

# Neural network fits of parton distributions

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### The NNPDF Collaboration:

Luigi Del Debbio, Stefano Forte, José I. Latorre,  
Andrea Piccione and Juan Rojo,  
(2007: +) Richard D. Ball, Alberto Guffanti and Maria Ubiali.

1. JHEP **02** (2002) 062 [arXiv:hep-ph/0204232].
2. JHEP **05** (2005) 080 [arXiv:hep-ph/0501067].
3. *Neural network determination of parton distributions: the nonsinglet case*, JHEP **07** (2007) 039 [arXiv:hep-ph/0701127].

## Introduction

## Methodological issues

Stopping criterion

Stability estimators

## The nonsinglet case

## Status of the singlet case

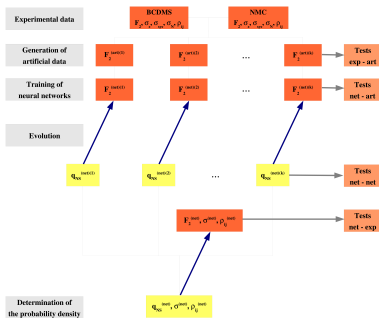
## Conclusions and outlook

# The neural Monte Carlo approach

Basic Idea: **Monte Carlo sampling** coupled to **Neural Network interpolation**

- ▶ Generate a set of Monte Carlo replicas  $\sigma^{(k)}(p_i)$  of the original data set  $\sigma^{(\text{data})}(p_i)$ , representation of  $\mathcal{P}[\sigma(p_i)]$  at discrete set of points  $p_i$
- ▶ Train a neural net for each pdf on each replica, obtaining a representation of the pdfs  $q_i^{(\text{net})}(k)$
- ▶ The set of neural nets is a representation of the probability density:

$$\langle \sigma[q_i] \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \sigma[q_i^{(\text{net})}(k)]$$

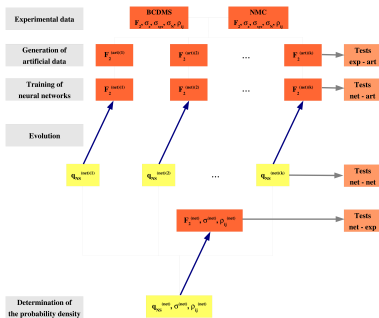


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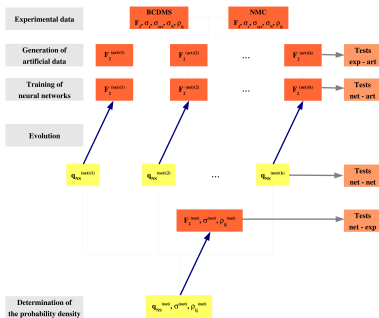


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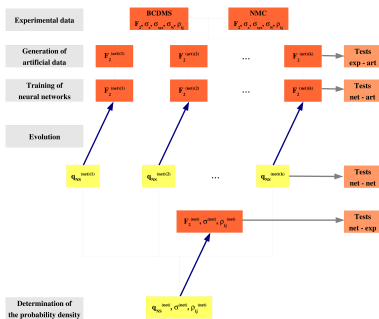


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Many different ingredients in the neural Monte Carlo approach to parton distributions: artificial data generation, neural network training, genetic minimization, preprocessing, result validation ...

Let us concentrate on a couple of the newest ones:

- ▶ Stopping criterion
- ▶ Stability estimators

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# When to stop a fit?

In a standard fit, look for minimum  $\chi^2$  for given parametrization. However ...

- ▶ If basis too large → convergence never reached
- ▶ If basis too small → parametrization bias

How can one obtain an unbiased compromise? For neural nets, smoothness decreases as fit quality improves...

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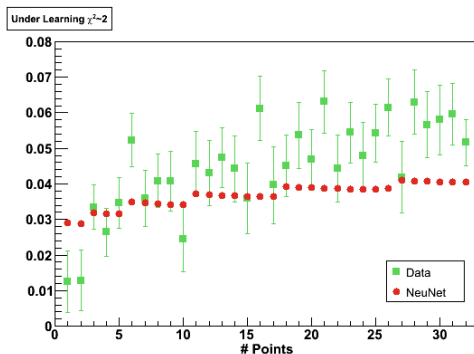
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### Underlearning

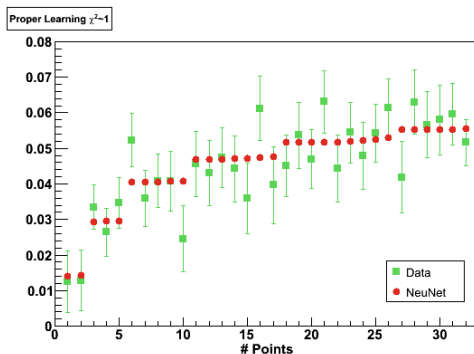


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### Proper learning

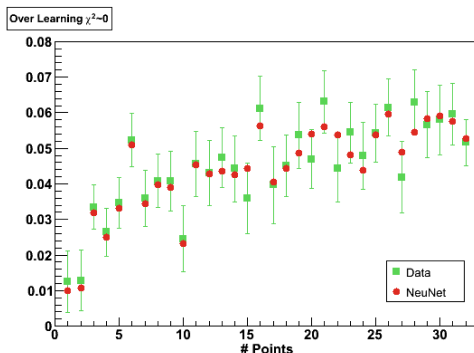


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## Overlearning



# The neural stopping criterion

So stop the fit before overlearning sets in ... How does this work in practice?

At each Genetic Algorithm generation,  $\chi^2$  either decreases or unchanged

1. Divide the data set into training and validation sets
2. Minimize  $\chi^2$  of training set, monitor  $\chi^2$  of validation set
3. Stop minimization when validation  $\chi^2$  begins to rise

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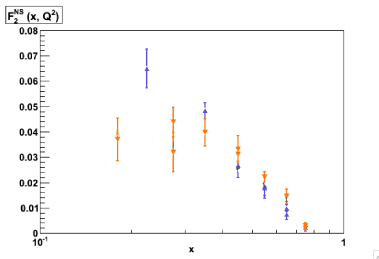
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# An explicit example

Let us see in practice how the **overlearning stopping criterion** works:

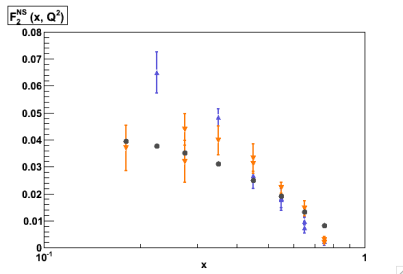
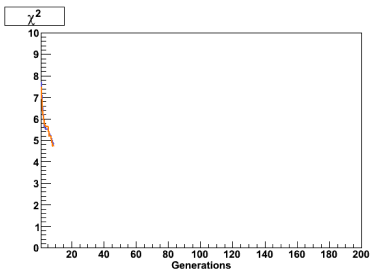
**On your marks, get ready ...**



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Go!

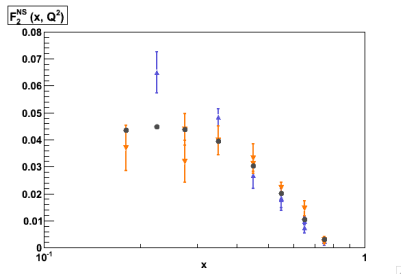
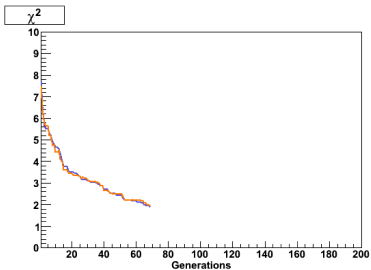


**Underlearning**, continue the minimization

# An explicit example

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**Stop!**

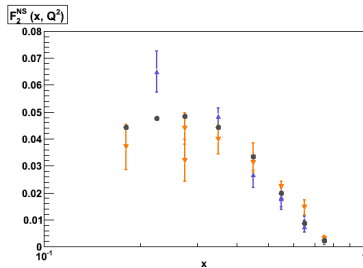
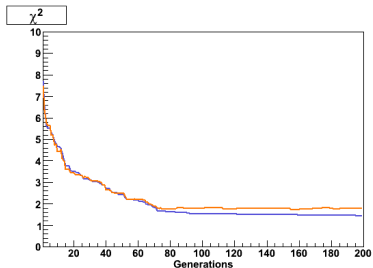


**Onset of overlearning, stop the minimization.**

# An explicit example

Let us see in practice how the **overlearning stopping criterion** works:

**Too late!**

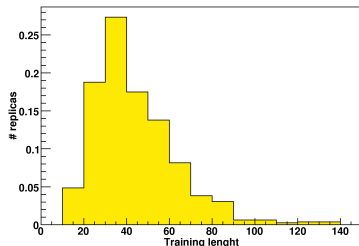


Deep in the **overlearning region**, fitting statistical noise ...

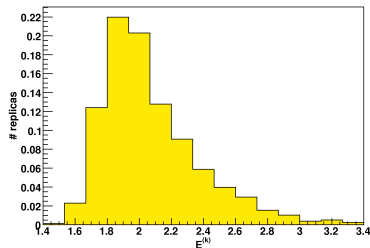
# Does it work?

## Distribution of $\chi^2$ of pseudo-data and training lengths

Distribution of training lengths



Distribution of  $E$

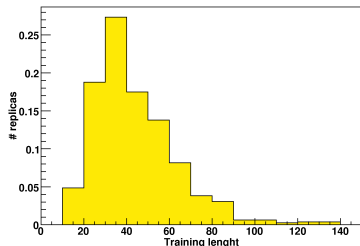


1. **Poissonian distribution of training lengths**
2. For best fit, average  $\chi^2$  of replicas  $\sim 2$ , while when averaging over replicas  $\chi^2 \sim 1$ .
3. Total training time is optimized (never underlearn nor overlearn)  $\rightarrow$  efficient neural fitting.

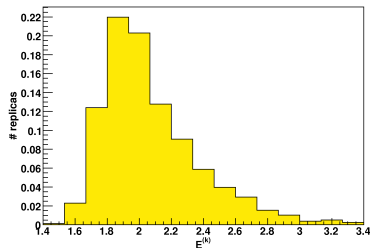
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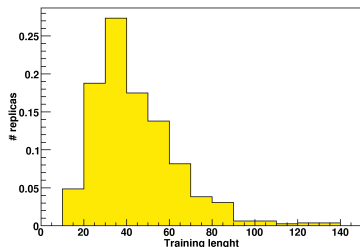
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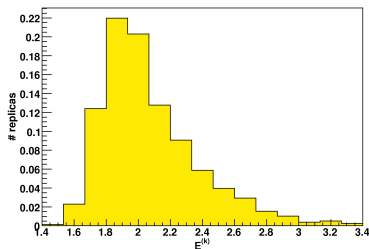
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# Stability

Check **stability and accuracy** of our results (both for central values and for errors) when parameters of fit modified

Define **RMS distance**

$$\langle d[q] \rangle = \sqrt{\left\langle \frac{(\langle q_i \rangle_{(1)} - \langle q_i \rangle_{(2)})^2}{\sigma^2[q_i^{(1)}] + \sigma^2[q_i^{(2)}]} \right\rangle_{\text{dat}}}$$

where  $\sigma[q_i] = \text{error on } \langle q_i \rangle = \text{error on } q_i / \sqrt{N_{\text{rep}}}$ .

Compute  $\langle d[q] \rangle$  and  $\langle d[\sigma] \rangle$  both in **data region** and in **extrapolation region**.

Statistical expectations:

- ▶ For statistically equivalent fits (different set of MC replicas, different net architecture) we expect  $\langle d[q] \rangle \sim 1$ ,  $\langle d[\sigma] \rangle \sim 1$ .
- ▶ For statistically nonequivalent fits (different perturbative order, different  $\alpha_s$  value) we expect (e.g. for central values)

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# Great expectations?

Indeed we observe the expected behavior:

Architecture	2-4-3-1 vs. 2-5-3-1	Perturbative order	LO vs. NLO
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$\langle d[q] \rangle_{\text{extra}}$	0.9	$\langle d[q] \rangle_{\text{extra}}$	1.2
$\langle d[\sigma_q] \rangle_{\text{dat}}$	0.9	$\langle d[\sigma_q] \rangle_{\text{dat}}$	2.2
$\langle d[\sigma_q] \rangle_{\text{extra}}$	1.4	$\langle d[\sigma_q] \rangle_{\text{extra}}$	1.3

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# The nonsinglet case

## Summary of the analysis

1. Data:  $F_2^p(x, Q^2) - F_2^d(x, Q^2)$  from NMC and BCDMS (483 points with  $Q^2 \geq 3 \text{ GeV}^2$ )
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$$F_2^{\text{NS}}(x, Q^2) = \frac{1}{6} x \int_x^1 \frac{dy}{y} \tilde{\Gamma}(y, \alpha_s(Q^2), \alpha_s(Q_0^2)) q_{\text{NS}}\left(\frac{x}{y}, Q_0^2\right).$$

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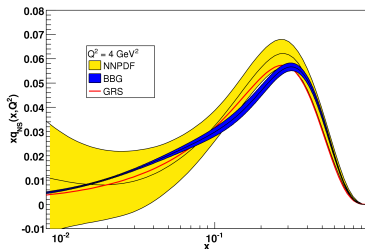
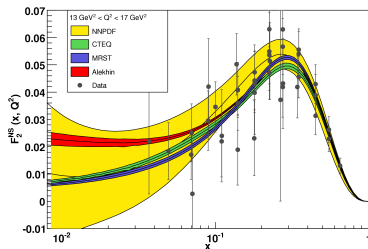
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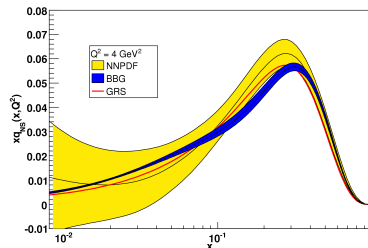
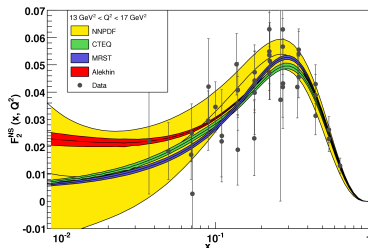
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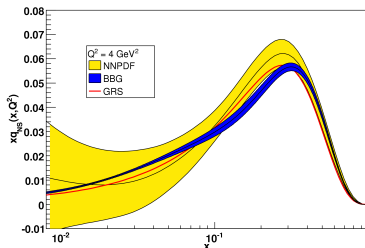
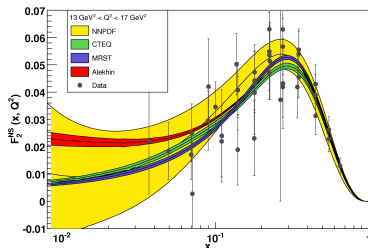
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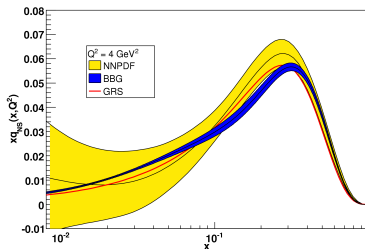
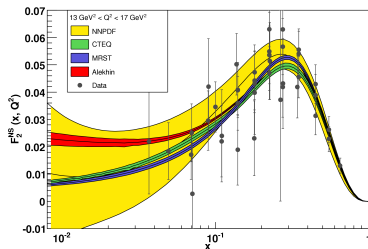
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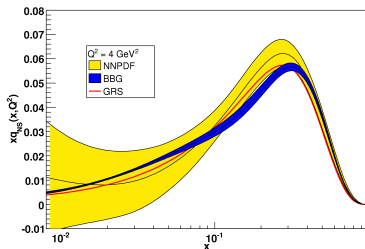
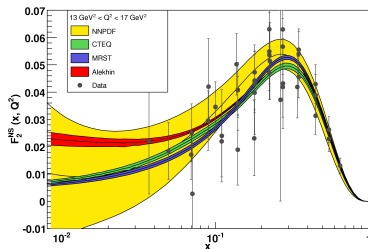
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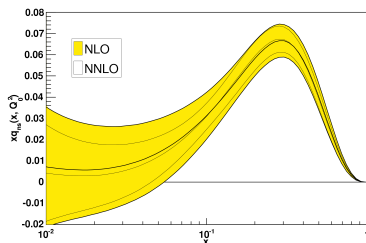
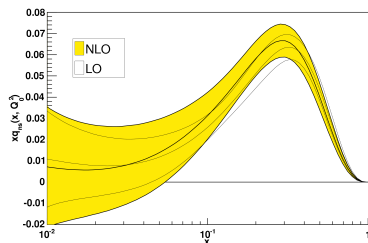
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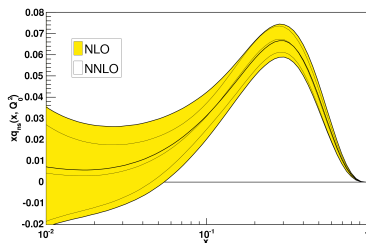
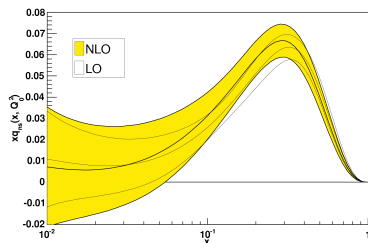


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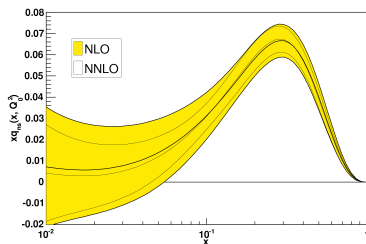
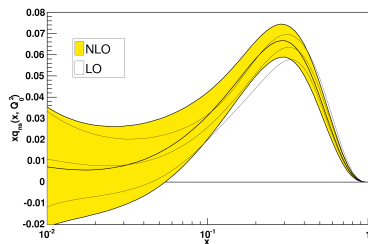
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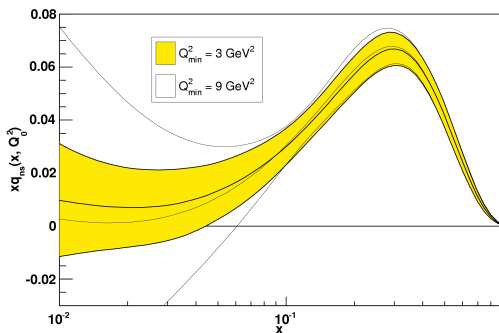
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# Kinematical cuts



Uncertainty in **extrapolation region** (small  $x$ ) increases when **kinematical cut in  $Q^2$**  is raised (as it should be!).

# Status of the singlet case

# The singlet case

- ▶ Data sets: SLAC, NMC, BCDMS structure function  $F_2(x, Q^2)$  and HERA reduced cross sections  $\tilde{\sigma}^{NC}(x, Q^2)$  and  $\tilde{\sigma}^{CC}(x, Q^2)$ .
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# EXTRA SLIDES

# Statistical estimators

	Total	NMC	BCDMS
$\chi_{\text{tot}}^2$	<b>0.75</b>	<b>0.72</b>	<b>0.78</b>
$\langle E \rangle$	2.27	1.99	2.52
$r [F_2^{\text{NS}}]$	0.81	0.66	0.95
$\langle \sigma^{(\text{exp})} \rangle_{\text{dat}}$	0.011	0.017	0.006
$\langle \sigma^{(\text{net})} \rangle_{\text{dat}}$	0.006	0.009	0.004
$r [\sigma^{(\text{net})}]_{\text{dat}}$	0.59	-0.04	0.86
$\langle \rho^{(\text{exp})} \rangle_{\text{dat}}$	0.11	0.39	0.16
$\langle \rho^{(\text{net})} \rangle_{\text{dat}}$	0.46	0.42	0.50
$r [\rho^{(\text{net})}]_{\text{dat}}$	0.15	0.25	0.04
$\langle \text{COV}^{(\text{exp})} \rangle_{\text{dat}}$	$8.6 \cdot 10^{-6}$	$1.0 \cdot 10^{-5}$	$7.2 \cdot 10^{-6}$
$\langle \text{COV}^{(\text{net})} \rangle_{\text{dat}}$	$2.1 \cdot 10^{-5}$	$3.8 \cdot 10^{-5}$	$6.9 \cdot 10^{-6}$
$r [\text{COV}^{(\text{net})}]_{\text{dat}}$	0.24	0.23	0.57

# Determination of $\alpha_s(M_Z^2)$

We do not fit  $\alpha_s(M_Z^2)$  with  $q_{\text{NS}}(x, Q_0^2)$  (although technically possible), but take it from **world average**:  $\alpha_s(M_Z^2)_{\text{NNLO}} = 0.115$ .

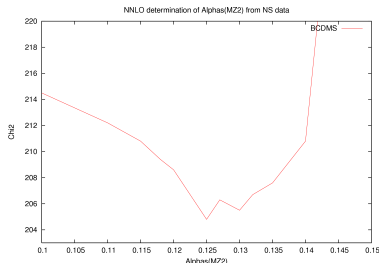
$\alpha_s(M_Z^2)$	0.116	0.118	0.120
$\chi^2$	0.743	0.750	0.744
$\langle d[q] \rangle_{\text{dat}}$	3.8	0	4.1
$\langle d[q] \rangle_{\text{extra}}$	0.8	0	0.7
$\langle d[\sigma_q] \rangle_{\text{dat}}$	1.6	0	2.4
$\langle d[\sigma_q] \rangle_{\text{extra}}$	1.4	0	1.5

Fit results **not sensible** to  $\alpha_s(M_Z^2)$  variation within world average uncertainty.

## Determination of $\alpha_s(M_Z^2)$ II

More dedicated NNLO analysis in preparation:

Repeat the NNLO determination of  $q_{NS}(x)$  scanning the  $\alpha_s(M_Z^2)$  space:



Preliminary results: rather large value  $\alpha_s(M_Z^2) = 0.124$  preferred by data, with large uncertainties (Similar conclusions in S. Forte et al., hep-ph/0205286).

# Higher Twist

No **evidence for Higher Twist** found in experimental data:

Fit	$Q_{\min}^2 = 3 \text{ GeV}^2 + \text{HT}$	$Q_{\min}^2 = 5 \text{ GeV}^2$	$Q_{\min}^2 = 5 \text{ GeV}^2 + \text{HT}$
$\chi^2$	0.76	0.79	0.78
$\langle d[q] \rangle_{\text{dat}}$	2.9	0.8	3.2
$\langle d[q] \rangle_{\text{extra}}$	1.4	0.8	0.9
$\langle d[\sigma_q] \rangle_{\text{dat}}$	1.2	1.5	1.9
$\langle d[\sigma_q] \rangle_{\text{extra}}$	1.3	1.8	2.3