Flavor physics

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Parameter counting

How many parameters we have?

How many parameters are physical?

- “Unphysical” parameters are those that can be set to zero by a basis rotation
- General theorem

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken})$$

  - $N(\text{Phys})$, number of physical parameters
  - $N(\text{tot})$, total number of parameters
  - $N(\text{broken})$, number of broken generators

Example: Zeeman effect

A hydrogen atom with weak magnetic field

- The magnetic field add one new physical parameter, B

$$V(r) = \frac{-e^2}{r} \qquad V(r) = \frac{-e^2}{r} + B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

- But there are 3 total new parameters
- The magnetic field breaks explicitly: $SO(3) \to SO(2)$
- 2 broken generators, can be “used” to define the z axis

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken}) \quad \Rightarrow \quad 1 = 3 - 2$$

Back to the flavor sector

Without the Yukawa interaction, a model with N copies of the same field has a $U(N)$ global symmetry

- It is just the symmetry of the kinetic term

$$\mathcal{L} = \bar{\psi}_i D_\mu \gamma^\mu \psi_i, \qquad i = 1, 2, ..., N$$

- $U(N)$ is the general rotation in N dimensional complex space
- $U(N) = SU(N) \times U(1)$ and it has N^2 generators

Lepton sector

First example, leptons in the "old" SM: L, E

- One Yukawa matrix: $Y\bar{L}\phi E$, $N_T = 18$
- Global symmetries: $U(3)_E \times U(3)_L$, 18 generators
- Exact accidental symmetries: $U(1)_e \times U(1)_\mu \times U(1)_\tau$, 3 generators
- Broken generators due to the Yukawa: $N_B = 18 - 3 = 15$
- Physical parameters: $N_P = 18 - 15 = 3$. They are the 3 charged lepton masses

Two generations of quarks

Q_L^i, D_R^i, U_R^i, $i = 1, 2$

Do it yourself...

$$\begin{aligned} N_T &= \\ N_G &= \\ N_U &= \\ N_B &= N_G - N_U = \end{aligned}$$

So we have

$$N_P = N_T - N_B =$$

how many are masses and how many are mixing angles?

The SM flavor sector

Back to the SM with three generations

- $N_T = 2 \times 18 = 36$
- $N_G = 3 \times 9 = 27$
- $N_U = 1$
- $N_B = 27 - 1 = 26$
- $N_P = 36 - 26 = 10$
- 6 quark masses, 3 mixing angles and one CPV phase

Remark: The broken generators are 17 Im and 9 Re. We have 18 to "start with" so the physical ones are $18 - 17 = 1$ and $18 - 9 = 9$

Homework

Consider a model with the same gauge symmetry and SSB as in the SM. The fermions, however, are

$$Q_L(3,2)_{1/6}, \quad S_L(3,1)_{-1/3}, \quad Q_R(3,2)_{1/6}, \quad S_R(3,1)_{-1/3}$$

- What is the spectrum of this model? That is, what are the quarks after SSB. Note that you can also have "bare masses" in this model. Also, there is no flavor index
- How many physical parameters there are, and what are they?
- Are there W exchange flavor changing interactions?
- Is there tree level FCNC in this model?

The CKM matrix

The flavor parameters

- The 6 masses. We kind of know them. There is a lot to discuss, but I will not do it in these lectures
- The CKM matrix has 4 parameters
  - 3 mixing angles (the orthogonal part of the mixing)
  - One phase (CP violating)
- We will concentrate on trying to find ways to determine the CKM three mixing angles and one phase. Here we will get into some details

The CKM matrix

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{U_L} V \gamma^\mu D_L W_\mu^+ + \text{h.c.}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- CKM is unitary

$$\sum V_{ij} V_{ik}^* = \delta_{jk}$$

- Experimentally, $V \sim 1$. Off diagonal terms are small
- Many ways to parametrize the matrix

CKM parametrization

- The standard parametrization

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

  where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.
- In general there are 5 entries that carry a phase
- Experimentally: (will explain later how these measurements were done)

$$|V| \approx \begin{pmatrix} 0.97383 & 0.2272 & 3.96\times10^{-3} \\ 0.2271 & 0.97296 & 4.221\times10^{-2} \\ 8.14\times10^{-3} & 4.161\times10^{-2} & 0.99910 \end{pmatrix}$$

The Wolfenstein parametrization

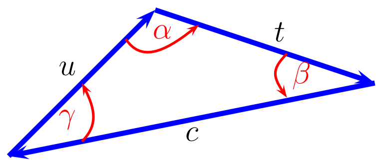
- Since $V \sim 1$ it is useful to expand it

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- One small parameter $\lambda \sim 0.2$, and three (A,ρ,η) that are roughly $O(1)$
- As always, be careful (unitarity...)
- Note that to this order only V_{13} and V_{31} have a phase

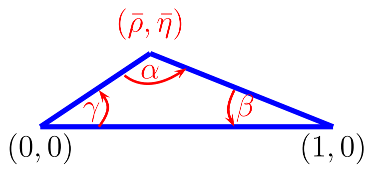
The unitarity triangle

A geometrical presentation of $V_{ub}^*V_{ud} + V_{tb}^*V_{td} + V_{cb}^*V_{cd} = 0$



Rescale by the c size and rotated

$$A\lambda^3\left[(\rho+i\eta)+(1-\rho-i\eta)+(-1)\right]=0$$



CKM determination

CKM determination

- Basic idea: Measure the 4 parameters in many different ways. Any inconstancy is a signal of NP
- Problems: Experimental errors and theoretical errors
- Have to be smart...
  - Smart theory to reduce the errors
  - Smart experiment to reduce the errors
- There are cases where both errors are very small

Classifications

Two classifications:

- Parameters
  - Sides of the UT (magnitudes of CKM elements)
  - Angles of the UT (relative phases between CKM elements)
  - Combination of those
- Amplitudes
  - Tree (mostly SM)
  - Loop (SM and maybe also NP)
  - Combination of those

Experimental issues

Just very brief

- Many times we look at very small rates or small asymmetries (we like to probe small couplings). Statistics is needed
- Very important to get the PID (like K/π separation)
- Flavor tagging: is it a B or a $\bar{B}$
- CP properties: the detector is made of matter

Theoretical uncertainties

Always: QCD

- We calculate with quark, but we measure hadrons
- The strong interaction is strong. No perturbation theory. Really a problem

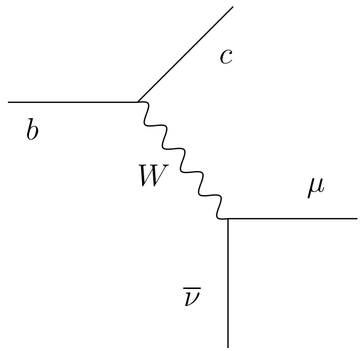
Solutions:

- We use some approximate symmetries: isospin, flavor SU(3), HQS
- There are cases where one can construct observables where the hadronic physics cancels

Measuring sides

Tree level decays are sensitive to absolute values of CKM element

$$\Gamma(B \to X_c \mu\nu) \propto |V_{cb}|^2$$



Measuring sides: problems

Not so simple...

$$\Gamma(b \to c\mu\nu) \propto m_b^5 |V_{cb}|^2$$

- Because the b is heavy, $m_b \gg \Lambda_{QCD}$ we can expand and we know that

$$\Gamma(b \to c\mu\nu) \approx \Gamma(B \to X_c \mu\nu)$$

- Not easy to get m_b the mass of the b quark. Again, we use HQS and use m_B, the B meson mass
- Using symmetries, and expanding around them we can get rather accurate determination

Always: Look for a process where we have sensitivity, and work our way around QCD

An aside: what is m_B?

$$\boxed{m_B = ?}$$

Other sides

Similar issues with other tree level decays

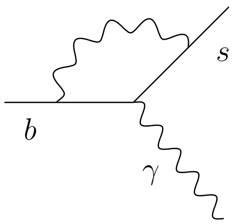
- β-decay, $d \to u e \bar{\nu} \propto V_{ud}$; Isospin
- K-decay, $s \to u e \bar{\nu} \propto V_{us}$; Isospin and SU(3)
- D-decay, $c \to q e \bar{\nu} \propto V_{cq}$ $q = d, s$; HQS
- B decays can be used also for V_{ub}. Harder
- Not easy with top. Cannot tag the final flavor, low statistics

Loop decays

- We have sensitivity to magnitude of CKM elements in loops
- More sensitive to V_{tq} that is harder to get in tree level decays
- But at the same time it may be modified by new heavy particles
- This is a general argument. NP is likely to include “heavy” particles, that can affect loop processes much more than tree level decays

Loop: example

$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^*$



What is $\sum V_{ib} V_{is}^*$?

GIM Mechanism

what we really have is

$$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^* f(m_i)$$

- Because the CKM is unitary, the m_i independent term in f vanishes
- Must be proportional to the mass (in fact, m_i^2) so the heavy fermion in the loop is dominant
- In Kaon decay this gives m_c^2/m_W^2 extra suppression. Numerically not important for b decays
- CKM unitarity and tree level Z exchange are related. (Is the diagram divergent?)

Meson mixing

Meson mixing

$$|f_1\rangle(t) = \exp\left[i\Delta Et/2\right]|1\rangle + \exp\left[-i\Delta Et/2\right]|2\rangle$$

The probability to measure flavor f_i at time t is

$$|\langle f_1|f_1\rangle|^2 = \frac{1+\cos\Delta Et}{2}$$
$$|\langle f_1|f_2\rangle|^2 = \frac{1-\cos\Delta Et}{2}$$

- Oscillations with frequency ΔE
- In the rest frame it is just Δm
- The relevant time scale is $x \equiv \Delta m/\Gamma$

Calculations of Δm

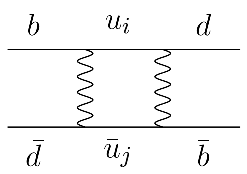
- There are 4 neutral mesons: $K(\bar{s}d)$, $B(\bar{b}d)$, $B_s(\bar{b}s)$, $D(c\bar{u})$
  - Why not charged mesons?
  - Why not the neutral pion?
  - Why not the K^*
  - Why not T mesons?
- The two flavor eigenstate B and $\bar{B}$ mix via the weak interactions. It is an FCNC process $m_{weak} = A(B \to \bar{B})$
- In the SM it is a loop process, and it gives an effect that is much smaller than the mass

$$M = \begin{pmatrix} m_B & m_{weak} \\ m_{weak} & m_B \end{pmatrix} \quad \Rightarrow \quad M_{H,L} = m_B \pm m_{weak}/2$$

$$\Delta M = m_{weak}$$

The box diagram

- In the SM the mixing is giving by the box diagram



- The result is

$$\Delta M \propto \sum_{i,j} V_{is}V_{id}^* V_{js}V_{jd}^* f(m_i, m_j)$$

- To leading order $f \sim m_i^2/m_W^2$ so for K mixing m_c^2/m_W^2 suppression

Meson mixing: remarks

- Mixing can be used to determine magnitude of CKM elements. The heavy fermion is the dominant one. For example B mixing is used to get $|V_{td}|$
- There are still hadronic uncertainties. We calculate at the quark level and we need the meson. Lattice QCD is very useful here
- My treatment was very simplistic, there are more effects
- Each meson has its own set of approximations

Meson mixing

In general we have also width different between the two eigenstates. They are due to common final states.

$$x \equiv \frac{\Delta m}{\Gamma} \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

K	$x \sim 1$	$y \sim 1$
D	$x \sim 10^{-2}$	$y \sim 10^{-2}$
B_d	$x \sim 1$	$y \sim 10^{-2}$
B_s	$x \sim 10$	$y \sim 10^{-1}$

Mixing measurements

How this is done?

- Need the flavor of the initial state. Usually the mesons are pair produced
  - Same side tagging ($D^* \to D\pi$)
  - Other side tagging (semileptonic B decays)
- The final flavor
  - Use time dependent (easier for highly boosted mesons)
  - Use time integrated signals
  - The final state may not be a flavor eigenstate, but we still can have oscillations as long as it is not a mass eigenstate

CPV

What is CP

- A symmetry between a particle and its anti-particle
- CP is violated if we have

$$\Gamma(A \to B) \neq \Gamma(\bar{A} \to \bar{B})$$

- It is a very small effect in Nature, and thus sensitive to NP
- In the SM it is closely related to flavor
- We do not discuss the strong CP problem that is not directly related to flavor
- We also do not discuss the need for CP for baryogenesis

How to find CPV

It is not easy to detect CPV

- Always need interference of two (or more) amplitudes
- CPT implies that the total widths of a particles and it anti-particles are the same, so we need at least two modes with CPV
- To see CPV we need 2 amplitudes with different weak and strong phases

All these phases

- Weak phase (CP-odd phase)
  - Phase in $\mathcal{L}$
  - In the SM they are only in the weak part so they are called weak phases

$$CP(Ae^{i\phi}) = Ae^{-i\phi}$$

Strong phase

- Strong phase (CP-even phase). Do not change under CP

$$CP(Ae^{i\delta}) = Ae^{i\delta}$$

  - Due to time evolution

$$\psi(t) = e^{iHt}\psi(0)$$

  - They are also due to intermediate real states, and have to do with "rescattering" of hadrons
  - Such strong phases are very hard to calculate

Why we need the two phases?

Intuitive argument

- If we have only one $|A|^2 = |\bar{A}|^2$
- Two but with a different of only weak phase

$$\left|A + be^{i\phi}\right|^2 = \left|A + be^{-i\phi}\right|^2$$

- When both are not zero it is not the same (do it for HW!)

CPV remarks

- The basic idea is to find processes where we can measure CPV
- In some cases they are clean so we get sensitivity to the phases of the UT (or of the CKM matrix)
- We can be sensitive to the CP phase without measuring CP violation
- Triple products and EDMs are also probes of CPV. I will not talk about that
- So far CPV was only found in meson decays, K_L, B_d and $B^\pm$, and we will concentrate on that

The three classes of CPV

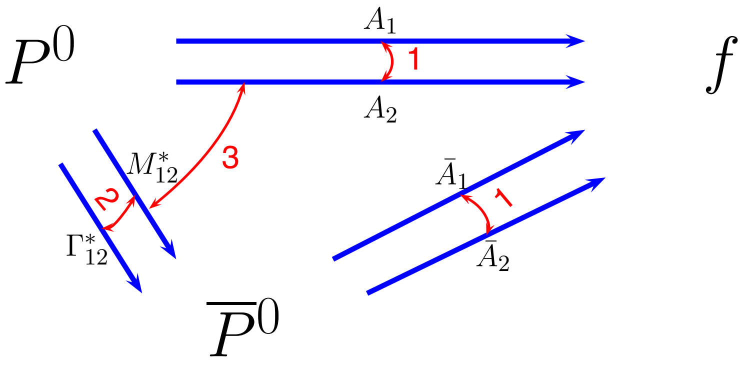
We need to find processes where we have two interfering amplitudes

- Two decay amplitudes
- Two oscillation amplitudes
- One decay and one oscillation amplitudes

Where the phases are coming from?

- Weak phases from the decay or mixing amplitudes (SM or NP)
- Strong phase is the time evolution (mixing) or the rescattering (decay)

The 3 classes



- 1: Decay 2: Mixing 3: Mixing and decay

Type 1: CPV in decay

Two decay amplitudes

$$|A(B \to f)| \neq |A(\bar{B} \to \bar{f})|$$

- The way to measure it is via

$$a_{CP} \equiv \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)} = \frac{|\bar{A}/A|^2 - 1}{|\bar{A}/A|^2 + 1}$$

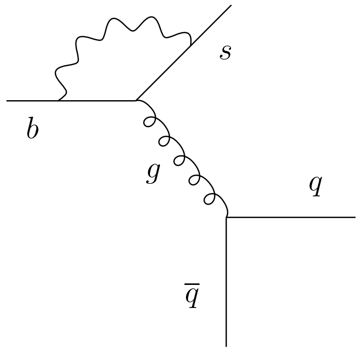
- If we write $A = A\,(1 + r\exp[i(\phi + \delta)])$

$$a_{CP} = r \sin\phi \sin\delta$$

- We like r, δ and ϕ to be large
- Work for decays of both charged and neutral hadrons

CPV in decay, example: $B \to K\pi$

(P) + (P_{EW})



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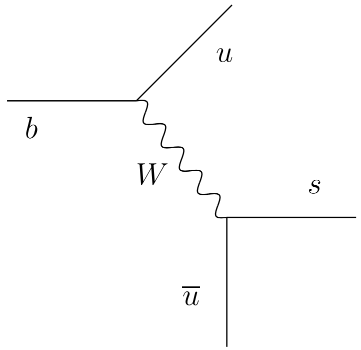


P is a loop amplitude, but due to CKM factors $P/T \sim 3$. We also have a strong phase difference

One more example: $B \to DK$

- A bit more "sophisticated" example of CPV in decay
- Theoretically by far the cleanest measurement of any CKM parameter

Mixing formalism with CPV

When there is CPV the mixing formalism is more complicated. Diagonalizing the Hamiltonian we get

$$B_{H,L} = p|B\rangle \pm q|\bar{B}\rangle$$

- In general B_H and B_L are not orthogonal
- This is because they are "resonances" not asymptotic states. Open system
- The condition for the non orthogonality is CPV

2: CPV in mixing

The second kind of CPV is when it is pure in the mixing

$$|q| \neq |p| \qquad (B_{H,L} = p|B\rangle \pm q|\bar{B}\rangle)$$

We measure it by semileptonic asymmetries

- It was measured in

$$\frac{\Gamma(K_L \to \pi\ell^+\nu) - \Gamma(K_L \to \pi\ell^-\bar{\nu})}{\Gamma(K_L \to \pi\ell^+\nu) + \Gamma(K_L \to \pi\ell^-\bar{\nu})} = (3.32 \pm 0.06) \times 10^{-3}$$

- This is so far the only way we can define the electron microscopically!

3: CPV in interference mixing & decay

Interference between decay and mixing amplitudes

$$A(B \to f_{CP}) \qquad A(B \to \bar{B} \to f_{CP})$$

- Best with one decay amplitude
- Very useful when f is a CP eigenstate
- In that case $|A(B \to f_{CP})| = |A(\bar{B} \to f_{CP})|$

Some definitions

$$\lambda \equiv \frac{q}{p}\frac{\bar{A}}{A}$$

In the case of a CP final state

- $\lambda \neq \pm 1 \;\Rightarrow\;$ CPV
  - $|\lambda| \neq 1$ because $|A| \neq |\bar{A}|$. CPV in decay
  - $|\lambda| \neq 1$ because $|q| \neq |p|$. CPV in mixing
  - The cleanest case $|\lambda| \approx 1$ and $Im(\lambda) \neq 0$. Interference between mixing and decay
- We can have several classes at the same time
- In the clean cases we have one dominant source

Formalism

B at $t = 0$ compared to a $\bar{B}$ and let them evolve

$$a_{CP}(t) \equiv \frac{\Gamma(B(t) \to f) - \Gamma(\bar{B}(t) \to f)}{\Gamma(B(t) \to f) + \Gamma(\bar{B}(t) \to f)}$$

Consider the case where $|\lambda| = 1$

$$A_{CP}(t) = -Im\lambda \sin \Delta m t$$

- We know Δm so we can measure $Im\lambda$
- $Im\lambda$ is the phase between mixing and decay amplitudes
- When we have only one dominant decay amplitude all the hadronic physics cancel (YES!!!)
- In some cases this phase is $O(1)$

Example: $B \to \psi K_S$

Reminder ψ is a $\bar{c}c$, K_S is s and d

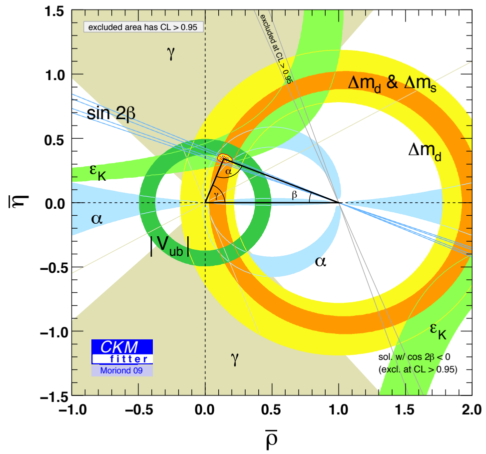
- One decay amplitude, tree level $A \propto V_{cb}V_{cs}^*$. In the standard parametrization it is real
- Very important: $|A| = |\bar{A}|$ to a very good approximation.
- In the standard parametrization $q/p = \exp(2i\beta)$ to a very good approximation
- We then get

$$Im\lambda = Im\left[\frac{q}{p}\frac{\bar{A}}{A}\right] = \sin 2\beta$$

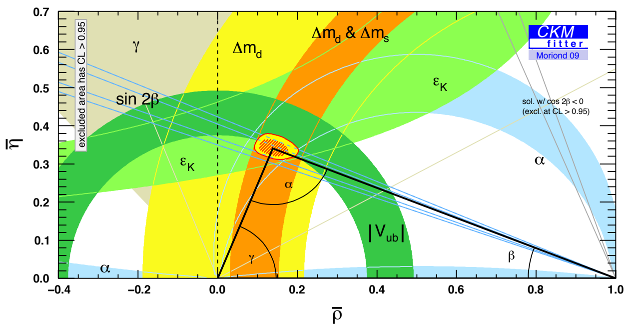
- For HW do some other decays: D^+D^-, $\pi^+\pi^-$, ϕK_S and $B_s \to \psi\phi$ (Ignore the subtleties)

Instead of summary

All together now



Zoom in



The NP flavor problem

The flavor problems

- "Problem" is not a problem. It is a hint for something more fundamental
- The SM flavor problems
  - Why there are 3 generations?
  - Why the mass ratios and mixing angles are small and hierarchical?
- The NP flavor problem is different

The SM is not perfect...

- We know the SM does not describe gravity
- At what scale it breaks down?

We parametrize the NP scale as the denominator of an effective higher dimension operator. The weak scale is roughly

$$\mathcal{L}_{\rm eff} = \frac{\mu e \nu \bar\nu}{\Lambda_W^2} \Rightarrow \Lambda_W \sim 100 \text{ GeV}$$

- The effective scale is roughly the masses of the new fields times unknown couplings
- Flavor bounds give $\Lambda \gtrsim 10^4$ TeV

Flavor and the hierarchy problem

There is tension:

- The hierarchy problem $\Rightarrow \Lambda \sim 1$ TeV
- Flavor bounds $\Rightarrow \Lambda \gtrsim 10^4$ TeV

Any TeV scale NP has to deal with the flavor bounds

$\Downarrow$

Such NP cannot have a generic flavor structure

Flavor is mainly an input to model building, not an output