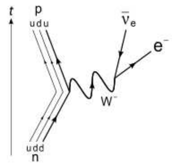
4. The electroweak sector of the Standard Model.

4.1. Gauge group and matter content.

Standard model = "unified" description of weak and electromagnetic interactions. From the Fermi theory of weak interactions



with $G_F/\sqrt{2} = g^2/8M_w^2$, we know that we need at least a charged gauge boson $W_m^{\pm}$ and the photon A_m.

Experimentally there also exists neutral currents (neutral massive gauge boson) and coloured strong interactions $\rightarrow$ gauge group

Gauge bosons : $G_m^A \qquad A_m^a \qquad B_m$

$$G \;=\; SU(3)_c \;\times SU(2)_L \;\times U(1)_Y \;.$$

- the Higgs mechanism generates the breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$.
- Only left-handed quarks/leptons interact with $SU(2)_L$ gauge fields. The SM lagrangian has the symbolic form

$$\mathcal{L}_{SM} \;=\; \mathcal{L}_{\text{kin}} \;-\; V(\Phi) \;+\; \mathcal{L}_{\text{Yuk.}}$$

where

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}F_{mn}^2 - \frac{1}{4}(F_{mn}^a)^2 + |D_m\Phi|^2$$
$$+\bar{\Psi}_L i\gamma^m D_m\Psi_L + \bar{\Psi}_R i\gamma^m D_m \Psi_R \;,$$

where

$$D_m\Psi_L = (\partial_m - ig\frac{\tau_a}{2}A_m - ig'\frac{Y_L}{2}B_m)\Psi_L$$
$$D_m\Psi_R = (\partial_m - ig'\frac{Y_R}{2}B_m)\Psi_R \;,$$
$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \;,$$

and $\mathcal{L}_{\text{Yuk.}}$ will be discussed later on. With our conventions

$$Q \;=\; T_3 + \frac{Y}{2} \;. \qquad (52)$$

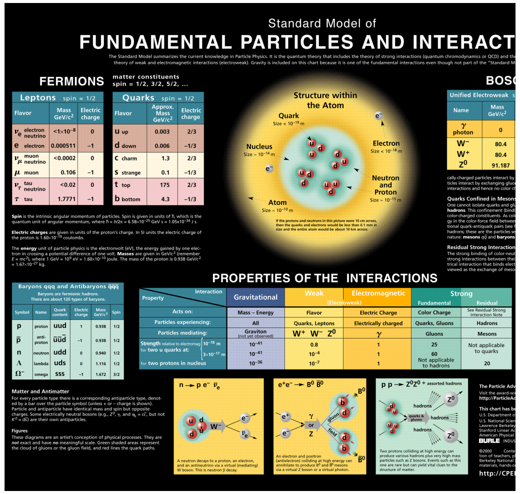
Matter content of the Standard Model:

Leptons : $l_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \;:\; (\mathbf{1},\mathbf{2})_{Y=-1} \quad , \quad e_{iR} \;:\; (\mathbf{1},\mathbf{1})_{Y=-2}$

Quarks : $q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \;:\; (\mathbf{3},\mathbf{2})_{Y=1/3}$

$u_{iR} \;:\; (\mathbf{3},\mathbf{1})_{Y=4/3} \qquad d_{iR} \;:\; (\mathbf{3},\mathbf{1})_{Y=-2/3}$

Higgs field : $\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} : \quad (\mathbf{1},\mathbf{2})_{Y=1}$



4.2. Weak mixing angles and gauge boson masses.

With an $SO(4)$ rotation, the Higgs vev can be written as

$$\Phi \;=\; \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \text{, where } \; v^2 = \frac{\mu^2}{\lambda} \simeq (246 \text{ GeV})^2 \quad \text{(exp.)} \tag{53}$$

Gauge boson masses arise from

$$\begin{aligned} |D_m\Phi|^2 &\to \frac{g^2v^2}{8}|A_m^{(1)} - iA_m^{(2)}|^2 + \frac{v^2}{8}|gA_m^{(3)} - g'B_m|^2 \\ &= \frac{g^2v^2}{4}W^{+,m}W_m^- + \frac{(g^2+g'^2)v^2}{8}Z^mZ_m \end{aligned}$$

where the definitions and masses of gauge bosons are

$$\begin{aligned} W_m^{\pm} &= \frac{1}{\sqrt{2}}(A_m^{(1)} \mp iA_m^{(2)}) \quad , \quad M_w = \frac{gv}{2} \\ Z_m &= \frac{gA_m^{(3)} - g'B_m}{\sqrt{g^2+g'^2}} \quad , \quad M_z = \frac{v}{2}\sqrt{g^2+g'^2} \\ A_m &= \frac{g'A_m^{(3)} + gB_m}{\sqrt{g^2+g'^2}} \quad , \quad M_A = 0 \end{aligned} \tag{54}$$

We introduce the electroweak angle

$$\cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}} = \frac{M_w}{M_z} \;,\; \tan\theta_w = \frac{g'}{g} \tag{55}$$

that rotates from the weak basis to the mass basis

$$\begin{pmatrix} Z_m \\ A_m \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} A_m^{(3)} \\ B_m \end{pmatrix} \tag{56}$$

Notice that

$$\rho \equiv \frac{M_w^2}{M_z^2\cos^2\theta_w} = 1 \text{ in the SM} \tag{57}$$

The electric charge is $e = g\sin\theta_w$.

4.3. Neutral and charged currents

Ex: With the definitions above, show that

$$\begin{aligned} D_m &= \partial_m - igA_m^a\frac{\tau_a}{2} - ig'\frac{Y}{2}B_m = \partial_m - ieQA_m \\ &- \frac{ig}{2\sqrt{2}}(W_m^+\tau_+ + W_m^-\tau_-) - \frac{ig}{\cos\theta_w}Z_m(T_3 - \sin^2\theta_w Q) \end{aligned}$$

We define the currents by

$$\mathcal{L} = \bar{\Psi}_i i\gamma^m\partial_m\Psi_i + g(W_m^+J_W^{m,+} + W_m^-J_W^{m,-} + Z_mJ_Z^m) + eA_mJ_{em}^m \tag{58}$$

Ex : Using the quantum numbers of the quarks/leptons, show that

$$J_W^{m,+} = \frac{1}{\sqrt{2}}(\bar\nu_L^i \gamma^m e_L^i + \bar u_L^i \gamma^m d_L^i) \ ,$$
$$J_W^{m,-} = \frac{1}{\sqrt{2}}(\bar e_L^i \gamma^m \nu_L^i + \bar d_L^i \gamma^m u_L^i) \ ,$$
$$J_{em}^m = -\bar e^i \gamma^m e^i + \frac{2}{3}\bar u^i \gamma^m u^i - \frac{1}{3}\bar d^i \gamma^m d^i \ ,$$
$$J_Z^m = \frac{1}{\cos\theta_w}\Big[\frac{1}{2}\bar\nu_L^i \gamma^m \nu_L^i + (-\frac{1}{2} + \sin^2\theta_w)\bar e_L^i \gamma^m e_L^i + \sin^2\theta_w \bar e_R^i \gamma^m e_R^i$$
$$+(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w)\bar u_L^i \gamma^m u_L^i - \frac{2}{3}\sin^2\theta_w \bar u_R^i \gamma^m u_R^i$$
$$(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_w)\bar d_L^i \gamma^m d_L^i + \frac{1}{3}\sin^2\theta_w \bar d_R^i \gamma^m d_R^i\Big] \ . \tag{59}$$

4.4. Fermion masses and the CKM matrix

Yukawas generate quarks/lepton masses after EWSB :

$$-\mathcal{L}_{\text{mass}} = m_{ij}^u \bar u_L^i u_R^j + m_{ij}^d \bar d_L^i d_R^j + m_{ij}^l \bar e_L^i e_R^j + \text{c.c.} \ , \tag{60}$$

where $m_{ij}^{u,d,l} = h_{ij}^{u,d,l} v/\sqrt{2}$. We use for compactness a matrix notation

$$-\mathcal{L}_{\text{mass}} = \bar u_L m^u u_R + \bar d_L m^d d_R + \bar e_L m^l e_R + \text{c.c.} \ . \tag{61}$$

Obs: No neutrino masses here, see lectures of B. Kayser.

We can define the mass eigenstate basis (as compared to the weak eigenstate basis) with the help of the 3×3 unitary transformations

$$u_{L,R} = V_{L,R}^u u'_{L,R} \ , \ d_{L,R} = V_{L,R}^d d'_{L,R} \ , \ e_{L,R} = V_{L,R}^e e'_{L,R} \ ,$$

such that

$$(V_L^u)^\dagger m^u V_R^u \ = \ diag\ (m_u, m_c, m_t) \ , \ \text{etc}$$

In the mass basis, the charged and e.m. currents remain the same, whereas the hadronic charged current becomes

$$(J_W^{m,+})_{\text{quarks}} \ \to \ \frac{1}{\sqrt{2}}\bar u'_L \gamma^m V_{CKM} d'_L \equiv \frac{1}{\sqrt{2}}\bar u'_L \gamma^m \tilde d_L$$

where $V_{CKM} = (V_L^u)^\dagger V_L^d$ is the (unitary) CKM matrix, 1973.

We also defined

$$\tilde d_L = V_{CKM} d'_L \ \leftrightarrow \ \begin{pmatrix} \tilde d_L \\ \tilde s_L \\ \tilde b_L \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}$$

There are therefore flavor changing transitions in the SM : $s \to uW^-$, etc. Experimental measurements give a hierarchical form of V_{CKM} of the type (Wolfenstein parametrization)

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{62}$$

where $\lambda = \sin\theta_c \simeq .0.22$ is the Cabibbo angle.

Cabibbo wrote first in 1962 the 2×2 version of the CKM matrix

$$\begin{pmatrix} \sin\theta_c & \cos\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \tag{63}$$

• V_{CKM} contain three rotation angles and a CP violating phase.

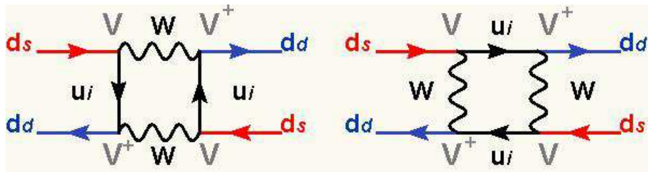
The unitarity of the CKM matrix

$$V_{ik}V^*_{jk} = \delta_{ij} \quad , \quad V^*_{ki}V_{kj} = \delta_{ij}$$

has various important consequences. One of them is the GIM mechanism (Glashow-Iliopoulos-Maiani, 1972).

The GIM mechanism

The FCNC (flavor changing neutral currents) effects were measure to be small. This was puzzling in the 1970's, but it is explained in the SM. Consider for ex. the $K^0 - \bar{K}^0$ mixing, which can arise at the loop-level :



In the limit of equal or vanishing quark masses, the amplitude vanishes due to the unitarity of V_{CKM} :

$$A_{K^0\bar{K}^0} \sim \frac{g^4}{M_W^4}(\sum_i V^*_{id}V_{is})(\sum_i V_{ss}V^*_{jd}) = 0 \tag{64}$$

The main contribution turn out to be proportional to $(m_c^2 - m_u^2)^2/M_W^4$ and is in excellent agreement with the experimental result.

HR : In 1972, only the u, d and s quarks were known. The GIM mechanism is considered to be the first proof of the existence of the charm quark.

Ex: Write down explicitly the diagrams for the $K^0 - \bar{K}^0$ mixing in the two generation case, with u and c quarks in the loop.

4.5. The custodial symmetry.

(Sikivie,Susskind,Voloshin,Zakharov,1980)

The tree-level relation $\rho = M_W^2/(M_Z^2 \cos^2\theta_w) = 1$ is the result of an (approximate) symmetry.

In any theory of electroweak interactions which conserves the electric charge and has an approximate global $SU(2)$ symmetry under which A^a_m transform as a triplet, $\rho = 1$ at tree-level.

Approximate : in the limit of $g' = 0$ and in the absence of the Yukawa couplings.

Proof: The gauge boson mass matrix is then of the form

$$\begin{pmatrix} M^2 & 0 & 0 & 0 \\ 0 & M^2 & 0 & 0 \\ 0 & 0 & M^2 & m_1^2 \\ 0 & 0 & m_1^2 & m_2^2 \end{pmatrix} \qquad (65)$$

No photon charge $\rightarrow$ $M^2 m_2^2 - m_1^4 = 0$. The $W_3 - A$ mass matrix is then of the form : Exercice :

$$\begin{pmatrix} M_W^2 & \pm M_W \sqrt{M_Z^2 - M_W^2} \\ \pm M_W \sqrt{M_Z^2 - M_W^2} & M_Z^2 - M_W^2 \end{pmatrix} \qquad (66)$$

It is then easy to check that $M_W = \cos\theta_w M_Z$.

The Higgs potential $V(\Phi^\dagger \Phi)$ is invariant under an $SO(4)$ symmetry. Indeed,

$$\Phi = \begin{pmatrix} \Phi_1 + i\Phi_2 \\ \Phi 3 + i\Phi_4 \end{pmatrix} \quad , \quad \Phi^\dagger \Phi = \sum_{i=1}^{4} \Phi^2 \qquad \rightarrow$$

$SO(4) = SU(2)_L \times SU(2)_R$ symmetry. The Higgs vev

$$\Phi = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{breaks } SO(4) \rightarrow SO(3) = SU(2)_D$$

Other Higgs representations ? Exercice :

Consider Higgs triplets. Show that the Higgs vev generate the breaking $SO(3) \rightarrow SO(2)$. In this case there is no custodial symmetry and $\rho \neq 1$.

A useful parametrization :

$$\mathcal{H} = \begin{pmatrix} i\tau_2 \Phi^* & \Phi \end{pmatrix} = \begin{pmatrix} \Phi_0^* & \Phi_+ \\ -\Phi_+^* & \Phi_0 \end{pmatrix} , \Phi^\dagger \Phi = Tr \mathcal{H}^\dagger \mathcal{H}$$

$V(\Phi^\dagger \Phi)$ is invariant under $\mathcal{H} \rightarrow U_L \mathcal{H} U_R^\dagger$, with $U_{L,R}$ unitary matrices implementing $SU(2)_L \times SU(2)_R$ transformations. Symmetry breaking

$$\langle \mathcal{H} \rangle = \frac{v}{\sqrt{2}} I_{2\times 2} \quad \text{breaks} \quad SU(2)_L \times SU(2)_R \rightarrow SU(2)_D$$

$U(1)_Y$ and Yukawas break the custodial symmetry. However

$$\mathcal{L}_{\text{Yuk}} = h \begin{pmatrix} \bar{t}_L & \bar{b}_L \end{pmatrix} \mathcal{H} \begin{pmatrix} t_R \\ b_R \end{pmatrix}$$

is invariant under $SU(2)_D$ (if $h_t = h_b$).

A one-loop computation in the SM gives

$$\delta\rho = \frac{3g^2(m_t^2 - m_b^2)}{64\pi^2 M_W^2} - \frac{3g^2}{32\pi^2} \ln \frac{m_H}{M_Z} + \cdots$$

where $\cdots$ are subleading contributions from the SM (or eventual new physics contributions, see lectures B. Dobrescu) are smaller than 10^{-3}.

5. QUANTUM CORRECTIONS AND RENORMALIZATION.

5.1. UV divergences and regularization.

Perturbation theory in QFT is plagued with UV divergences. We have to keep an UV cutoff Λ in computing physical quantities. There are three cases that arise :

- Super-renormalizable theories : only a finite number of Feynman diagrams diverge.
- Renormalizable theories : a finite number of amplitudes diverge. Divergences at all orders in pert. theory.
- Non-renormalizable theories : All amplitudes are divergent at a certain order in perturbation theory.

• In (super)renormalizable theories, UV divergences can be absorbed into rescaling of fields and redefinitions of the various couplings and masses. Taking the couplings/masses from experience, the UV cutoff disappears from physical quantities $\rightarrow$ the theory is predictive at any energy scale.

• In non-renormalizable theories, we need an infinite number of couplings and masses in order to absorbe UV divergences. We would need an infinite amount of experimental data to determine all these couplings $\rightarrow$ at high-energies $E > \Lambda$ the theory looses its predictive power. At low-energy the theory is perfectly predictive.

- 5.2. Relevant, marginal and irrelevant couplings

Consider a scalar theory of the form

$$S_\Lambda = \int d^4x \left(\frac{1}{2}(\partial\phi)^2 + \frac{m^2\phi^2}{2} + \sum_n \lambda_n \phi^n \right) \quad , \quad (67)$$

where S_Λ is the euclidian action defined with a cutoff Λ. The couplings λ_n have (classical) mass dimensions $[\lambda_n] = 4 - n$. Let us consider the theory with two different maximal euclidian momenta/cutoffs:

i) $0 < p < \Lambda$

ii) $0 < p < \Lambda' = \epsilon \Lambda$, where $\epsilon < 1$.

The theory ii) has therefore a lower cutoff.

It is interpreted as a theory where the high-momenta of theory i) were integrated out. The theory i) has the action (67). In the theory ii) the cutoff can be redefined to be the same as in i) with the help of a scale transformation

$$x' = \epsilon \, x \quad , \quad p' = \epsilon^{-1} p \quad , \quad \phi' = \epsilon^{-1}\phi \qquad (68)$$

In terms of the rescaled field and coordinates, the action of theory ii) become

$$S_{\Lambda'} = \int d^4x' \left(\frac{1}{2}(\partial'\phi')^2 + \frac{m'^2(\phi')^2}{2} + \sum_n \lambda'_n (\phi')^n \right) , \qquad (69)$$

where

$$m'^2 = \frac{1}{\epsilon^2} m^2 \quad , \quad \lambda'_n = \epsilon^{n-4} \lambda_n \tag{70}$$

Notice that the new mass and couplings scale with their classical dimension. We see therefore that the mass and couplings with positive dimension grow in the IR, whereas couplings with negative dimension decrease in the IR. It is said that

$$[\lambda_n] > 0 \quad \rightarrow \quad \text{relevant coupling}$$
$$[\lambda_n] = 0 \quad \rightarrow \quad \text{marginal coupling}$$
$$[\lambda_n] < 0 \quad \rightarrow \quad \text{irrelevant coupling}$$

5.3. (Non)renormalizability and couplings dims.

There is a straight connection between renormalizability and the three type of couplings above:

- relevant couplings $\rightarrow$ super-renormalizability.
- marginal couplings $\rightarrow$ renormalizability.
- irrelevant couplings $\rightarrow$ non-renormalizability.

It is easy to argue for this by dimensional arguments. Take some simple examples.

a) - Relevant coupling

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2\phi^2}{2} - \lambda_3\phi^3 \,. \tag{71}$$

The coupling has dimension $[\lambda_3] = +1$, so it is relevant.

At one-loop, the UV divergent terms lead to (Ex:)

$$\delta\mathcal{L}_1 \sim \lambda_3\Lambda^2\phi + \lambda_3^2\phi^2 \ln\Lambda \,,$$

which are both of super-renormalizable type. The first lead to mass renormalization, whereas the second leads to a scalar tadpole.

At two loops, the only UV divergences are a cosmological constant and a scalar tadpole. At three loops, there is only a log UV divergence in the cosmological constant. No UV divergences exist at higher loops.

Dim. argument : The highest UV divergent term in

the coupling is the three-loop vacuum energy

$$\lambda_3^4 \ln\Lambda \tag{72}$$

Higher loops have higher powers in λ_3 and cannot contribute to the UV divergent terms in the effective lagrangian

Obs: $1/m^2$ terms are IR, not UV contributions).

b) - Irrelevant coupling

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2\phi^2}{2} - \lambda_6\phi^6 \,. \tag{73}$$

The coupling has dimension $[\lambda_6] = -2$, so it is irrelevant. At one-loop, the UV divergent terms in the eight-point amplitude lead to (Ex:)

$$\Gamma^{(8)}_{1-\text{loop}}(p_i) \sim c\ \lambda_6^2 \ln\Lambda + \cdots\ .$$

To cancel this divergence, one has to add a new coupling to the original action

$$\delta\mathcal{L}_1 \sim \lambda_8 \phi^8\ ,$$

and to adjust the coupling λ_8 such that

$$\lambda_8\ +\ c\ \lambda_6^2 \ln\Lambda\ =\ \text{finite}$$

At two-loops, we get new new UV divergences, like the one in the six-point amplitude, prop. to

$$\Gamma^{(6)}_{2-\text{loops}}(p_i)\ \sim\ c'\ (p_i p_j)\lambda_6^2 \ln\Lambda\ ,$$

which can be canceled by adding another coupling

$$\delta\mathcal{L}_2\ \sim\ \lambda_8'\ \phi^4(\partial\phi)^2\ ,$$

such that

$$\lambda_8'\ +\ c'\ \lambda_6^2 \ln\Lambda\ =\ \text{finite}$$

The UV divergences proliferate at higher loop orders, generating an infinite tower of operators of higher and higher dimension.

Dimensional argument: Terms of the type $\lambda_6^n \phi^{4+2n} \ln\Lambda$, $\lambda_6^n (\partial\phi)^2 \phi^{2n} \ln\Lambda$ have the correct dimension to be generate for any n. Predictivity at high-energy is lost.

• However, let us define $\lambda_6 \sim 1/M^2$. Then :

In the IR $E < M$, the effect of non-renormalizable operators on physical quantities is prop. to some power or E/M and/or m/M, so their effects is negligible. Effective theories with cutoff Λ (ex. General relativity, $\Lambda = M_P$) are predictive at energies $E << \Lambda$.

Another viewpoint: for $\mathcal{L}_{\text{int}} = \sum_n \lambda_n \phi^n$, leading cross-section for $2 \to 2$ particle scattering is

$$\sigma\ =\ \sum_n c_n \lambda_n^2 E^{2n-10}\ \sim \frac{1}{E^2} \sum_n c_n (\frac{E}{M})^{2n}$$

for $\lambda_n \sim 1/M^{n-4} \to$ predictive power lost for $E \geq M$.

Ex : Coupling renormalization for ϕ^4 theory.

Consider the ϕ^4 theory

$$\mathcal{L}\ =\ \frac{1}{2}(\partial\phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{4!}\phi^4$$

and compute the four-point function at one-loop

$$\Gamma(k_1k_2k_3k_4) = -i\lambda_0 + \frac{(-i\lambda_0)^2}{2} \times$$
$$\int \frac{d^4p}{(2\pi)^4}\ \frac{i}{p^2 - m_0^2}\ \frac{i}{(p-k_1-k_2)^2 - m_0^2} + \text{two crossing terms}$$

After the Wick rotation to euclidian momenta

$$\Gamma(k_1k_2k_3k_4) = -i\lambda_0 + \frac{i\lambda_0^2}{2}\int \frac{d^4p}{(2\pi)^4}\ \frac{1}{p^2+m_0^2}\ \frac{1}{(p-k_1-k_2)^2 + m_0^2}$$
$$+\ \text{two crossing terms}$$

The integral is log divergent in the UV. There are various ways to "renormalize" the integral. Here is a simple way : Define

$$V(s) \equiv \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2+m_0^2} \frac{1}{(p-k_1-k_2)^2+m_0^2}$$
$$= \int_{p^2 \geq \mu^2}^{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} + \text{finite} \ ,$$

where the energy scale μ is arbitrary. We find

$$\Gamma(k_1k_2k_3k_4) = -i\lambda_0 + \frac{3i\lambda_0^2}{16\pi^2} \ln \frac{\Lambda}{\mu} + \text{finite} = -i\lambda(\mu) + \text{finite}$$

What is the physical interpretation of this manipulation?

i) λ_0 is not a physical parameter. It can be chosen to depend on Λ such that

$$\lambda(\mu) = \lambda_0(\Lambda) - \frac{3\lambda_0^2}{16\pi^2} \ln \frac{\Lambda}{\mu}$$

is independent of Λ.

ii) Any value of μ leads to the same physical result. λ_0 is independent of μ? Therefore

$$\frac{d\lambda}{d \ln \mu} = \frac{3\lambda^2}{16\pi^2} = \beta(\lambda) \tag{74}$$

describes the renormalization group equation (RGE) of λ at one-loop. (74) is then a differential eq., whose solution is

$$\lambda(\mu) = \lambda(\mu_0) \ + \ \frac{\lambda(\mu_0)}{1 - \frac{3\lambda(\mu_0)}{16\pi^2} \ln \frac{\mu}{\mu_0}}$$

There is an equivalent prescription : add a local "counterterm" to the lagrangian

$$\mathcal{L} \ + \ \delta\mathcal{L} \ = \ \mathcal{L}_0 \ ,$$

which cancels the UV divergence.

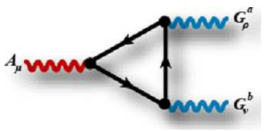
In renormalizable theories, a finite number of counterterms are needed in order to render the theory UV finite. In non-renormalizable theories, an infinite number of counterterms are needed.

5.4. Quantum anomalies*

Symmetries of the classical action can have anomalies at the quantum level. They are generate by one-loop triangle diagrams.

- For global symmetries, this does not creates problems. Ex: $\pi^0 \to \gamma\gamma$ is related to the axial $U(1)_A$ anomaly.

- For gauge symmetries, if present, they generate inconsistencies.



The corresponding currents are of chiral type

$$J_m^A \ = \ \bar{\Psi} \ \gamma_m \ \gamma_5 \ T^a \ \Psi \ = \ \bar{\Psi}_R \gamma_m T^a \Psi_R \ - \ \bar{\Psi}_L \gamma_m T^a \Psi_L$$

The resulting gauge anomaly that has to vanish is

$$A^{ABC} \ = \ tr \ [\{T^A, T^B\} T^C]_L \ - \ tr \ [\{T^A, T^B\} T^C]_R \ = \ 0 \ ,$$

where the trace is taken over all the fermions. For the SM, the only possible anomalies are (To check:) $SU(2)_L^2 U(1)_Y$, $U(1)_Y^3$ and $SU(3)_c^2 U(1)_Y$. The results in the SM are

$$tr \ [\{\frac{\tau^a}{2}, \frac{\tau^b}{2}\} Y]_L \ = \ \frac{1}{2}\delta^{ab} (trY)_L = 3 \times (N_c \times \frac{1}{3} - 1) = 0 \ ,$$
$$tr \ [\{Y, Y\} Y]_{L-R} = \cdots = 6(-2N_c + 6) = 0$$
$$tr \ [\{\frac{\lambda^A}{2}, \frac{\lambda^B}{2}\} Y]_{L-R} \ = \ \frac{1}{3}\delta^{AB} (trY)_{L-R} = \cdots = 0$$

- Anomaly cancelation happens precisely for three colors $N_c = 3$!
- Anomaly cancelation provides a deep connection between quarks and leptons in the SM, maybe a hint towards Grand Unified Theories ? (Bogdan lectures ?)

6. The Higgs / Symmetry breaking sector of the Standard Model.

6.1.1 Perturbativity bounds

The RGE for the Higgs self-coupling in the SM is

$$16\pi^2 \frac{d\lambda}{d \ln \mu} = 24\lambda^2 - (3g'^2 + 3g^2 - 12h_t^2) \ \lambda$$
$$+ \frac{3}{8}(g'^4 + 2g^2 g'^2 + 3g^4) - 6h_t^4 \ + \ \cdots \ ,$$

where $\cdots$ denote smaller Yukawas. In the large Higgs mass limit $\lambda >> g^2, h_t^2$, it reduces to

$$\frac{d\lambda}{\lambda^2} = \frac{3}{2\pi^2} \, d \ln \mu \ \to \ \frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \ln \frac{\Lambda}{\mu} \ .$$

This can be interpreted in two alternative ways :

i) If the Higgs mass is known, SM has a Landau pole (non-pert. regime) $\lambda(\Lambda) >> 1$ for

$$\Lambda \ = \ v \ e^{\frac{2\pi^2}{3\lambda}} \ = \ v \ e^{\frac{4\pi^2 v^2}{3M_h^2}}$$

ii) Conversely, asking for perturbativity up to scale Λ (say M_{GUT}), we obtain an upper bound on the Higgs mass

$$M_h^2 \ \leq \ \frac{4\pi^2 v^2}{3 \ln \frac{\Lambda}{v}} \ .$$

6.1.2 Stability bounds

SM has another instability in the small Higgs mass limit, since λ can become negative at high-energy.

If $\lambda << h_t^2$, the leading RGE's are

$$16\pi^2 \frac{d\lambda}{d\ln\mu} = -6h_t^4 \; , \; 16\pi^2 \frac{dh_t}{d\ln\mu} = \frac{9h_t^3}{2}$$

which integrates to

$$\lambda(\mu) = \lambda(\Lambda) + \frac{\frac{3h_t^4(\Lambda)}{8\pi^2}\ln\frac{\Lambda}{\mu}}{1+\frac{9h_t^2(\Lambda)}{16\pi^2}\ln\frac{\Lambda}{\mu}} \, ,$$

$$h_t^2(\mu) = \frac{h_t^2}{1+\frac{9h_t^2(\Lambda)}{16\pi^2}\ln\frac{\Lambda}{\mu}} \, .$$

This can be interpreted in two ways :

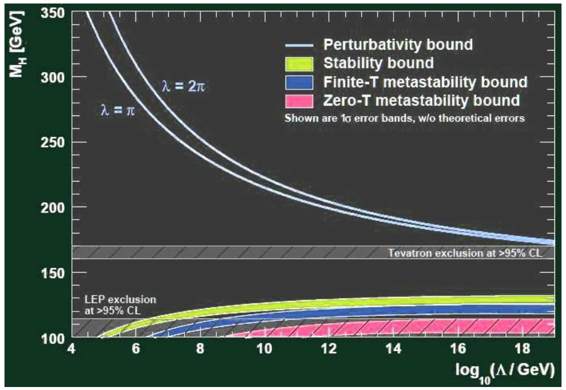
i) For a fixed, known value of the Higgs mass : take $\mu = v$. Then, new physics should show up before the scale Λ where $\lambda(\Lambda) = 0$

$$\Lambda \le v\, e^{\frac{8\pi^2\lambda}{3h_t^4}} = v\, e^{\frac{4\pi^2 M_h^2}{3h_t^4 v^2}}$$

ii) For a fixed Λ, we get a lower bound on the Higgs mass

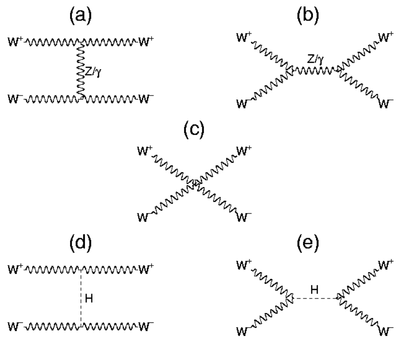
$$M_h^2 \ge \frac{3h_t^4 v^2}{4\pi^2}\ln\frac{\Lambda}{v} = \frac{3m_t^4}{\pi^2 v^2}\ln\frac{\Lambda}{v}$$

These theoretical Higgs mass limits are summarized in the following plot



- 6.2. $W\ W$ scattering and unitarity.

Let us consider the longitudinal $W_L W_L \to W_L W_L$ scattering



For a massive gauge particle of momentum k and mass M_W, $A_m = \epsilon_m\ e^{ikx}$, the three polarizations satisfy $\epsilon_m \epsilon^m = -1$, $k_m \epsilon^m = 0$. For $k^m = (E, 0, 0, k)$, they are

$$\text{transverse} \ : \ \epsilon_1^m = (0,1,0,0) \quad , \quad \epsilon_2^m = (0,0,1,0) \ ,$$
$$\text{longitudinal} \ : \ \epsilon_L^m = (\frac{k}{M_W}, 0, 0, \frac{E}{M_W}) \sim \frac{k^m}{M} + \mathcal{O}(\frac{E}{M_W}) \ .$$

Since the longitudinal polarization is proportional to the energy, we expect a tree-level amplitude behaving as

$$\mathcal{A} \ = \ \mathcal{A}^{(4)}(\frac{E}{M_W})^4 \ + \ \mathcal{A}^{(2)}(\frac{E}{M_W})^2 + \cdots$$

Actually, the diagrams a),b) and c) give $\mathcal{A} = g^2(\frac{E}{M_W})^2$. On the other hand, unitarity constrains the amplitude to stay small enough at any energy.

Start with the unitarity of the S-matrix $S^\dagger S = 1$. Then

$$S = 1 + i\mathcal{A} \quad \rightarrow \quad i(\mathcal{A} - \mathcal{A}^\dagger) + \mathcal{A}^\dagger \mathcal{A} = 0$$

Let us sandwich this eq. between a two-particle state $|i>$:

$$i(\mathcal{A} - \mathcal{A}^\dagger)_{ii} + \sum_f |\mathcal{A}_{fi}|^2 = 0 \qquad (75)$$

which is the optical theorem : the imaginary part of the forward amplitude of the process $i \rightarrow i$ is proportional to the total cross section of $i \rightarrow$ anything.

Let us decompose the scattering amplitude into partial waves

$$\mathcal{A} \ = \ \sum_{l=0}^{\infty} (2l+1)\ P_l(\cos\theta) a_l \ ,$$

where a_l are partial wave amplitudes of elastic scattering of two particles. Projecting (75) into the partial wave l gives $\quad Im\ a_l = |a_l|^2$. $\quad$ This is only possible if

$$|Re\ a_l| \le 1/2 \ , \ 0 \le Im\ a_l \le 1 \quad \rightarrow \quad |a_l|^2 \le 5/4 \ ,$$

which is the unitarity bound we were searching for.

- For the SM without the Higgs boson

$$a_0 = \frac{g^2 E^2}{M_W^2} \quad \rightarrow \quad \text{unitarity breaks down for} \sqrt{s} \sim 1.2\ TeV$$

With the Higgs boson, amplitudes d),e) cancel the raising energy term, such that

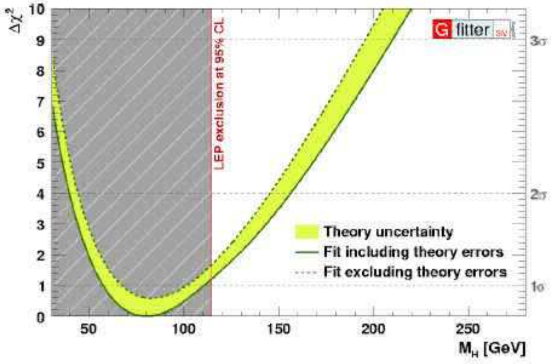
$$a_0 = \frac{g^2 M_H^2}{4 M_W^2} \quad \rightarrow \quad \text{unitarity breaks down unless } M_H \le 1.2\ TeV$$

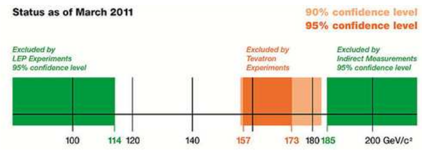
By considering other channels, one get the stronger bound $M_H \le 800 GeV$.

Intepretation :

- If LHC finds no Higgs with a mass $M_H \le 800 GeV$, unitarity of S-matrix will be violated ! New light degrees of freedom should exist in order to restore unitarity $\rightarrow$ the no-loose "theorem" for LHC.

Most theories have a biased towards a light Higgs, since it provides a better fit for the SM precision tests.





Latest news ("Lepton-Photon", august 2011): Both ATLAS+CMS exclude the SM Higgs at 95 % CL for 145 $\leq M_H \leq 446\ GeV$ except $288 - 296\ GeV$

M. Peskin (LP2011) "There is therefore strong evidence that either :

- Higgs is light, compatible with electroweak precision tests and theoretical prejudice, or
- the Higgs boson is very heavy and strongly self-coupled".