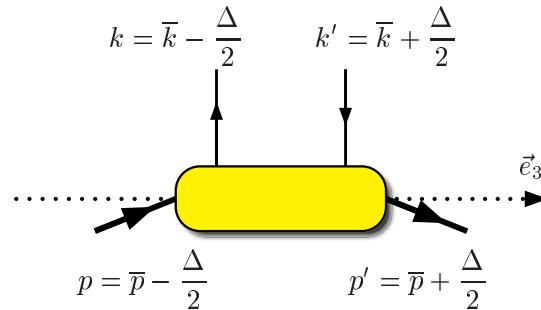
lead.  $\ln(1/x_{\text{Bj}})$  appr.: (Brodsky et al):  $x \simeq x' \simeq x_{\text{Bj}}$ ; GPD  $\rightarrow$  PDF

# Generalized Parton Distributions

D. Müller et al (94), Ji(97), Radyushkin (97)

$$\xi = \frac{(p - p')^+}{(p + p')^+} \quad \bar{x} = \frac{\bar{k}^+}{\bar{p}^+}$$

$$x = \frac{\bar{x} + \xi}{1 + \xi} \quad x' = \frac{\bar{x} - \xi}{1 - \xi}$$



$$\int \frac{dz^-}{\pi} e^{i \bar{x} \bar{p}^+ z^-} \langle p' | G^{+\mu}(-\bar{z}/2) G_\mu^+(\bar{z}/2) | p \rangle = \\ \bar{u}(p') \gamma^+ u(p) H^g(\bar{x}, \xi; t) + \bar{u}(p') i \sigma^{+\alpha} \frac{\Delta_\alpha}{2m} u(p) E^g(\bar{x}, \xi; t)$$

(gauge  $A^+ = 0$ ;  $\bar{z} = [0, z^-, \mathbf{0}_\perp]$ )  $\tilde{G}_\mu^+ \longrightarrow \tilde{H}^g, \tilde{E}^g$  (quarks analogously)

reduction formulas:

$$H^g(\bar{x}, 0; 0) = \bar{x} g(\bar{x}) \quad \tilde{H}^g(\bar{x}, 0; 0) = \bar{x} \Delta g(\bar{x})$$

sum rules, universality, polynomiality, evolution, positivity constraints

$$\gamma^* p \rightarrow V p$$

gluon subprocess dominant at large  $Q^2$ ,  $W$  and small  $x_{\text{Bj}} (\lesssim 0.2)$ ,  $t$ ;  
kinematics fixes skewness:  $\xi \simeq \frac{x_{\text{Bj}}}{2-x_{\text{Bj}}} [1 + m_V^2/Q^2] \simeq x_{\text{Bj}}/2 + \text{m.m.c.}$

$$M_{0+,0+}^{V(g)} = \frac{e}{2} \mathcal{C}_V \int_0^1 \frac{d\bar{x}}{(\bar{x} + \xi)(\bar{x} - \xi + i\epsilon)} \times \\ \left\{ \sum_{\lambda} \mathcal{H}_{0\lambda,0\lambda}^{V(g)} [H^g - \frac{\xi^2}{1-\xi^2} \textcolor{red}{E^g}] + \sum_{\lambda} \lambda \mathcal{H}_{0\lambda,0\lambda}^{V(g)} [\tilde{H}^g - \frac{\xi^2}{1-\xi^2} \textcolor{red}{\tilde{E}^g}] \right\}$$

$$M_{0-,0+}^{V(g)} = \dots \frac{\sqrt{-t}}{2m} \dots E^g + \dots \xi \tilde{E}^g$$

( $\mathcal{C}_V$  flavor factor, quarks analogously)

**Unpolarized protons:** no flip-nonflip interference (expectation  $|E(\tilde{E})| \lesssim |H(\tilde{H})|$ )

$$|M_{0-,0+}|^2 \propto t/m^2 \quad \text{neglected}$$

**parity conservation:**  $\sum \lambda \mathcal{H}_{0\lambda,0\lambda}^{V(g)} = 0$

**Electroprod.** with unpolarized protons at small  $x_{\text{Bj}}$  probes  $H^g$  and  $H^q$

(pseudoscalar mesons:  $\sum \mathcal{H}_{0\lambda,0\lambda}^{P(q)} = 0$  only  $\tilde{H}^q$  contributes)

# Double distributions

integral representation ( $i = \text{valence, sea quarks, gluons}$ )

$$H_i(\bar{x}, \xi, t') = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - \bar{x}) f_i(\beta, \alpha, t') + D_i \Theta(\xi^2 - \bar{x}^2)$$

$f_i$  double distributions      Mueller *et al* (94), Radyushkin (99)

advantage - polynomiality automatically satisfied

$D_i(\bar{x}, t)$  ( $i = \text{gluon, sea}$ ) additional free function, support  $-\xi < \bar{x} < \xi$

useful ansatz with relation to PDFs

$$f_i(\beta, \alpha, t') = h_i(\beta, t) \frac{\Gamma(2n_i + 2)}{2^{2n_i + 1} \Gamma^2(n_i + 1)} \frac{[(1 - |\beta|)^2 - \alpha^2]^{n_i}}{(1 - |\beta|)^{2n_i + 1}}$$

$$h_g(t = 0) = |\beta| g(|\beta|), \quad n_g = 2$$

$$h_{\text{sea}}^q(t = 0) = q(|\beta|) \text{sign}(\beta), \quad n_{\text{sea}} = 2$$

$$h_{\text{val}}^q(t = 0) = q_{\text{val}}(\beta) \Theta(\beta), \quad n_{\text{val}} = 1$$

sea quarks mix with gluons under evolution

# The $t$ dependence

parameterization of PDFs:

$\delta_i$  Regge intercepts

$$h_i(\beta) = \beta^{-\delta_i} (1 - \beta)^{2n_i + 1} \sum_j c_{ij} \beta^{j/2}$$

Landshoff *et al* (71), Feynman (72)

$$\alpha_i(t) = \alpha_i(0) + \alpha'_i t$$

valence quarks:  $\delta_{\text{val}} = \alpha_{\text{val}}(0) \simeq 0.48$ ,  
 $\alpha'_{\text{val}} \simeq 0.9 \text{ GeV}^{-2}$

gluon and sea trajectory  
(Pomeron - diffraction)

$$\sigma_L \propto W^{4\delta_g(Q^2)} \text{ (HERA data)}$$

$$\delta_g = \alpha_g(0) - 1 \simeq 0.10 + 0.06 \ln \frac{Q^2}{4 \text{ GeV}^2}$$

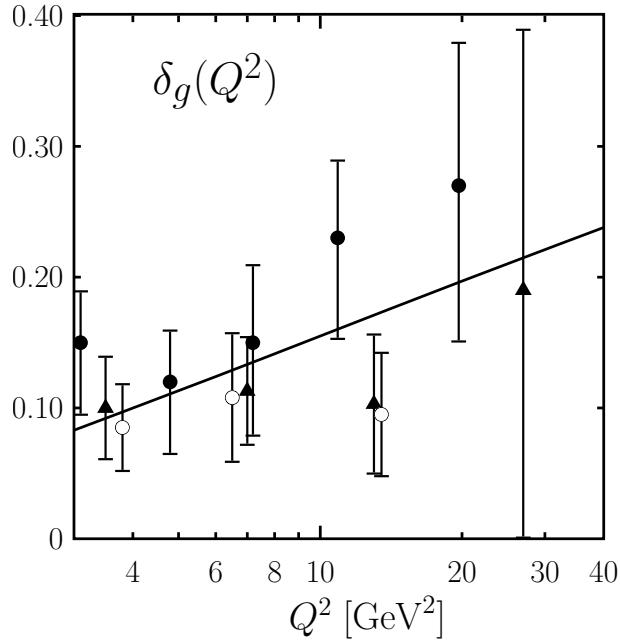
$$\delta_{\text{sea}} = \alpha_g(0)$$

$$\alpha'_g \simeq 0.1 - 0.2 \text{ (photoprod. of } J/\Psi)$$

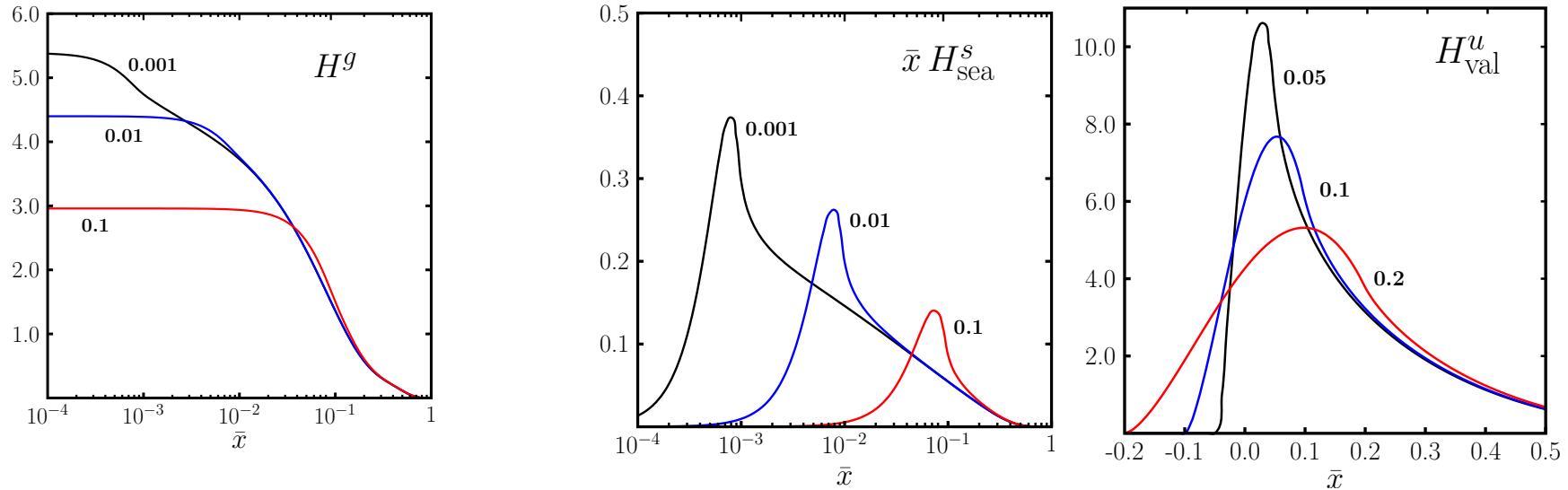
Reggeized GPDs (small  $-t$ ):  $h_i(\beta, t) = \exp [(b_i + \alpha'_i \ln(1/\beta))t] h_i(\beta, t=0)$

$b_i$ :  $t$  dependence of **Regge residue** (taken from experiment)

GPD integral can be executed analytically



## The GPDs at $t = 0$



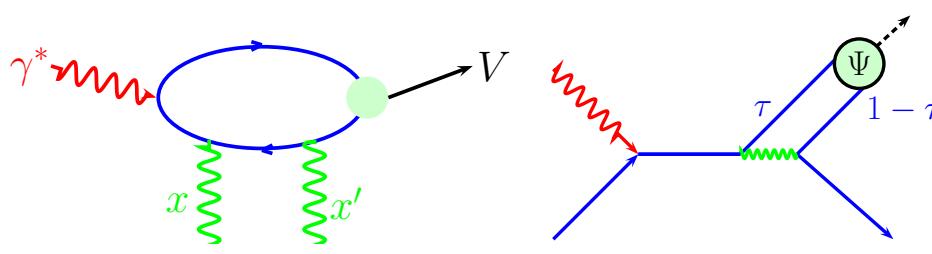
various values of  $\xi$ ,  
**INPUT:** NLO CTEQ6M

$\xi \ll \bar{x}$ : GPD→PDF up to corrections of  $\mathcal{O}(\xi^2)$   
at scale  $4\text{ GeV}^2$

$$H_{\text{val}}^d \simeq H_{\text{val}}^u / 2 \quad \quad H_{\text{sea}}^u \simeq H_{\text{sea}}^d \simeq \kappa_s H_{\text{sea}}^s \; (\kappa_s \simeq 2 \text{ at } 4 \text{ GeV}^2; \rightarrow 1 \text{ for } Q^2 \rightarrow \infty)$$

polynomiality and reduction formulas respected by construction  
positivity bounds respected as checked numerically

# $\gamma^* p \rightarrow Vp$ to leading-twist order



$$I_g = 2 \int_0^1 d\bar{x} \frac{\xi H^g(\bar{x}, \xi, t)}{(\bar{x} + \xi)(\bar{x} - \xi + i\epsilon)}$$

$$I_{\text{sea}} = 2 \int_0^1 d\bar{x} \frac{\bar{x} H_{\text{sea}}^s(\bar{x}, \xi, t)}{(\bar{x} + \xi)(\bar{x} - \xi + i\epsilon)}$$

$$I_{\text{val}}^a = 2 \int_{-\xi}^1 d\bar{x} \frac{\bar{x} H_{\text{val}}^a(\bar{x}, \xi, t)}{(\bar{x} + \xi)(\bar{x} - \xi + i\epsilon)},$$

pole terms:  $\text{Im} I_i = -\pi H_i(\xi, \xi, t)$

Regge phases for  $\xi \rightarrow 0$

$$\mathcal{M}_\phi = e \frac{8\pi\alpha_s}{N_c Q} f_\phi \langle 1/\tau \rangle_\phi \frac{-1}{3} \left\{ \frac{1}{2\xi} I_g + C_F I_{\text{sea}} \right\}$$

$$\mathcal{M}_\rho = e \frac{8\pi\alpha_s}{N_c Q} f_\rho \langle 1/\tau \rangle_\rho \frac{1}{\sqrt{2}} \left\{ \frac{1}{2\xi} I_g + \kappa_s C_F I_{\text{sea}} + \frac{1}{3} C_F I_{\text{val}}^u + \frac{1}{6} C_F I_{\text{val}}^d \right\}$$

$t$  dependence only in GPD considered

(scaled by soft parameter, actually slope of diffraction peak)

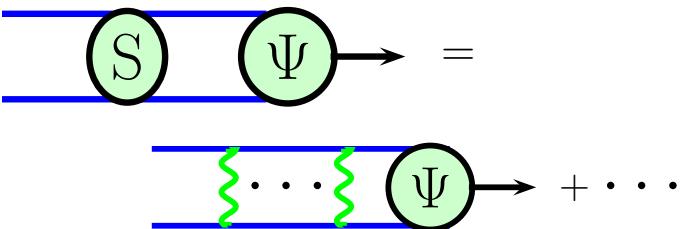
$\mathcal{H}$  provides power corrections of order  $t/Q^2$ , neglected

- leading-twist results too large
- by order of magnitude at  $Q^2 \simeq 4 \text{ GeV}^2$   
less at larger values of  $Q^2$
- transverse resolution power of  $\gamma^*$  cannot be neglected  
as compared with trans. size of meson
- suppression through quark transverse momentum  
required

# The modified perturbative approach

Sterman et al (92): quark transverse momenta and gluonic radiative corrections (Sudakov) are taken into account **in subprocess**

suppresses end-point regions ( $\tau \rightarrow 0, 1$ ) where  $q, \bar{q}$  separated by large transv. distances  
 (bears similarities to treatment in lead.  $\ln(1/x_{\text{Bj}})$  appr. e.g. Frankfurt et al (95))  
**in axial gauge:** modification of wf.



$$S = \frac{8}{3\beta_0} \ln \frac{\tau Q}{\sqrt{2}\Lambda_{\text{QCD}}} \ln \left( \frac{\ln(\tau Q/\sqrt{2}\Lambda_{\text{QCD}})}{-\ln(b\Lambda_{\text{QCD}})} \right) + \text{NLL-terms}$$

resummation  $\implies \exp[-S]$

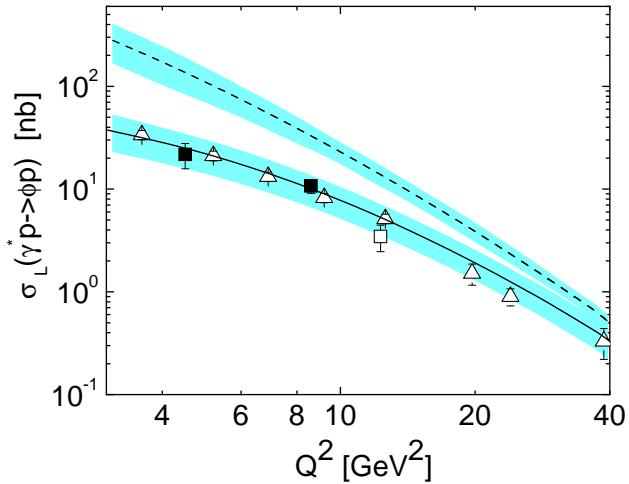
FT  $\vec{k}_\perp \rightarrow \vec{b}$ :

$$\mathcal{H}_L^V = \int d\tau d^2 b \hat{\Psi}_{VL}(\tau, b) \hat{T}_{LL} \exp[-S(\tau, b, Q^2)]$$

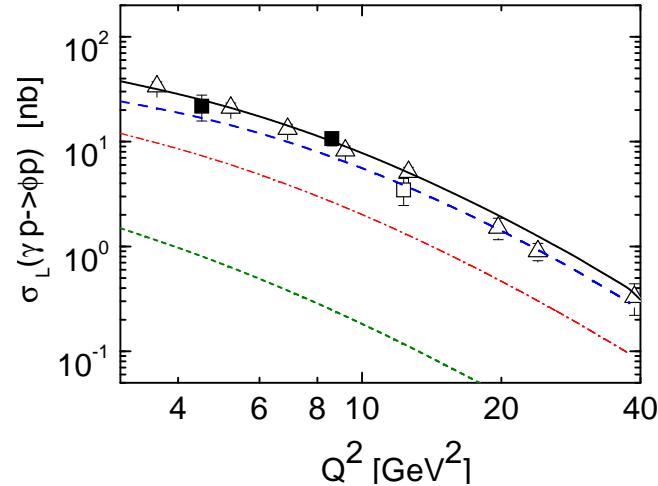
Gaussian wavefunction:  $\Psi_{VL} \sim f_{VL} \exp[-a_{VL}^2 k_\perp^2 / (\tau(1-\tau))]$

**parameters:**  $f_{\rho L} = 0.216 \text{ GeV}$  ( $V \rightarrow e^+ e^-$ )     $a_{\rho L} = 0.75 \text{ GeV}^{-1}$  (fit)  
 $f_{\phi L} = 0.237 \text{ GeV}$      $a_{\phi L} = 0.70 \text{ GeV}^{-1}$

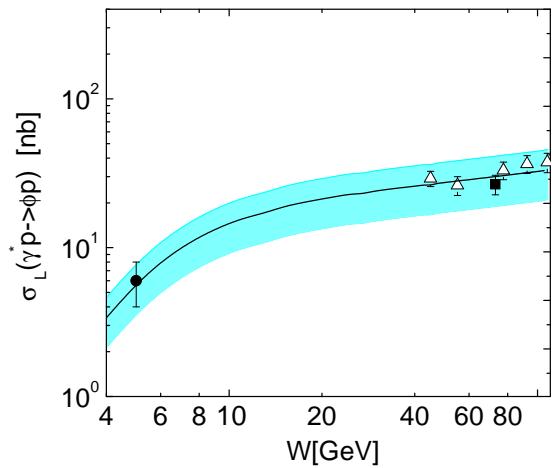
$$\sigma_L(\gamma^* p \rightarrow \phi p)$$



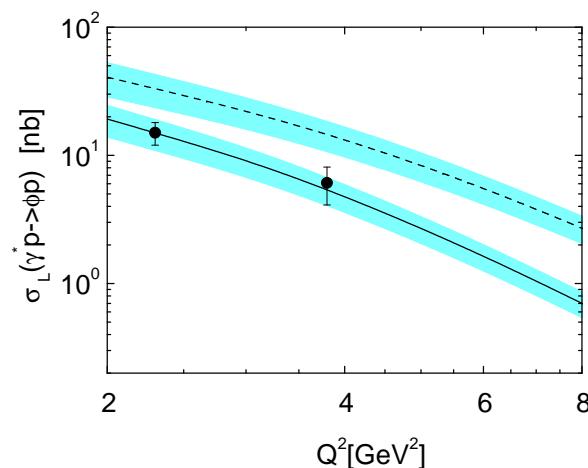
$W = 75 \text{ GeV}$



gluon, gluon-sea interf., sea

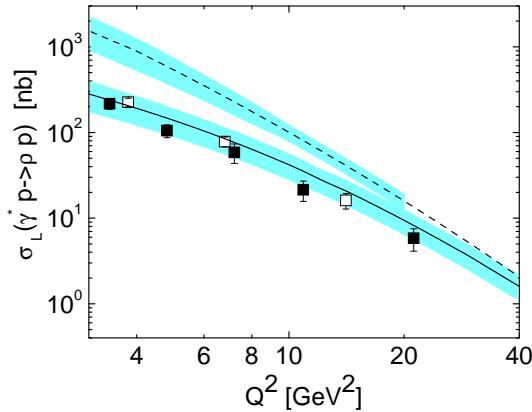


$Q^2 = 4 \text{ GeV}^2$ , data: HERMES, H1, ZEUS,

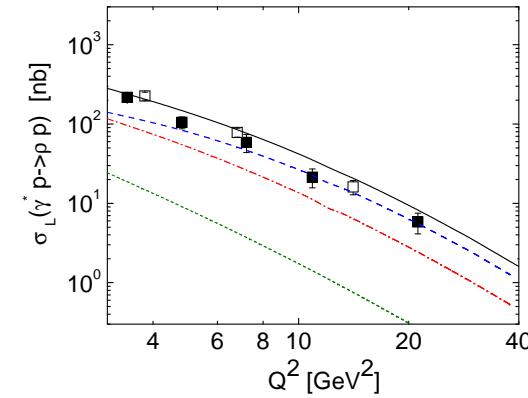


$W = 5(10) \text{ GeV}$  solid (dashed)

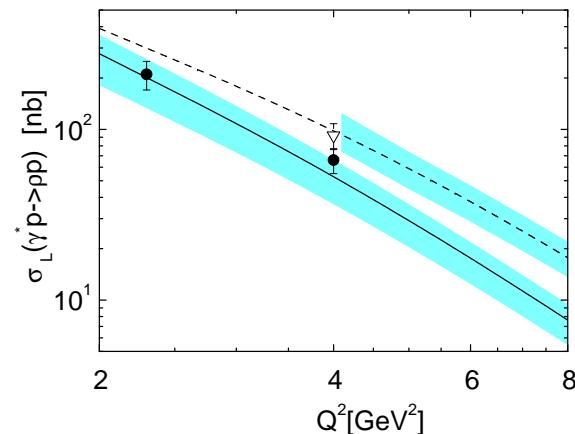
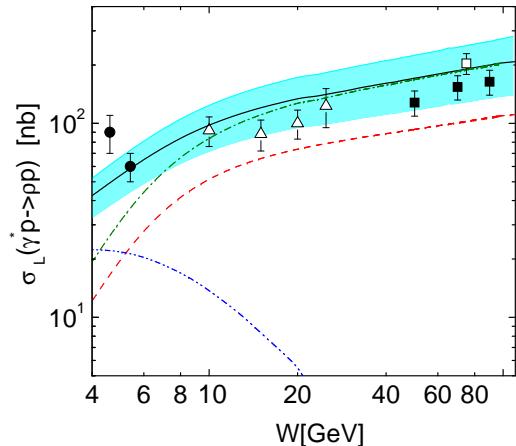
$$\sigma_L(\gamma^* p \rightarrow \rho p)$$



$W = 75 \text{ GeV}$  data: H1, ZEUS

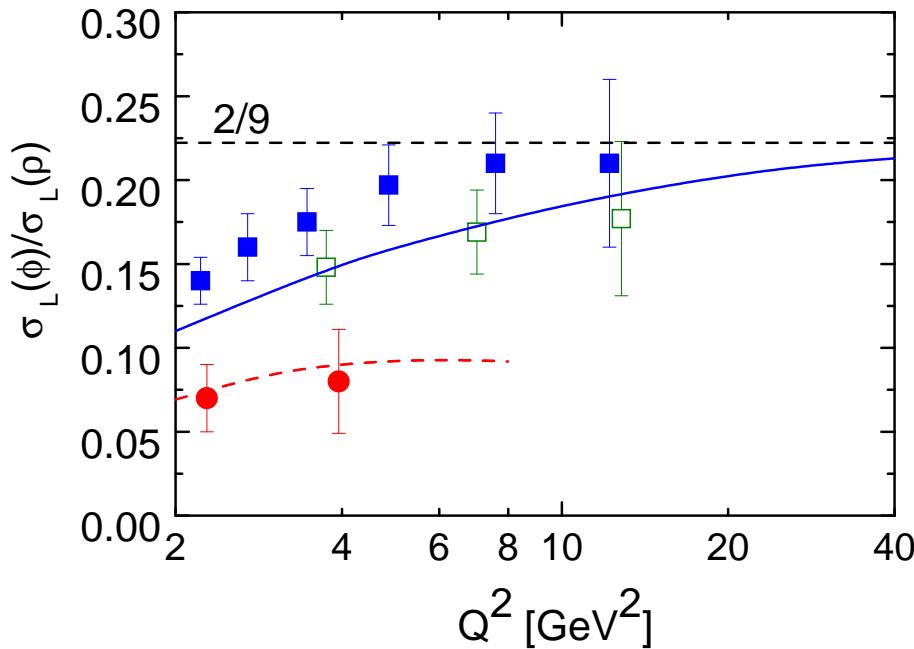


valence quarks negligible



$Q^2 = 3.8 \text{ GeV}^2$ , data: H1, ZEUS, E665, HERMES (dashed blue: valence )

# The $\phi$ - $\rho$ ratio



HERA ( $W = 75$  GeV): close to exp. at large  $Q^2$

HERMES ( $W = 5$  GeV): clear deviation

need for valence quarks

## Generalization to other $\gamma^* \rightarrow V$ transitions

IR singularities in  $V_T$  ampl. regularized by  $k_\perp$ :  $[\tau^2 Q^2]^{-1} \rightarrow \tau^{-1} [\tau Q^2 + k_\perp^2]^{-1}$

spin wavefct for  $V_T$ :  $\frac{-1}{\sqrt{2}M_V}\{\not{q}, \gamma_\perp\}\not{k}_\perp$  (instead of  $\frac{1}{\sqrt{2}}\not{q}$ )

with one unit of orbital angular momentum,  $M_V$  soft parameter of order  $m_V$

$$\mathcal{H}_T^V = \int d\tau d^2 b b^2 \hat{\Psi}_{VT} \hat{T}_{TT} \exp[-S]$$

second Gaussian wavefunction:  $\Psi_{VT} \sim f_{VT} \exp[-a_{VT}^2 k_\perp^2 / (\tau(1-\tau))]$

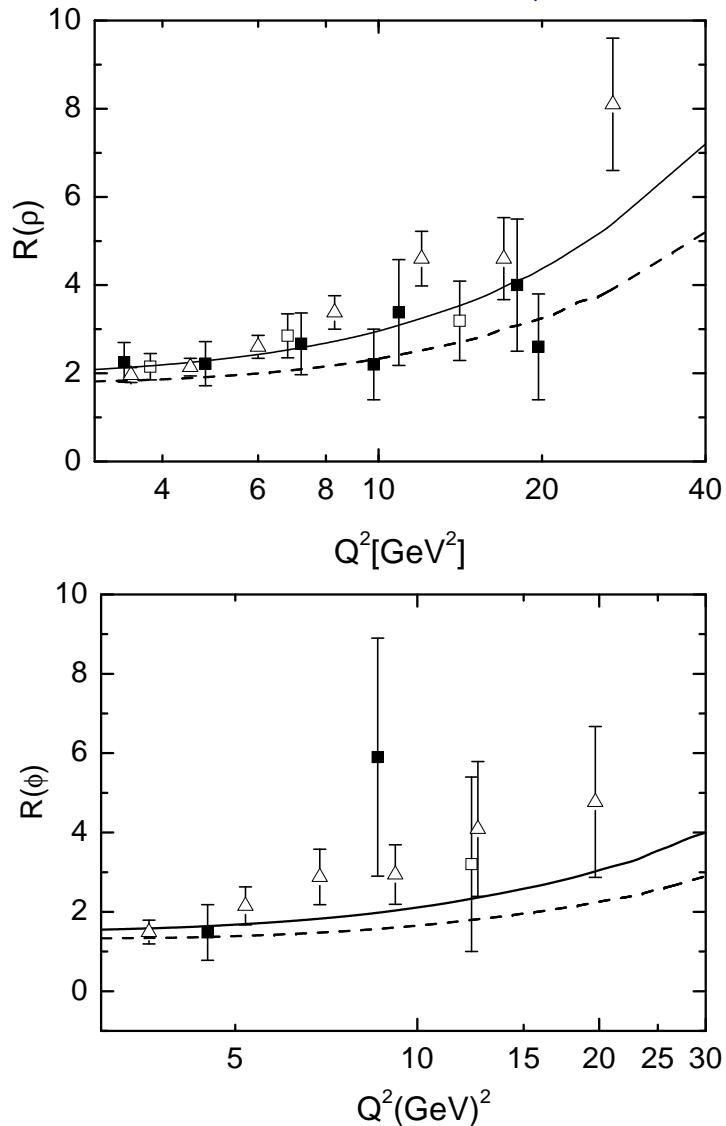
$T \rightarrow T$  amplitude sensitive to  $f_{VT}/M_V$  (suppressed by  $\langle k_\perp \rangle/Q$ )

e.g.  $M_V = m_V/2$ :  $f_{\rho T} = 0.170 \text{ GeV}$   $a_{\rho T} = 0.65 \text{ GeV}^{-1}$   
 $f_{\phi T} = 0.190 \text{ GeV}$   $a_{\phi T} = 0.60 \text{ GeV}^{-1}$

QCD sum rules (Ball et al):  $f_{VT}/f_{VL} \simeq 0.8$

Lattice QCD (Braun et al, Becirevic et al) similar values

# $R = \sigma_L/\sigma_T$ for $\rho$ and $\phi$ production



$W \simeq 75 \text{ GeV}$   
 filled (open) symbols:  
 H1 (ZEUS) data

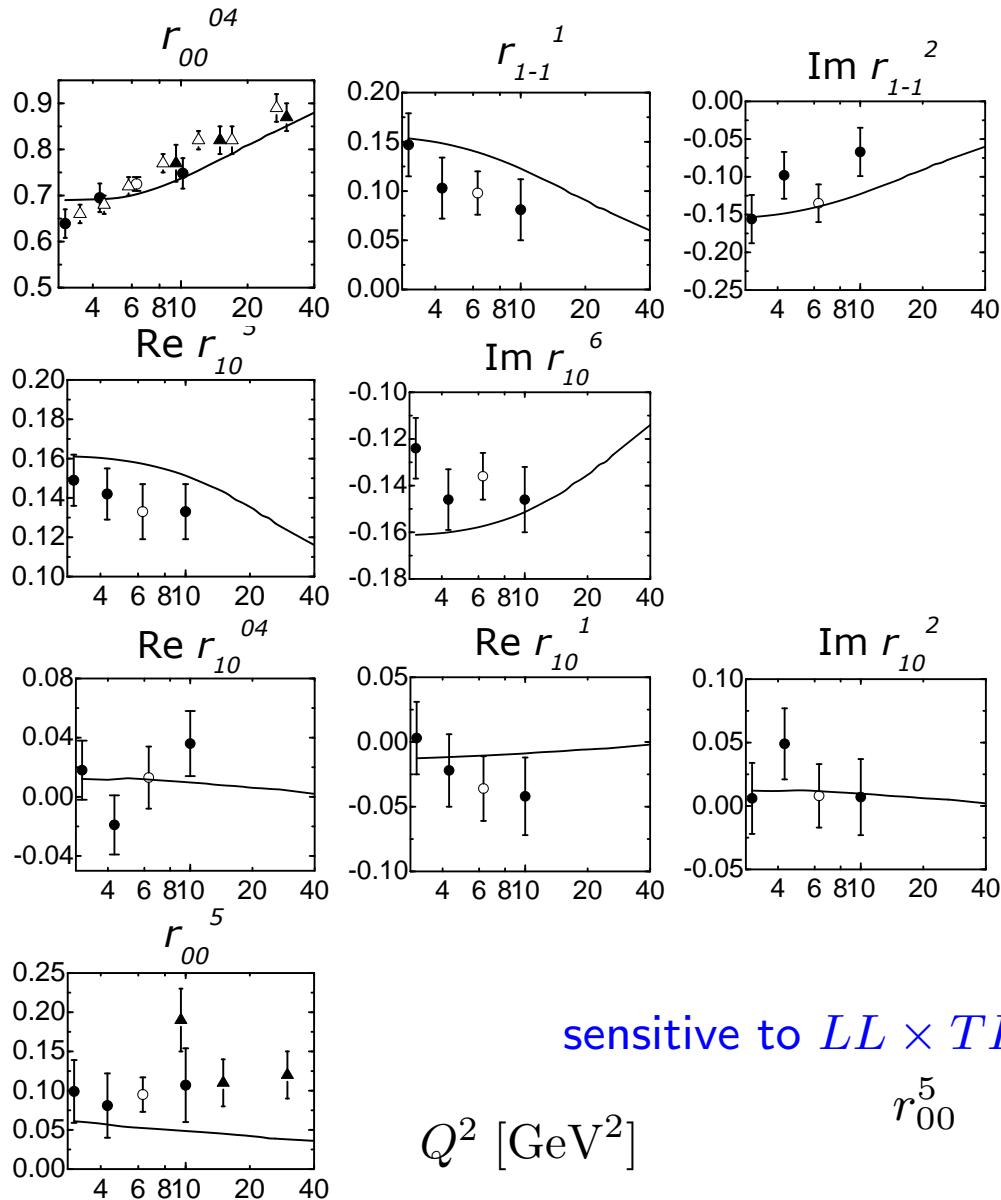
$$R = \sigma_L/\sigma_T: \propto Q^2$$

solid line:  $B_{TT}^V = B_{LL}^V$   
 (since same GPDs)

dashed line:  $B_{TT}^V = B_{LL}^V/2$   
 extreme examp. for comparison  
 not excluded by exp.

# SDME of the $\rho$

$W \simeq 75 \text{ GeV}$   $t \simeq -0.15 \text{ GeV}^2$ ; filled (open) symbols: H1 (ZEUS) data



only gluons as yet

sensitive to  $R$ :

$$r_{00}^{04}, r_{1-1}^1 = -\text{Im } r_{1-1}^2$$

sensitive to  $LL \times TT$

$$\text{Re } r_{10}^5 = -\text{Im } r_{10}^6$$

sensitive to  $TT \times TL$

$$\begin{aligned} \text{Re } r_{10}^{04} &= -\text{Re } r_{10}^1 \\ &= \text{Im } r_{10}^2 \end{aligned}$$

$L \rightarrow T, T \rightarrow -T$ :

$$\begin{aligned} r_{1-1}^{04} &= r_{11}^1 = r_{11}^5 \\ &= r_{1-1}^5 = \text{Im } r_{1-1}^6 = 0 \end{aligned}$$

# Summary

- phenomenology of DVME (DVES) within the handbag approach is complicated, still a lot of work to be done
- GPDs are modeled through reggeized double distr. ( $\alpha_g, \alpha_{\text{val}}$ )  
 $t$  dependence of valence quark GPDs not well probed as yet  
accurate  $t$  dependent data ( $d\sigma_{L,T}/dt$ , SDME) required
- fair agreement with exp. (HERA, E665, HERMES) found for LL transitions predictions for COMPASS energies  
subprocess calculated within modified pert.approach  
the gluonic GPD  $H^g$  dominates  
valence quarks only important at HERMES kinematics
- extension to TT transitions only for gluons as yet  
also fair agreement with data on  $R$  and SDME for HERA energies
- **in progress:** calculation of TT transitions for sea and valence quarks  
estimate of effects from GPDs  $\tilde{H}$  and  $E$  (pol. beams and/or targets)