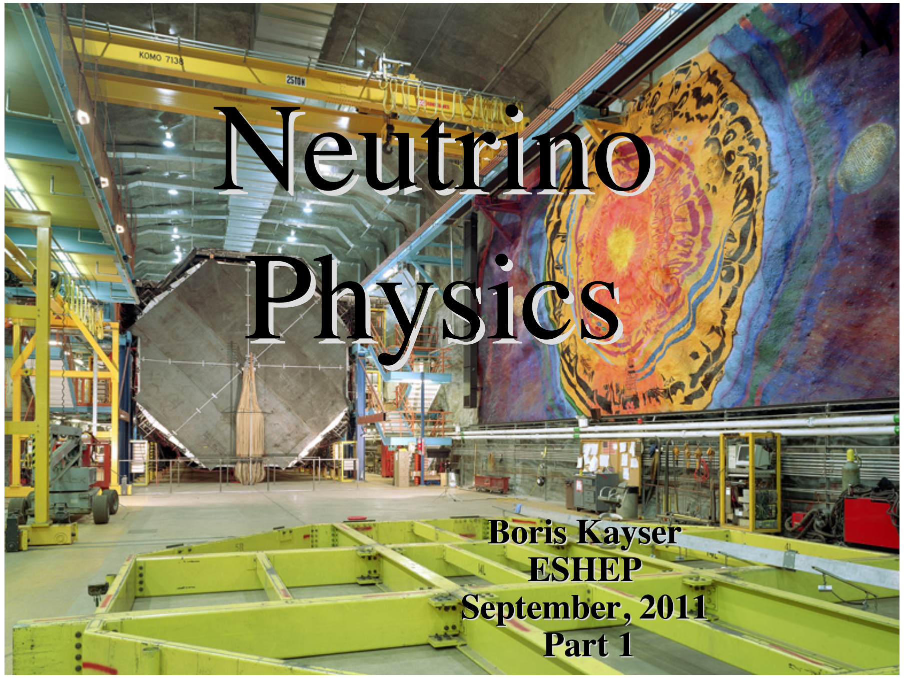

Neutrino Physics

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ESHEP
September, 2011
Part 1

What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

$$p + p \rightarrow d + e^{+} + \nu$$

Spin: $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……

The Neutrinos

Neutrinos and photons are by far the most abundant elementary particles in the universe. There are 340 neutrinos/cc.

The neutrinos are spin – 1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that their interactions with other matter have very low strength.
Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!

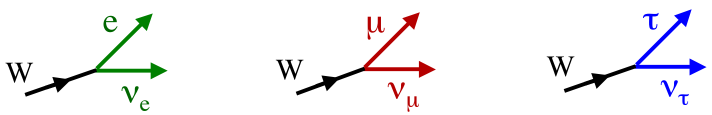
Neutrino masses suggest, via the See-Saw picture, new physics far above the LHC energy scale.

The discovery of neutrino masses and leptonic mixing has come from the observation of ***neutrino flavor change (neutrino oscillation)***.

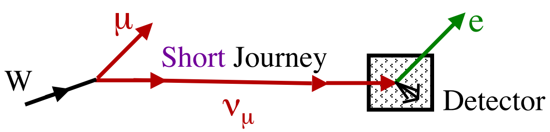
The Physics of Neutrino Oscillation — Preliminaries

The Neutrino Flavors

We ***define*** the three known flavors of neutrinos, ν_e, ν_μ, ν_τ, by W boson decays:



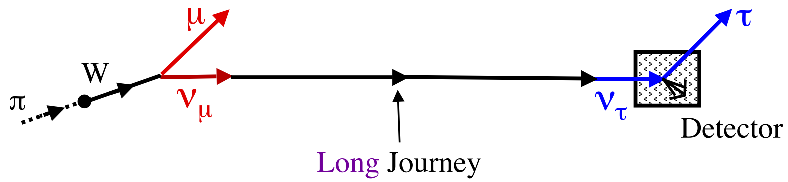
As far as we know, neither



nor any other change of flavor in the $\nu \to \ell$ ***interaction*** ever occurs. With $\alpha = e, \mu, \tau$, ν_α makes only ℓ_α ($\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$).

Neutrino Flavor Change

If neutrinos have masses, and leptons mix, we can have —



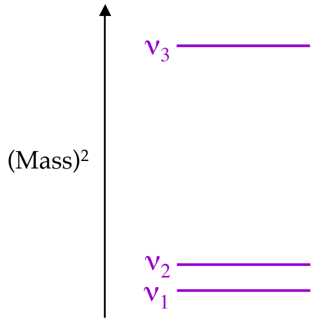
Give ν time to change character

$$\nu_\mu \longrightarrow \nu_\tau$$

The last decade has brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum of neutrino mass eigenstates ν_i:



Mass $(\nu_i) \equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

($W \rightarrow e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau$)

must be superpositions of the mass eigenstates:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\, |\nu_i\rangle \quad .$$

Neutrino of flavor $\alpha = e, \mu,$ or τ

Neutrino of definite mass m_i

PMNS Leptonic Mixing Matrix

There must be ***at least 3*** mass eigenstates ν_i, because there are 3 orthogonal neutrinos of definite flavor ν_α.

This *mixing* is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

Semi-weak coupling

Left-handed

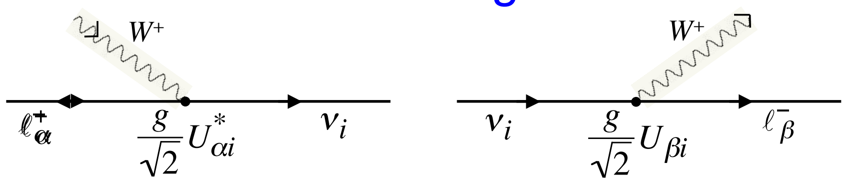
$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_{L\alpha}\gamma^\lambda \nu_{L\alpha} W_\lambda^- + \bar{\nu}_{L\alpha}\gamma^\lambda \ell_{L\alpha} W_\lambda^+ \right)$$

$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha}\gamma^\lambda U_{\alpha i} \nu_{Li} W_\lambda^- + \bar{\nu}_{Li}\gamma^\lambda U_{\alpha i}^* \ell_{L\alpha} W_\lambda^+ \right)$$

Taking mixing into account

If neutrino *masses* are described by an extension of the SM, and there are no new leptons, U is *unitary*.

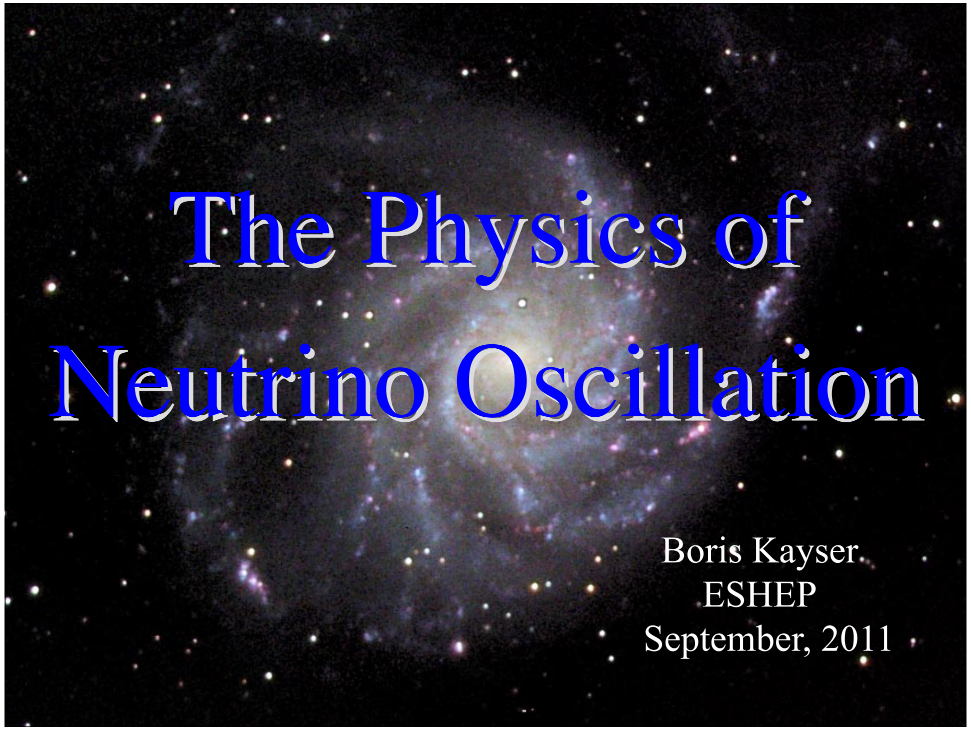
The Meaning of *U*



$$U = \begin{array}{c c} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \end{array}$$

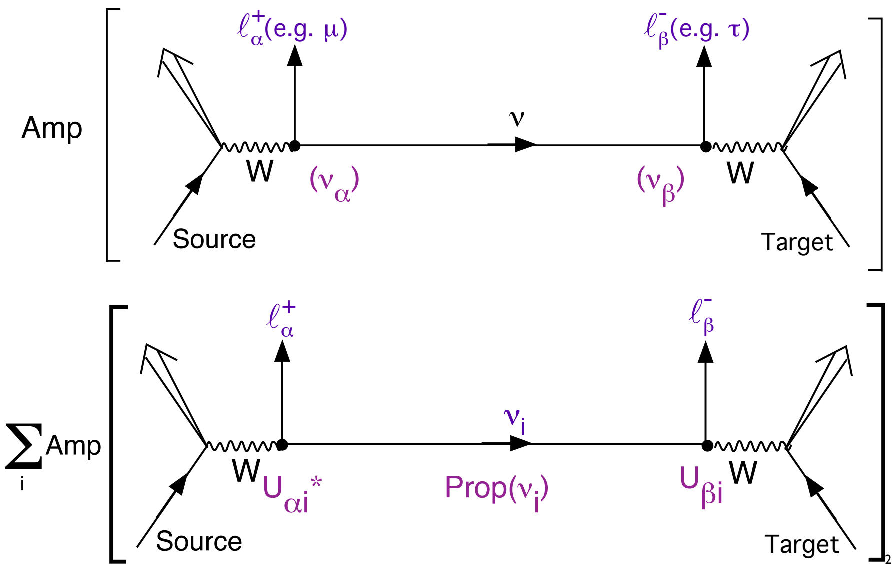
The e row of U: The linear combination of neutrino mass eigenstates that couples to e.

The ν_1 column of U: The linear combination of charged-lepton mass eigenstates that couples to ν_1 .



Neutrino Flavor Change (Oscillation) in Vacuum

(Approach of B.K. & Stodolsky)



Amp [$\nu_\alpha \rightarrow \nu_\beta$] = $\Sigma U_{\alpha i}^*$ Prop(ν_i)$U_{\beta i}$

What is Propagator (ν_i) ≡ Prop(ν_i)?

In the ν_i rest frame, where the proper time is τ_i,

$$i\frac{\partial}{\partial \tau_i}|\nu_i(\tau_i)>= m_i|\nu_i(\tau_i)> \quad .$$

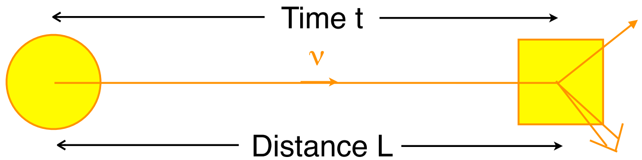
Thus,

$$|\nu_i(\tau_i)>= e^{-im_i\tau_i}|\nu_i(0)> \quad .$$

Then, the amplitude for propagation for time τ_i is —

$$\text{Prop}(\nu_i) \equiv <\nu_i(0)|\nu_i(\tau_i)>= e^{-im_i\tau_i} \quad .$$

In the laboratory frame —



The experimenter chooses L and t.

They are common to all components of the beam.

For each ν_i, by Lorentz invariance,

$$(E_i, p_i) \times (t, L) = m_i\tau_i = E_i t - p_i L \; .$$

Neutrino sources are ~ constant in time.

Averaged over time, the

$e^{-iE_1 t} - e^{-iE_2 t}$ interference

is —

$$< e^{-i(E_1-E_2)t} >_t \; = \; 0$$

$$unless \; E_2 = E_1 \; .$$

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ,

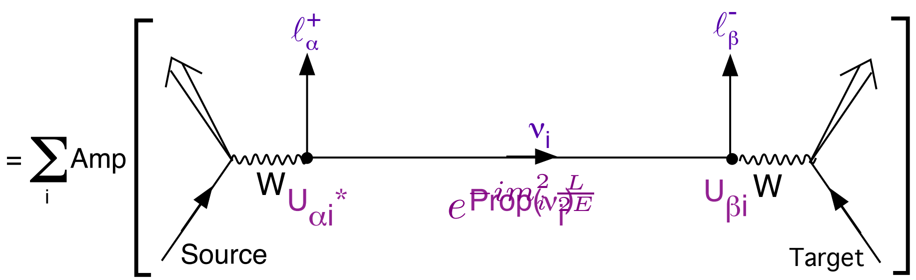
$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \quad .$$

Then the phase in the ν_i propagator $exp[-im_i\tau_i]$ is

$$m_i\tau_i = E_i t - p_i L \cong Et - (E - m_i^2/2E)L$$

$$= \underbrace{E(t-L)}_{\text{Irrelevant overall phase}} + m_i^2 L/2E \quad .$$

Amp [$\nu_\alpha \to \nu_\beta$]



$$= \sum_i U_{\alpha i}^* e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E})$$

$$+2\sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E})$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$

For Antineutrinos –

We assume the world is CPT invariant.

Our formalism assumes this.

$$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \stackrel{CPT}{=} P(\nu_\beta \to \nu_\alpha) = P(\nu_\alpha \to \nu_\beta; U \to U^*)$$

Thus,

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$\underset{(-)}{+}2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

A complex U would lead to the CP violation
$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \neq P(\nu_\alpha \to \nu_\beta)$.

Must we assume all mass eigenstates have the same *E*?

No, we can take entanglement into account, and use energy conservation.

The oscillation probabilities are still the same.

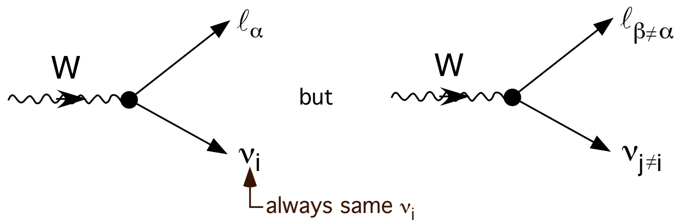
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$$

Flavor *change* ⇒ ν Mass

2. If there is no mixing,



$\Rightarrow U_{\alpha i} U_{\beta\neq\alpha,i} = 0$, so that $P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$.

Flavor *change* ⇒ Mixing

3. One can detect ($\nu_\alpha \to \nu_\beta$) in two ways:

See $\nu_{\beta \neq \alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

4. Including $\hbar$ and c

$$\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2(\mathrm{eV}^2) \frac{L(\mathrm{km})}{E(\mathrm{GeV})}$$

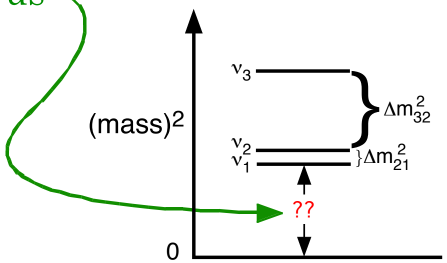
$\sin^2[1.27\Delta m^2(\mathrm{eV})^2 \frac{L(\mathrm{km})}{E(\mathrm{GeV})}]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

$$\Delta m^2(\mathrm{eV}^2) \gtrsim \frac{E(\mathrm{GeV})}{L(\mathrm{km})} \quad .$$

5. Flavor change in vacuum oscillates with L/E. Hence the name "neutrino oscillation". {The L/E is from the proper time τ.}

6. P ($\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta$) depends only on squared-mass splittings. Oscillation experiments cannot tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

$$\sum_{\text{All } \beta} P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = 1$$

But some of the flavors β≠α could be sterile.

Then some of the *active* flux disappears:

$$\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}}$$

Important Special Cases

Three Flavors

For $\beta \neq \alpha$,

$$e^{-im_1^2 \frac{L}{2E}} \mathrm{Amp}^*(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i} U^*_{\beta i} e^{im_i^2 \frac{L}{2E}} e^{-im_1^2 \frac{L}{2E}}$$

$$= U_{\alpha 3} U^*_{\beta 3} e^{2i\Delta_{31}} + U_{\alpha 2} U^*_{\beta 2} e^{2i\Delta_{21}} \underbrace{-(U_{\alpha 3} U^*_{\beta 3} + U_{\alpha 2} U^*_{\beta 2})}_{\text{Unitarity}}$$

$$= 2i[U_{\alpha 3} U^*_{\beta 3} e^{i\Delta_{31}} \sin \Delta_{31} + U_{\alpha 2} U^*_{\beta 2} e^{i\Delta_{21}} \sin \Delta_{21}]$$

$$\text{where } \Delta_{ij} \equiv \Delta m^2_{ij} \frac{L}{4E} \equiv (m_i^2 - m_j^2) \frac{L}{4E} \quad .$$

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = \left| e^{-im_1^2 \frac{L}{2E}} \, \mathrm{Amp}^*(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \right|^2$$

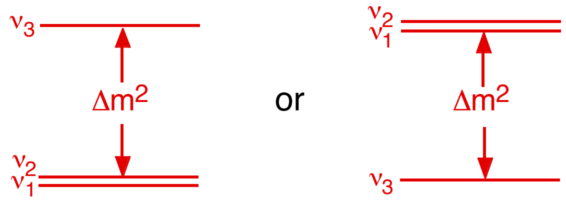
$$= 4[|U_{\alpha 3}U_{\beta 3}|^2 \sin^2 \Delta_{31} + |U_{\alpha 2}U_{\beta 2}|^2 \sin^2 \Delta_{21}$$

$$+2|U_{\alpha 3}U_{\beta 3}U_{\alpha 2}U_{\beta 2}| \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \overset{+}{\underset{(-)}{}} \delta_{32})] \quad .$$

Here $\delta_{32} \equiv \arg(U_{\alpha 3}U^*_{\beta 3}U^*_{\alpha 2}U_{\beta 2})$, a CP – violating phase.

Two waves of different frequencies,
and their ~~CP~~ interference.

When the Spectrum Is—



$\Big\{$ Invisible if $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$.

For $\beta \neq \alpha$,

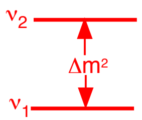
$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \cong 4|U_{\alpha 3}U_{\beta 3}|^2 \sin^2(\Delta m^2 \frac{L}{4E}) \quad .$

For no flavor change,

$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) \cong 1 - 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) \sin^2(\Delta m^2 \frac{L}{4E}) \quad .$

Experiments with $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$ can determine the flavor content of ν_3.

When There are Only Two Flavors and Two Mass Eigenstates



Majorana ~~CP~~ phase

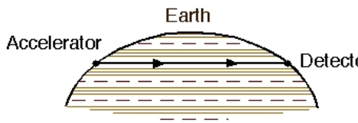
$$U = \begin{bmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} e^{i\xi} & 0 \\ 0 & 1 \end{bmatrix}$$

Mixing angle

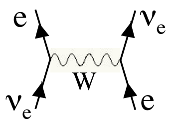
For $\beta \neq \alpha$, $\quad P(\overset{(-)}{\nu}_\alpha \leftrightarrow \overset{(-)}{\nu}_\beta) = \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E}) \quad .$

For no flavor change, $P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.

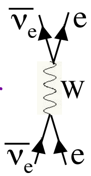
Neutrino Flavor Change In Matter



involves



or



Coherent forward scattering via this
W-exchange interaction leads to
an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2}G_F N_e, & \nu_e \\ -\sqrt{2}G_F N_e, & \overline{\nu_e} \end{cases}$$

Fermi constant — Electron density

This raises the effective mass of ν_e, and lowers that of $\overline{\nu_e}$.

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —

$$\overbrace{[\sqrt{2}G_F N_e]}^{\text{Interaction energy}} \; / \; \overbrace{[\Delta m^2/2E]}^{\text{Vacuum energy}} \equiv x \; .$$

The matter effect —

— Grows with neutrino energy E

— Is sensitive to Sign(Δm^2)

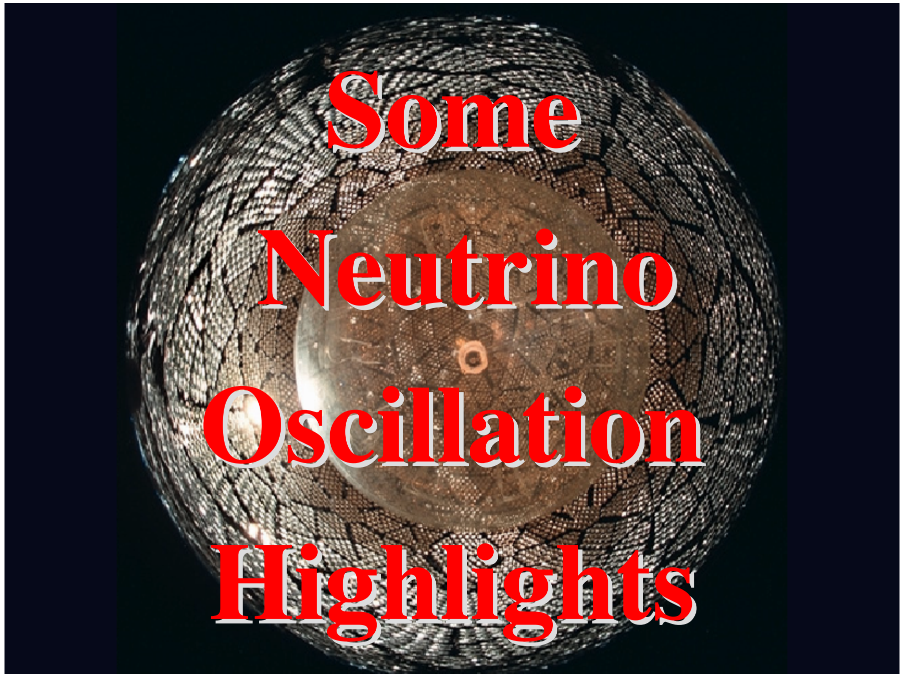
— Reverses when ν is replaced by $\bar{\nu}$

This last is a "fake CP violation", but the matter effect is negligible when $x << 1$.

Evidence For Flavor Change

Neutrinos	Evidence of Flavor Change
Solar	Compelling
Reactor (L ~ 180 km)	Compelling
Atmospheric	Compelling
Accelerator (L = 250 and 735 km)	Compelling
Stopped μ^+ Decay (LSND L ≈ 30 m)	Does MiniBooNE see this too??

Very recent evidence to be discussed soon.



Solar Neutrinos

History –

Nuclear reactions in the core of the sun produce ν_e. Only ν_e.

Theorists, especially John Bahcall, calculated the produced ν_e flux vs. energy E.

Ray Davis' Homestake experiment measured the higher-E part of the ν_e flux ϕ_{ν_e} that arrives at earth.

The Homestake experiment could detect only ν_e. It found:

$$\frac{\phi_{\nu_e}\,(\text{Homestake})}{\phi_{\nu_e}\,(\text{Theory})} = 0.34 \pm 0.06$$

The Possibilities:

The theory was wrong.

The experiment was wrong.

Both were wrong.

Neither was wrong. Two thirds of the ν_e flux morphs into a flavor or flavors that the Homestake experiment could not see.

The Resolution —

Sudbury Neutrino Observatory (SNO) measures, for the high-energy part of the solar neutrino flux:

$$\nu_{sol}\, d \rightarrow e\, p\, p \;\Rightarrow \phi_{\nu_e}$$

$$\nu_{sol}\, d \rightarrow \nu\, n\, p \;\Rightarrow \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$$

From the two reactions,

$$\frac{\phi_{\nu_e}}{\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}} = 0.301 \pm 0.033$$

Clearly, $\phi_{\nu_\mu} + \phi_{\nu_\tau} \neq 0$. Neutrinos change flavor.

$$P(\nu_e \rightarrow \nu_e) = 0.3$$

Change of flavor does not change the total number of neutrinos.

The total flux, $\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$, should agree with Bahcall's prediction.

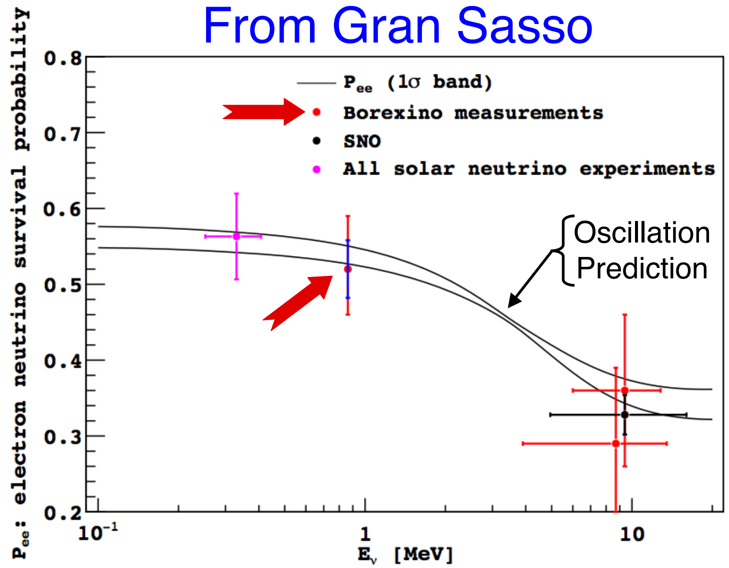
SNO: $\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} = (5.54 \pm 0.32 \pm 0.35) \times 10^6/\mathrm{cm}^2\mathrm{sec}$

Theory*: $\phi_{total} = (5.69 \pm 0.91) \times 10^6/\mathrm{cm}^2\mathrm{sec}$

*Bahcall, Basu, Serenelli

John Bahcall and Ray Davis both stuck to their results for several decades, and both were right all along.

Recent Evidence From Gran Sasso



Reactor (Anti)Neutrinos In KamLAND

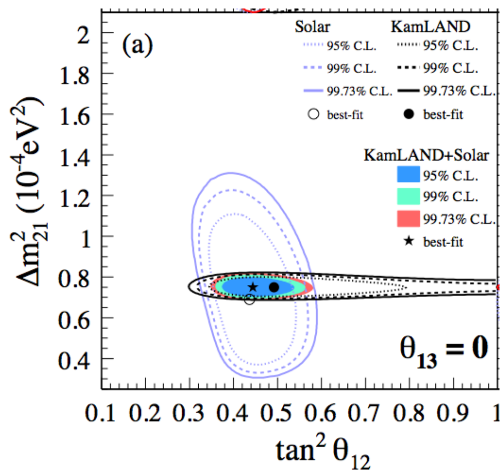
The KamLAND detector studies $\overline{\nu}_e$ produced by Japanese nuclear power reactors ~ 180 km away.

Our understanding of solar neutrino behavior implies that a considerable fraction of these reactor $\overline{\nu}_e$ should disappear before reaching KamLAND .

KamLAND does see a disappearance of about 1/3 of the $\overline{\nu}_e$.

The solar and KamLAND data are both described by the same, single set of neutrino parameters:

$$\Delta m^2_{sol} = 7.50 \times 10^{-5} \mathrm{eV}^2 \quad , \quad \tan^2\theta_{sol} = 0.44$$



Analysis by KamLAND

KamLAND Evidence for Oscillatory Behavior

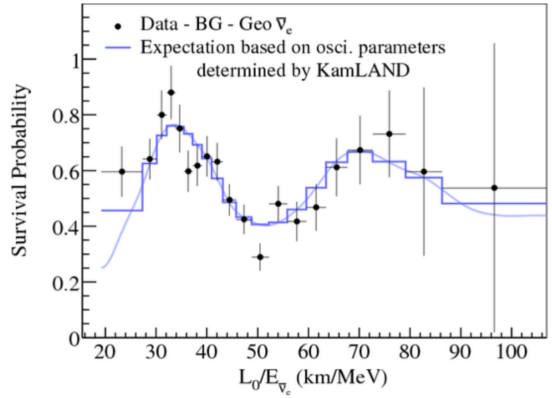
For KamLAND, $x_{Matter} < 10^{-2}$. Matter effects are negligible.

The $\bar{\nu}_e$ survival probability, $P(\bar{\nu}_e \to \bar{\nu}_e)$, should oscillate as a function of L/E following the vacuum oscillation formula.

In the two-neutrino approximation, we expect —

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2\left[1.27\Delta m^2\left(eV^2\right)\frac{L(km)}{E(GeV)}\right] .$$

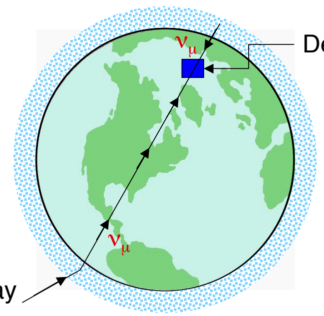
Survival probability $P(\bar{\nu}_e \to \bar{\nu}_e)$ of reactor $\bar{\nu}_e$



L_0 = 180 km is a flux-weighted average travel distance.

$P(\bar{\nu}_e \to \bar{\nu}_e)$ actually oscillates!

Atmospheric Neutrinos

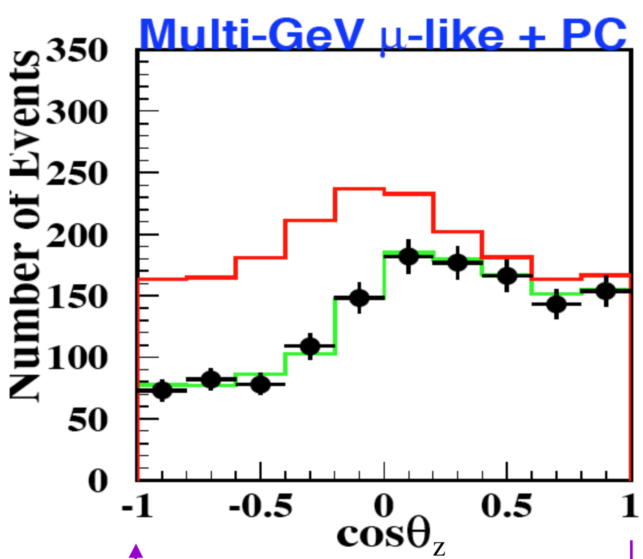


Isotropy of the $\gtrsim$ 2 GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Rightarrow \frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 1 .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV

$$\frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} \cong 1/2 .$$



(Super-K)

Voluminous atmospheric neutrino data are well described by —

$$\nu_\mu \longrightarrow \nu_\tau$$

with —

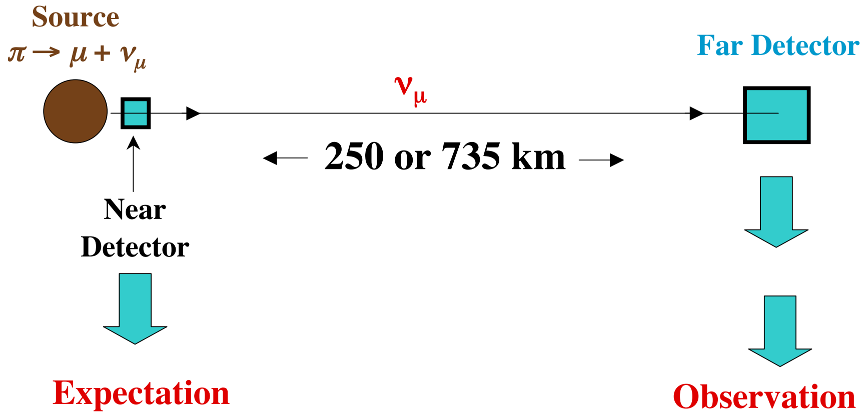
$$\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ \mathrm{eV}^2$$

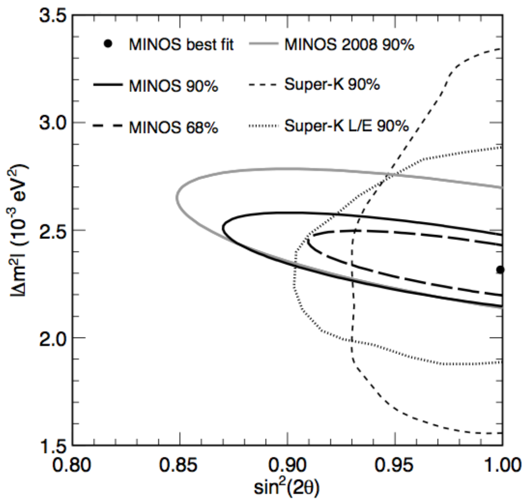
and —

$$\sin^2 2\theta_{atm} \cong 1$$

Accelerator Neutrinos

Two experiments: K2K and MINOS





Region of the "atmospheric" oscillation parameters allowed by MINOS and Super-Kamiokande

(From MINOS paper 1103.0340)

A single pair of parameters, with ~ maximal mixing, fits both the atmospheric and accelerator neutrino data.