GPDs: General Formalism and a few examples

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- General Overview $\Rightarrow$ Standard OPE and Ji's sumrule

- Exclusive electroproduction of π^+-π^- pairs

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- DVCS in NNLO

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- Photon diffractive dissociation

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- Conclusions

In QCD hadron structure is described by correlators of various type

$\left\langle P(p)\middle|\bar{q}(x)\gamma_\mu D_{\mu_1}...D_{\mu_n}q(x)\middle|P(p)\right\rangle$	momentum distribution of quarks
$\left\langle P(p')\middle|\bar{q}(x)\gamma_\mu q(x)\middle|P(p)\right\rangle$	form factors of a proton
$\left\langle P(p)\middle|\bar{q}(x)\Gamma_\mu q(x)\bar{q}'(x)\Gamma'_\nu q'(x)\middle|P(p)\right\rangle$	diquark correlations in a proton
$\left\langle P(p,s)\middle|\bar{q}(x)\gamma_\mu\tilde{G}_{\nu\lambda}(x)q(x)\middle|P(p,s)\right\rangle$	color magnetic field in a proton
$\left\langle 0\middle|\bar{d}(-z)\not{z}[-z,z]u(z)q(x)\middle|\rho^+(p,s)\right\rangle$	ρ distribution amplitude
$\left\langle 0\middle|\bar{u}(z)u(z)\middle|0\right\rangle$	vacuum condensates

Operator **P**roduct **E**xpansion is the art of linking such correlators to physical observables.

The physics of GPDs is just a natural extension of this framework

$$\left\langle P(p,s) \middle| \bar{q}(-z)\gamma_\mu[-z,z]q(z) \middle| P(p',s') \right\rangle$$

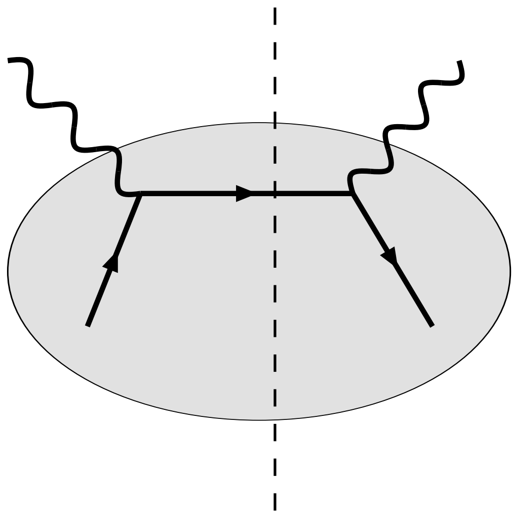
which can be treated with the same rigor.

Be aware: There is no reason to believe that GPDs are any simpler than e.g. distribution amplitudes, d_2, or diquark correlations. Do not trust anybody who sells fast and easy solutions.

The good news: you can stand on the shoulders of giants. The whole arsenal of QCD techniques developed during the last 30 years is at your disposal. However, climbing up there is very tedious, indeed.

To set the stage I will start with standard textbook OPE and Ji's sumrule.

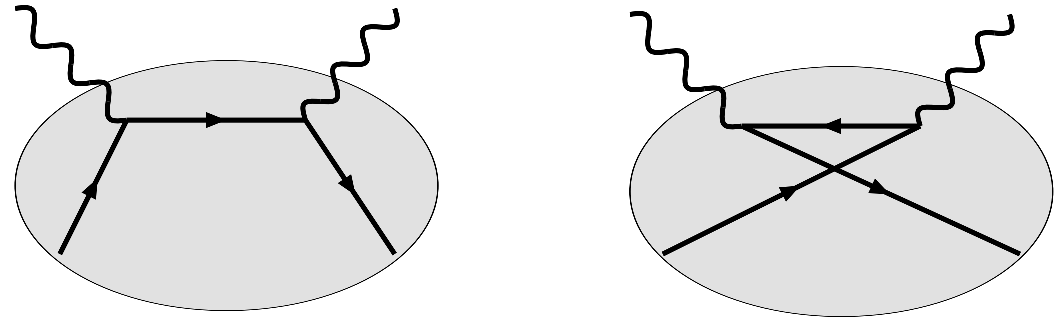
An example from standard OPE: The hadron scattering tensor



$$
\begin{aligned}
4\pi W^{\mu\nu} &= \frac{1}{2}\sum_{S_i} \sum\!\!\!\!\!\!\!\int_X (2\pi)^4 \delta^4(P_i + q - P_X) \left\langle p(P_i, S_i) \right| \hat{J}^{\mu\dagger}_{\text{em}}(0) \left| X(P_X, S_X) \right\rangle \\
&\times \left\langle X(P_X, S_X) \right| \hat{J}^{\nu}_{\text{em}}(0) \left| p(P_i, S_i) \right\rangle \\
&= \frac{1}{4\pi} \int d^4y \; e^{\mathrm{i}q\cdot y} \left\langle p(P_i, S_i) \right| \left[\hat{J}^{\mu\dagger}_{\text{em}}(y), \hat{J}^{\nu}_{\text{em}}(0) \right]_- \left| p(P_i, S_i) \right\rangle
\end{aligned}
$$

We will derive the optical theorem for $W^{\mu\nu}$.

Forward scattering amplitude



$$T^{\mu\nu}(P_i, S_i, q) \;=\; \mathrm{i}\int d^4y\; e^{\mathrm{i}q\cdot y}\left\langle p(P_i,S_i)\right|\,\mathcal{T}\left\{\hat{J}^{\mu\dagger}_{\mathrm{em}}(y)\hat{J}^{\nu}_{\mathrm{em}}(0)\right\}\left|p(P_i,S_i)\right\rangle$$

$$T^{\mu\nu}(P_i, S_i, q) \;=\; -(2\pi)^3\,\frac{1}{2}\sum_{S_i}\sum\!\!\!\!\!\!\int_X$$

$$\times \quad \left\{\left\langle p(P_i,S_i)\right|\,\hat{J}^{\mu\dagger}_{\mathrm{em}}(0)\left|X\right\rangle\left\langle X\right|\,\hat{J}^{\nu}_{\mathrm{em}}(0)\left|p(P_i,S_i)\right\rangle\frac{\delta^3(\vec{q}+\vec{P}_i-\vec{P}_X)}{q^0+P_i^0-P_X^0+\mathrm{i}\epsilon}\right.$$

$$\left. - \quad \left\langle p(P_i,S_i)\right|\,\hat{J}^{\nu}_{\mathrm{em}}(0)\left|X\right\rangle\left\langle X\right|\,\hat{J}^{\mu\dagger}_{\mathrm{em}}(0)\left|p(P_i,S_i)\right\rangle\frac{\delta^3(\vec{q}-\vec{P}_i+\vec{P}_X)}{q^0-P_i^0+P_X^0-\mathrm{i}\epsilon}\right\}$$

with

$$\Theta(y^0) = \frac{\mathrm{i}}{2\pi}\int d\omega\,\frac{1}{\omega+\mathrm{i}\epsilon}\,e^{-\mathrm{i}\omega y^0}$$

.

$$\mathcal{I}m \quad T^{\mu\nu}(P_i, S_i, q) \;=\; \pi(2\pi)^3 \, \frac{1}{2} \sum_{S_i} \sum\!\!\!\!\!\!\int_X$$

$$\times \quad \Big\{ \delta^4(q + P_i - P_X) \left\langle p(P_i, S_i) \right| \hat{J}^{\mu\dagger}_{\text{em}}(0) \left| X \right\rangle \left\langle X \right| \hat{J}^{\nu}_{\text{em}}(0) \left| p(P_i, S_i) \right\rangle$$

$$+ \quad \delta^4(q - P_i + P_X) \left\langle p(P_i, S_i) \right| \hat{J}^{\nu}_{\text{em}}(0) \left| X \right\rangle \left\langle X \right| \hat{J}^{\mu\dagger}_{\text{em}}(0) \left| p(P_i, S_i) \right\rangle \Big\}$$

The last term vanishes, because there is no hadron state with the nucleon quantum numbers and lower energy, and one has

The optical theorem

$$\boxed{\mathcal{I}m \; T^{\mu\nu}(P_i, S_i, q) \;=\; 2\pi \; W^{\mu\nu}}$$

The derivation of, e.g., Ji's sum rule is completely analogous, with the energy-momentum tensor $\mathbf{T}^{\mu\nu}$ playing the role of the hadron scattering tensor $W^{\mu\nu}$.

$$
\begin{aligned}
\mathcal{L}_{QCD} \quad &\rightarrow \quad \mathbf{T}_{\mu\nu} = \frac{1}{2}[\bar{q}\gamma^{(\mu} i \overleftrightarrow{D}^{\nu)} q + \bar{q}\gamma^{(\mu} i \overleftrightarrow{D}^{\nu)} q] + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}{}_{\alpha} \\
J^i_{q,g} \quad &= \quad \frac{1}{2}\, \epsilon^{ijk} \int d^3x (\mathbf{T}^{0k}_{q,g} x^j - \mathbf{T}^{0j}_{q,g} x^k) \\
&\rightarrow \quad \text{Lorentz decomposition for} \quad \left\langle P_2 \middle| \mathbf{T}_{\mu\nu} \middle| P_1 \right\rangle \\
\left\langle P_2 \middle| \mathbf{T}^{\mu\nu}_{q,g} \middle| P_1 \right\rangle \quad &= \quad \bar{N}(P_2)\Big[A_{q,g}(\Delta^2)\gamma^{(\mu} P^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha/2M \\
&\quad + C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \bar{C}_{q,g}(\Delta^2) g^{\mu\nu} M \Big] N(P_1) \\
P^\mu \quad &= \quad (P_2^\mu + P_1^{\mu\prime})/2, \qquad \Delta^\mu = P_1^\mu - P_2^\mu \\
\left\langle J^i_{q,g} \right\rangle \quad &= \quad \frac{1}{2}\,[A_{q,g}(0) + B_{q,g}(0)]
\end{aligned}
$$

The DVCS amplitude:

$$T^{\mu\nu}(P,\Delta,t) \quad = \quad \mathrm{i}\int d^4y\; e^{\mathrm{i}(q+q')\cdot\frac{y}{2}}\,\left\langle P_2,S_2\right|\,\mathcal{T}\left\{\hat{J}^{\mu\dagger}_{\mathrm{em}}\left(\frac{y}{2}\right)\hat{J}^{\nu}_{\mathrm{em}}\left(\frac{y}{2}\right)\right\}\left|P_1,S_1\right\rangle$$

$$M_{\lambda'\mu',\lambda\mu} \quad := \quad \epsilon_\alpha T^{\alpha\beta}\epsilon_\beta$$ helicity amplitudes

$$M_{++,--} \quad = \quad \sqrt{1-\xi^2}\left(\mathcal{H}+\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}+\tilde{\mathcal{E}})\right)$$ Compton formfactors

$$M_{-+,-+} \quad = \quad \sqrt{1-\xi^2}\left(\mathcal{H}-\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}-\tilde{\mathcal{E}})\right)$$ etc.

$$\mathcal{H}(\rho,\xi,t) \quad = \quad \sum_q e_q^2\int_{-1}^{1}dx\; H_q(x,\xi,t)\left(\frac{1}{\rho-x-\mathrm{i}\epsilon}-\frac{1}{\rho+x-\mathrm{i}\epsilon}\right)\;+\;O(\alpha_s)$$

$$\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle P_2|\,\bar{q}(-\frac{1}{2}z)\,\gamma^+q(\frac{1}{2}z)\,|P_1\rangle\Big|_{z^+=0,\;z_\perp=0}$$

$$= \quad \frac{1}{P^+}\left[H_q(x,\xi,t)\,\bar{N}(P_2)\gamma^+N(P_1)+E_q(x,\xi,t)\,\bar{N}(P_2)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}N(P_1)\right]$$

Where can one find the same matrix elements in $T^{\mu\nu}$ as in $\left\langle P_2 \middle| \mathbf{T}^{\mu\nu} \middle| P_1 \right\rangle$?
$\rightarrow$ OPE:

$$\mathcal{O}_q^{\mu\mu_1\ldots\mu_n} \quad := \quad \textbf{Sym}\ \bar{q}(x)\gamma^\mu \mathrm{i} \overleftrightarrow{D}^{\mu_1} \ldots \mathrm{i} \overleftrightarrow{D}^{\mu_n} q(x) \qquad \text{local operators}$$

$$\begin{aligned}
\left\langle P_2 \middle| \mathcal{O}_q^{\mu\mu_1\ldots\mu_n} \middle| P_1 \right\rangle \quad = \quad & \textbf{Sym}\ \bar{N}(P_2)\gamma^\mu N(P_1) \sum_{i=0,even}^{n} \\
& \qquad A^q_{n+1,i}(t)\Delta^{\mu_1}\ldots\Delta^{\mu_i}P^{\mu_{i+1}}\ldots P^{\mu_n} \\
+ \quad & \textbf{Sym}\ \bar{N}(P_2)\frac{\mathrm{i}\sigma^{\mu\alpha}\Delta_\alpha}{2M}N(P_1) \sum_{i=0,even}^{n} \\
& \qquad B^q_{n+1,i}(t)\Delta^{\mu_1}\ldots\Delta^{\mu_i}P^{\mu_{i+1}}\ldots P^{\mu_n} \\
+ \quad & \textbf{Sym}\ \bar{N}(P_2)\frac{\Delta_\mu}{M}N(P_1)\ C^q_{n+1}(t)\mathrm{mod}(n,2)\Delta^{\mu_1}\ldots\Delta^{\mu_n}
\end{aligned}$$

The $A's$ in $\left\langle P_2 \middle| \mathbf{T}^{\mu\nu} \middle| P_1 \right\rangle$ correspond to $A^q_{2,0}$.

$$\boxed{\left\langle J^3_q \right\rangle = \frac{1}{2}[A^q_{2,0}(0) + B^q_{2,0}(0)] \qquad \text{Ji's sumrule}}$$

$$\begin{aligned}
\int_{-1}^{1} dx\, x^{n-1}\, H(x,\xi,t) &= \sum_{\substack{k=0\\ \text{even}}}^{n-1} (2\xi)^k\, A_{n,k}(t) + \mathrm{mod}(n+1,2)\,(2\xi)^n C_n(t) \\
\int_{-1}^{1} dx\, x^{n-1}\, E(x,\xi,t) &= \sum_{\substack{k=0\\ \text{even}}}^{n-1} (2\xi)^k\, B_{n,k}(t) - \mathrm{mod}(n+1,2)\,(2\xi)^n C_n(t)
\end{aligned}$$

The following moments have so far been calculated on the lattice

N:

$$A_{10}^q,\ \ A_{20}^q,\ \ A_{30}^q,\ \ A_{32}^q,\ \ B_{10}^q,\ \ B_{20}^q,\ \ B_{30}^q,\ \ B_{32}^q,\ \ C_{20}^q,$$

$$\tilde{A}_{10}^q,\ \ \tilde{A}_{20}^q,\ \ \tilde{A}_{30}^q,\ \ \tilde{A}_{32}^q,\ \ \tilde{B}_{10}^q,\ \ \tilde{B}_{20}^q,\ \ \tilde{B}_{30}^q,\ \ \tilde{B}_{32}^q,$$

$$A_{T10}^q,\ \ A_{T20}^q,\ \ \bar{B}_{T10}^q = B_{T10}^q + 2\tilde{A}_{T10}^q,\ \ \bar{B}_{T20}^q,\ \ \tilde{A}_{T10}^q,\ \ \tilde{A}_{T20}^q,\ \ \tilde{B}_{T21}^q$$

π:

$$A_{10}^q,\ \ A_{20}^q,\ \ C_{20}^q,\ \ B_{T10}^q,\ \ B_{T20}^q$$

Formal Definition of GPDs

We use the notation of X.Ji and name the momenta according to: $h(P_1)+\Gamma^*(q_1) \rightarrow h(P_2) + \Gamma(q_2)$ for any hadron h and define $\Delta_\mu = q_{2\mu} - q_{1\mu}$, $t = \Delta^2$, $P_\mu = (P_{1\mu} + P_{2\mu})/2$; $n_\mu = (1,0,0,-1)/\sqrt{2}P^+ \rightarrow n\cdot P = 1$ and $\xi = -Q^2/2P\cdot q$.

Spin $\frac{1}{2}$ - the nucleon

$$
\begin{aligned}
&\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2|\, \bar{q}(-\frac{1}{2}z)\, \gamma^+ q(\frac{1}{2}z)\, |P_1\rangle \Big|_{z^+=0,\, z_\perp=0} \\
=\ & \frac{1}{P^+}\left[H_q(x,\xi,t)\, \bar{N}(P_2)\gamma^+ N(P_1) + E_q(x,\xi,t)\, \bar{N}(P_2)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} N(P_1) \right]
\end{aligned}
$$

$$
\begin{aligned}
&\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\, \bar{q}(-\frac{1}{2}z)\, \gamma^+ \gamma_5\, q(\frac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0} \\
=\ & \frac{1}{P^+}\left[\tilde{H}_q(x,\xi,t)\, \bar{N}(P_2)\gamma^+\gamma_5 N(P_1) + \tilde{E}_q(x,\xi,t)\, \bar{N}(P_2)\frac{\gamma_5\Delta^+}{2M} N(P_1) \right]
\end{aligned}
$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle P_2 | \, G^{+\mu}(-\frac{1}{2}z) \, G_\mu{}^+(\frac{1}{2}z) \, | P_1 \rangle \Big|_{z^+=0,\, z_\perp=0}$$

$$= \frac{1}{2} \left[H_g(x,\xi,t) \, \bar{N}(P_2) \gamma^+ N(P_1) + E_g(x,\xi,t) \, \bar{N}(P_2) \frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} N(P_1) \right]$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle P_2 | \, G^{+\mu}(-\frac{1}{2}z) \, \tilde{G}_\mu{}^+(\frac{1}{2}z) \, | P_1 \rangle \Big|_{z^+=0,\, z_\perp=0}$$

$$= \frac{i}{2} \left[\tilde{H}_g(x,\xi,t) \, \bar{N}(P_2) \gamma^+ \gamma_5 N(P_1) + \tilde{E}_g(x,\xi,t) \, \bar{N}(P_2) \frac{\gamma_5 \Delta^+}{2M} N(P_1) \right]$$

and many more

Some aspects:

- relation to form factors and distribution functions

$$H_q(x,0,0) = q(x) \qquad \int_{-1}^{1} dx H_q(x,\xi,t) = F_{1q}(t)$$

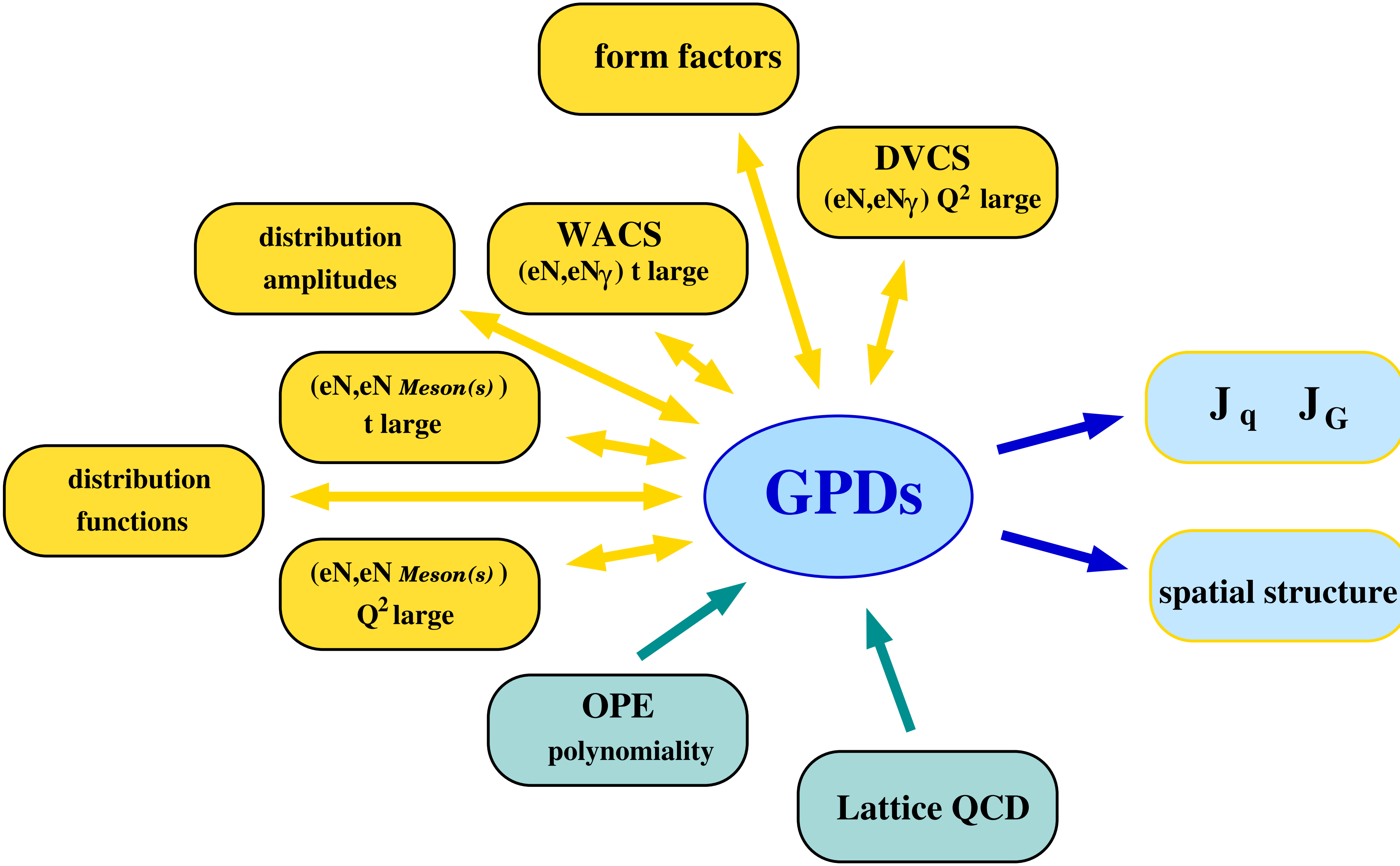
$$\tilde{H}_q(x,0,0) = \Delta q(x) \qquad \int_{-1}^{1} dx H_q(x,\xi,t) = g_{Aq}(t)$$

- polynomiality

$$\int dx x^n H(x,\xi,t) = \sum_{i \;\; \text{even}}^{n} (2\xi)^i A^q_{n,i}(t) + \mathrm{mod}(n,2)(2\xi)^{n+1} C^q_n(t)$$

- GPDs give information on the transverse structure of hadrons in the impact parameter plane. The transverse mass is $\sqrt{q_{\|}^2 + m^2}$. Therefore a probabilistic interpretation makes sense.

$$H_q(x,0,\mathbf{b}_\perp^2) = \frac{1}{(2\pi)^2} \int d^2\mathbf{\Delta}_\perp \, \mathrm{e}^{\mathrm{i}\mathbf{b}_\perp \mathbf{\Delta}_\perp} H_q(x,0,\mathbf{\Delta}_\perp^2)$$

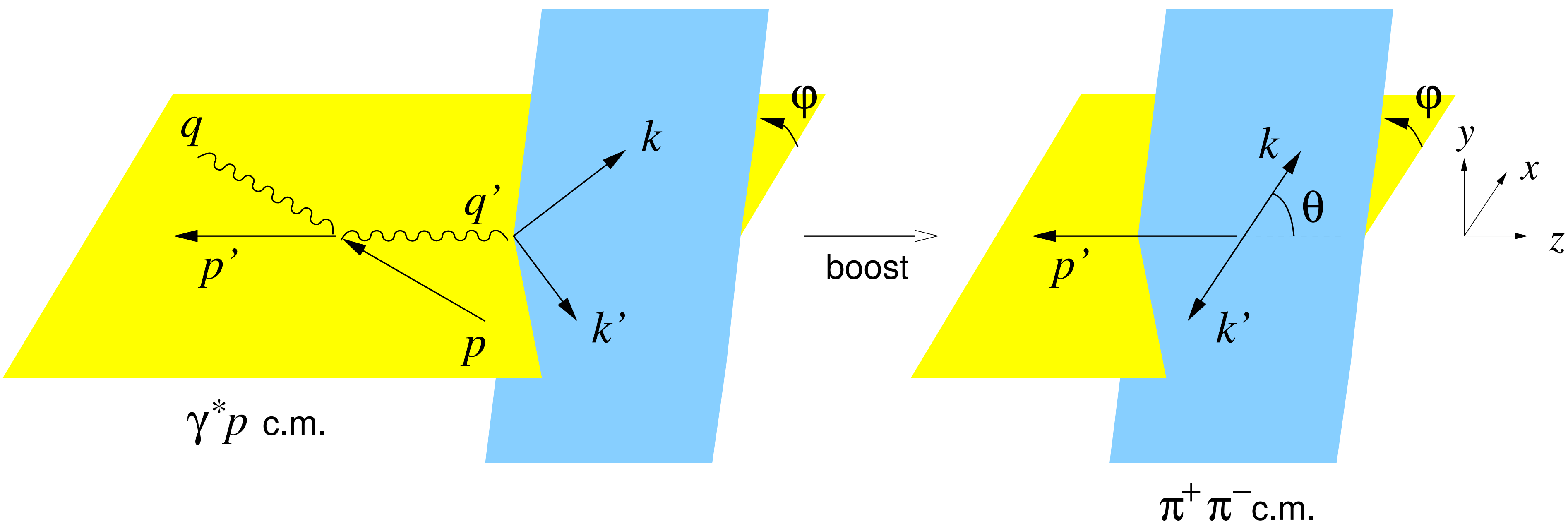

form factors
DVCS
(eN,eNγ) Q2 large
distribution
amplitudes
WACS
(eN,eNγ) t large
(eN,eN Meson(s))
t large
J q J G
distribution
functions
GPDs
(eN,eN Meson(s))
Q2 large
spatial structure
OPE
polynomiality
Lattice QCD

Exclusive electro production of pion pairs

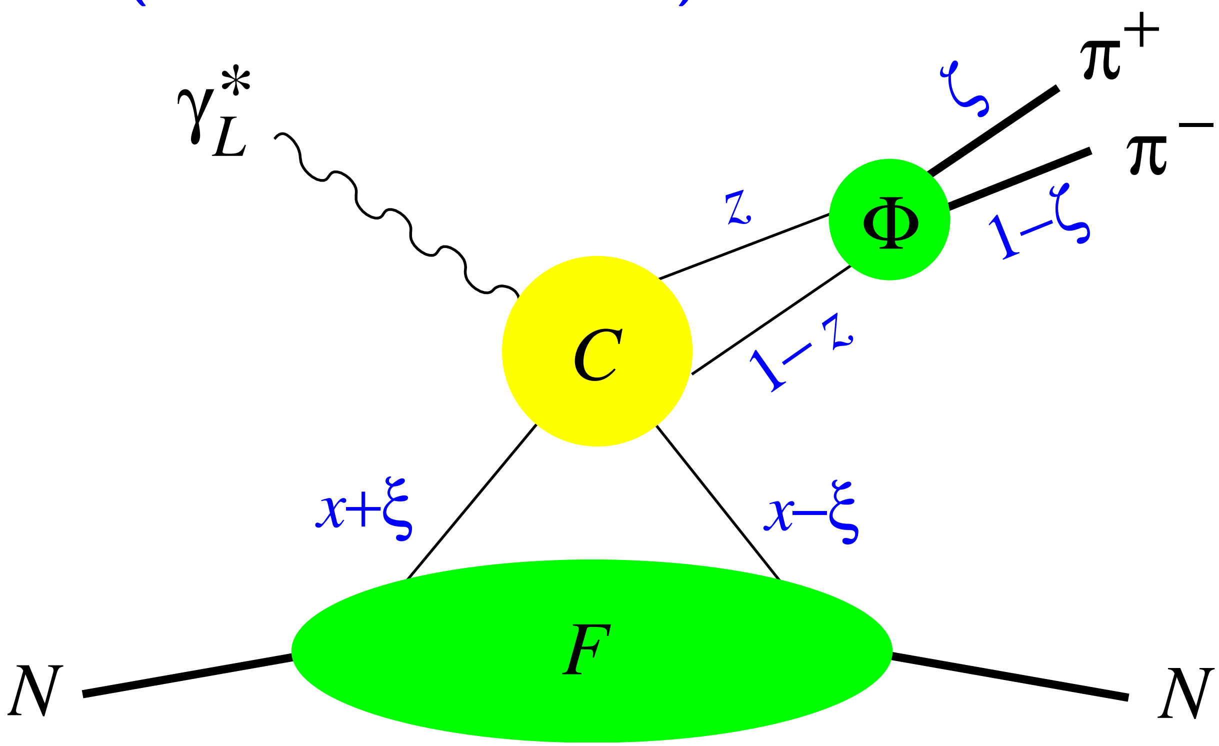
N. Warkentin, M. Diehl, D.Yu. Ivanov, and AS

hep-ph/0703148

$$\gamma^*(q) + N(p) \rightarrow \pi^+(k) + \pi^-(k') + N(p')$$



The factorization (Collins et al., Freund)



F: GPDs

C: Hard kernel in NLO

Φ: Two-pion distribution amplitudes

We will find that the main source of uncertainties are not due to the core QCD calculation but due to a lack of understanding of 2 pion phase shifts **!**

Two-pion distribution amplitudes

$$\begin{aligned}
\Phi^q(z,\zeta,s_\pi) &= \int \frac{d\lambda}{2\pi} e^{-iz\lambda(q'\cdot n)} \left\langle \pi^+(k)\pi^-(k') \middle| \bar{q}(\lambda n) \not{n} q(0) \middle| 0 \right\rangle \\
\Phi^g(z,\zeta,s_\pi) &= \frac{1}{q'\cdot n} \int \frac{d\lambda}{2\pi} e^{-iz\lambda(q'\cdot n)}\, n_\alpha n_\beta \left\langle \pi^+(k)\pi^-(k') \middle| G^{\alpha\mu}(\lambda n) G_\mu{}^\beta(0) \middle| 0 \right\rangle
\end{aligned}$$

$$\zeta = \frac{k\cdot n}{q'\cdot n}, \qquad \beta\cos\theta = 2\zeta - 1, \qquad \beta = \sqrt{1 - \frac{4m_\pi^2}{s_\pi}}$$

states with definite C parity:

$$\Phi^{q(\pm)}(z,\zeta,s_\pi) = \frac{1}{2}\big[\Phi^q(z,\zeta,s_\pi) \pm \Phi^q(z,1-\zeta,s_\pi)\big]$$

Q^2 evolution of distribution amplitudes is described by ERBL-equations.
Gegenbauer polynomials are eigenfunctions of the evolution kernel
($C_n^{3/2}$ for spin 1/2 and $C_n^{5/2}$ for spin 1).
$\rightarrow$ It is natural to expand GDAs in terms of Gegenbauer polynomials.

$$\Phi^{q(-)}(z,\zeta,s_\pi) \;=\; 6z(1-z)\sum_{\substack{n=0\\ \text{even}}}^{\infty}\sum_{\substack{l=1\\ \text{odd}}}^{n+1} B_{nl}^{q(-)}(s_\pi)\, C_n^{3/2}(2z-1)\, P_l(2\zeta-1)\,,$$

$$\Phi^{q(+)}(z,\zeta,s_\pi) \;=\; 6z(1-z)\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{nl}^{q(+)}(s_\pi)\, C_n^{3/2}(2z-1)\, P_l(2\zeta-1)\,,$$

$$\Phi^{g}(z,\zeta,s_\pi) \;=\; 9z^2(1-z)^2\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{nl}^{g}(s_\pi)\, C_{n-1}^{5/2}(2z-1)\, P_l(2\zeta-1)\,,$$

HERMES analyzed only $\langle P_1(\cos\theta)\rangle$ and $\langle P_3(\cos\theta)\rangle$ $\longrightarrow$ We only need B_{10} and B_{12}.

The case $s_\pi = 0+$ is fixed by our GPD OPE analysis for $t = 0-$.

$$\left\langle \pi^+(p') \middle| \mathbf{Sym}\ \bar{q}(0)\gamma^\mu \mathrm{i} \overleftrightarrow{D}^{\mu_1} ... \mathrm{i} \overleftrightarrow{D}^{\mu_n} q(0) \middle| \pi^+(p) \right\rangle$$

$$= \mathbf{Sym}\ 2 \sum_{i=0,even}^{n} A^q_{n,i}(t) \Delta^{\mu_1} ... \Delta^{\mu_i} P^{\mu_{i+1}} ... P^{\mu_n}$$

$$B^q_{12}(0) = \frac{10}{9} A^q_{20}(0) \qquad \text{etc.}$$

The case s_π, m_π small, can be treated by chPT in NLO.
M. Diehl, A. Manashov and AS, Phys. Lett. **B622**, 69 (2005), hep-ph/0505269.

$$B_{10}(s_\pi) = -B^{(0)}_{12} \left\{ 1 + c^{(m)}_{10} m^2_\pi + c^{(s)}_{10} s_\pi + \frac{m^2_\pi - 2s_\pi}{2\Lambda^2_\chi} \left[\ln \frac{m^2_\pi}{\mu^2_\chi} + \frac{4}{3} - \frac{s_\pi + 2m^2_\pi}{s_\pi} J(\beta) \right] \right\} + O\left(\Lambda^{-4}_\chi\right)$$

$$B_{12}(s_\pi) = B^{(0)}_{12} \left\{ 1 + c^{(m)}_{12} m^2_\pi + c^{(s)}_{12} s_\pi \right\} + O\left(\Lambda^{-4}_\chi\right),$$

The elastic region $4m_\pi^2 \leq s_\pi \leq 4m_K^2 \sim 1\mathrm{GeV}^2$, can be treated using Watson's theorem.

The $\mathcal{S}$-matrix elements in the 2π channel: $\eta_l e^{2\mathrm{i}\delta_l}$

The $\mathcal{T}$-matrix elements in the 2π channel: $\dfrac{\eta_l e^{2\mathrm{i}\delta_l} - 1}{2\mathrm{i}}$

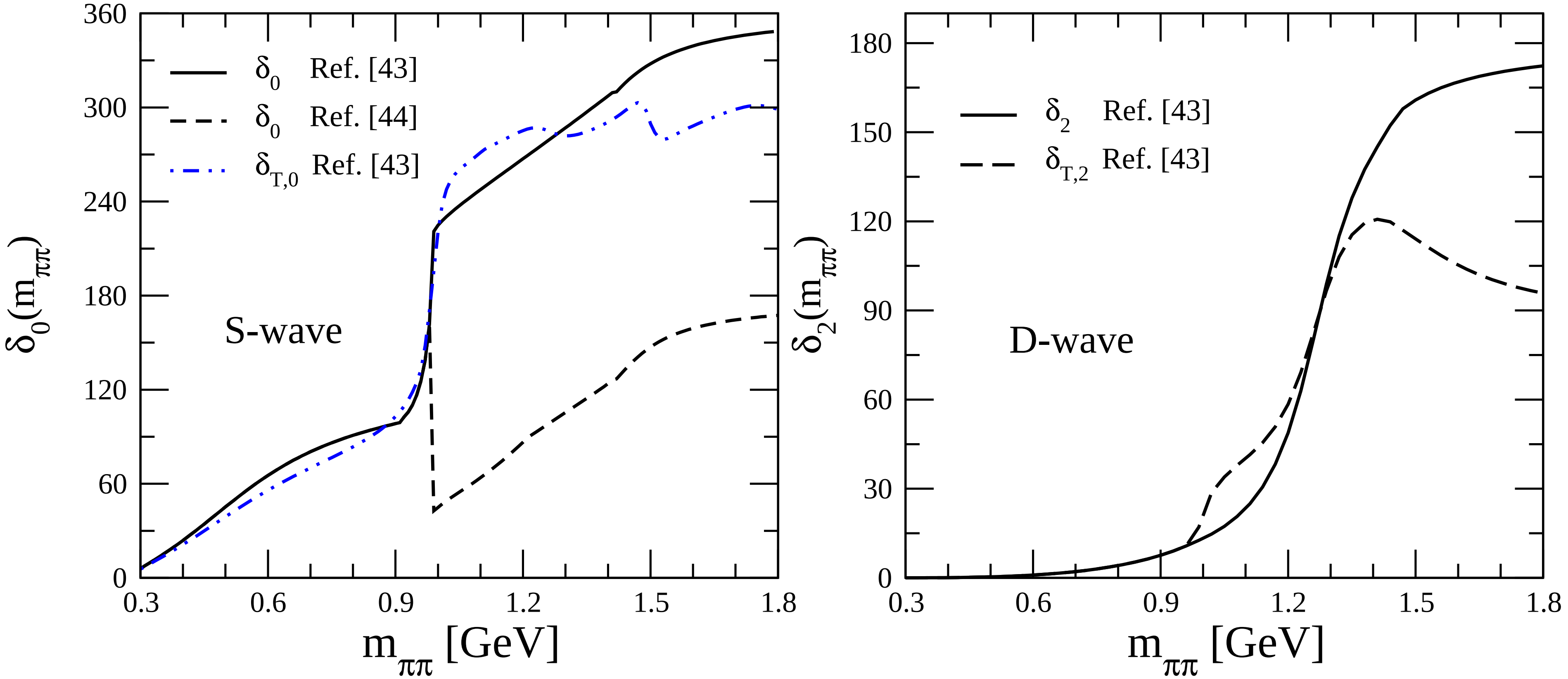
For elastic scattering $\eta_l = 1$ the phases of both are equivalent.

$$B_{10}(s_\pi) + B_{12}(s_\pi)\, P_2(2\zeta - 1) = \tilde{B}_{10}(s_\pi) + \tilde{B}_{12}(s_\pi)\, P_2(\cos\theta)\,,$$

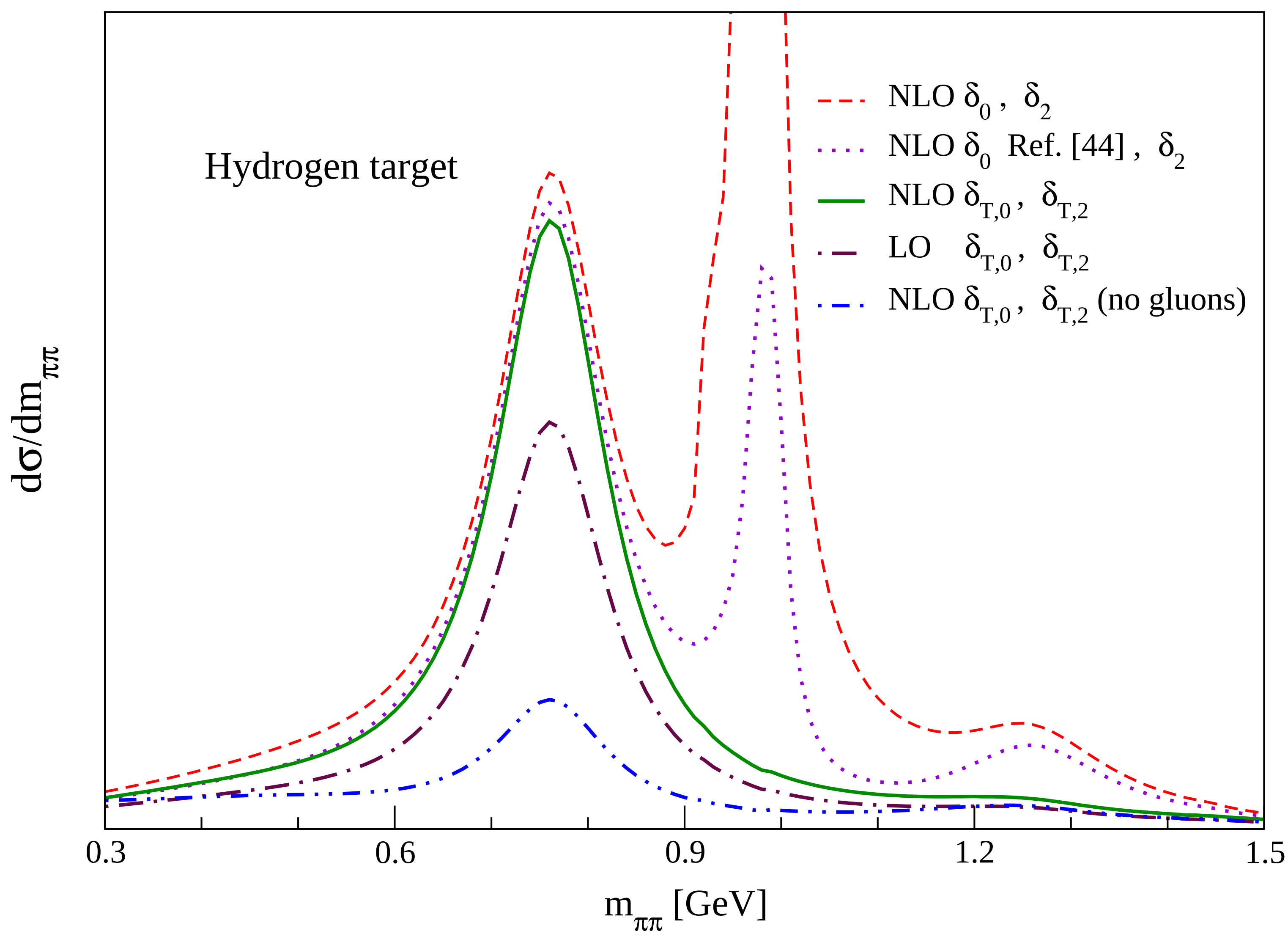
$$\Rightarrow \qquad \tilde{B}^*_{nl}(s_\pi) = \tilde{B}_{nl}(s_\pi)\exp\big[-2i\delta_l(s_\pi)\big]$$

For inelastic scattering $\eta_l < 1$ the situation becomes confusing.

The $\pi\pi$ phase shifts



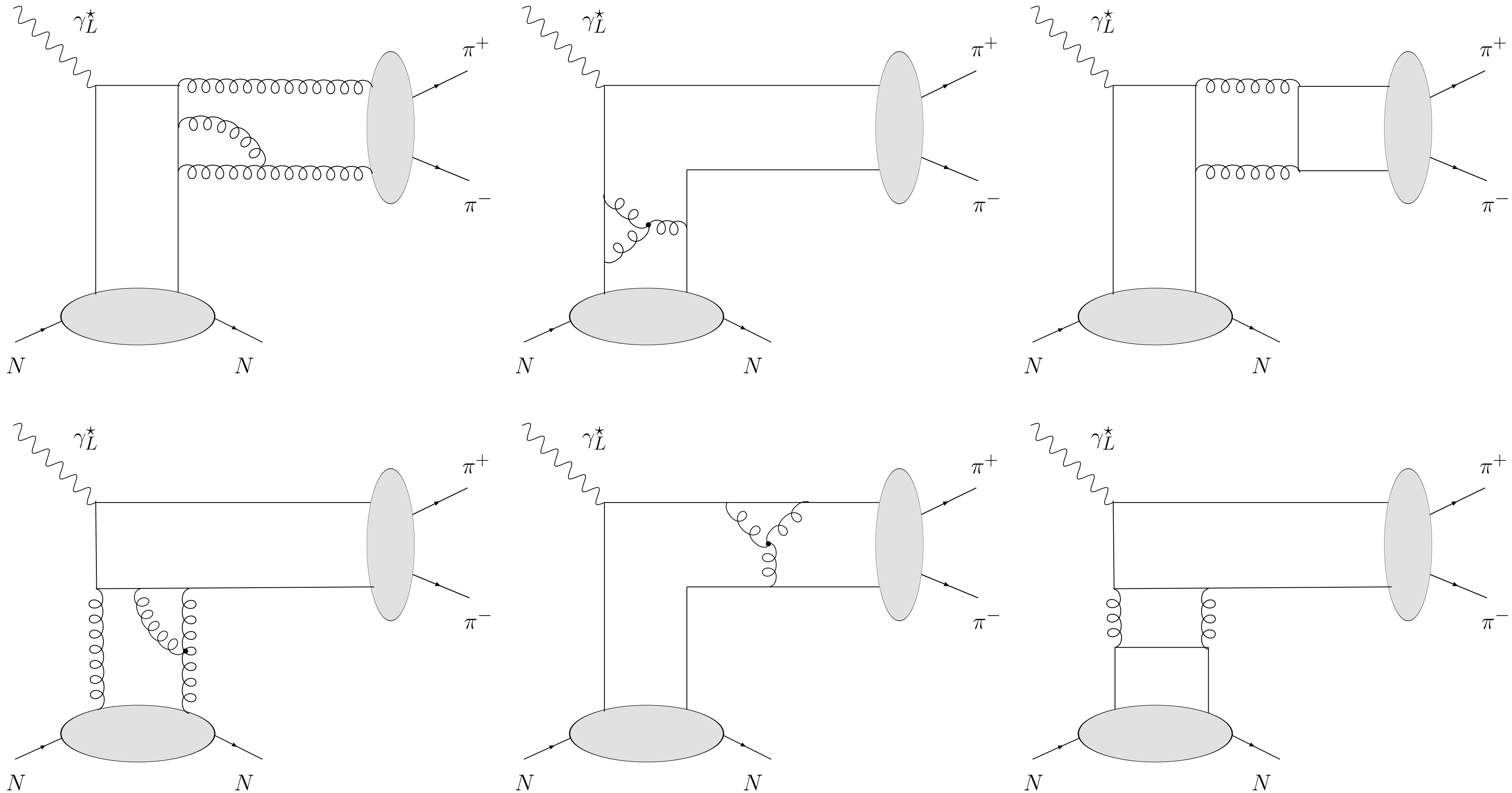
Phase shifts δ_l of $\pi\pi$ scattering in the isoscalar channel obtained by Kamiński et al. [43] and by Bugg [44]. $\delta_{T,l}$ is the $\mathcal{T}$-matrix phase. Our signal is primarily sensitive to the derivative.



Two-pion invariant mass spectrum (in arbitrary units) for $\gamma^* + p \to \pi^+\pi^- + p$

In the end we use a once subtracted dispersion integral representation for the s_π-dependence, for details see our paper.

The NLO hard cross section:



$\Rightarrow$ lengthy expressions, see paper

GPD parametrisation

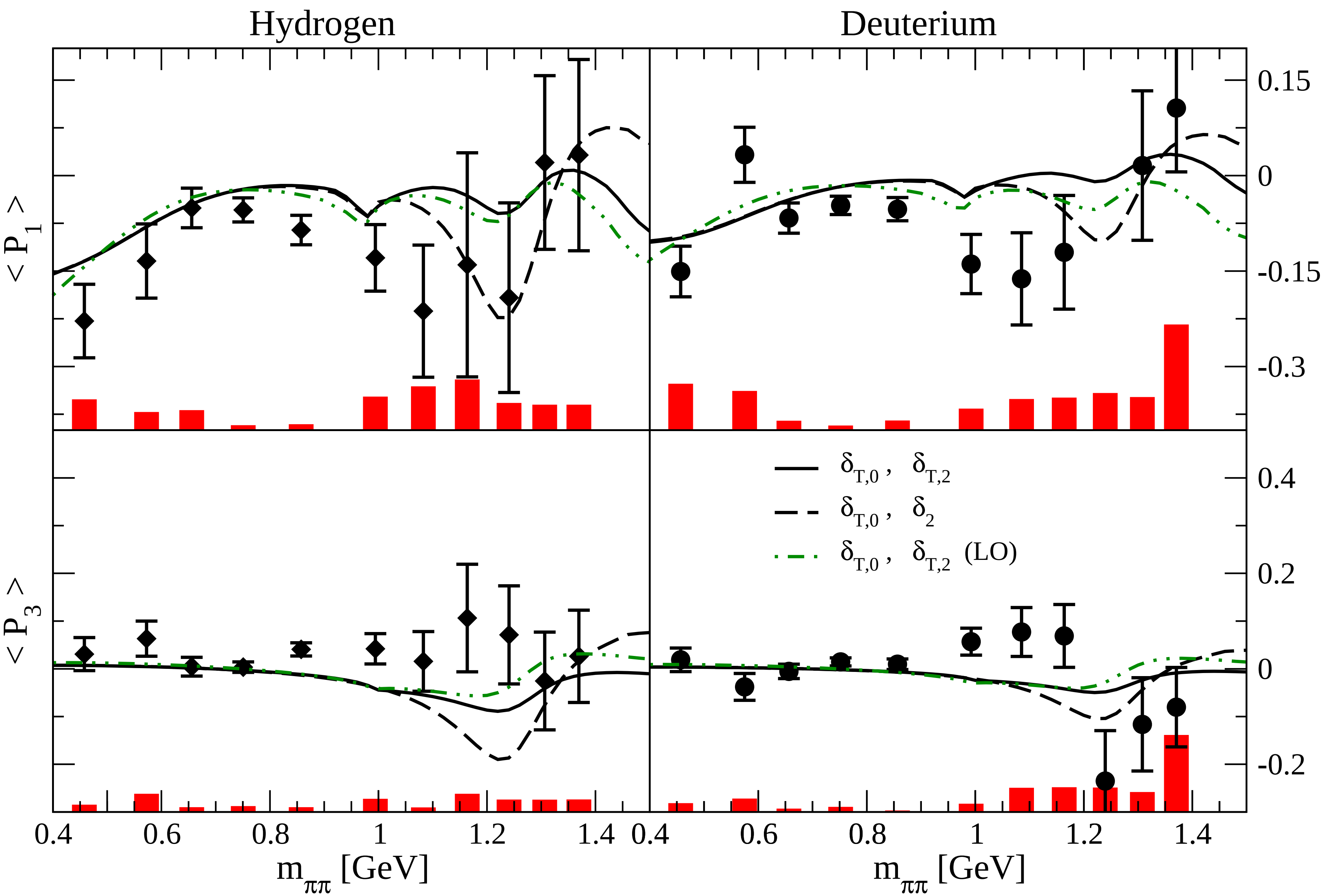
The precision of the HERMES data is insufficient to fit the GPDs. We use as place-keeper: A factorized Radyushkin's double distribution ansatz plus Polyakov-Weiss term.

$$
\begin{aligned}
H^q(x,\xi,0) &= H^q_{DD}(x,\xi,0) + \frac{1}{3}\theta\big(\xi - |x|\big)\, D\left(\frac{x}{\xi}\right) \\
H^q_{DD}(x,\xi,0) &= \int_{-1}^{1} dx' \int_{-1+|x'|}^{1-|x'|} d\alpha\, \delta(x - x' - \xi\alpha)\left[\theta(x')\, q(x')\; - \theta(-x')\, \bar{q}(-x')\right] \\
&\times\; h^q(x',\alpha)
\end{aligned}
$$

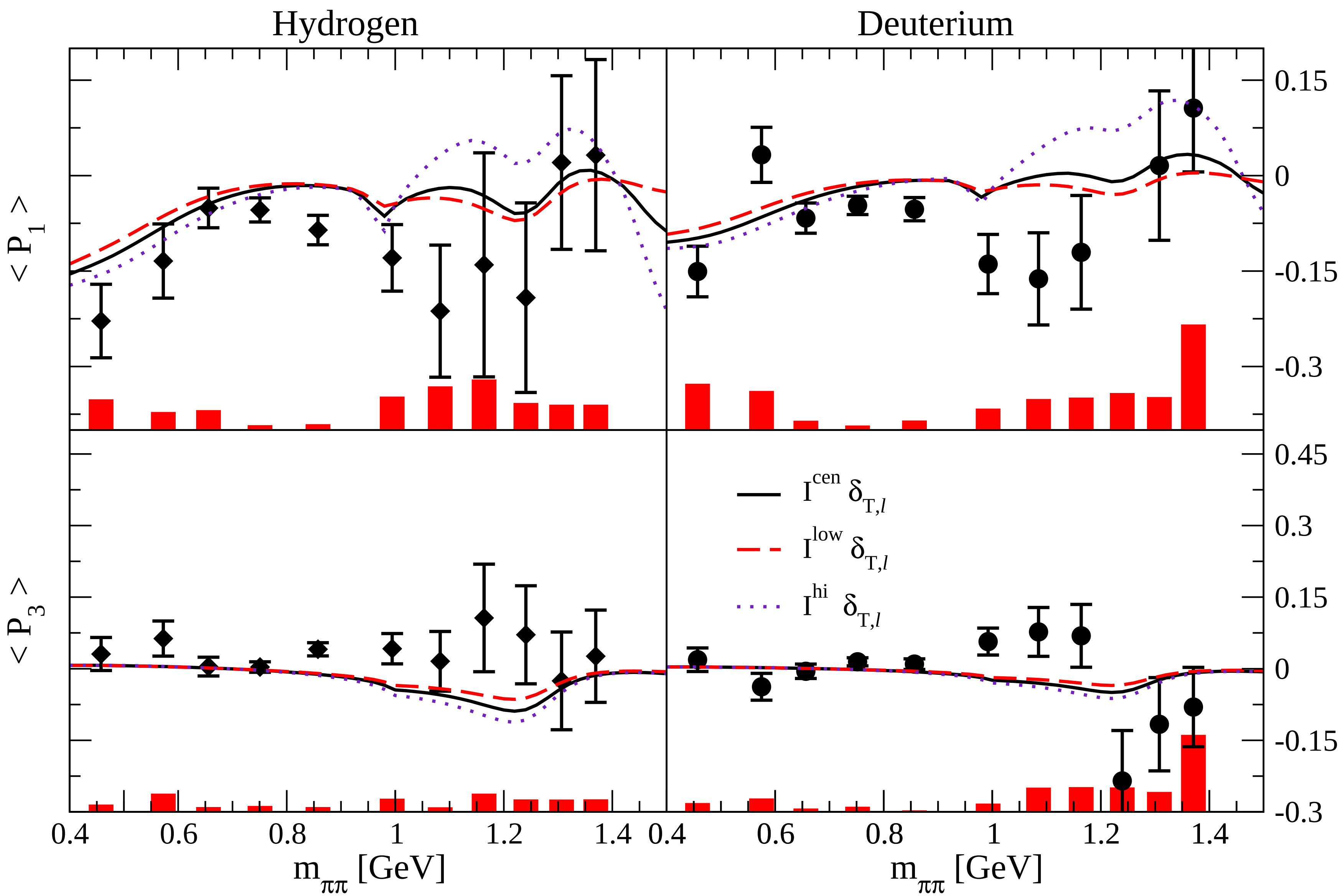
$$
h^q(x',\alpha) = \frac{3}{4}\frac{[(1-|x'|)^2 - \alpha^2}{(1-|x'|)^3}
$$

with MRST 2004 parton distributions and D from the quark-soliton model

$$
D(x) = -4.0\,(1-x^2)\left[C_1^{3/2}(x) + 0.3\, C_3^{3/2}(x) + 0.1\, C_5^{3/2}(x)\right]
$$



Legendre moments $\langle P_1 \rangle$ and $\langle P_3 \rangle$ for different two-pion phase shifts.



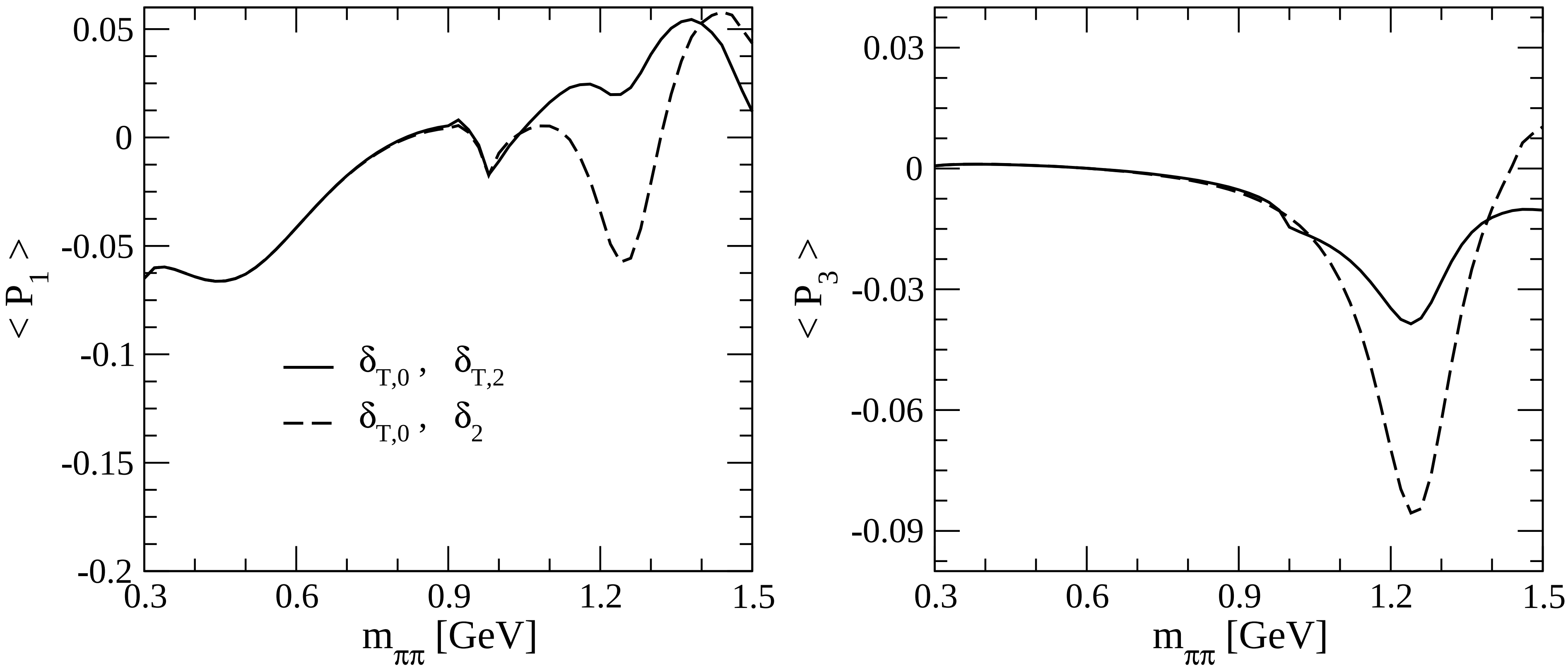
Legendre moments for $\delta_{T,l}$ and different choices for a free constant of our dispersion relation.

Interpretation:

• For s_π <1 GeV^2 things seem to be under control. More precise data is needed.

• For s_π >1 GeV^2 large uncertainties due to a lack of understanding of the 2π-system.

$\Rightarrow$ A nice topic for COMPASS-hadron + COMPASS-muon ?



Predictions for a deuterium target and COMPASS kinematics, $x_B = 0.08$,
$t = -0.27$ GeV^2 and $Q^2 = 7$ GeV^2.

DVCS in NNLO

• My firm belief: To achieve the precision needed for GPD physics onemust go to NNLO. Models with never reach the accuracy needed.

• However, NNLO for GPDs is much harder than for PDFs etc. New concepts are needed. D. Müller was able to solve this problem for DVCS (with a little help from some friends). Kumerički, Müller and Passek-Kumerički, hep-ph/0703179
This is real tour de force of far reaching importance !
Technically, however, it is too difficult for this talk. I shall only sketch a few elements.

Main Idea: Formulate OPE in a new manner, based on **conformal symmetry**, which you should anyway learn about in view of AdS/QCD.
A very good introduction to start with:
'The Uses of conformal symmetry in QCD.'
V.M. Braun, G.P. Korchemsky and Dieter Mueller
Prog.Part.Nucl.Phys.51:311-398,2003, hep-ph/0306057

Conformal symmetry

Poincaré group: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu \rightarrow ds^2$ $\quad g'_{\mu\nu} = g_{\mu\nu}$

Conformal group: $ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu \rightarrow ds^2$ $\quad g'_{\mu\nu}(x) = \omega(x) g_{\mu\nu}(x)$

$\omega(x)$ can be re-absorbed into x $\rightarrow$ scaling transformation, e.g. the special conformal transformation

$$x_\mu \quad \rightarrow \quad x'_\mu \; = \; \frac{x_\mu + a_\mu x^2}{1 + 2a \cdot x + a^2 x^2}$$

There are 15 generators:

4 translation operators:	P_μ	6 rotation/boost operators:	$M_{\mu\nu}$
1 dilatation operators:	D	4 special conformal operators:	K_μ

The corresponding orthogonal polynomials are the Jacobi polynomials $P_n^{k,k'}$.
At twist-2 level one needs only $P_n^{k,k} \sim$ Gegenbauer polynomials C_n^j.

$\overline{MS} \rightarrow \overline{CS}$

If QCD were a conformal theory, conformal operators would not mix under $Q^2 := -(q_1+q_2)^2/4$-evolution. In real life $\beta \neq 0$ and mixing occurs.
The 'modified conformal scheme': The Wilson coefficients:

$$C_j\Big(\frac{\eta}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s \mu\Big) = \sum_{k=j}^{\infty} C_k\Big(\frac{\eta}{\xi}, 1, \alpha_s Q\Big) \times \mathcal{P}\exp\Bigg\{\int_{Q^2}^{\mu^2} \frac{d\mu'}{\mu'}\Big[\gamma_j(\alpha_s(\mu'))\delta_{kj} + \Big(\frac{\eta}{\xi}\Big)^{k-j}\frac{\beta}{g}\Delta_{kj}(\alpha_s(\mu'))\Big]\Bigg\}$$

The $C_k\Big(\frac{\eta}{\xi}, 1, \alpha_s Q\Big)$ and $\gamma_j(\alpha_s(\mu'))$ can be obtained from the NNLO results of Vermaseren, Moch, Vogt, van Neerven. The Δ_{kj} are basically unknown, their calculation is a major outstanding task. However, for $\mu \approx Q$ and $\frac{\eta}{\xi} = \frac{\Delta \cdot q}{q^2}$ not too large the corrections should be small. This solves the basic problem in principle, but not in practice, as standard expansions in Gegenbauer polynomials etc. do not converge.

The Mellin-Barnes representation

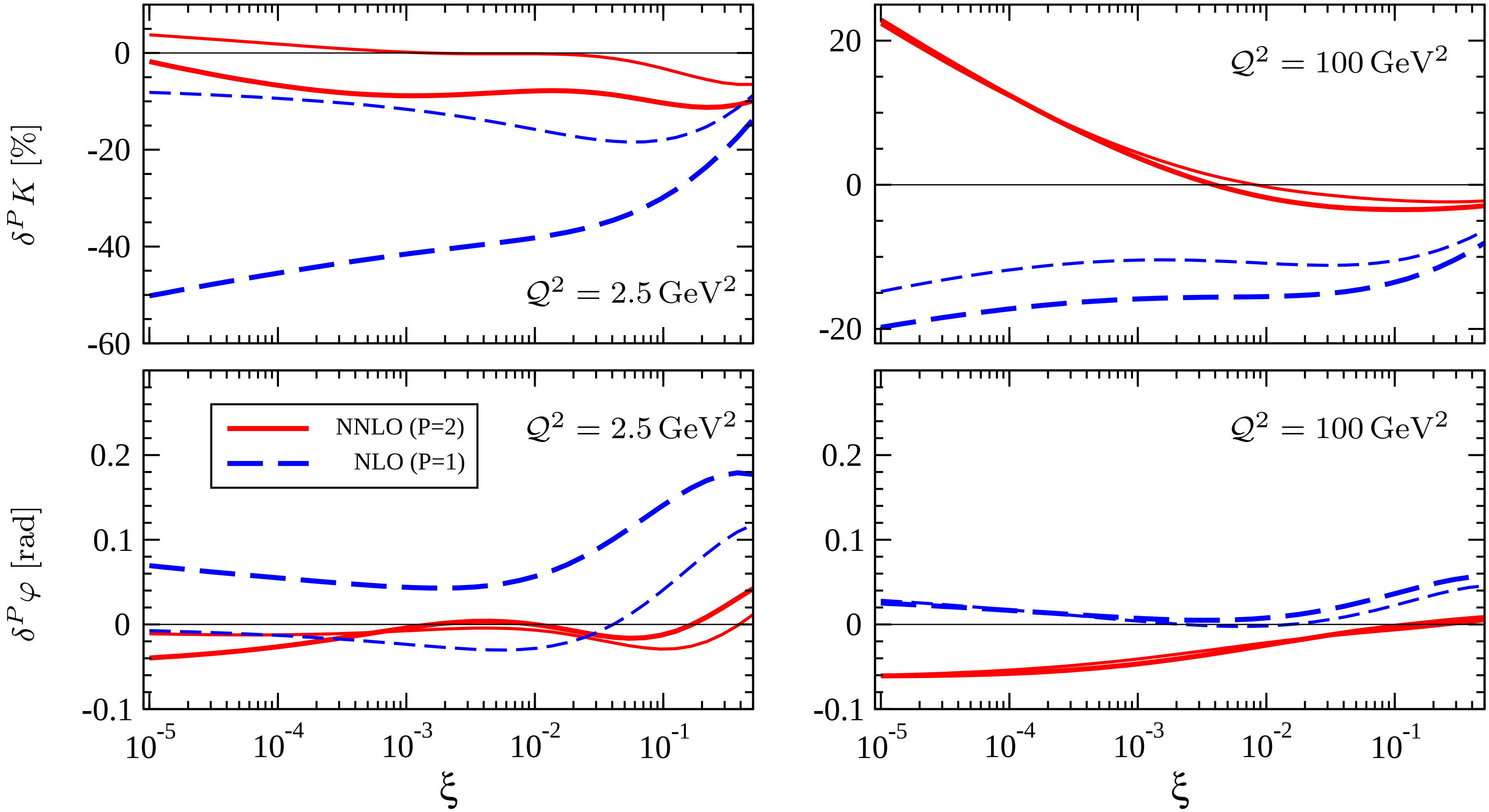
$\mathcal{F}(\xi, \eta/\xi =: \theta, \Delta^2, Q^2)$ stands for a generic Compton form factor, e.g. $\mathcal{H}$ and we suppress NS, $S+$, $S-$ indices.

$$
\begin{aligned}
\mathcal{F}(\xi, \theta, \Delta^2, Q^2) &= \frac{1}{2\mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} dj\; \xi^{-j-1} \left[\mathrm{i} \pm \left\{ \begin{matrix} \tan \\ \cot \end{matrix} \right\} \left(\frac{\pi j}{2} \right) \right] \mathcal{F}_j(\theta, \Delta^2, Q^2) \\
&+ \mathcal{C}(\theta, \Delta^2, Q^2) \\
\mathcal{C}(\theta, \Delta^2, Q^2)[\mathcal{H} + \mathcal{E}] &= 0 \\
\mathcal{C}(\theta, \Delta^2, Q^2)[\mathcal{E}] &= 2 \lim_{j \to -1} \left[\mathcal{E}_j(\theta, \Delta^2, Q^2) - R_j \mathcal{E}_j(\theta = 0, \Delta^2, Q^2) \right] \\
\lim_{j \to -1} R_j &:= \lim_{j \to -1} \frac{C_j(\theta, \Delta^2/\mu^2, \alpha_s(\mu))}{C_j(\theta = 0, \Delta^2/\mu^2, \alpha_s(\mu))} = 1 \\
\mu \frac{d}{d\mu} F_j &= -\gamma_j(\alpha_s(\mu)) F_j - \frac{\beta(\alpha_s(\mu))}{g(\mu)} \sum_{k=0}^{j-2} (\theta\xi)^{j-k} \Delta_{jk}(\alpha_s(\mu)) F_k
\end{aligned}
$$

Results

$$\Delta K(\mathcal{Q}^2 = -q_1^2, \mathcal{Q}_0^2) \;=\; \frac{\left|\mathcal{H}(\mathcal{Q}^2)\right|}{\left|\mathcal{H}(\mathcal{Q}_0^2)\right|} - 1$$

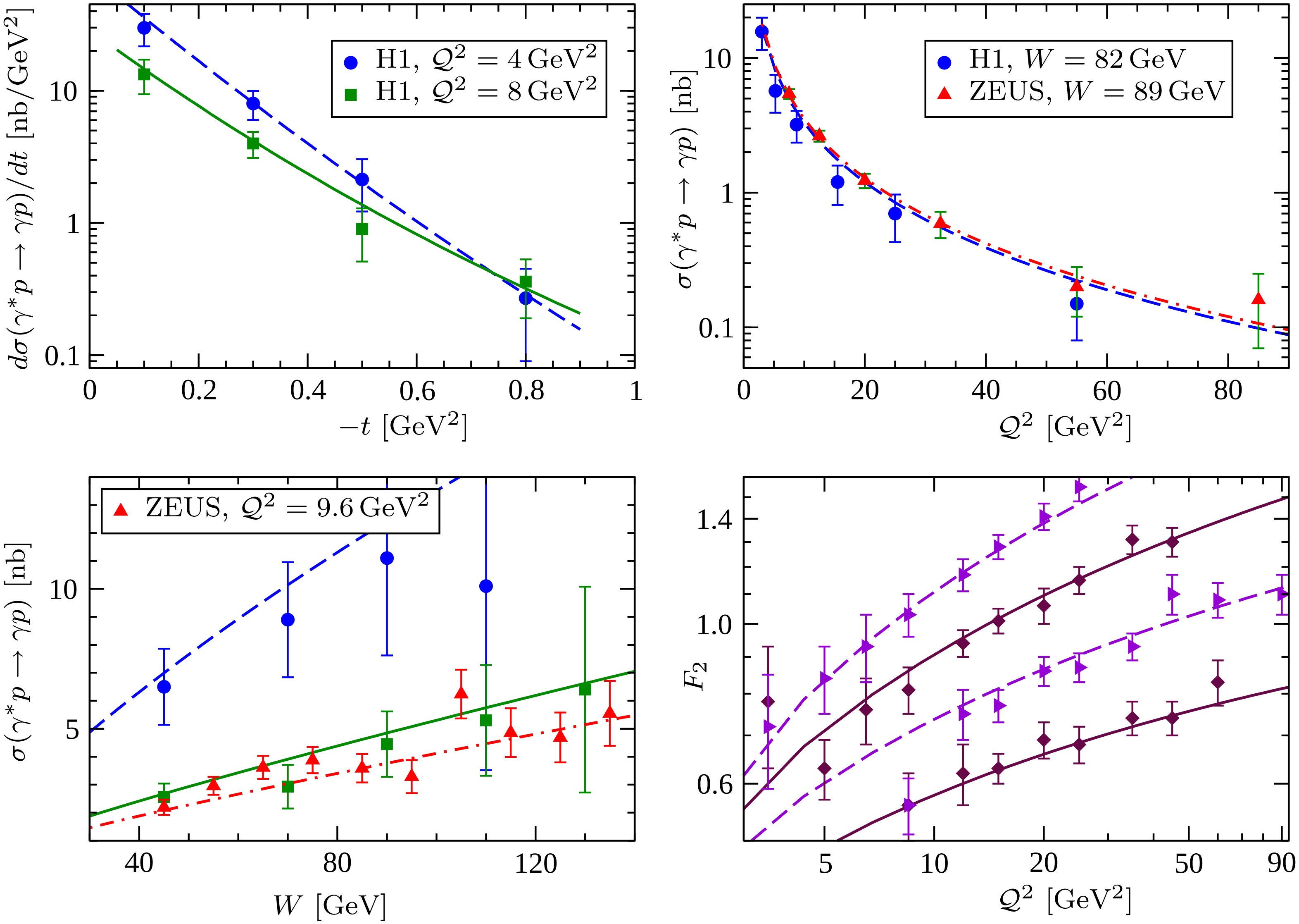
$$\Delta\phi(\mathcal{Q}^2, \mathcal{Q}_0^2) \;=\; \arg\left(\frac{\mathcal{H}(\mathcal{Q}^2)}{\mathcal{H}(\mathcal{Q}_0^2)}\right)$$



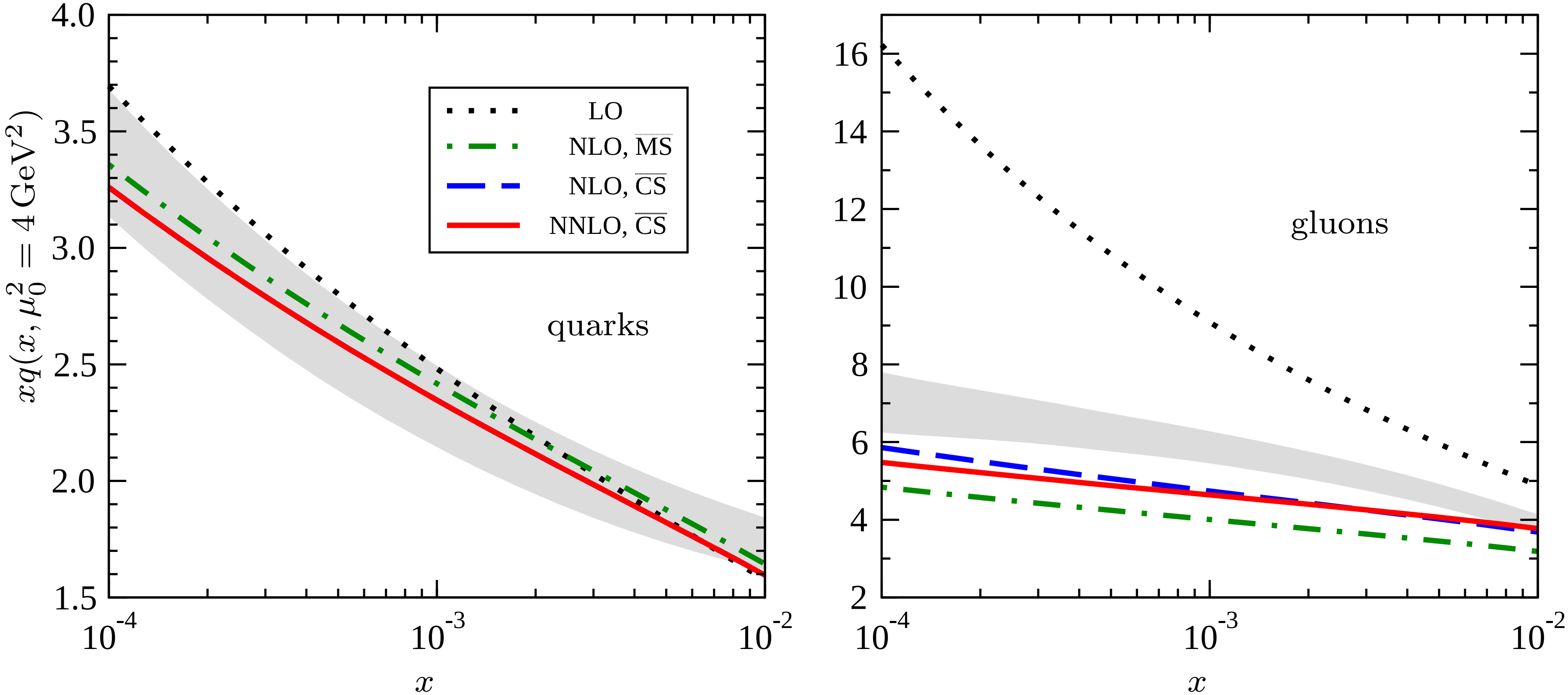
soft gluons: thin lines hard gluons: thick lines

The μ_r dependence of ${}^S\mathcal{H}$ for $\mu_r^2 = \mathcal{Q}^2/2 \leftrightarrow \mu_r^2 = 2\mathcal{Q}^2$ in percent

soft gluon	$\xi = 10^{-5}$	10^{-4}	10^{-3}	0.01	0.1	0.25	0.5
NLO	2.4	2.8	3.5	5.0	5.8	4.4	2.4
NNLO	-1.6	-0.6	0.3	0.6	2.2	3.5	3.7
hard gluon							
NLO	24.9	21.0	18.1	15.8	10.9	7.1	3.9
NNLO	3.4	5.6	6.5	5.7	6.7	6.9	5.9



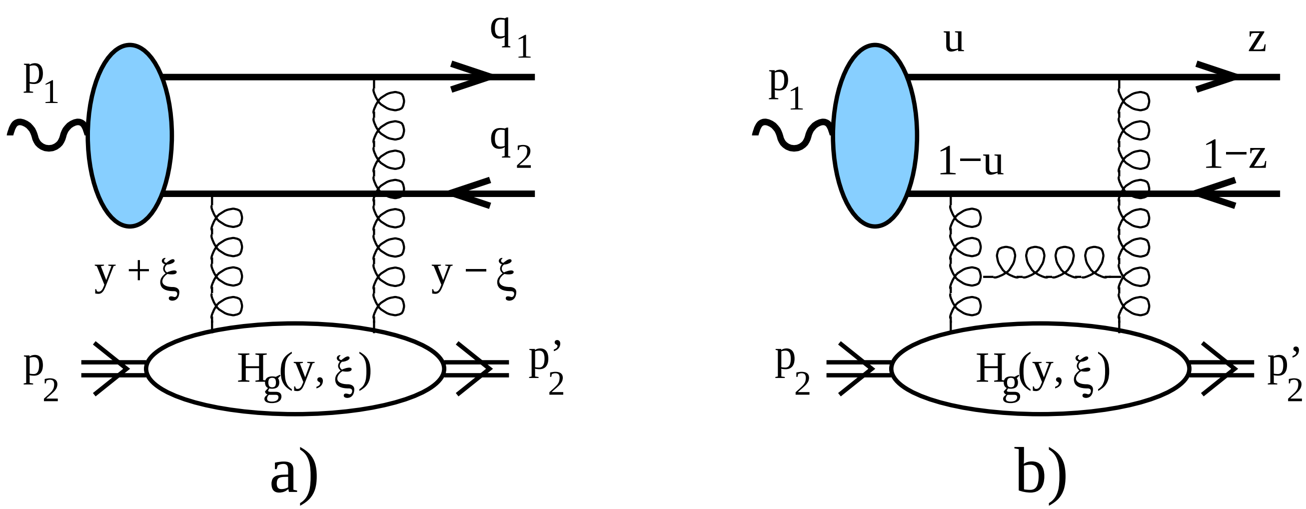
NNLO fit to DVCS and DISdata in the $\overline{CS}$ scheme. Lower right:
$x_{\mathrm{Bj}} = 8 \cdot 10^{-3}, 3.2 \cdot 10^{-3}, 1.3 \cdot 10^{-3}, 5 \cdot 10^{-4}$.



Resulting pdf's. The band is Alekhin's NLO fit.

Photon Diffractive Dissociation

V. Braun et al., PRL **89** (2002) 172001; hep-ph/0206305



Chirality Conserving contribution (CC):

$$\left\langle 0 \middle| \bar{q}(0)\gamma_+ \frac{1 \pm \gamma_5}{2} q(x_-, \mathbf{r}) \middle| \gamma^{(\lambda)}(q) \right\rangle$$

Chirality Violating contribution (CV):

$$\langle 0|\bar{q}(0)\sigma_{\alpha\beta}q(x)|\gamma^{(\lambda)}(q)\rangle = i\, e_q\, \chi\langle\bar{q}q\rangle \left(e_\alpha^{(\lambda)} q_\beta - e_\beta^{(\lambda)} q_\alpha\right) \int_0^1 du\, e^{-iu(qx)}\, \Phi_\gamma(u, \mu)$$

A very interesting channel, as χ is the magnetic susceptibility of the quark condensate (Ioffe, Smilga 1983).

$$\langle 0|\bar{q}\sigma_{\alpha\beta}q|0\rangle_F = e_q\,\chi\,\langle\bar{q}q\rangle\,F_{\alpha\beta}$$

CC and CV contributions do not interfere and CC contribution is of order $1/q_\perp^6$ while CV is of order $1/q_\perp^8$.

Everything looks fine, but it is not !

The Chirality Violating Amplitude

$$
\begin{aligned}
\mathcal{J}_{CV} &= -\frac{1}{\pi}\int\limits_{-1}^{1} dy \int\limits_{0}^{1} du \mathcal{H}_g(y,\xi)\frac{\Phi_\gamma(u)}{u\bar{u}}\Bigg\{C_F\left(\frac{2\xi}{(y-\xi+i\epsilon)^2}-\frac{1}{y-\xi+i\epsilon}\right) \\
&+ \left(C_F\left(\frac{z\bar{z}}{u\bar{u}}+1\right)\right. \\
&+ \left.\frac{1}{2N_c}\left(\frac{z}{u}+\frac{\bar{z}}{\bar{u}}\right)\right)\frac{z\bar{u}+u\bar{z}}{y(z-u)-\xi(z\bar{u}+u\bar{z})+i\epsilon} \\
&- \left(C_F\frac{z\bar{z}}{u\bar{u}}+\frac{1}{2N_c}\left(\frac{z}{u}+\frac{\bar{z}}{\bar{u}}\right)\right)\frac{1}{(y-\xi-i\epsilon)}\Bigg\}
\end{aligned}
$$

is logarithmically divergence at the end-points $u \to 0, 1$

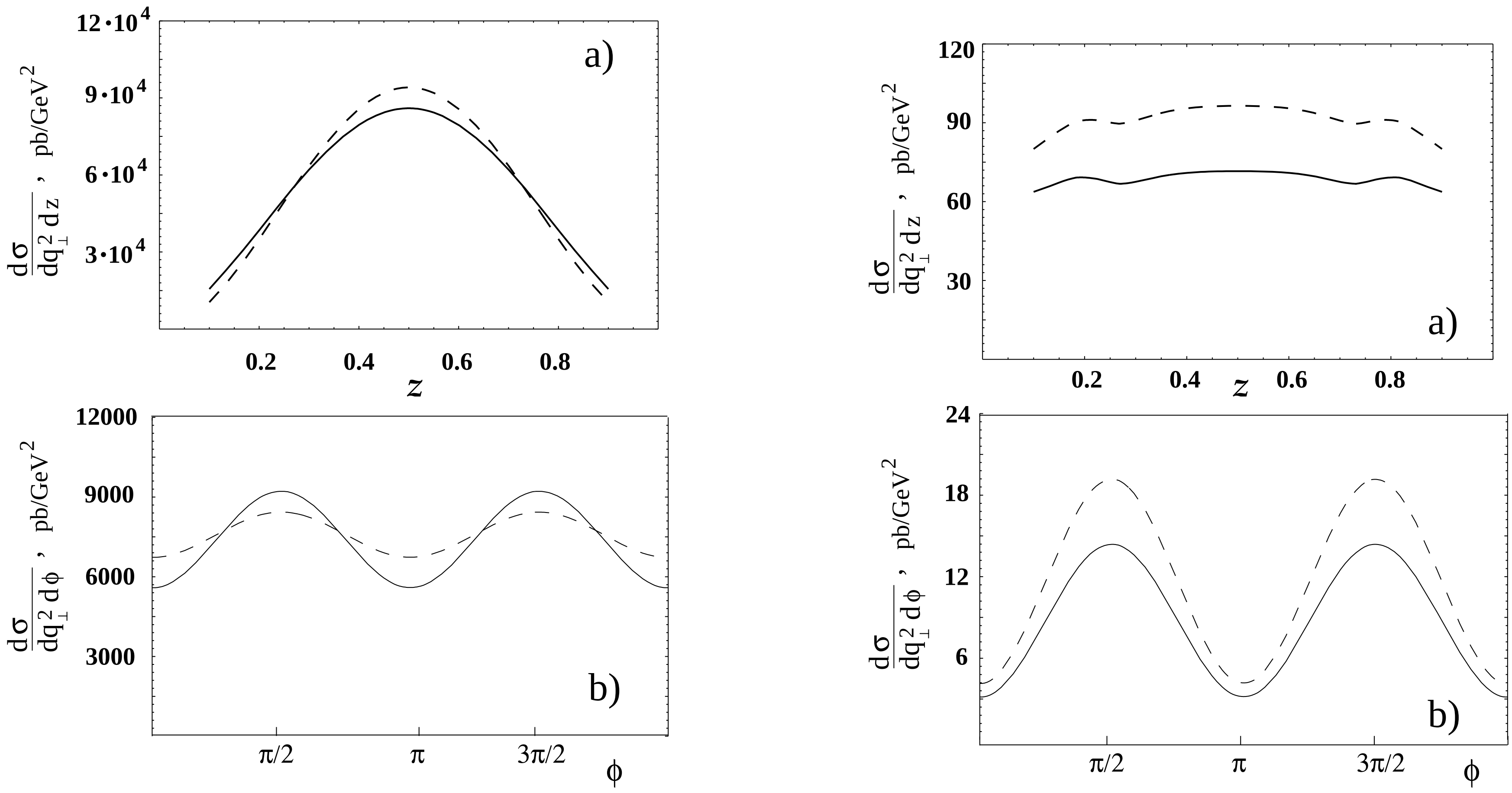
$\longrightarrow$ collinear factorization is violated !

In this case things can be patched up with an IR-cutoff $u_{\min} = \mu_{\mathrm{IR}}^2/q_\perp^2$

$$\mathcal{J}_{CV}^{IR} = 2i\mathcal{H}_g(\xi,\xi)\left(N_c z\bar{z} + \frac{z^2+\bar{z}^2}{2N_c}\right)\int_{u_{\min}}^1 \frac{du}{u^2}\,\Phi_\gamma(u)$$

Also, the z and ϕ dependence is characteristically different for CV and CC, which allows to disentangle them phenomenologically. ϕ is the angle between the photon polarization and the transverse jet momentum.

However, the lesson to learn is that 'physical intuition' can be very misleading.



solid: $\mu_{IR} = 500$ MeV; dashed $\mu_{IR} = 350$ MeV

$q_\perp = 2$ GeV CV dominates

$q_\perp = 5$ GeV CC dominates

Conclusions

• GPDs are a fascinating topic. Unfortunately I did not have the time to speak about their rich phenomenology. $\longrightarrow$ P. Hägler's and P. Kroll's talks.

• Over the last ten years very substantial theoretical progress was made in our understanding of GPDs: pQCD in NLO and NNLO, lattice QCD, chPT, dispersion integral analysis, ...

• The message is mixed: Some reactions get under control, like DVCS, some need further phenomenological input, like exclusive $\pi^+ - \pi^-$ electro production, some are problematic, like photon diffraction, and some are a complete mess, like exclusive J/ψ production.

• There is no alternative to state-of-the-art QCD calculations for each reaction channel, to find out.