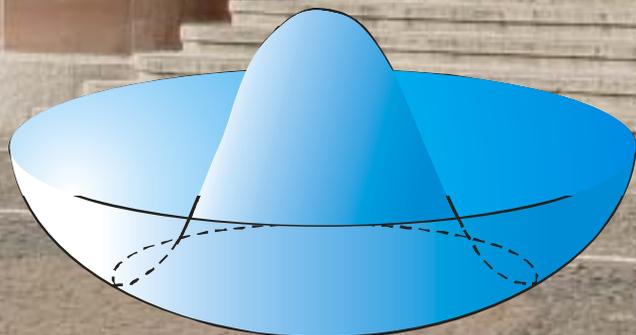


Hadron Structure and Dynamics at the QCD Scale



Freiburg 2007

Low energy hadron physics



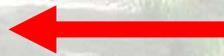
Ancient history

LHC collisions



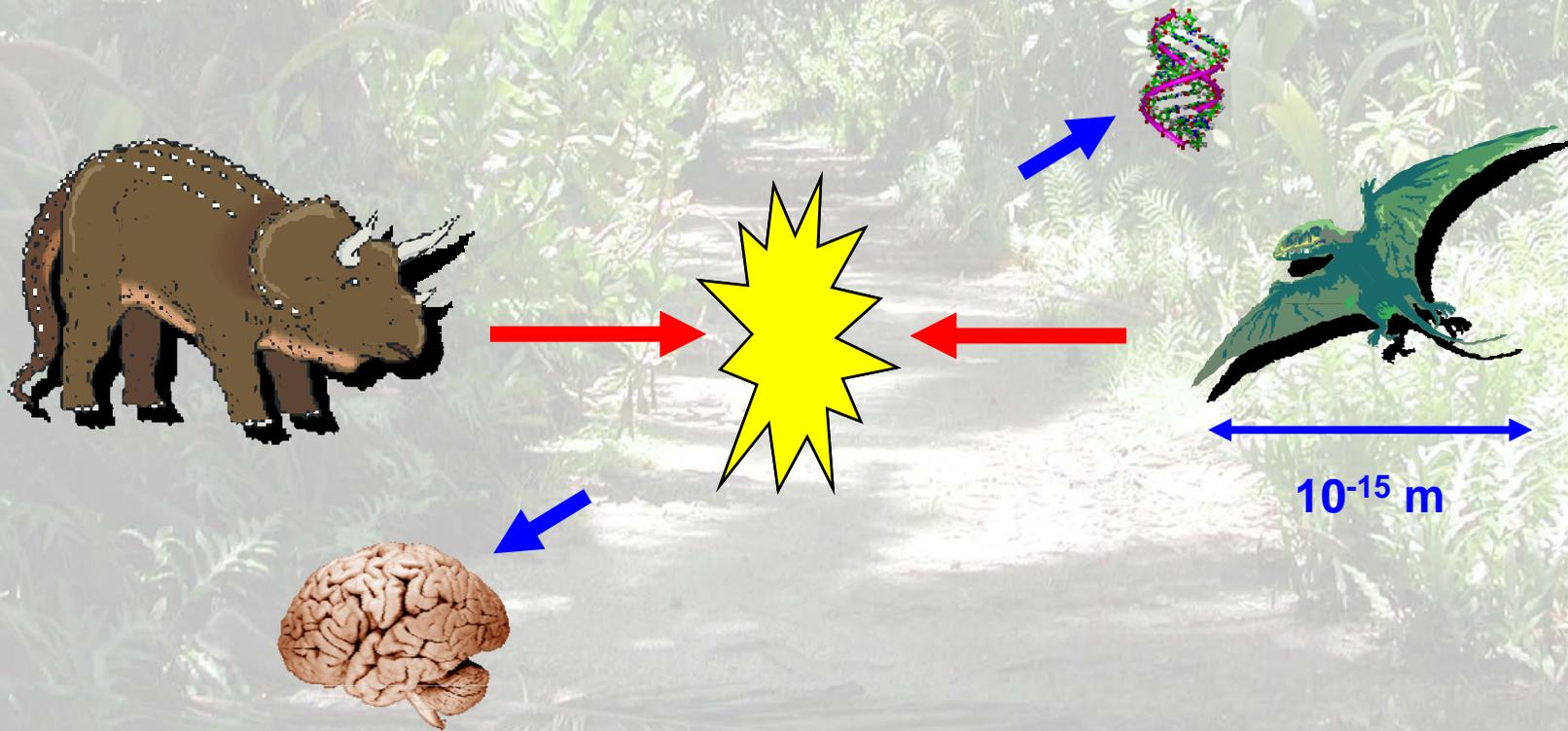
10^{-15} m

SPS collisions

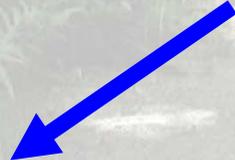
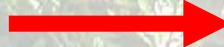


10^{-15} m

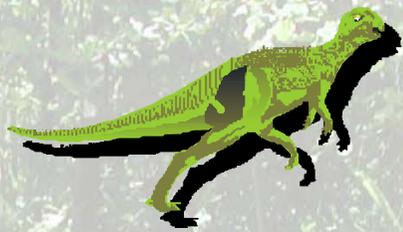
LHC collisions



SPS collisions



10^{-15} m



QCD

1971

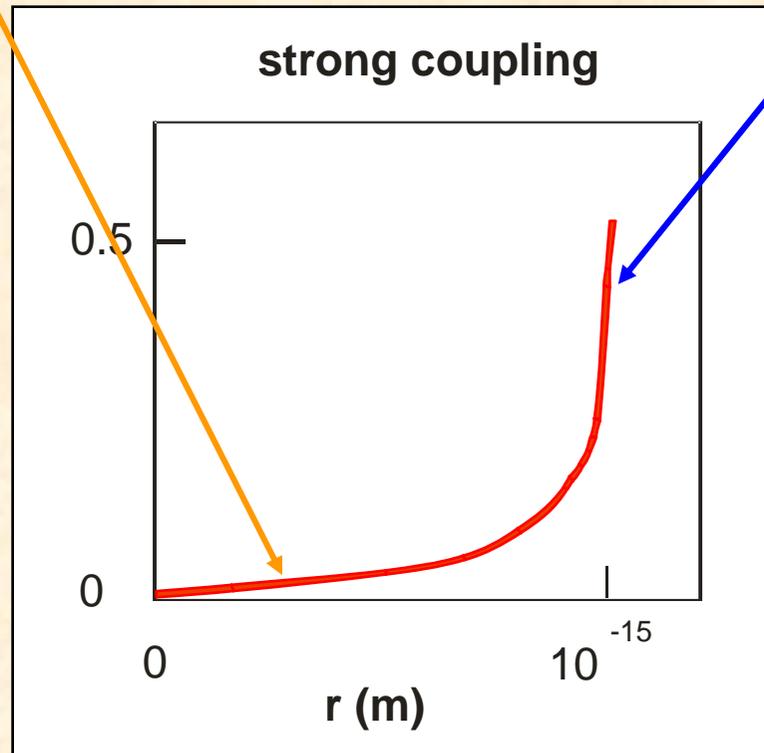
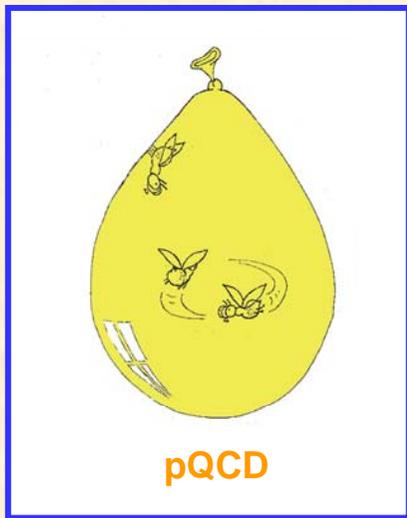


$$\mathcal{L}_{\text{QCD}} = \sum_{q=u,d,s,c,b} \bar{q} (i\gamma_{\mu} D^{\mu} - m_q) q - \frac{1}{4} G^{\mu\nu} G_{\mu\nu}$$

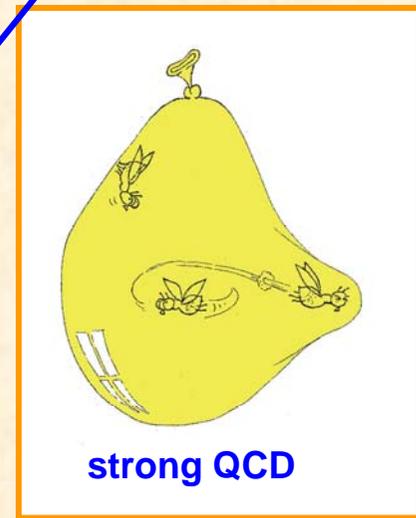


QCD

asymptotic freedom

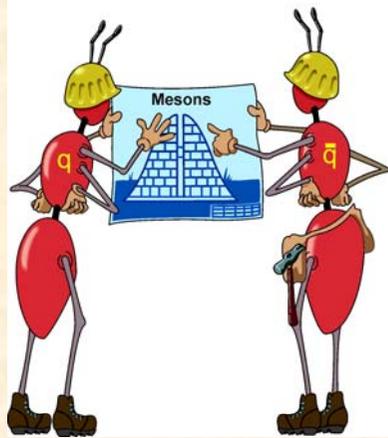


confinement

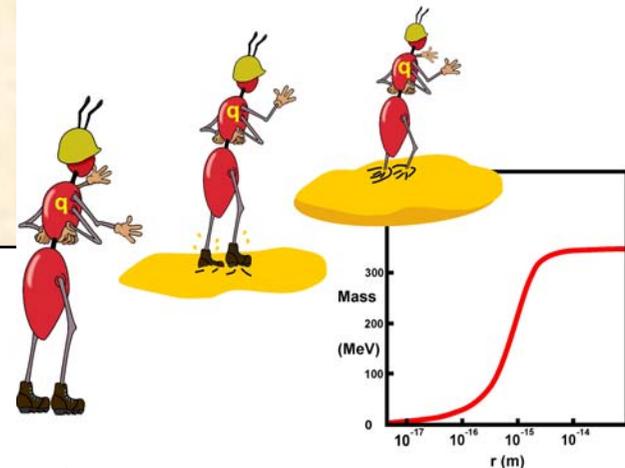


Strong physics problems

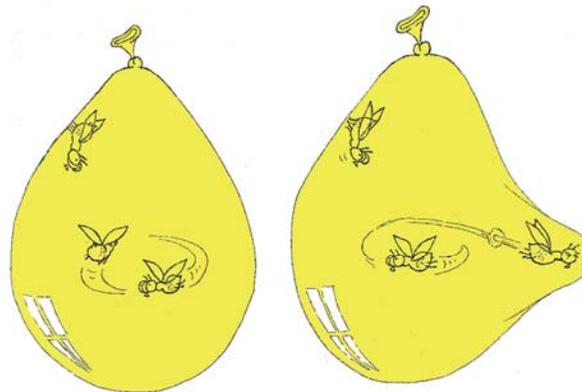
bound states



mass generation



confinement



perturbative QCD

non-perturbative QCD

Hadron masses ²

Mass²
(GeV²)

1.0

0.5

0

π



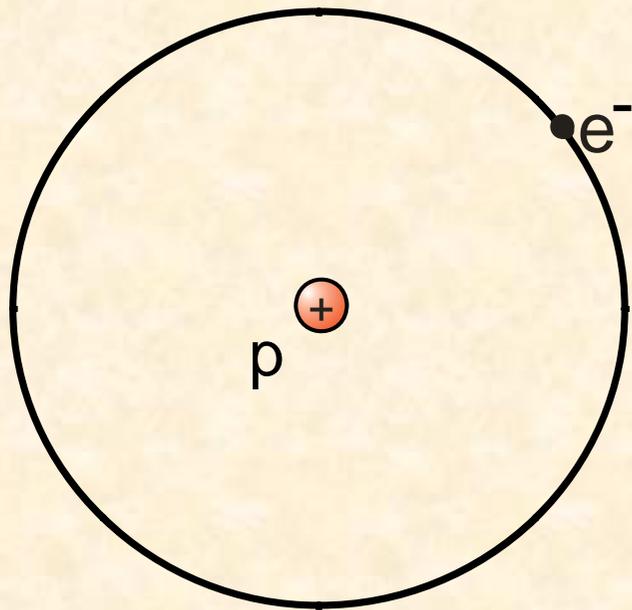
ρ



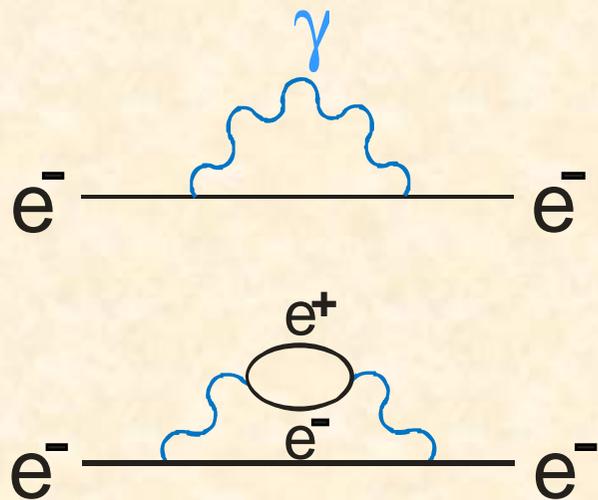
N



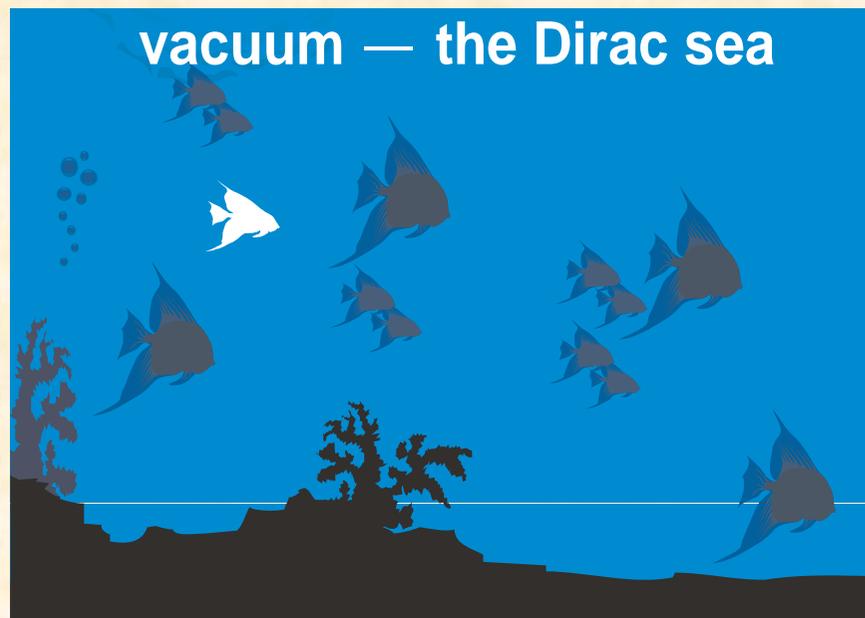
Hydrogen Atom



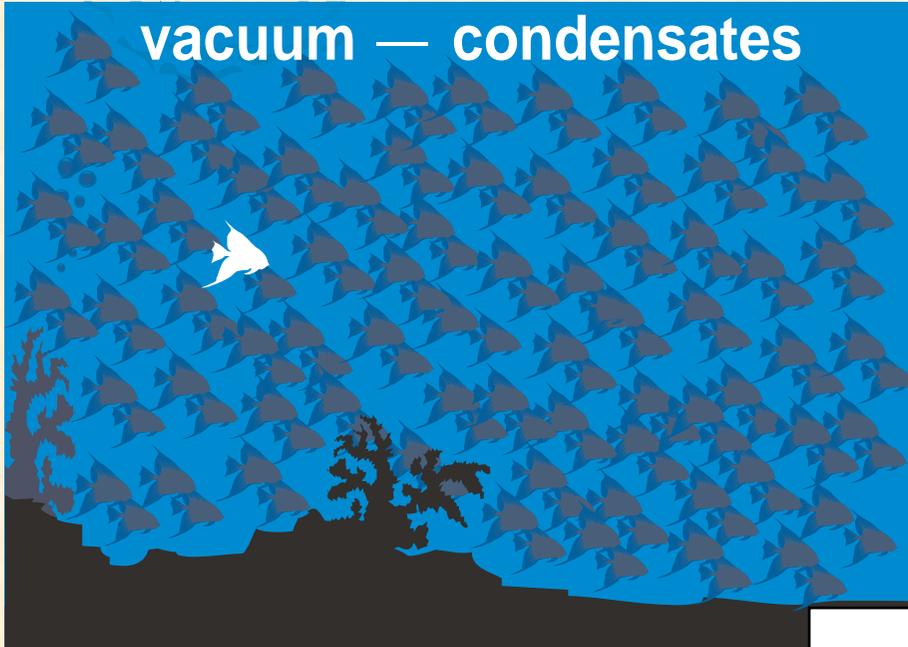
vacuum



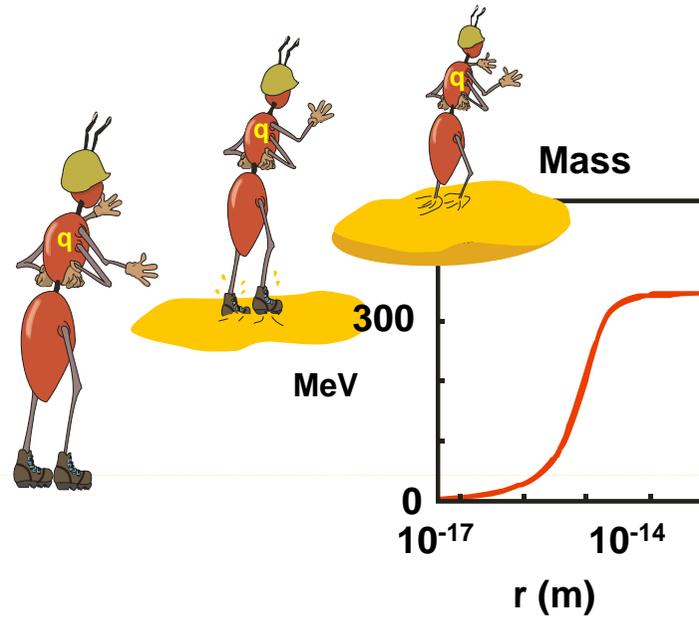
vacuum — the Dirac sea



vacuum — condensates



u/d quarks propagating



quarks, gluons

hadrons

QCD

effective Lagrangian

SU(N_f)

SU(N_f)

$$\mathcal{L}_{\text{QCD}} - \sum_{q=u,d} \bar{q} (i \gamma_{\mu} D^{\mu} - m_q) q$$

$$m_u, m_d \approx 1-5 \text{ MeV} \ll \Lambda_{\text{QCD}}$$

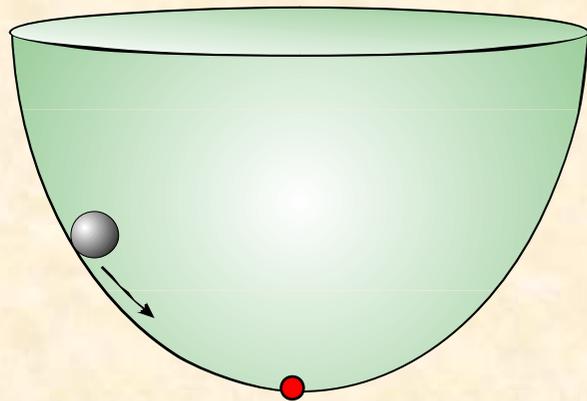
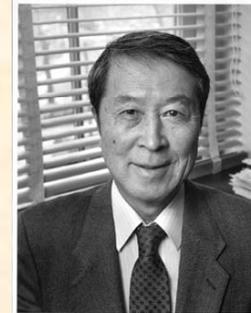
$$- \frac{1}{4} G^{\mu\nu} G_{\mu\nu}$$

SU(N_f) x SU(N_f)

SU(N_f) x SU(N_f)

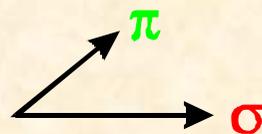
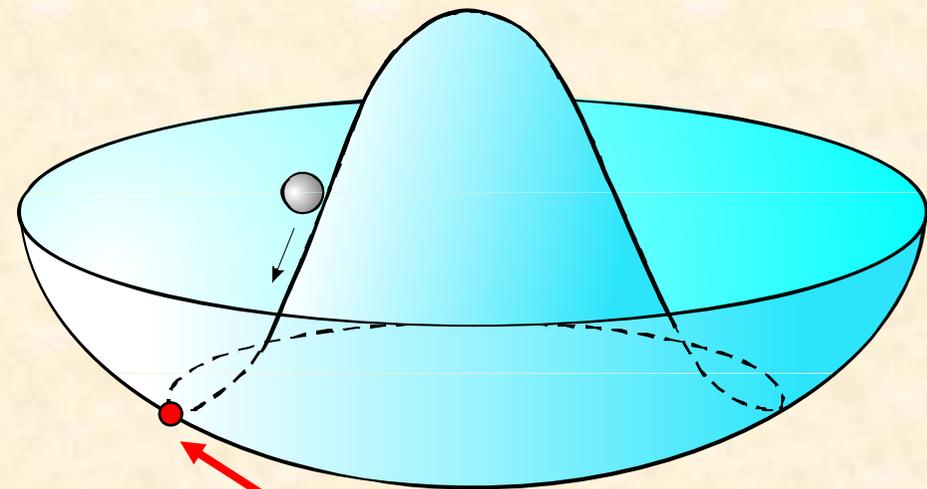
spontaneous χ SB

Ground State – Vacuum



vacuum

Chiral symmetry breaking



vacuum

quarks, gluons

hadrons

QCD

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SU(N_f) x SU(N_f)

SU(N_f) x SU(N_f)

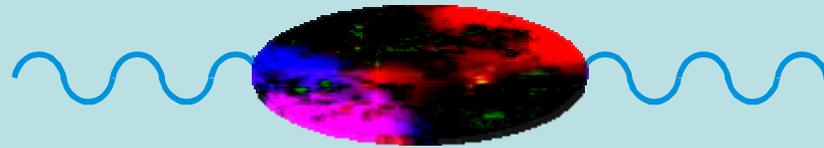
dynamical χ SB

spontaneous χ SB

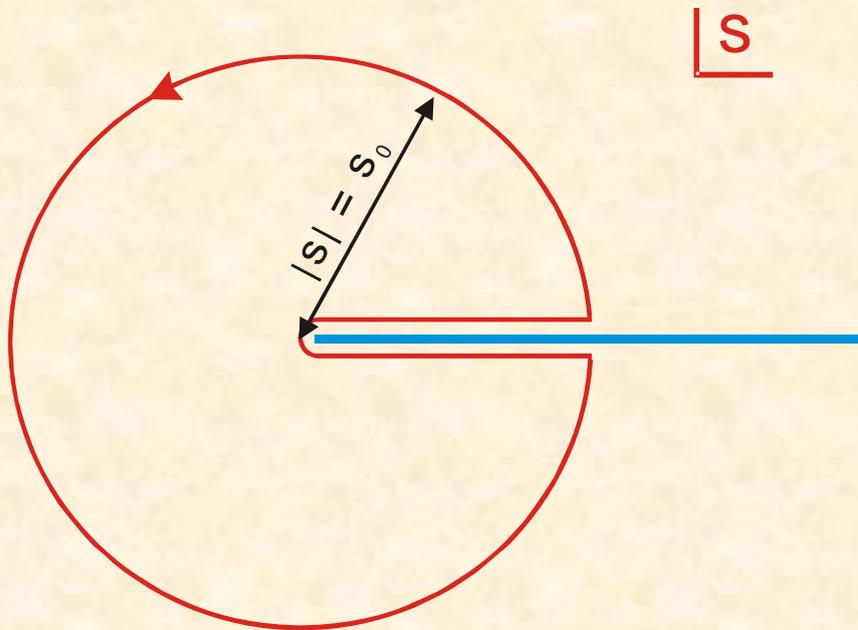
$\langle q \bar{q} \rangle, \langle q G \bar{q} \rangle$

$\langle \sigma \rangle$

QCD sum rules

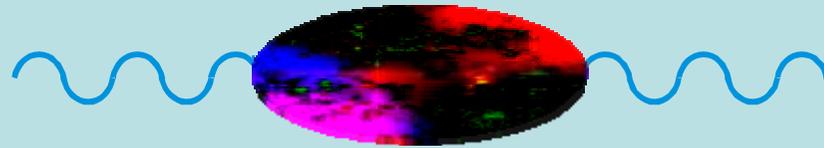


current correlator

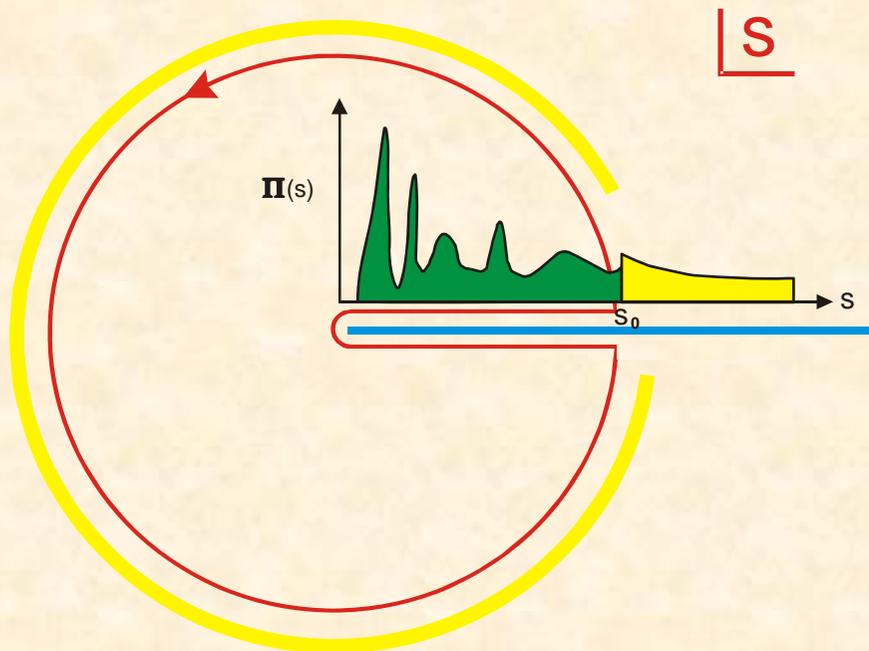


$$\oint ds \omega(s) \Pi(s) = 0$$

QCD sum rules



current correlator



$$\langle \bar{q}q \rangle_0 \sim - (240 \text{ MeV})^3$$

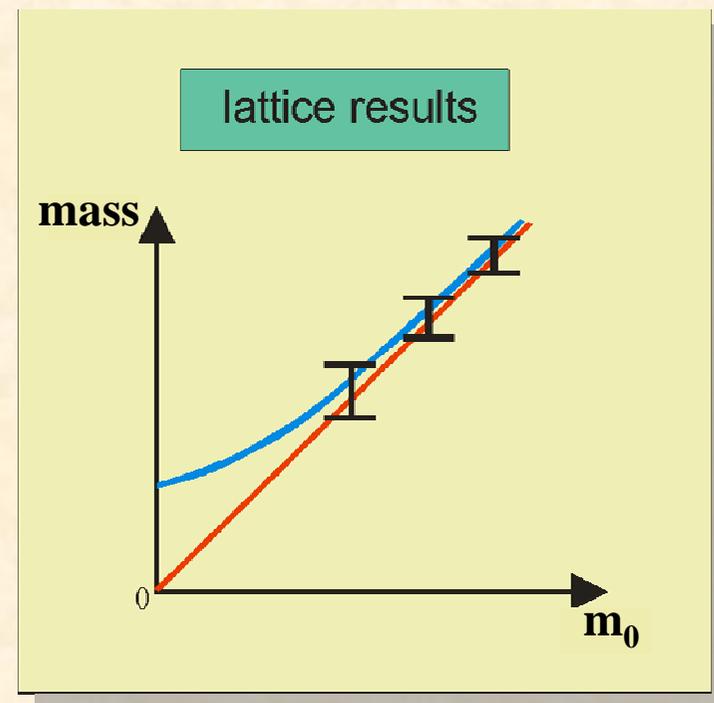
Masses from Nothing



perturbative

$$m(p) = m_0 \left[1 + C \alpha \ln p^2 / \mu^2 + O(\alpha^2) \right]$$

$m(p) \neq 0$ when $m_0 \rightarrow 0$?



masses without explicit Higgs

Schwinger-Dyson Equations

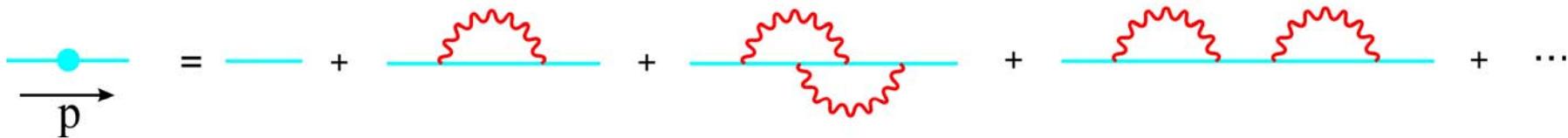
$$S_F(p) = S_F^0(p) + S_F^0(p) \Sigma(p) S_F(p)$$



$$S_F(p)^{-1} = S_F^0(p)^{-1} - \Sigma(p)$$

Schwinger-Dyson Equations

$$S_F(p) = S_F^0(p) + \dots$$



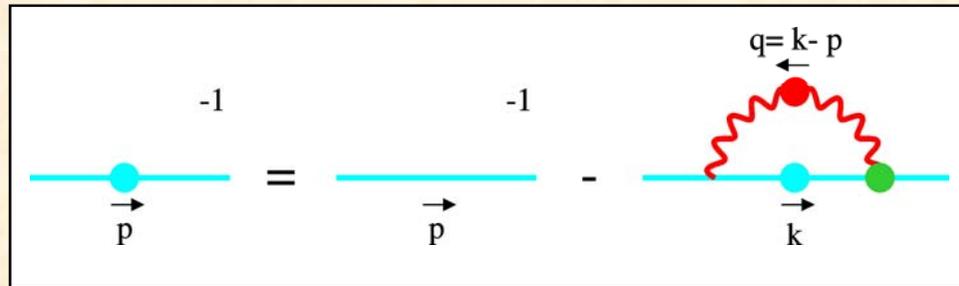
$$\Sigma(p) \equiv \text{[Diagram of a self-energy loop: a blue line with a blue dot, a red wavy loop with a red dot, and a blue line with a green dot.]}$$



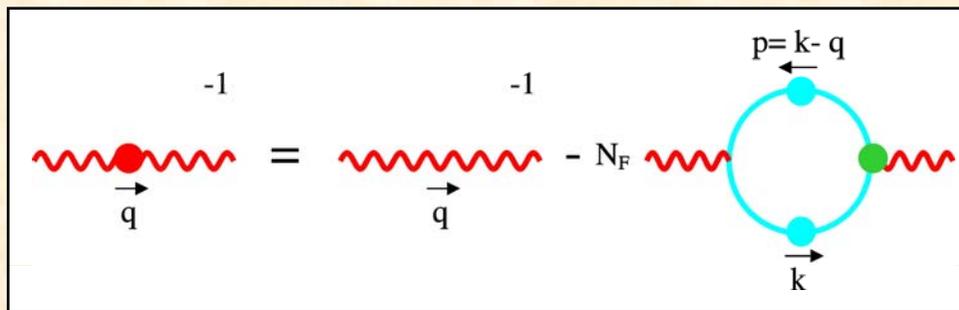
$$S_F(p) = S_F^0(p) + S_F^0(p)\Sigma(p)S_F^0(p) + S_F^0(p)\Sigma(p)S_F^0(p)\Sigma(p)S_F^0(p) + \dots$$

Schwinger-Dyson Equations

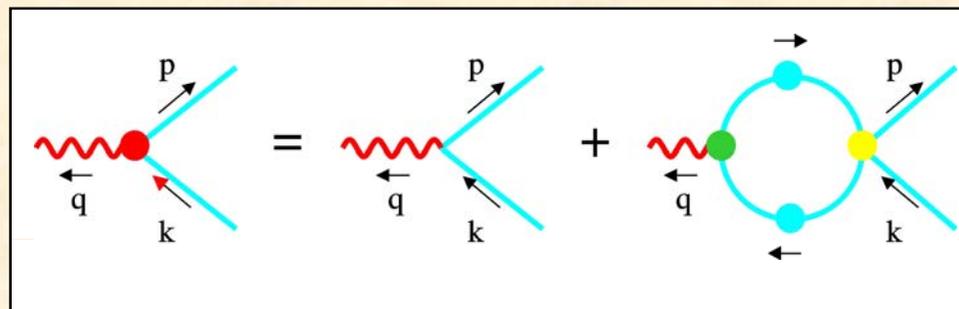
QED



2 equations

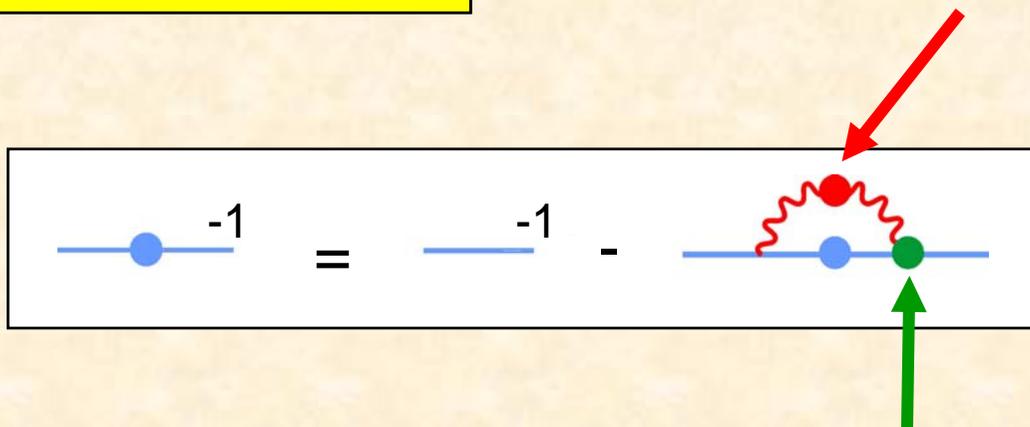


2 equations



12 equations

Fermion Schwinger-Dyson Equation



Fermion Schwinger-Dyson Equation

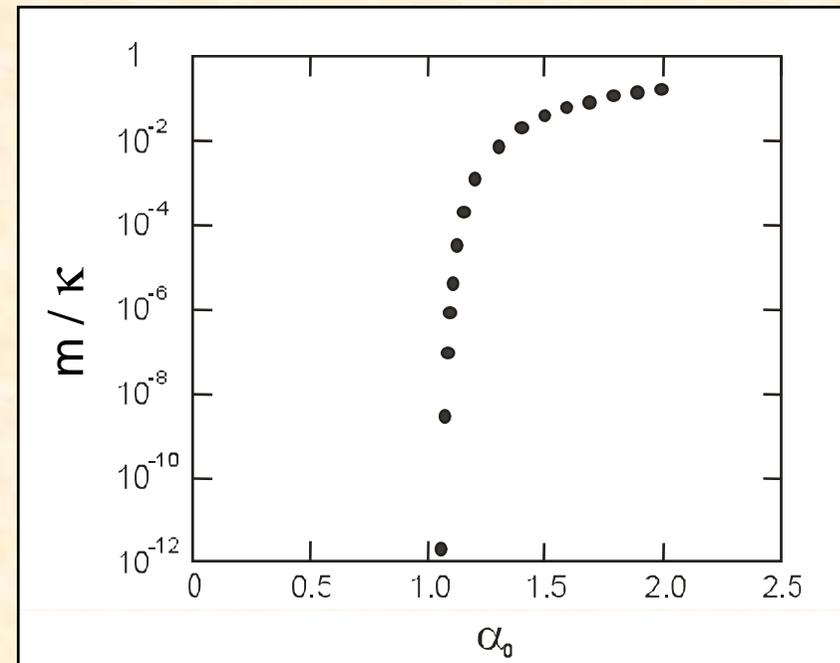
quenched
rainbow

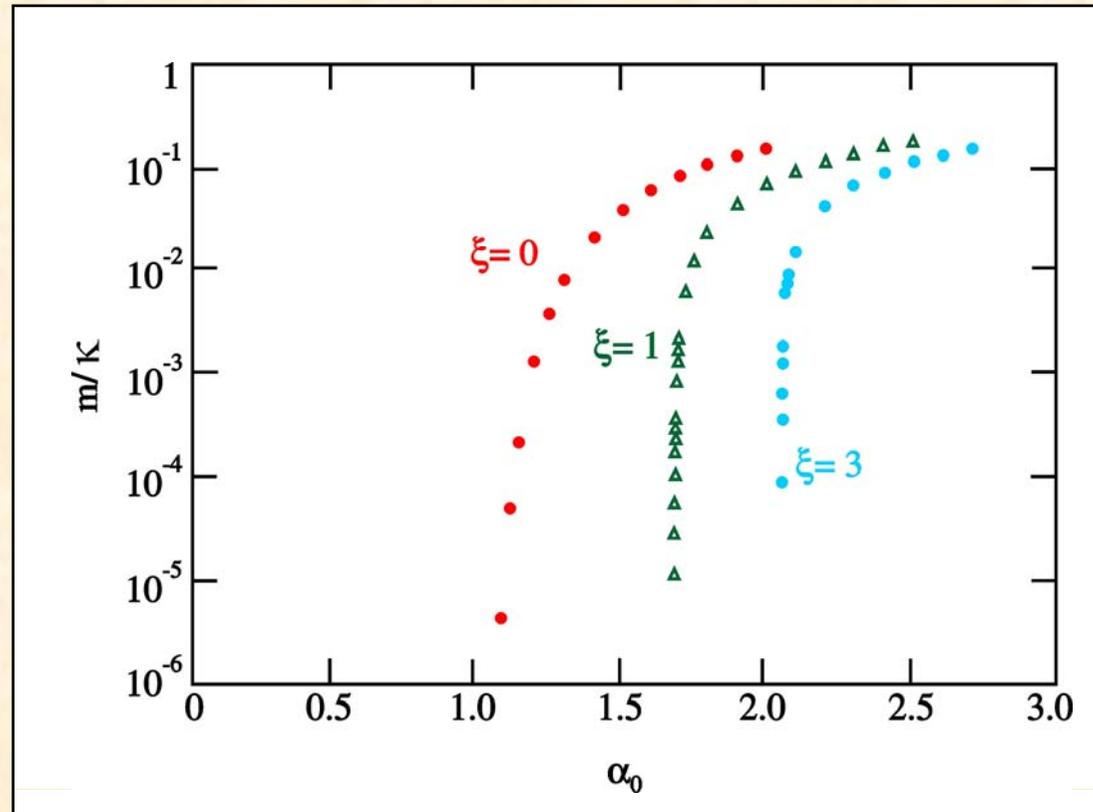
$$\text{Feynman Diagram 1}^{-1} = \text{Feynman Diagram 2}^{-1} - \text{Feynman Diagram 3}$$

bifurcation

$$\alpha_0 > \pi / 3$$

κ = cutoff



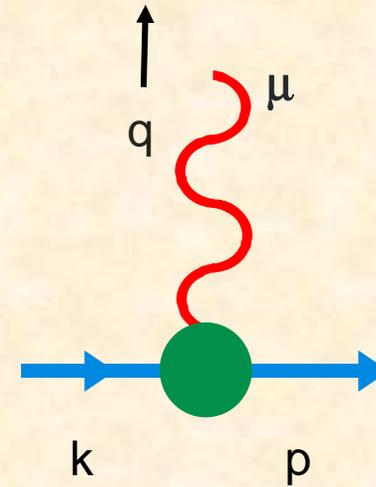
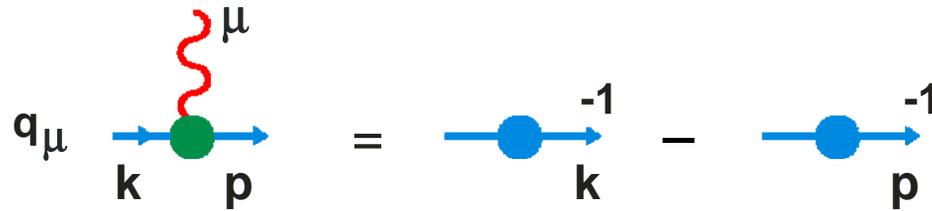


K = cutoff

gauge dependent

Ward – Green –Takahashi

$$q_\mu \Gamma^\mu(k, p) = S_F^{-1}(k) - S_F^{-1}(p)$$



$$S_F^{-1}(p) = A(p^2) \not{p} + B(p^2)$$

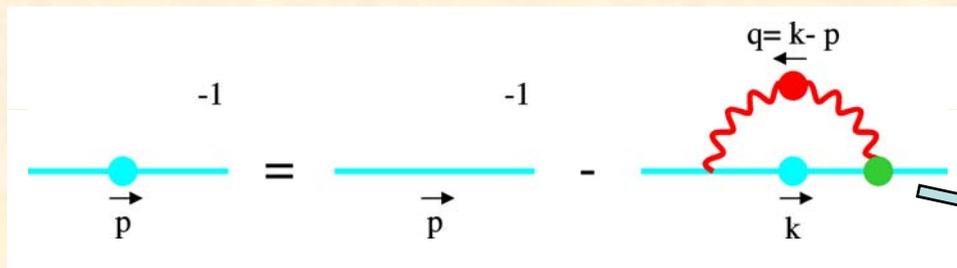
Ball-Chiu
construction

$$\begin{aligned} \Gamma^\mu(k, p) = & \frac{1}{2} (A(k^2) + A(p^2)) \gamma^\mu \\ & + \frac{1}{2} \frac{(A(k^2) - A(p^2))}{k^2 - p^2} (k + p) (k^\mu + p^\mu) \\ & + \frac{(B(k^2) - B(p^2))}{k^2 - p^2} (k^\mu + p^\mu) + \Gamma_T^\mu(k, p) \end{aligned}$$

Transverse components at $O(\alpha)$ for $k^2 \gg p^2$

$$\Gamma_T^\mu(k, p) = -\frac{\alpha\xi}{8\pi} \ln \frac{k^2}{p^2} \left[\gamma^\mu - \frac{k^\mu \not{k}}{k^2} \right]$$

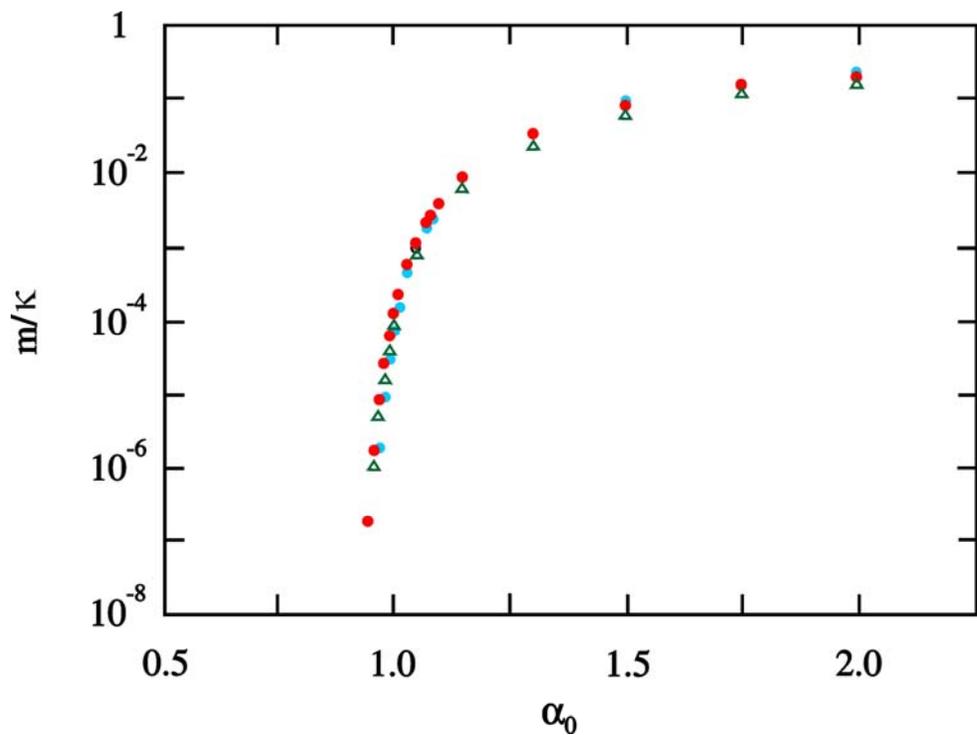
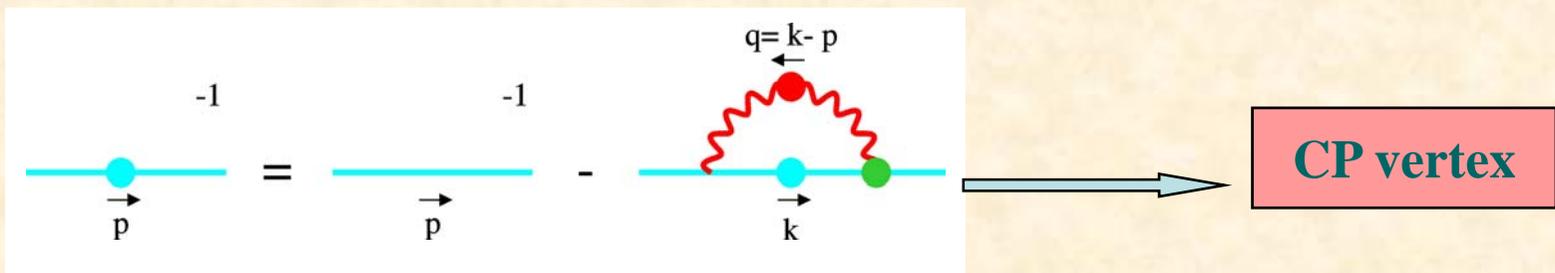
$$\Gamma_T^\mu(k^2 \gg p^2) = \frac{1}{2} \left(\frac{1}{\mathcal{F}(k)} - \frac{1}{\mathcal{F}(p)} \right) \frac{(k^2 + p^2)}{(k^2 - p^2)^2} T_6^\mu$$



CP vertex

Gauge Invariance and Multiplicative Renormalizability

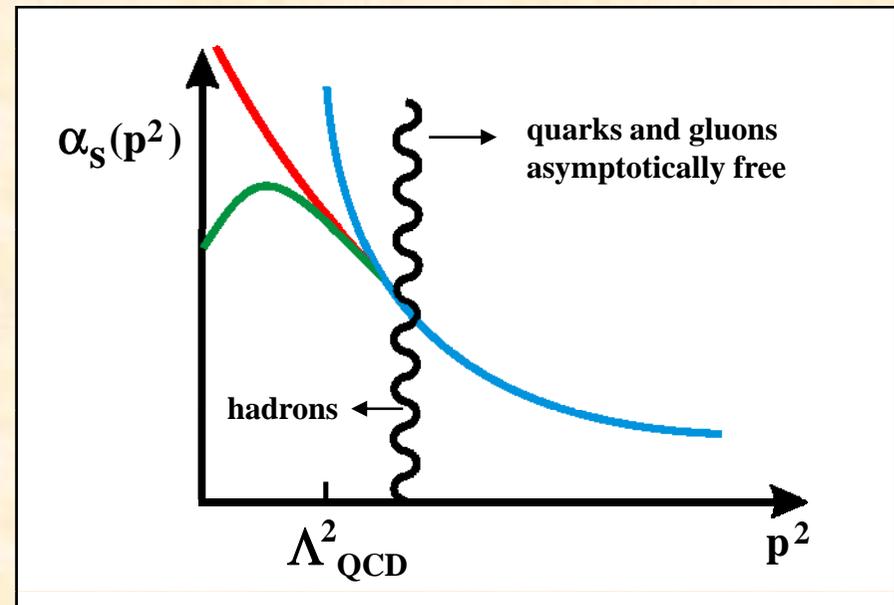
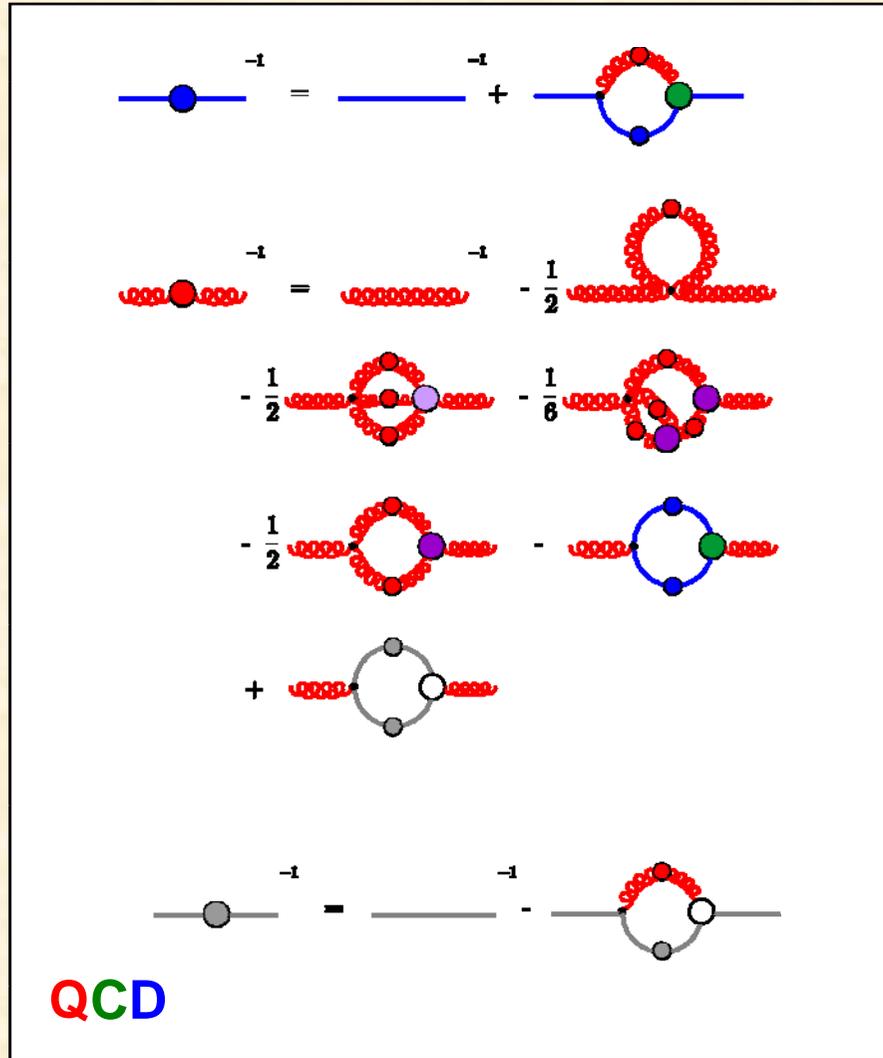
Gauge Invariance and Multiplicative Renormalizability



κ = cutoff

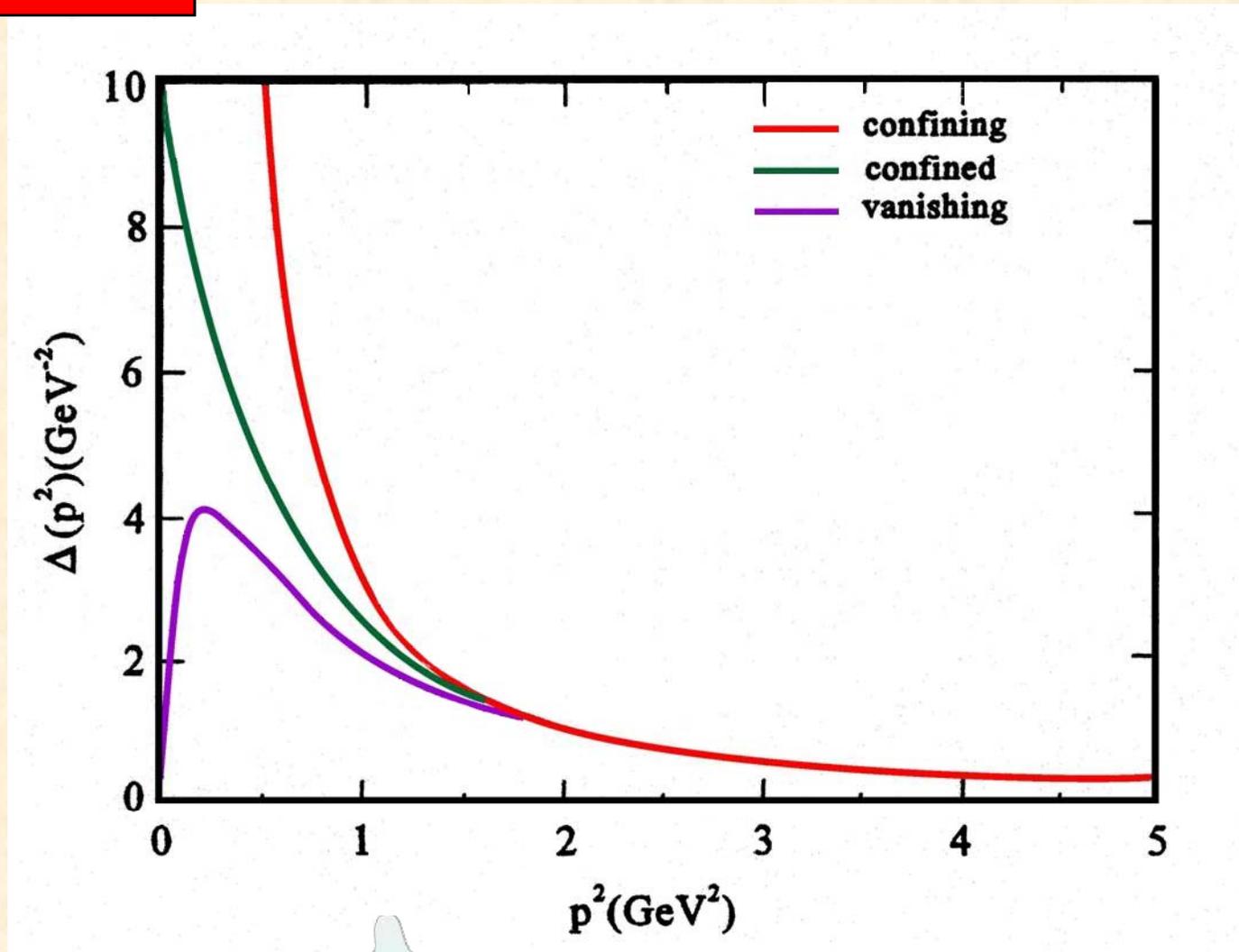
almost gauge independent

Schwinger-Dyson Equations



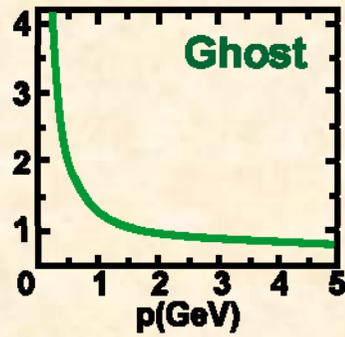
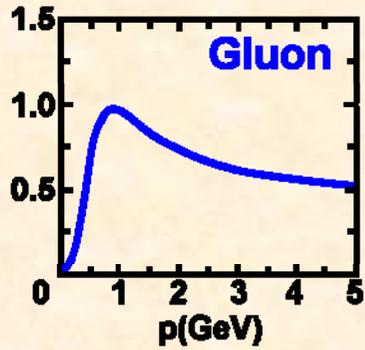
gluon propagator

$$\Delta(\mathbf{p}) \equiv \Delta_{00}(\mathbf{p})$$

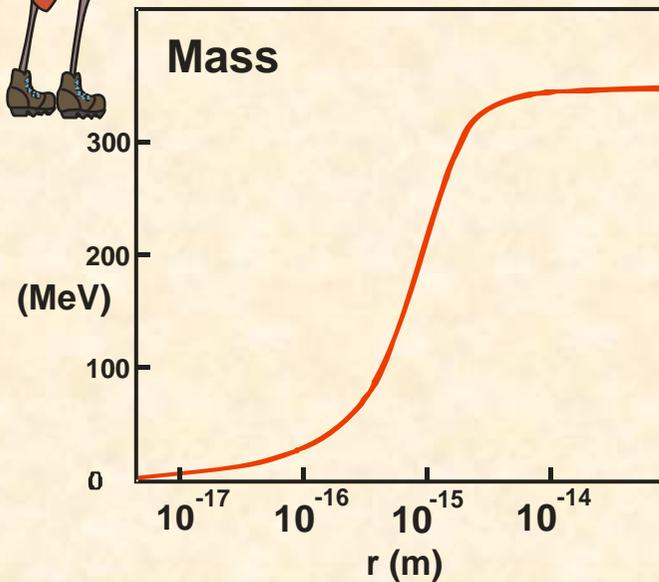
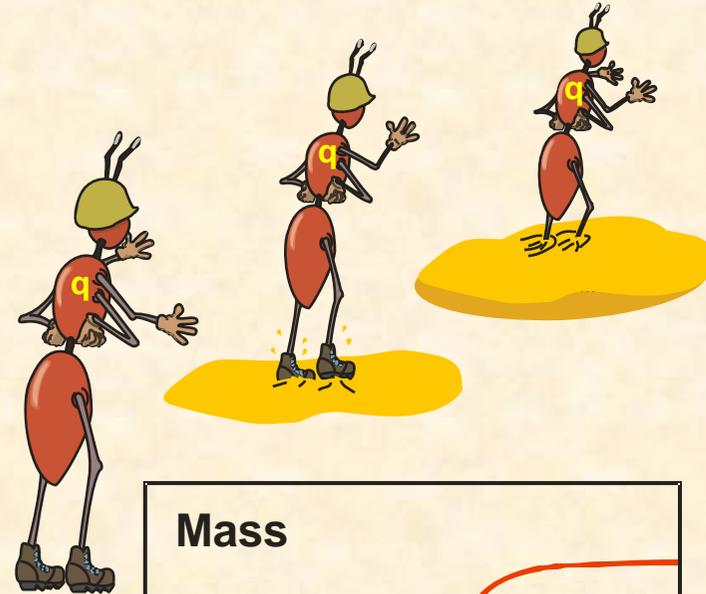
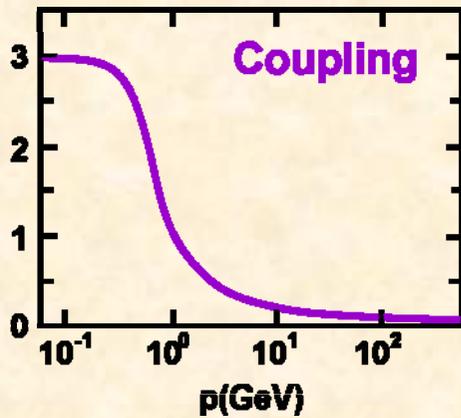


Tübingen group: Alkofer et al.

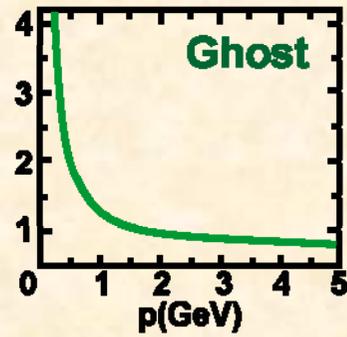
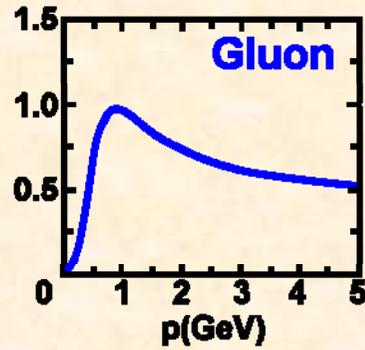
calculating the masses of **u/d** quarks



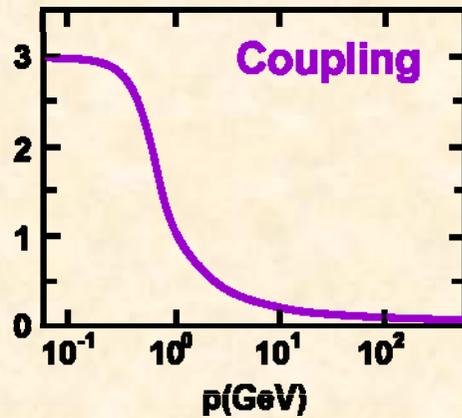
Alkofer et al.



calculating the masses of **u/d** quarks



Alkofer et al.



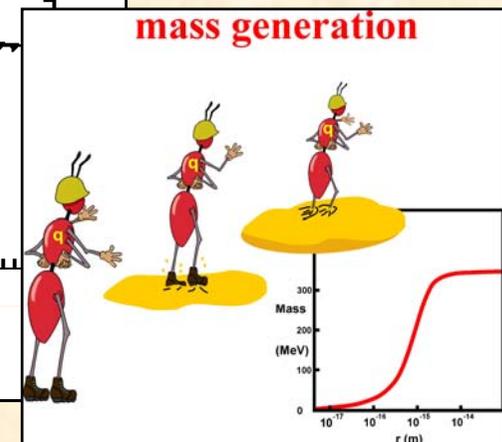
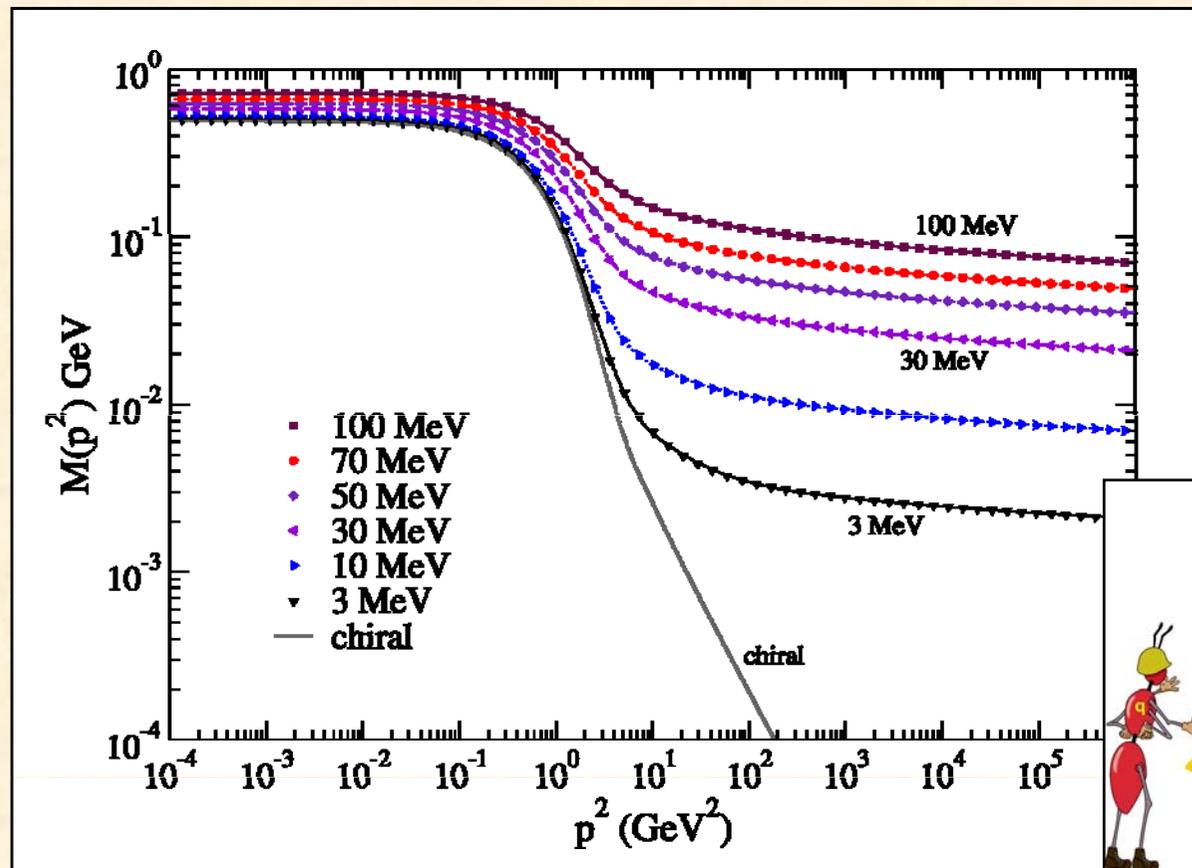
QCD running coupling

$$\alpha(Q^2) > 1 \quad \text{for} \quad Q^2 < Q_0^2 \approx \Lambda^2$$

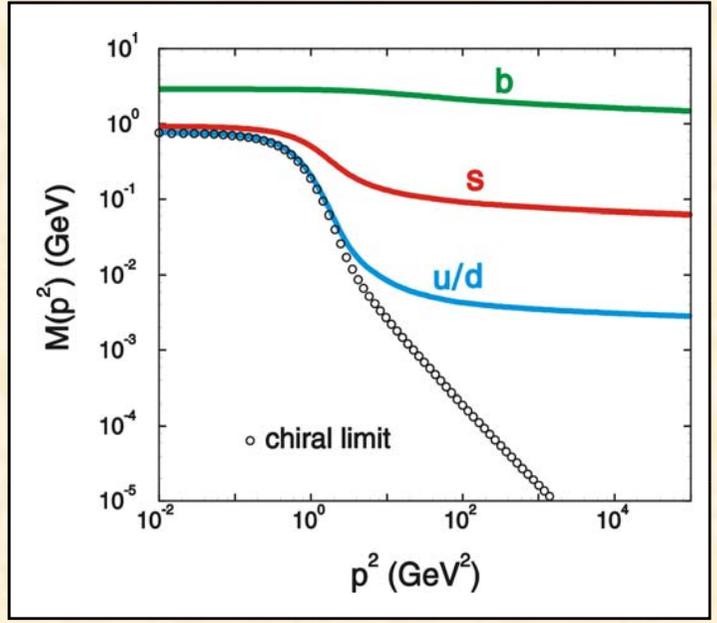
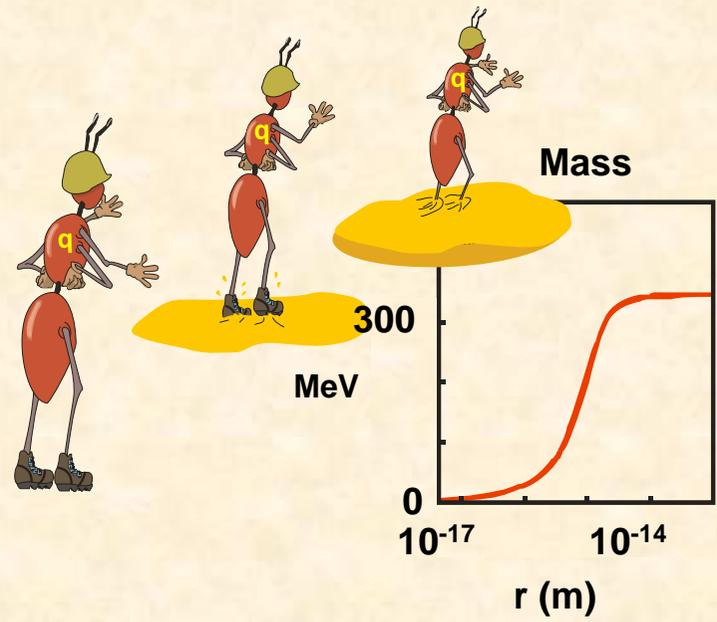


χ SB

Williams,
Fischer,
P



Mass generation



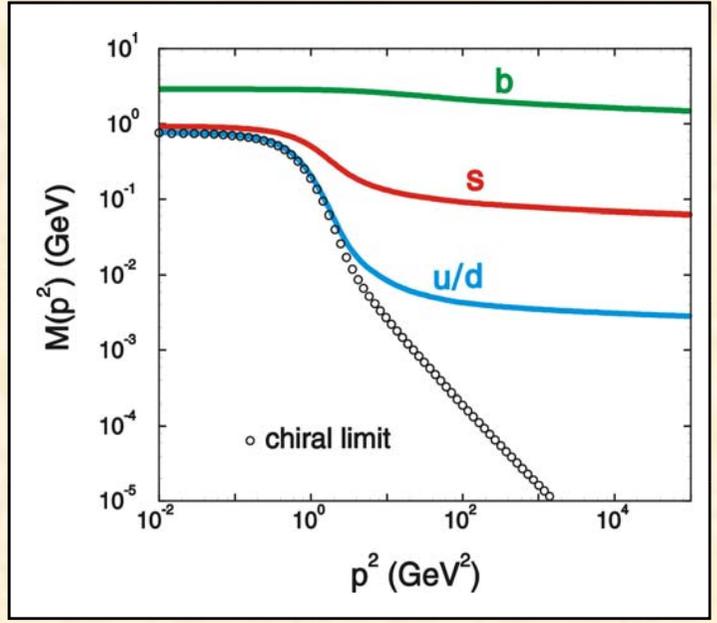
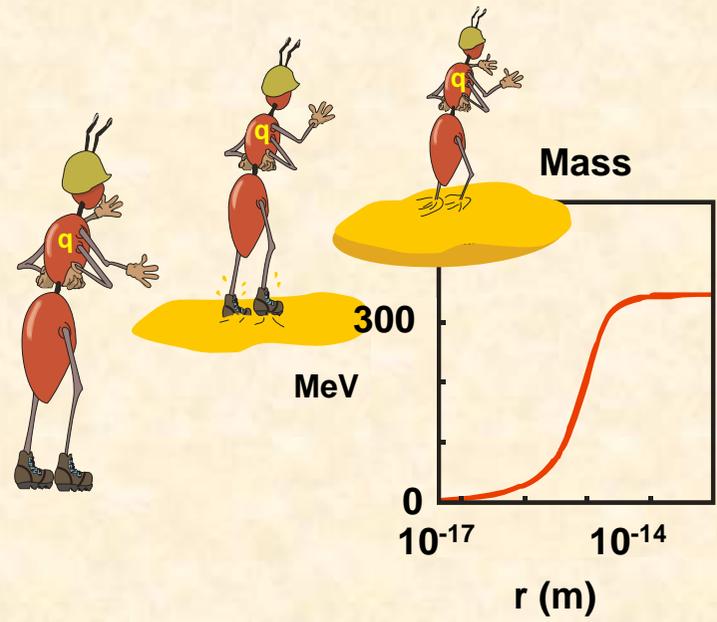
$$\langle \bar{q}q \rangle_0 \sim - (240 \text{ MeV})^3$$

QCD sum rules

current correlator

$$\mathcal{M}(p) \simeq m_0 [\ln p/\Lambda_{QCD}]^d + C \frac{-\langle \bar{q}q \rangle}{p^2 [\ln p/\Lambda_{QCD}]^{d+1}}$$

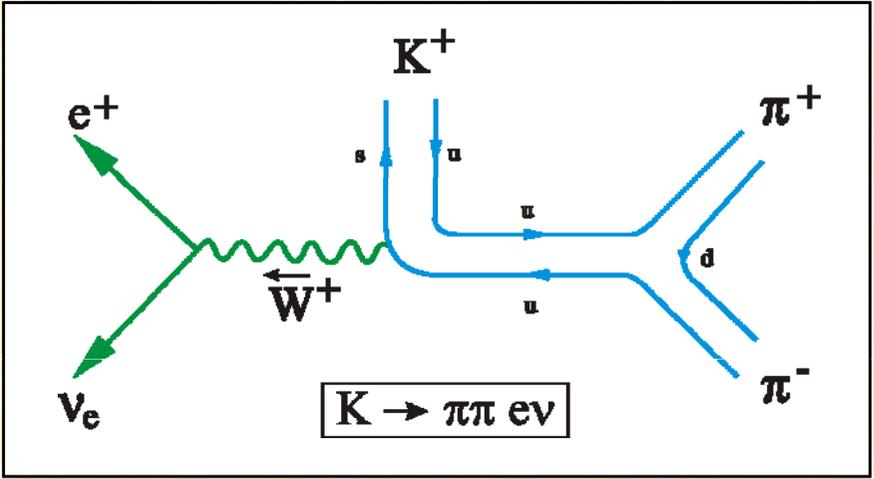
Mass generation



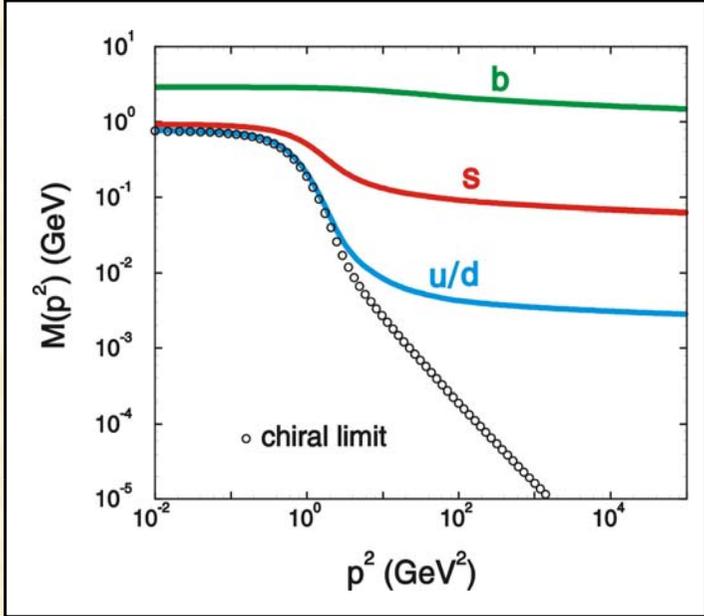
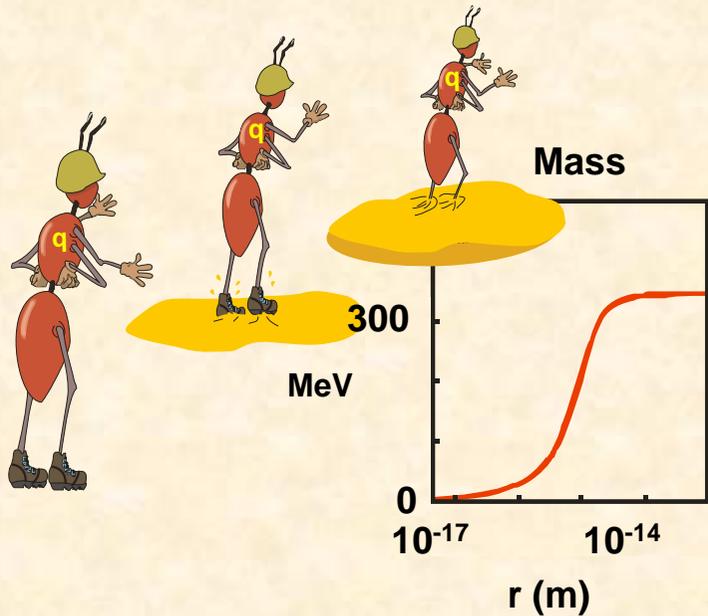
$\langle \bar{q}q \rangle_0 \sim - (240 \text{ MeV})^3$

QCD sum rules

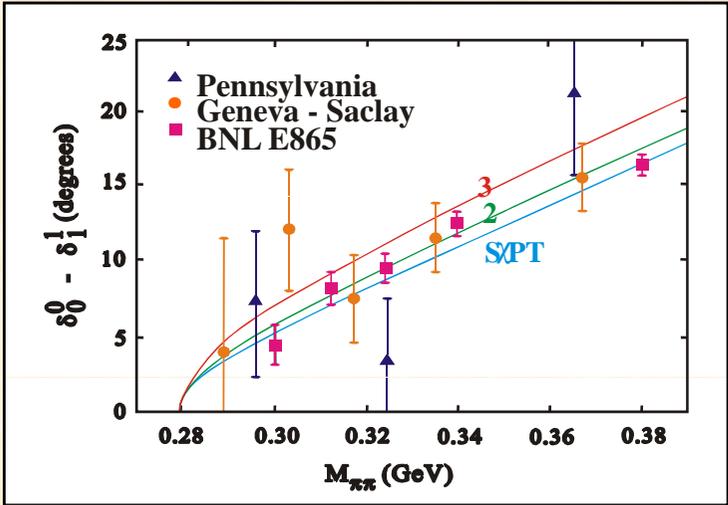
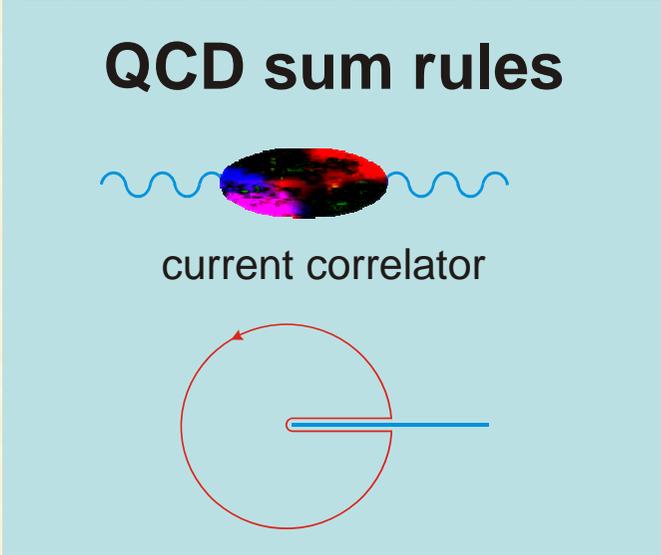
The diagram shows a 'current correlator' represented by a wavy line connected to a colorful sphere. Below it is a quark loop diagram consisting of a circle with a horizontal line through the center, representing a quark propagator.



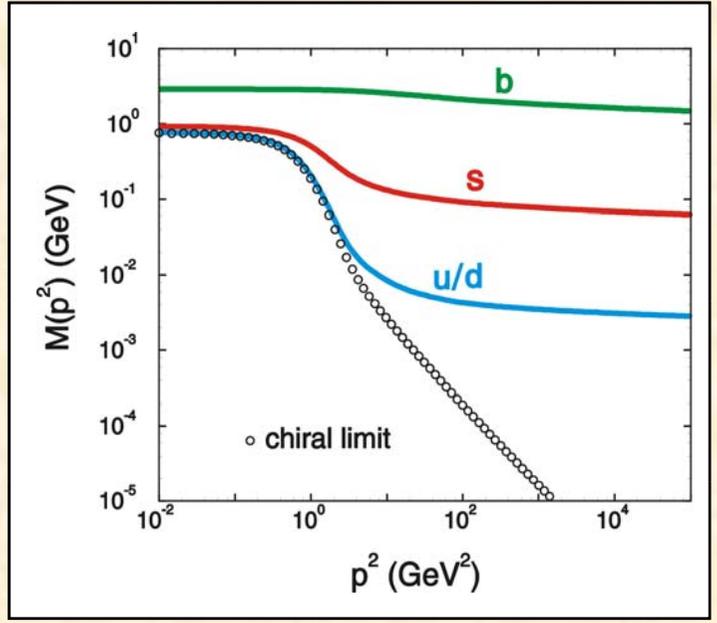
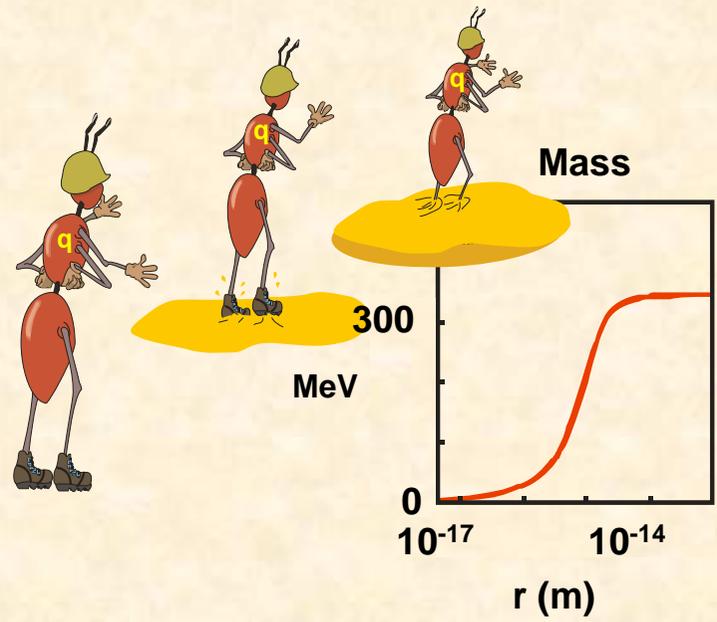
Mass generation



$\langle \bar{q}q \rangle_0 \sim - (240 \text{ MeV})^3$



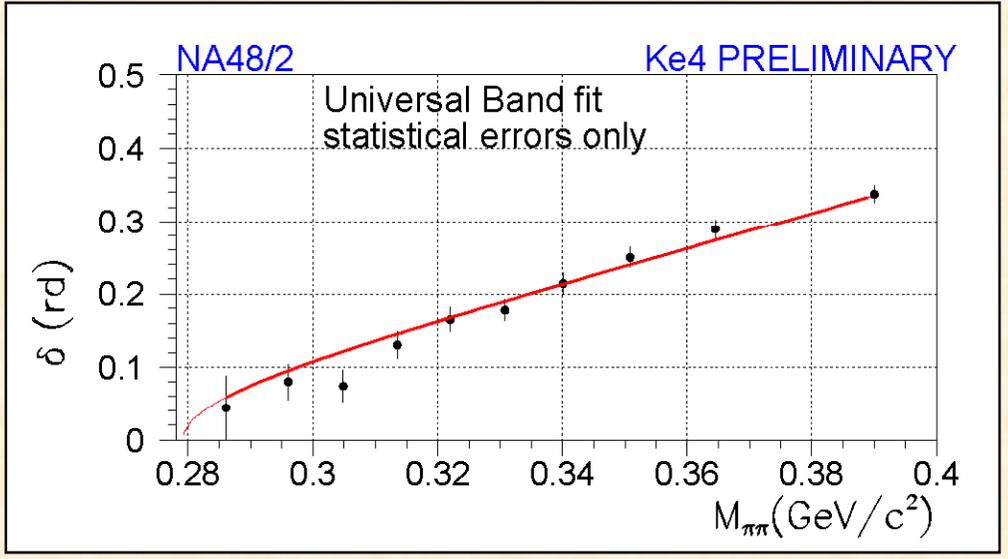
Mass generation



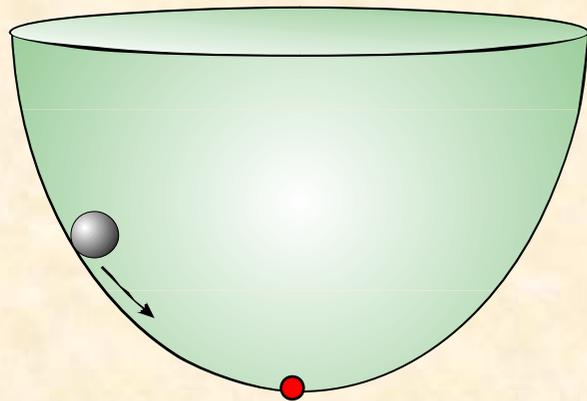
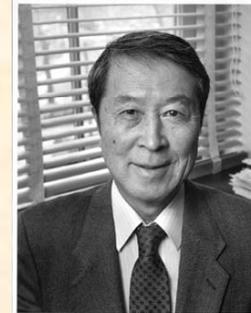
$\langle \bar{q}q \rangle_0 \sim - (240 \text{ MeV})^3$

QCD sum rules

current correlator

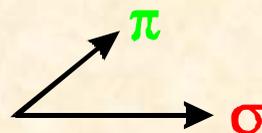
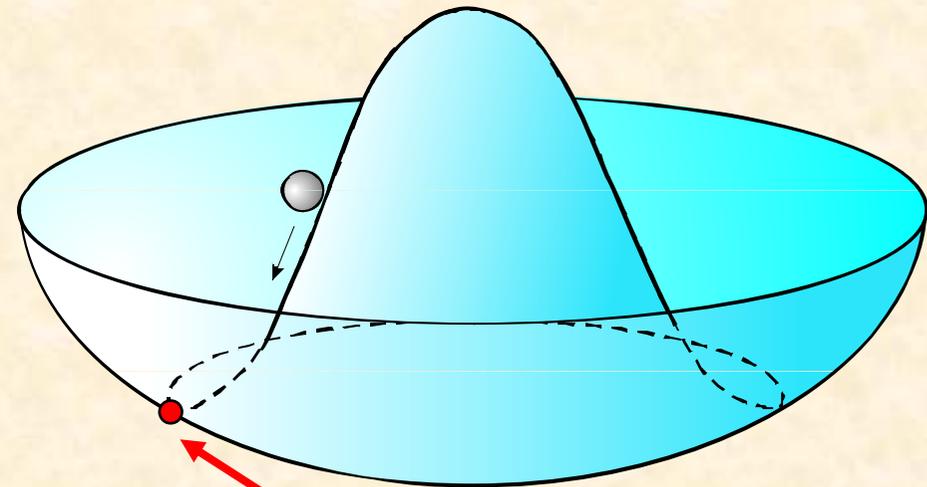


Ground State – Vacuum



vacuum

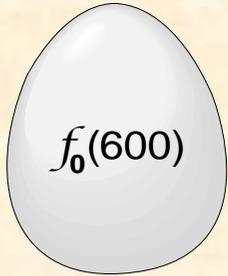
Chiral symmetry breaking



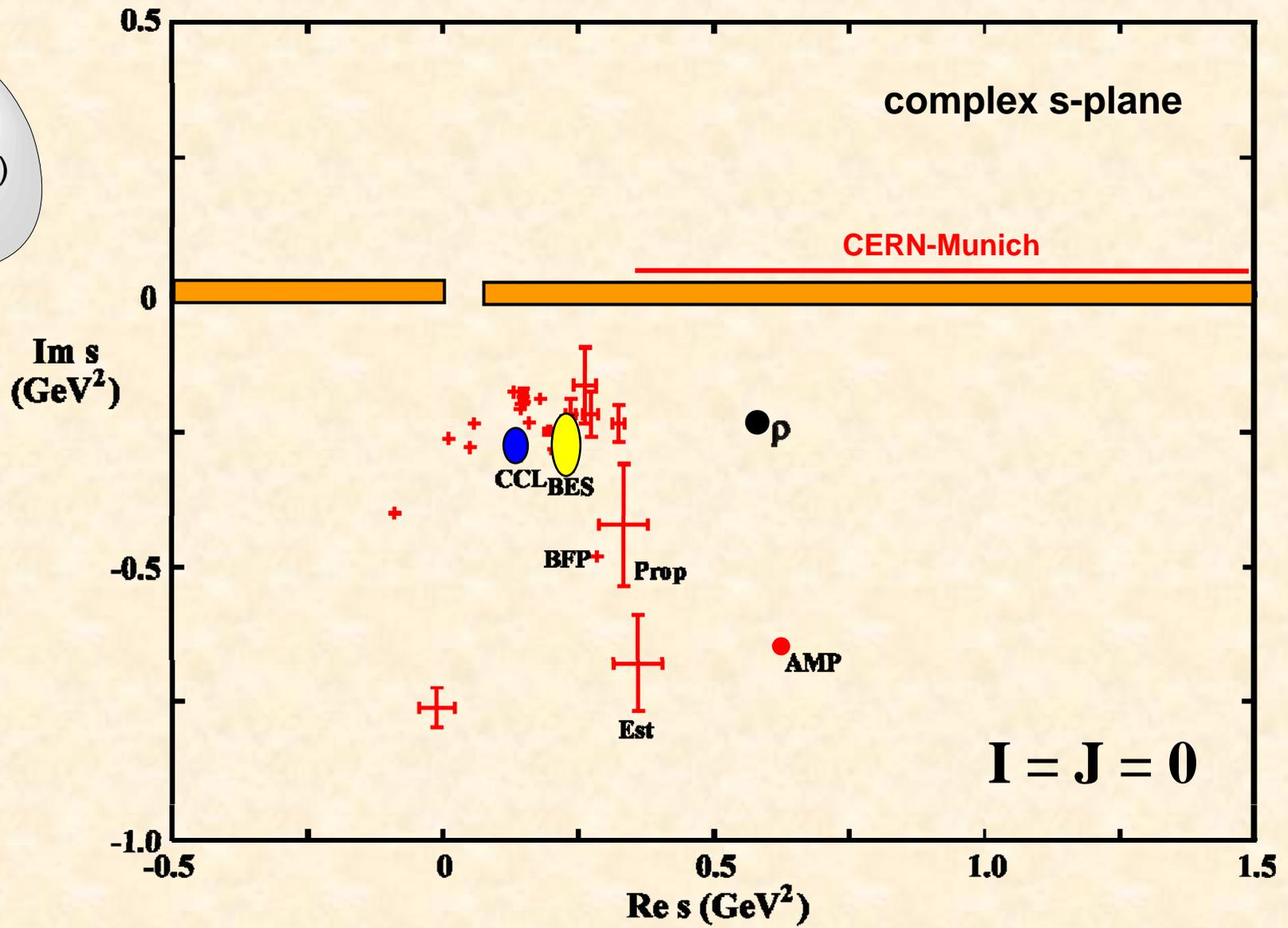
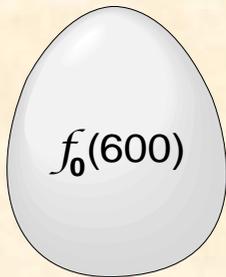
vacuum

ferromagnet

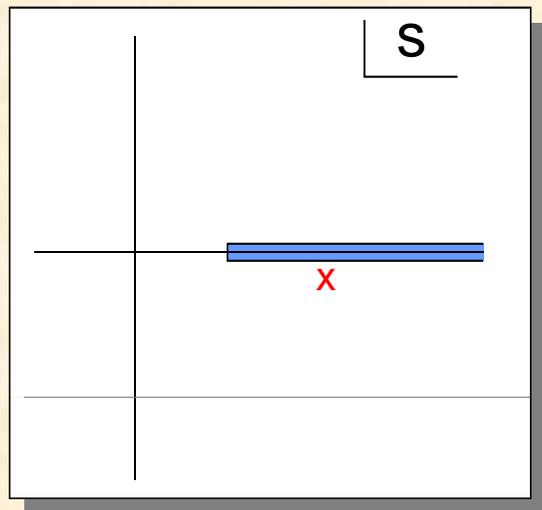
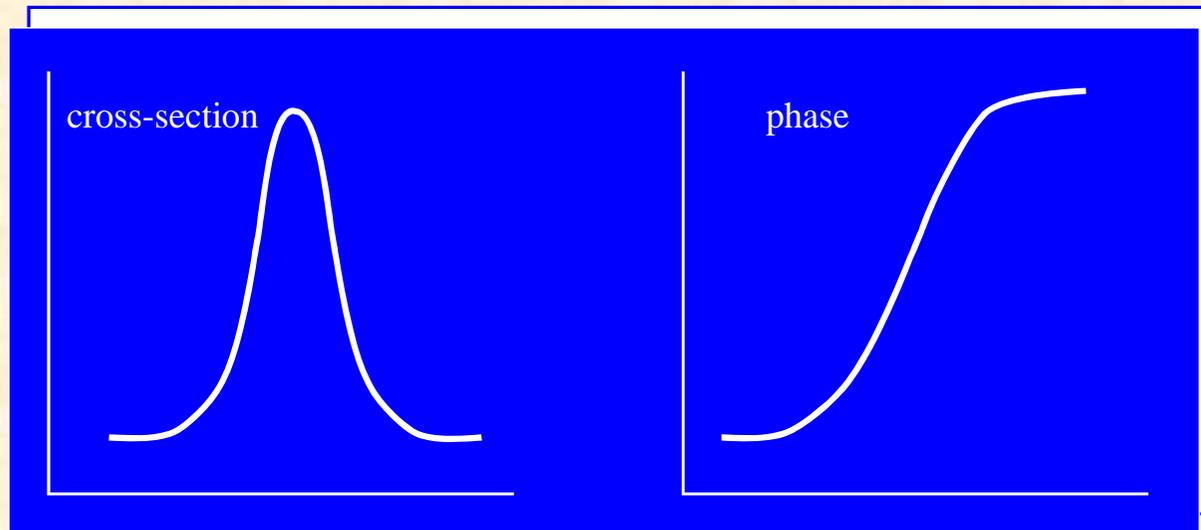
Scalar mesons



Scalar mesons



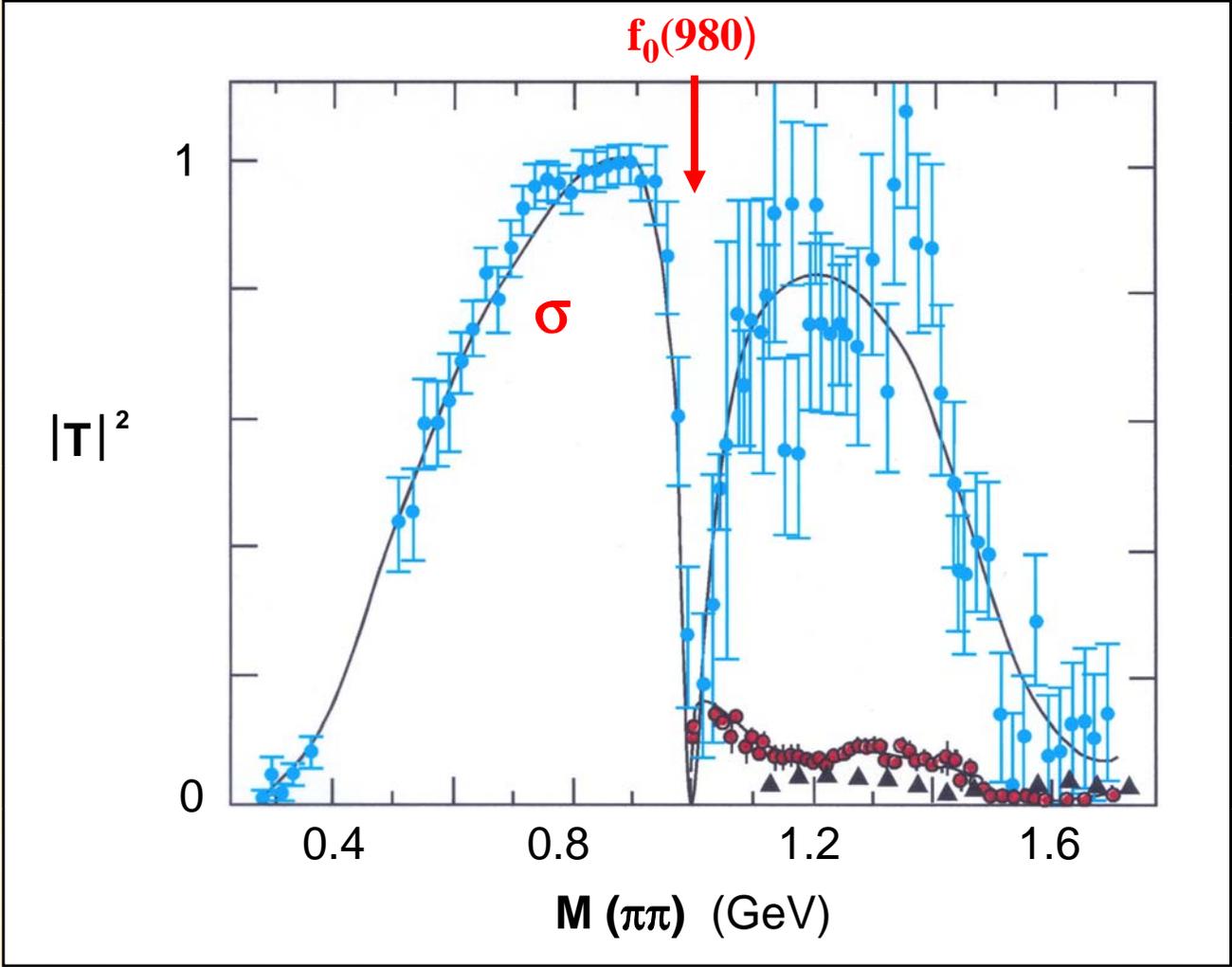
Hadron states



BW

$$\frac{1}{M^2 - s - iM\Gamma}$$

$I = J = 0$

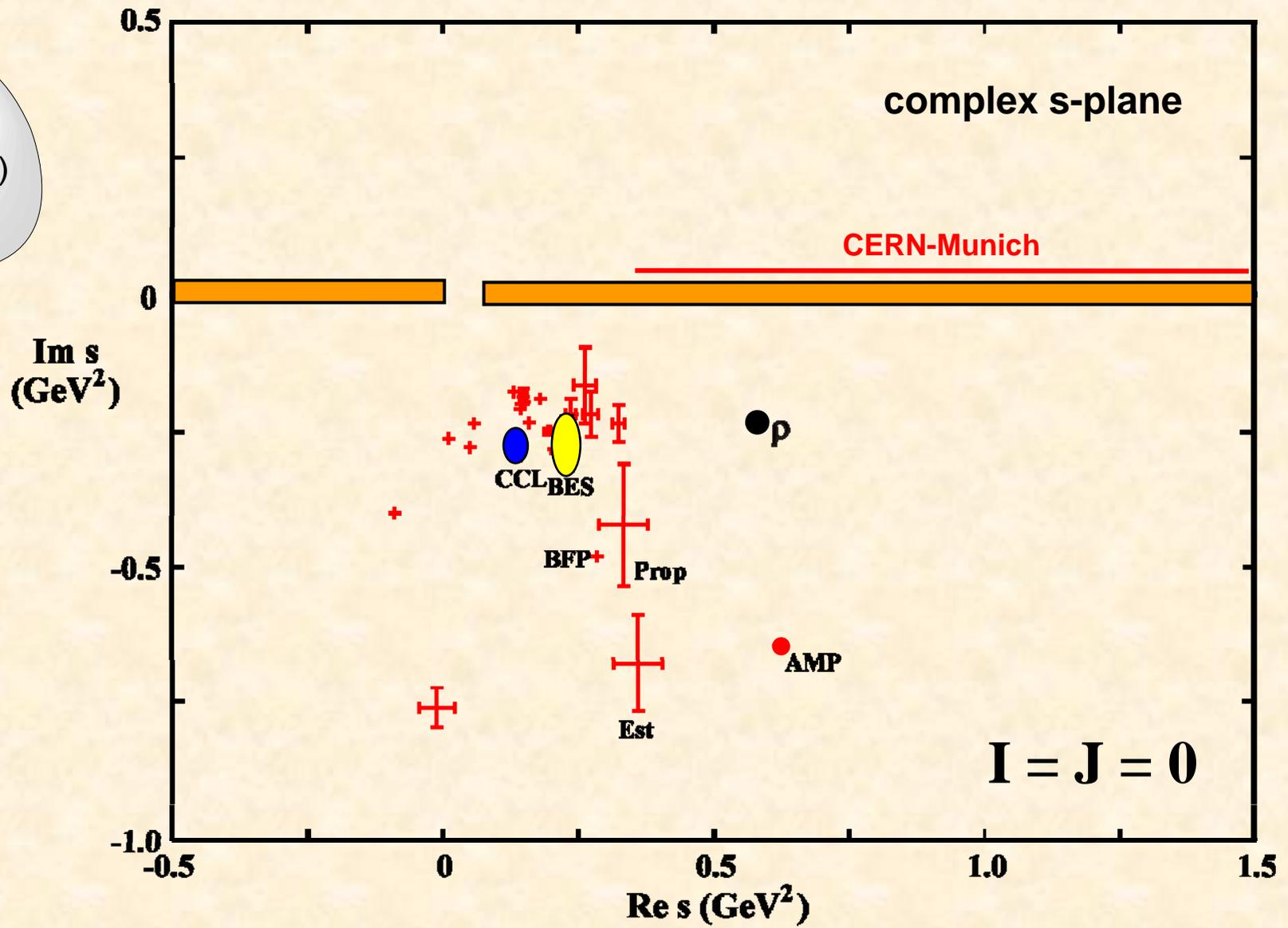
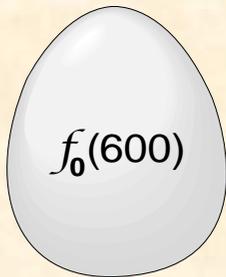


● $\pi\pi \rightarrow \pi\pi$

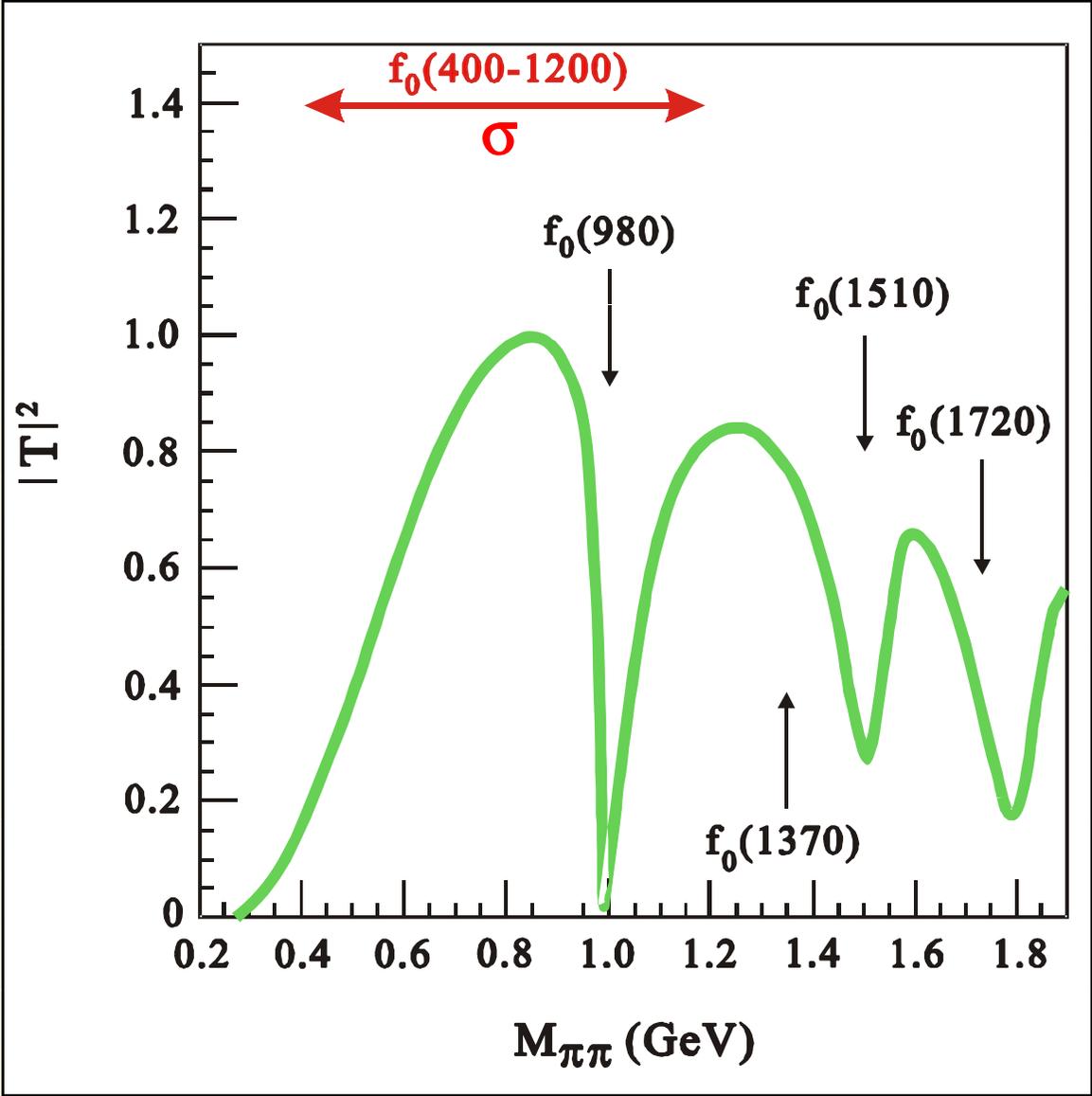
● $\pi\pi \rightarrow \bar{K}K$

▲ $\pi\pi \rightarrow \eta\eta$

Scalar mesons

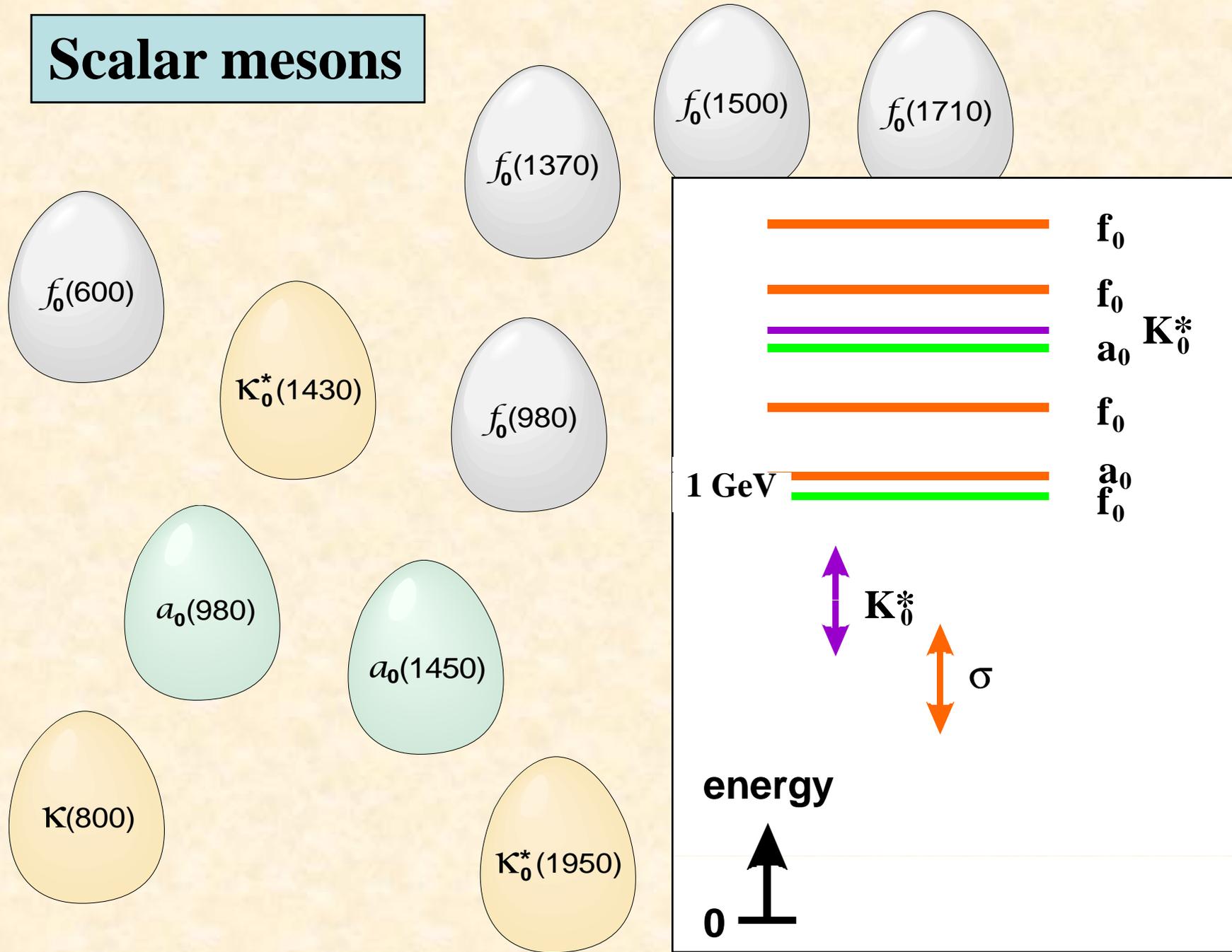


I = J = 0

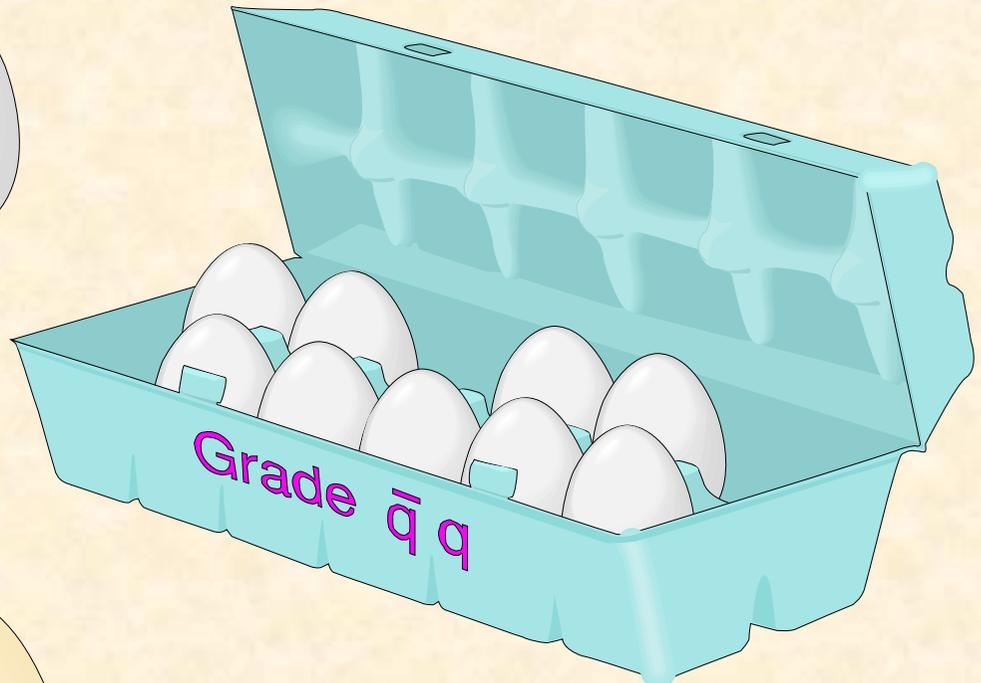
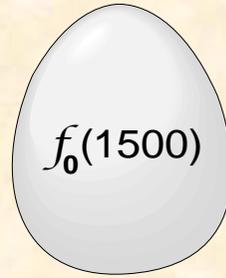
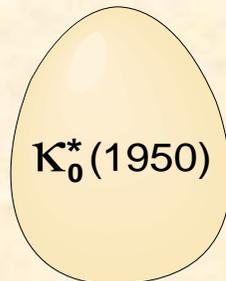
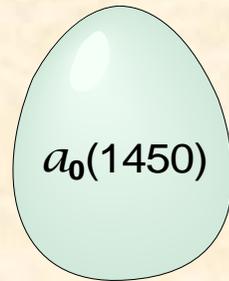
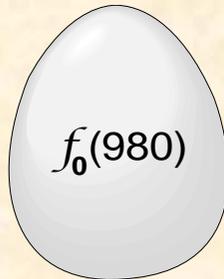
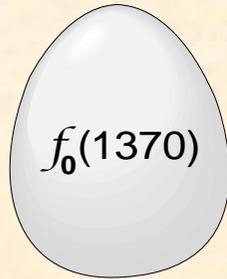
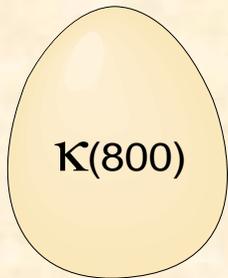
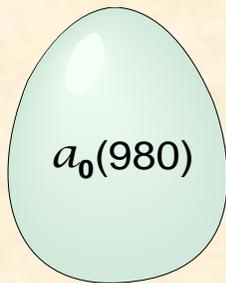
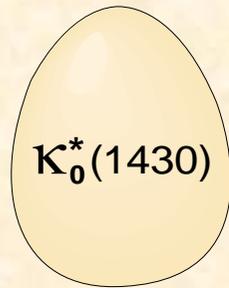
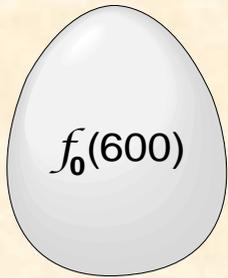


Zou

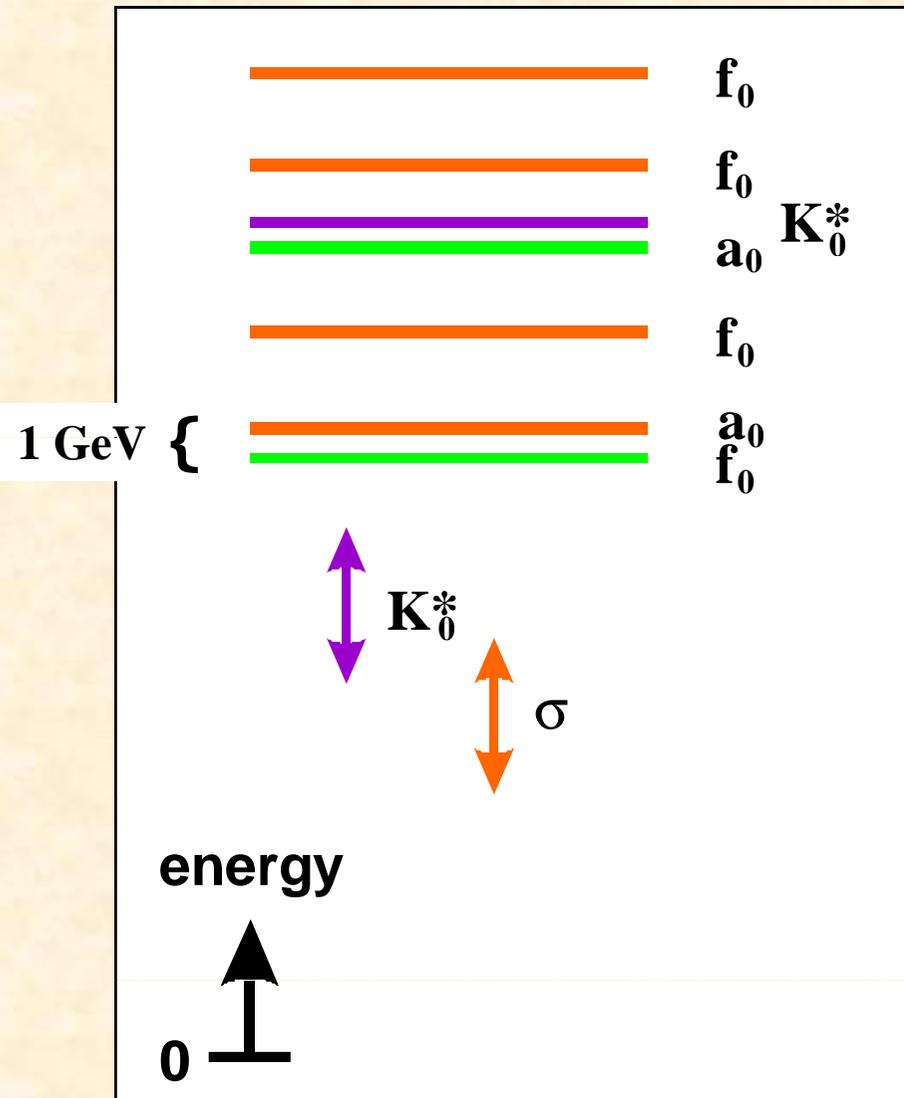
Scalar mesons



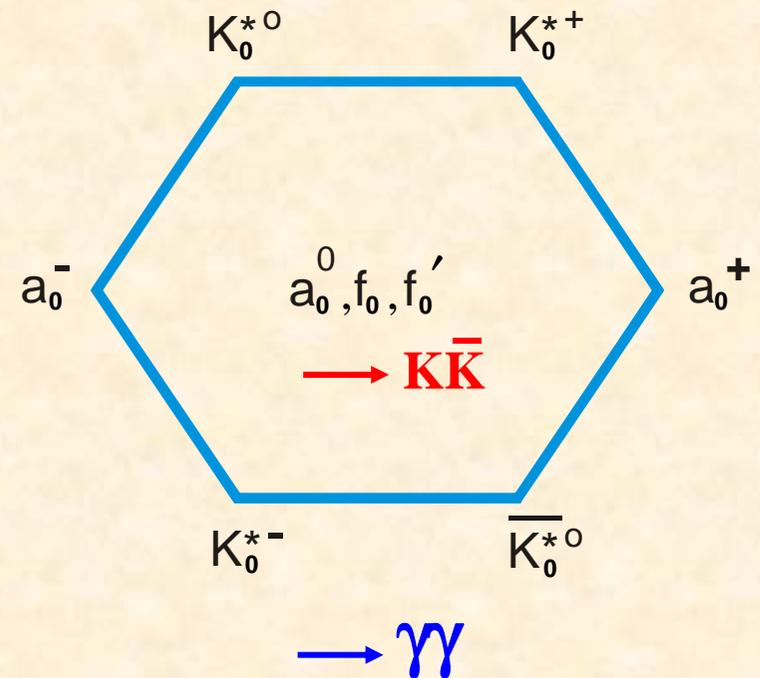
Scalar mesons



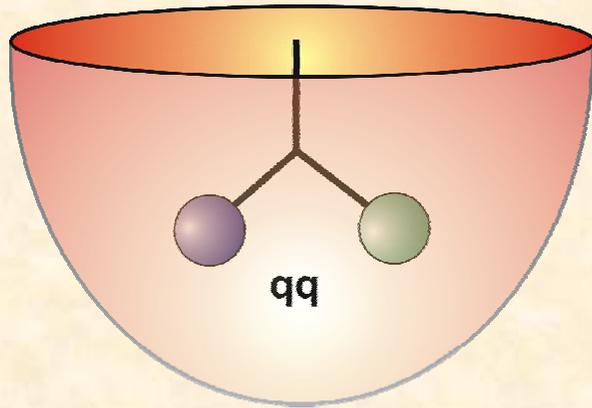
Scalar multiplet



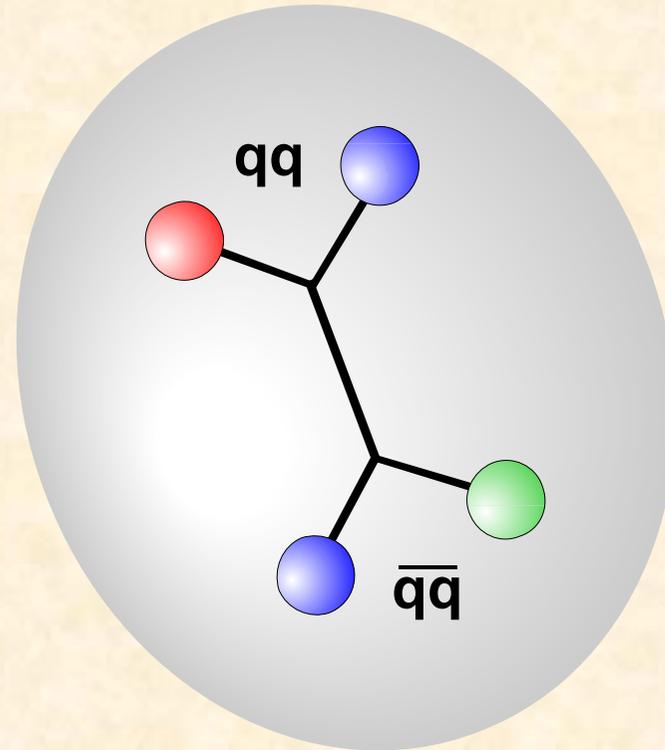
$$J^{PC} = 0^{++}$$



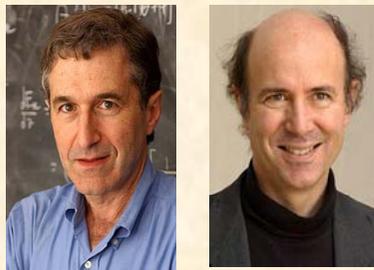
diquarks: colour



tetraquark



Jaffe & Wilczek



Scalar diquarks

[ud] [us] [ds]
[cd] [cu] [cs]

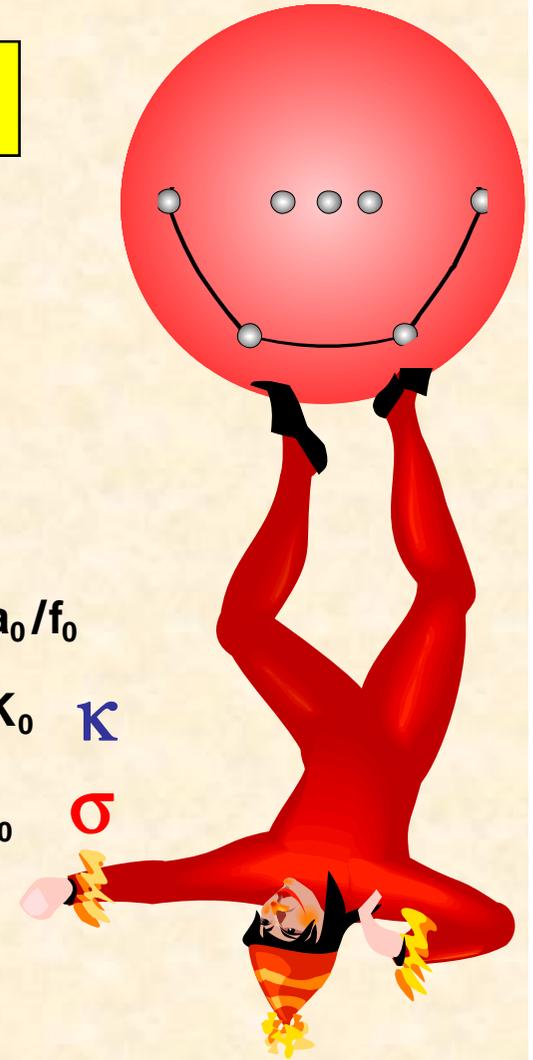
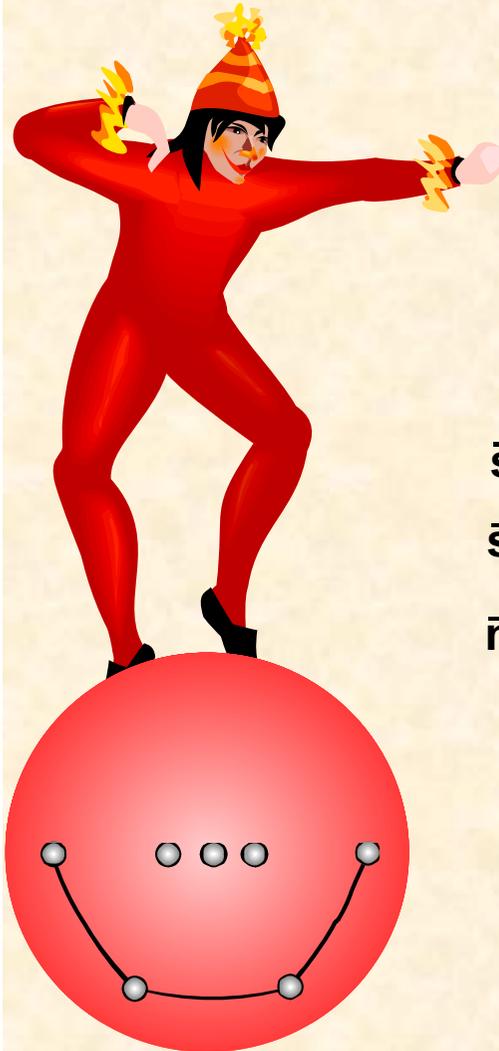
Scalar meson multiplets

$q\bar{q}$

$q\bar{q}q\bar{q}$

$\bar{s}s$ ————— f_0
 $\bar{s}n$ ————— K_0
 $\bar{n}n$ ————— a_0/f_0

$\bar{s}s\bar{n}n$ ————— a_0/f_0
 $\bar{s}n\bar{n}n$ ————— K_0 κ
 $\bar{n}n\bar{n}n$ ————— f_0 σ



Scalar meson multiplets

$q\bar{q}$

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$\bar{s}s$ ——— f_0
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$\bar{s}s\bar{n}n$ ——— a_0/f_0
 $\bar{s}n\bar{n}n$ ——— K_0 \mathcal{K}
 $\bar{n}n\bar{n}n$ ——— f_0 σ

Jaffe

Maiani, Piccinini, Polosa, Riquer
Black, Schechter et al

Scalar meson multiplets

$q\bar{q}$

$q\bar{q}q\bar{q}$

$\bar{s}s$ ————— f_0
 $\bar{s}n$ ————— K_0
 $\bar{n}n$ ————— a_0/f_0

$\bar{s}s\bar{n}n$ ————— a_0/f_0
 $\bar{s}n\bar{n}n$ ————— K_0 κ
 $\bar{n}n\bar{n}n$ ————— f_0 σ

N_c large \rightarrow stable

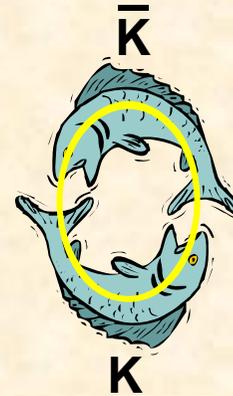
N_c large \rightarrow meson continuum

quark model = hadron world?

ϕ



+



$$\frac{1}{m_0^2 - s} \longrightarrow \frac{1}{M^2 - s - iM\Gamma}$$

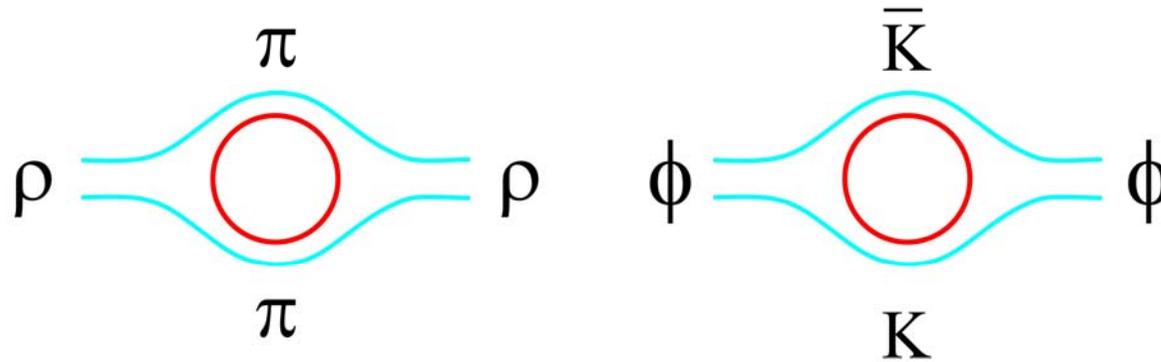
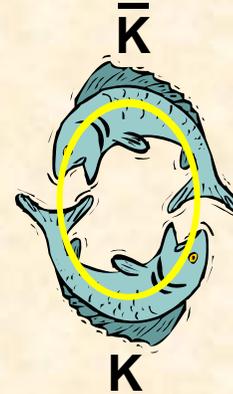
quark model = hadron world?

ϕ



+

$\frac{1}{N_c}$

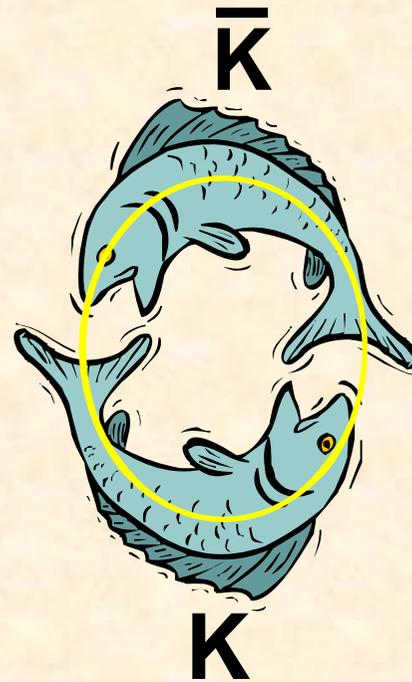


unquenching unimportant

f_0

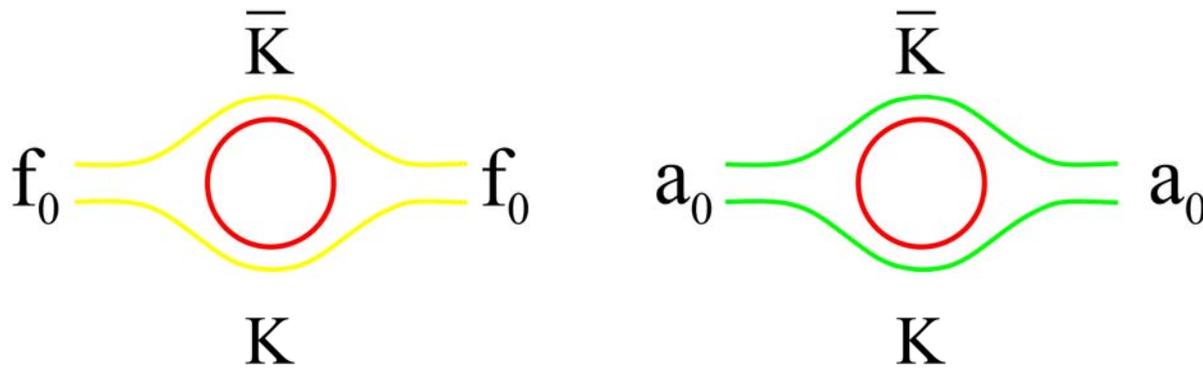


+



40%

Tornqvist
van Beveren

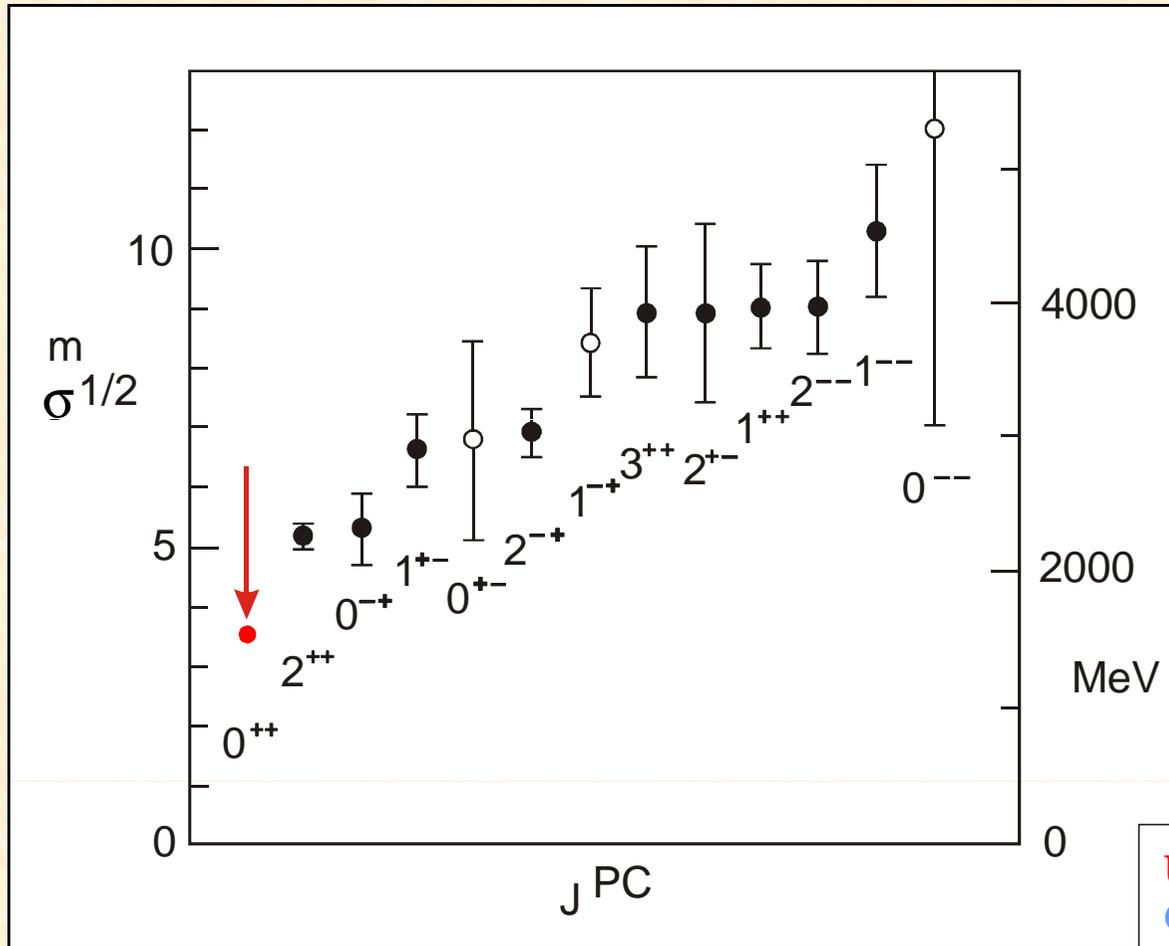


unquenching important

Lattice QCD



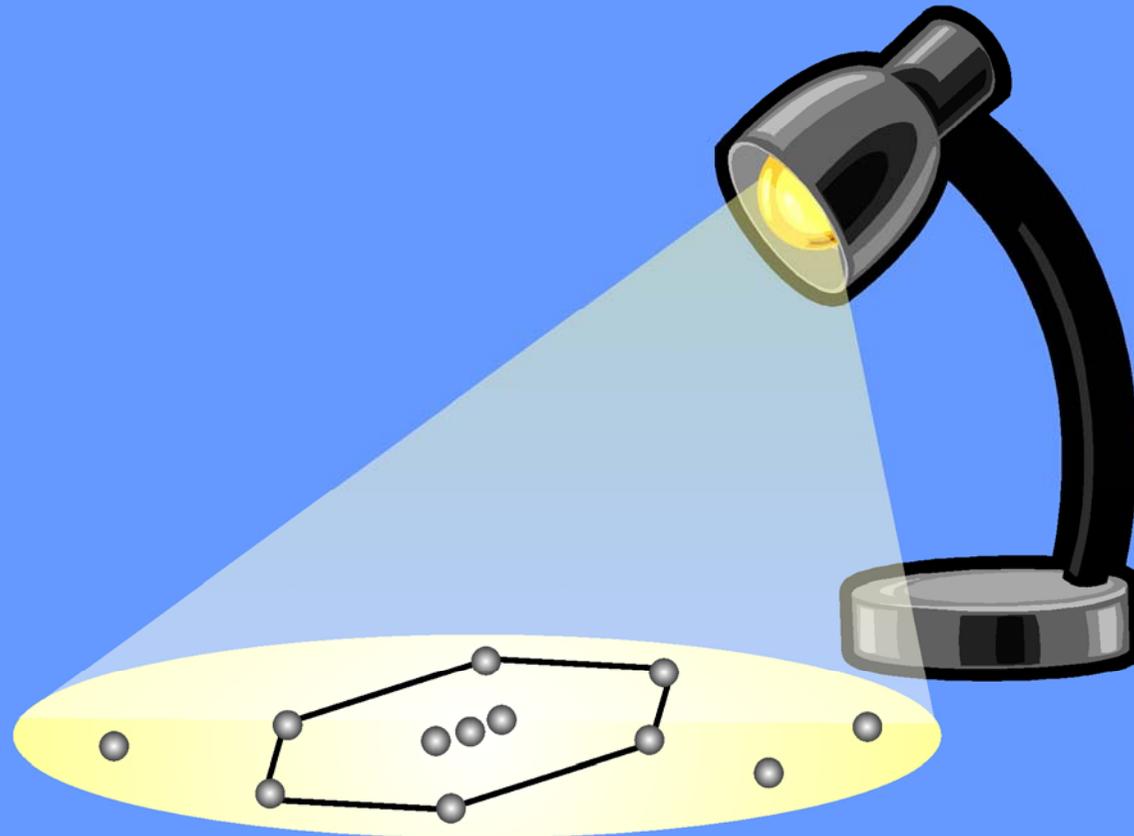
glueball spectrum in a world without quarks



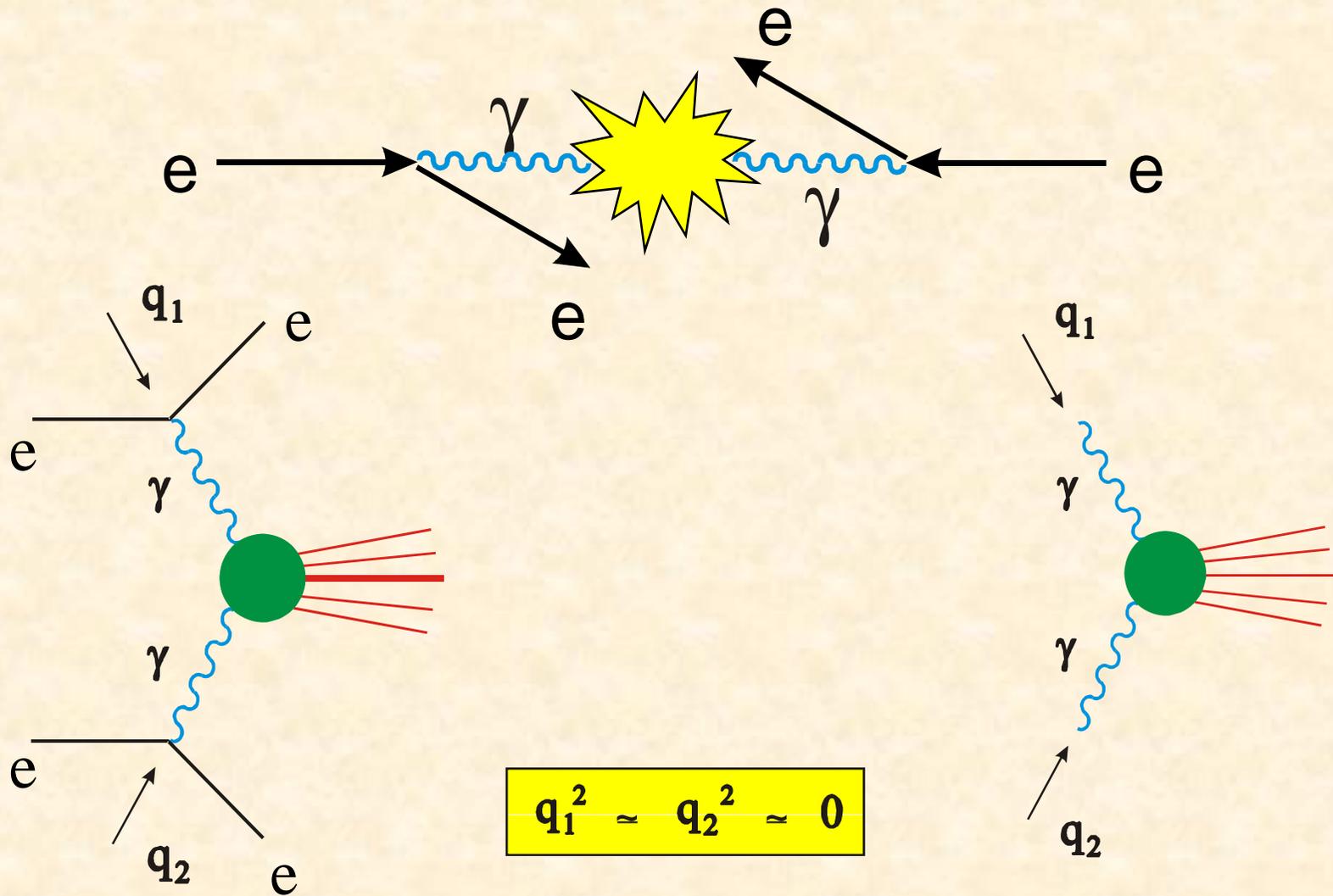
UKQCD	m =	1568 ± 89
GF11		1740 ± 71
MP		1630 ± 100
GF11 (reanal)		1648 ± 58

Two photon processes

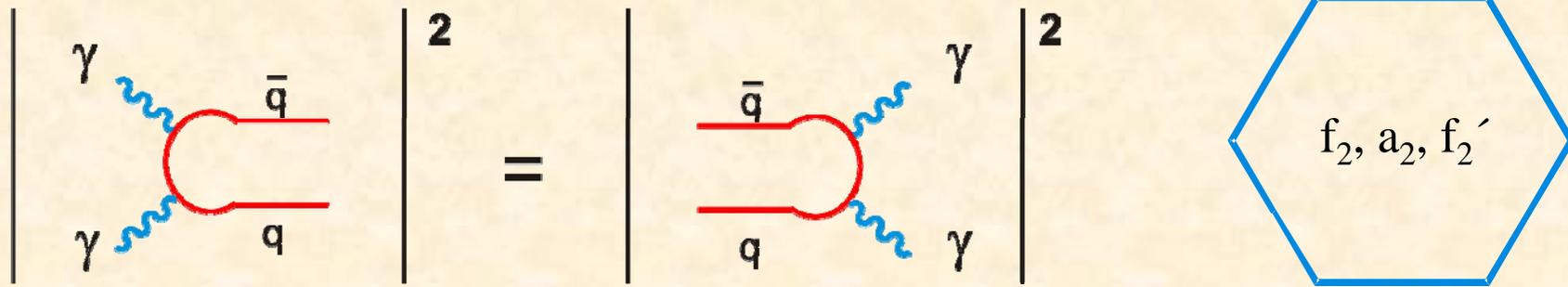
Shedding light on scalar mesons



Light by light scattering



Exclusive: $f_2, a_2, f_2' \rightarrow \gamma\gamma$

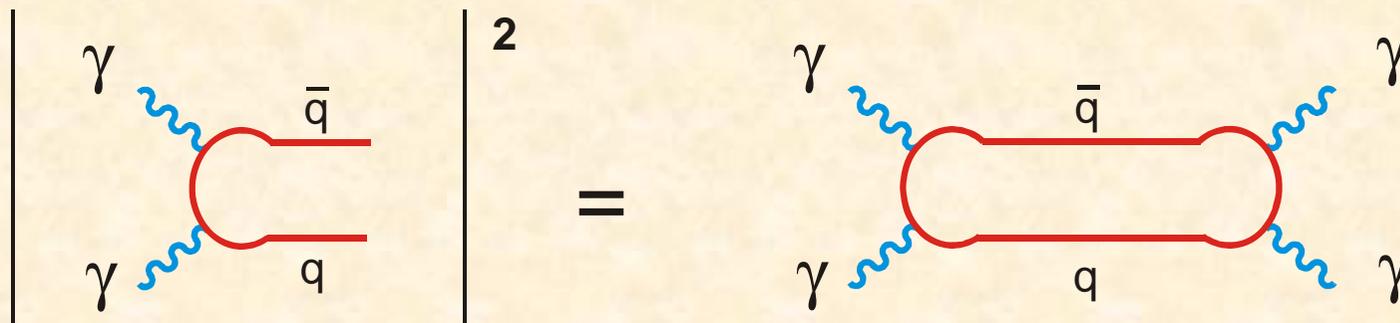


$$\Gamma(T \rightarrow \gamma\gamma) = \alpha^2 \langle e_q^2 \rangle^2 \Pi_R$$

$$|\psi(0)|^2$$

	f_2	:	a_2	:	f_2'
ideal mixing	25	:	9	:	2
experiment	25	:	10 ± 2	:	1 ± 0.2

Exclusive: $\gamma\gamma \rightarrow \pi^0, \eta, \eta'$

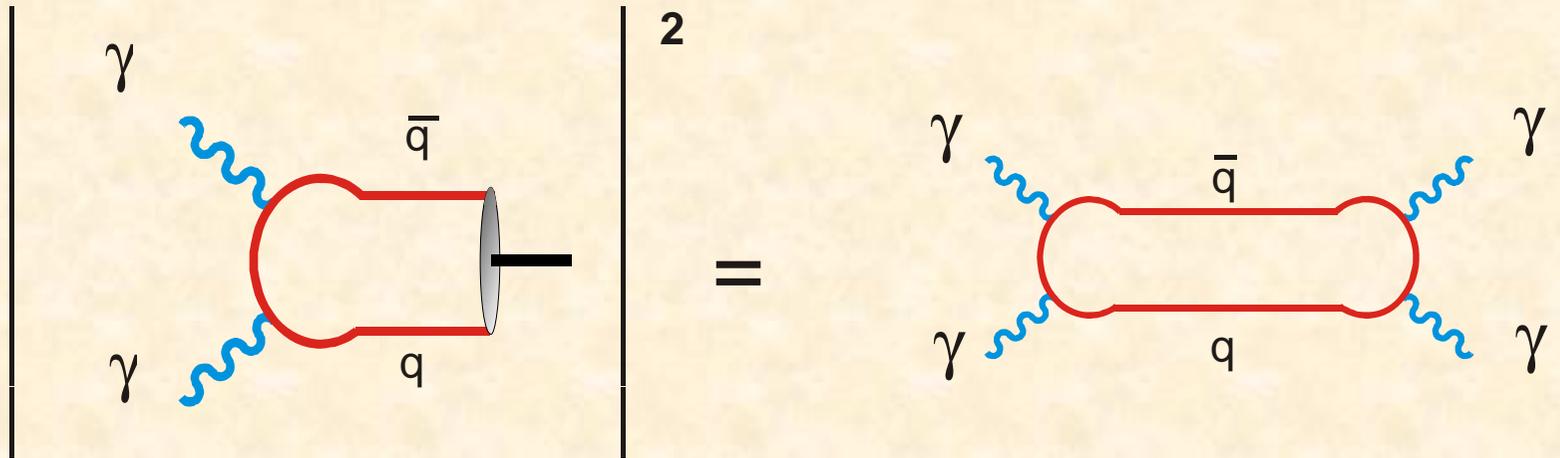


$$\left(\sum_q \langle e_q^2 \rangle \right)^2 \Pi_R$$

$$|\psi(0)|^2$$

$$\Gamma(\pi^0) : \Gamma(\eta) : \Gamma(\eta') \sim 1 : 60 : 500$$

Exclusive: $\gamma\gamma \rightarrow \pi^0, \eta, \eta'$



Hayne & Isgur: M^3

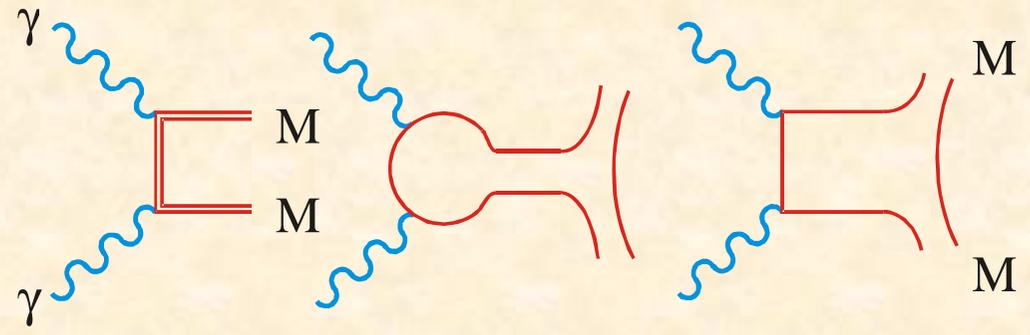
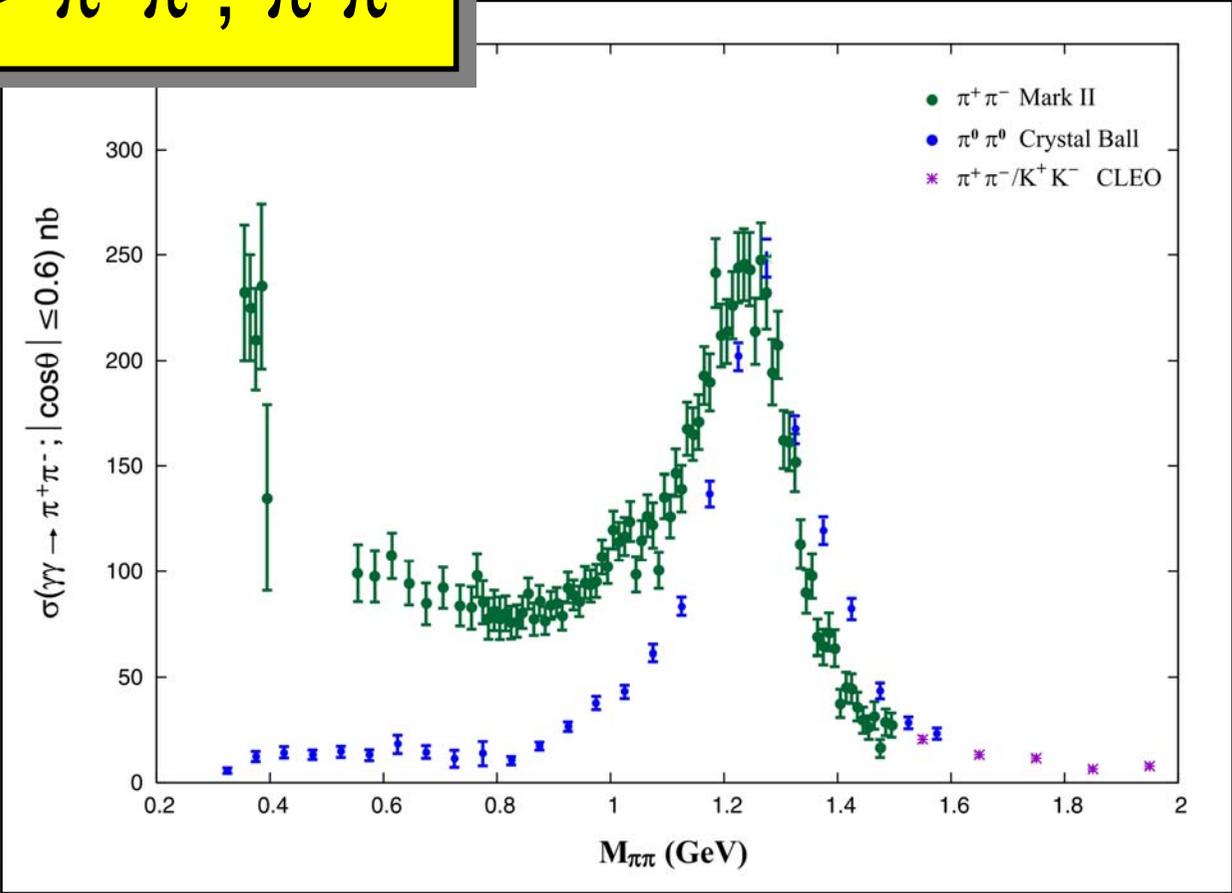
$$\left(\sum_q \langle e_q^2 \rangle \right)^2 \Pi_R$$

$|\psi(0)|^2$

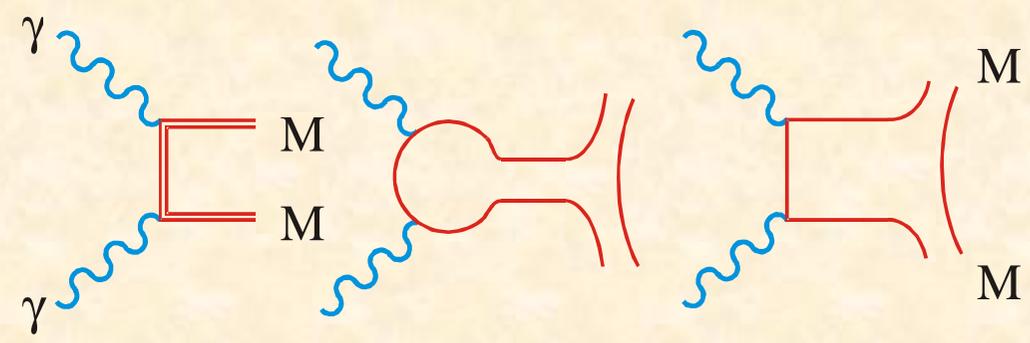
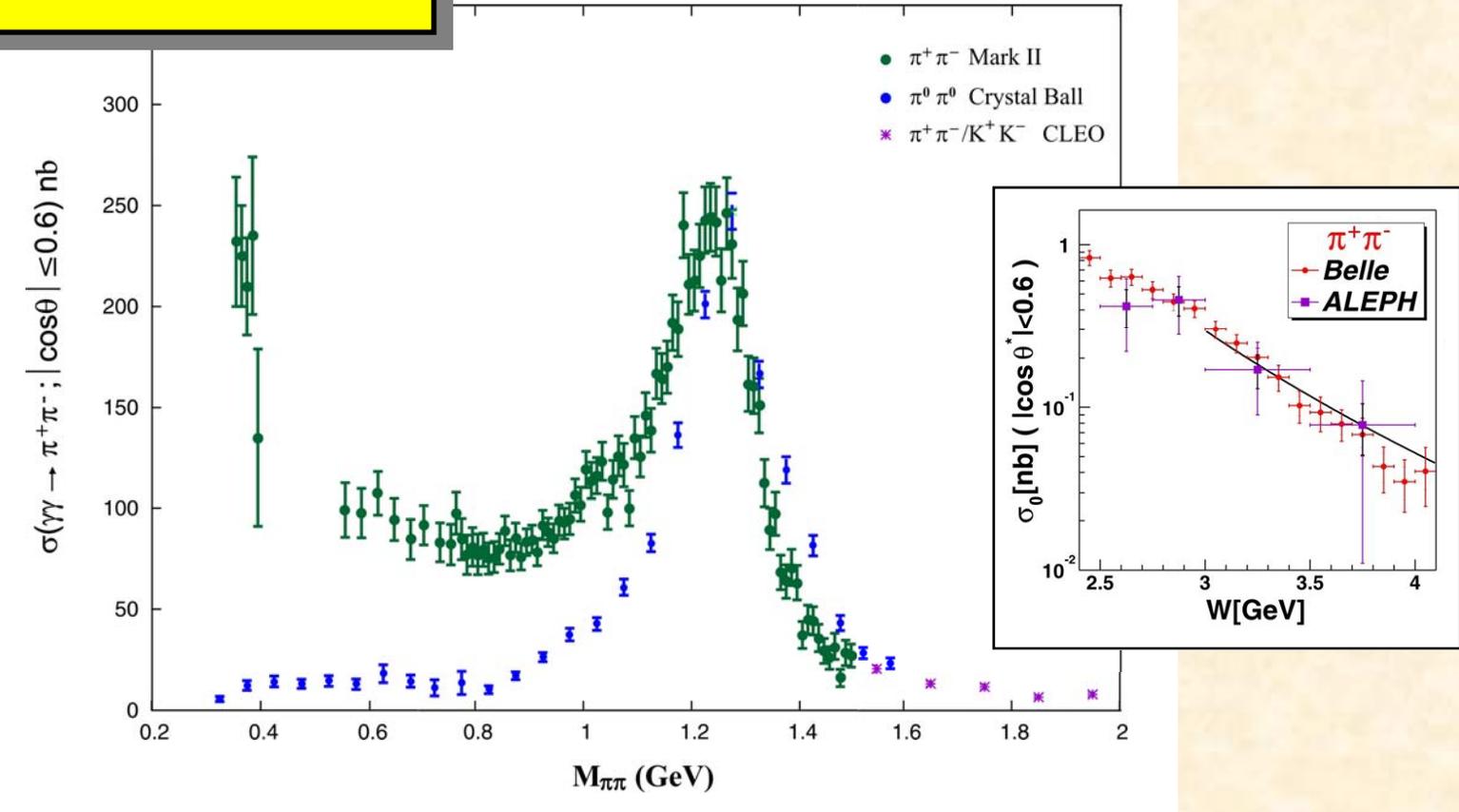
→

$\Gamma(\pi^0) : \Gamma(\eta) : \Gamma(\eta') \sim 1 : 60 : 500$

$$\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0$$



$$\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0$$

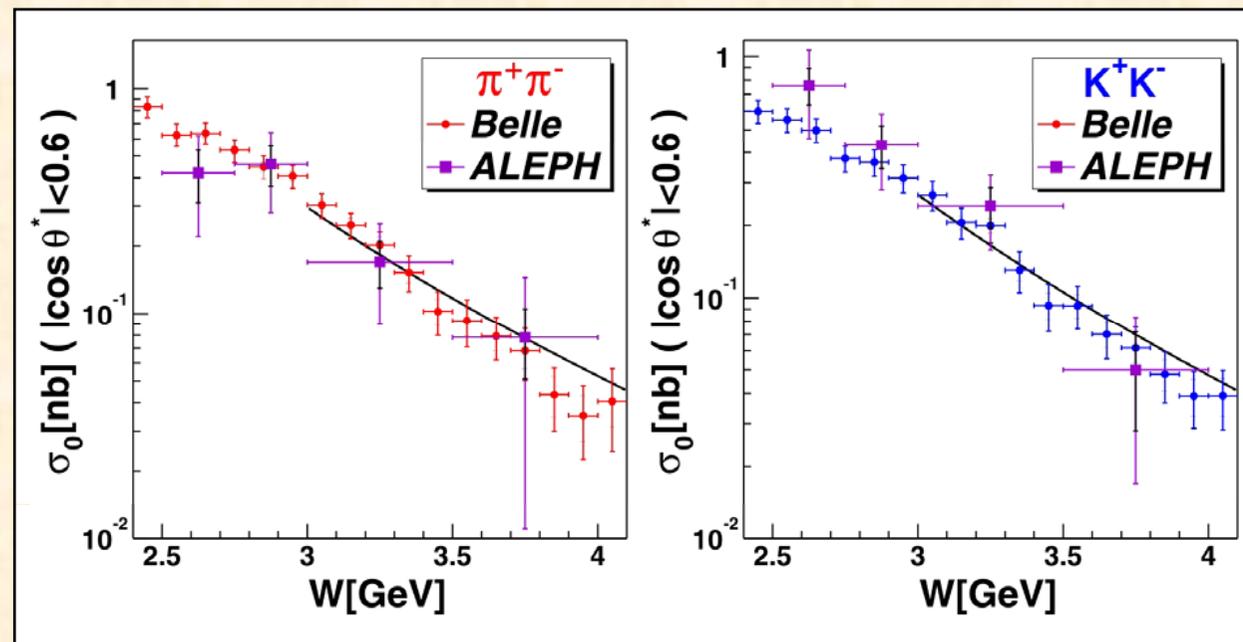
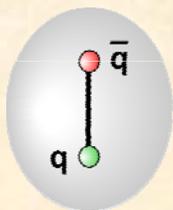
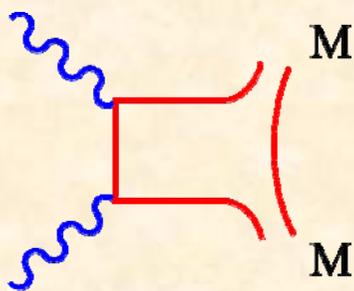




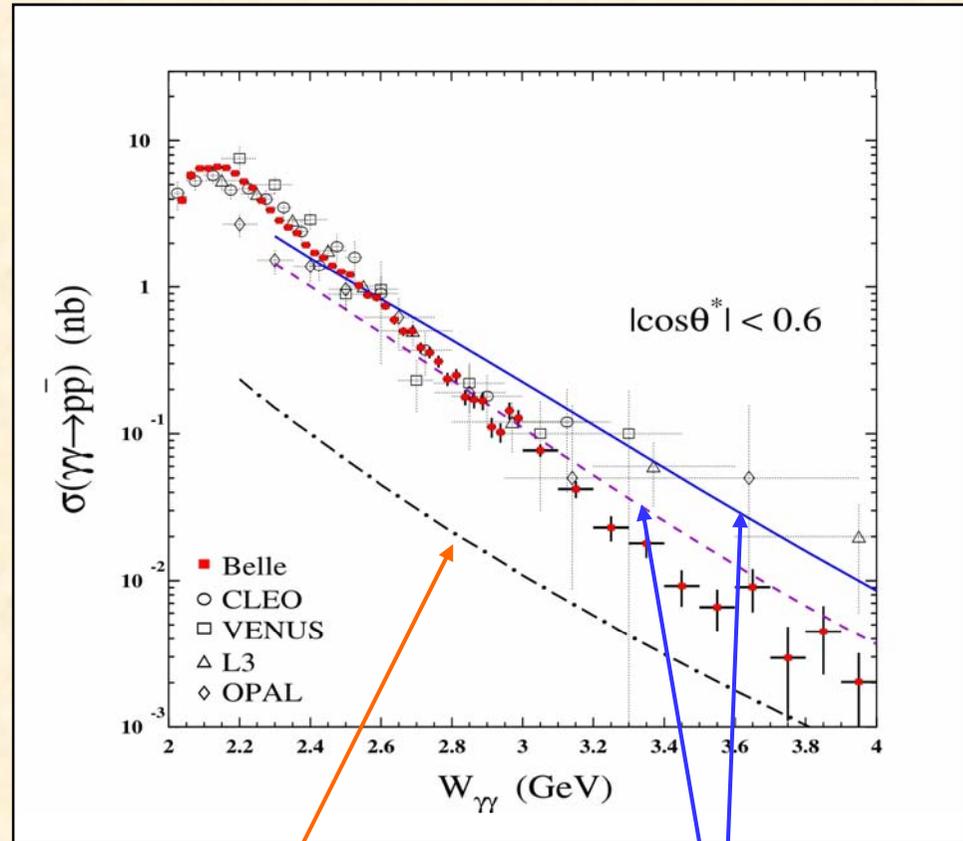
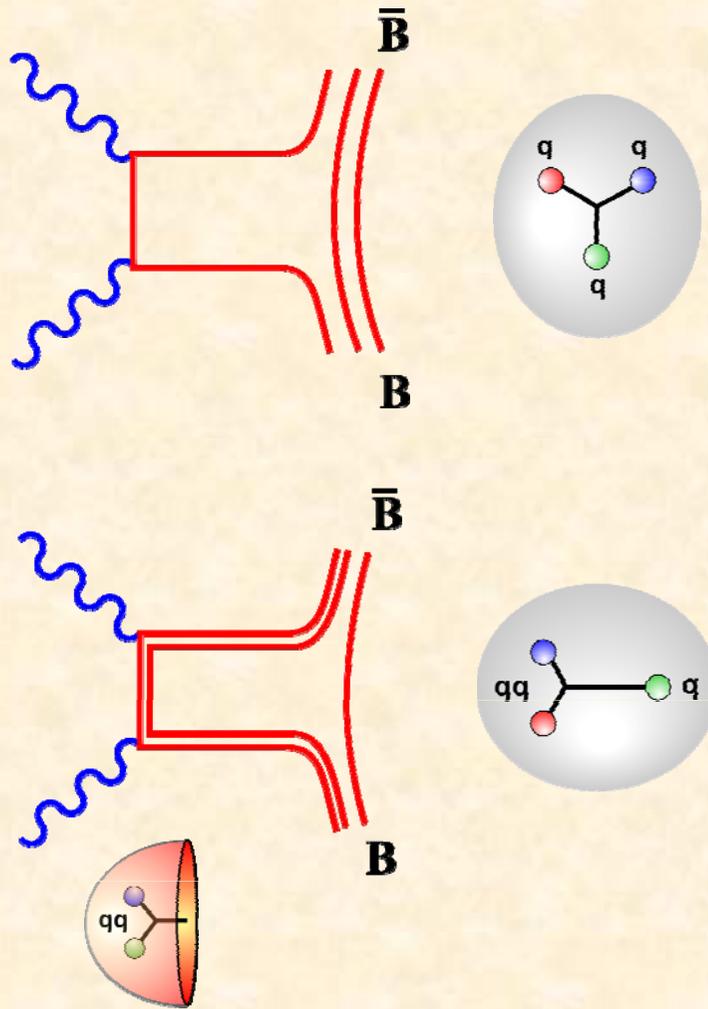
$$\gamma\gamma \longrightarrow \pi^+\pi^-, K^+K^-$$

$$\frac{d\sigma}{d|\cos\theta^*|} (\gamma\gamma \rightarrow M^+M^-) \simeq \frac{16\pi\alpha^2}{s} \frac{|F_M(s)|^2}{\sin^4\theta^*}$$

Brodsky, Lepage



$$\gamma\gamma \longrightarrow \bar{p}p$$

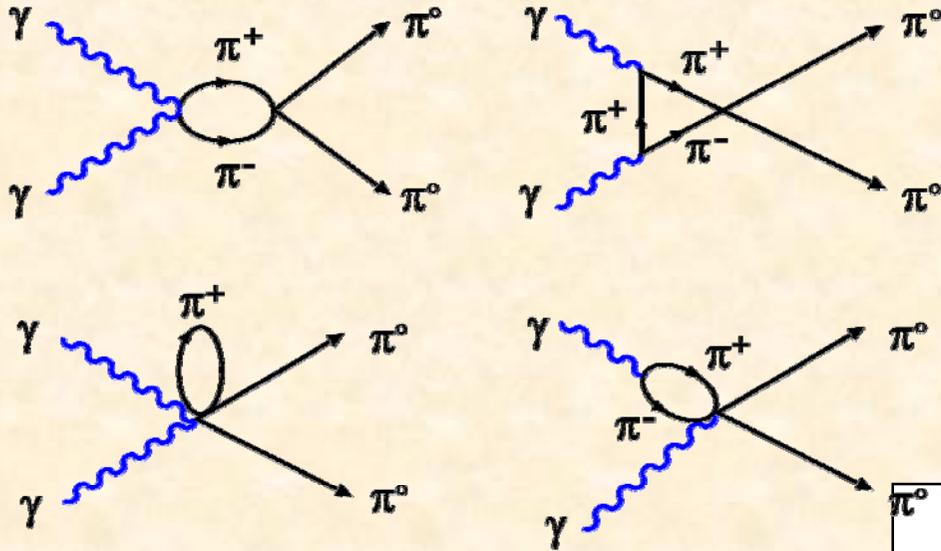


Farrar, Maina & Neri

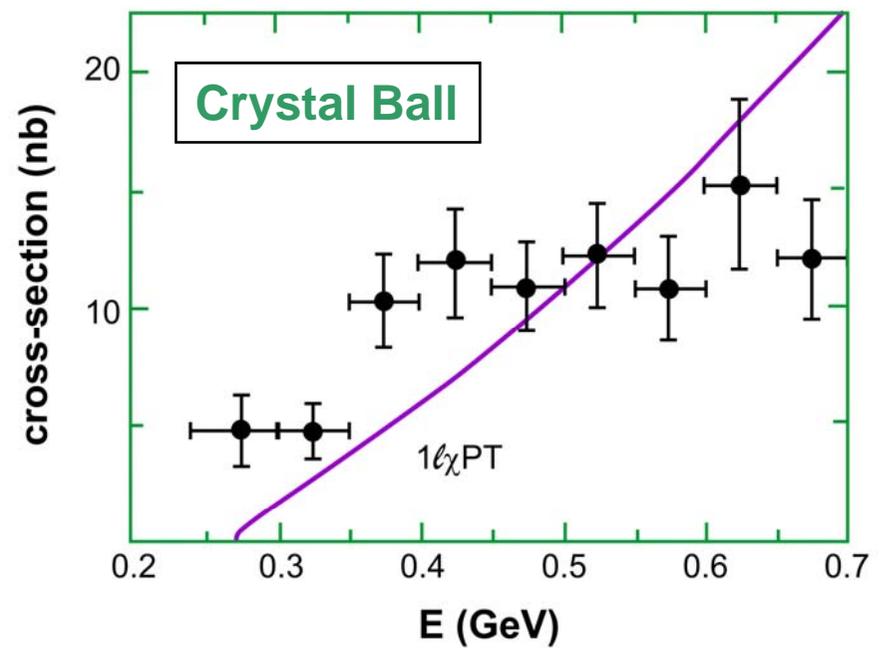
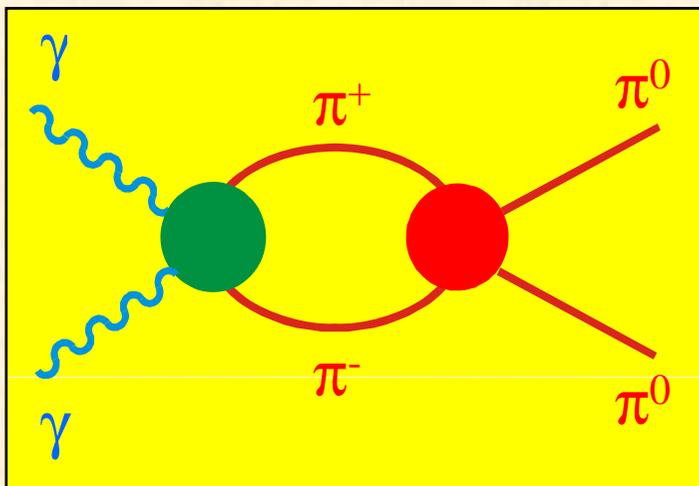
Berger & Schweiger

$$\gamma\gamma \longrightarrow \pi^0\pi^0$$

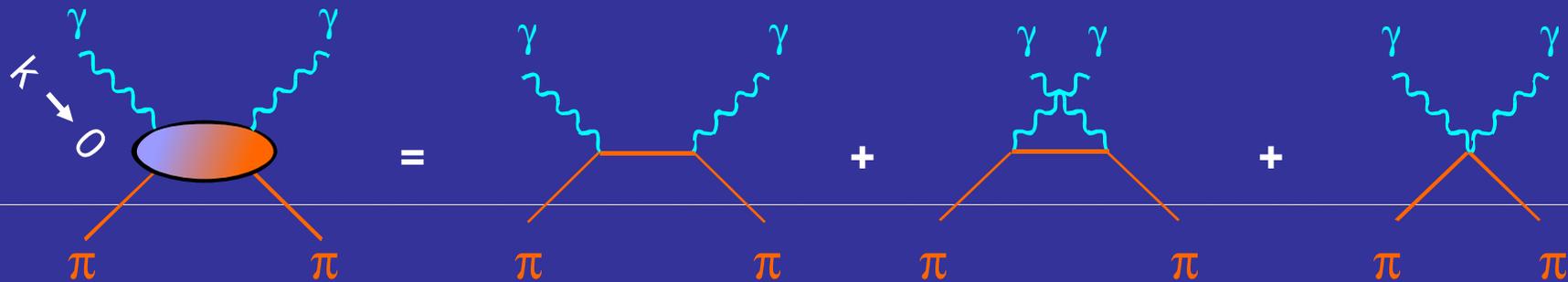
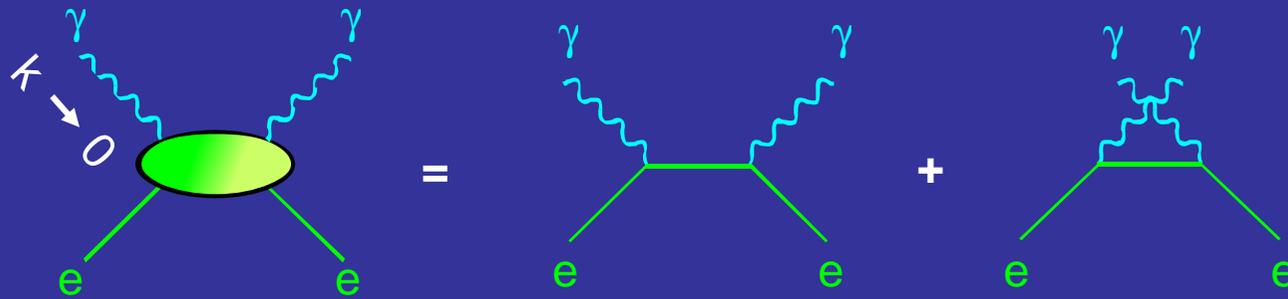
Chiral loops



Maiani:
“gold plated test of χ PT”

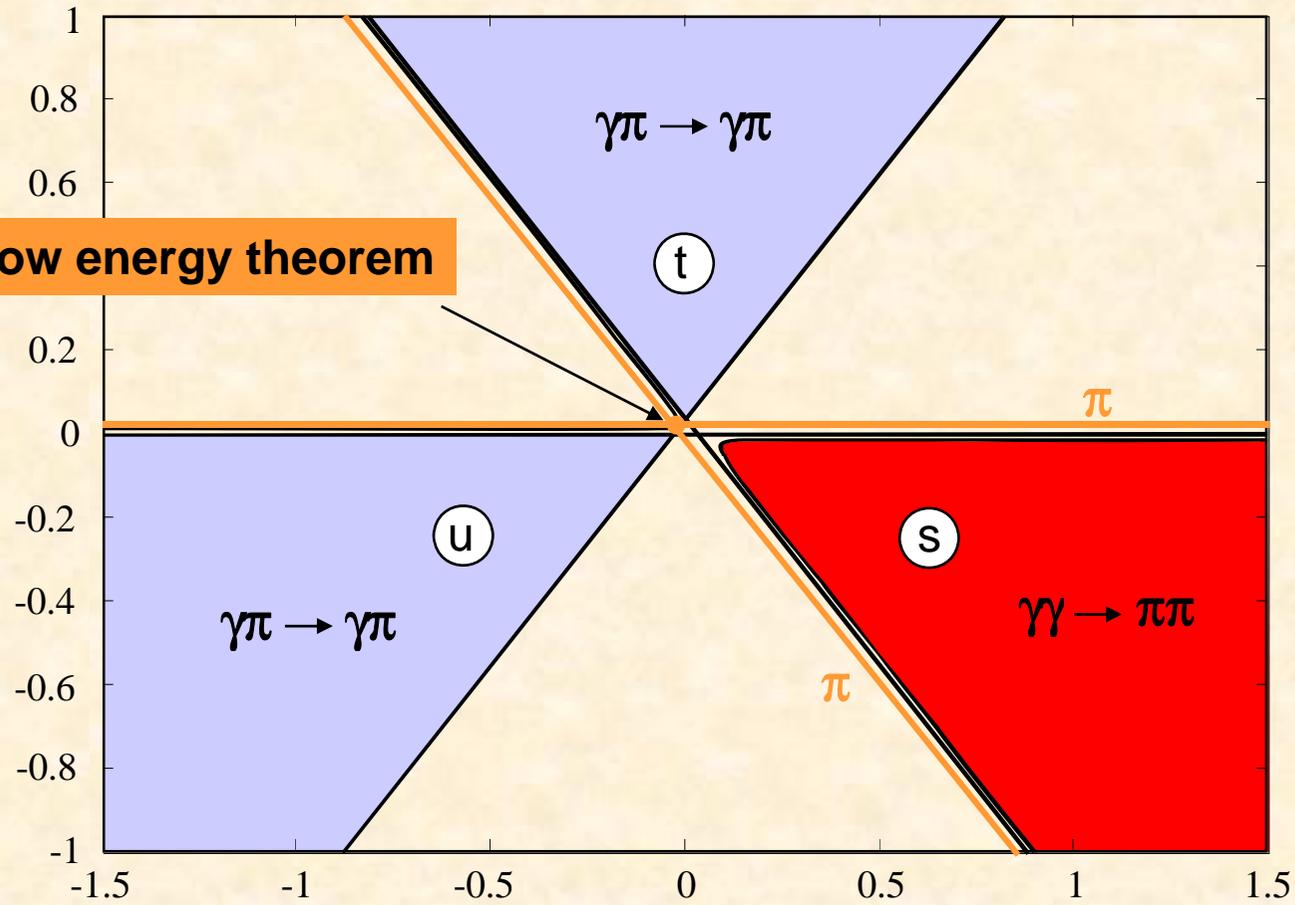


Low energy theorems

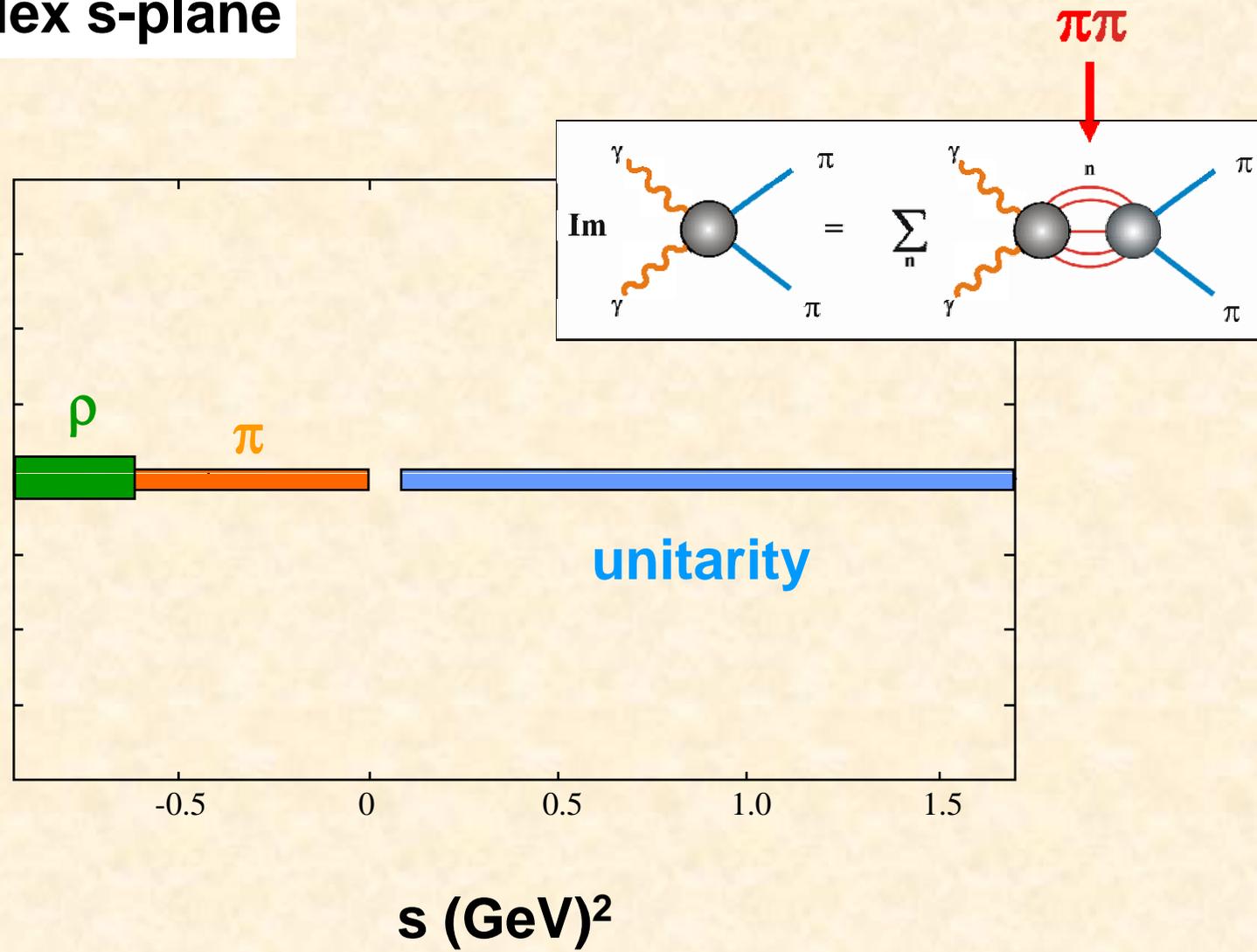


Mandelstam plane

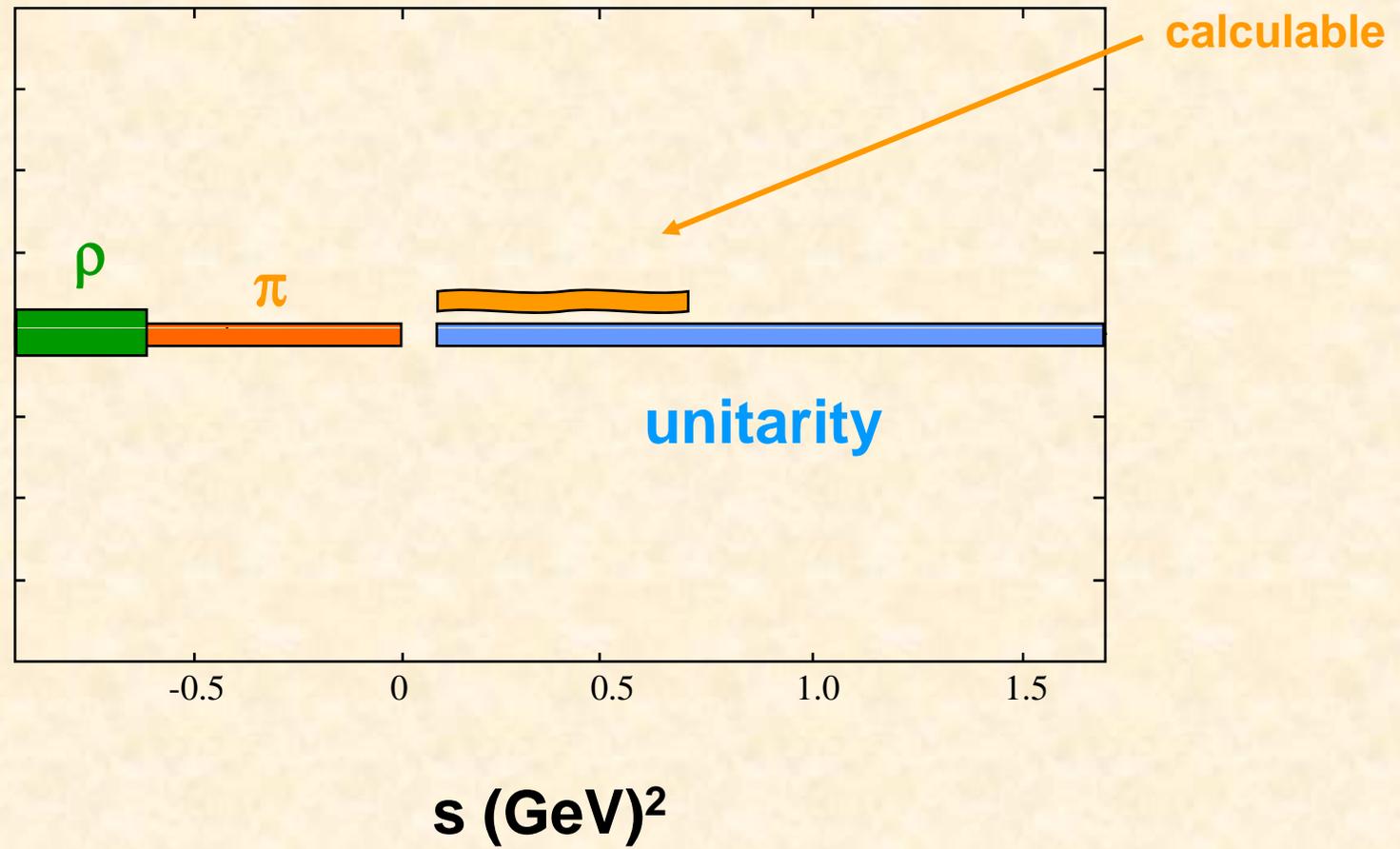
Low's low energy theorem



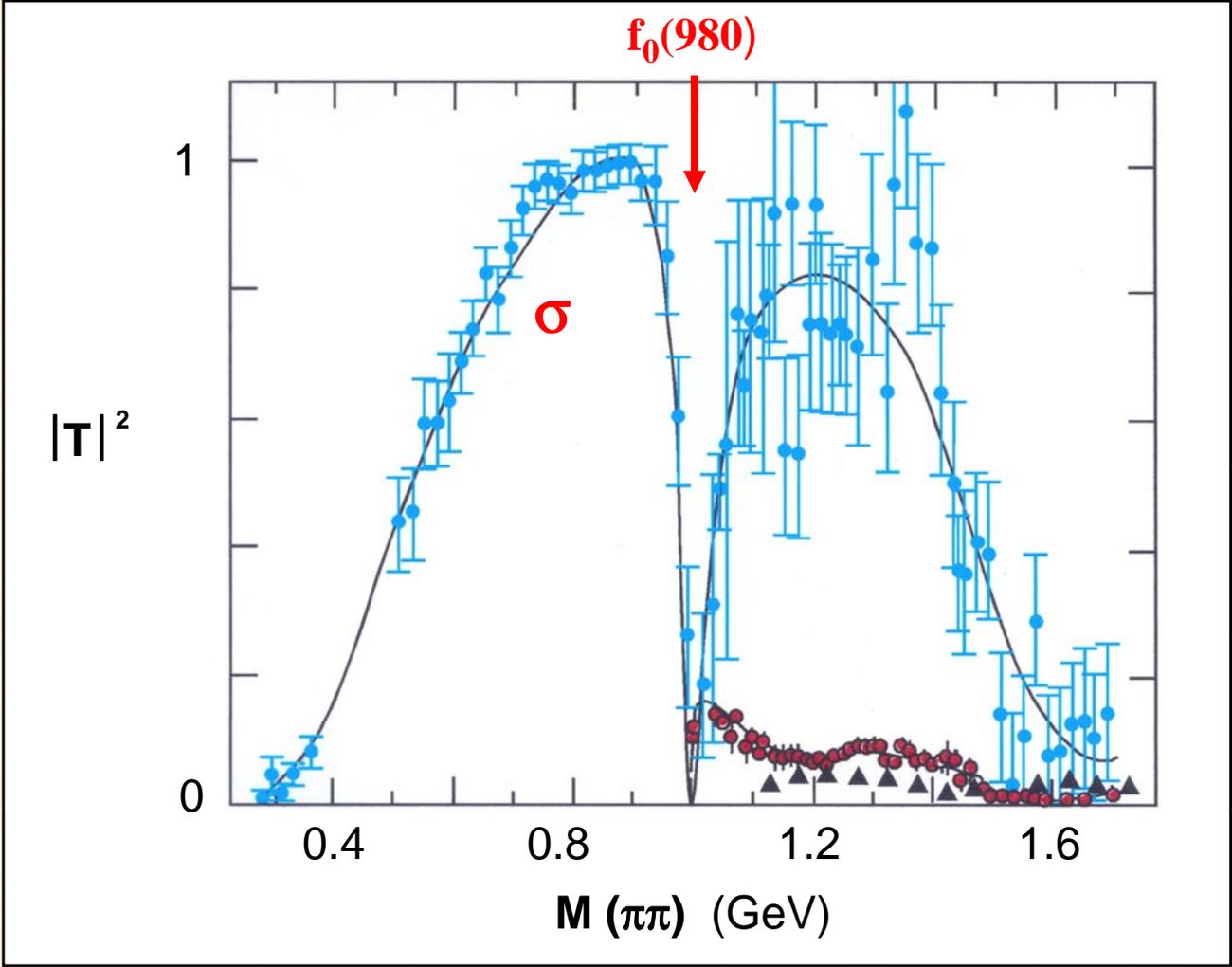
Complex s-plane



Complex s-plane



$I = J = 0$



● $\pi\pi \rightarrow \pi\pi$

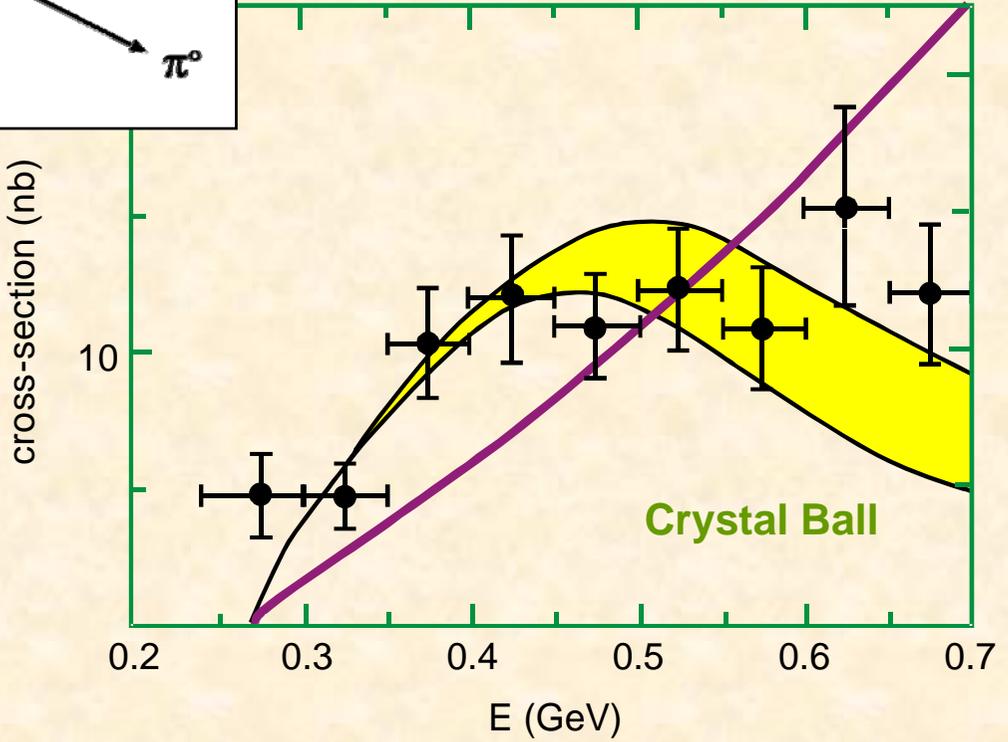
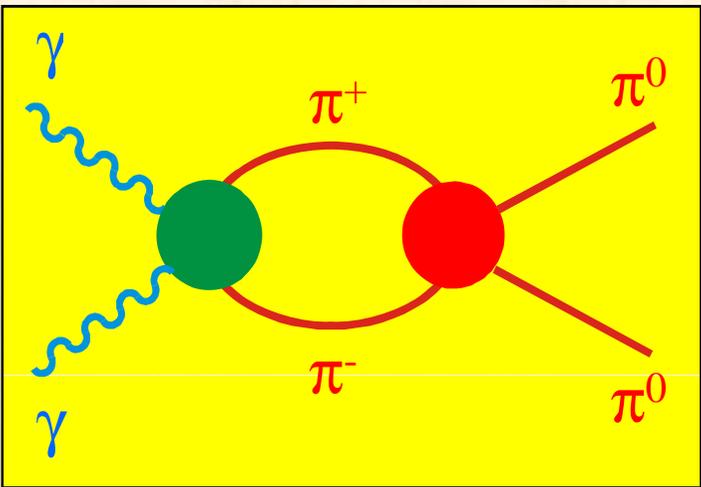
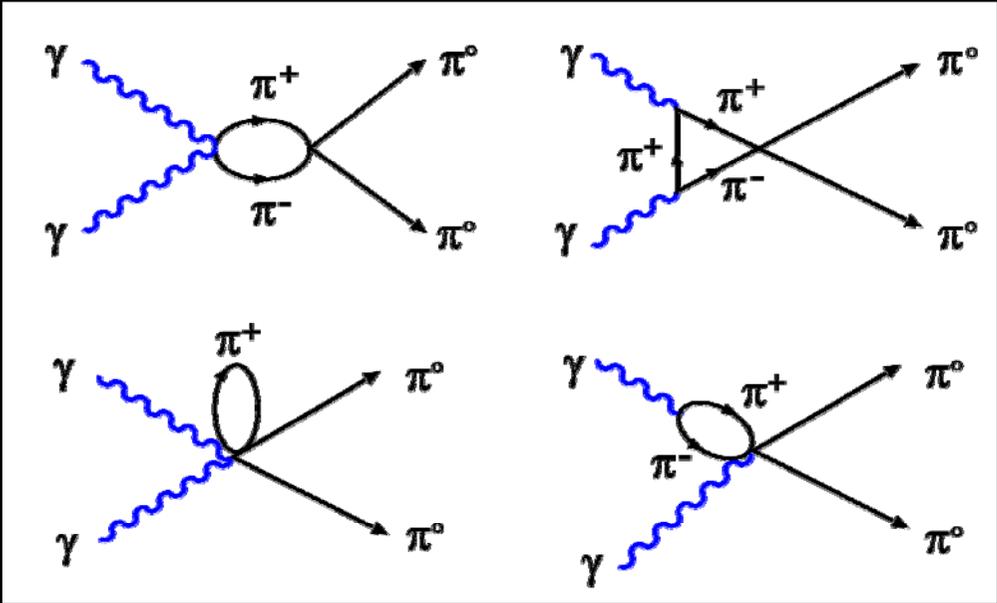
● $\pi\pi \rightarrow \bar{K}K$

▲ $\pi\pi \rightarrow \eta\eta$

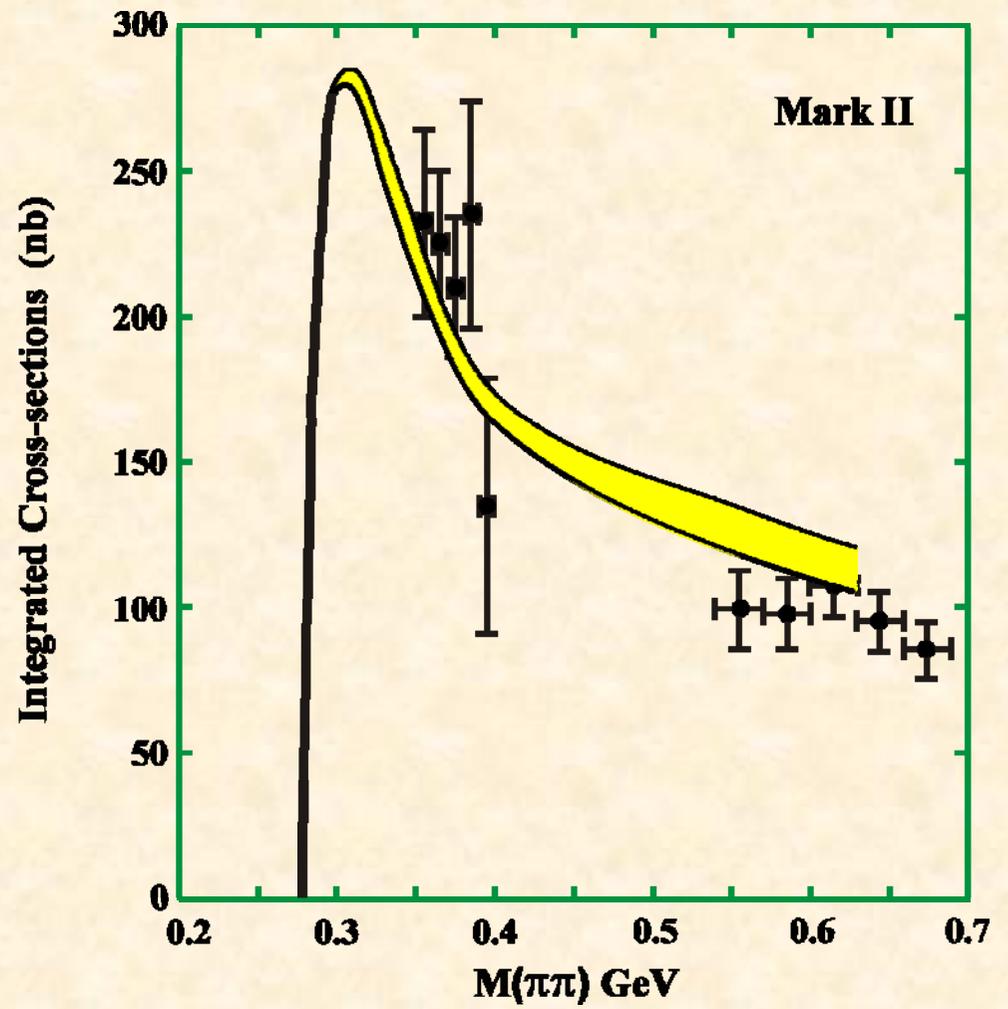
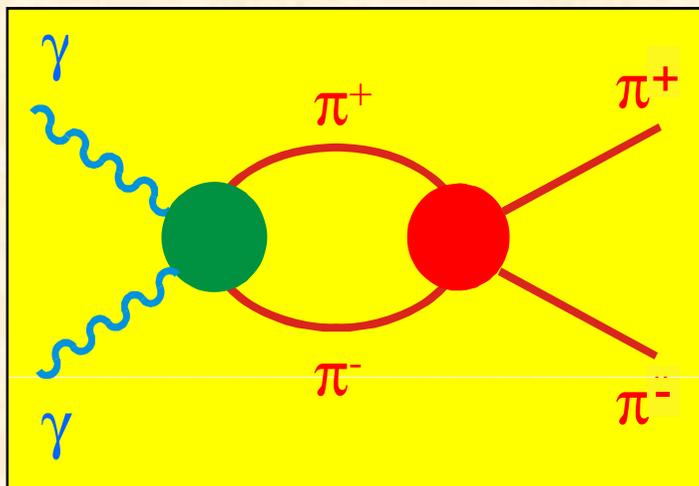
$$\gamma\gamma \rightarrow \pi^0\pi^0$$

Chiral loops

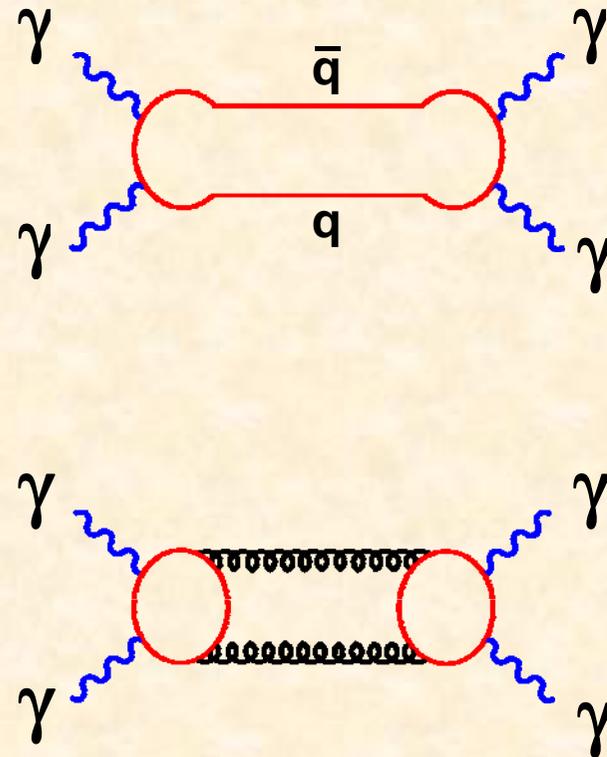
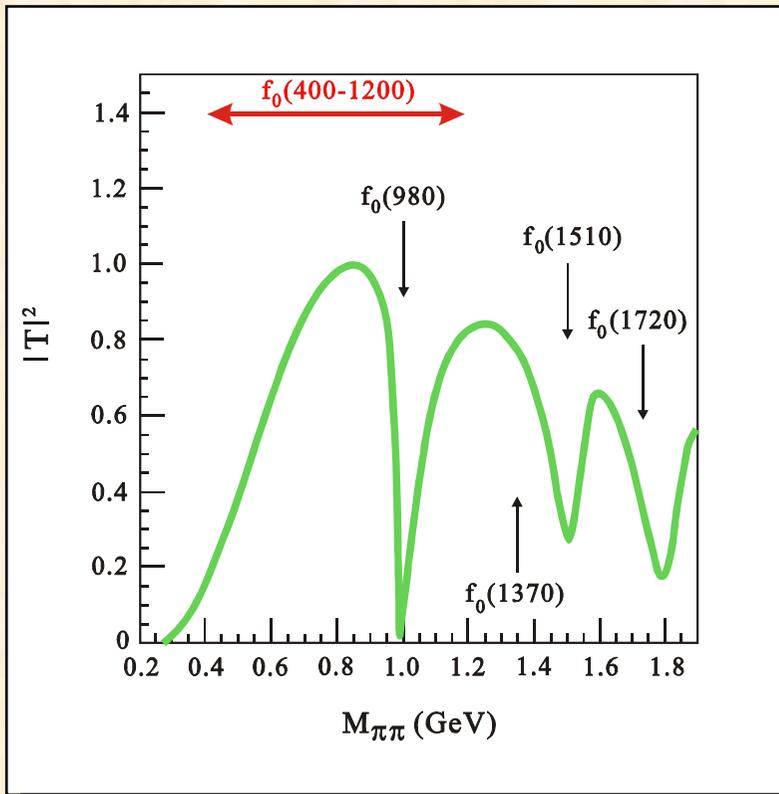
Maiani:
“gold plated test of χ PT”



$$\gamma\gamma \rightarrow \pi^+\pi^-$$

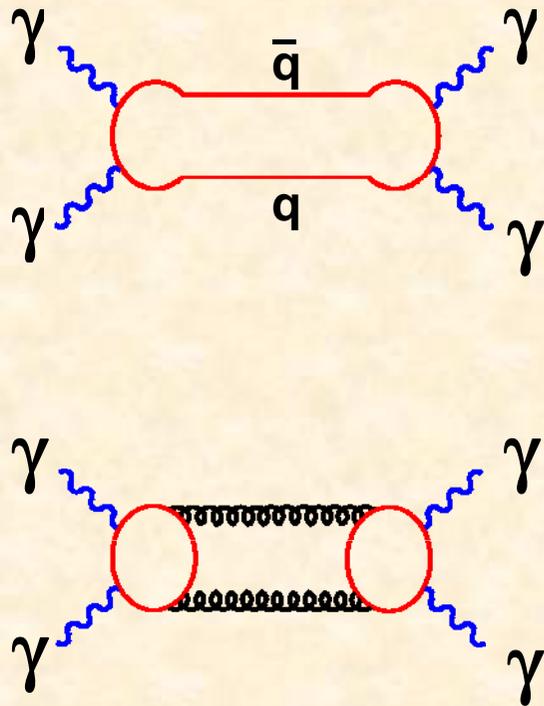


Resonance physics



$q\bar{q}$, $qq\bar{q}\bar{q}$, $K\bar{K}$, gg ?
 $\gamma\gamma$ has the answer

Resonance physics



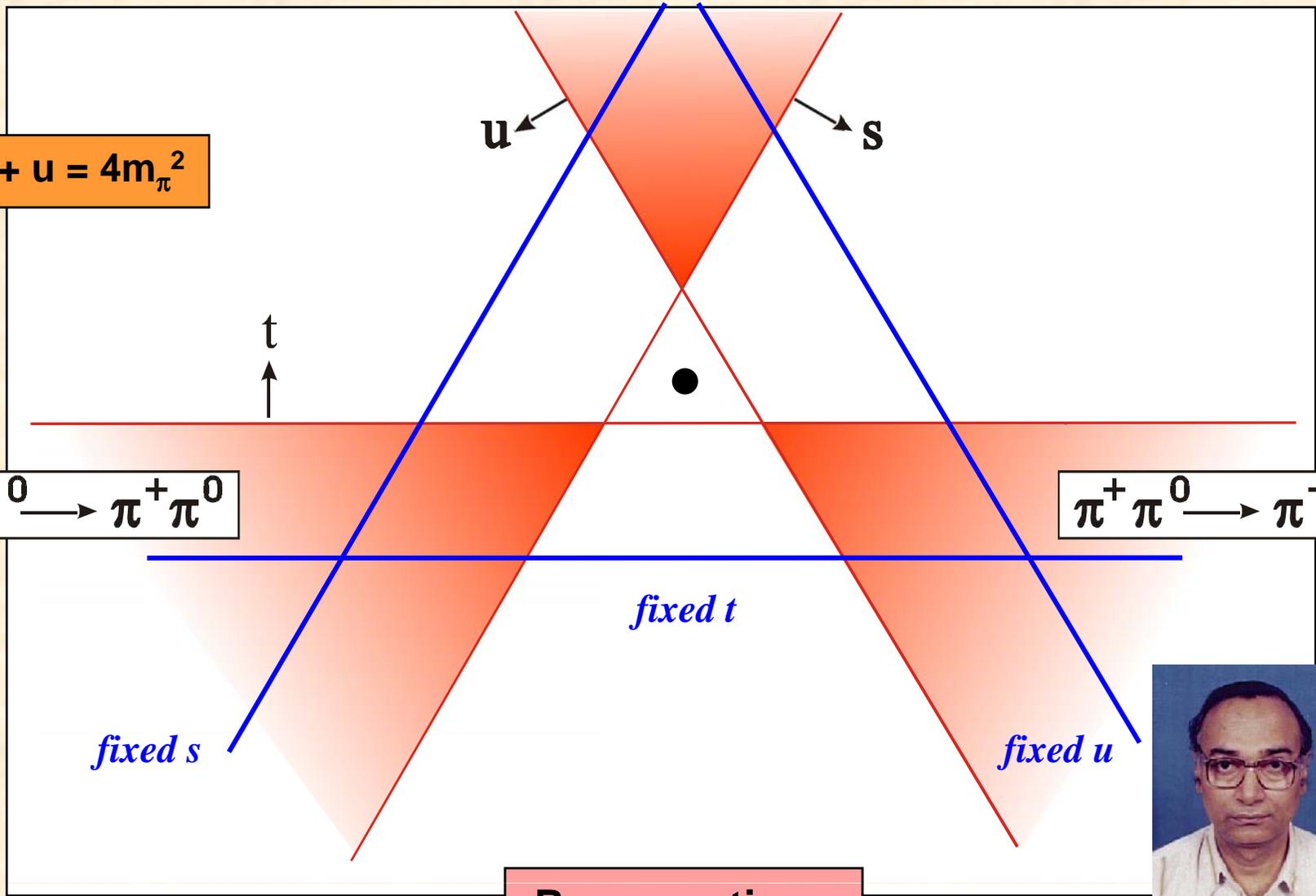
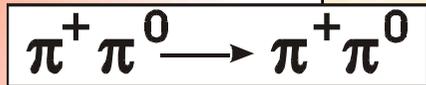
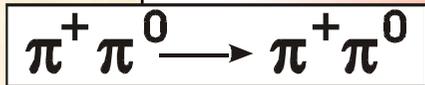
$\Gamma(0^{++} \rightarrow \gamma\gamma)$ keV

$(\bar{u}u + \bar{d}d)/\sqrt{2}$	$\bar{s}s$	$[\bar{n}s][ns], n = (u,d)$	$\bar{K}K$
4	0.2	0.27	0.6
Babcock & Rosner	Barnes	Achasov <i>et al</i>	Achasov

$q\bar{q}$, $qq\bar{q}\bar{q}$, $K\bar{K}$, gg ?
 $\gamma\gamma$ has the answer



$$s + t + u = 4m_\pi^2$$

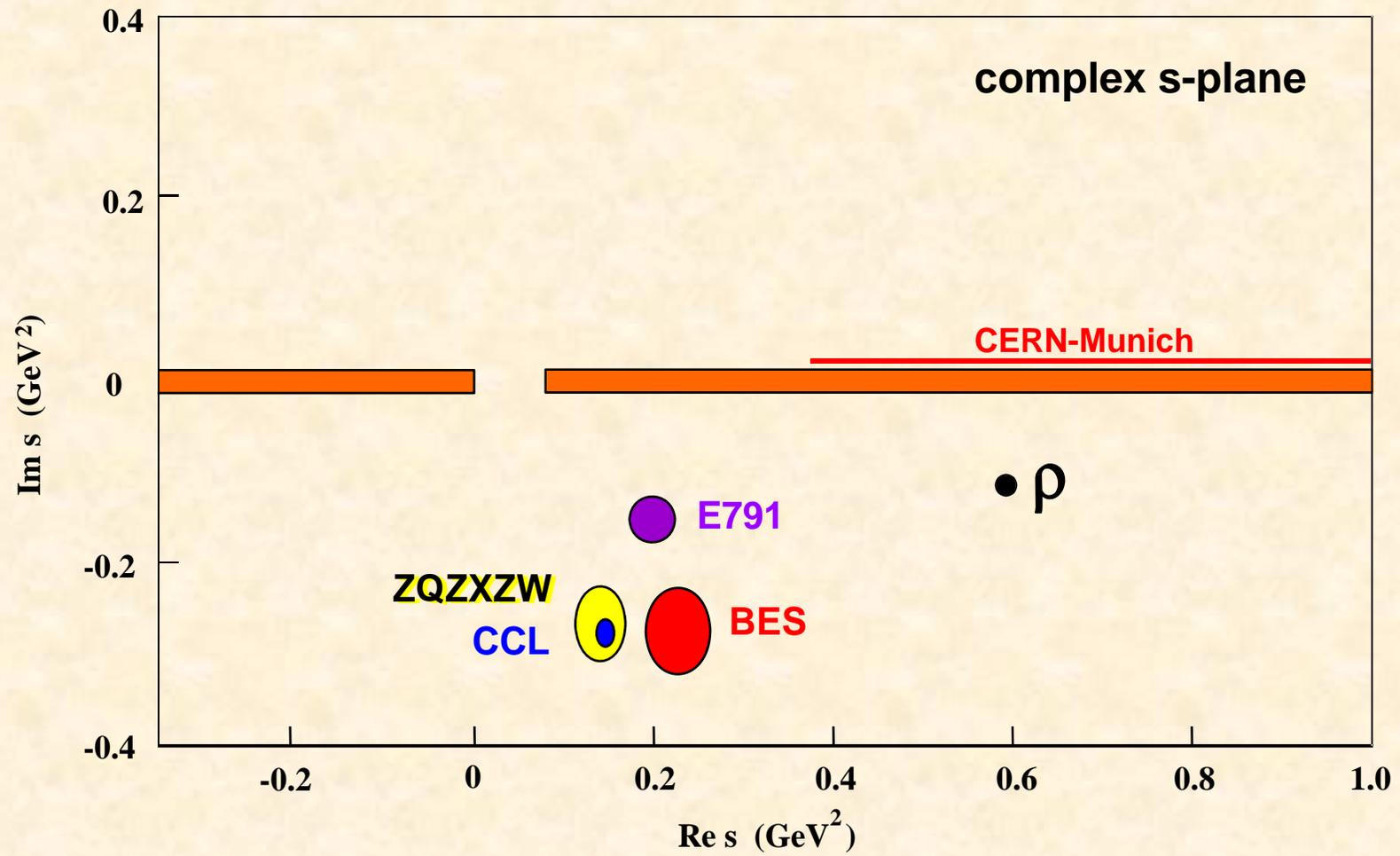


Roy equations



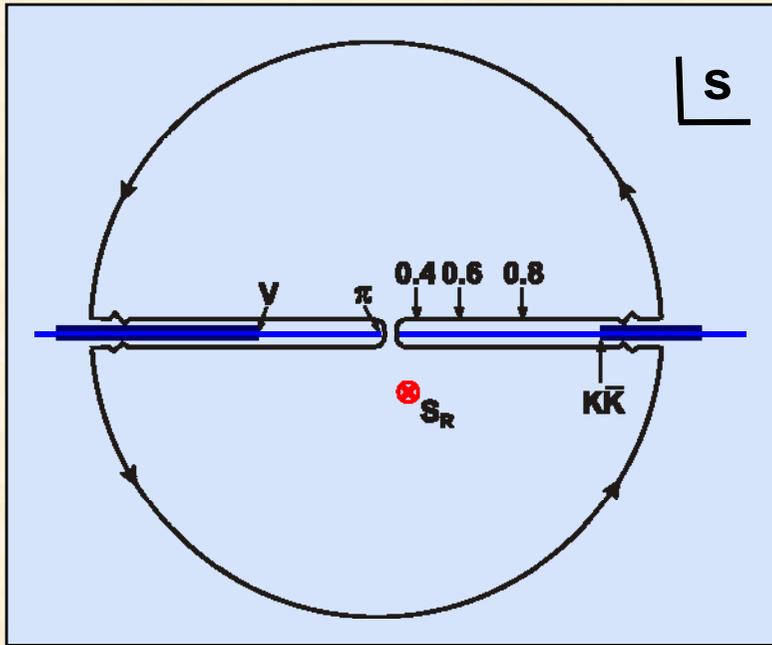
S M Roy

$\pi\pi : I = 0, J = 0$



Zhou, Qin, Zhang, Xiao, Zheng & Wu

Caprini, Colangelo, & Leutwyler

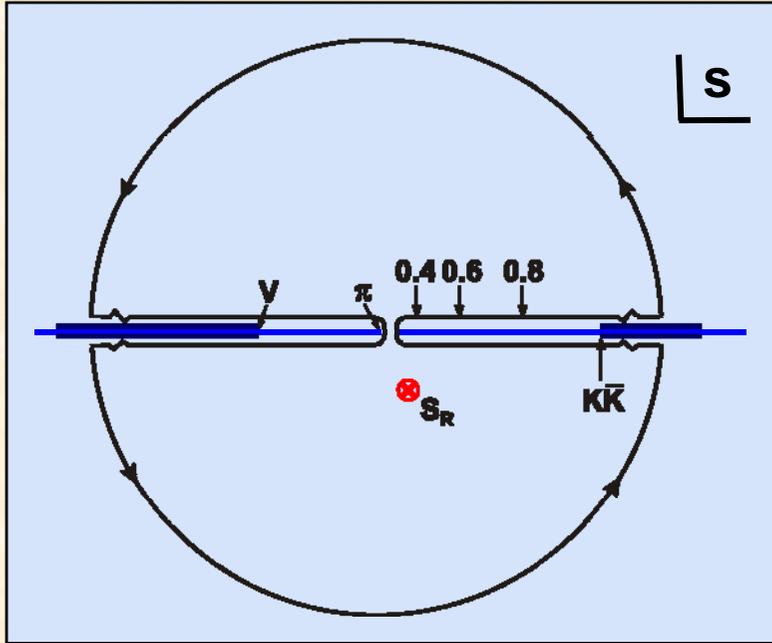


Into the complex plane

$$E_R = 441 - i 272 \text{ MeV}$$

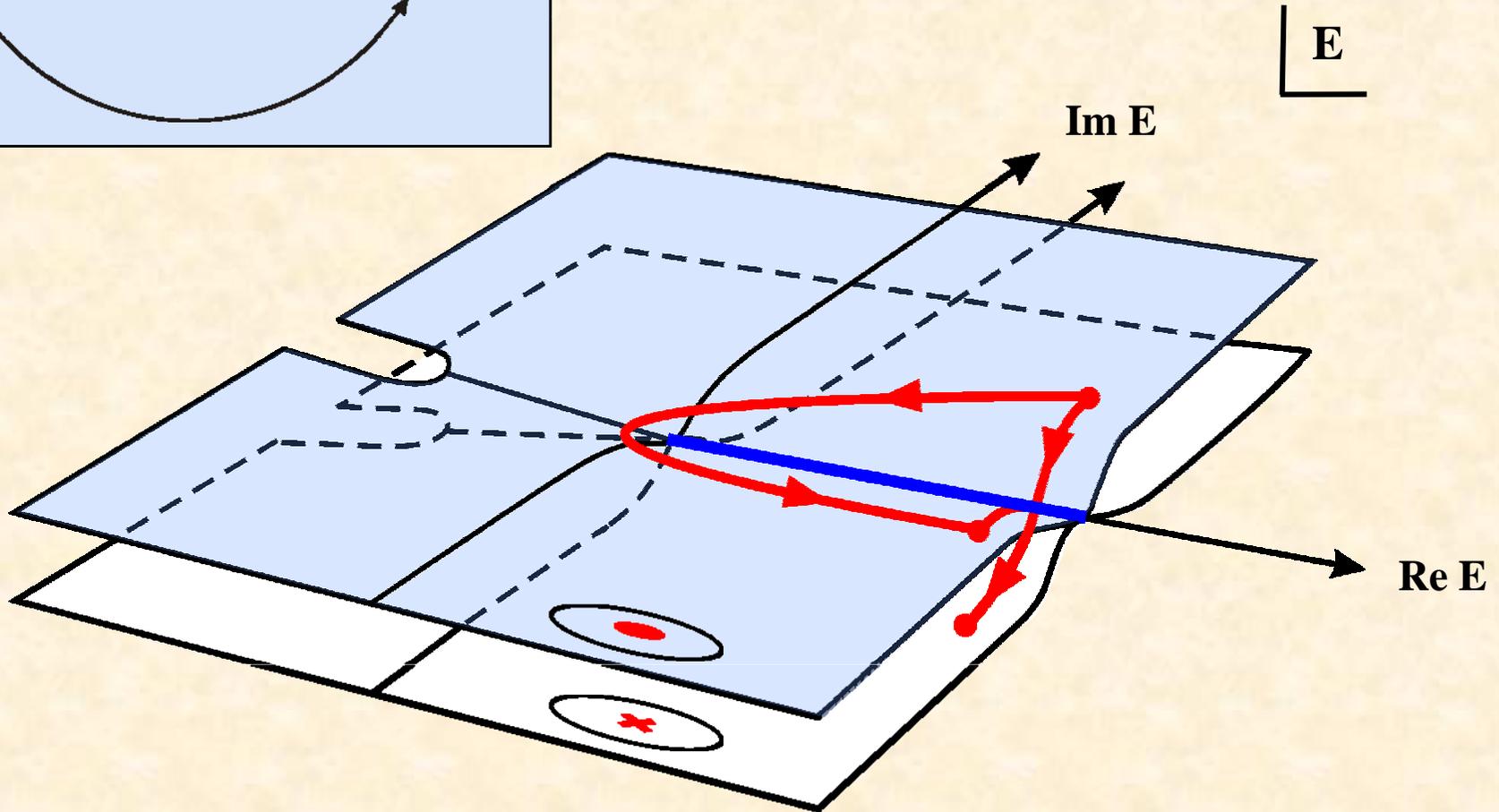
Caprini, Colangelo, & Leutwyler





Into the complex plane

$\Gamma(\sigma \rightarrow \gamma\gamma) = 4.1 \pm 0.3 \text{ keV}$



$$\text{If } \sigma = (u\bar{u} + d\bar{d}) / \sqrt{2}$$

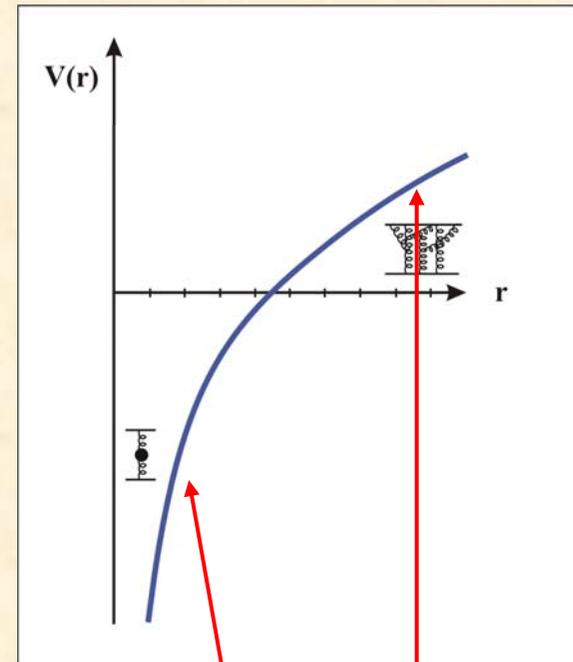
$$\frac{\Gamma(\sigma \rightarrow \gamma\gamma)}{\Gamma(f_2 \rightarrow \gamma\gamma)} = \frac{15}{4} \left(\frac{m_\sigma}{m_{f_2}} \right)^n$$

Goble, Rosenfeld & Rosner

$$\Gamma(f_2 \rightarrow \gamma\gamma) \approx 3 \text{ keV}$$



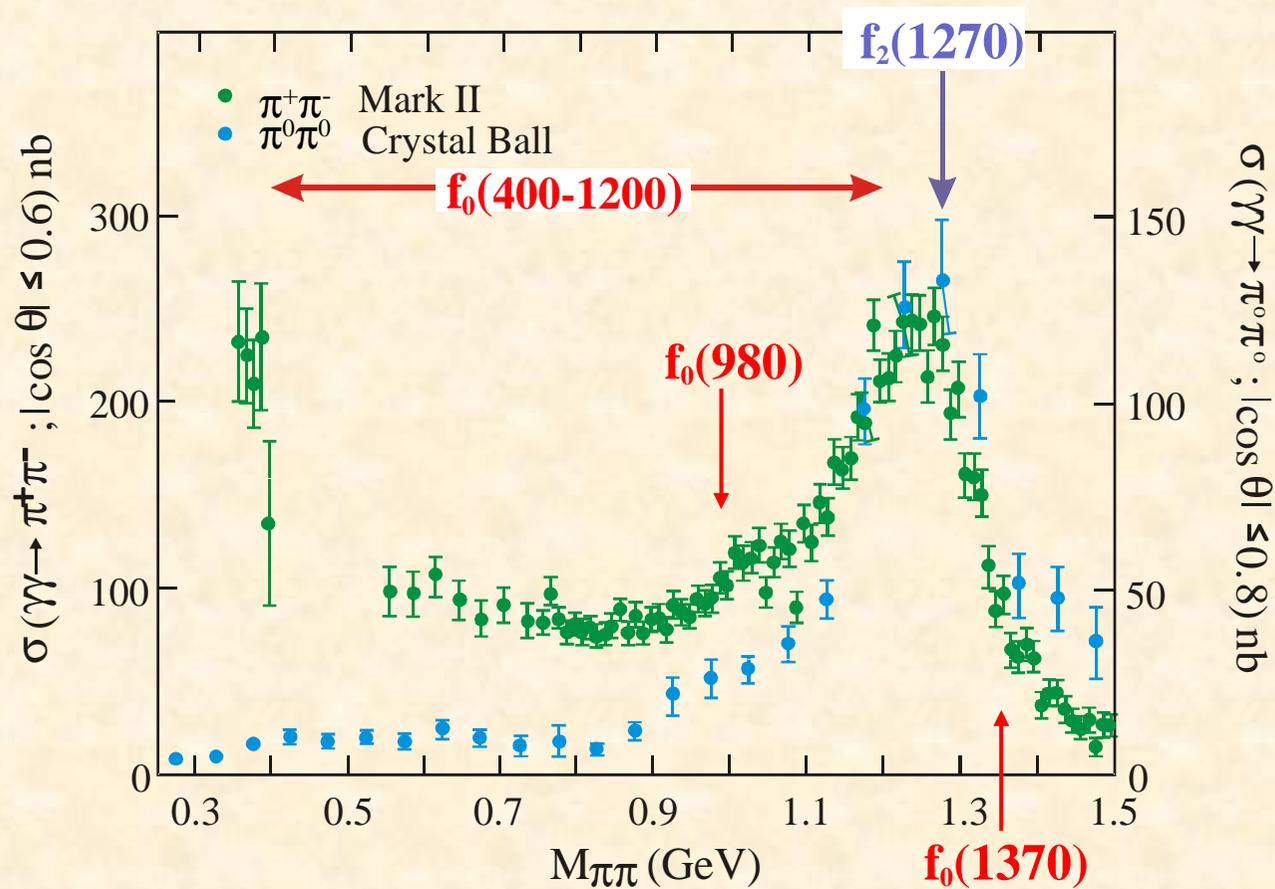
$$\Gamma(\sigma \rightarrow \gamma\gamma) \approx 4 \text{ keV}$$



n = 3

n = 1

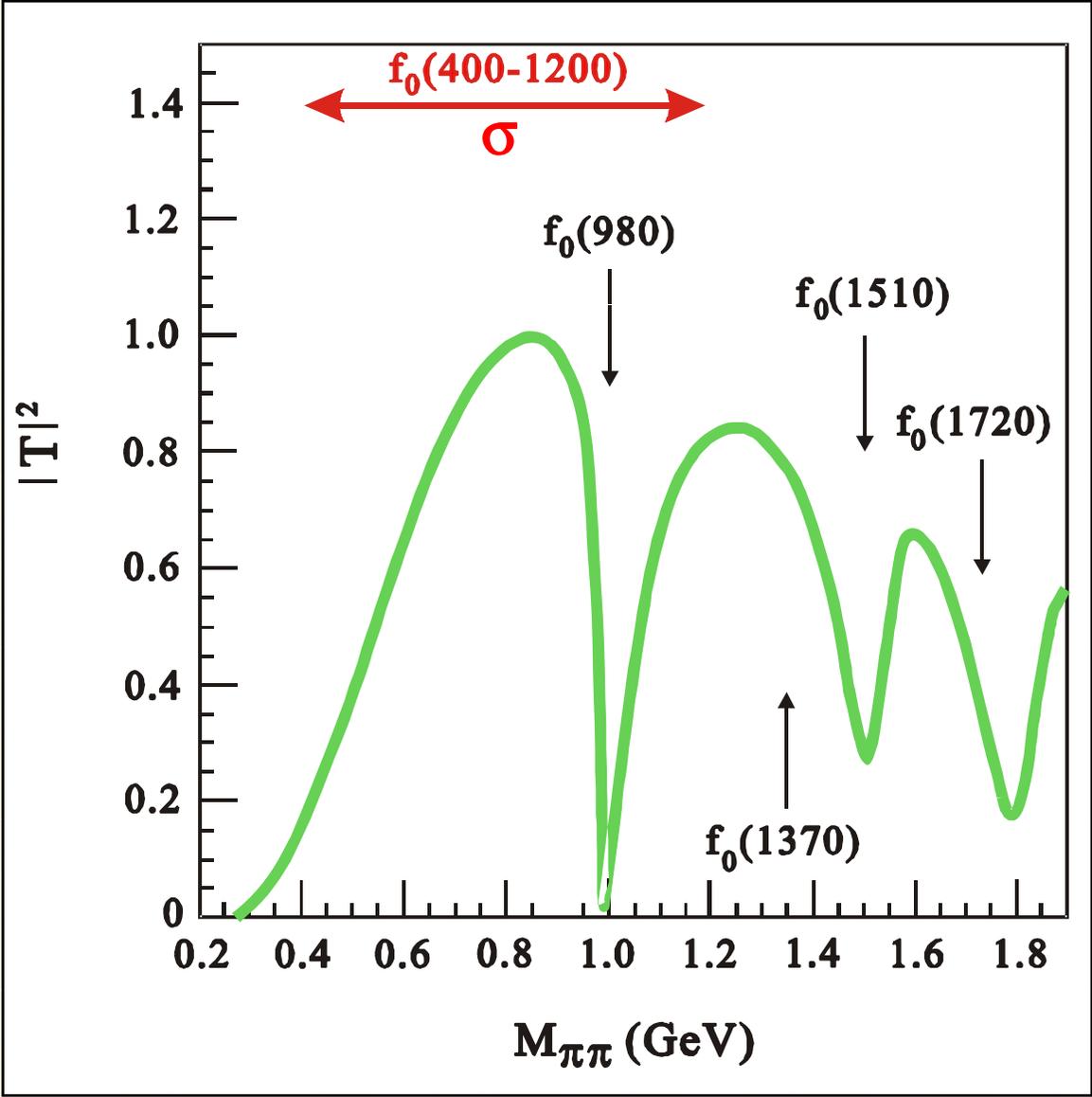
$\gamma\gamma$ couplings



- limited angular coverage
- no polarisation

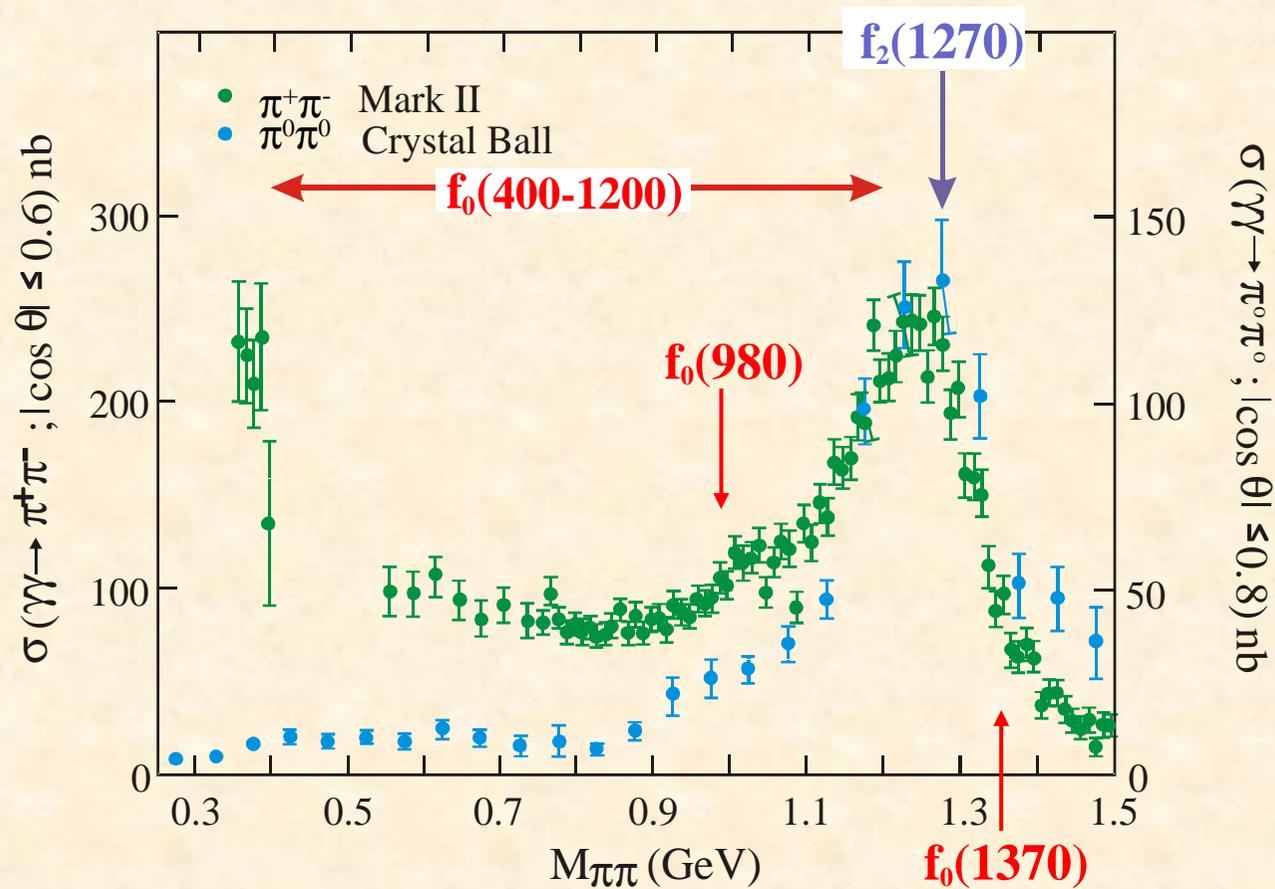
– **Amplitude analysis**

I = J = 0



Zou

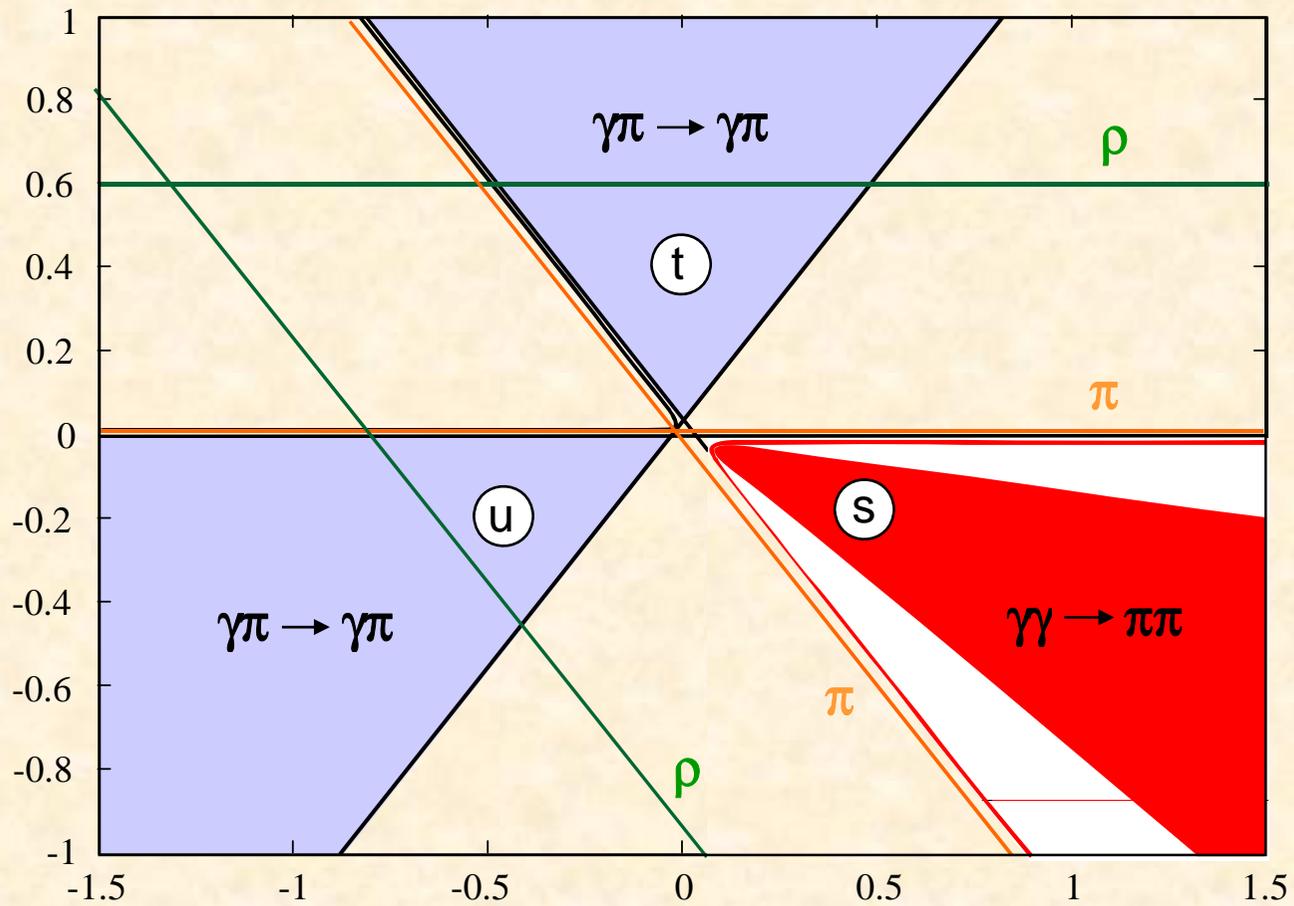
$\gamma\gamma$ couplings

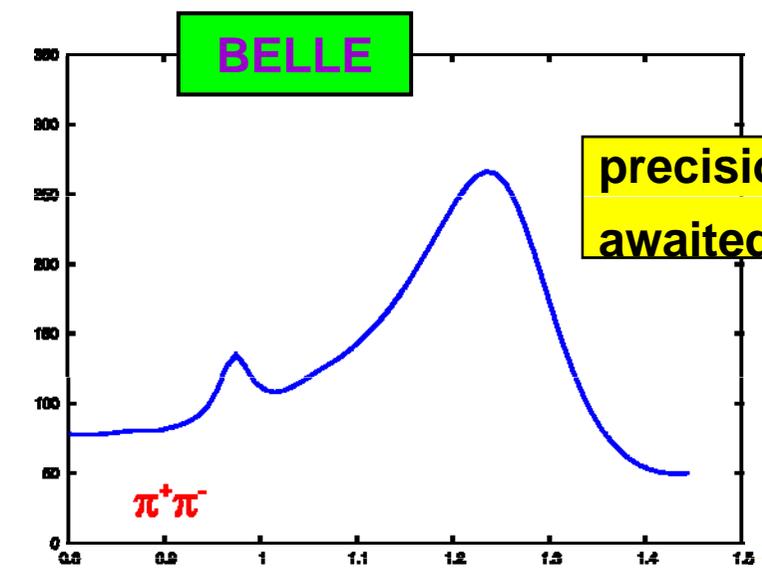
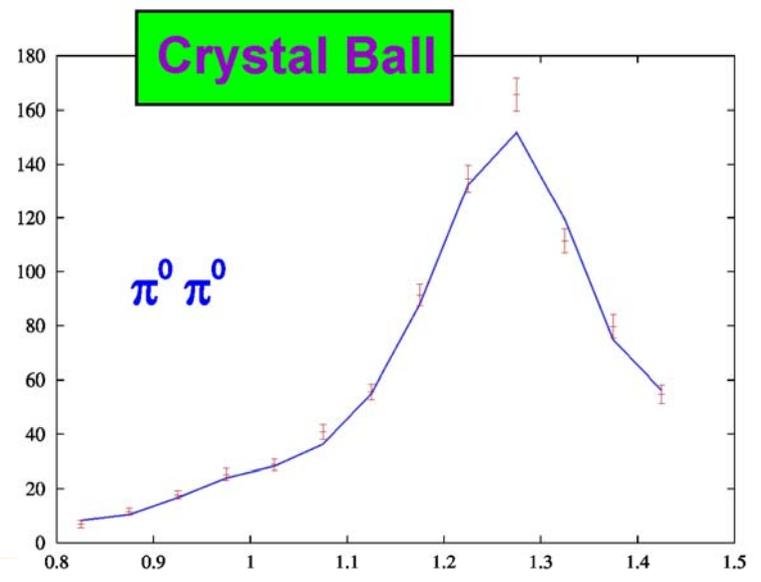
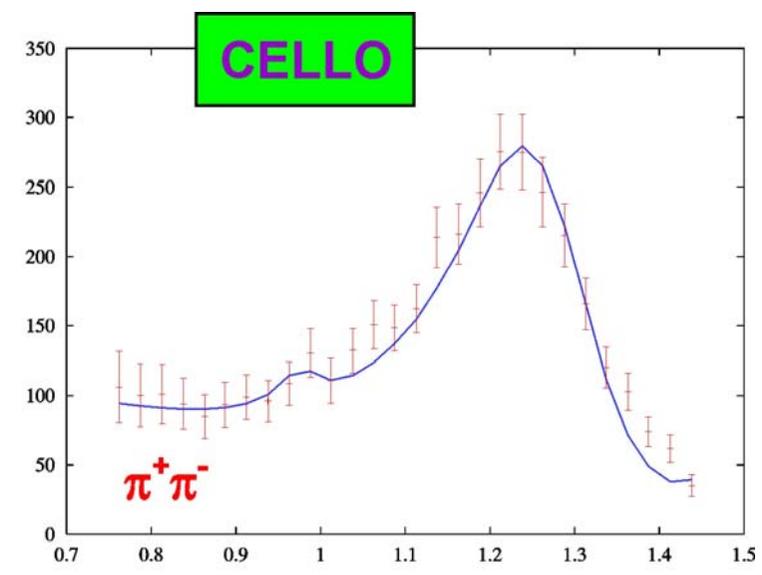
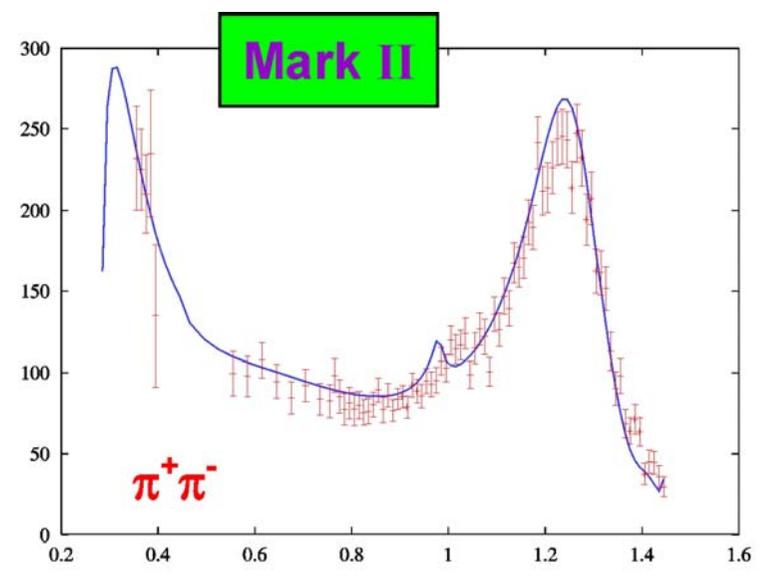


- limited angular coverage
- no polarisation

– **Amplitude analysis**

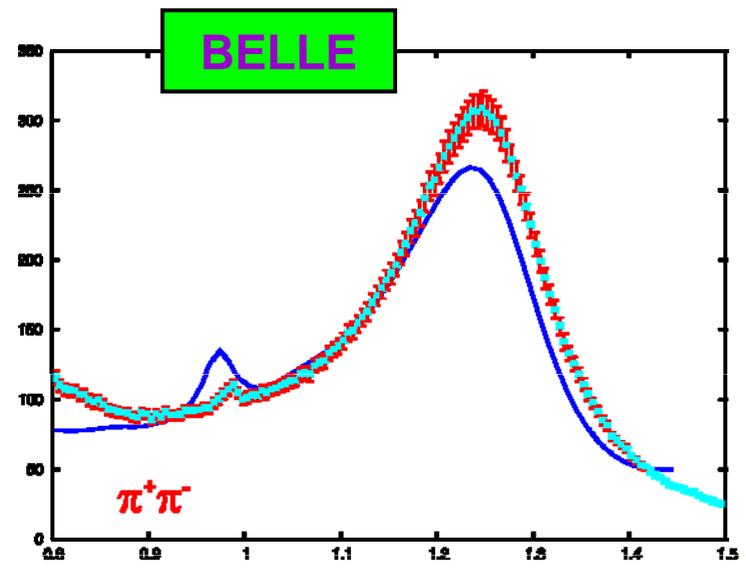
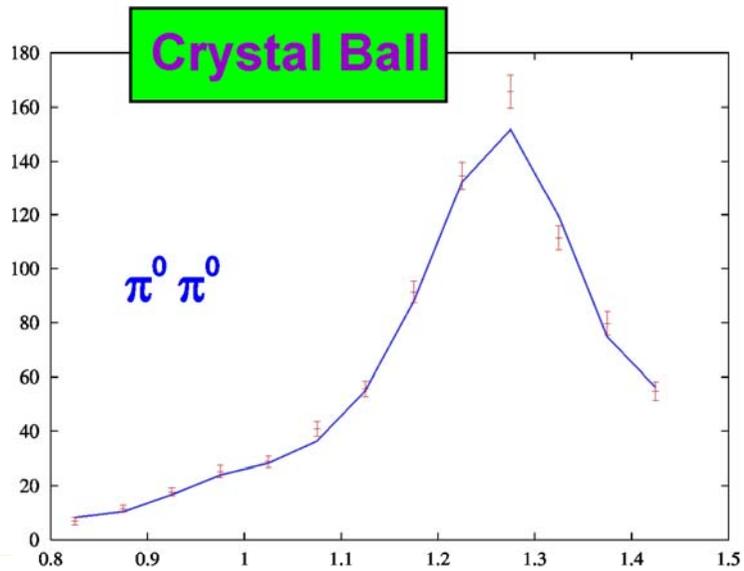
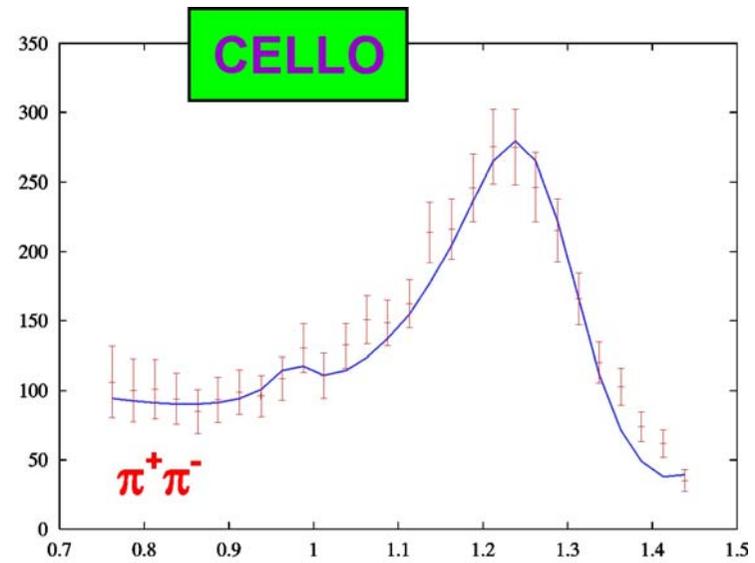
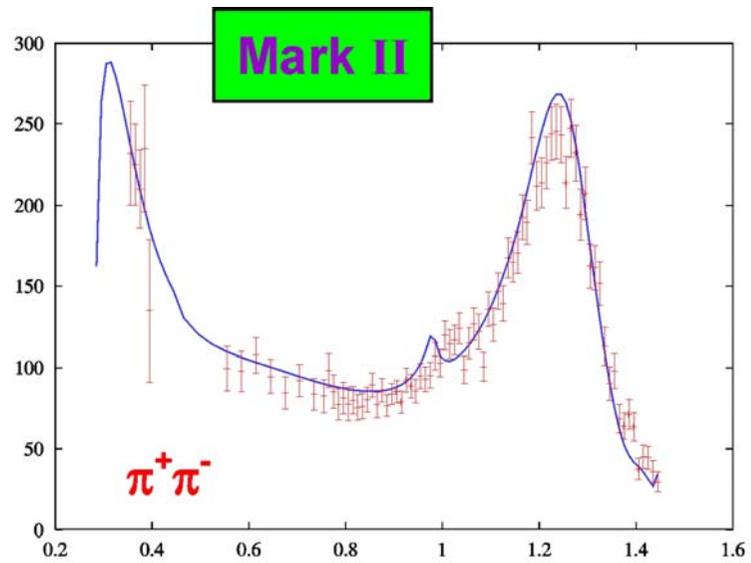
Mandelstam plane





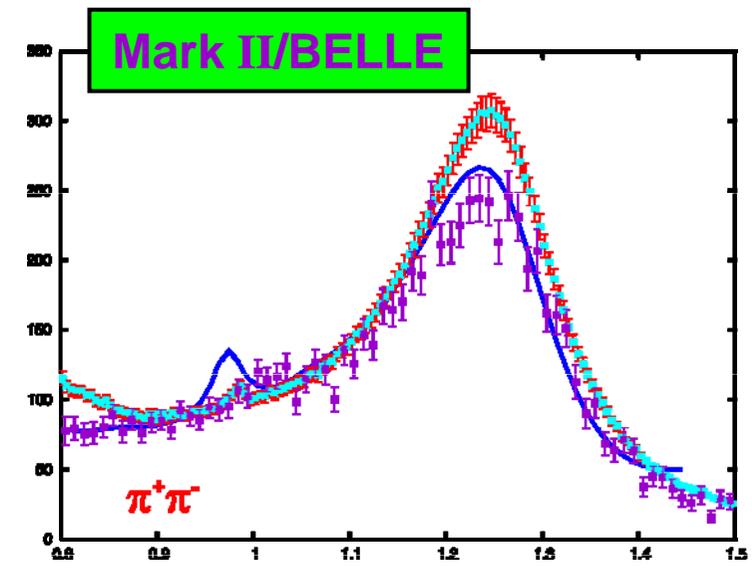
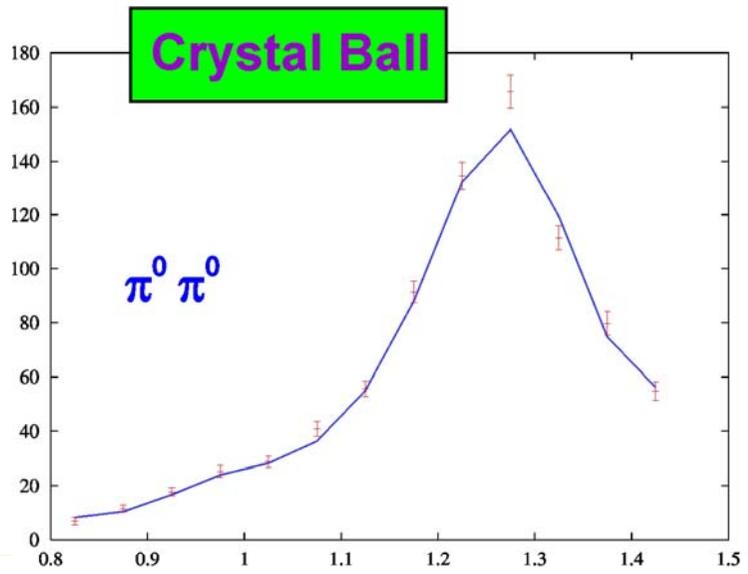
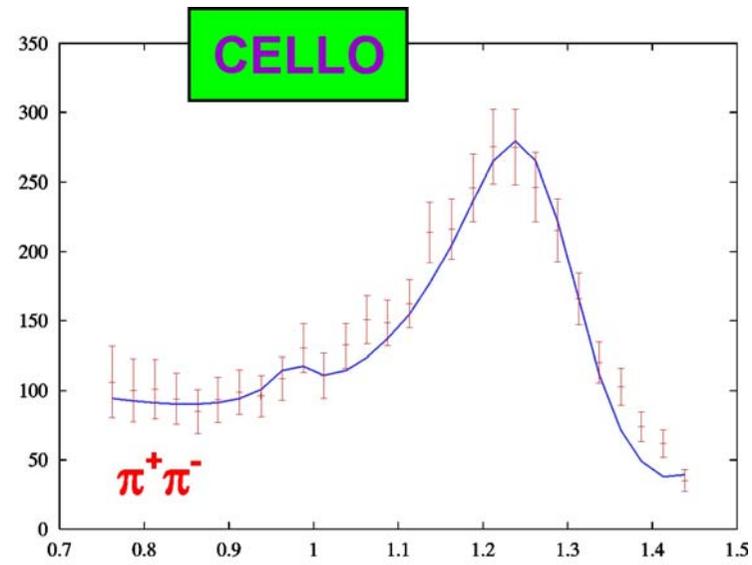
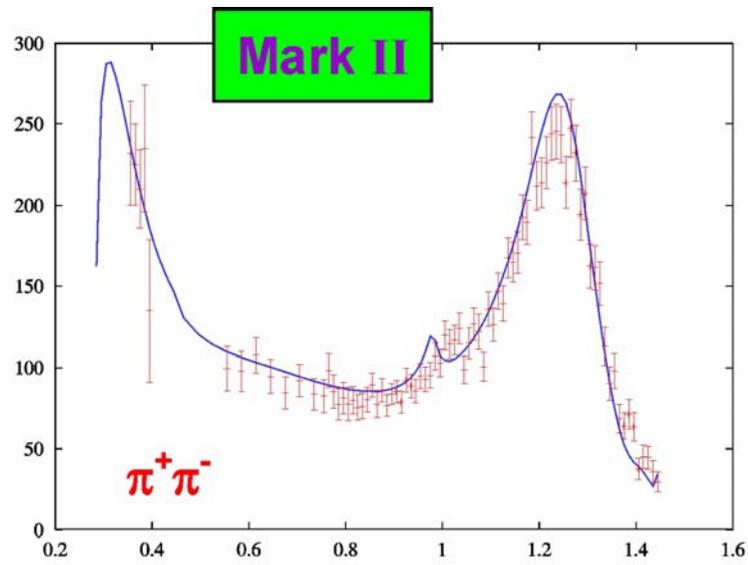
M ($\pi\pi$) GeV

M ($\pi\pi$) GeV



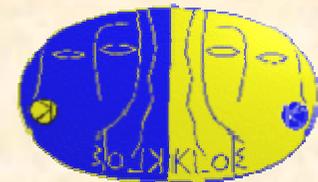
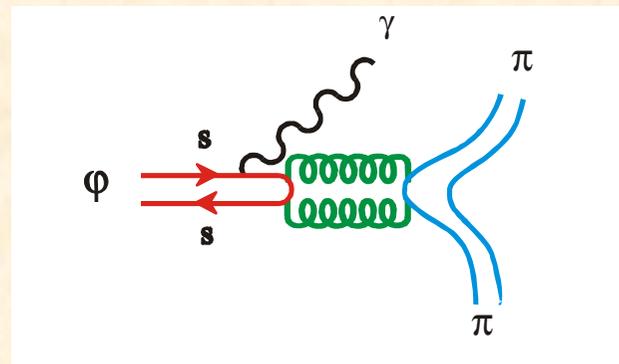
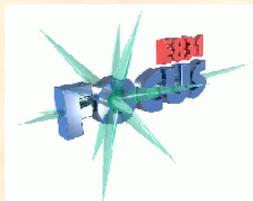
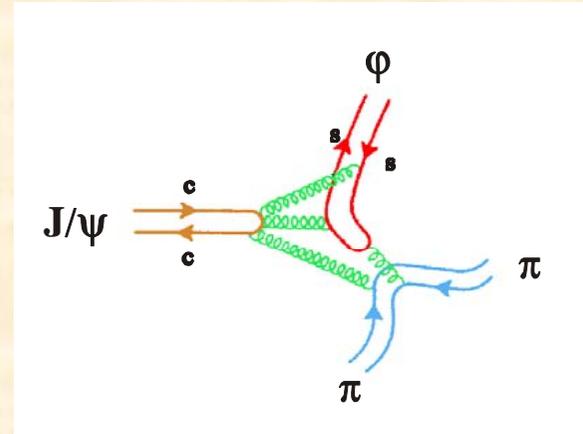
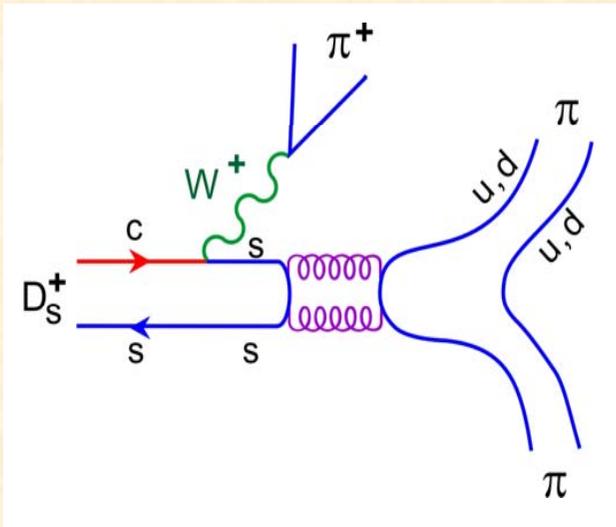
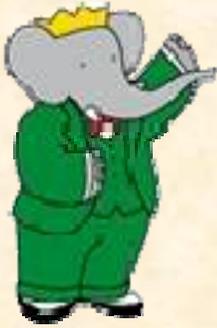
M ($\pi\pi$) GeV

M ($\pi\pi$) GeV

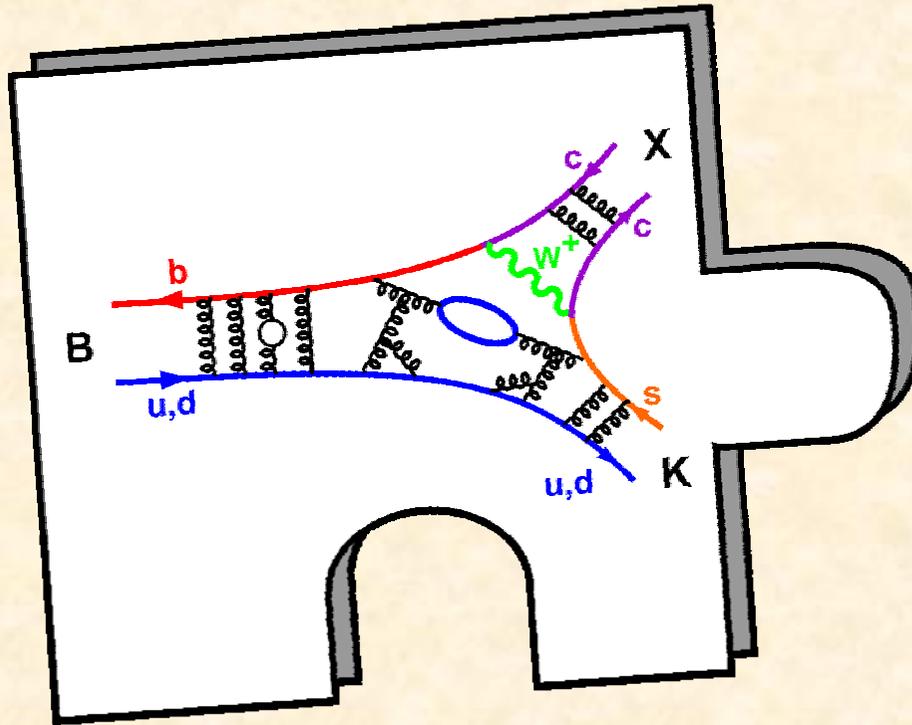


M ($\pi\pi$) GeV

M ($\pi\pi$) GeV

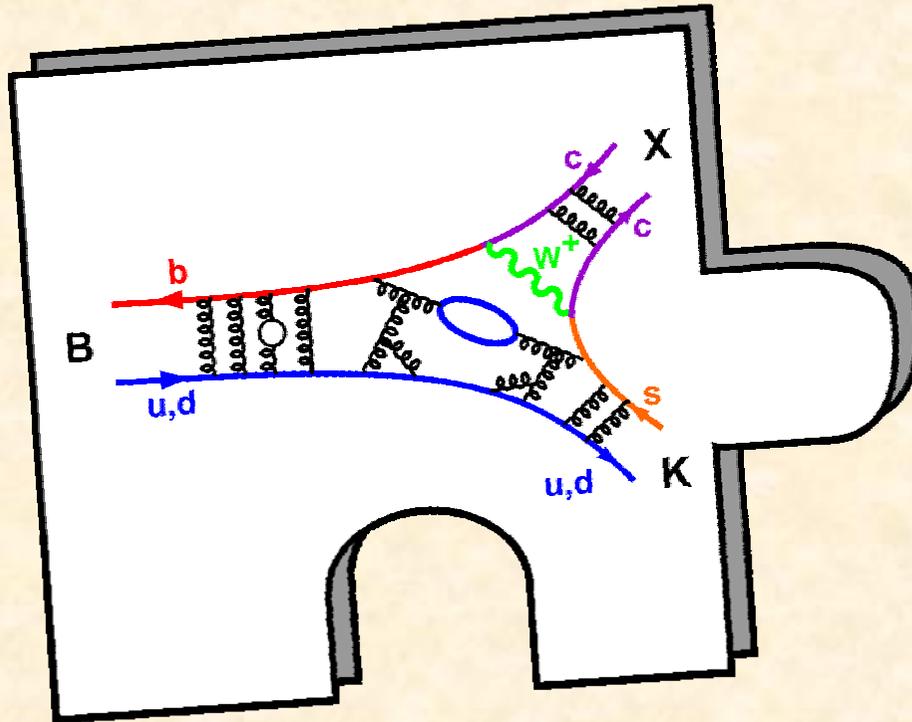


Physics Puzzle

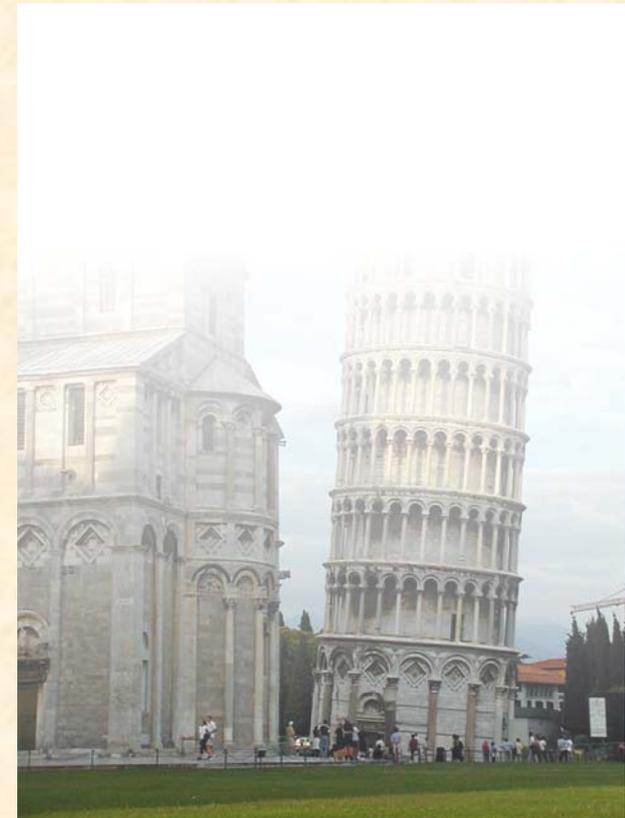


~~CP~~

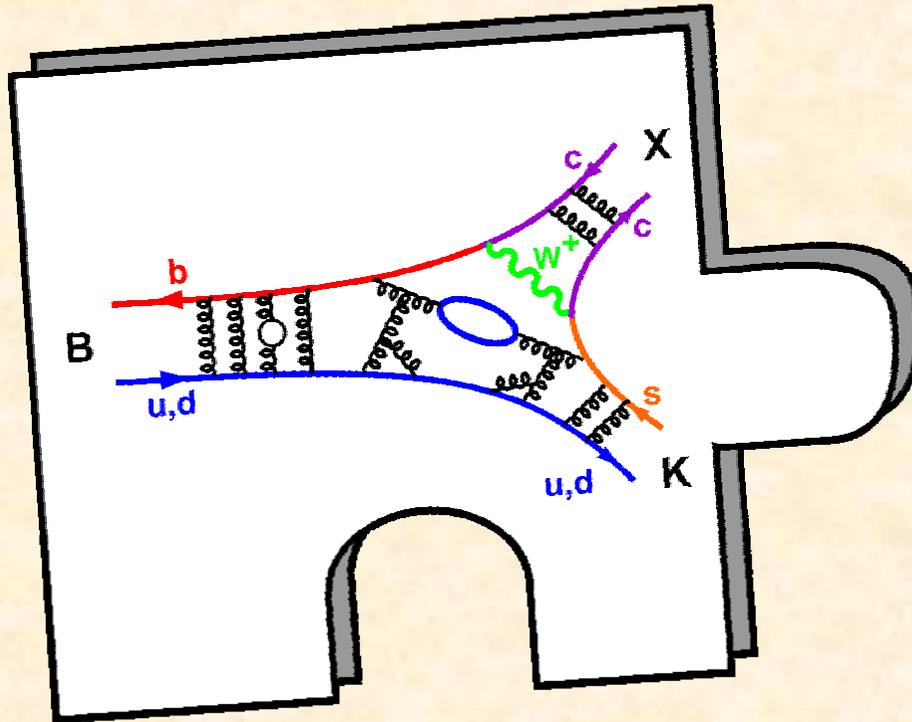
Physics Puzzle



~~CP~~



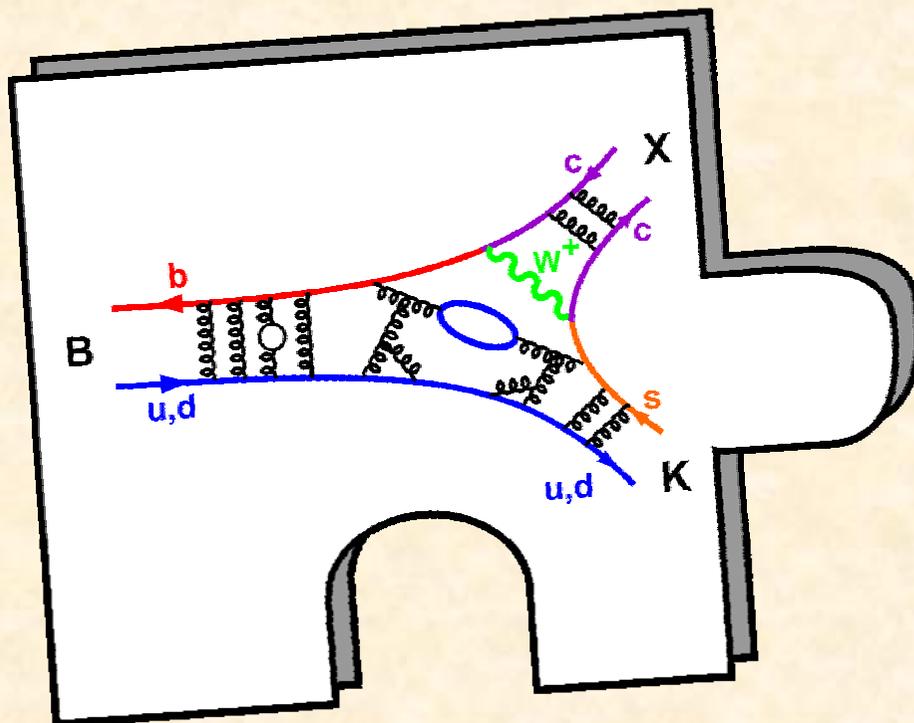
Physics Puzzle



~~CP~~



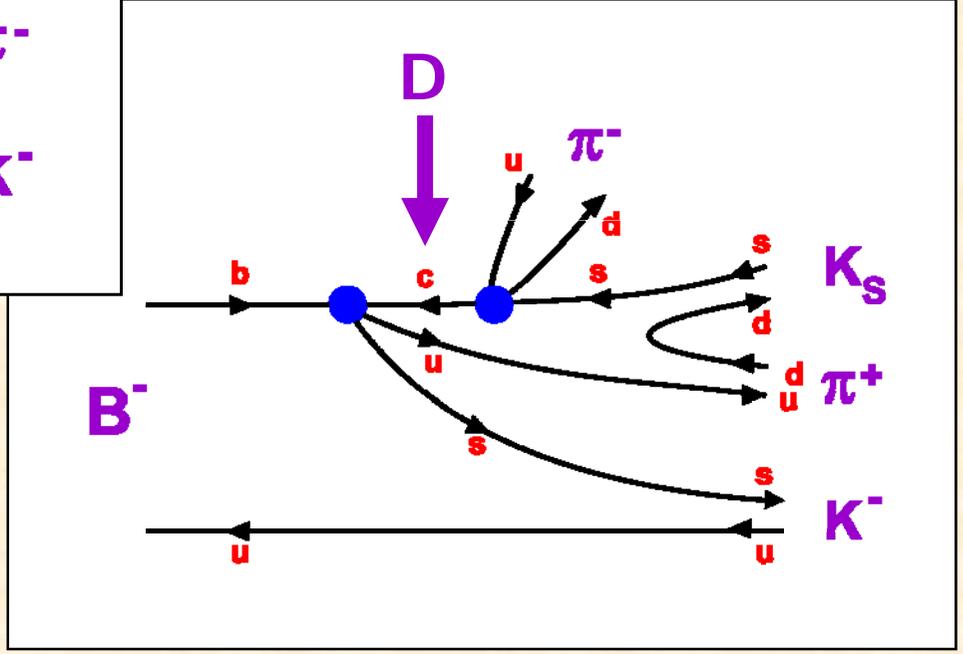
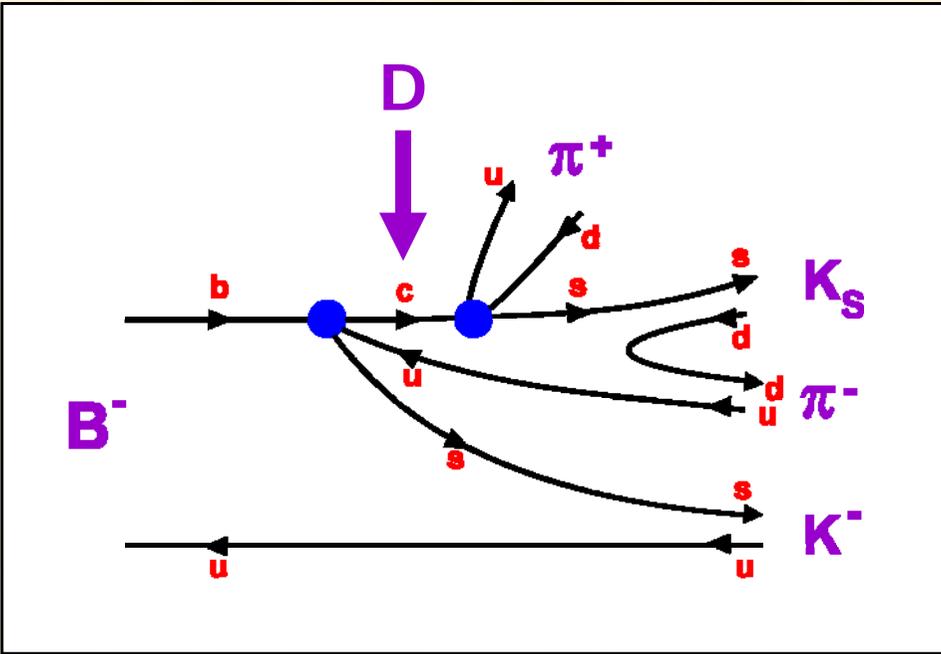
Physics Puzzle



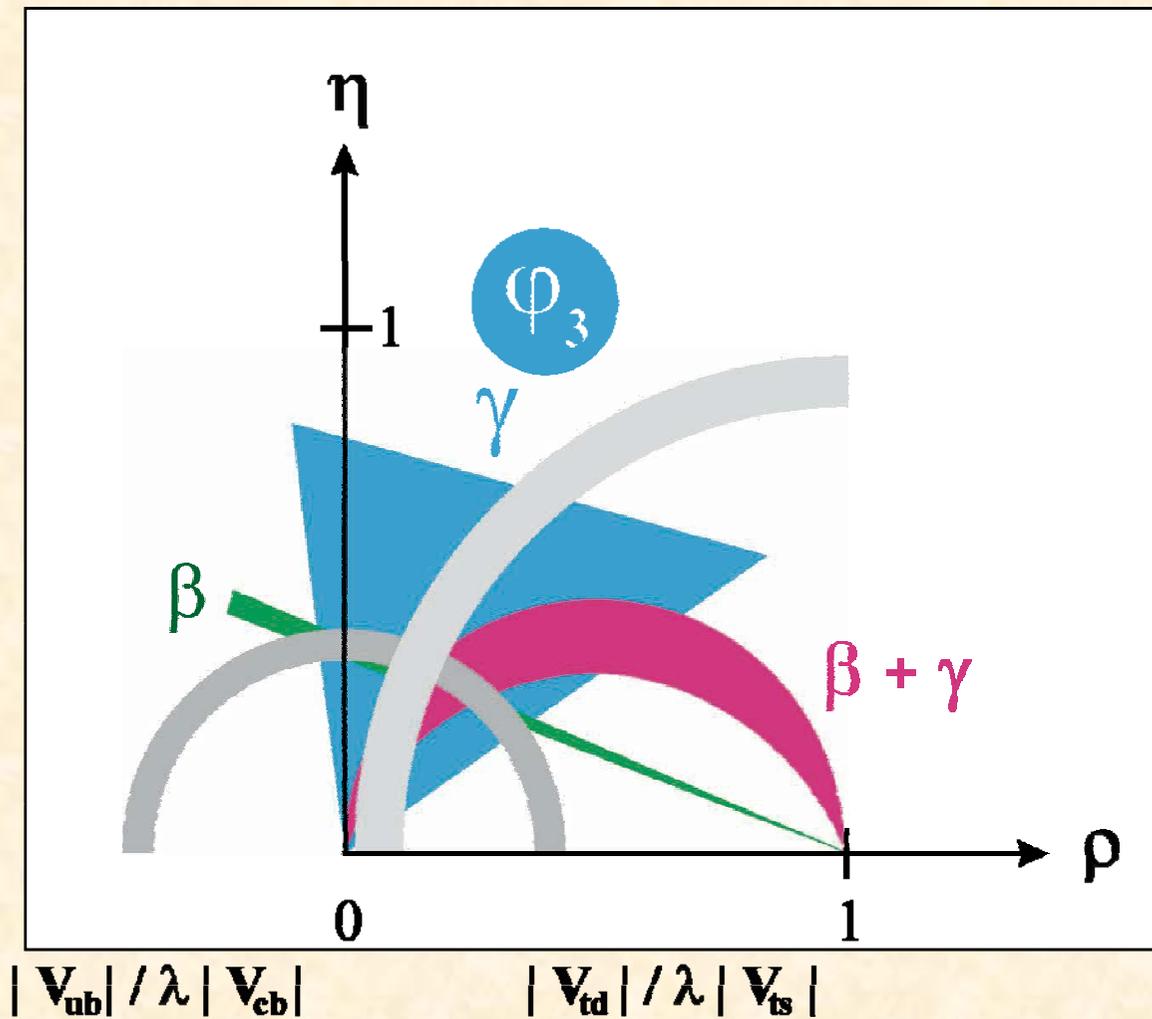
~~CP~~



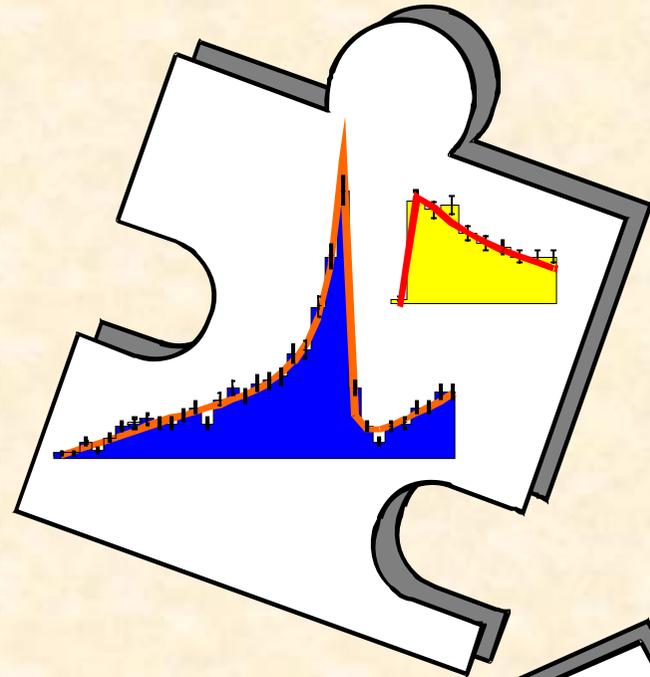
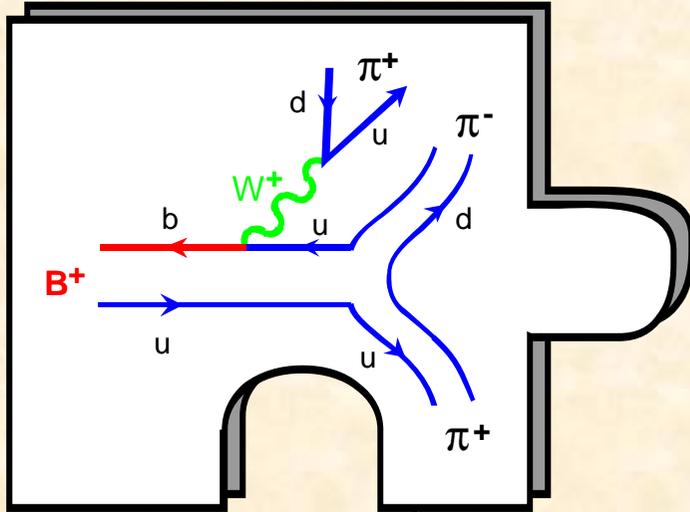
$B \rightarrow D\bar{K} \rightarrow K\bar{K}\pi\pi$



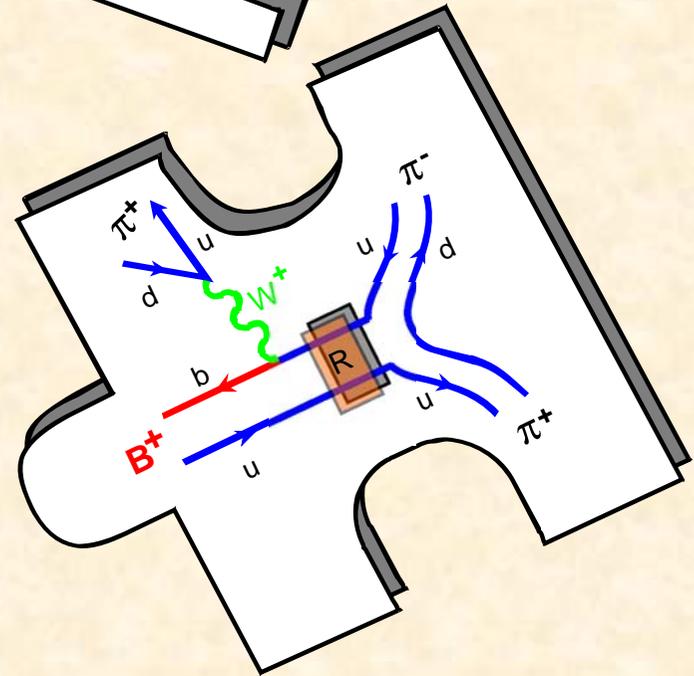
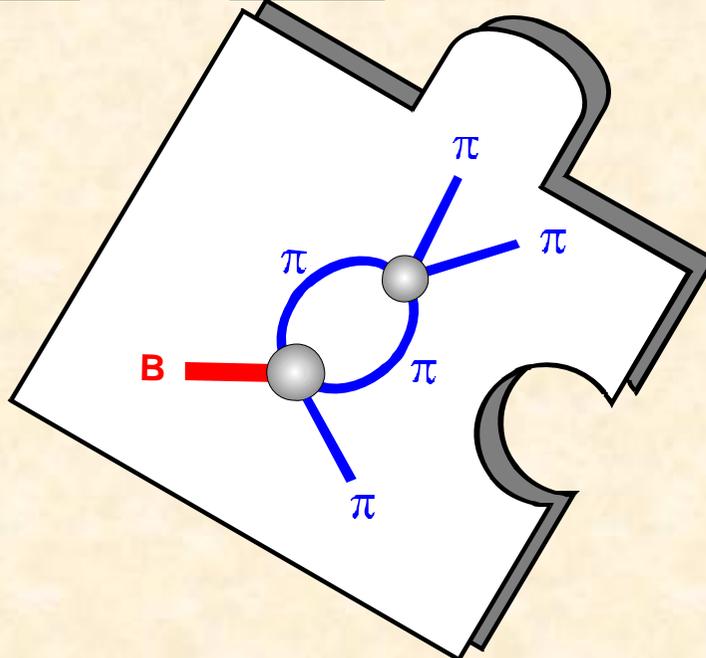
CKM angles



Physics Puzzle



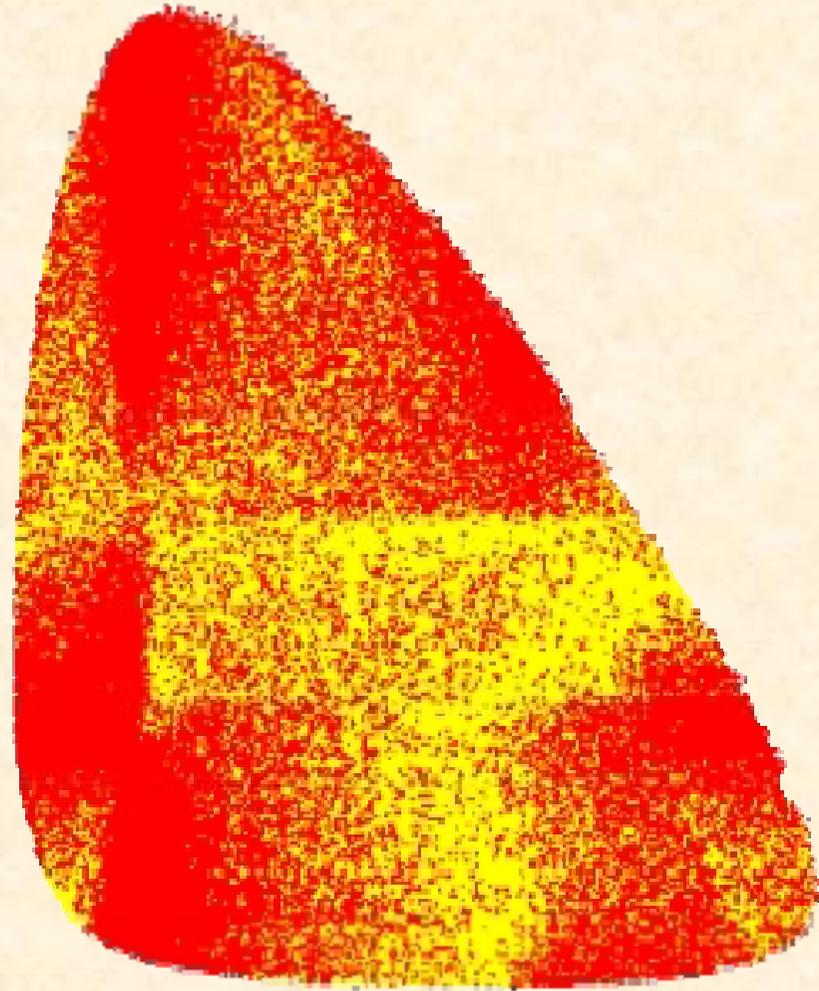
~~CP~~



R.H. Dalitz

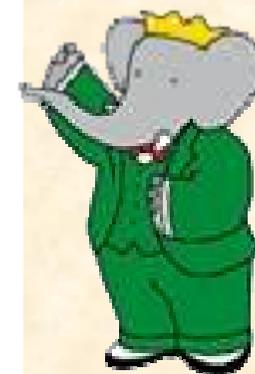
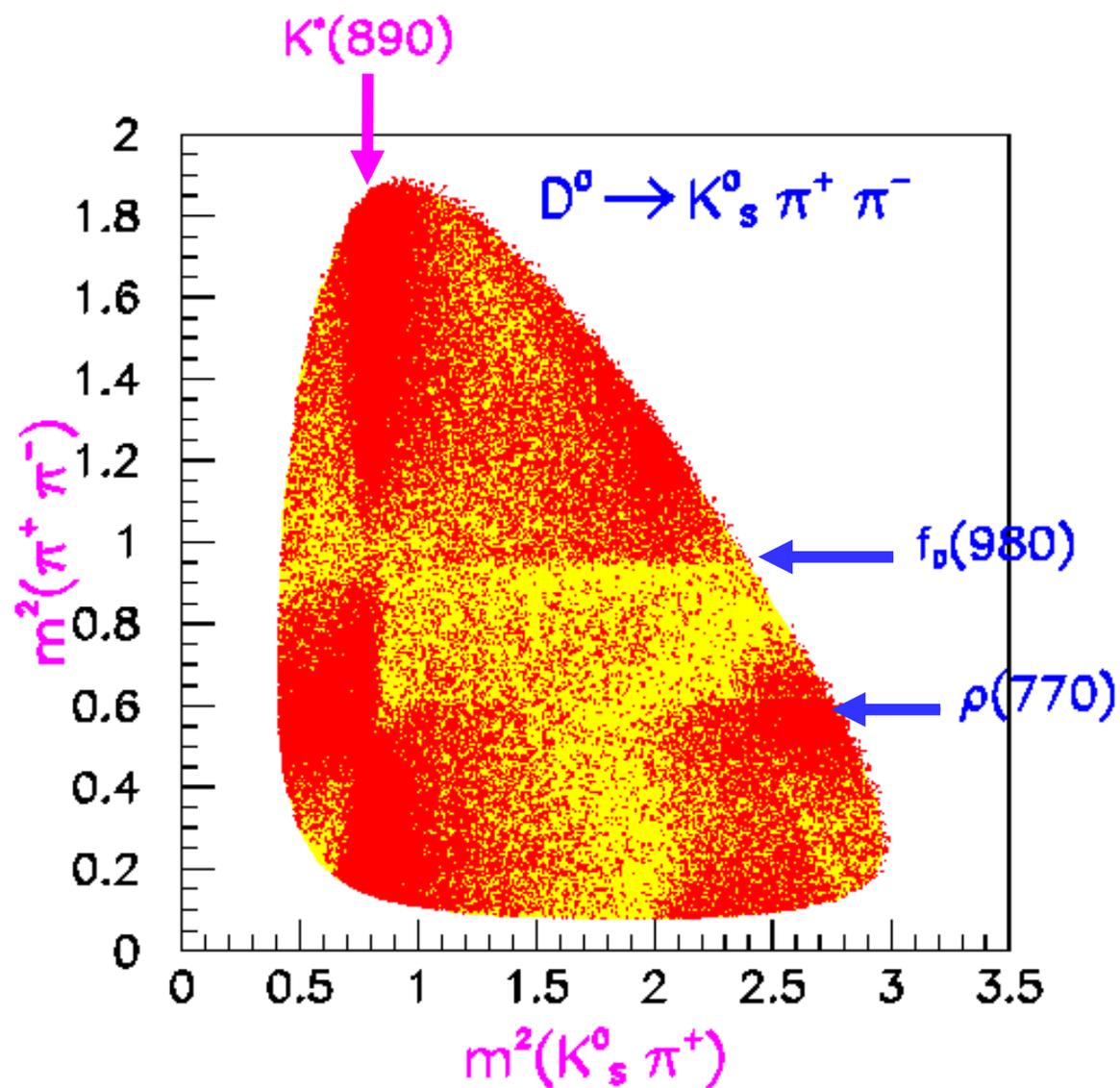


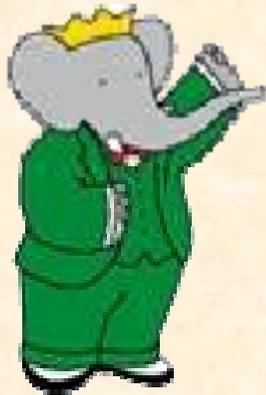
1925 - 2006



Dalitz plot

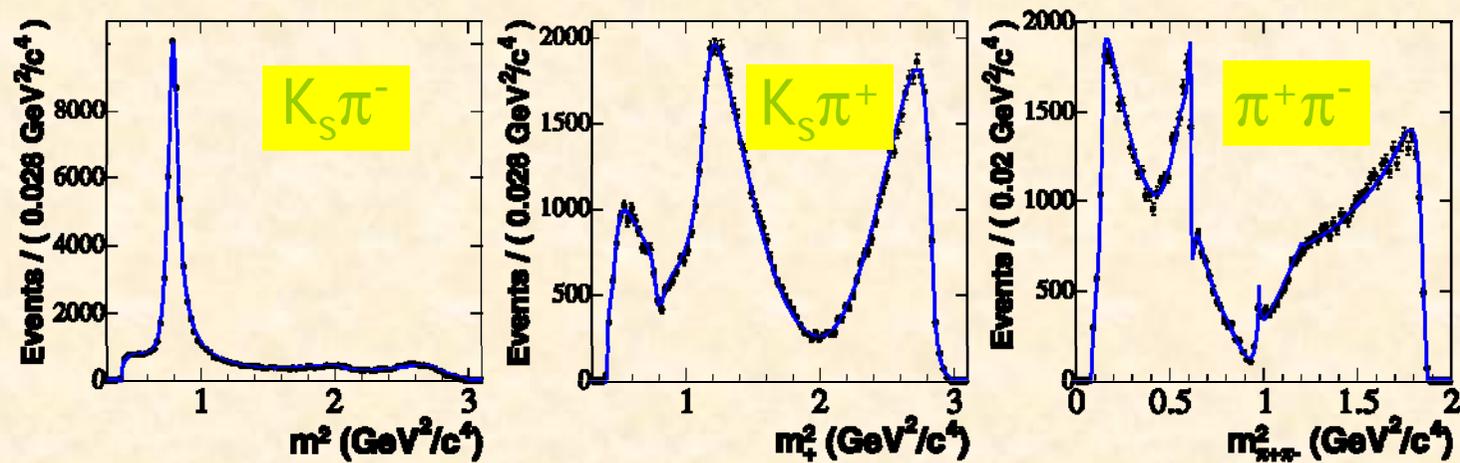
Dalitz plot of $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$.





Dalitz plot of $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$.

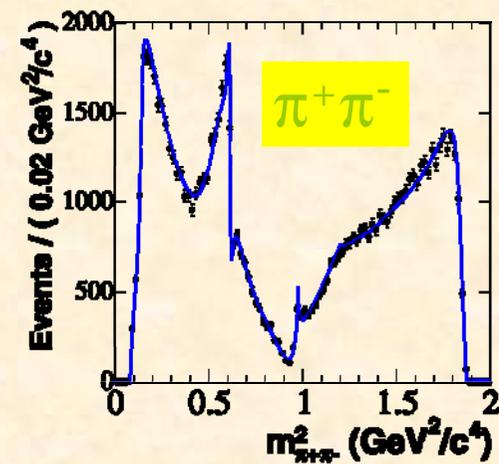
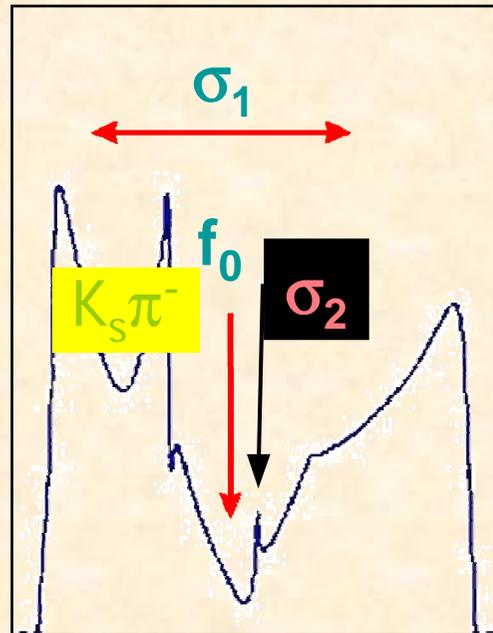
3 mass projections



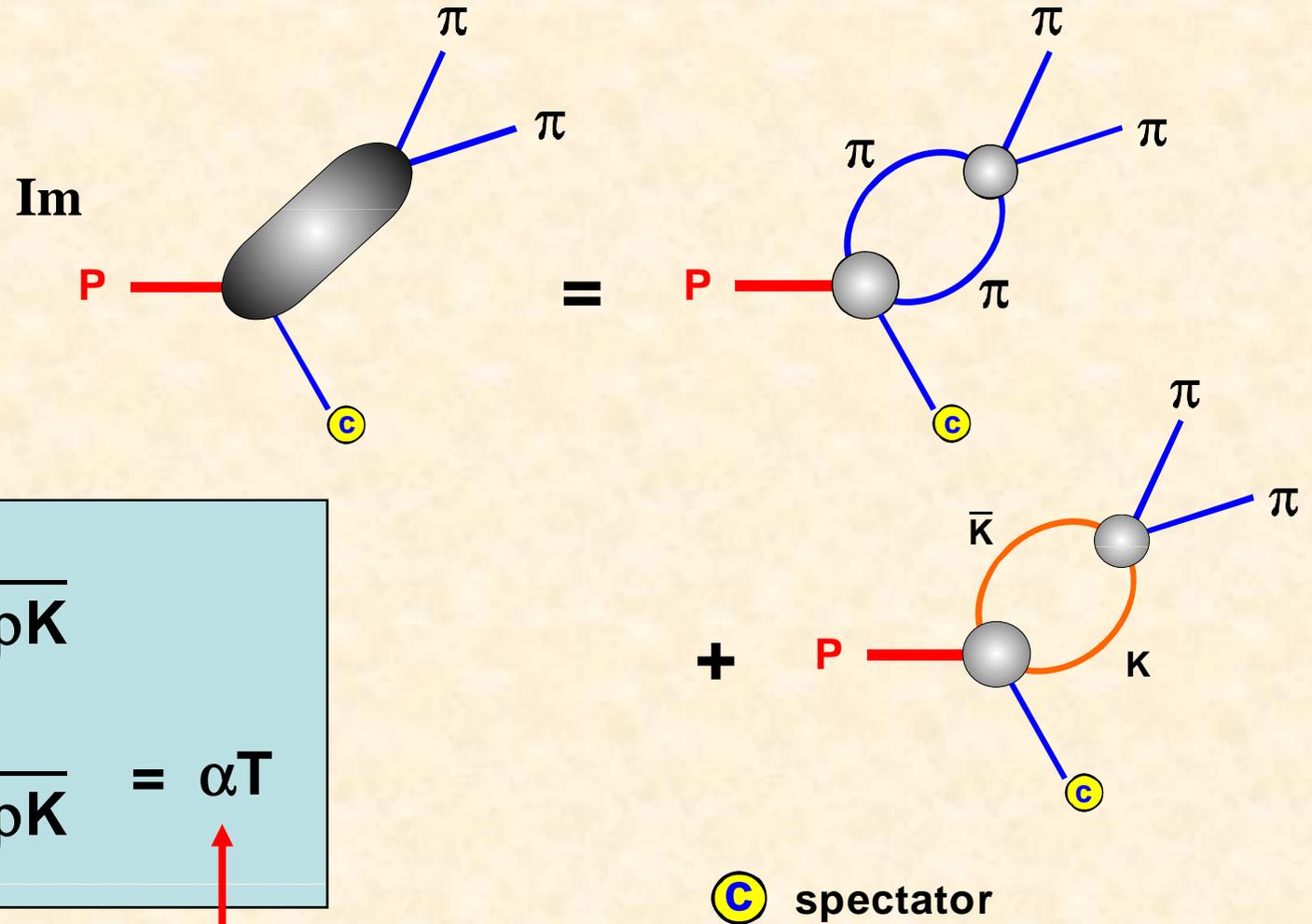


Dalitz plot of $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$.

$\pi^+ \pi^-$



UNITARITY : decays in spectator picture

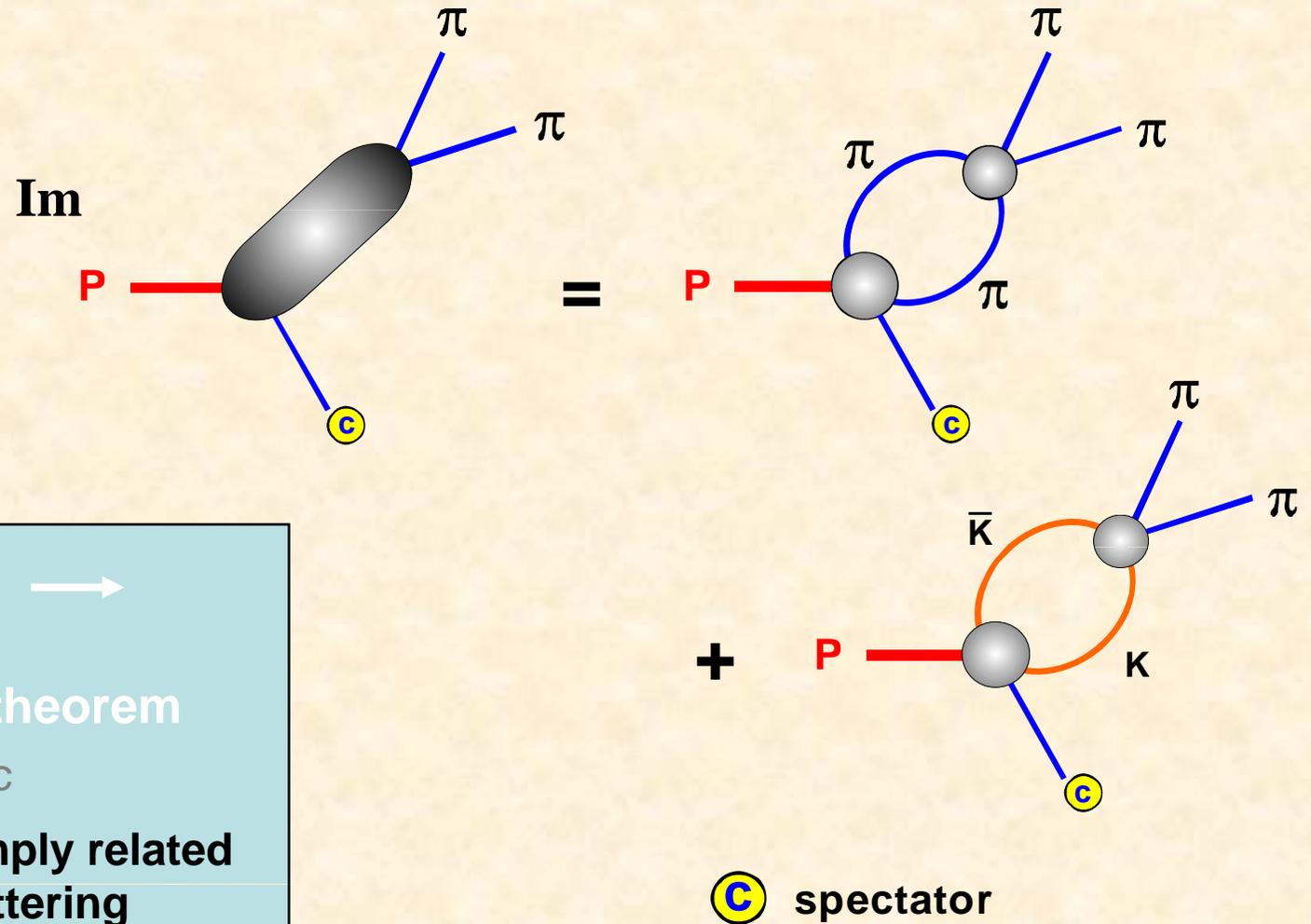


$$T = \frac{K}{1 - i\rho K}$$

$$F = \frac{P}{1 - i\rho K} = \alpha T$$

↑
coupling function

UNITARITY : decays in spectator picture



$$F = \alpha T \longrightarrow$$

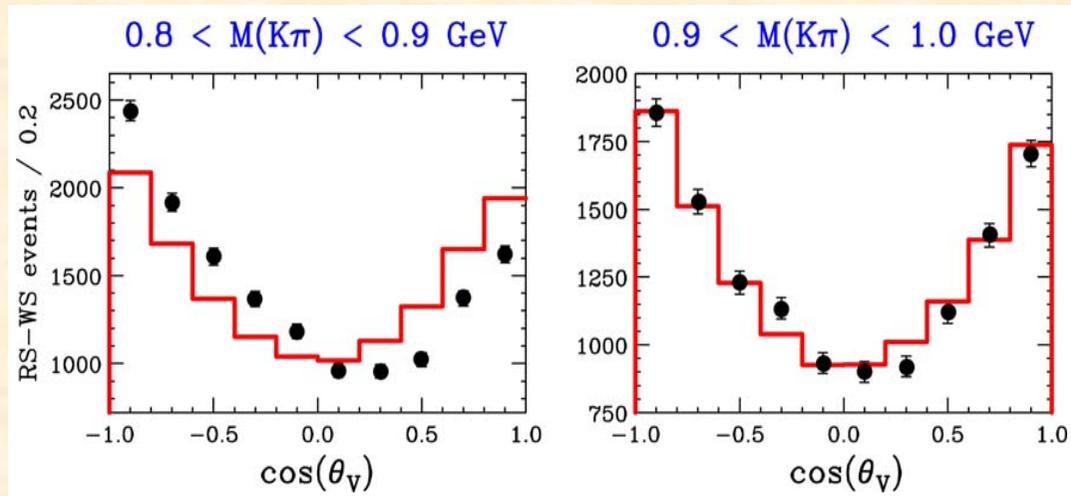
Watson's theorem

elastic

phases simply related
if no rescattering

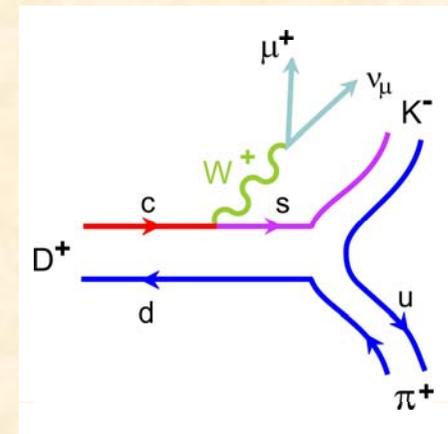


$K^*(892)$ region

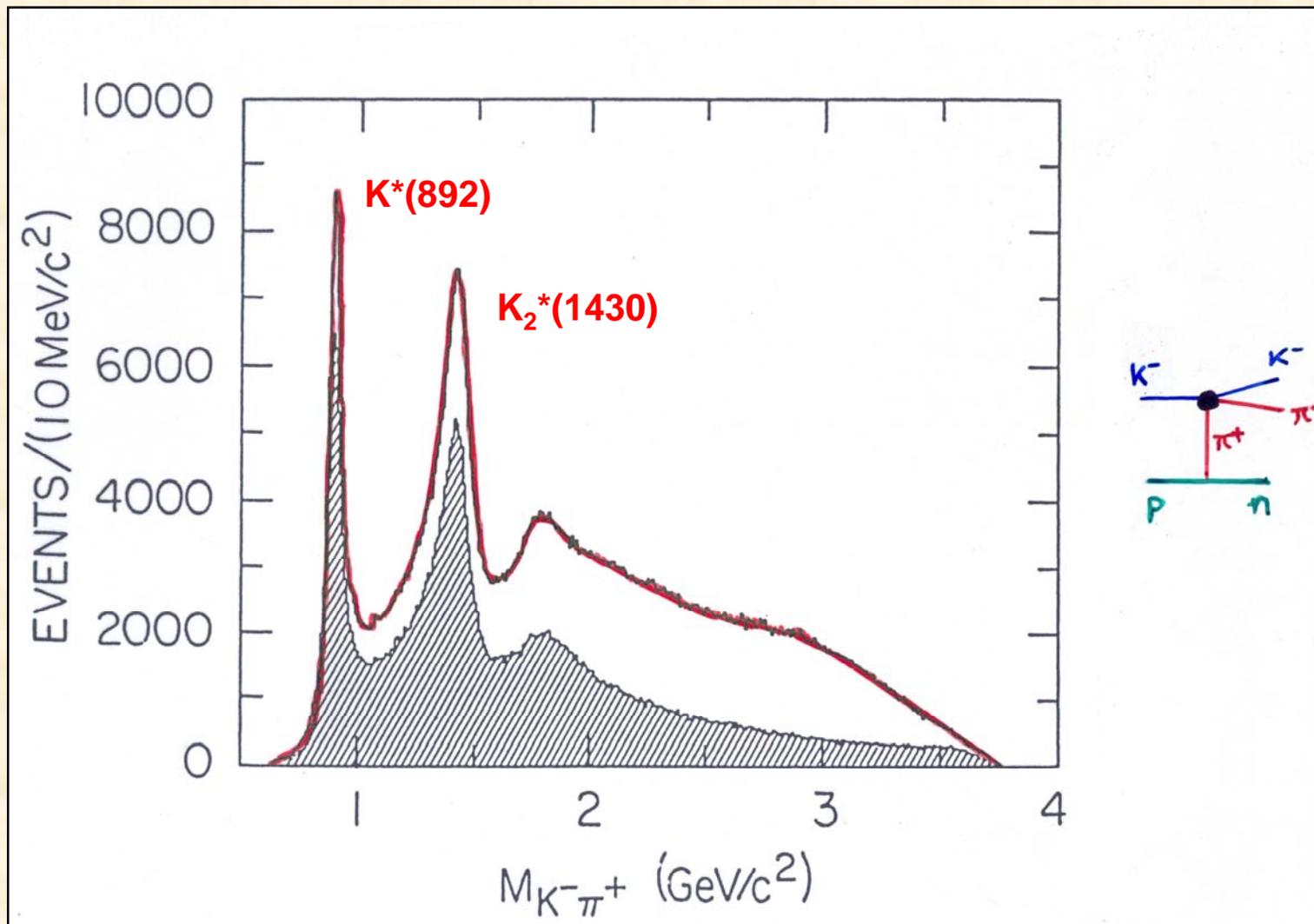


$$\mathcal{F}(D \rightarrow (K\pi)\mu\nu; s) = \mathcal{F}_{sl}^{1/2}(s) + \mathcal{F}_{sl}^{3/2}(s)$$

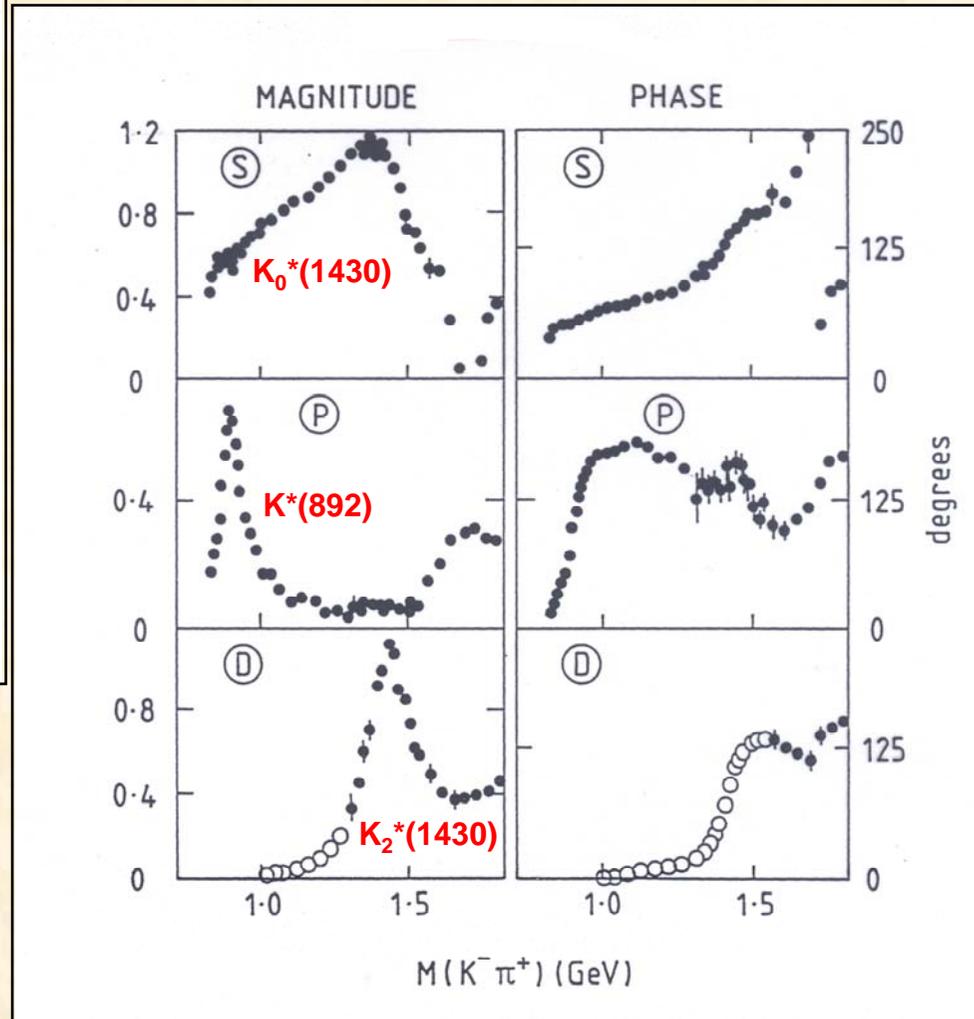
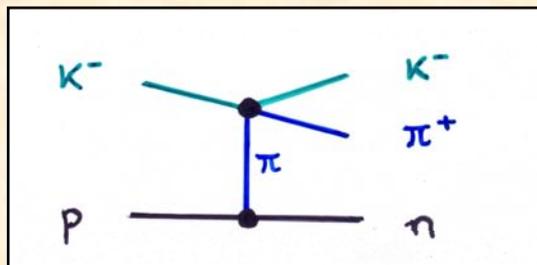
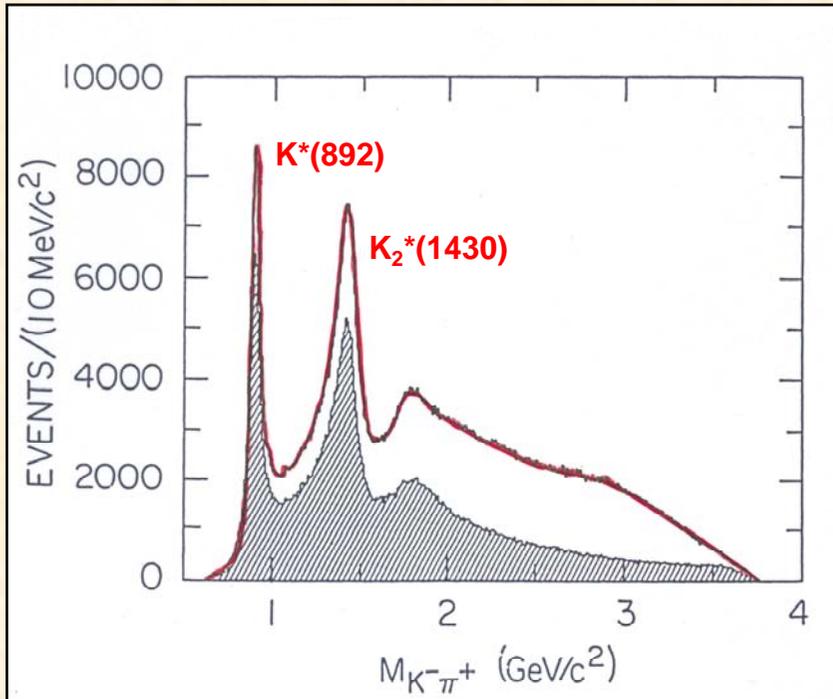
$$\mathcal{F}_{sl}^I(s) = |\mathcal{F}_{sl}^I(s)| \exp[i\delta^I(s)]$$

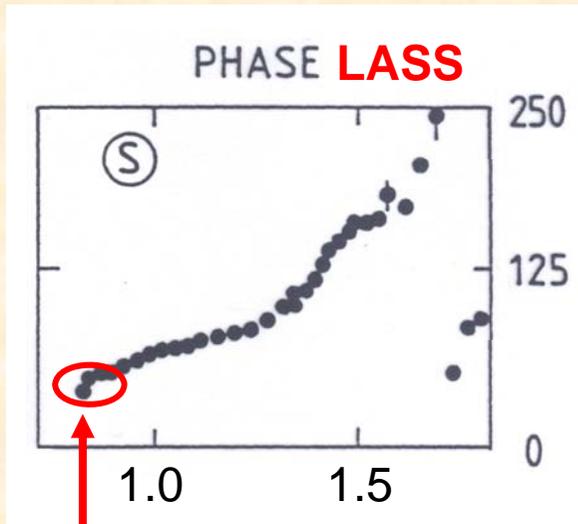
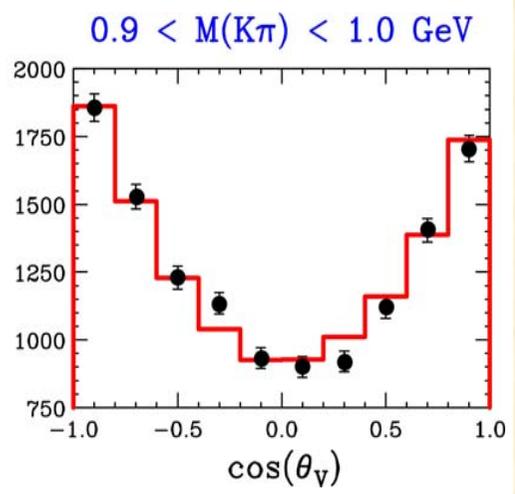
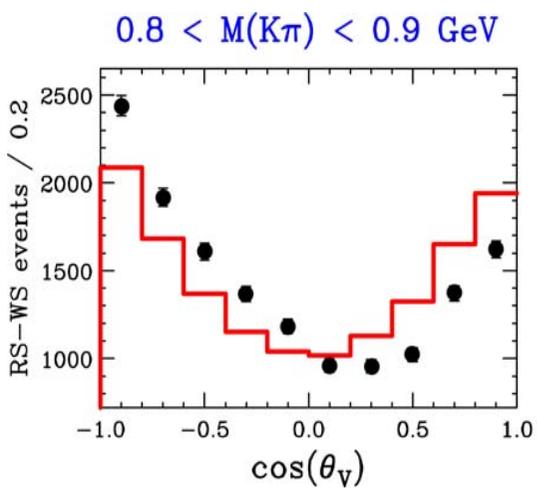
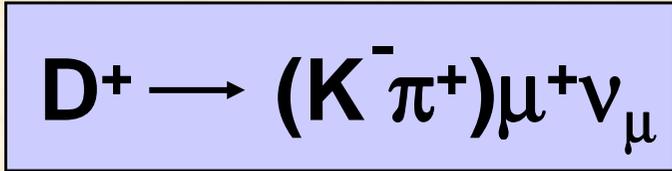


LASS: $K^-p \rightarrow K^-\pi^+n$

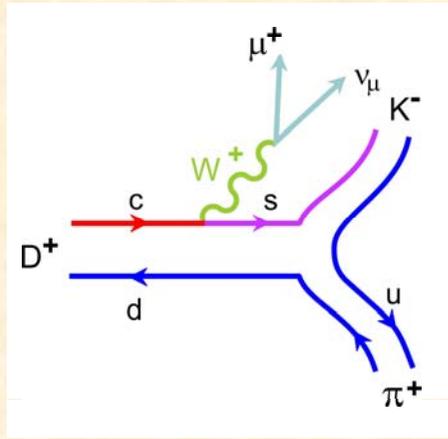
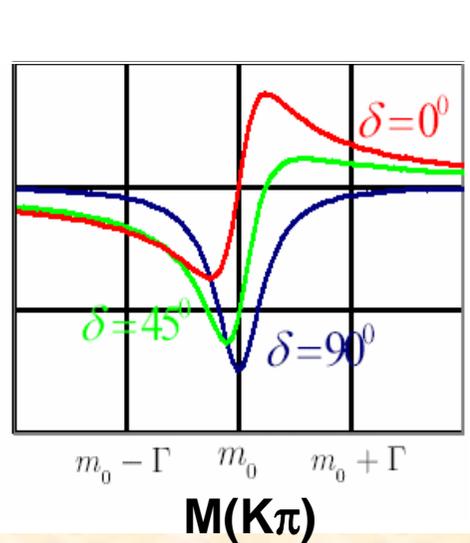
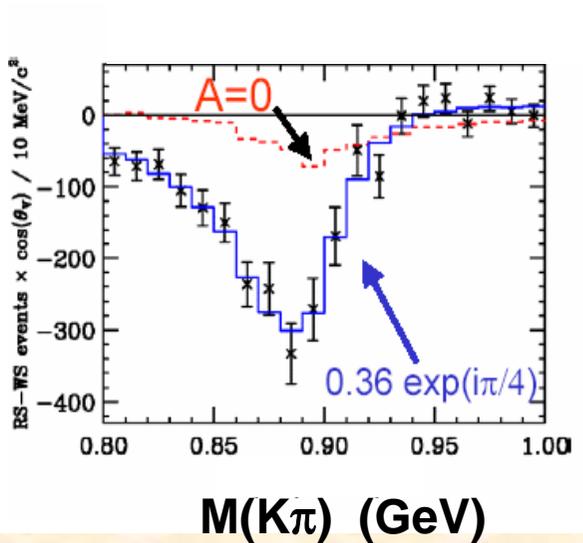


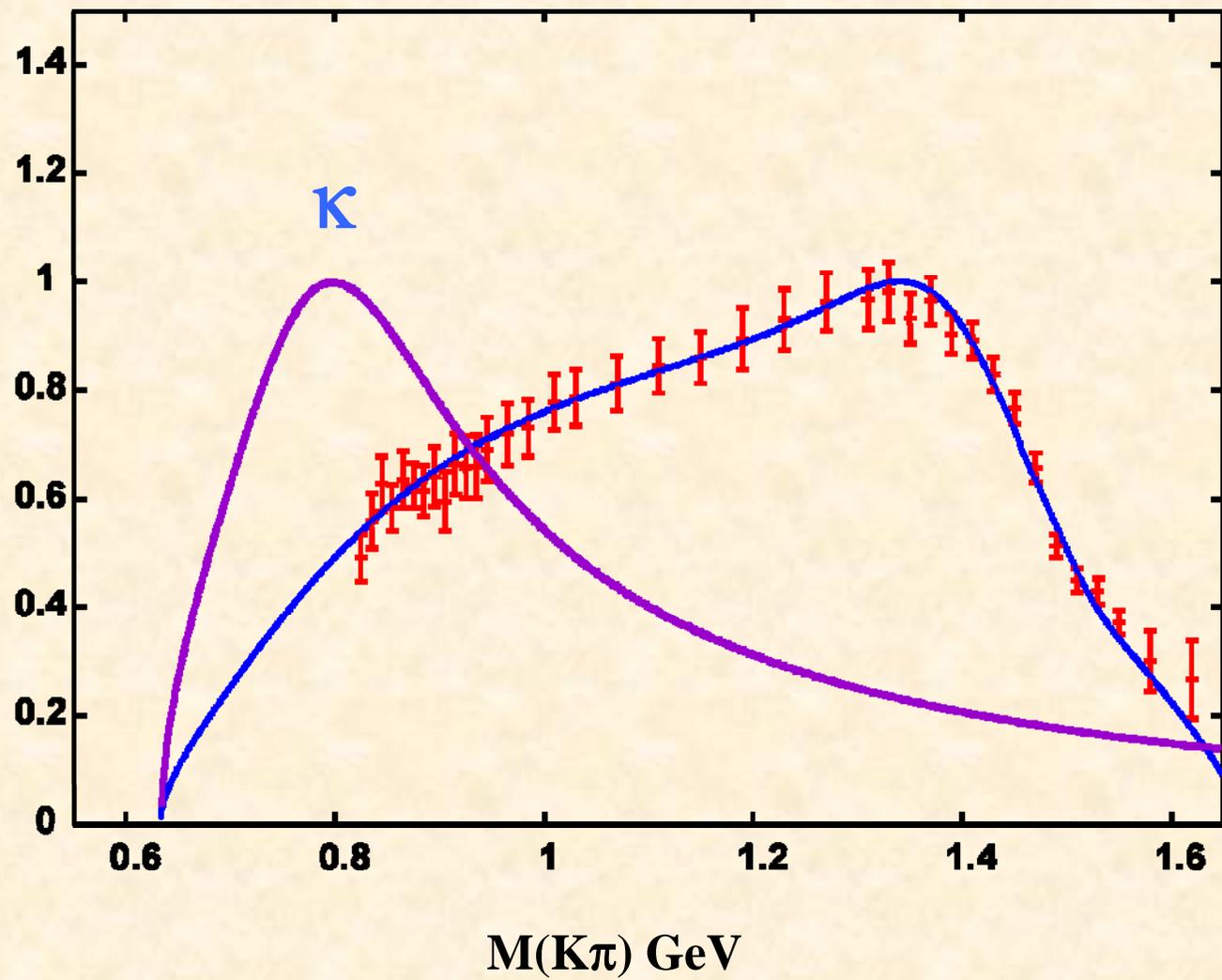
LASS: $K^-p \rightarrow K^- \pi^+ n$



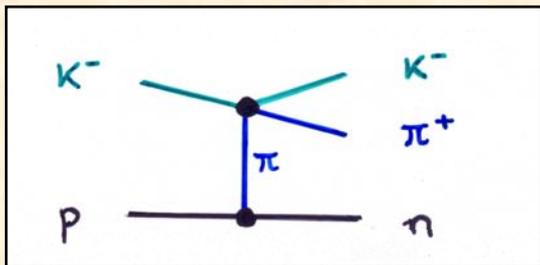
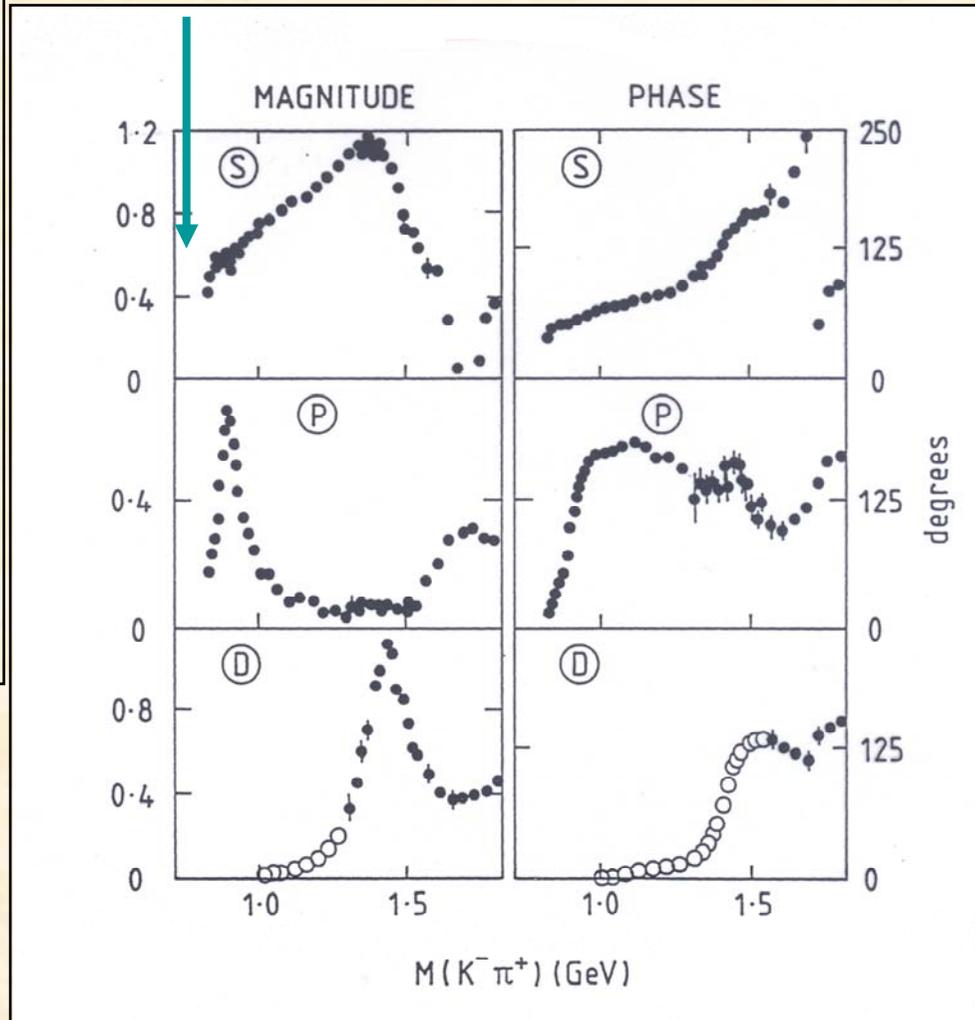
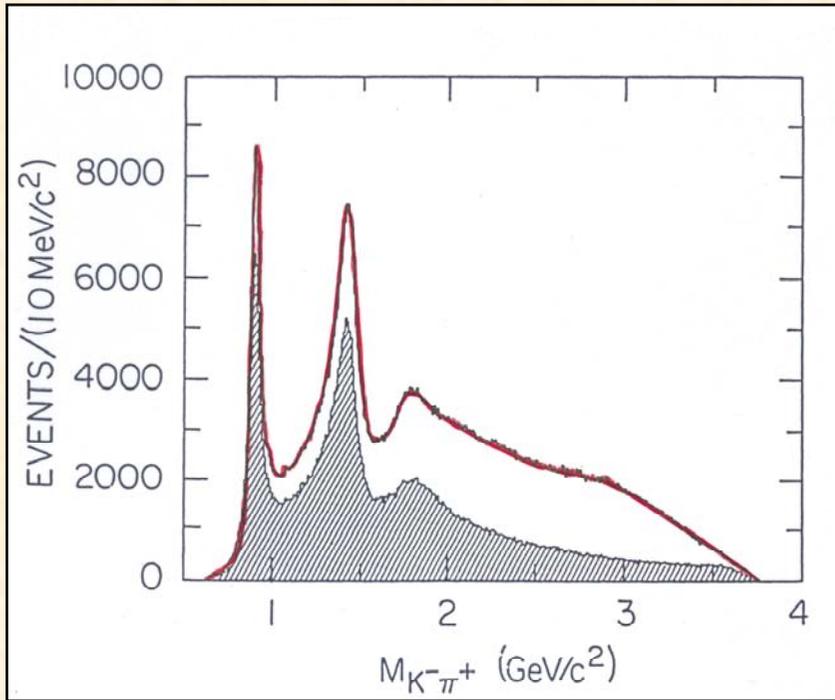


M(Kπ) (GeV)





LASS: $K^-p \rightarrow K^- \pi^+ n$

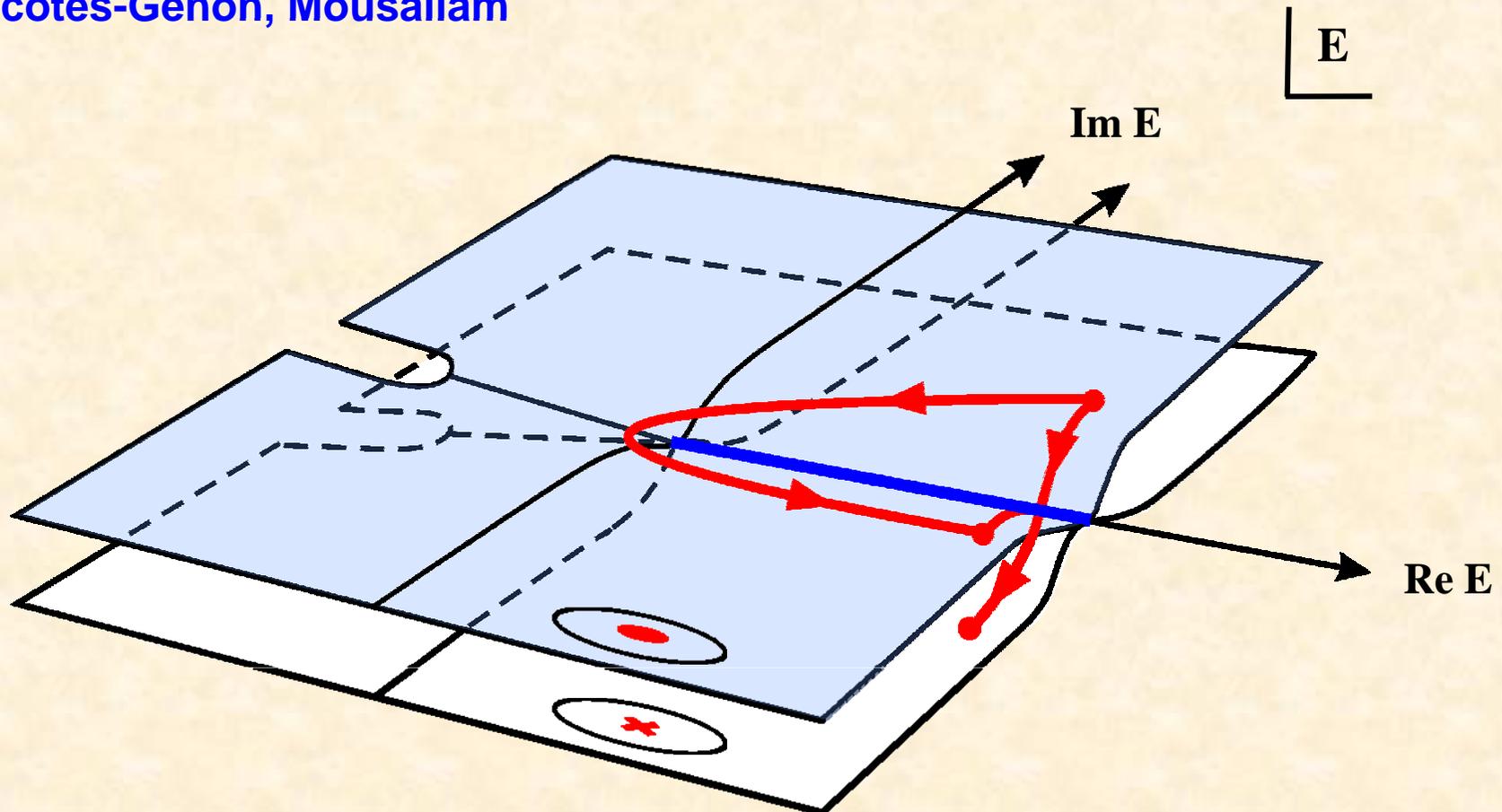


$K\pi$ scattering into the complex plane

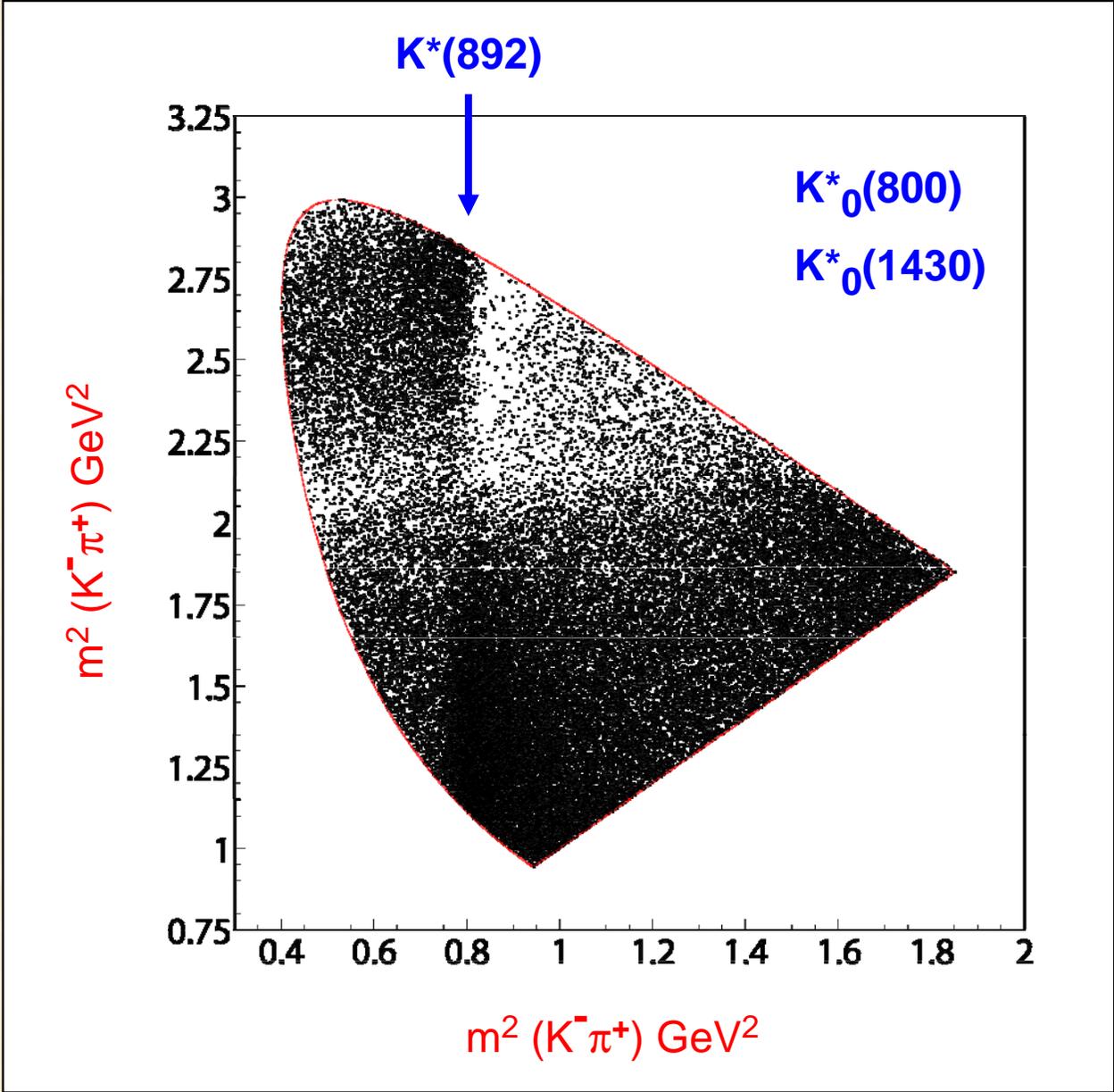


$$E_R = 658 - i 289 \text{ MeV}$$

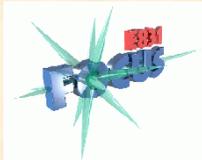
Descotes-Genon, Mousallam



Dalitz plot: $D^+ \rightarrow K^- \pi^+ \pi^+$



Malvezzi, P

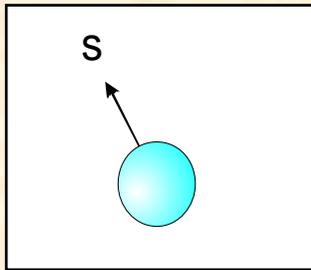


States beyond the quark model

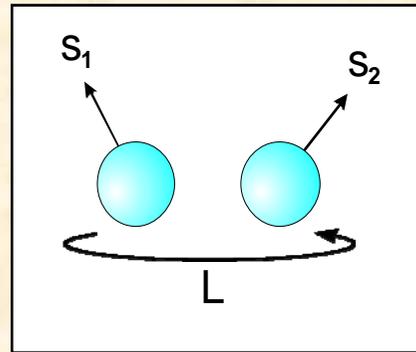


the search continues

Quark model quantum numbers



$$S = \frac{1}{2}$$



$$S = S_1 +$$

$$S_2 = L + S$$

$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

conventional mesons

J^{PC} quantum numbers

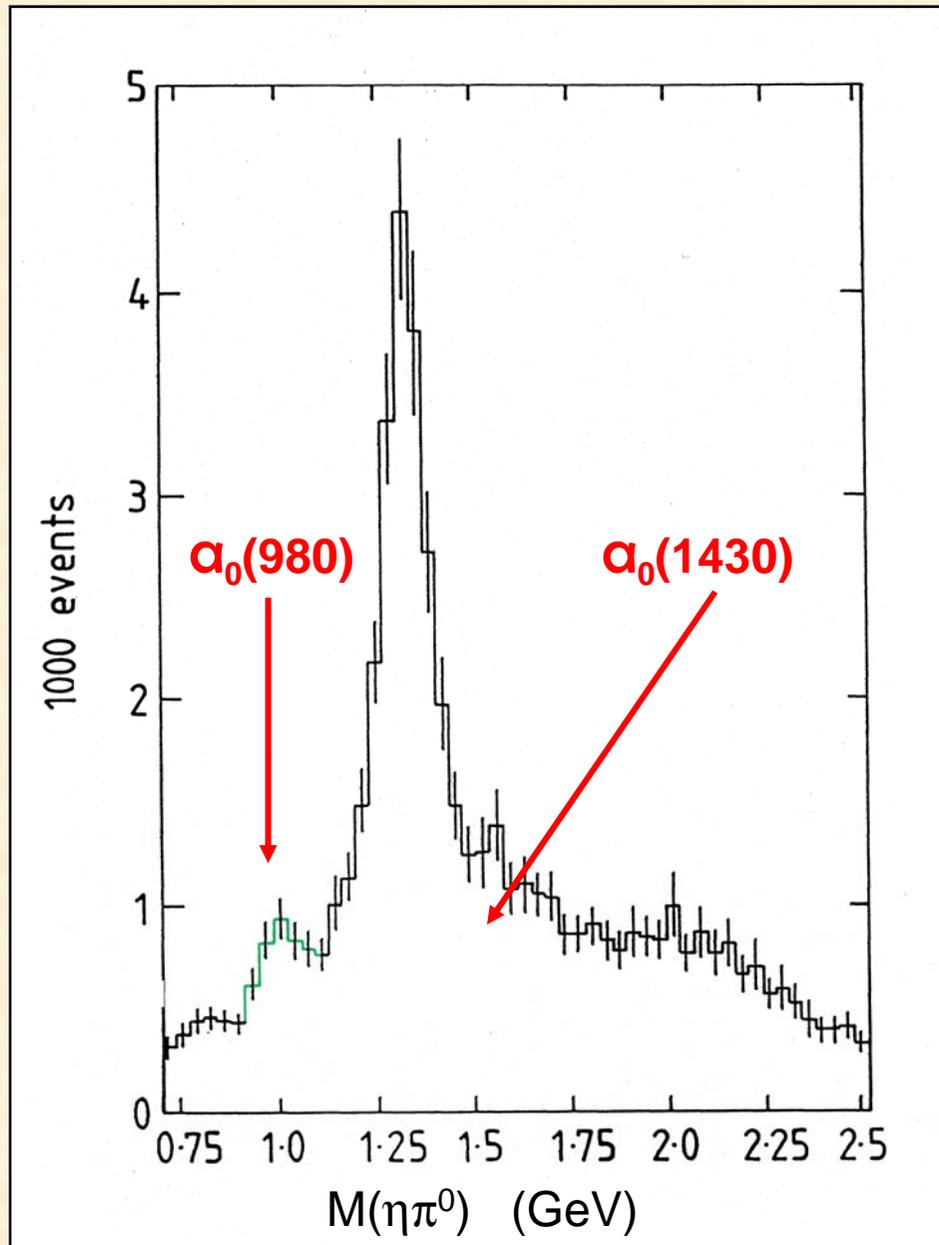
non $\bar{q}q$

0^{++} 0^{+-} 0^{-+} 0^{--}

1^{++} 1^{+-} 1^{-+} 1^{--}

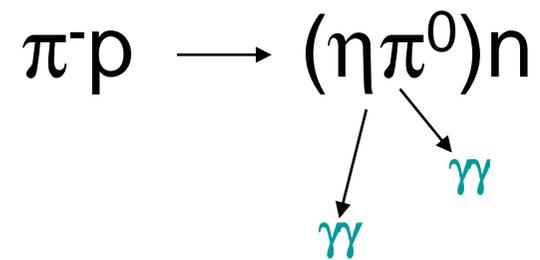
2^{++} 2^{+-} 2^{-+} 2^{--}

.....



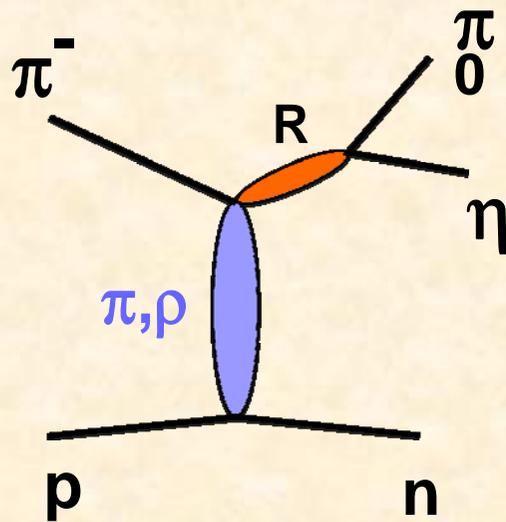
GAMS

Alde et al.

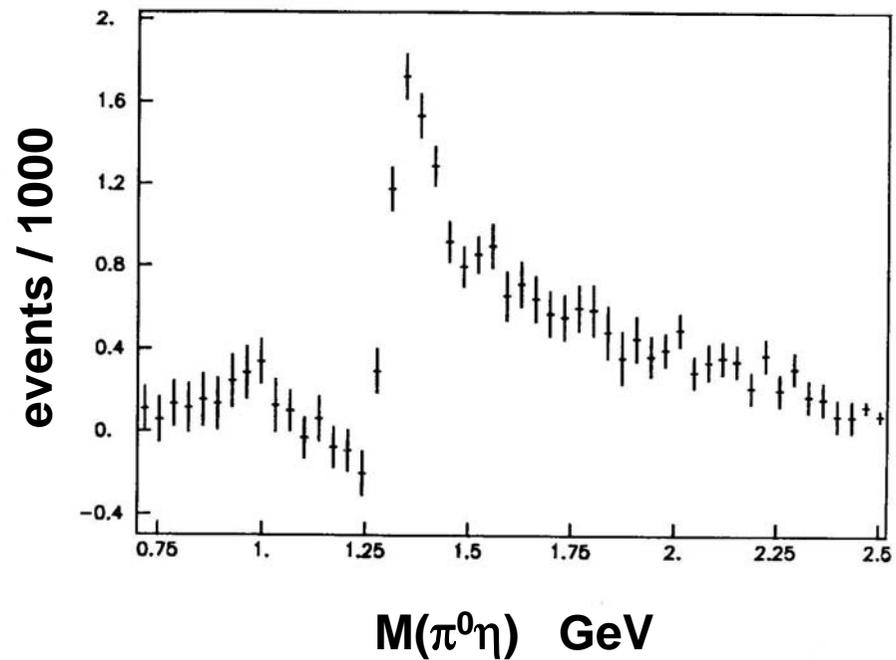


GAMS: $\pi^- p \rightarrow (\pi^0 \eta) n$

Alde et al.



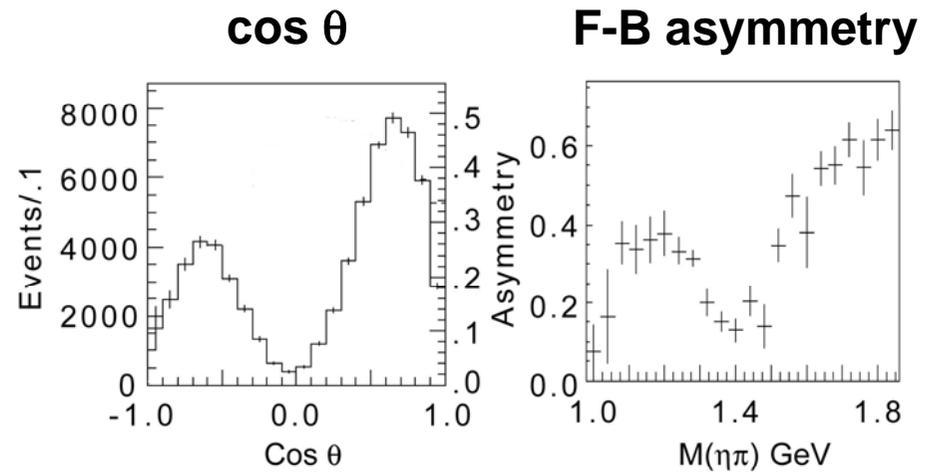
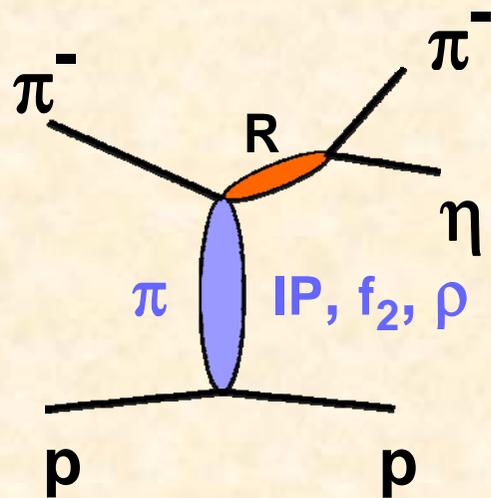
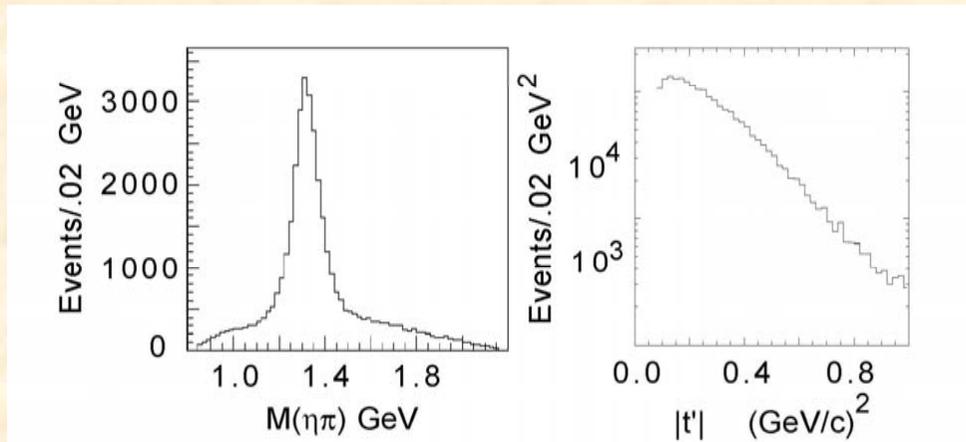
Forward-backward asymmetry



Gottfried-Jackson frame

BNL-E852: $\pi^- p \rightarrow (\pi^- \eta) p$

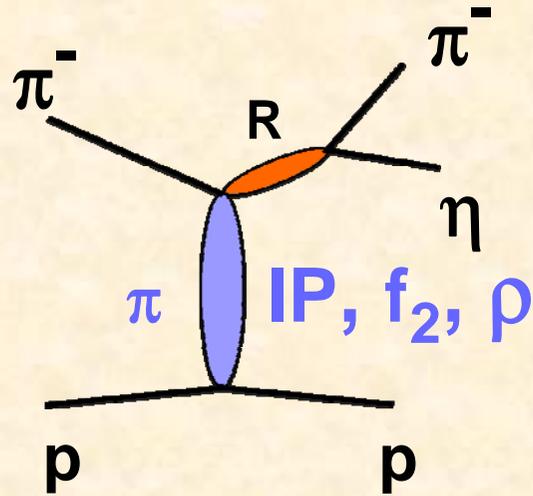
Thompson et al.



Gottfried-Jackson frame

BNL-E852: $\pi^- p \rightarrow (\pi^- \eta) p$

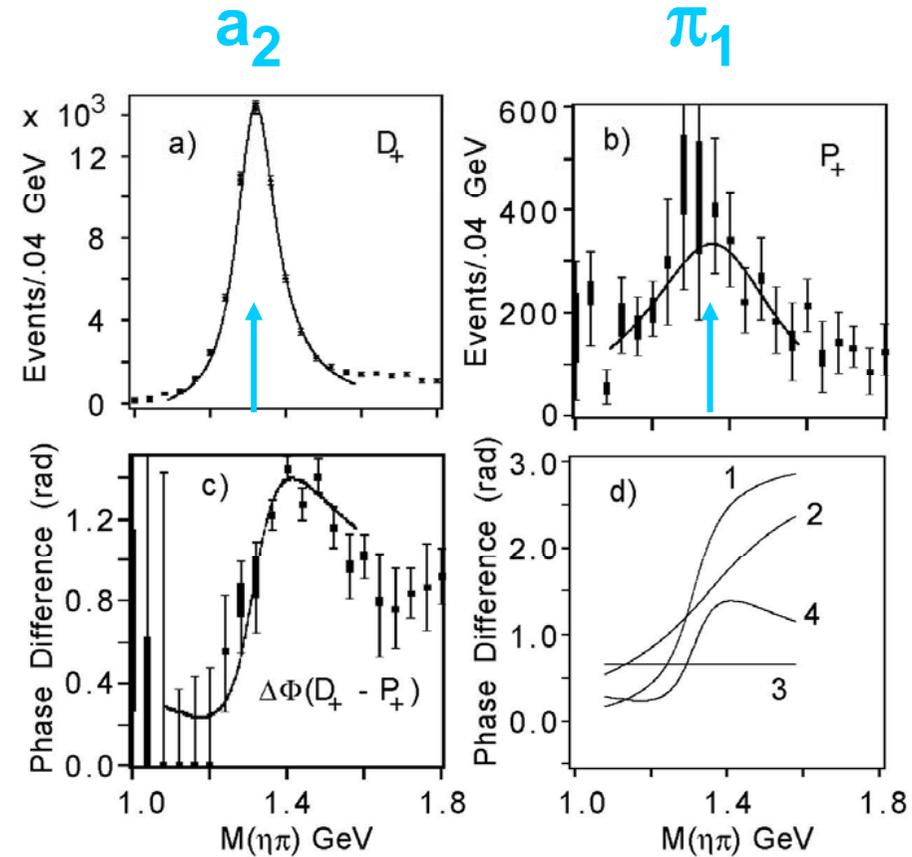
Thompson et al.



$\pi_1(1400)$

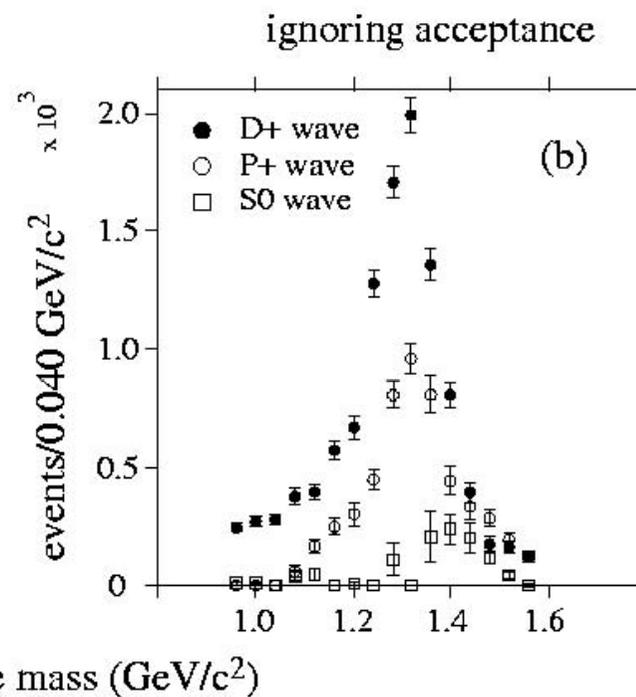
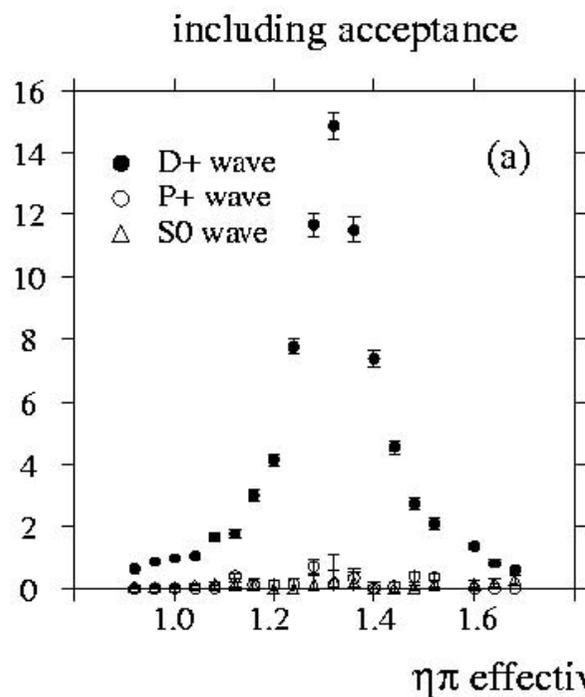
$M = (1370 \pm 16 \pm 40) \text{ MeV}$

$\Gamma = (385 \pm 40 \pm 85) \text{ MeV}$



BNL-E852: $\pi^- p \rightarrow (\pi^- \eta) p$

Dzierba et al.



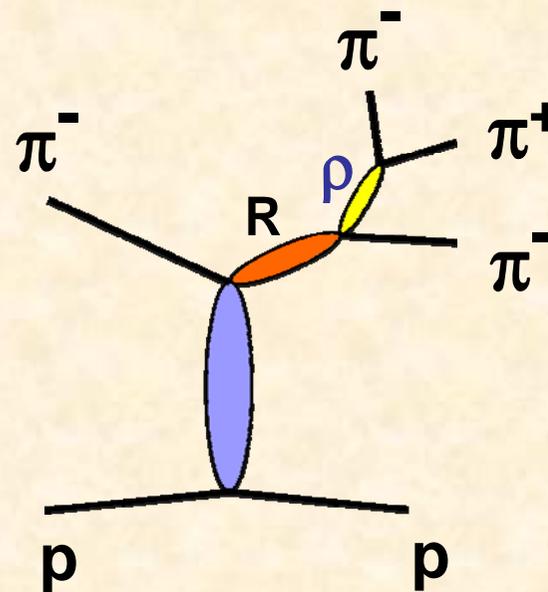
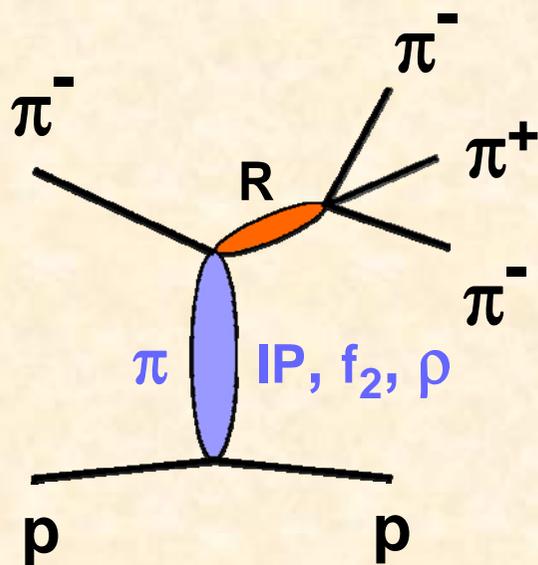
1^{-+} hybrid decay pattern



S + P mesons: $b_1\pi$, $f_1\pi$

$b_1\pi : f_1\pi : \rho\pi = 170 : 60 : 10$ MeV

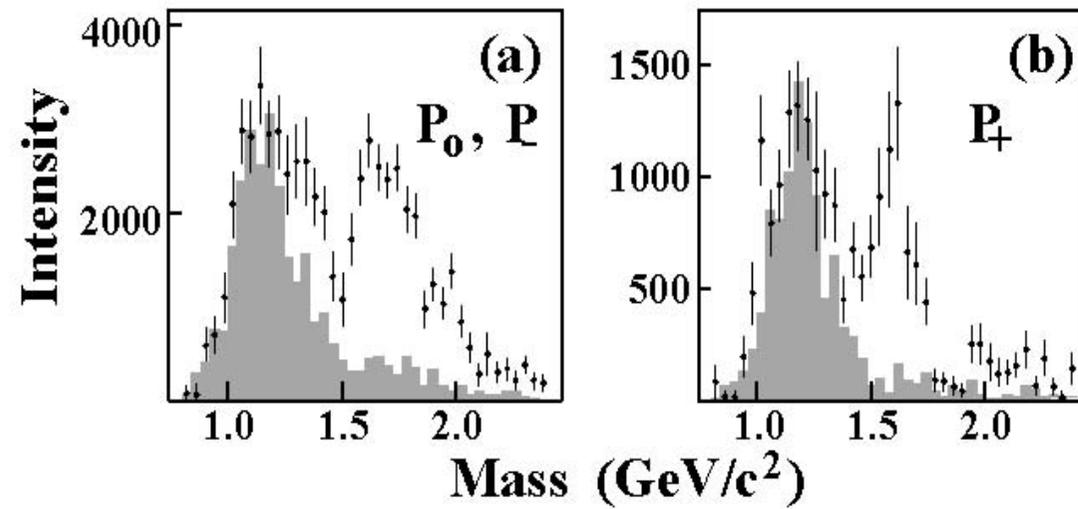
Close & Page



BNL-E852: $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$



MC with NO 1^-+

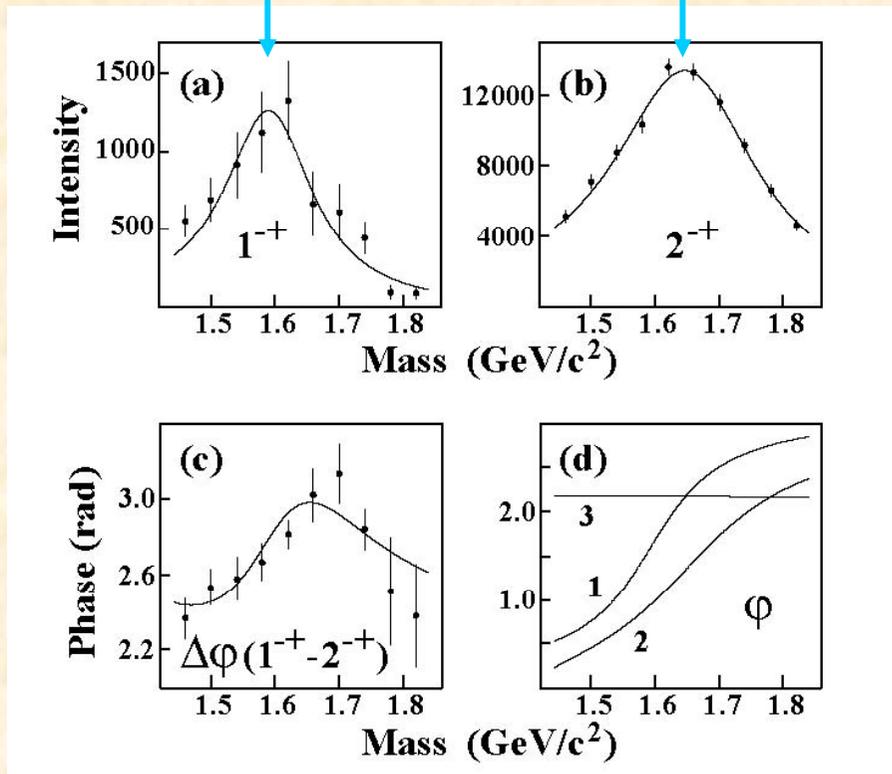


BNL-E852: $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$

Adams et al.
Chung et al.

$\pi_1(1600)$

$\pi_2(1670)$



$\pi_2(1670) \rightarrow f_2(1270)\pi$

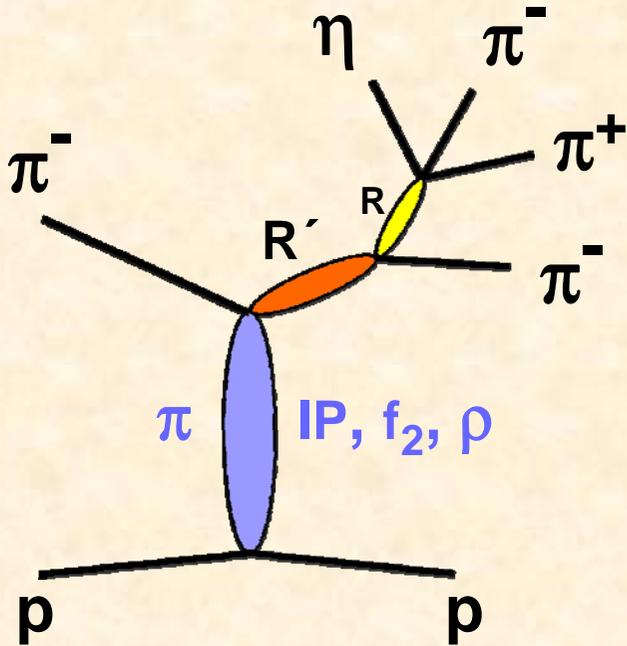
$\pi_1(1600)$

$M = (1593 \pm 8 \pm 38) \text{ MeV}$

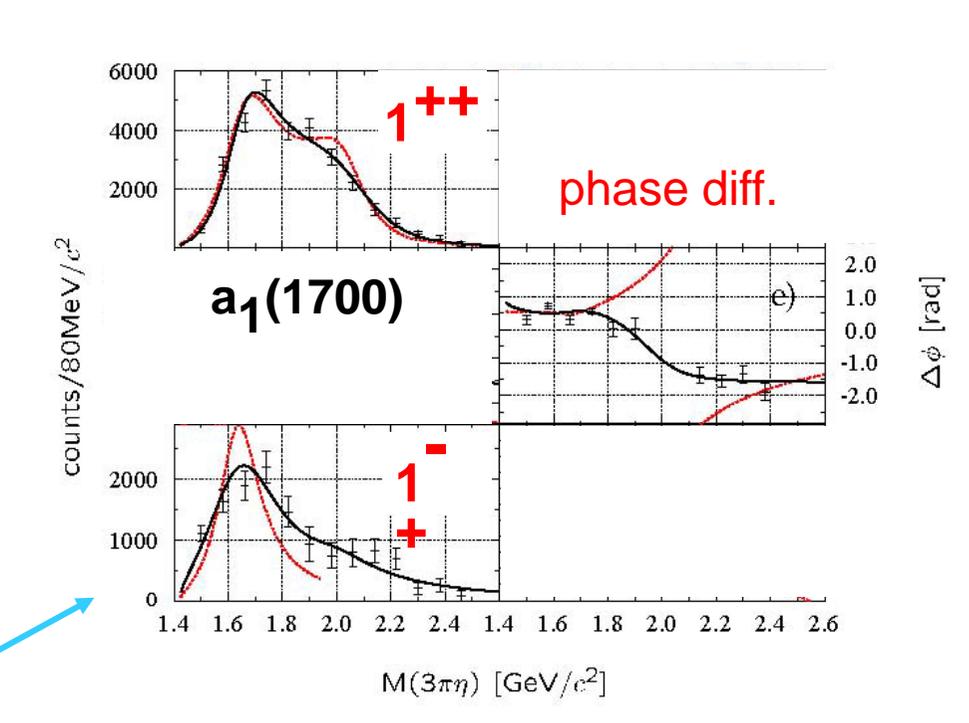
$\Gamma = (168 \pm 20^{+150}_{-12}) \text{ MeV}$

BNL-E852: $\pi^- p \rightarrow \eta \pi^+ \pi^- \pi^- p$

Kuhn et al. 69k events



53 partial waves



$\pi_1(1600) \rightarrow f_1(1285) \pi, \eta(1295) \pi$

$\pi_1(1600) : M = (1709 \pm 24 \pm 41) \text{ MeV} \quad \Gamma = (403 \pm 80 \pm 115) \text{ MeV}$

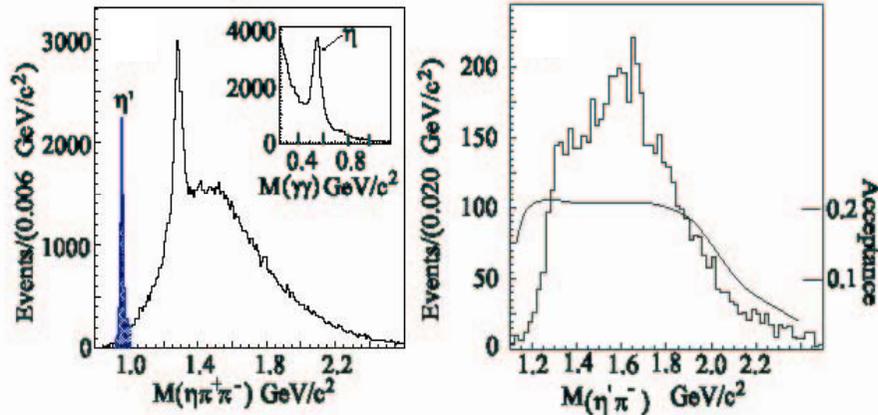
$\pi_1(2000)$

BNL-E852: $\pi^- p \rightarrow \eta' \pi^- p$

18 GeV/c

Ivanov et al.

5.8k events



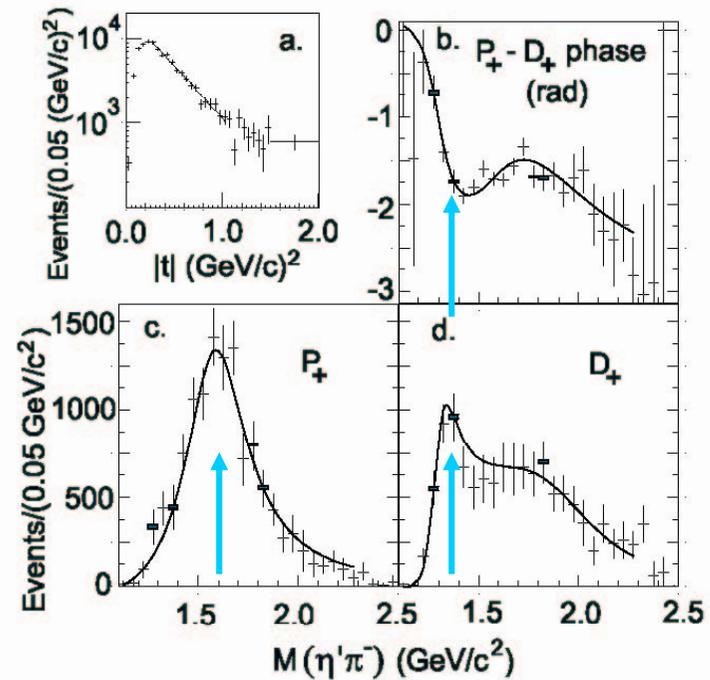
S, P, D, G waves

dominated by natural parity exchange

1^{-+} , 2^{++} , 4^{++}
 $\pi_1(1600)$, $a_2(1320)$, $a_4(2040)$

$M = 1597 \pm 40 \text{ MeV}$

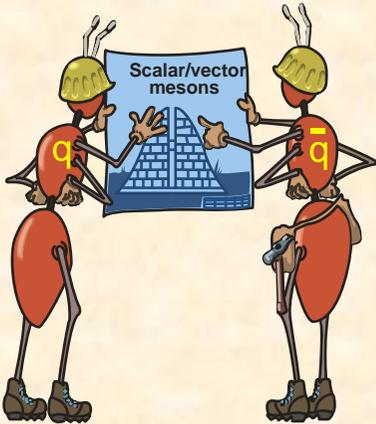
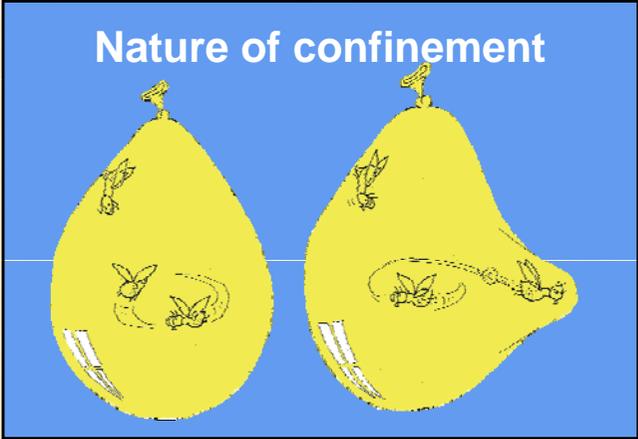
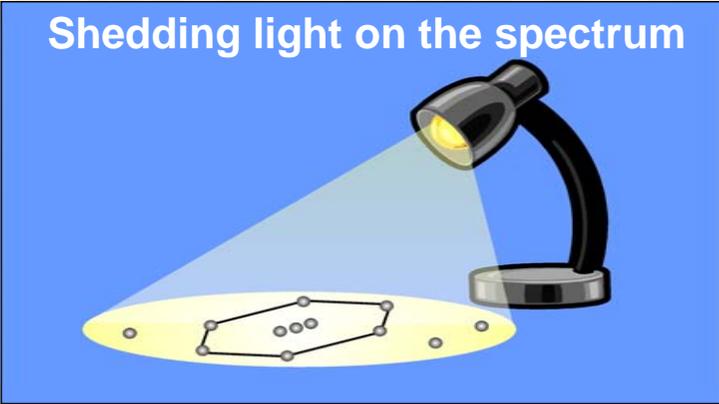
$\Gamma = 340 \pm 65 \text{ MeV}$





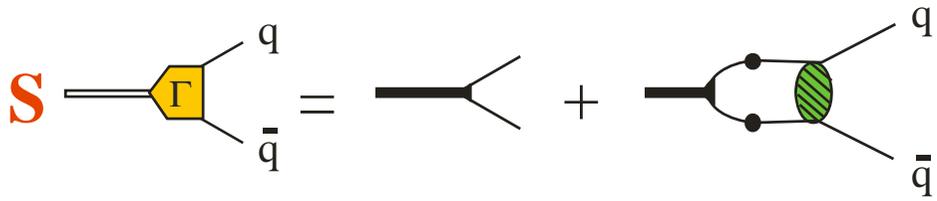
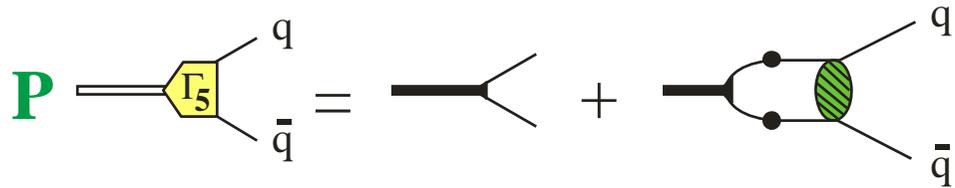
To learn about hadron structure and dynamics at the QCD scale

demands global analyses of J/ψ , B,D decays, $\gamma\gamma$, $\bar{p}p$...
and reliable strong coupling calculations

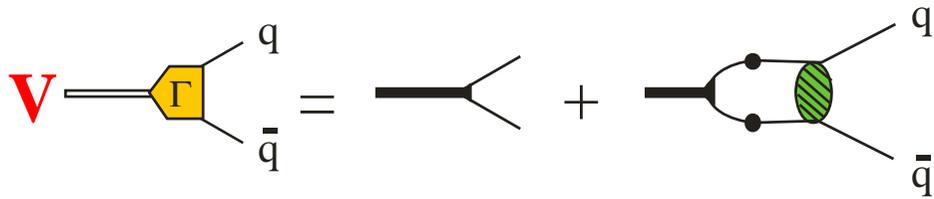
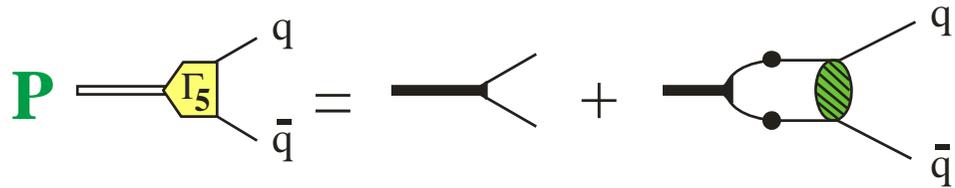


**EXTRA
SLIDES**

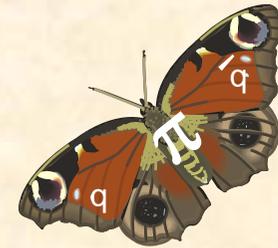
π, σ BSE



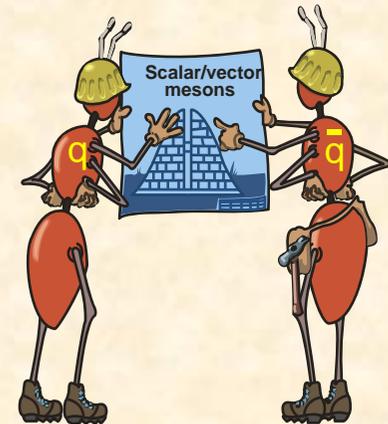
π, ρ BSE



$M_\pi \approx 140$

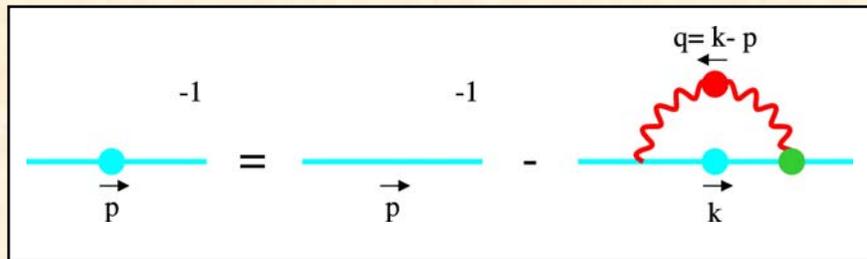


$M_\rho \approx 770$

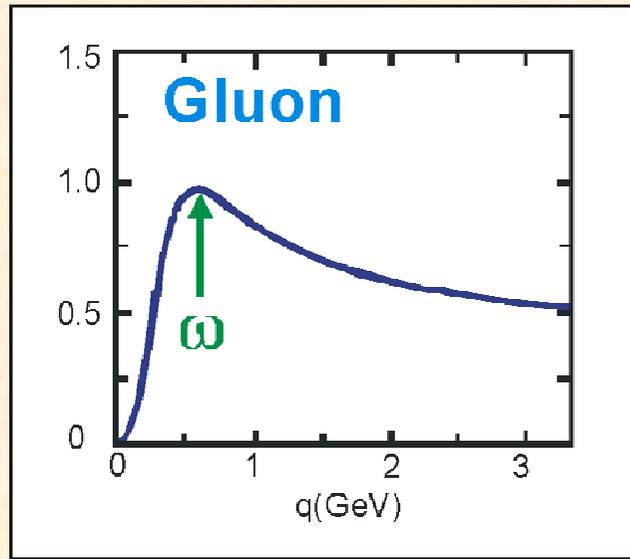


calculating hadron masses

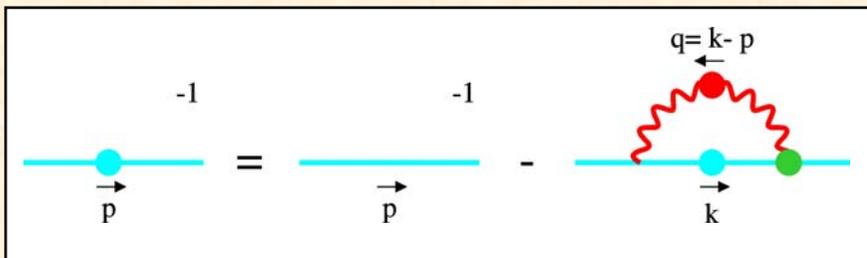
SDE/BSE-Tübingen



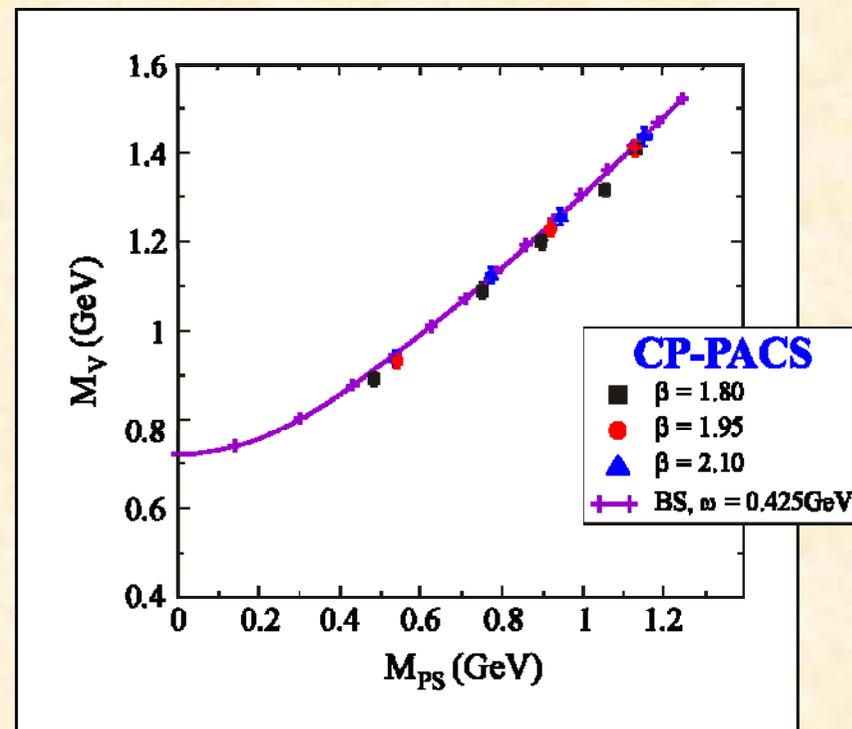
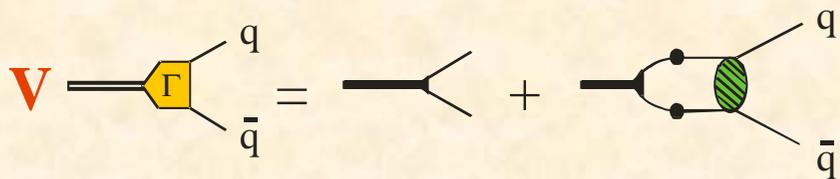
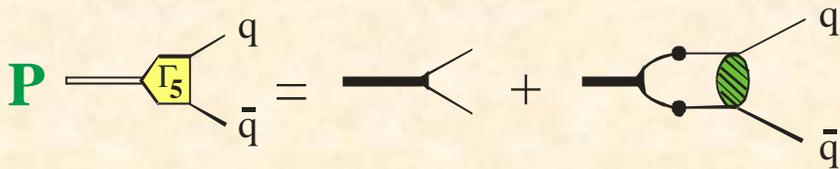
$$g^2 \Delta_{\mu\nu}(q) = C T_{\mu\nu}(q) \frac{q^2}{\omega^2} \exp\left(-\frac{q^2}{\omega^2}\right)$$



SDE/BSE-Tübingen

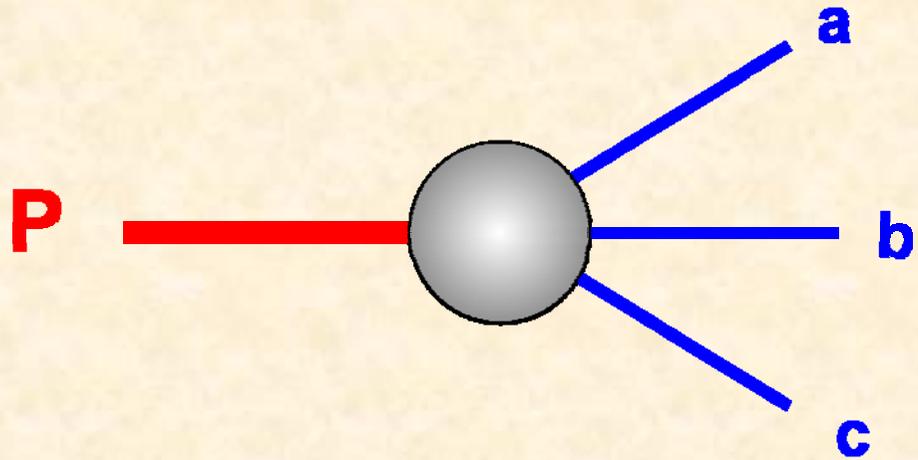


$$g^2 \Delta_{\mu\nu}(q) = C T_{\mu\nu}(q) \frac{q^2}{\omega^2} \exp\left(-\frac{q^2}{\omega^2}\right)$$

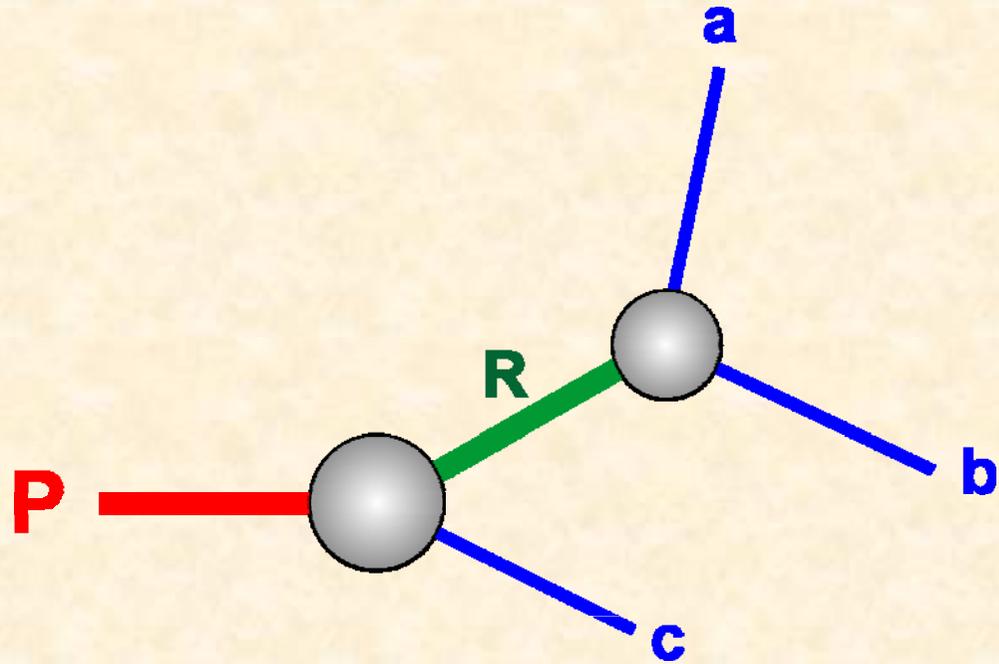


Watson, Benhaddou, P

P → **a b c**

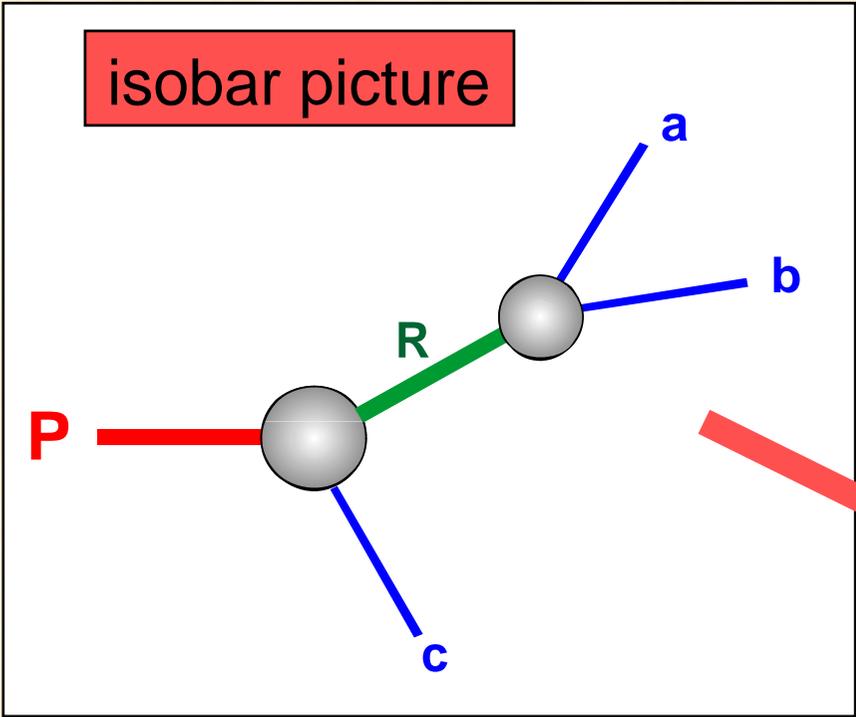


isobar picture

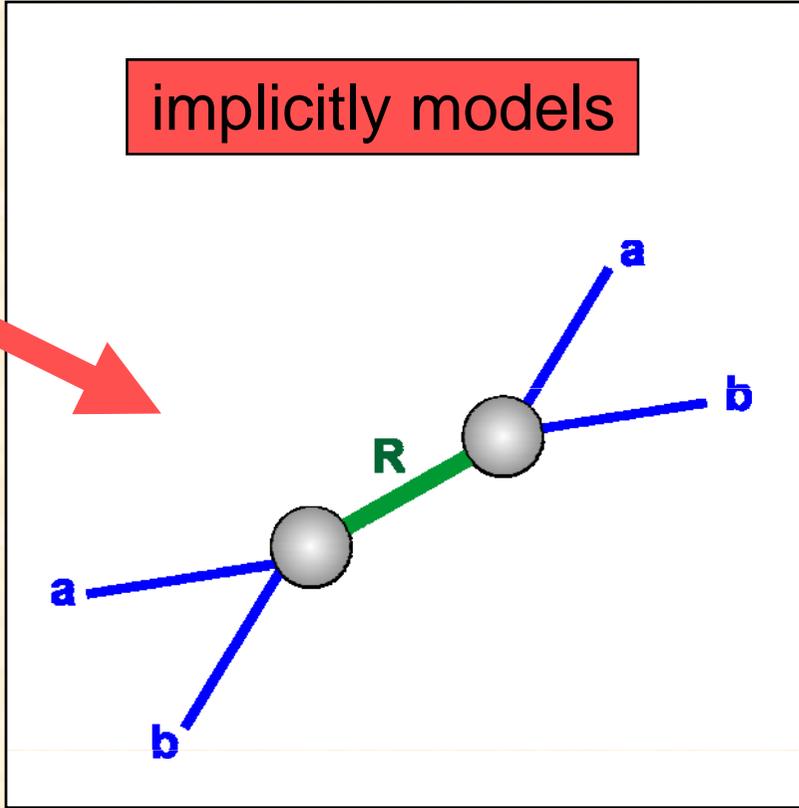


P → **a b c**

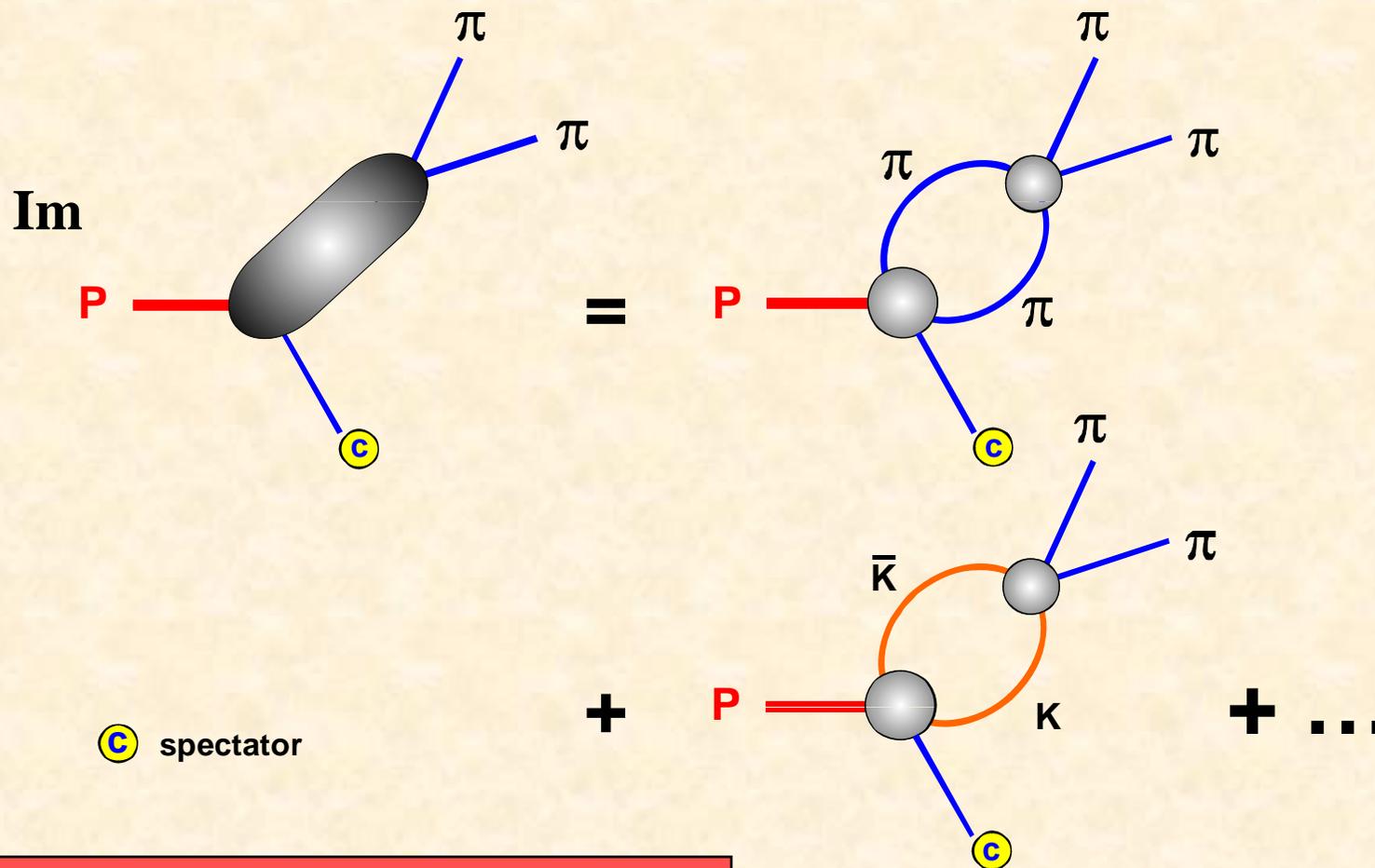
isobar picture



implicitly models

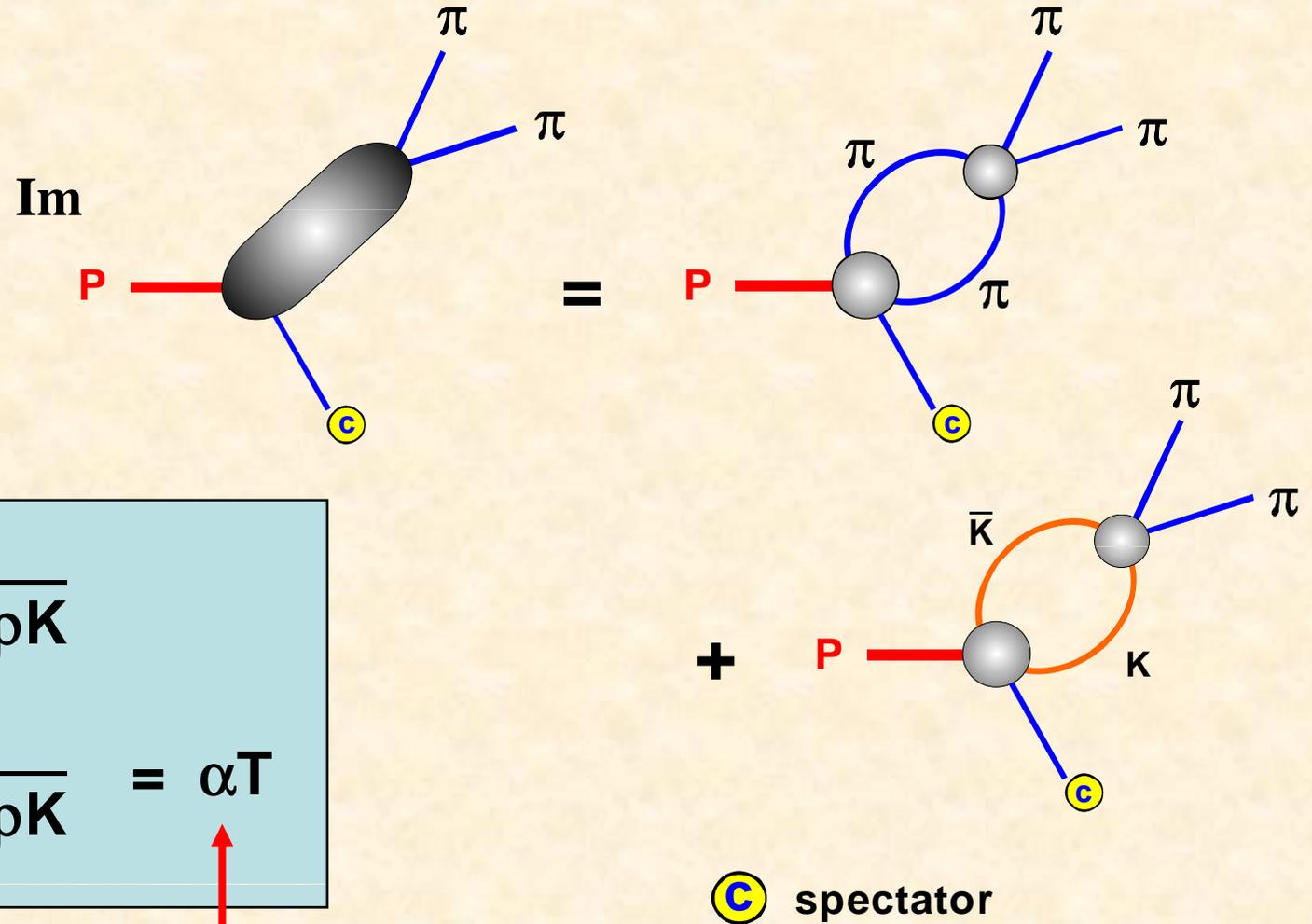


Unitarity for $P \rightarrow \pi\pi (c)$



unitarity connects to hadronic scattering

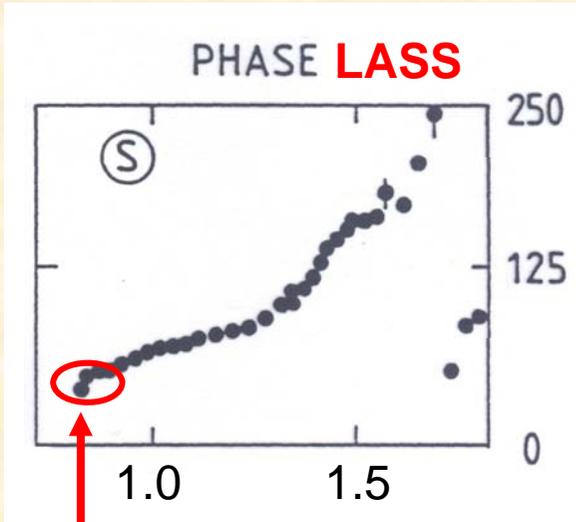
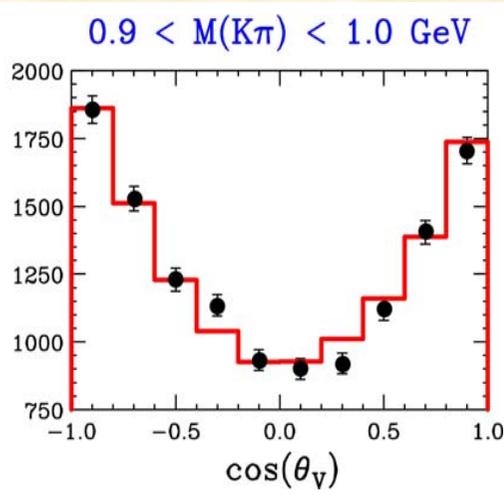
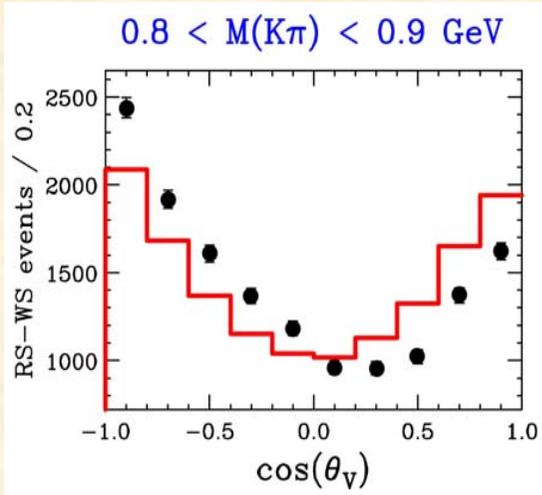
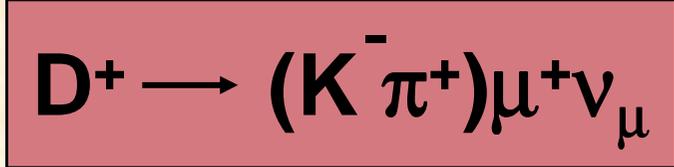
UNITARITY : decays in spectator picture



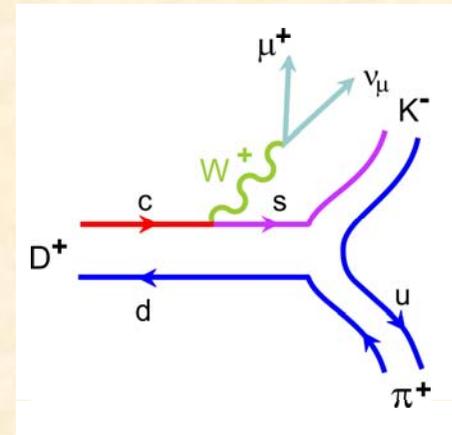
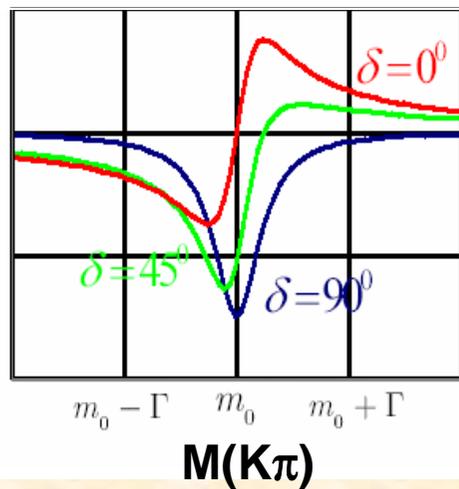
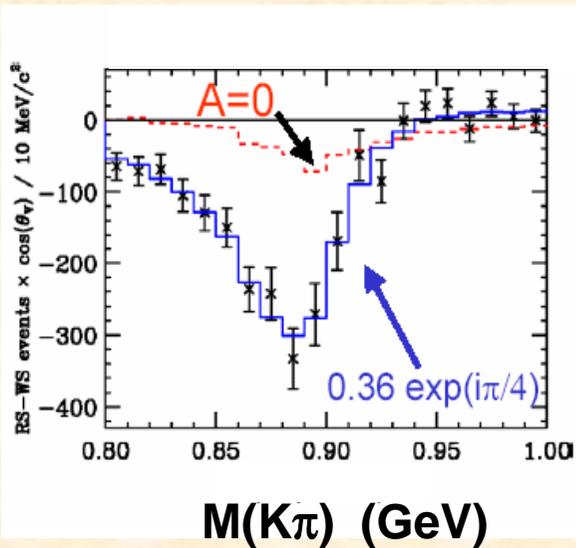
$$T = \frac{K}{1 - i\rho K}$$

$$F = \frac{P}{1 - i\rho K} = \alpha T$$

↑
coupling function



M(Kπ) (GeV)



s -wave from $D^+ \rightarrow K^-\pi^+\pi^+$ Dalitz Plot?

- Divide $m^2(K^-\pi^+)$ into slices
- Find s -wave amplitude in each slice (two parameters)
 - Use remainder of Dalitz plot as an interferometer

$$\frac{d^2\Gamma}{ds_{12}ds_{13}} \propto |S + (P + D)|^2$$



- For s -wave:

- Interpolate between (c_k, γ_k) points:

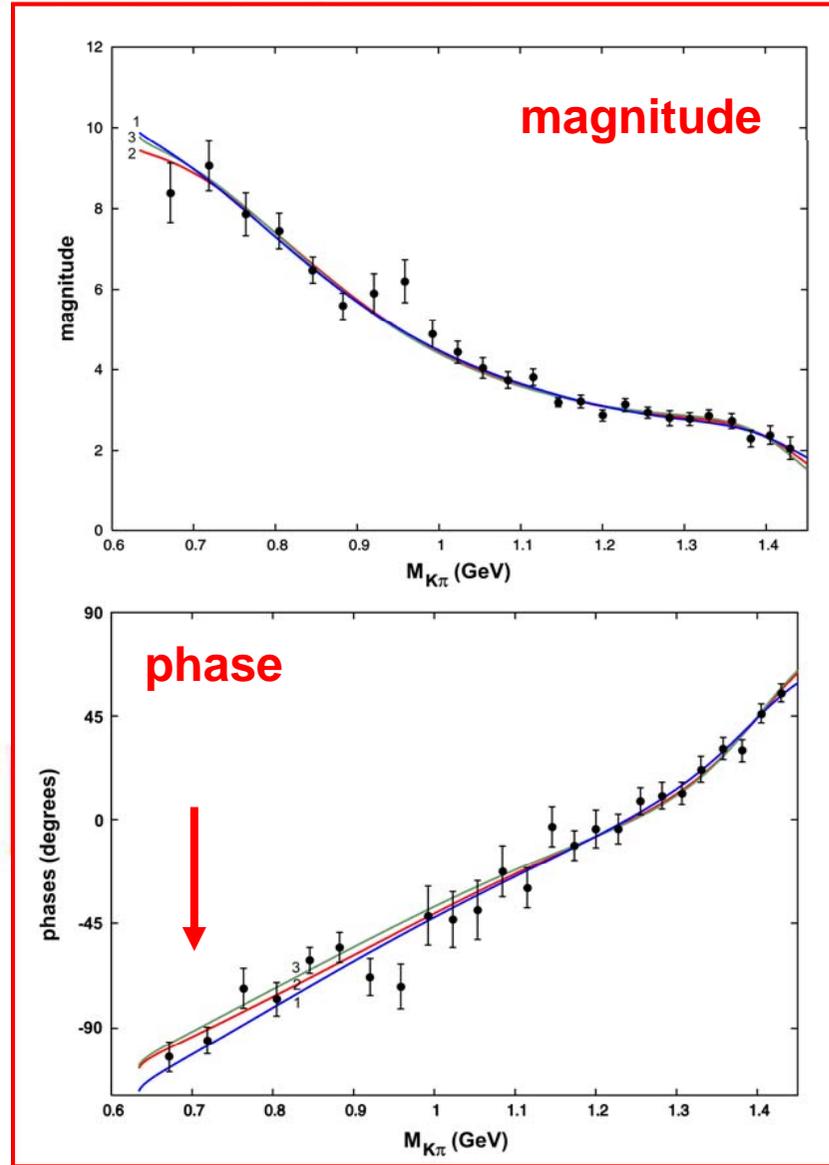
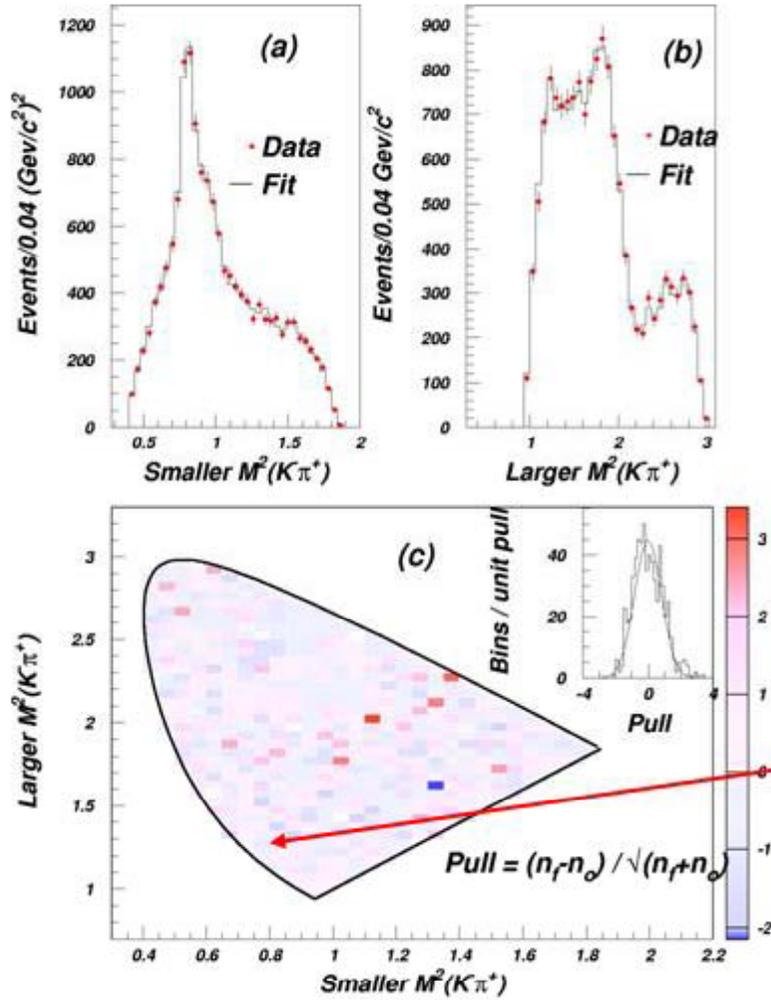
$$S = \text{Interp}(c_k e^{i\gamma_k}) \times F_0^D(q, r_D) F_0^R(p, r_R)$$

- Model P and D

S ("partial wave")

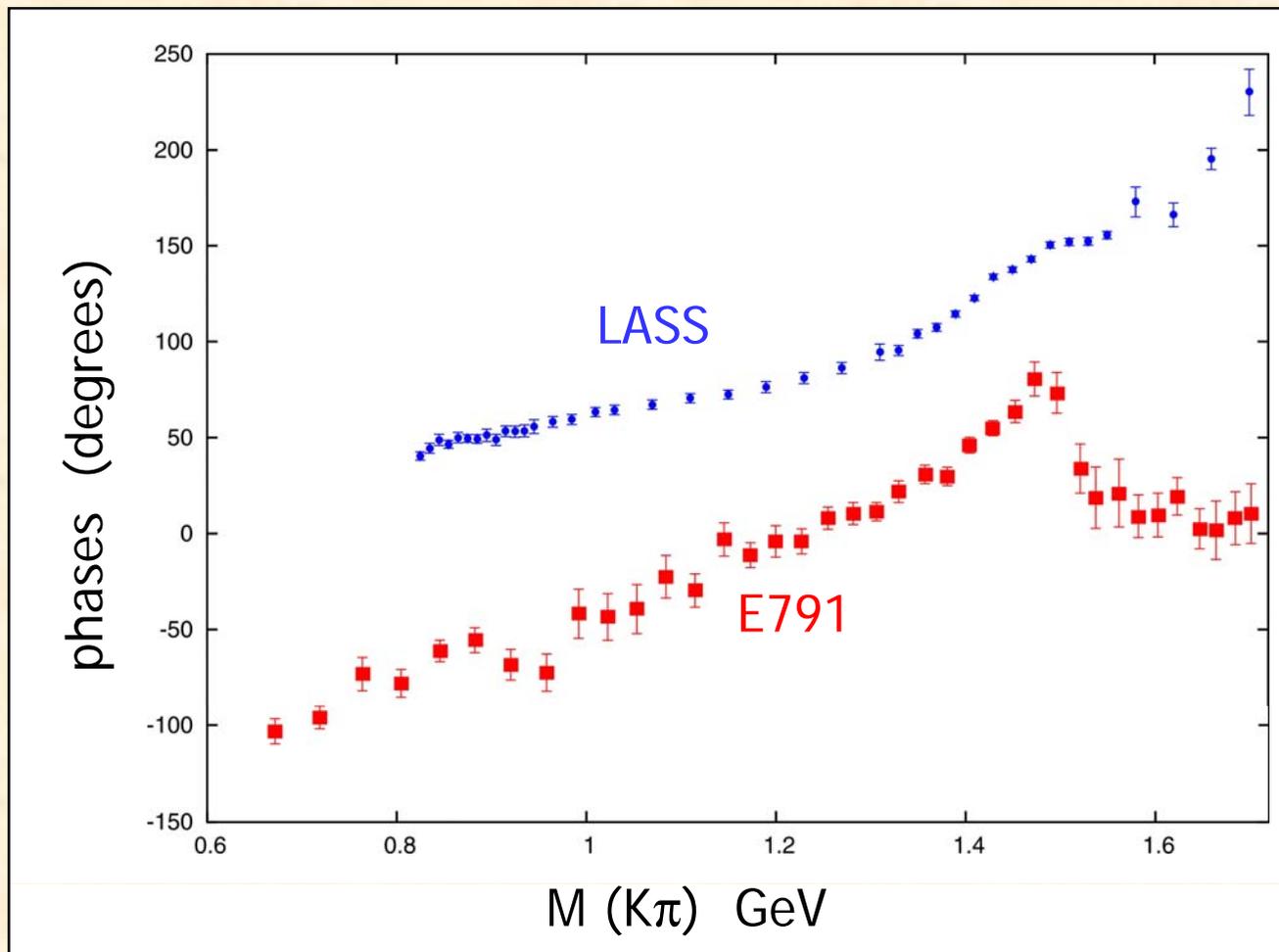
Brian Meadows

Comparison with Data



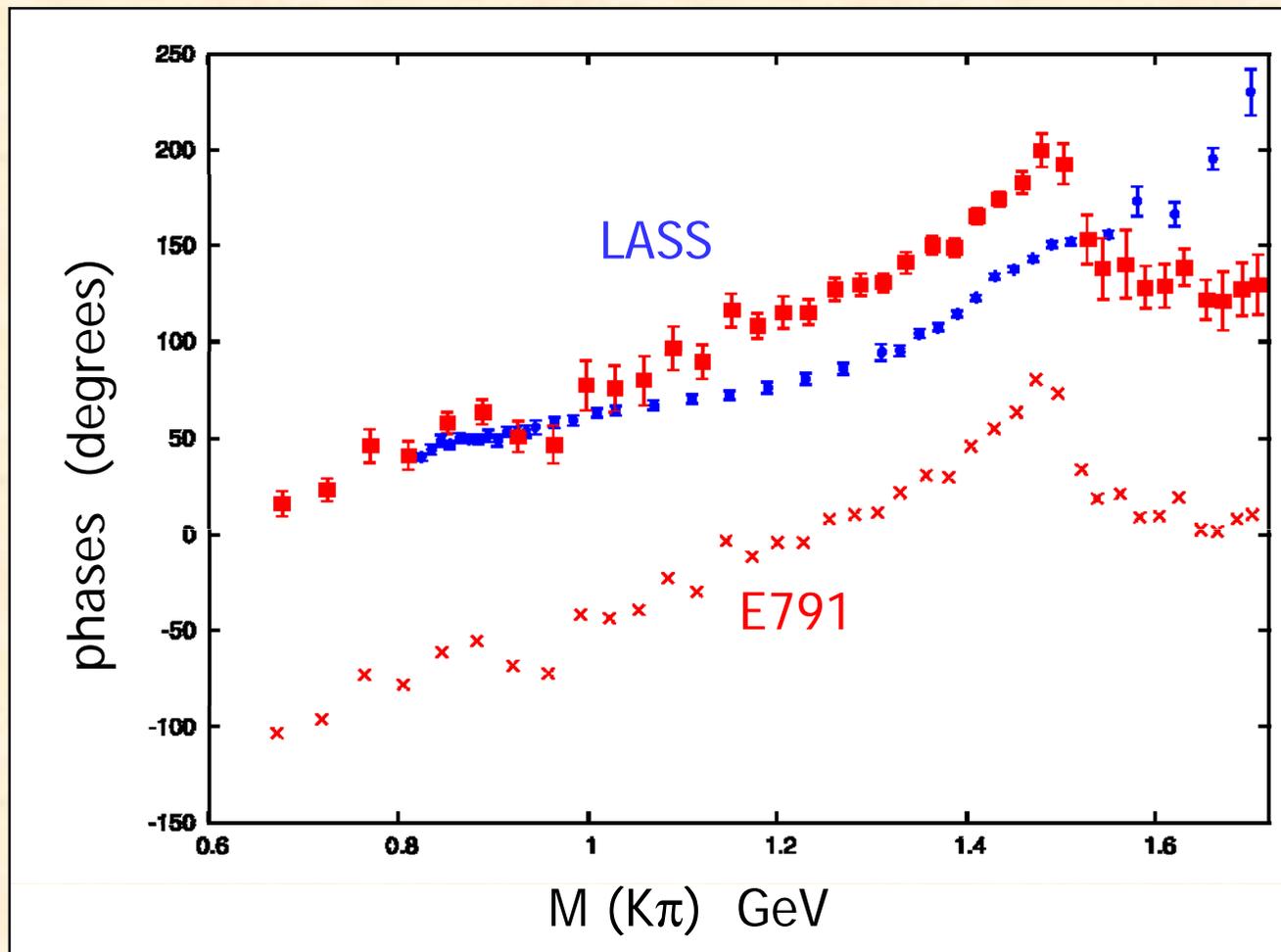
Brian Meadows

E791 v elastic scattering (LASS)

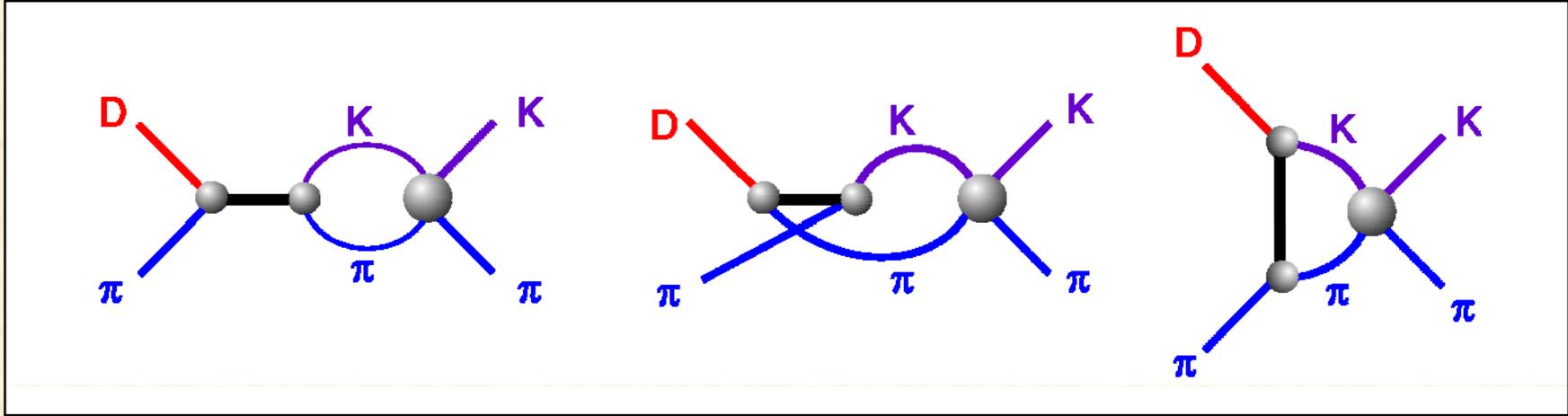
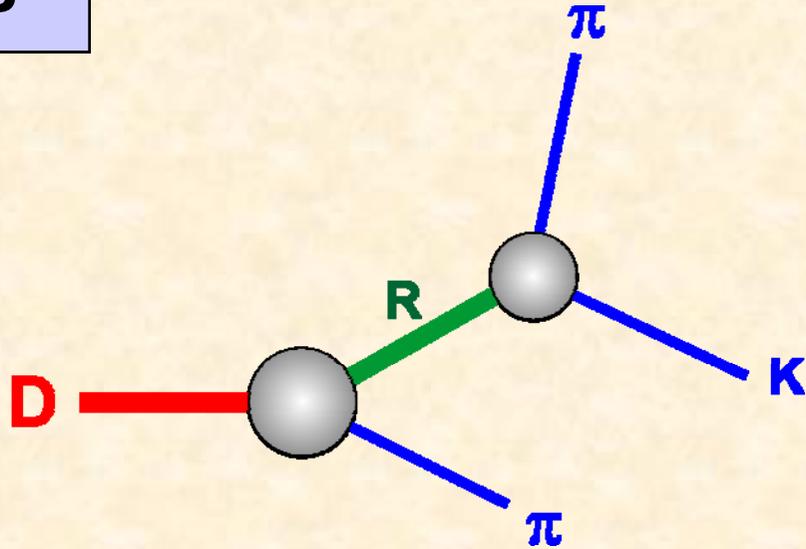


E791 relative phase

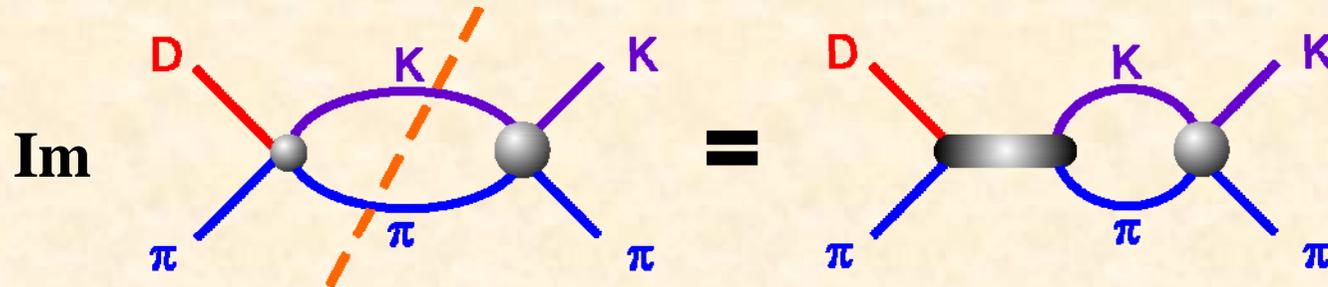
E791 ν elastic scattering (LASS)



Rescattering



Rescattering : Unitarity



Watson's theorem

elastic

phases simply related
if no rescattering

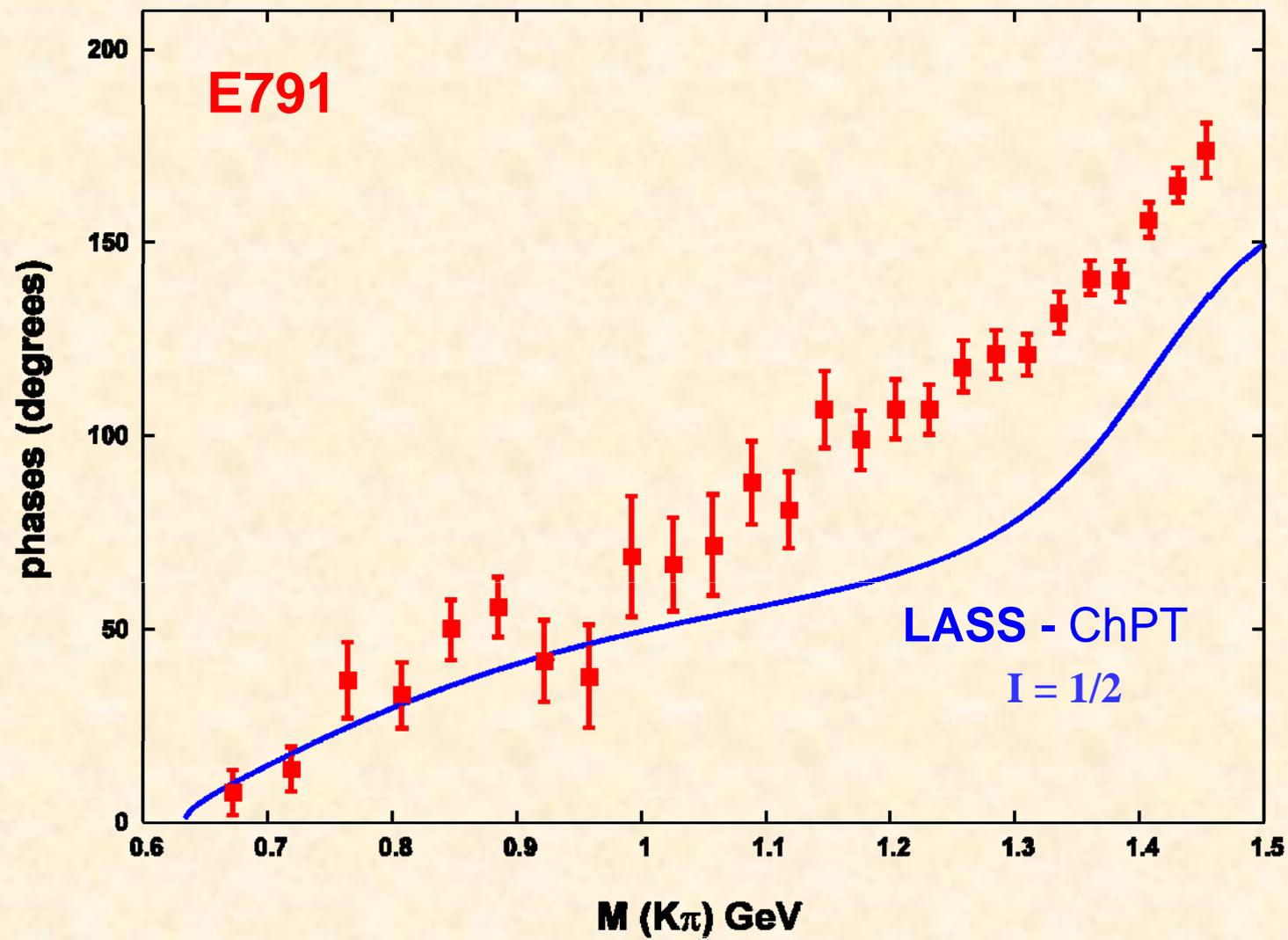
Rescattering : Unitarity

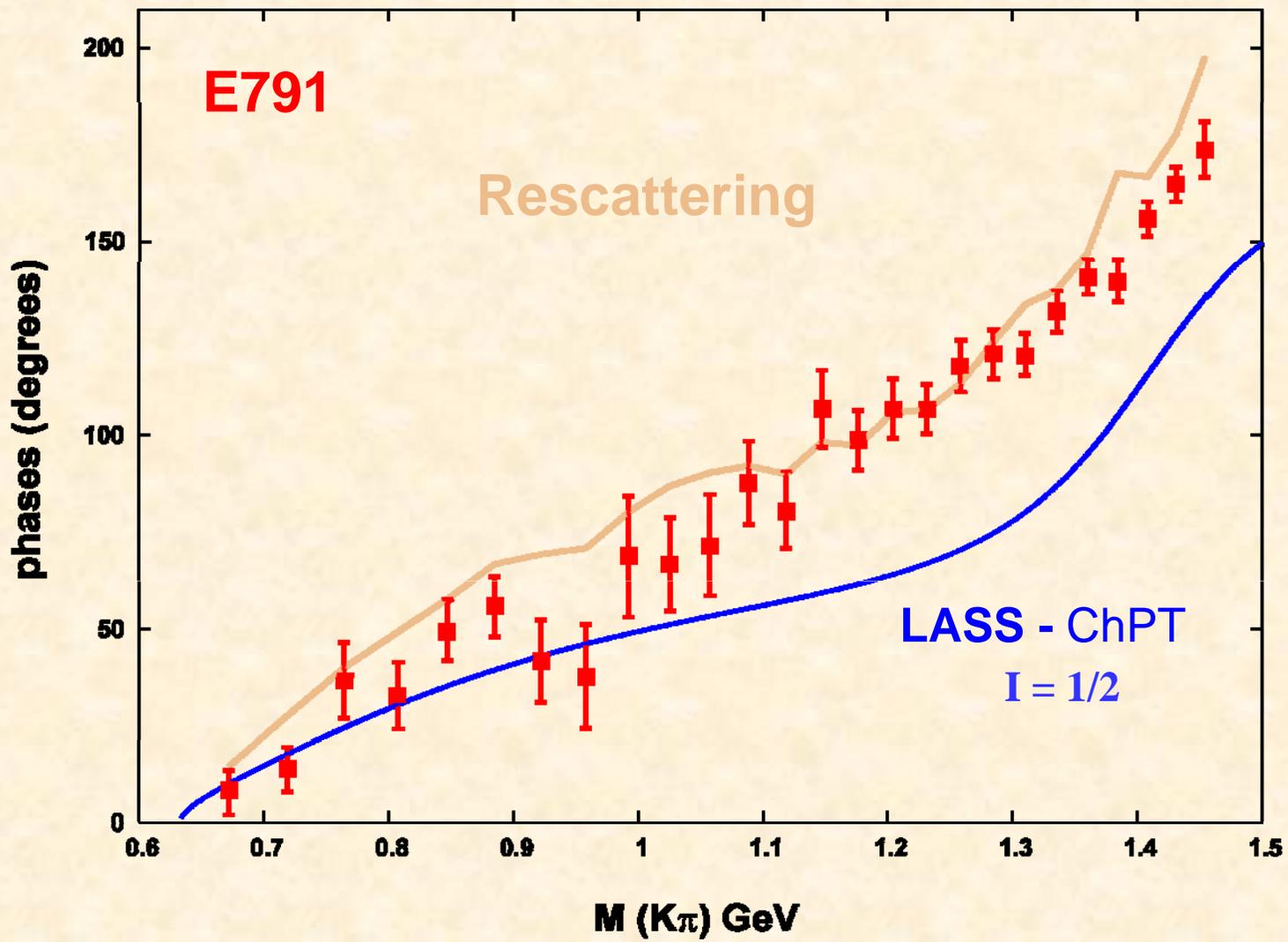
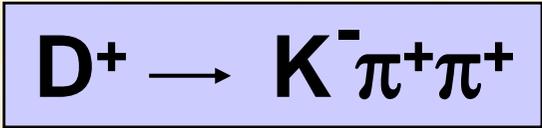
Im

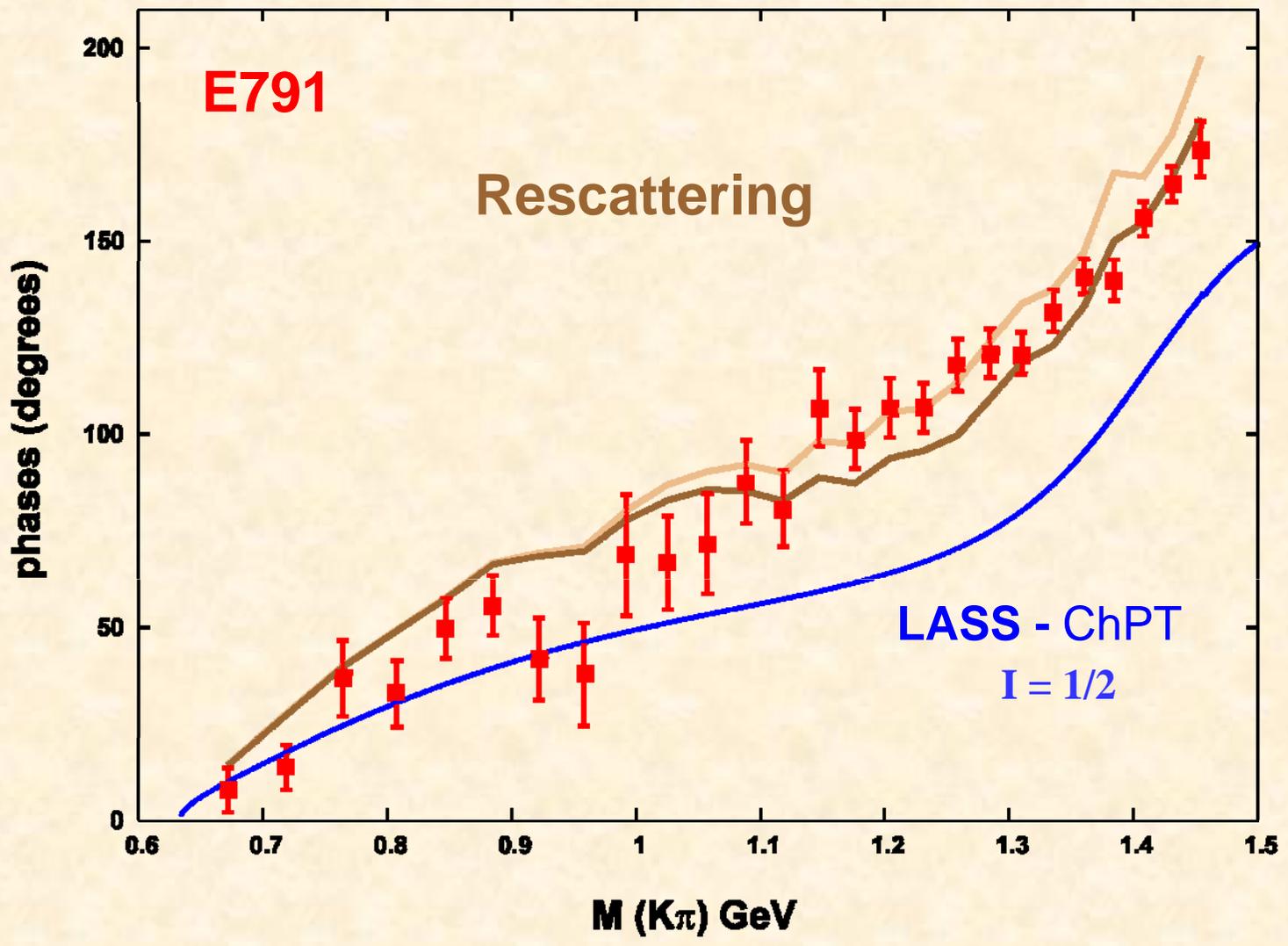
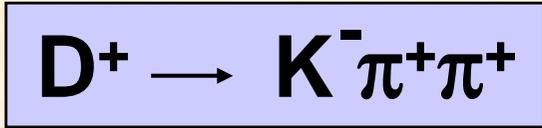
Watson's theorem

modified by rescattering

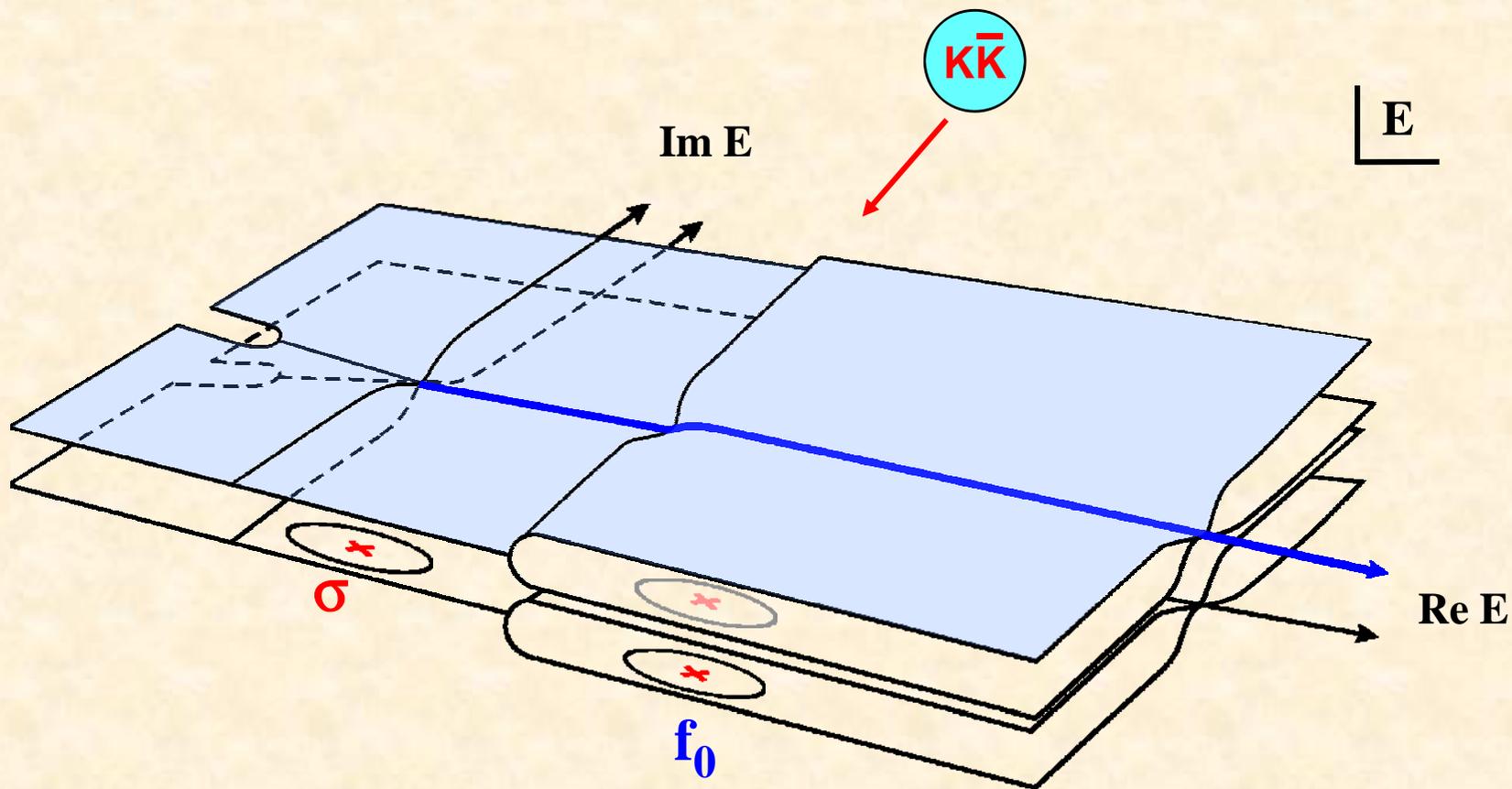
Caprini





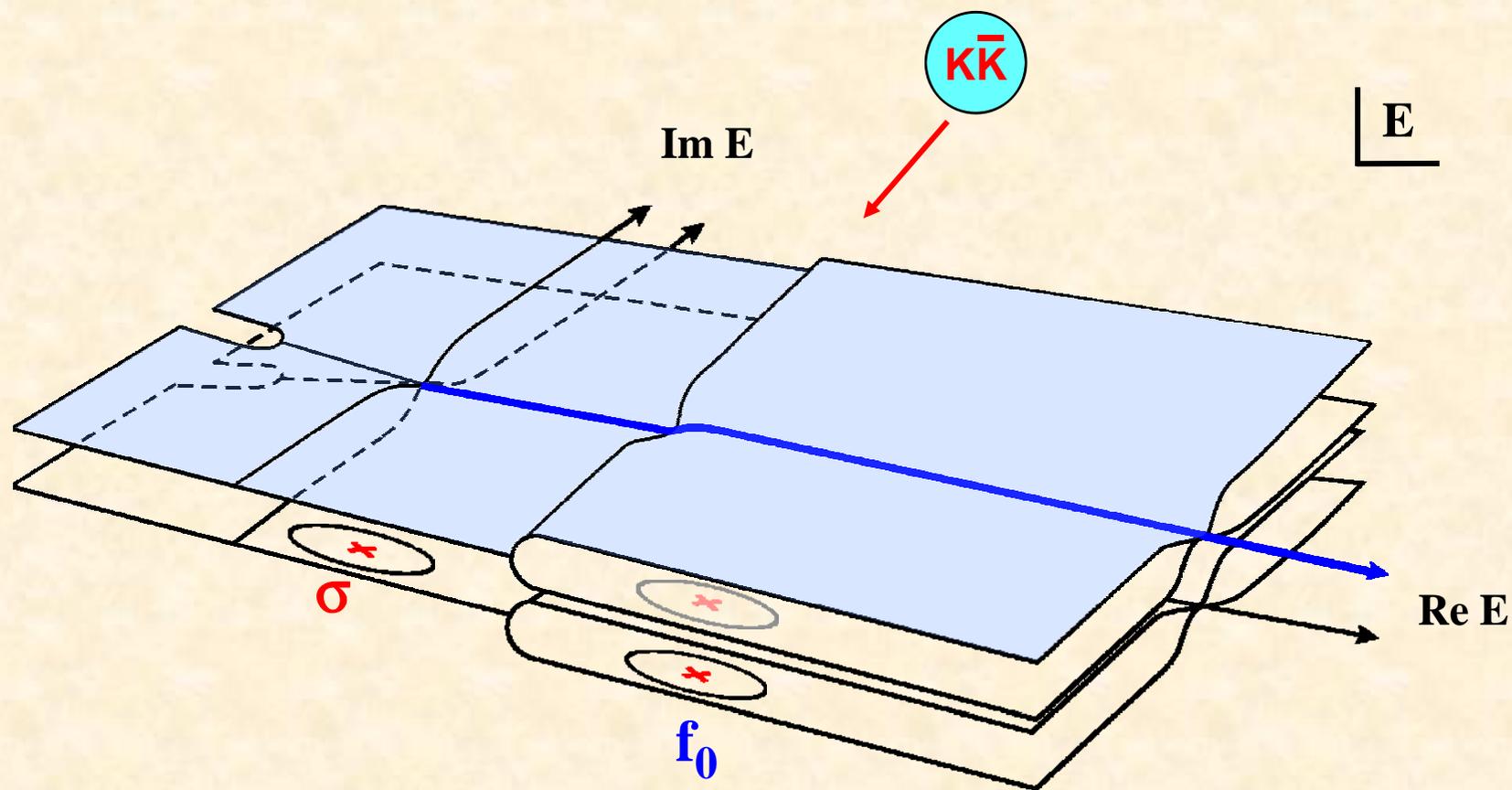


Quark state v Molecule



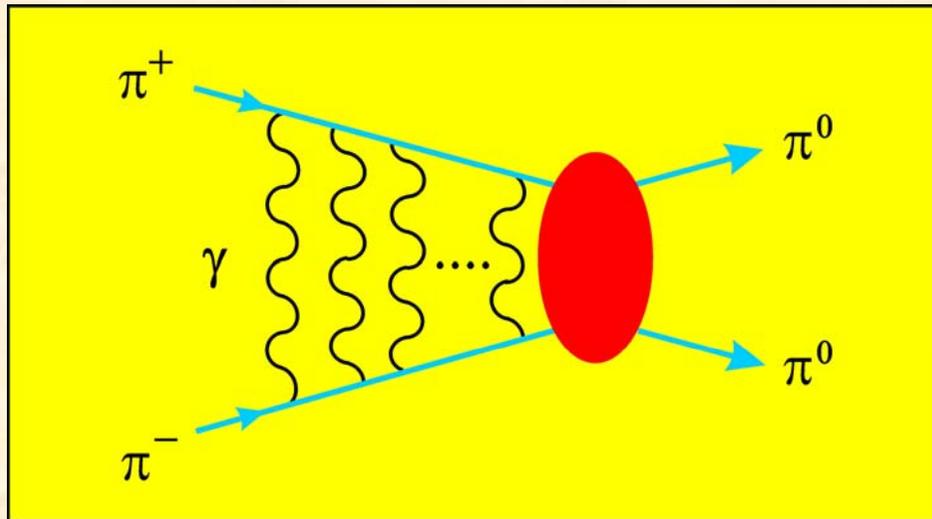
Weinberg
Morgan

Quark state v Molecule



Weinberg
Morgan

DIRAC experiment

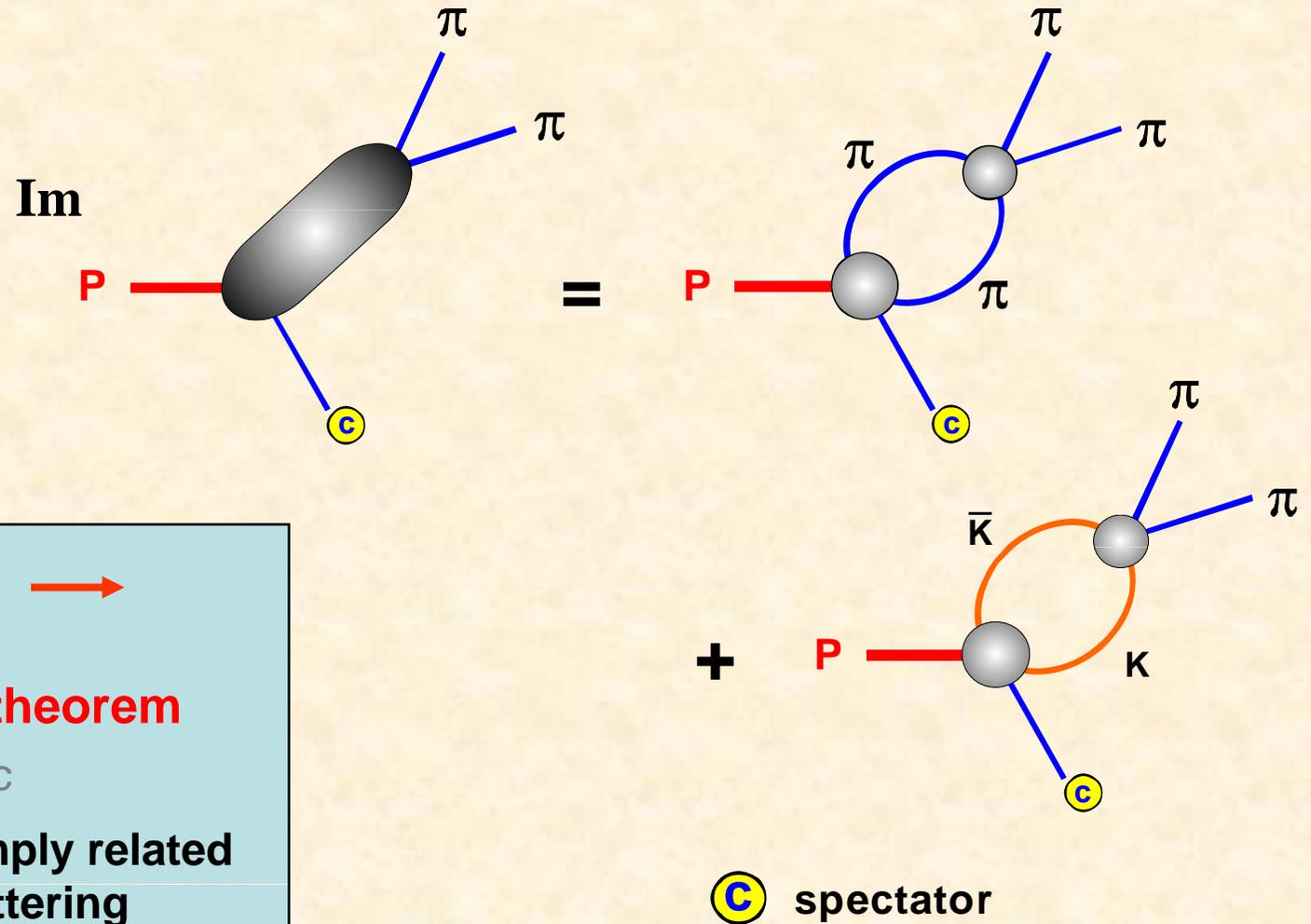


**2005
results**

$$\tau_{1S} = \left[2.91^{+0.49}_{-0.62} \right] 10^{-15} \text{ s}$$

→ $a_0 - a_2 = 0.264^{+0.033}_{-0.020}$

UNITARITY : decays in spectator picture



$$F = \alpha T \rightarrow$$

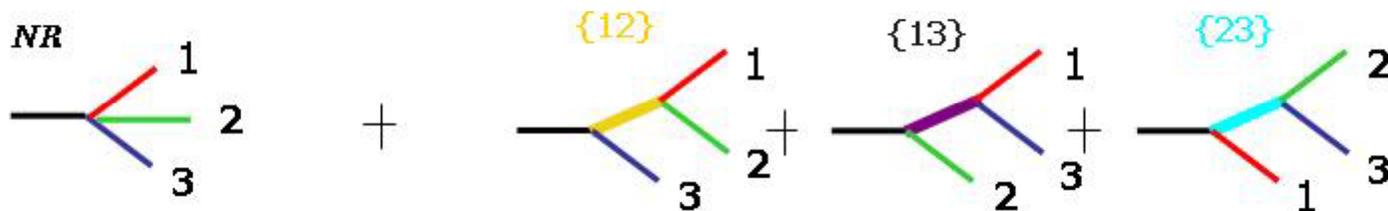
Watson's theorem

elastic

phases simply related
if no rescattering

“Traditional” Dalitz Plot Analyses

- The “isobar model” has been widely used, with Breit-Wigner resonant terms, over the past 15 years.



- Amplitude for channel $\{ij\}$:

$$\mathcal{A}_{ij} = \underbrace{d_0 e^{i\delta_0}}_{\substack{NR \\ \text{Constant}}} + \sum_R d_R e^{i\delta_R} A(s_{ij}) \times \underbrace{F_0^D(q, r_D)}_{\substack{D \text{ form} \\ \text{factor}}} \underbrace{F_J^R(p, r_R)}_{\substack{R \text{ form} \\ \text{factor}}} \underbrace{M_J(p, q)}_{\substack{\text{spin} \\ \text{factor}}}$$

- Each resonance “R” (mass M_R , width Γ_R) assumed to have form

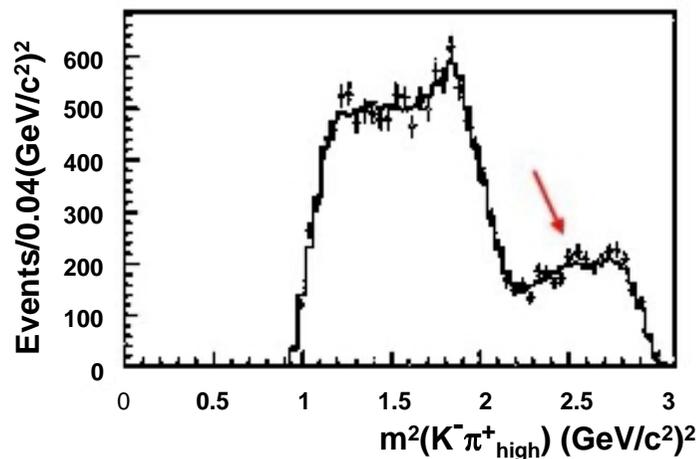
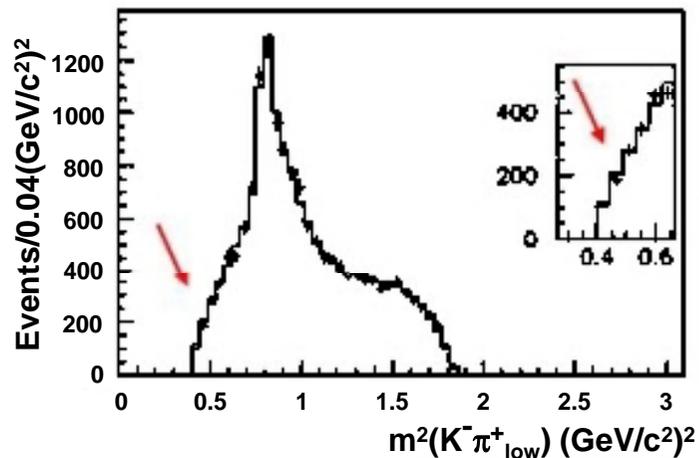
$$A_R(s_{ij}) = [m_R^2 - s_{ij} - im_R \Gamma(p, r_R)]^{-1}$$

p, q are momenta in ij rest frame

r_D, r_R meson radii

Brian Meadows

E791 $D^+ \rightarrow K^- \pi^+ \pi^+$



D^+

\rightarrow

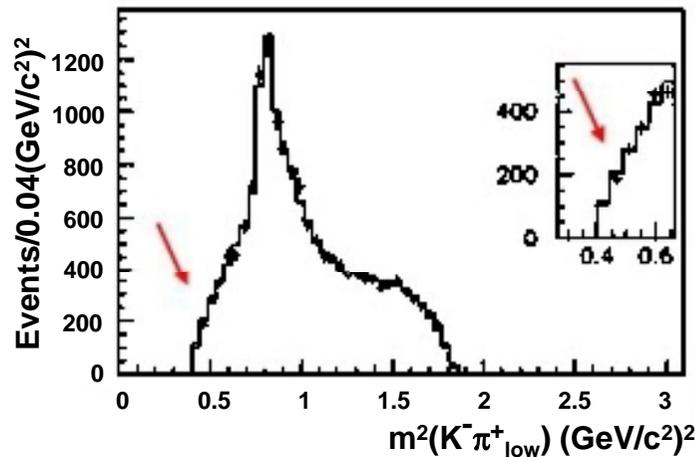
non resonant	$90.0 \pm 2.6\%$	0° (fixed)
$K^*(890)\pi^+$	$13.8 \pm 0.5\%$	$54 \pm 2^\circ$
$K^*_0(1430)\pi^+$	$30.6 \pm 1.6\%$	$109 \pm 2^\circ$
$K^*_2(1430)\pi^+$	$0.4 \pm 0.1\%$	$33 \pm 8^\circ$
$K^*_1(1680)\pi^+$	$3.2 \pm 0.3\%$	$66 \pm 3^\circ$
<hr/>		
$\sim 138\%$		

$\chi^2/\text{d.o.f.} = 2.7$

non-resonant dominates

Brian Meadows

E791 $D^+ \rightarrow K^- \pi^+ \pi^+$

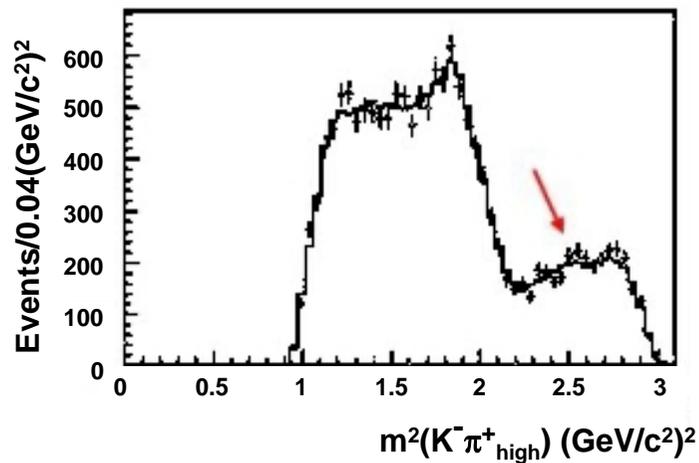


D^+

→

non resonant	$13.0 \pm 5.8 \pm 2.6\%$	$349 \pm 14 \pm 8^\circ$
" κ " π^+	$47.8 \pm 12.1 \pm 3.7\%$	$187 \pm 8 \pm 17^\circ$
$K^*(890)\pi^+$	$12.3 \pm 1.0 \pm 0.9\%$	0° (fixed)
$K_0^*(1430)\pi^+$	$12.5 \pm 1.4 \pm 0.4\%$	$48 \pm 7 \pm 10^\circ$
$K_2^*(1430)\pi^+$	$0.5 \pm 0.1 \pm 0.2\%$	$306 \pm 8 \pm 6^\circ$
$K_1^*(1680)\pi^+$	$2.5 \pm 0.7 \pm 0.2\%$	$28 \pm 13 \pm 15^\circ$

$\sim 89\%$



$\chi^2/\text{d.o.f.} = 0.73$
(95 %)

Probability

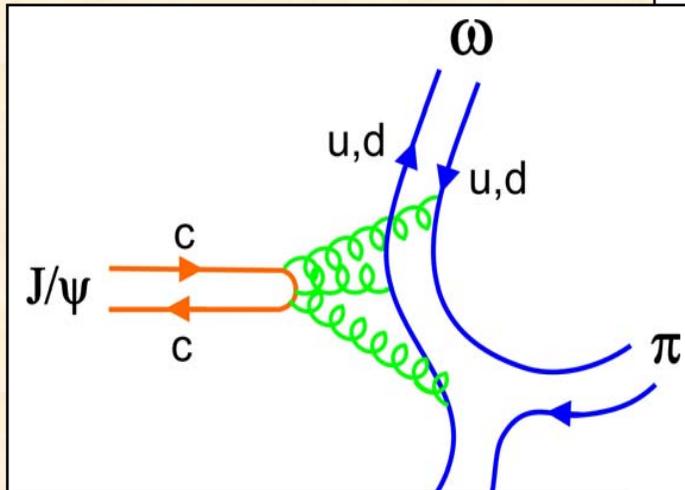
κ

$M_\kappa = 797 \pm 19 \pm 42 \text{ MeV}/c^2$
 $\Gamma_\kappa = 410 \pm 43 \pm 85 \text{ MeV}/c^2$

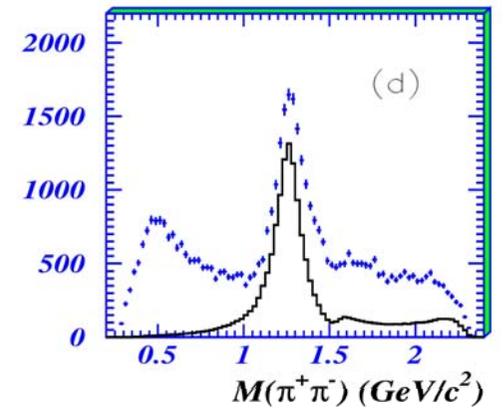
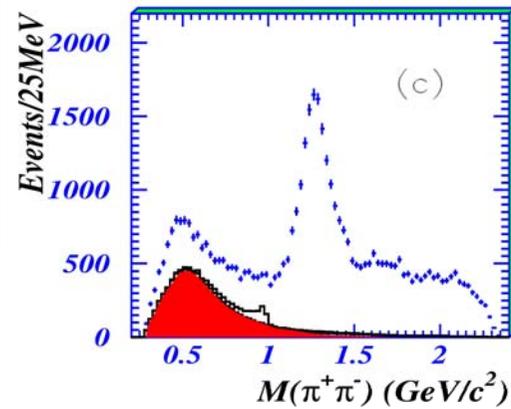
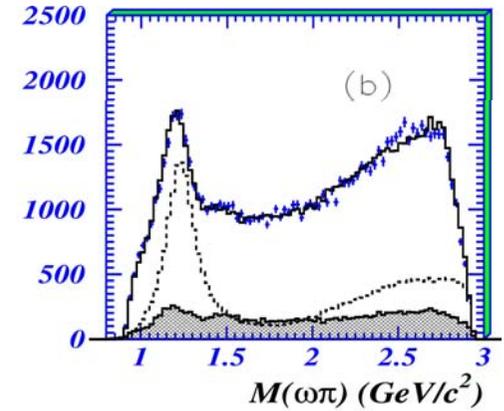
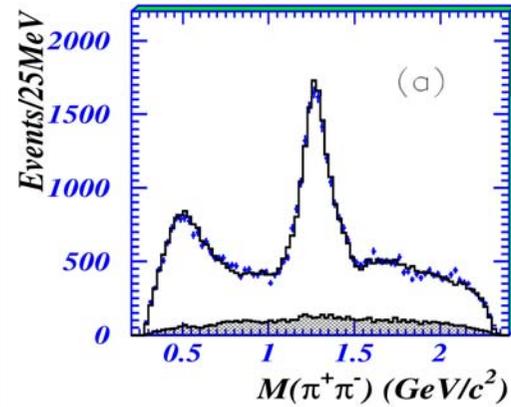
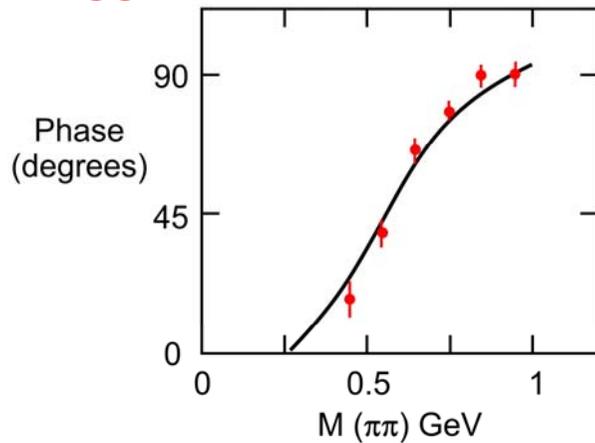
Brian Meadows

$$J/\psi \rightarrow \omega \pi^+ \pi^-$$

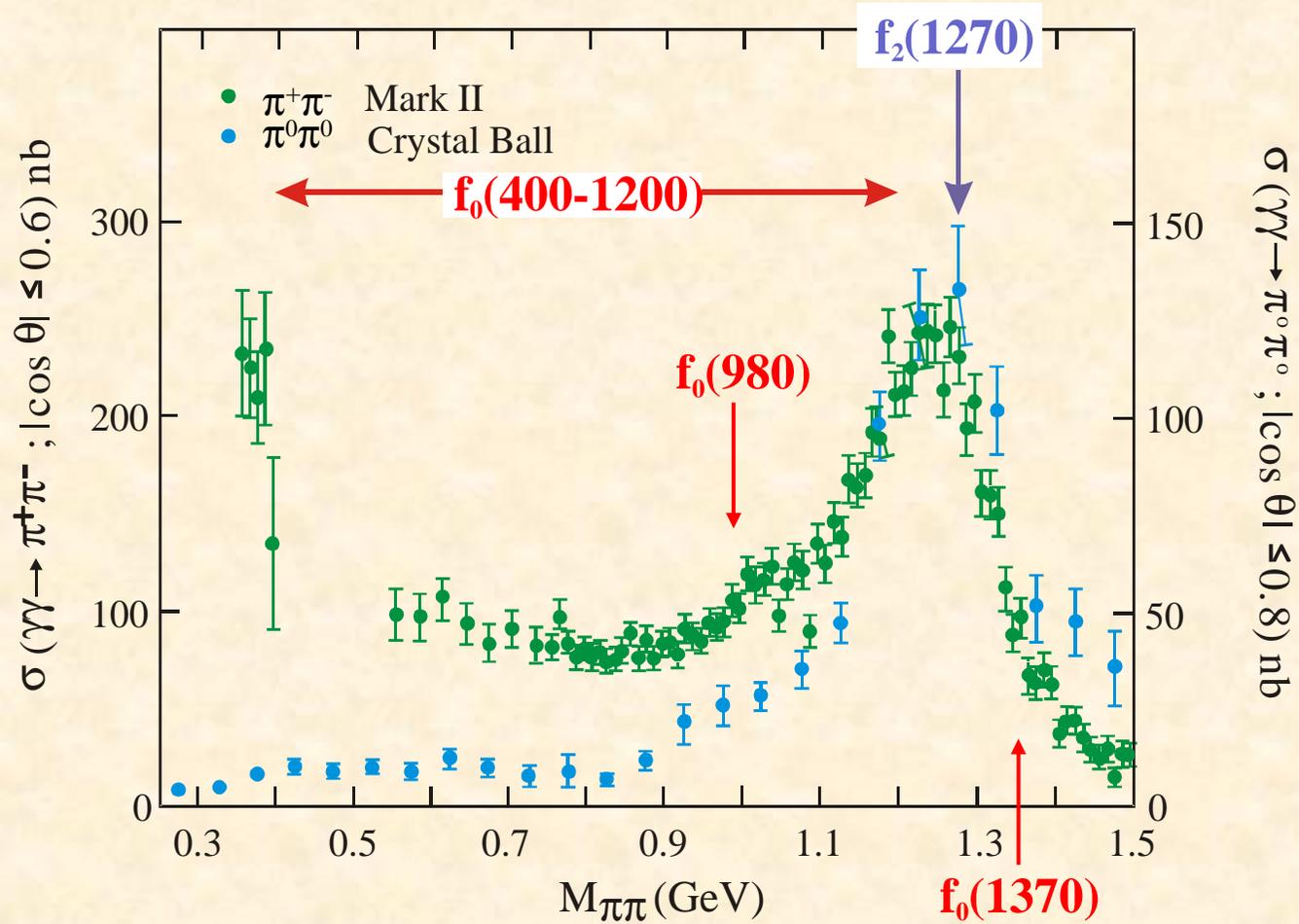
BES



Bugg



$\gamma\gamma$ couplings



Old PDG
values

$\Gamma(\gamma\gamma)$
keV

$f_0(980)$
0.13 – 0.36

$f_0(400-1200)$
3 – 5