

## Standard Model

### Beyond the Standard Model

#### Lecture 3

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#### Outline:

- Electroweak symmetry breaking (Lecture 1)
- Quark and lepton masses; vectorlike quarks (Lecture 2)
- **New gauge bosons (Lecture 3)**
- WIMPs and cascade decays (Lecture 4)
- How to search for new phenomena (Lecture 5)

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## Anomaly cancellation

Gauge symmetries may be broken by quantum effects.

Cure: sums over fermion triangle diagrams must vanish.



Standard Model: anomalies cancel within each generation

$$[SU(3)]^2 U(1): \quad 2(1/3) + (-4/3) + (2/3) = 0$$

$$[SU(2)]^2 U(1): \quad 3(1/3) + (-1) = 0$$

$$[U(1)]^3: \quad 3[2(1/3)^3 + (-4/3)^3 + (2/3)^3] + 2(-1)^3 + (-2)^3 = 0$$

$$U(1)\text{-gravitational}: \quad 2(1/3) + (-4/3) + (2/3) = 0$$

Fermion and scalar gauge charges:

	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$
quark doublet: $q_L^i = (u_L^i, d_L^i)$	3	2	1/3
right-handed up-type quark: $u_R^i$	3	1	4/3
right-handed down-type quark: $d_R^i$	3	1	-2/3
lepton doublet: $l_L^i = (\nu_L^i, e_L^i)$	1	2	-1
right-handed charged lepton: $e_R^i$	1	1	-2
Higgs doublet: $H$	1	2	+1

$i = 1, 2, 3$  labels the fermion generations.

## $Z'$ bosons

$Z'$  = any new electrically-neutral gauge boson (spin 1).

Consider an  $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_Z$  gauge symmetry spontaneously broken down to  $SU(3)_C \times U(1)_{\text{em}}$  by the VEVs of a doublet  $H$  and an  $SU(2)_W$ -singlet scalar,  $\varphi$ .

The mass terms for the three electrically-neutral gauge bosons,  $W^{3\mu}$ ,  $B_Y^\mu$  and  $B_Z^\mu$ , arise from the kinetic terms for the scalars:

$$\frac{v_H^2}{8} (gW^{3\mu} - g_Y B_Y^\mu - z_H g_Z B_Z^\mu) (gW_\mu^3 - g_Y B_{Y\mu} - z_H g_Z B_{Z\mu})$$

$$+ \frac{v_\varphi^2}{8} g_Z^2 B_Z^\mu B_{Z\mu}$$

Mass-square matrix for  $B_Y^\mu$ ,  $W^{3\mu}$  and  $B_z^\mu$ :

$$\mathcal{M}^2 = \frac{g^2 v_H^2}{4 \cos^2 \theta_w} U^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -z_H t_z \cos \theta_w \\ 0 & -z_H t_z \cos \theta_w & (r + z_H^2) t_z^2 \cos^2 \theta_w \end{pmatrix} U$$

where  $t_z \equiv g_z/g$ ,  $\tan \theta_w = g_Y/g$ ,  $r = v_\phi^2/v_H^2$

$$U = \begin{pmatrix} \cos \theta_w & \sin \theta_w & 0 \\ -\sin \theta_w & \cos \theta_w & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{relates the neutral gauge bosons to the physical states in the case } z_H = 0.$$

The relation between the neutral gauge bosons and the corresponding mass eigenstates can be found by diagonalizing  $\mathcal{M}^2$ :

$$\begin{pmatrix} B_Y^\mu \\ W^{3\mu} \\ B_z^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \cos \theta' & \sin \theta_w \sin \theta' \\ \sin \theta_w & \cos \theta_w \cos \theta' & -\cos \theta_w \sin \theta' \\ 0 & \sin \theta' & \cos \theta' \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix}$$

$A^\mu$  is the photon field

$Z^\mu$  is the field associated with the observed  $Z$  boson

$Z'^\mu$  is a neutral gauge boson, not discovered yet.

The mixing angle  $-\pi/4 \leq \theta' \leq \pi/4$  satisfies

$$\tan 2\theta' = \frac{2z_H t_z \cos \theta_w}{(r + z_H^2) t_z^2 \cos^2 \theta_w - 1}$$

The  $Z$  and  $Z'$  masses are given by

$$M_{Z,Z'} = \frac{g v_H}{2 \cos \theta_w} \left[ \frac{1}{2} \left( (r + z_H^2) t_z^2 \cos^2 \theta_w + 1 \right) \mp \frac{z_H t_z \cos \theta_w}{\sin 2\theta'} \right]^{1/2}$$

$Z'$  is heavier than  $Z$  when  $(r + z_H^2) t_z^2 \cos^2 \theta_w > 1$ .

The mass and couplings of the  $Z'$  boson are described by the following parameters:

- gauge coupling  $g_z$
- VEV  $v_\phi$
- $U(1)_z$  charge of the Higgs doublet,  $z_H$
- fermion charges under  $U(1)_z$  – constrained by anomaly cancellation conditions and requirement of fermion mass generation

## Nonexotic $Z'$

Nonanomalous  $U(1)_z$  gauge symmetry *without new fermions charged under  $SU(3)_C \times SU(2)_W \times U(1)_Y$*

Allow an arbitrary number of  $\nu_R$ 's

Assume: • generation-independent charges,  
• quark and lepton masses from standard model Yukawa couplings

Fermion and scalar gauge charges:

	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	$U(1)_z$
$q_L^i$	3	2	1/3	$z_q$
$u_R^i$	3	1	4/3	$z_u$
$d_R^i$	3	1	-2/3	$2z_q - z_u$
$l_L^i$	1	2	-1	$-3z_q$
$e_R^i$	1	1	-2	$-2z_q - z_u$
$\nu_R^k, k = 1, \dots, n$	1	1	0	$z_k$
$H$	1	2	+1	$-z_q + z_u$
$\varphi$	1	1	0	1

$[SU(3)_C]^2 U(1)_z, [SU(2)_W]^2 U(1)_z, U(1)_Y [U(1)_z]^2$  and  
 $[U(1)_Y]^2 U(1)_z$  anomalies cancel

Gravitational- $U(1)_z$  and  $[U(1)_z]^3$  anomaly cancellation conditions:

$$\frac{1}{3} \sum_{k=1}^n z_k = -4z_q + z_u$$

$$\left( \sum_{k=1}^n z_k \right)^3 = 9 \sum_{k=1}^n z_k^3$$

• For  $n \leq 2$ :

$z_1 = -z_2 \Rightarrow z_u = 4z_q \Rightarrow$  trivial or  $Y$ -sequential  $U(1)_z$ -charges

• For  $n \geq 3$ :

$U(1)_{B-L}$  charges:  $z_1 = z_2 = z_3 = -4z_q + z_u$

or  $z_1 = z_2 = -(4/5)z_3 = -16z_q + 4z_u = -4$

$\nu$  masses: three LH Majorana,  
two dimension-7 and one dimension-12 Dirac operators,  
RH Majorana ops. of dimension ranging from 4 to 13

or ...

LEP I requires  $\theta' \lesssim 10^{-3} \Rightarrow M_{Z'} \gtrsim 2 \text{ TeV}$

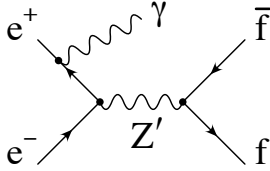
Special case:  $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_{B-L}$

$$z_q = z_u = z_d = -\frac{z_l}{3} = -\frac{z_e}{3} = -\frac{z_\nu}{3} \implies z_H = 0$$

No  $Z_{B-L}$ - $Z$  mixing at tree level ( $\theta' = 0$ )!

Best bounds on  $z_{lgz}$  come from limits on direct production at the LHC, Tevatron and LEP II.

Initial state radiation at LEP for a narrow  $Z_{B-L}$  resonance at  $M_{Z'} < \sqrt{s}$ :



$$\sigma(e^+e^- \rightarrow \gamma Z_{B-L}) \text{Br}(Z_{B-L} \rightarrow \mu^+\mu^-) \approx \frac{3\alpha}{74} (z_l g_z)^2 \frac{s^2 + M_{Z'}^4}{s^2 (s - M_{Z'}^2)} \ln\left(\frac{s}{m_e^2}\right)$$

LEP II has run at  $\sqrt{s} \approx 130, 136, 161, 172, 183, 189, 192 - 209$  GeV

For a  $Z_{B-L}$  with  $M_{Z'} \sim 140$  GeV:

Number of  $\mu^+\mu^-$  events at  $\sqrt{s} \approx 161$  GeV due to  $Z_{B-L}$ :

$$N(Z_{B-L}) \approx 3 \times 10^4 (z_l g_z)^2$$

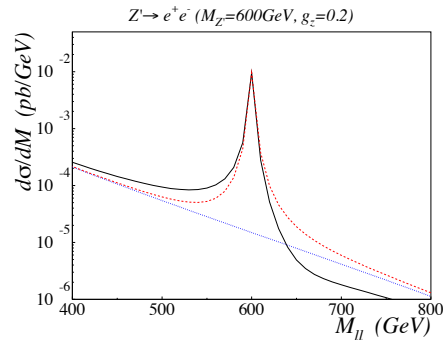
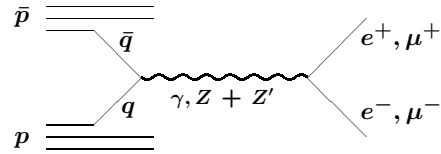
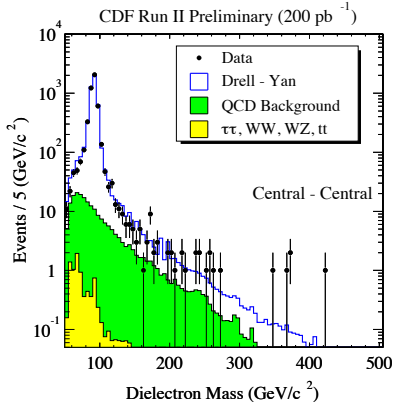
Main background:  $e^+e^- \rightarrow \gamma^*\gamma, Z^*\gamma \rightarrow \mu^+\mu^-\gamma$   
( $\sim 6.4$  events in an energy bin of 5 GeV)

At the 95% confidence-level:  $z_l g_z \lesssim 10^{-2}$

If  $\sqrt{s} = M_{Z'}$ : no need for initial state radiation.

Strongest bound for  $M_{Z'} \sim 189$  GeV:  $z_l g_z < 10^{-3}$

$Z'$  searches at the Tevatron

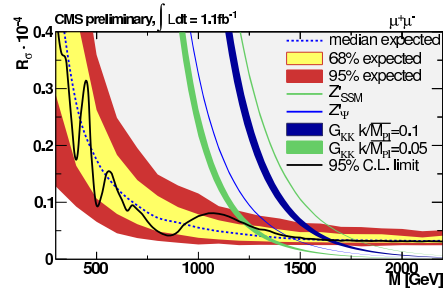
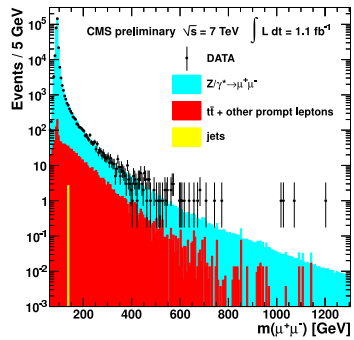


More general charges are allowed in the presence of **new fermions**:

	$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_{B-xL}$	$U(1)_{q+xu}$	$U(1)_{10+x5}$	$U(1)_{d-xu}$
$q_L$	3	2	1/3	1/3	1/3	1/3	0
$u_R$	3	1	4/3	1/3	$x/3$	-1/3	$-x/3$
$d_R$	3	1	-2/3	1/3	$(2-x)/3$	$-x/3$	1/3
$l_L$	1	2	-1	$-x$	-1	$x/3$	$(-1+x)/3$
$e_R$	1	1	-2	$-x$	$-(2+x)/3$	-1/3	$x/3$
$\nu_R$	1	1	0	-1	$(-4+x)/3$	$(-2+x)/3$	$-x/3$
$\nu'_R$				.	.	$-1-x/3$	.
$\psi_L^l$	1	2	-1	-1	.	$-(1+x)/3$	$-2x/5$
$\psi_R^l$				$-x$	.	2/3	$(-1+x/5)/3$
$\psi_L^e$	1	1	-2	-1	.	.	.
$\psi_R^e$				$-x$	.	.	.
$\psi_L^d$	3	1	-2/3	.	.	-2/3	$(1-4x/5)/3$
$\psi_R^d$				.	.	$(1+x)/3$	$x/15$

Homework 3.1:

Identify the couplings of the  $Z'$  arising from the  $SO(10) \rightarrow SU(5)$  GUT breaking.

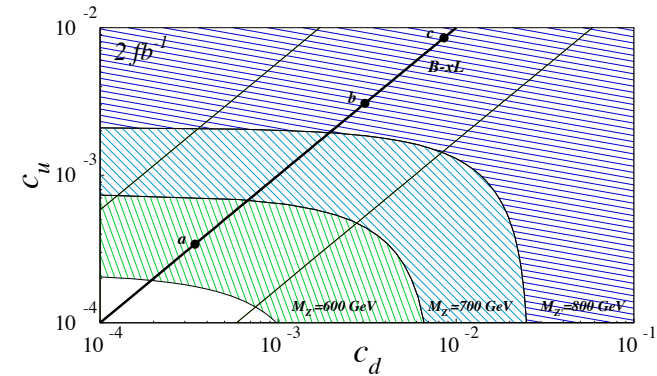


A user-friendly parametrization (hep-ph/0408098):

$$\sigma(p\bar{p} \rightarrow Z'X \rightarrow l^+l^-X) = \frac{\pi}{48s} \left[ c_u w_u \left( \frac{M_{Z'}^2}{s}, M_{Z'} \right) + c_d w_d \left( \frac{M_{Z'}^2}{s}, M_{Z'} \right) \right]$$

All the information about charges is contained in:

$$c_{u,d} = g_z^2 (z_q^2 + z_{u,d}^2) \text{Br}(Z' \rightarrow l^+l^-)$$



$$\sigma(pp \rightarrow Z'X \rightarrow l^+l^-X) = \frac{\pi}{48s} \left[ c_u w'_u \left( \frac{M_{Z'}^2}{s}, M_{Z'} \right) + c_d w'_d \left( \frac{M_{Z'}^2}{s}, M_{Z'} \right) \right]$$

$w'_u$  and  $w'_d$  contain all the information about QCD:  
values at the LHC are different than at the Tevatron

$\Rightarrow c_u$  and  $c_d$  can be determined independently if a  $Z'$  is  
observed at both the Tevatron and the LHC.

More information about  $Z'$  couplings ( $U(1)_z$  charges) can be extracted from angular distributions, etc.

Homework # 3.2:

What are the analytical formulas for  $w'_{u,d}$  at LO in  $\alpha_s$ ?

### "Gluon-prime": a heavy spin-1 color-octet particle

Gauge extension of QCD ("Topcolor", C. Hill 1991):

$SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c$  spontaneously broken by the VEV  
of a scalar  $\Sigma$  transforming as  $(3, \bar{3})$

$G_\mu^a$  - massless gluon of QCD, with  $g_s = \frac{h_1 h_2}{\sqrt{h_1^2 + h_2^2}}$  ( $h_{1,2}$  are the  $SU(3)_{1,2}$   
gauge couplings)

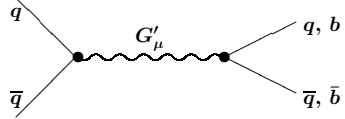
$G_\mu'^a$  - massive "gluon-prime"

("topgluon" or "coloron" depending on its couplings to  $t, b$ )

Interactions:  $g_s r G_\mu'^a \bar{q} \gamma^\mu T^a q$  where  $r = h_1/h_2$ .

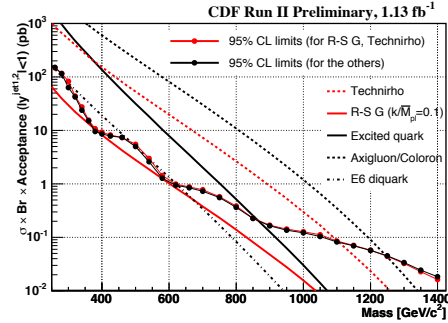
$G'_\mu$  production (in the narrow width approximation):

$$\sigma(p\bar{p} \rightarrow G'_\mu X) \approx \frac{16\pi^2\alpha_s r^2}{9s} \sum_q \int_{M^2/s}^1 \frac{dx}{x} \left[ q(x) q\left(\frac{M^2}{xs}\right) + \bar{q}(x) \bar{q}\left(\frac{M^2}{xs}\right) \right]$$



Limit on dijet resonance of

$\sim 300$  GeV:  $r^2 \mathcal{B}(G'_\mu \rightarrow jj) < 0.04$



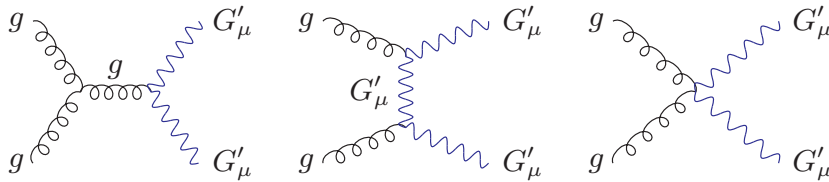
## Model “independent” search for Gluon-prime

work with KC Kong and Rakhi Mahbubani hep-ph/0709.2378

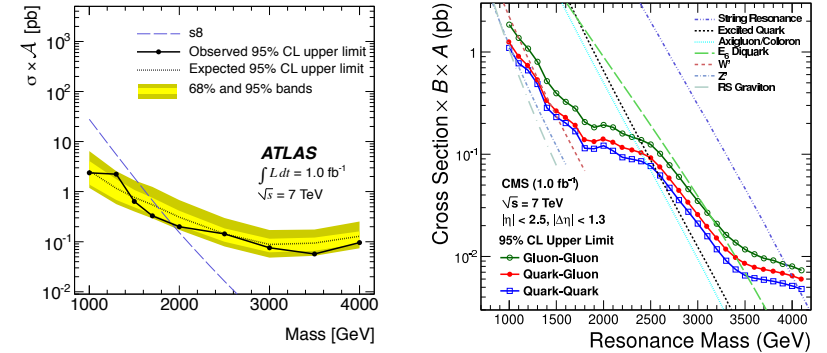
$G'_\mu$  couplings to quarks are model dependent: if they are small enough, the mass limits from dijet resonance searches are evaded.

$G'_\mu$  couplings to gluons are fixed by gauge invariance.  $G'_\mu$  couples only in pairs to the gluon.

Pair production of heavy gluons from gluon-gluon initial state:



Dijet resonance searches at the LHC 7:



My wishes: extend the search to lower masses;  
show theory predictions for several choices of the coupling ( $r$ );

A pair of gluon-primes decays to 2 pairs of dijets (or  $j\bar{j}b\bar{b}$ ,  $b\bar{b}b\bar{b}$ ).

Dominant background: QCD 4-jet production.

Background simulated at tree level using MadGraph (checked with NJETS), taking the b-tagging efficiency to be 50%.

Production and decay of gluon-primes ( $G'_\mu$ ) at the LHC:

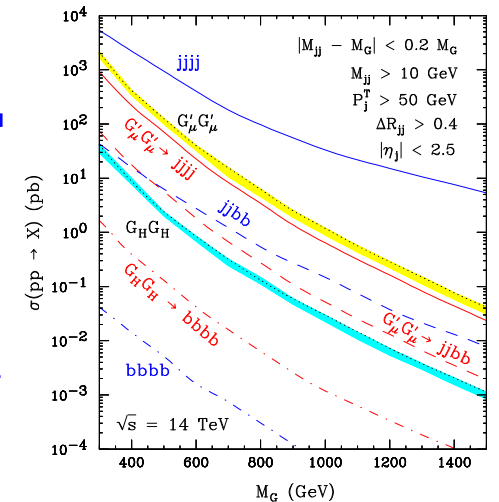
The jets reconstruct (in pairs) resonances of equal mass.

Cutting around the resulting peak decreases the background dramatically.

A spin-0 color octet ( $G_H$ ) has similar properties, except it decays into the heaviest quarks.

Signal:  $p\bar{p} \rightarrow G_H G_H \rightarrow (b\bar{b})(b\bar{b})$

LHC mass reach for a  $G'_\mu$  is  $M_G \lesssim 1$  TeV with  $1 \text{ fb}^{-1}$



Could there exist  
massless gauge bosons  
other than the photon?

$U(1)_{B-L}$  is the only global symmetry of the standard model that can be gauged and **unbroken**.

$Z_{B-L}$  coupling to ordinary matter:  $N_n g_z$   
( $N_n$  = number of neutrons)

To avoid deviations from Newton's law:

$$g_z \ll \frac{m_n}{M_{\text{Pl}}} \sim 10^{-19}$$

Tests of the equivalence principle:  $g_z < 10^{-24}$

But even when  $z_q = z_u = z_d = z_l = z_e = 0$

there can still be interactions of the standard model fields with the new massless gauge boson:

**higher-dimensional operators!**

$\gamma'$  couplings to leptons:

$$\frac{1}{M^2} P_{\mu\nu} (\bar{l}_L \sigma^{\mu\nu} F_e H e_R + \text{h.c.})$$

$F_e$ :  $3 \times 3$  matrix in flavor space,  
dimensionless parameters

$\gamma'$  couplings to quarks: similar dimension-6 operators

In the mass eigenstate basis  $F_e \rightarrow F'_e = U_L^e F_e U_R^{e\dagger}$

$U_L^e$  and  $U_R^e$  are the unitary matrices that diagonalize the masses of the electrically-charged leptons.

- Interactions of the mass-eigenstate leptons with  $P^\mu$   
(chirality-flip operators  $\sim v_h \approx 174$  GeV) :

$$\frac{v_h}{M^2} P_{\mu\nu} \overline{e'} \sigma^{\mu\nu} (\text{Re} F'_e + i \text{Im} F'_e \gamma_5) e'$$

→ magnetic-like and electric-like dipole moments

**Kinetic mixing of  $U(1)_Y \times U(1)_z$  gauge bosons:**

$c_0 B^{\mu\nu} P_{\mu\nu}$  dimension-four operator!

Holdom 1985:

Kinetic terms can be diagonalized and canonically normalized by a  $SL(2, R)$  transformation.

Global  $SO(2)$  symmetry: linear combination of  $U(1)$  fields that couples to hypercharge is the real  $B^\mu$ .

Orthogonal combination (“paraphoton” =  $\gamma'$ ) does not have any renormalizable couplings to standard model fields.

*Conclusion:*

*kinetic mixing has no effect on the standard model fields other than a renormalization of the hypercharge gauge coupling.*

$\text{Re}(F'_e)^{ij}, \text{Im}(F'_e)^{ij}$  could have any value  $\lesssim 4\pi$ , but:

chirality-flip operators → probably  $|F_e^{ij}| \lesssim |\lambda_e^{ij}|$

$$\Rightarrow |F_e^{ij}| \lesssim \frac{m_\tau}{v_h} \approx 10^{-2}$$

$|F_e'^{11}|$  may naturally be below  $m_e/v_h \approx 3 \times 10^{-6}$

**Bosonic interactions of the paraphoton:**

$$\frac{1}{M^2} H^\dagger H \left( c_1 B_{\mu\nu} + \tilde{c}_1 \tilde{B}_{\mu\nu} + c_2 P_{\mu\nu} + \tilde{c}_2 \tilde{P}_{\mu\nu} \right) P^{\mu\nu}$$

- renormalize the  $U(1)$  gauge couplings
- include vertices with two  $U(1)$  gauge bosons and Higgs bosons.

*These are all operators of  $d \leq 6$  involving both*

*$\gamma'$  and SM fields*



The strength of the  $\gamma'$  interaction with the electrons depends on

$$c_e \equiv \frac{v_h}{m_e} |(C'_e)_{11}| \lesssim O(1)$$

Similar parameters defined for interactions such as  $\mu^+ \mu^- \gamma'$ ,  $\mu^\pm e^\mp \gamma'$ , ...

Various measurements set limits on these parameters.

Spin-dependent long-range forces:

$$V(r) = -\frac{c_e^2 m_e^2}{\pi M^4} \frac{1}{r^3} [3(\sigma_1 \cdot r)(\sigma_2 \cdot r) - \sigma_1 \cdot \sigma_2]$$

Measurements of  $e - e$  long range forces impose that

$$\frac{M}{\sqrt{c_e}} \gtrsim 3 \text{ GeV}$$

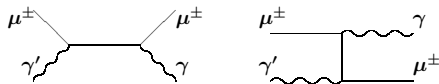
## Primordial Nucleosynthesis

Constraints on new particles with mass below several MeV.

Maximum number of new relativistic degrees of freedom:

$$\Delta g_*^{\max} = \frac{7}{8} \Delta N^{\max}; \quad \text{at the } 2\sigma \text{ level: } \Delta N_\nu^{\max} \approx 0.6$$

$\gamma'$  must go out of equilibrium at  $T_P > T_{\text{BBN}} \approx 1 \text{ MeV}$



Measurements of light element abundances set a limit on the effective mass scale:

$$M \gtrsim 1.5 \text{ TeV} \times \sqrt{c_\mu}$$

## Star cooling

Effective coupling of  $\gamma'$  to electrons:  $g_{\gamma'e} = \frac{c_e}{M^2} m_e^2$

Red giant stars:  $\gamma'$  emission via Bremsstrahlung & Compton-like processes

$$g_{\gamma'e}^2 / 4\pi < 2.5 \times 10^{-27} \quad \Rightarrow \quad \frac{M}{\sqrt{c_e}} \gtrsim 3.2 \text{ TeV}$$

For supernovae:  $\nu$  emission rate  $\gg \gamma'$  emission rate  
 $\Rightarrow$  no useful bound on electron- $\gamma'$  coupling  
 (strong bound on quark- $\gamma'$  couplings)

## Flavor-changing neutral currents

Chirality-flip transition:  $\Gamma(\mu \rightarrow e\gamma') = c_{e\mu}^2 \frac{m_\mu^5}{8\pi M^4}$

Standard model:  $\Gamma(\mu \rightarrow e\nu\bar{\nu}) = \frac{m_\mu^5 G_F^2}{192\pi^3} \approx 3.2 \times 10^{-10} \text{eV}$

$$\text{Br}(\mu \rightarrow e\gamma') < 3 \times 10^{-5} \quad \Rightarrow \quad \frac{M}{\sqrt{c_{e\mu}}} \gtrsim 15 \text{ TeV}$$

### Conclusions so far

The LHC explores the TeV scale (= “terra incognita”).  
Many possibilities for what can be discovered:

- extended Higgs sectors (Lecture 1)
- Vectorlike fermions (Lecture 2)
- New gauge bosons ( $Z'$ ,  $G'$ , ...) (this lecture)

New very light (even massless) particles may also exist.

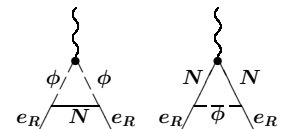
E.g., a  $\gamma'$  may couple to quarks and leptons via dimension-6 operators suppressed by the TeV scale!

An UV origin of the dimension-6 operators:

	$l_L$	$e_R$	$N_L$	$N_R$	$H$	$\phi$
$SU(2)_W$	2	1	1	1	2	1
$U(1)_Y$	-1	-2	0	0	+1	-2
$U(1)_D$	0	0	+1	+1	0	-1

Yukawa interaction:  $\lambda_\phi^j \bar{e}_R^j N_L \phi + \text{h.c.}$

Contribution to the  $iP_{\mu\nu}\bar{e}_R\gamma^\mu\partial^\nu e_R$  operator ( $\sim P_{\mu\nu}\bar{l}_L\sigma^{\mu\nu}e_R H$ ):



$$\Rightarrow (C_e)_{ij} = \frac{g_p}{192\pi^2} \sum_k \left( \lambda_\phi^i \lambda_\phi^{*k} \lambda_e^{kj} + \lambda_e^{ik} \lambda_\phi^k \lambda_\phi^{*j} \right)$$