

Transversity, Collins and Sivers Effects from COMPASS, HERMES and BELLE Data: New Global Analysis

Alexei Prokudin

Università di Torino and INFN Sezione di Torino

International Workshop on Structure and Spectroscopy



In collaboration with M. Anselmino, M. Boglione, U. D'Alesio,
F. Murgia, A. Kotzinian and C.Turk

Outline of this talk

1 Introduction

2 Collins effect in SIDIS and e^+e^- annihilation

- The model for Collins FF and transversity
- Description of the data & Predictions

3 Sivers effect in SIDIS

- The model for the Sivers function
- Description of the data & Predictions

4 Conclusions

The fundamental distributions of partons inside a nucleon

Unpolarised Distribution

$f_1(x)$ or $q(x)$



Distribution of unpolarised partons in an unpolarised nucleon.
Well known

Helicity Distribution

$g_1(x)$ or $\Delta q(x)$



Distribution of longitudinally polarised partons in a longitudinally polarised nucleon.

Known

Transversity Distribution

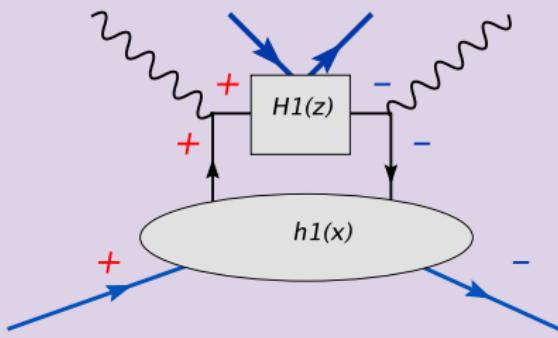
$h_1(x)$ or $\Delta_T q(x)$



Distribution of transversely polarised quarks in a transversely polarised nucleon.
Little known!
HERMES and COMPASS
first experimental measurements

Transversity in SIDIS

Transversity in Semi inclusive Deep Inelastic Scattering $IN \rightarrow l'hX$



Transversely polarised quark fragments into an unpolarised hadron:

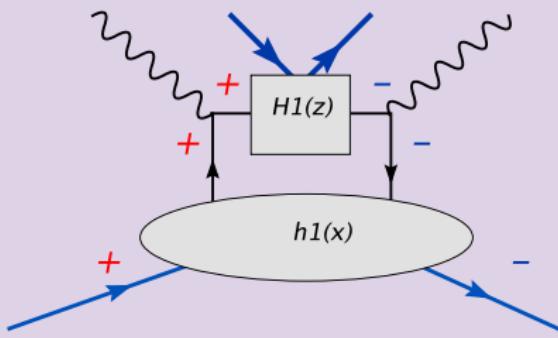
$$D_{h/q^\uparrow}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{1}{2} S_{q'} \cdot (\hat{\mathbf{p}}_{q'} \times \hat{\mathbf{p}}_\perp) \Delta^N D_{h/q^\uparrow}(z, |\mathbf{p}_\perp|),$$

where $\Delta^N D_{h/q^\uparrow}(z, |\mathbf{p}_\perp|)$ is so called Collins fragmentation function.



Transversity in SIDIS

Transversity in Semi inclusive Deep Inelastic Scattering $IN \rightarrow l'hX$



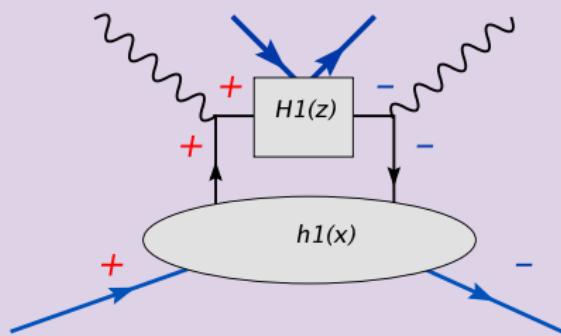
Transversely polarised quark fragments into an unpolarised hadron:

$$D_{h/q^\uparrow}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{1}{2} S_{q'} \cdot (\hat{\mathbf{p}}_{q'} \times \hat{\mathbf{p}}_\perp) \Delta^N D_{h/q^\uparrow}(z, |\mathbf{p}_\perp|),$$

where \mathbf{p}_\perp is transverse momentum of produced hadron with respect to fragmenting quark \rightarrow non-perturbative effect.

Transversity in SIDIS

Transversity in Semi inclusive Deep Inelastic Scattering $IN \rightarrow I'hX$



Transversely polarised quark fragments into an unpolarised hadron:

$$D_{h/q^\dagger}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{1}{2} S_{q'} \cdot (\hat{\mathbf{p}}_{q'} \times \hat{\mathbf{p}}_\perp) \Delta^N D_{h/q^\dagger}(z, |\mathbf{p}_\perp|),$$

The effect vanishes if $\mathbf{p}_\perp \rightarrow 0$.

Collins FF

Collins Fragmentation Function

There are two different notations for Collins FF:

$$D_{h/q^\dagger}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{1}{2} S_{q'} \cdot (\hat{p}_{q'} \times \hat{\mathbf{p}}_\perp) \Delta^N D_{h/q^\dagger}(z, |\mathbf{p}_\perp|)$$

and

$$D_{h/q^\dagger}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{S_{q'} \cdot (\hat{p}_{q'} \times \mathbf{p}_\perp)}{z M_\pi} H_1^{\perp q}(z, |\mathbf{p}_\perp|),$$

both $\Delta^N D_{h/q^\dagger}(z, |\mathbf{p}_\perp|)$ and $H_1^{\perp q}(z, |\mathbf{p}_\perp|)$ refer to Collins FF



Collins FF

Collins Fragmentation Function

There are two different notations for Collins FF:

$$D_{h/q^\dagger}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{1}{2} S_{q'} \cdot (\hat{\mathbf{p}}_{q'} \times \hat{\mathbf{p}}_\perp) \Delta^N D_{h/q^\dagger}(z, |\mathbf{p}_\perp|)$$

and

$$D_{h/q^\dagger}(z, \mathbf{p}_\perp) = D_{h/q}(z, |\mathbf{p}_\perp|) + \frac{S_{q'} \cdot (\hat{\mathbf{p}}_{q'} \times \mathbf{p}_\perp)}{zM_\pi} H_1^{\perp q}(z, |\mathbf{p}_\perp|),$$

Relation

$$\Delta^N D_{h/q^\dagger}(z, |\mathbf{p}_\perp|) = \frac{2|\mathbf{p}_\perp|}{zM_\pi} H_1^{\perp q}(z, |\mathbf{p}_\perp|).$$

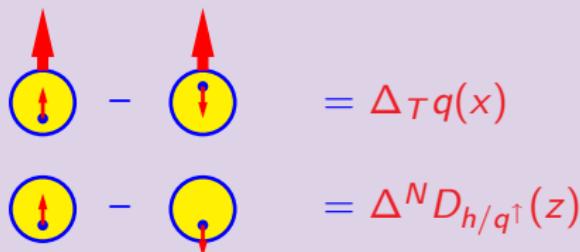
Trento conventions: A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Phys. Rev. **D70**, 117504 (2004).

Collins effect

Collins effects $A \propto \sin(\phi_h + \phi_S)$

The azimuthal asymmetry arises due to modulation in fragmentation function, the Collins function $\Delta^N D_{h/q^\uparrow}(z, |\mathbf{p}_\perp|)$ couples to transversity $\Delta_T q(x)$

$$A_N \sim \sin(\phi_h + \phi_S) \cdot \Delta_T q(x) \otimes \Delta^N D_{h/q^\uparrow}(z, |\mathbf{p}_\perp|)$$



J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

Collins effect

Collins effects $A \propto \sin(\phi_h + \phi_S)$

$$A_{UT}^{\sin(\phi_h + \phi_S)}(x, z) \sim \frac{\sum_q e_q^2 x \Delta_T q(x) \Delta^N D_{h/q^\uparrow}(z)}{\sum_q e_q^2 x f_q(x) D_{h/q}(z)},$$

Positivity constraints :

$$|\Delta^N D_{h/q^\uparrow}(z, \mathbf{p}_\perp)| \leq 2 D_{h/q}(z, \mathbf{p}_\perp)$$

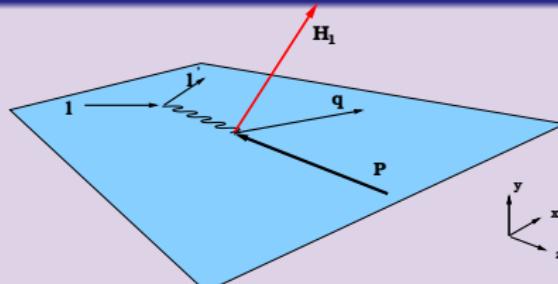
Soffer bound :

$$|\Delta_T q(x)| \leq \frac{1}{2} [f_{q/p}(x) + \Delta q(x)]$$

J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

SIDIS and e^+e^- annihilation

SIDIS $IN \rightarrow l' H_1 X$



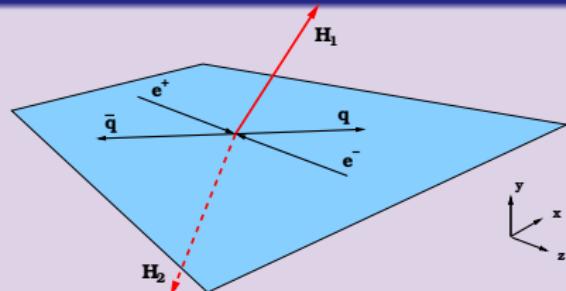
Collins effect gives rise to azimuthal Single Spin Asymmetry

$$\begin{array}{c} \uparrow \\ \text{---} \\ \uparrow \end{array} - \begin{array}{c} \uparrow \\ \text{---} \\ \downarrow \end{array} = \Delta_T q(x, Q^2)$$

$$\begin{array}{c} \uparrow \\ \text{---} \\ \downarrow \end{array} - \begin{array}{c} \downarrow \\ \text{---} \\ \uparrow \end{array} = \Delta^N D_{h/q^\uparrow}(z, Q^2)$$

J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

$e^+e^- \rightarrow H_1 H_2 X$



Collins effect gives rise to azimuthal asymmetry, q and \bar{q} Collins functions are present in the process:

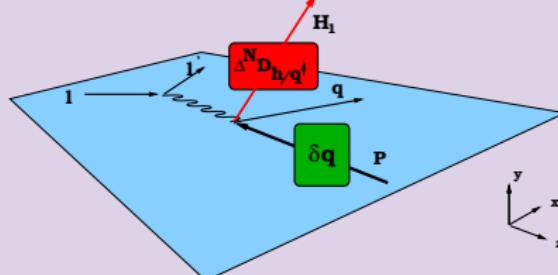
$$\Delta^N D_{h/q^\uparrow}(z_1, Q^2)$$

$$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2)$$

D. Boer, R. Jacob and P. J. Mulders *Nucl. Phys.* **B504** (1997) 345

SIDIS and e^+e^- annihilation

SIDIS $IN \rightarrow I'H_1X$

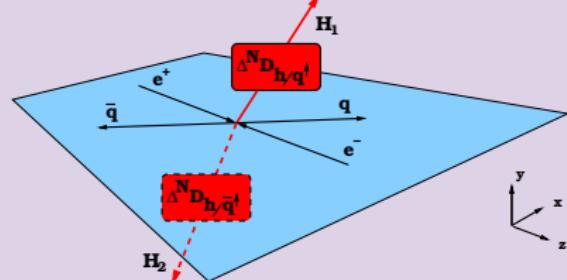


Cross Section $\sim \sin(\phi_H + \phi_S) \cdot \Delta_T q(x, Q^2) \otimes \Delta^N D_{h/q^\dagger}(z, Q^2)$

?

$\Delta_T q(x, Q^2) \neq 0 ?$
 $\Delta^N D_{h/q^\dagger}(z, Q^2) \neq 0 ?$

$e^+e^- \rightarrow H_1H_2X$



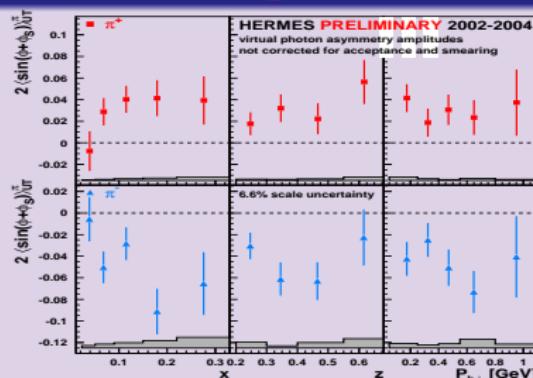
Cross Section $\sim \cos(\phi_{H_1} + \phi_{H_2}) \cdot \Delta^N D_{h/q^\dagger}(z_1) \otimes \Delta^N D_{h/\bar{q}^\dagger}(z_2)$

?

$\Delta^N D_{h/q^\dagger}(z_1, Q^2) \neq 0 ?$
 $\Delta^N D_{h/\bar{q}^\dagger}(z_2, Q^2) \neq 0 ?$

SIDIS and e^+e^- annihilation

SIDIS $\mathcal{N} \rightarrow l' H_1 X$



HERMES, proton target,
 $p_{lab} = 27.5$ (GeV)

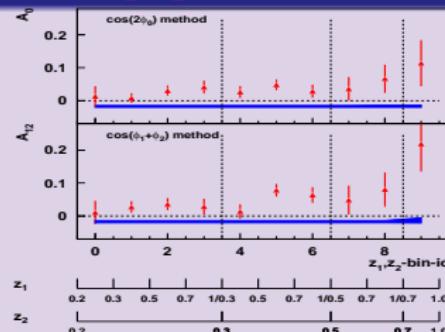
HERMES

$$\Delta_T q(x, Q^2) \neq 0 !$$

$$\Delta^N D_{h/q^\uparrow}(z, Q^2) \neq 0 !$$

HERMES Collaboration, A. Airapetian et al. Phys. Rev. Lett. **94** 94 (2005) 012002

$e^+e^- \rightarrow H_1 H_2 X$



BELLE, $\sqrt{s} = 10.52$ (GeV),

BELLE

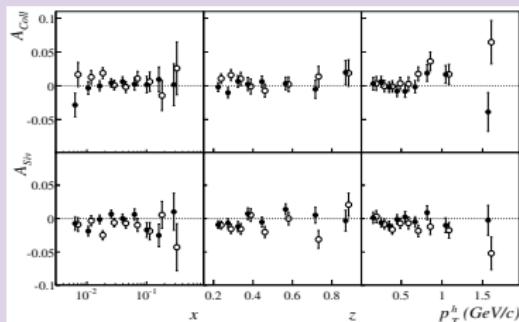
$$\Delta^N D_{h/q^\uparrow}(z_1, Q^2) \neq 0 !$$

$$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2) \neq 0 !$$

Belle Collaboration,
K. Abe et al., Phys. Rev. Lett. **96** (2006) 232002

SIDIS and e^+e^- annihilation

SIDIS $\text{IN} \rightarrow l'H_1X$



COMPASS, deuteron target

$p_{lab} = 160$ (GeV)

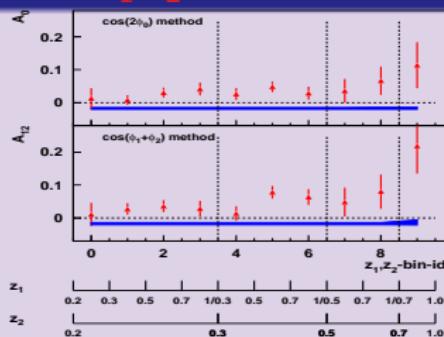
COMPASS

$\Delta_T q(x, Q^2) \neq 0 ?$

$\Delta^N D_{h/q^\uparrow}(z, Q^2) \neq 0 ?$

COMPASS Collaboration, E. S. Ageev *et al.*,
 Nucl. Phys. **B765**, 31 (2007).

$e^+e^- \rightarrow H_1H_2X$



BELLE, $\sqrt{s} = 10.52$ (GeV),

BELLE

$\Delta^N D_{h/q^\uparrow}(z_1, Q^2) \neq 0 !$

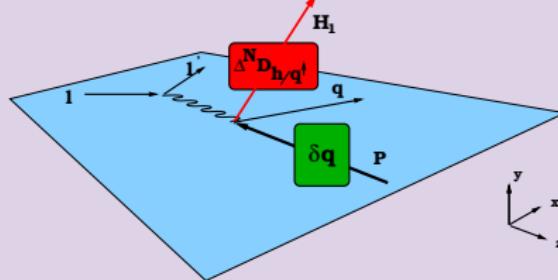
$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2) \neq 0 !$

Belle Collaboration,

K. Abe *et al.*, Phys. Rev. Lett. **96** (2006) 232002

SIDIS and e^+e^- annihilation

SIDIS $IN \rightarrow I'H_1X$

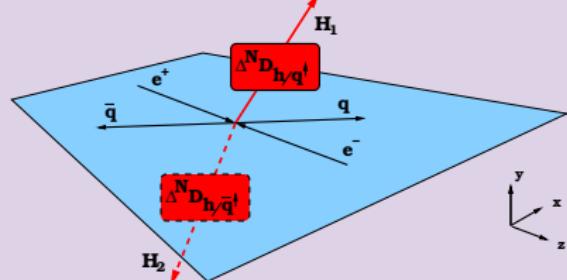


?

Are HERMES and COMPASS data compatible?

Fit HERMES & BELLE and check if we describe COMPASS data.

$e^+e^- \rightarrow H_1H_2X$



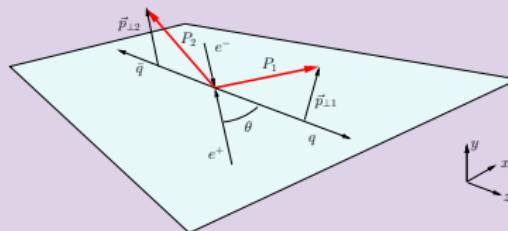
?

$\Delta^N D_{h/q^\dagger}^{SIDIS}(z) = \Delta^N D_{h/\bar{q}^\dagger}^{e^+e^-}(z)$?

Fit simultaneously HERMES, COMPASS and BELLE data sets.

$\cos(2\varphi_0)$ and $\cos(\varphi_1 + \varphi_2)$ method of BELLE

$\cos(2\varphi_0)$ (Matthias Grosse - Perdekamp talk)



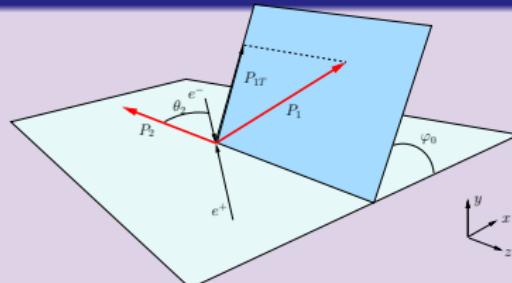
$$\frac{d\sigma^{e^+e^- \rightarrow h_1h_2X}}{dz_1 dz_2 d\cos\theta d(\varphi_1 + \varphi_2)} =$$

$$\frac{3\pi\alpha^2}{2s} \sum_q e_q^2 \left\{ (1 + \cos^2\theta) D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2) \right.$$

$$\left. + \frac{1}{4} \sin^2\theta \Delta^N D_{h_1/q^\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2) \cos(\varphi_1 + \varphi_2) \right\}.$$

$\cos(2\varphi_0)$ and $\cos(\varphi_1 + \varphi_2)$ method of BELLE

$\cos(2\varphi_0)$ (Matthias Grosse - Perdekamp talk)



$$\frac{d\sigma^{e^+e^- \rightarrow h_1h_2X}}{dz_1 dz_2 d\cos\theta_2 d(\varphi_0)} =$$

$$\frac{3\pi\alpha^2}{2s} \sum_q e_q^2 \left\{ (1 + \cos^2\theta_2) D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2) \right.$$

$$\left. + \frac{z_1 z_2}{\pi(z_1^2 + z_2^2)} \sin^2\theta_2 \Delta^N D_{h_1/q^\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2) \cos(2\varphi_0) \right\}.$$

$\cos(2\varphi_0)$ and $\cos(\varphi_1 + \varphi_2)$ method of BELLE

The same combination of Collins FF $\Delta^N D_{h_1/q^\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)$ enters into the asymmetry. Two different methods are used, thus we must use them separately in our fit procedure.

Unpolarised distribution and fragmentation functions.

$f_{q/p}(x, k_\perp)$ and $D_{h/q}(z, p_\perp)$ TMD distribution and fragmentation functions are used.

We assume the k_\perp and p_\perp dependences to be factorized in a Gaussian form

$$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$\langle k_\perp^2 \rangle = 0.25 \text{ (GeV}^2\text{)}$$

$$\langle p_\perp^2 \rangle = 0.2 \text{ (GeV}^2\text{)}$$

M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin,
Phys. Rev. **D71**, 074006 (2005).

Unpolarised distribution and fragmentation functions.

$f_{q/p}(x, k_\perp)$ and $D_{h/q}(z, p_\perp)$ TMD distribution and fragmentation functions are used.

We assume the k_\perp and p_\perp dependences to be factorized in a Gaussian form

Distribution functions:

$f_{q/p}(x)$ GRV LO 1998

M. Gluck, E. Reya, and A. Vogt, Eur. Phys. J. **C5**, 461 (1998).

Fragmentation functions:

$D_{h/q}(z)$ Kretzer

S. Kretzer, Phys. Rev. **D62**, 054001 (2000).

Collins function

Model for Collins FF

$\Delta^N D_{h/q^\uparrow}(z, |p_\perp|) \Rightarrow$ we use factorization of z and p_\perp
and Gaussian dependence on p_\perp

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = 2 \mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle},$$

with

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma + \delta)^{(\gamma + \delta)}}{\gamma^\gamma \delta^\delta}$$

$$h(p_\perp) = \sqrt{2e} \frac{p_\perp}{M} e^{-p_\perp^2/M^2},$$

where N_q^C , γ , δ , and M are parameters.

Collins function

Model for Collins FF

$\Delta^N D_{h/q^\uparrow}(z, |p_\perp|) \Rightarrow$ we use factorization of z and p_\perp
and Gaussian dependence on p_\perp

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = 2 \mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle},$$

with

$$\begin{aligned} \mathcal{N}_q^C(z) &\leq 1 \\ h(p_\perp) &\leq 1 \end{aligned}$$

positivity constraint $|\Delta^N D_{h/q^\uparrow}(z, p_\perp)| \leq 2 D_{h/q}(z, p_\perp)$ is fulfilled.

Transversity

$$\Delta_T q(x, \mathbf{k}_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) [f_{q/p}(x) + \Delta q(x)] \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T},$$

where

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha + \beta)^{(\alpha + \beta)}}{\alpha^\alpha \beta^\beta},$$

N_q^T , α , β and $\langle k_\perp^2 \rangle_T$ are parameters.

$$\mathcal{N}_q^T(x) \leq 1$$

thus Soffer bound

$$|\Delta_T q(x)| \leq \frac{1}{2} [f_{q/p}(x) + \Delta q(x)]$$

is fulfilled.

Description of $A_{UT}^{\sin(\phi_h+\phi_s)}$

We use HERMES and COMPASS data sets on $A_{UT}^{\sin(\phi_h+\phi_s)}$ in the fitting procedure, we use one of the two sets of data from BELLE corresponding to either $\cos(\varphi_1 + \varphi_2)$ or $\cos(2\varphi_0)$ extraction method.

Favored and unfavored fragmentation functions are defined as follows:

$$D^{fav}(z) \equiv D^{u \rightarrow \pi^+}(z) = D^{d \rightarrow \pi^-}(z) = D^{\bar{u} \rightarrow \pi^-}(z) = D^{\bar{d} \rightarrow \pi^+}(z)$$
$$D^{unfav}(z) \equiv D^{u \rightarrow \pi^-}(z) = D^{d \rightarrow \pi^+}(z) = D^{\bar{u} \rightarrow \pi^+}(z) = D^{\bar{d} \rightarrow \pi^-}(z)$$

HERMES Collaboration, L. Pappalardo *et al.*, in the proceedings of the XIV International Workshop on Deep Inelastic Scattering, Tsukuba city, Japan, April 20th - April 24th. (2006).

COMPASS Collaboration, E. S. Ageev *et al.*, Nucl. Phys. **B765**, 31 (2007).

Belle Collaboration, R. Seidl *et al.*, Phys. Rev. Lett. **96**, 232002 (2006).

Description of the data (Anselmino *et al* hep-ph/0701006)

Table: Best values of the free parameters for the u and d transversity distribution functions and for the favored and unfavored Collins fragmentation functions ($\cos(\varphi_1 + \varphi_2)$ BELLE data set.)

| FIT I (A_{12}) | | $\chi^2/\text{d.o.f.} =$ | 0.81 |
|------------------------------------|-------------------------------|--------------------------|----------------------|
| Transversity distribution function | N_u^T | = | 0.48 ± 0.09 |
| | α | = | 1.14 ± 0.68 |
| | $\langle k_\perp^2 \rangle_T$ | = | 0.25 GeV^2 |
| Collins FF | N_{fav}^C | = | 0.35 ± 0.16 |
| | γ | = | 1.14 ± 0.38 |
| | $\langle p_\perp^2 \rangle$ | = | 0.20 GeV^2 |
| | | | GeV^2 |

Description of the data (Anselmino *et al* hep-ph/0701006)

Table: Best values of the free parameters for the u and d transversity distribution functions and for the favored and unfavored Collins fragmentation functions ($\cos(\varphi_0)$ BELLE data set.)

| FIT II (A_0) | | $\chi^2/\text{d.o.f.} =$ | 0.77 |
|------------------------------------|-------------------------------|--------------------------|----------------------|
| Transversity distribution function | N_u^T | = | 0.42 ± 0.09 |
| | α | = | 1.20 ± 0.83 |
| | $\langle k_\perp^2 \rangle_T$ | = | 0.25 GeV^2 |
| Collins FF | N_{fav}^C | = | 0.41 ± 0.10 |
| | γ | = | 0.81 ± 0.40 |
| | $\langle p_\perp^2 \rangle$ | = | 0.20 GeV^2 |
| | | | GeV^2 |

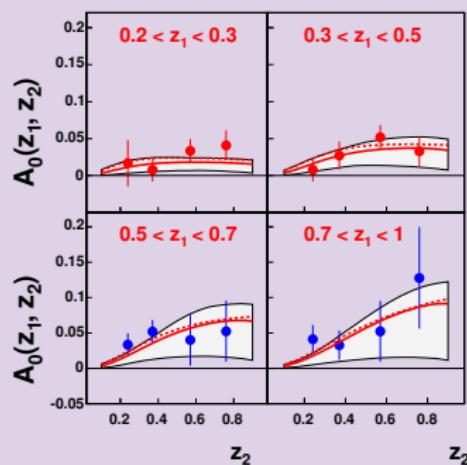
Description of the data (Anselmino *et al* hep-ph/0701006)

Table: FIT I $\cos(\varphi_1 + \varphi_2)$ and FIT II $\cos(\varphi_0)$ are within 1σ

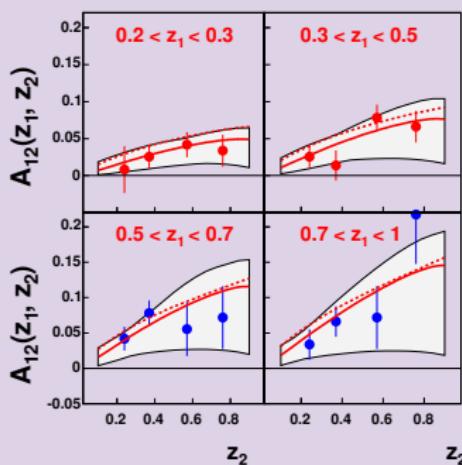
| Transversity | | | | | |
|--------------|--------------------|---|--------------------------|--------------------|--------------------|
| FIT I | N_u^T | = | 0.48 ± 0.09 | N_d^T | = -0.62 ± 0.30 |
| FIT II | N_u^T | = | 0.42 ± 0.09 | N_d^T | = -0.53 ± 0.28 |
| FIT I | α | = | 1.14 ± 0.68 | β | = 4.74 ± 5.45 |
| FIT II | α | = | 1.20 ± 0.83 | β | = 5.09 ± 5.87 |
| Collins FF | | | | | |
| FIT I | N_{fav}^C | = | 0.35 ± 0.16 | N_{unf}^C | = -0.85 ± 0.36 |
| FIT II | N_{fav}^C | = | 0.41 ± 0.10 | N_{unf}^C | = -0.99 ± 1.24 |
| FIT I | γ | = | 1.14 ± 0.38 | δ | = 0.14 ± 0.36 |
| FIT II | γ | = | 0.81 ± 0.40 | δ | = 0.02 ± 0.37 |
| FIT I | M^2 | = | 0.70 ± 0.65 GeV 2 | | |
| FIT II | M^2 | = | 0.88 ± 1.15 GeV 2 | | |

Description of BELLE data

BELLE $\cos(\varphi_0)$



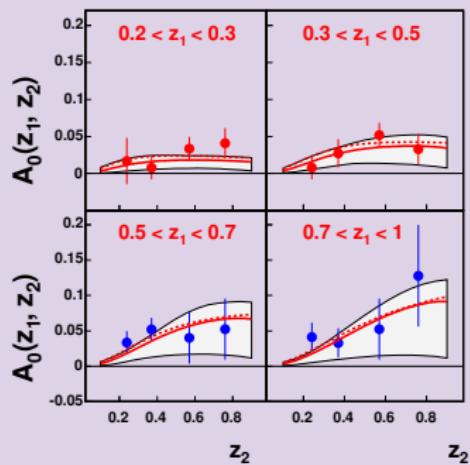
BELLE $\cos(\varphi_1 + \varphi_2)$



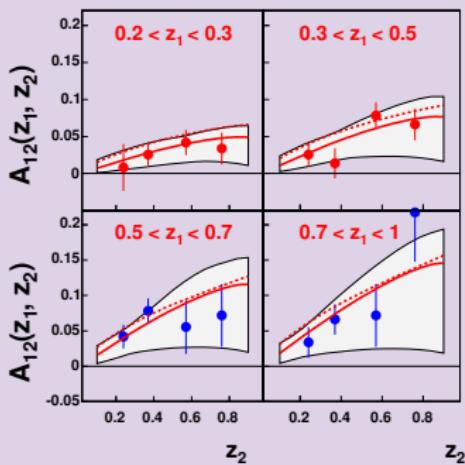
Solid line corresponds to FIT II, dashed line corresponds to FIT I

Description of BELLE data

BELLE $\cos(\varphi_0)$



BELLE $\cos(\varphi_1 + \varphi_2)$

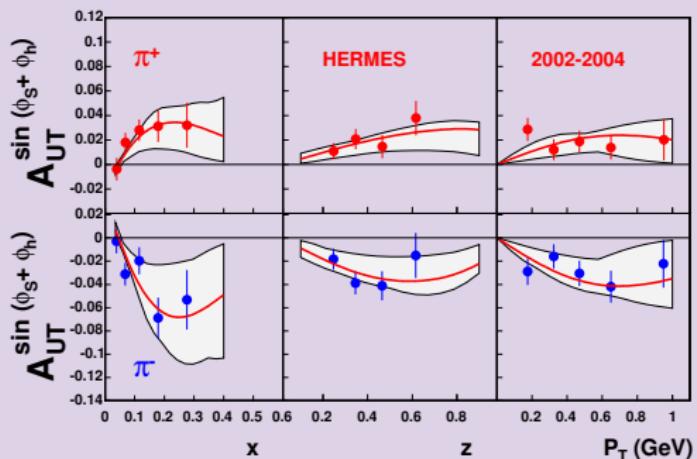


FIT I and FIT II are compatible

Description of HERMES data $A_{UT}^{\sin(\phi_h+\phi_s)}$

HERMES $A_{UT}^{\sin(\phi_h+\phi_s)}$

$ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.

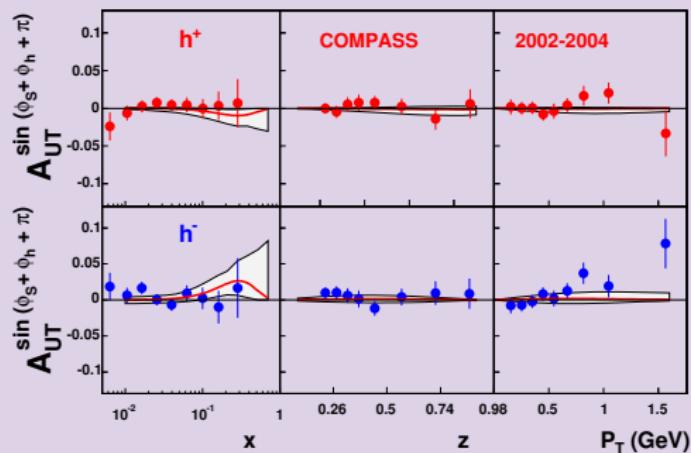


HERMES Collaboration, L. Pappalardo *et al.*, in the proceedings of the XIV International Workshop on Deep Inelastic Scattering, Tsukuba city, Japan, April 20th - April 24th. (2006).

Description of COMPASS data $A_{UT}^{\sin(\phi_h + \phi_s + \pi)}$

COMPASS $A_{UT}^{\sin(\phi_h + \phi_s + \pi)}$

$\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.

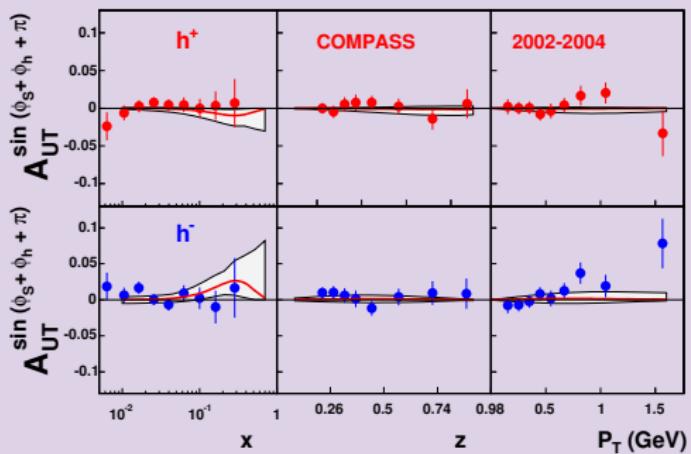


COMPASS Collaboration, E. S. Ageev *et al.*, Nucl. Phys. **B765**, 31 (2007).

Description of COMPASS data $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$

COMPASS $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$

$\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.

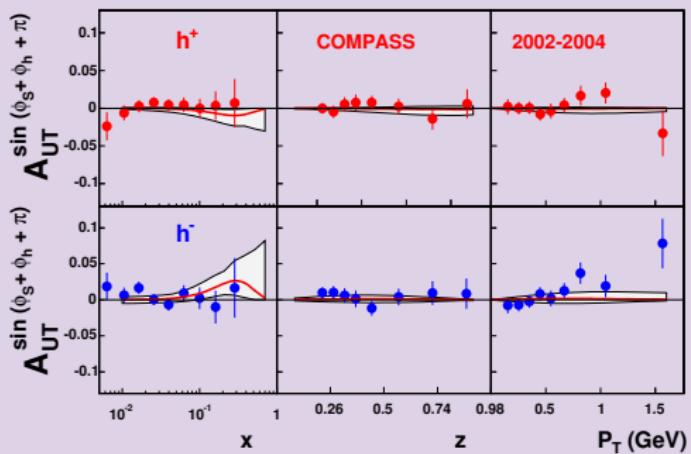


Why $A_{UT}^{\sin(\phi_h + \phi_S + \pi)} \sim 0$? One of the reasons is that $\langle x \rangle \sim 0.03$ ($\langle x \rangle_{HERMES} \sim 0.1$) is very small and $\Delta_T q(x) \rightarrow 0$.

Description of COMPASS data $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$

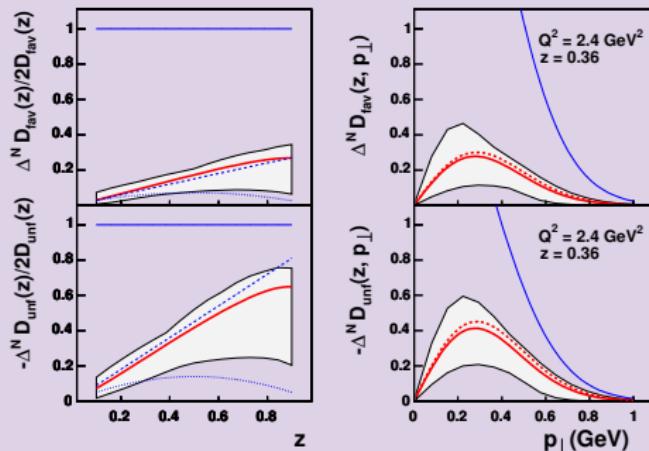
COMPASS $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$

$\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.



But deuteron target allows us to fit $\Delta_T d(x)$ as combination of $\Delta_T u(x) + \Delta_T d(x)$ enters into the asymmetry.

Collins fragmentation function

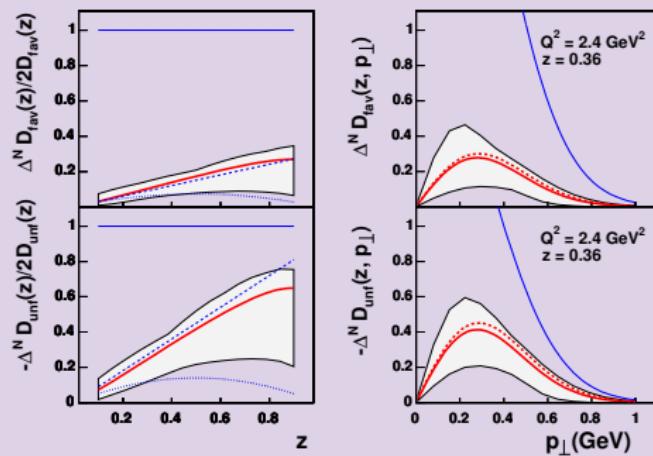


compared to Ref. [1] (dashed line) and Ref. [2] (dotted line)

[1] A. V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. **D73**, 094025 (2006).

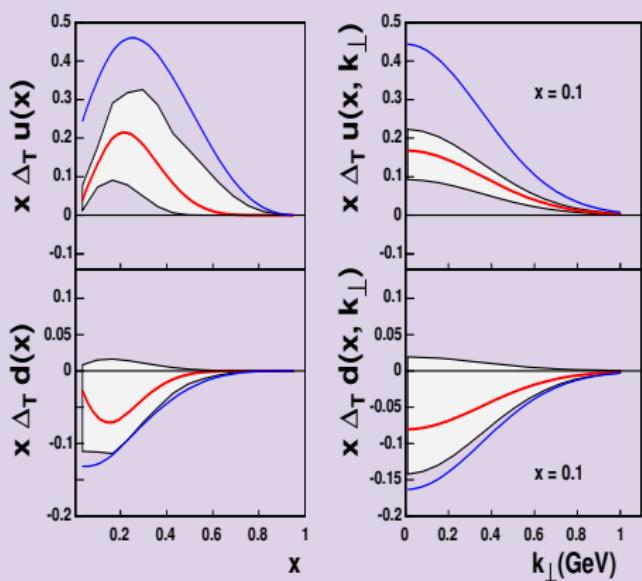
[2] W. Vogelsang and F. Yuan, Phys. Rev. **D72**, 054028 (2005).

Collins fragmentation function



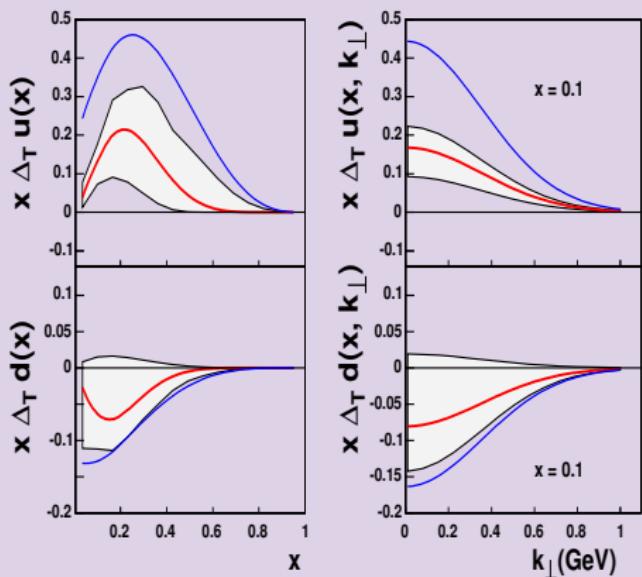
Right panel: solid line corresponds to **FIT II**, dashed line corresponds to **FIT I**

Transversity



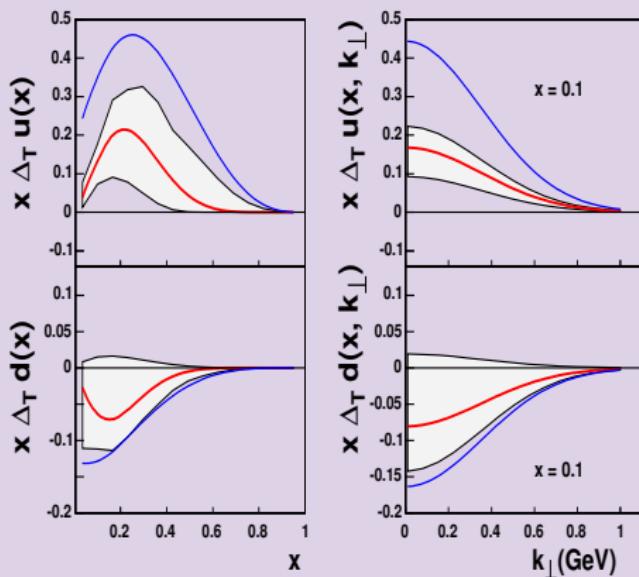
- This is the first extraction of **transversity** from experimental data.
 - $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
 - Both $\Delta_T u(x)$ and $\Delta_T d(x)$ do not saturate Soffer bound.
 - HERMES data alone fixes well $\Delta_T u(x)$ while HERMES+COMPASS allows us to extract $\Delta_T d(x)$.

Transversity



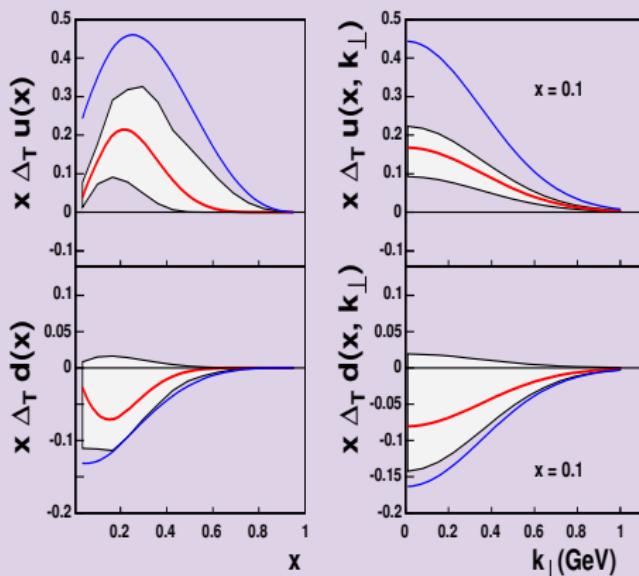
- This is the first extraction of **transversity** from experimental data.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Both $\Delta_T u(x)$ and $\Delta_T d(x)$ do not saturate Soffer bound.
- HERMES data alone fixes well $\Delta_T u(x)$ while HERMES+COMPASS allows us to extract $\Delta_T d(x)$.

Transversity



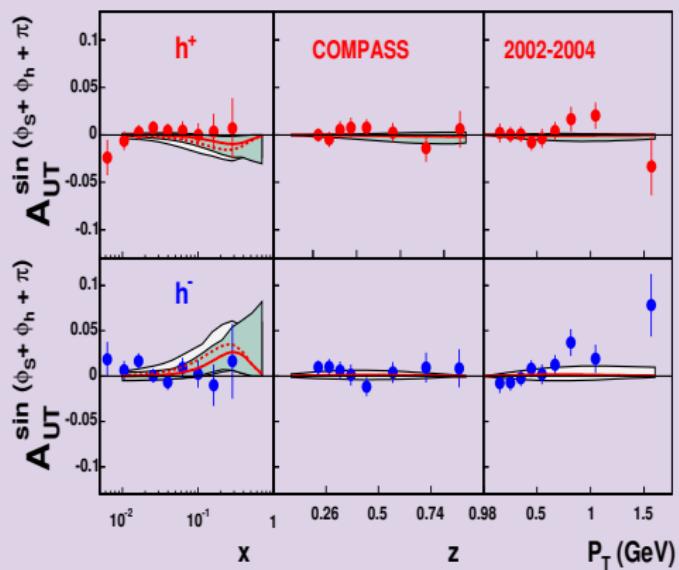
- This is the first extraction of **transversity** from experimental data.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Both $\Delta_T u(x)$ and $\Delta_T d(x)$ do not saturate Soffer bound.
- HERMES data alone fixes well $\Delta_T u(x)$ while HERMES+COMPASS allows us to extract $\Delta_T d(x)$.

Transversity



- This is the first extraction of **transversity** from experimental data.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Both $\Delta_T u(x)$ and $\Delta_T d(x)$ do not saturate Soffer bound.
- **HERMES** data alone fixes well $\Delta_T u(x)$ while **HERMES+COMPASS** allows us to extract $\Delta_T d(x)$.

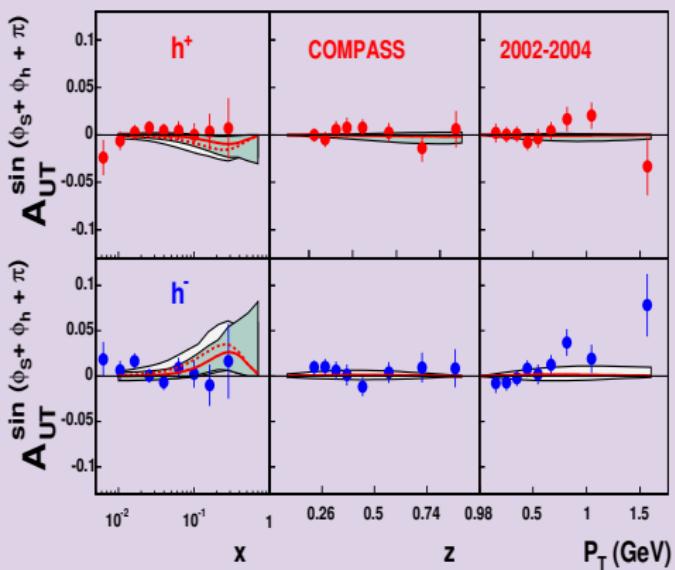
Impact of COMPASS data on the results



What happens if COMPASS data are excluded from the fit?

- Description of COMPASS remains very good: dashed line – without COMPASS solid line – with COMPASS
- Thus we can conclude that HERMES and COMPASS data sets are compatible

Impact of COMPASS data on the results

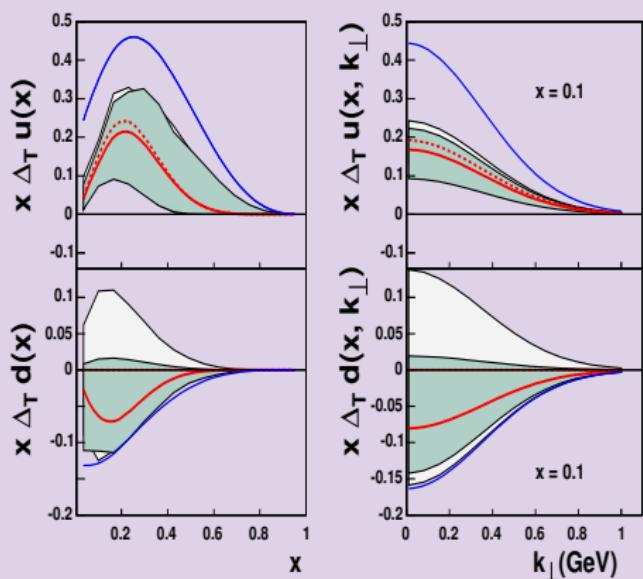


What happens if COMPASS data are excluded from the fit?

- Description of COMPASS remains very good: dashed line – without COMPASS solid line – with COMPASS
- Thus we can conclude that HERMES and COMPASS data sets are compatible

Impact of COMPASS data on transversity

What happens if COMPASS data are excluded from the fit?

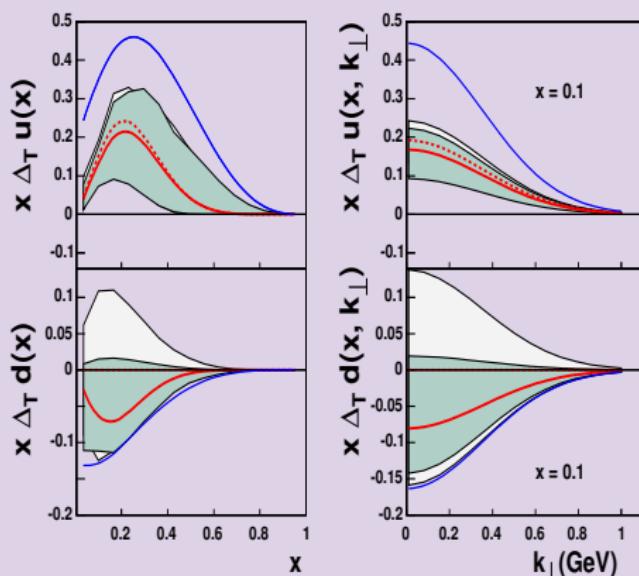


- Extraction of $\Delta_T u(x)$ remains very good: dashed line – without COMPASS
solid line – with COMPASS
- Extraction of $\Delta_T d(x)$ is significantly improved when COMPASS data set is included. Indeed without it both sign and the magnitude of $\Delta_T d(x)$ are undefined.

Impact of COMPASS data on transversity

What happens if COMPASS data are excluded from the fit?

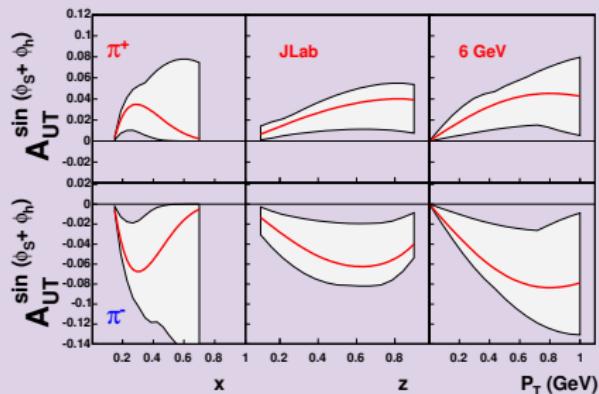
- Extraction of $\Delta_T u(x)$ remains very good: dashed line – without COMPASS
solid line – with COMPASS
- Extraction of $\Delta_T d(x)$ is significantly improved when COMPASS data set is included. Indeed without it both sign and the magnitude of $\Delta_T d(x)$ are undefined.



PREDICTIONS

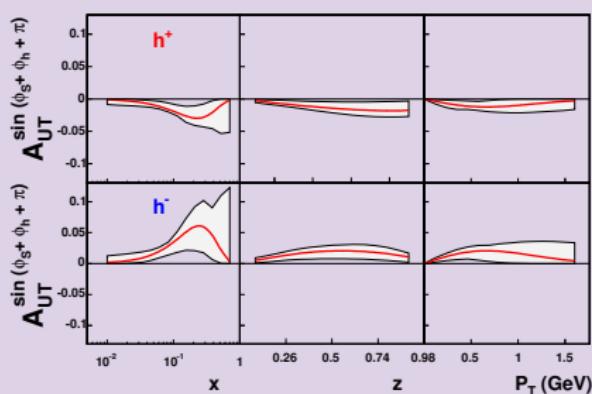
JLab

$ep \rightarrow e\pi X$, $p_{lab} = 6$ GeV.



COMPASS

$\mu p \rightarrow \mu h X$, $p_{lab} = 160$ GeV.

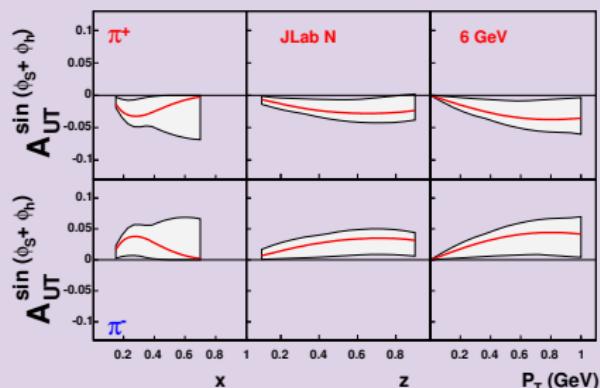


JLab can improve our knowledge of transversity in high x region.
COMPASS operating on proton target is expected to measure 5% asymmetry at $x \sim 0.2$

PREDICTIONS

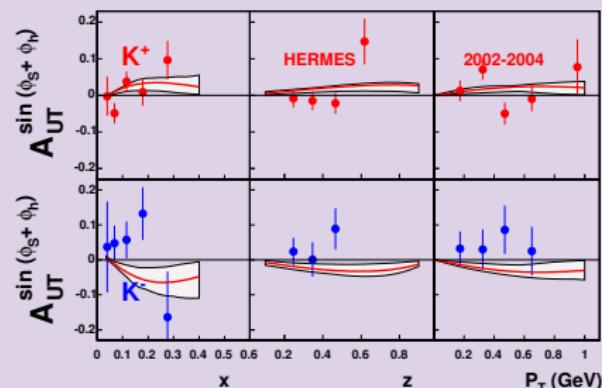
JLab

$eN \rightarrow e\pi X$, $p_{lab} = 6$ GeV.



HERMES

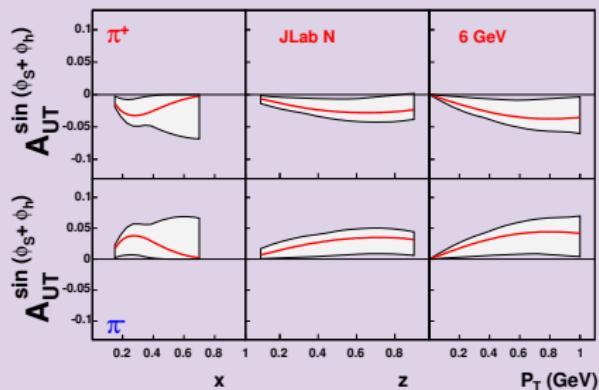
$ep \rightarrow eKX$, $p_{lab} = 27.57$ GeV.



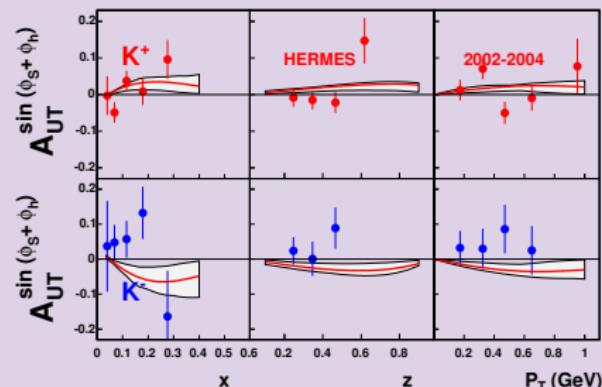
JLab can improve our knowledge of $\Delta_T d(x)$ transversity using neutron target. Prediction of the model are compatible with Kaon data from HERMES.

PREDICTIONS

JLab

 $eN \rightarrow e\pi X, p_{lab} = 6 \text{ GeV.}$


HERMES

 $ep \rightarrow eKX, p_{lab} = 27.57 \text{ GeV.}$


$$A_{UT}^{sin(\phi_h + \phi_S)}|_{proton} \sim 4\Delta_T u(x)\Delta^N D_{h/u^\dagger}(z) + \Delta_T d(x)\Delta^N D_{h/d^\dagger}(z)$$

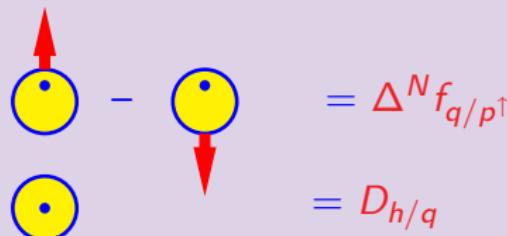
$$A_{UT}^{sin(\phi_h + \phi_S)}|_{neutron} \sim 4\Delta_T d(x)\Delta^N D_{h/u^\dagger}(z) + \Delta_T u(x)\Delta^N D_{h/d^\dagger}(z)$$

Sivers effect

$$\text{Sivers effect } A \propto \sin(\phi_h - \phi_s)$$

The azimuthal asymmetry arises due to modulation in parton density, the so called Sivers function $\Delta^N f_{q/p^\uparrow}$ is the difference of parton distributions in a polarized hadron.

$$A_N \sim \sin(\phi_h - \phi_s) \cdot \Delta^N f_{q/p^\uparrow}(x, k_\perp) \otimes D_{h/q}(z)$$



D. Sivers, *Phys. Rev. D* **41**(1990) 83

Sivers effect

Sivers effect $A \propto \sin(\phi_h - \phi_S)$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, z) \sim \frac{\sum_q e_q^2 x z \Delta^N f_{q/p^\uparrow}(x) D_{h/q}(z)}{\sum_q e_q^2 x f_q(x) D_{h/q}(z)},$$

Positivity constraints :

$$|\Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp)| \leq 2f_q(x, \mathbf{k}_\perp)$$

Two different notations:

$$\begin{aligned} f_{q/p^\uparrow}(x, \mathbf{k}_\perp) &= f_{q/p}(x, \mathbf{k}_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp) \\ &= f_{q/p}(x, \mathbf{k}_\perp) - f_{1T}^{\perp q}(x, \mathbf{k}_\perp) \frac{\mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \mathbf{k}_\perp)}{m_p}, \end{aligned}$$

Sivers effect

Sivers effect $A \propto \sin(\phi_h - \phi_S)$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, z) \sim \frac{\sum_q e_q^2 x z \Delta^N f_{q/p^\uparrow}(x) D_{h/q}(z)}{\sum_q e_q^2 x f_q(x) D_{h/q}(z)},$$

Positivity constraints :

$$|\Delta^N f_{q/p^\uparrow}(x, k_\perp)| \leq 2f_q(x, k_\perp)$$

Two different notations:

Relation

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = -\frac{2|k_\perp|}{m_p} f_{1T}^{\perp q}(x, k_\perp).$$

Trento conventions: A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Phys. Rev. **D70**, 117504 (2004).

Sivers function

Model for Sivers function

$\Delta^N f_{q/p^\uparrow}(x, k_\perp) \Rightarrow$ we use factorization of x and k_\perp
and Gaussian dependence on k_\perp

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2 \mathcal{N}_q(x) f_q(x) h(k_\perp) \frac{e^{-p_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle},$$

with

$$\mathcal{N}_q(x) = N_q x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}$$

$$h(k_\perp) = \sqrt{2e} \frac{k_\perp}{M'} e^{-k_\perp^2/M'^2},$$

where N_q , a_q , b_q , and M' are parameters.

Sivers function

Model for Sivers function

$\Delta^N f_{q/p^\uparrow}(x, k_\perp) \Rightarrow$ we use factorization of x and k_\perp
and Gaussian dependence on k_\perp

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2 \mathcal{N}_q(x) f_q(x) h(k_\perp) \frac{e^{-p_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle} ,$$

with

$$\mathcal{N}_q(x) \leq 1$$

$$h(k_\perp) \leq 1$$

positivity constraint $|\Delta^N f_{q/p^\uparrow}(x, k_\perp)| \leq 2 f_q(x, k_\perp)$ is fulfilled.

Description of $A_{UT}^{\sin(\phi_h - \phi_s)}$

We use HERMES and COMPASS data sets on $A_{UT}^{\sin(\phi_h - \phi_s)}$ in the fitting procedure.

u , d and sea Sivers functions are fitted.

We assume that

$$\Delta^N f_{\bar{u}/p^\uparrow}(x, k_\perp) = \Delta^N f_{\bar{d}/p^\uparrow}(x, k_\perp) = \Delta^N f_{s,\bar{s}/p^\uparrow}(x, k_\perp) \equiv \\ \Delta^N f_{sea/p^\uparrow}(x, k_\perp).$$

HERMES Collaboration, L. Pappalardo *et al.*, in the proceedings of the XIV International Workshop on Deep Inelastic Scattering, Tsukuba city, Japan, April 20th - April 24th. (2006).

COMPASS Collaboration, E. S. Ageev *et al.*, Nucl. Phys. **B765**, 31 (2007).

Description of the data

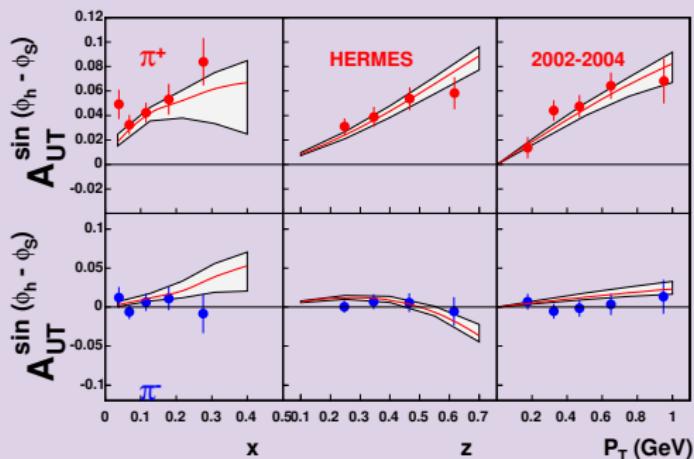
Table: Best values of the free parameters for the u , d and sea Sivers functions.

| | | $\chi^2/\text{d.o.f.} =$ | 1.14 |
|--------------------|-----------------------------|--------------------------|--------------------------------------|
| u | N_u | $= 0.41 \pm 0.07$ | |
| Sivers function | a_u | $= 0 \pm 0.4$ | $b_u = 0.49 \pm 0.88$ |
| d | N_d | $= -0.96 \pm 1.17$ | |
| Sivers function | a_d | $= 2 \pm 0.54$ | $b_d = 8.6 \pm 3.5$ |
| sea | N_{sea} | $= -0.52 \pm 0.17$ | |
| Sivers function | a_{sea} | $= 0.4 \pm 0.1$ | $b_{sea} = 0 \pm 6$ |
| | $\langle k_\perp^2 \rangle$ | $= 0.25 \text{ GeV}^2$ | $M'^2 = 0.99 \pm 1.23 \text{ GeV}^2$ |

Description of HERMES data $A_{UT}^{\sin(\phi_h - \phi_s)}$

HERMES $A_{UT}^{\sin(\phi_h - \phi_s)}$

$ep \rightarrow e\pi X, p_{lab} = 27.57 \text{ GeV.}$

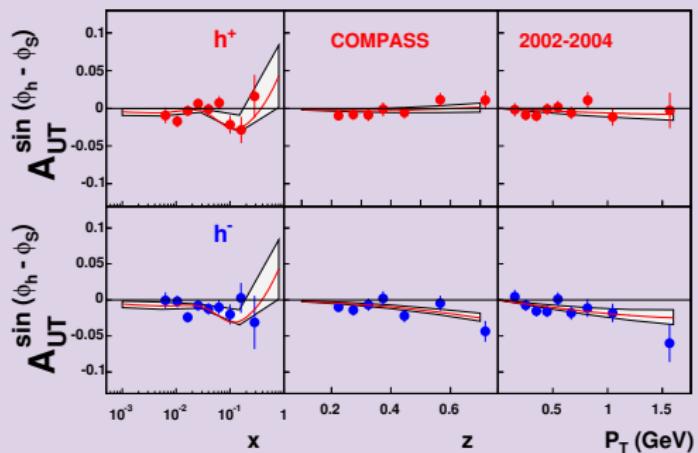


HERMES Collaboration, L. Pappalardo *et al.*, in the proceedings of the XIV International Workshop on Deep Inelastic Scattering, Tsukuba city, Japan, April 20th - April 24th. (2006).

Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$

COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$

$\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.

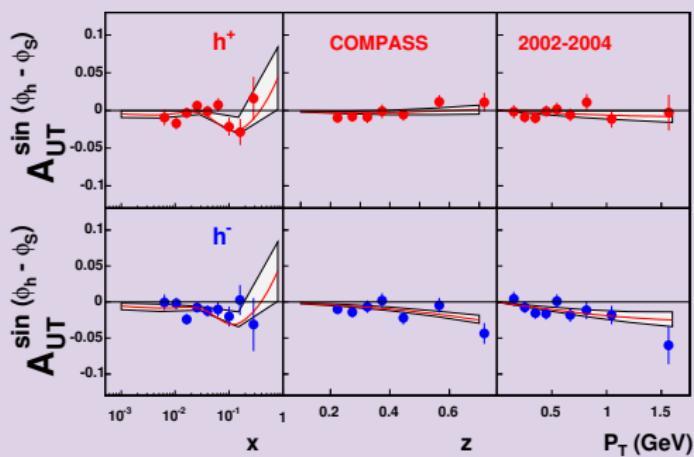


COMPASS Collaboration, E. S. Ageev *et al.*, Nucl. Phys. **B765**, 31 (2007).

Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_s)}$

COMPASS $A_{UT}^{\sin(\phi_h - \phi_s)}$

$\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.

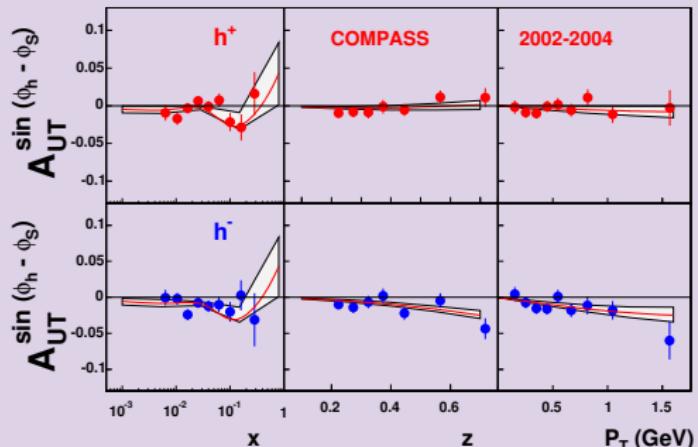


Why $A_{UT}^{\sin(\phi_h - \phi_s)} \sim 0$?

Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$

COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$

$\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.

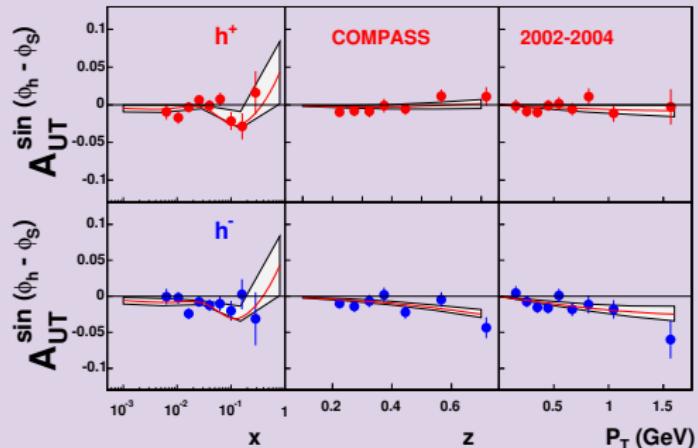


$$\left(A_{UT}^{\sin(\phi_h - \phi_S)} \right)_{\text{hydrogen}} \sim 4 \Delta N f_{u/p^\uparrow} D_u^h + \Delta N f_{d/p^\uparrow} D_d^h$$

Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$

COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$

$\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.

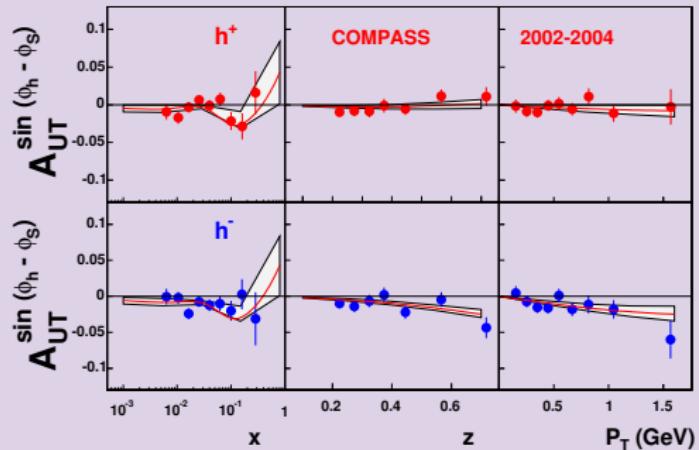


$$\left(A_{UT}^{\sin(\phi_h - \phi_S)} \right)_{\text{deuterium}} \sim \left(\Delta^N f_{u/p^\uparrow} + \Delta^N f_{d/p^\uparrow} \right) \left(4 D_u^h + D_d^h \right)$$

Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_S)}$

COMPASS $A_{UT}^{\sin(\phi_h - \phi_S)}$

$\mu D \rightarrow \mu h X$, $p_{lab} = 160$ GeV.

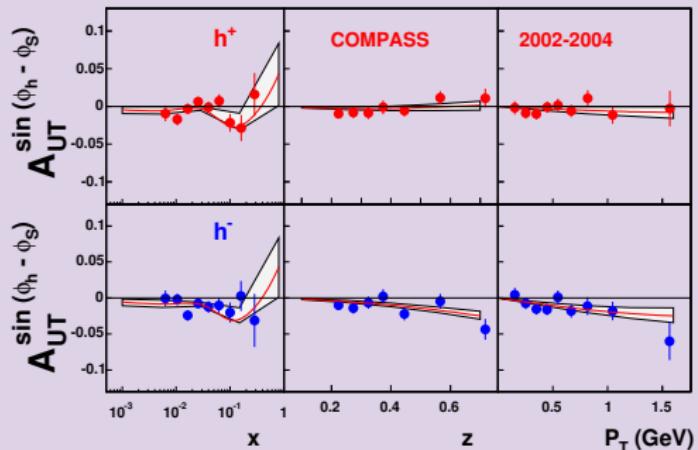


$$\left(A_{UT}^{\sin(\phi_h - \phi_S)} \right)_{\text{deuterium}} \sim \left(\Delta N f_{u/p^\uparrow} + \Delta N f_{d/p^\uparrow} \right) \sim 0$$

Description of COMPASS data $A_{UT}^{\sin(\phi_h - \phi_s)}$

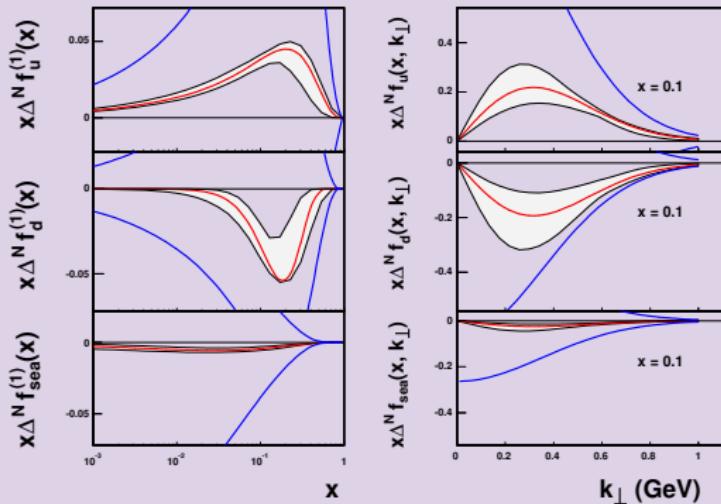
COMPASS $A_{UT}^{\sin(\phi_h - \phi_s)}$

$\mu D \rightarrow \mu hX$, $p_{lab} = 160$ GeV.



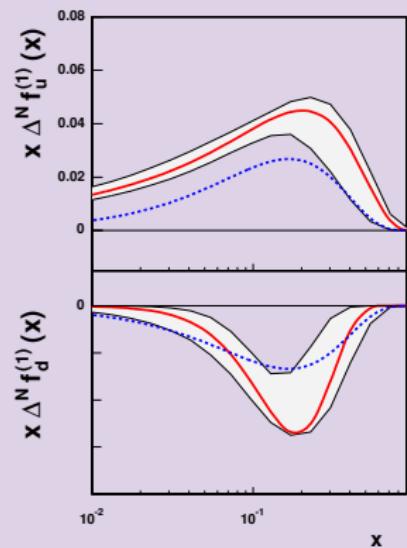
But deuteron target allows us to fit better $\Delta N f_{d/p^\uparrow}$ as combination of $\Delta N f_{u/p^\uparrow} + \Delta N f_{d/p^\uparrow}$ enters into the asymmetry.

Sivers function



$$\Delta^N f_q^{(1)}(x) \equiv \int d^2 \mathbf{k}_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\perp}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x).$$

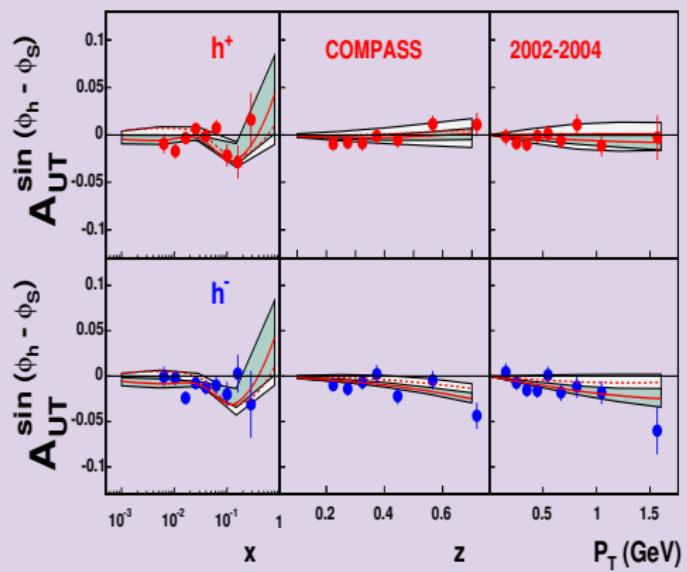
Sivers function



compared to Ref. [1] (dashed line)

[1] A. V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. D73, 094025 (2006).

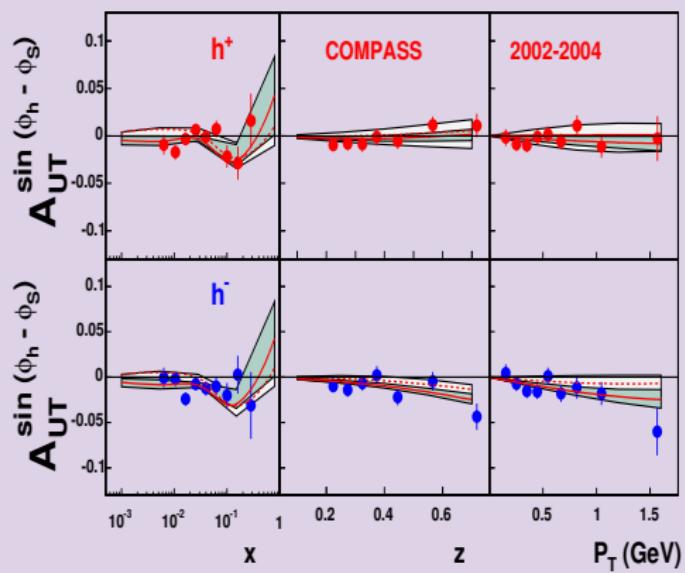
Impact of COMPASS data on the results



What happens if COMPASS data are excluded from the fit?

- Description of COMPASS data remains very good: dashed line – without COMPASS solid line – with COMPASS
- Thus we can conclude that HERMES and COMPASS data sets are compatible

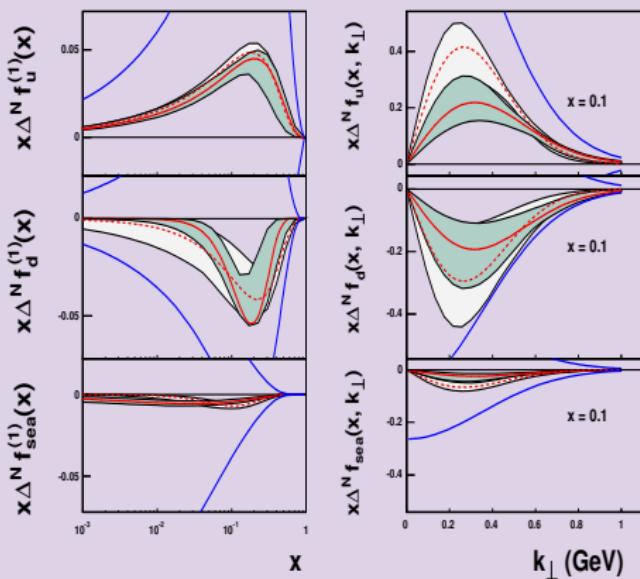
Impact of COMPASS data on the results



What happens if COMPASS data are excluded from the fit?

- Description of COMPASS data remains very good: dashed line – without COMPASS solid line – with COMPASS
- Thus we can conclude that HERMES and COMPASS data sets are compatible

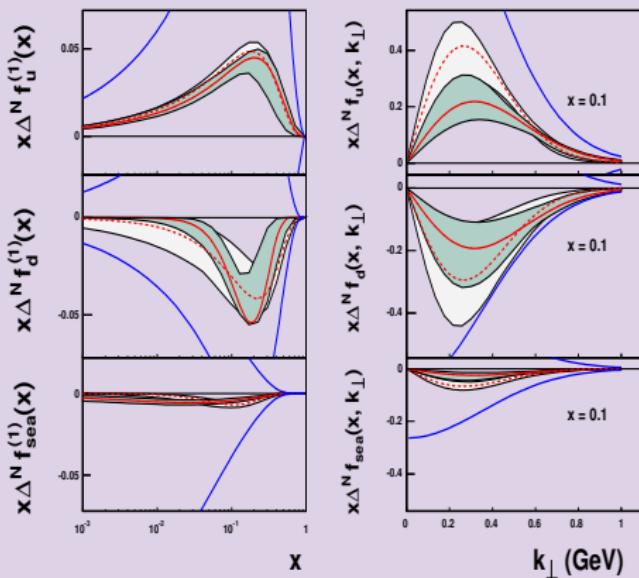
Impact of COMPASS data on Sivers function



What happens if COMPASS data are excluded from the fit?

- Extraction of $\Delta N f_{u/p^\uparrow}$ remains very good:
dashed line – without COMPASS
solid line – with COMPASS
- Extraction of $\Delta N f_{d/p^\uparrow}$ is significantly improved when COMPASS data set is included.

Impact of COMPASS data on Sivers function



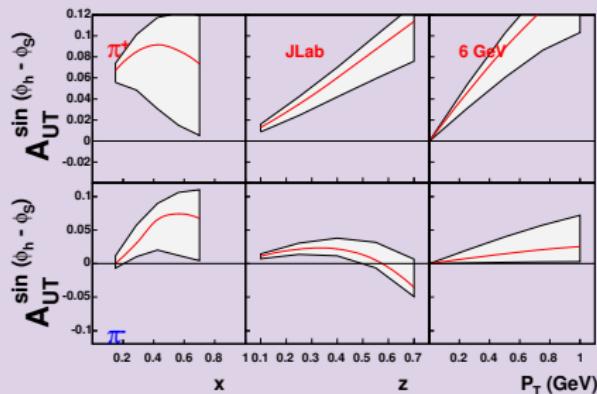
What happens if COMPASS data are excluded from the fit?

- Extraction of $\Delta N f_{u/p^\uparrow}$ remains very good:
dashed line – without COMPASS
solid line – with COMPASS
- Extraction of $\Delta N f_{d/p^\uparrow}$ is significantly improved when COMPASS data set is included.

PREDICTIONS

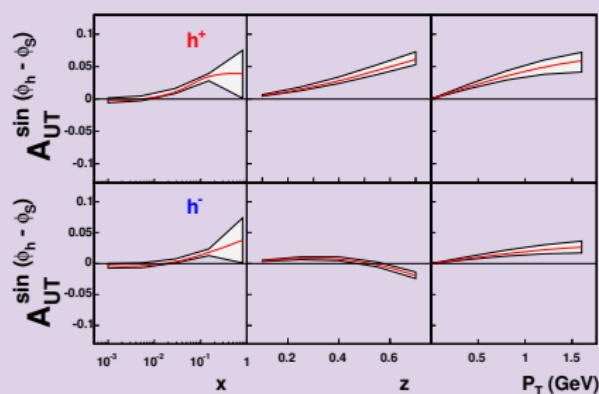
JLab

$ep \rightarrow e\pi X$, $p_{lab} = 6$ GeV.



COMPASS

$\mu p \rightarrow \mu h X$, $p_{lab} = 160$ GeV.

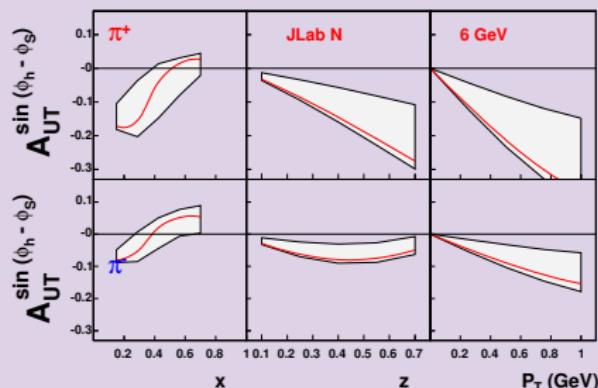


JLab can improve our knowledge of Sivers function in high x region. COMPASS operating on proton target is expected to measure 5% asymmetry for h^+ .

PREDICTIONS

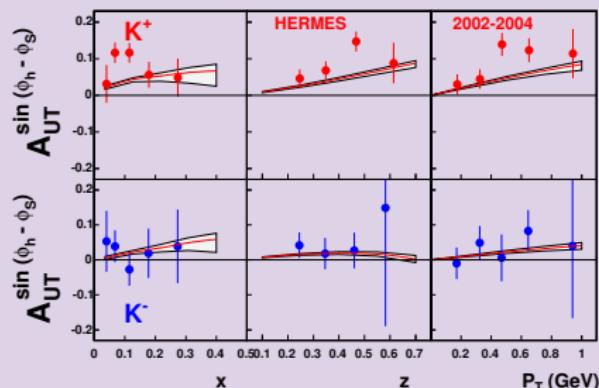
JLab

$eN \rightarrow e\pi X$, $p_{lab} = 6$ GeV.



HERMES

$ep \rightarrow eKX$, $p_{lab} = 27.57$ GeV.



JLab can improve our knowledge of $\Delta N_{f_d/p^\uparrow}$ using neutron target.
Prediction of the model are compatible with Kaon data from
HERMES.

CONCLUSIONS

- First extraction of transversity for u and d quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- Transversity $\Delta_T q(x)$ is found not to saturate Soffer bound $(q(x) + \Delta q(x))/2$.
 $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Estimates of the Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
 $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for u , d and sea quarks are extracted from HERMES and COMPASS data.
- Predictions for Collins and Sivers asymmetries at JLab and COMPASS (with the proton target) are presented and expected to be sizable.

CONCLUSIONS

- First extraction of transversity for u and d quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- Transversity $\Delta_T q(x)$ is found not to saturate Soffer bound $(q(x) + \Delta q(x))/2$.
 $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Estimates of the Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
 $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for u , d and sea quarks are extracted from HERMES and COMPASS data.
- Predictions for Collins and Sivers asymmetries at JLab and COMPASS (with the proton target) are presented and expected to be sizable.

CONCLUSIONS

- First extraction of transversity for u and d quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- Transversity $\Delta_T q(x)$ is found not to saturate Soffer bound $(q(x) + \Delta q(x))/2$.
 $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Estimates of the Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
 $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for u , d and sea quarks are extracted from HERMES and COMPASS data.
- Predictions for Collins and Sivers asymmetries at JLab and COMPASS (with the proton target) are presented and expected to be sizable.

CONCLUSIONS

- First extraction of transversity for u and d quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- Transversity $\Delta_T q(x)$ is found not to saturate Soffer bound $(q(x) + \Delta q(x))/2$.
 $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Estimates of the Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
 $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for u , d and sea quarks are extracted from HERMES and COMPASS data.
- Predictions for Collins and Sivers asymmetries at JLab and COMPASS (with the proton target) are presented and expected to be sizable.

CONCLUSIONS

- First extraction of transversity for u and d quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- Transversity $\Delta_T q(x)$ is found not to saturate Soffer bound $(q(x) + \Delta q(x))/2$.
 $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Estimates of the Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
 $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for u , d and sea quarks are extracted from HERMES and COMPASS data.
- Predictions for Collins and Sivers asymmetries at JLab and COMPASS (with the proton target) are presented and expected to be sizable.

CONCLUSIONS

- First extraction of transversity for u and d quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- Transversity $\Delta_T q(x)$ is found not to saturate Soffer bound $(q(x) + \Delta q(x))/2$.
 $\Delta_{Tu}(x) > 0$ and $\Delta_{Td}(x) < 0$

DANKE SCHÖN!

- $\Delta^N D_h^{tav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unt}(z, |p_\perp|) < 0$
- Sivers functions for u , d and sea quarks are extracted from HERMES and COMPASS data.
- Predictions for Collins and Sivers asymmetries at JLab and COMPASS (with the proton target) are presented and expected to be sizable.

Backup slides: $A_{UT}^{\sin(\phi_h - \phi_S)}$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x_B, z_h, P_T) \simeq \frac{\Delta\sigma_{\text{siv}}}{\sigma_0},$$

$$\Delta\sigma_{\text{siv}}(x_B, y, z_h, P_T) = \frac{2\pi\alpha^2}{x_B y^2 s} \sum_q e_q^2 2N_q(x_B) f_q(x_B) D_q^h(z_h) [1 + (1 - y)^2]$$

$$\cdot z_h P_T \frac{\sqrt{2e} \widehat{\langle k_\perp^2 \rangle}^2}{M \widehat{\langle P_T^2 \rangle}^2 \langle k_\perp^2 \rangle} \exp\left(-\frac{P_T^2}{\widehat{\langle P_T^2 \rangle}}\right),$$

$$\sigma_0(x_B, y, z_h, P_T) = 2\pi \frac{2\pi\alpha^2}{x_B y^2 s} \sum_q e_q^2 f_q(x_B) D_q^h(z_h) [1 + (1 - y)^2] \\ \cdot \frac{1}{\pi \langle P_T^2 \rangle} \exp\left(-\frac{P_T^2}{\langle P_T^2 \rangle}\right),$$

where

$$\widehat{\langle k_\perp^2 \rangle} = \frac{M^2 \langle k_\perp^2 \rangle}{M^2 + \langle k_\perp^2 \rangle}, \quad \widehat{\langle P_T^2 \rangle} = \langle p_\perp^2 \rangle + z^2 \widehat{\langle k_\perp^2 \rangle}.$$

Backup slides: $A_{UT}^{\sin(\phi_h - \phi_S)}$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x_B, z_h, P_T) \simeq \frac{\Delta\sigma_{\text{siv}}}{\sigma_0},$$

$$\Delta\sigma_{\text{siv}}(x_B, y, z_h, P_T) = \frac{2\pi\alpha^2}{x_B y^2 s} \sum_q e_q^2 2N_q(x_B) f_q(x_B) D_q^h(z_h) [1 + (1 - y)^2]$$

$$\cdot z_h P_T \frac{\sqrt{2e} \widehat{\langle k_\perp^2 \rangle}^2}{M \widehat{\langle P_T^2 \rangle}^2 \langle k_\perp^2 \rangle} \exp\left(-\frac{P_T^2}{\widehat{\langle P_T^2 \rangle}}\right),$$

$$\sigma_0(x_B, y, z_h, P_T) = 2\pi \frac{2\pi\alpha^2}{x_B y^2 s} \sum_q e_q^2 f_q(x_B) D_q^h(z_h) [1 + (1 - y)^2]$$

$$\cdot \frac{1}{\pi \langle P_T^2 \rangle} \exp\left(-\frac{P_T^2}{\langle P_T^2 \rangle}\right),$$

$A_{UT}^{\sin(\phi_h - \phi_S)} \propto z_h P_T$ and $A_{UT}^{\sin(\phi_h - \phi_S)} = 0$ when $z_h = 0$ or $P_T = 0$.

Backup slides

$$A_{UT}^{\sin(\phi_h + \phi_s)} \equiv \frac{2\langle \sin(\phi_h + \phi_s) \rangle}{D_{NN}}, \quad D_{NN} = \frac{2(1-y)}{1+(1-y)^2}$$

This is the definition of the measured asymmetry. Using this definition we can calculate $A_{UT}^{\sin(\phi_h + \phi_s)}$ as a function of x , z and P_T .

$$A_{UT}^{\sin(\phi_h + \phi_s)}(P_T) = \frac{-\frac{sx}{Q^4} \sum_q e_q^2 \frac{\langle p_C^2 \rangle}{2} \Delta_T q(x) H(z) \frac{P_T}{\pi \langle P_{T1}^2 \rangle^2} e^{-\frac{P_T^2}{\langle P_{T1}^2 \rangle}}}{\frac{sx}{Q^4} \sum_q e_q^2 q(x) D_q(z) \frac{1}{\pi \langle P_T^2 \rangle} e^{-\frac{P_T^2}{\langle P_T^2 \rangle}}}$$

where $\langle p_C^2 \rangle = \frac{\langle p_\perp^2 \rangle \langle p_{\perp 1}^2 \rangle}{\langle p_\perp^2 \rangle + \langle p_{\perp 1}^2 \rangle}$, $\langle P_{T1} \rangle = \langle p_C^2 \rangle + z^2 \langle k_{\perp h_1}^2 \rangle$, and
 $\langle P_T \rangle = \langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle$.

Backup slides

$$A_{UT}^{\sin(\phi_h + \phi_s)} \equiv \frac{2\langle \sin(\phi_h + \phi_s) \rangle}{D_{NN}}, \quad D_{NN} = \frac{2(1-y)}{1+(1-y)^2}$$

This is the definition of the measured asymmetry. Using this definition we can calculate $A_{UT}^{\sin(\phi_h + \phi_s)}$ as a function of x , z and P_T .

$$A_{UT}^{\sin(\phi_h + \phi_s)}(P_T) = \frac{-\frac{sx}{Q^4} \sum_q e_q^2 \frac{\langle p_C^2 \rangle}{2} \Delta_T q(x) H(z) \frac{P_T}{\pi \langle P_{T1}^2 \rangle^2} e^{-\frac{P_T^2}{\langle P_{T1}^2 \rangle}}}{\frac{sx}{Q^4} \sum_q e_q^2 q(x) D_q(z) \frac{1}{\pi \langle P_T^2 \rangle} e^{-\frac{P_T^2}{\langle P_T^2 \rangle}}}$$

We use either $\Delta_T q(x) \equiv (q(x) + \Delta_T q(x))/2$ or
 $\Delta_T q(x) = \Delta_{-} q(x)$

Backup slides

$$A_{UT}^{\sin(\phi_h + \phi_s)} \equiv \frac{2\langle \sin(\phi_h + \phi_s) \rangle}{D_{NN}}, \quad D_{NN} = \frac{2(1-y)}{1+(1-y)^2}$$

This is the definition of the measured asymmetry. Using this definition we can calculate $A_{UT}^{\sin(\phi_h + \phi_s)}$ as a function of x , z and P_T .

$$A_{UT}^{\sin(\phi_h + \phi_s)}(P_T) = \frac{-\frac{sx}{Q^4} \sum_q e_q^2 \frac{\langle p_C^2 \rangle}{2} \Delta_T q(x) H(z) \frac{P_T}{\pi \langle P_{T1}^2 \rangle^2} e^{-\frac{P_T^2}{\langle P_{T1}^2 \rangle}}}{\frac{sx}{Q^4} \sum_q e_q^2 q(x) D_q(z) \frac{1}{\pi \langle P_T^2 \rangle} e^{-\frac{P_T^2}{\langle P_T^2 \rangle}}}$$

$A_{UT} \sim \langle p_C^2 \rangle$, thus $A_{UT} \rightarrow 0$ if $\langle p_C^2 \rangle \rightarrow 0$

Backup slides

$$A_{UT}^{sin(\phi_h + \phi_S)}(x, z) = \frac{-\frac{sx}{Q^4} \sum_q e_q^2 \frac{\sqrt{\pi} \langle p_C^2 \rangle}{2\sqrt{\langle p_C^2 \rangle + z^2 \langle k_{\perp h1}^2 \rangle}} \Delta_T q(x) H(z)}{\frac{sx}{Q^4} \sum_q e_q^2 q(x) D_q(z)}$$

Collins fragmentation function is parametrized in the form

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp^2) \equiv \frac{2|\mathbf{p}_\perp|}{zm_\pi} H_1^{\perp q}(z, p_\perp^2),$$

$$\Delta^N D_{h/q^\uparrow}(z, \mathbf{p}_\perp) \equiv 2h(p_\perp) H(z) D_{h/q}(z, \mathbf{p}_\perp),$$

$$h(p_\perp) \equiv p_\perp \sqrt{\frac{2e}{\langle p_{\perp 1}^2 \rangle}} \exp(-p_\perp^2 / \langle p_{\perp 1}^2 \rangle) \leq 1,$$

$$|H(z)| \equiv |N_q z^{\alpha_q} (1-z)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}| \leq 1.$$