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New type of parameterizations for parton distributions

OUTLINE

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1. Introduction

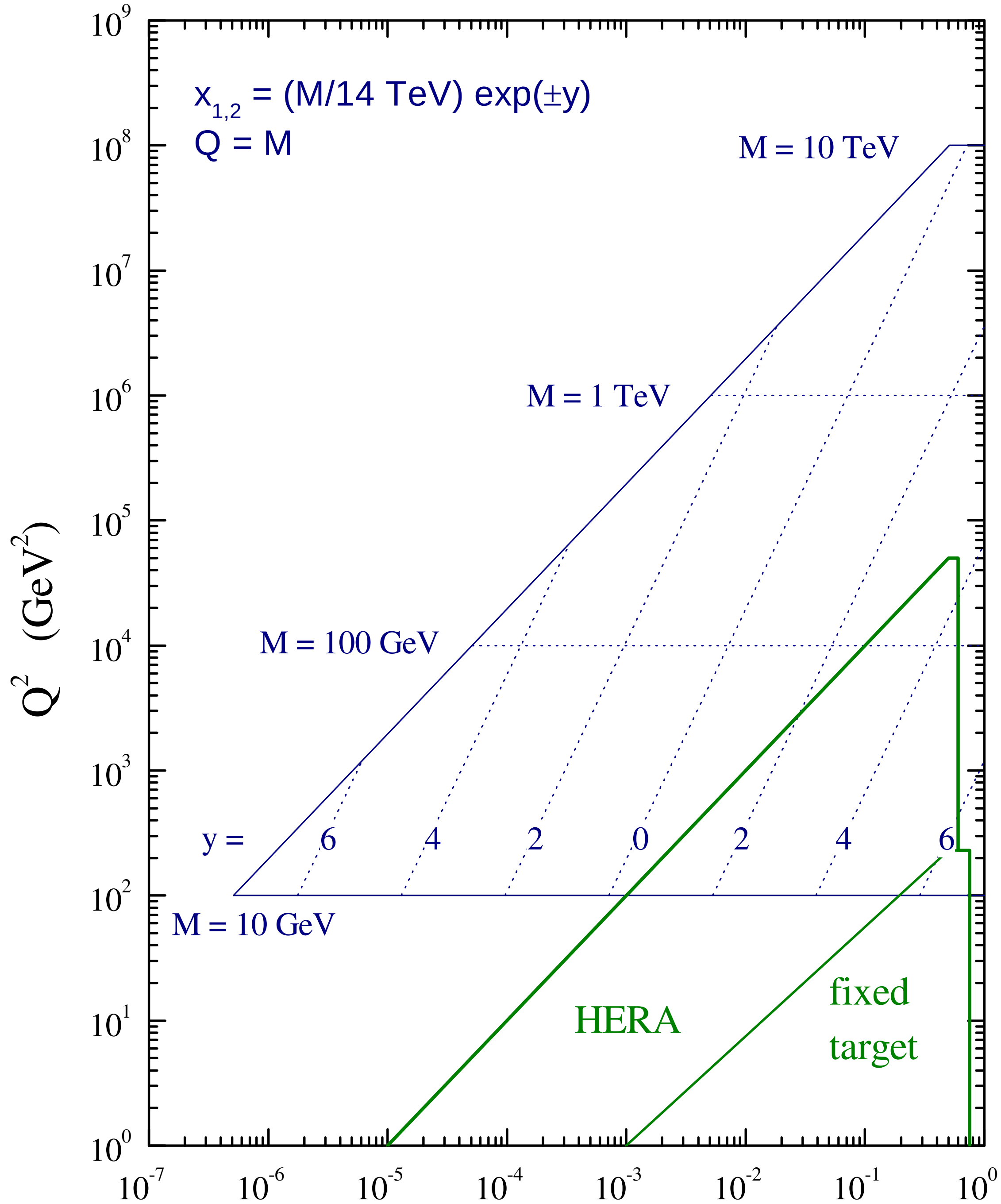
A. The overlape between (x, Q^2) range of parton densities contributed for LHC processes and parton densities fitted at HERA and fixed target experiments, is not completly same (see, for example, fig. from (R.S.Thorne et al, 2005). So, direct application of modern sets of parton distributions may be not so correct.

B. The larger uncertanties for many processes at LHC came from restricted knowledge of parton distributions.

C. There is an idea to solve approximately DGLAP equations for parton densities at low and large x values and to approach a combination of the two solutions for full range of x.

It is not new idea. There is a similar parameterizations give by F.J. Yndurain et al.

LHC parton kinematics



2. Introduction to DIS

A. Deep-inelastic scattering (DIS) cross-section:

$$\sigma \sim L^{\mu\nu} F^{\mu\nu}$$

Hadron part $F^{\mu\nu}$ ($Q^2 = -q^2 > 0$, $x = Q^2/[2(pq)]$):

$$F^{\mu\nu} = (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) F_1(x, Q^2) - (p^\mu - \frac{(pq)}{q^2} q^\mu)(p^\nu - \frac{(pq)}{q^2} q^\nu) \frac{2x}{q^2} F_2(x, Q^2) + ...,$$

where $F_k(x, Q^2)$ ($k = 1, 2, 3, L$) - are DIS structure functions (SF) and q and p are photon and hadron (parton) momentums.

B. Wilson operator expansion: Mellin moments $M_k(j,Q^2)$ of DIS SF $F_k(x,Q^2)$ can be represented as sum

$$M_k(j,Q^2) = \sum_{a=NS,SI,G} \underbrace{C_k^a(j,Q^2/\mu^2)}_{\text{Coeff. function}} A_a(j,\mu^2),$$

where $A_a(j,\mu^2) =< N|\mathcal{O}^a_{\mu_1,\ldots,\mu_j}|N >$ are matrix elements of the Wilson operators $\mathcal{O}^a_{\mu_1,\ldots,\mu_j}$.

C. The matrix elements $A_a(j,\mu^2)$ are Mellin moments of the unpolarized and polarized parton densities $f_a(j,\mu^2)$ and $\tilde{f}_a(j,\mu^2)$.

DGLAP equations:

$$\frac{d}{d\ln Q^2} f_a(x,Q^2) = \int_x^1 \frac{dy}{y} \sum_b W_{b\to a}(x/y)\, f_b(y,Q^2)\,,$$
$$\frac{d}{d\ln Q^2} \tilde{f}_a(x,Q^2) = \int_x^1 \frac{dy}{y} \sum_b \tilde{W}_{b\to a}(x/y)\, \tilde{f}_b(y,Q^2)\,. \qquad (1)$$

The anomalous dimensions (AD) $\gamma_{ab}(j)$ of the twist-2 Wilson operators $\mathcal{O}^a_{\mu_1,\ldots,\mu_j}$ (hereafter $a_s = \alpha_s/(4\pi)$)

$$\gamma_{ab}(j) = \int_0^1 dx\; x^{j-1} W_{b\to a}(x) = \sum_{m=0}^{\infty} \gamma_{ab}^{(m)}(j) a_s^m,$$
$$\tilde{\gamma}_{ab}(j) = \int_0^1 dx\; x^{j-1} \tilde{W}_{b\to a}(x) = \sum_{m=0}^{\infty} \tilde{\gamma}_{ab}^{(m)}(j) a_s^m.$$

All parton densities are multiplies by x, t.e.

structure function $=$ combination of parton densities.

3. Large x and low x asymptotics

A. Large x asymptotics.

Singlet quark density $f_{SI}(x,Q^2)$ contains the valent part $f_V(x,Q^2)$ and the sea part $f_S(x,Q^2)$.

The large x asymptotics (D.I.Gross, 1974), (C.Lopez and F.J. Yndurain, 1980,1981)

$$f_i \longrightarrow \overline{B}_i(s)(1-x)^{\beta_i(s)} \quad (i = NS,V,S,G)$$

where $(j = NS,V,S)$

$$s = \ln\left(\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}\right), \quad \beta_i(s) = \beta_i(0) + \hat{d}_i s,$$

$$\hat{d}_j = \frac{32}{3}\frac{1}{2\beta_0}, \quad \hat{d}_G = \frac{24}{2\beta_0},$$

$$\overline{B}_i(s) = B_i(0)\frac{e^{-p_i s}}{\Gamma(1+\beta_i(s))},$$

$$p_j = \hat{d}_j \left(\gamma_E - \frac{3}{4}\right) s, \quad p_G = \hat{d}_G \left(\gamma_E - \frac{\beta_0}{12}\right) s.$$

Here $\beta_0 = 11 - 2f/3$ is the first term of QCD β-function (f is the number of active quarks) and γ_E is Euler constant. The constants $\beta_i(0)$ can be taken from quark counting rules (V.A.Matveev et al., 1973), (S.J.Brodsky et al., 1973,1995):

$$\beta_V(0) \sim \beta_{NS}(0) \sim 3,$$
$$\beta_G(0) \sim \beta_{NS}(0) + 1 \sim 4,$$
$$\beta_S(0) \sim \beta_G(0) + 1 \sim 5$$

B. **Low x asymptotics.**

1. Valent and nonsinglet parts are $(i = V, NS)$ (F.Martin, 1979), (C.Lopez and F.J. Yndurain, 1980,1981), (A.V.K., 1994)

$$f_i(x) \to A_i(s)\, x^{\lambda_i} \qquad (2)$$

where

$$\overline{A}_i(s) = \overline{A}_i(0) e^{-d_i(1-\lambda_i)s},$$

$$d_i(n) = \frac{32}{3}\frac{1}{2\beta_0}\left[\Psi(n+1) + \gamma_E - \frac{3}{4} - \frac{1}{2n(n+1)}\right],$$

where $\Psi(n+1)$ is Euler Ψ-function

and $\lambda_i \sim Const$ ($\neq$ function of s): $\lambda_i \sim 0.5 \div 0.7$

2. Gluon density $f_G(x,Q^2)$ and sea quark part $f_S(x,Q^2)$ are mixed during the DGLAP evolution.

After diagonalization they have at $x \to 0$

$$f_G(x,Q^2) = f_G^+(x,Q^2) + f_G^-(x,Q^2),$$
$$f_S(x,Q^2) = f_S^+(x,Q^2) + f_S^-(x,Q^2),$$

where (L.Mankiewicz et al, 1997), (A.V.K. and G.Parente, 1998)

$$f_G^+(x,Q^2) \to \left[A_G(0) + \frac{4}{9}A_S(0)\right] I_0(\sigma) e^{-\overline{d}_+(1)s},$$
$$f_S^+(x,Q^2) \to \frac{f}{9}\frac{\rho I_1(\sigma)}{I_0(\sigma)} f_G^+(x,Q^2),$$
$$f_S^-(x,Q^2) \to A_S(0)\, e^{-\overline{d}_-(1)s},$$
$$G^-(x,Q^2) \to -\frac{4}{9}(1-x)\, S^-(x,Q^2),$$

where (R.D.Ball and S.Forte, 1994)

$$\sigma = 2\sqrt{\frac{12}{\beta_0} s \ln\frac{1}{x}}, \quad \rho = \frac{\sigma}{2\ln\frac{1}{x}}$$

are Ball-Forte variables and

$$\overline{d}_-(1) = \frac{16f}{27\beta_0}, \quad \overline{d}_+(1) = 1 + \frac{20f}{27\beta_0},$$

3. Fits of HERA data

At low x, the structure function $F_2(x, Q^2)$ is related to parton densities as (A.V.K. and G.Parente, 1998)

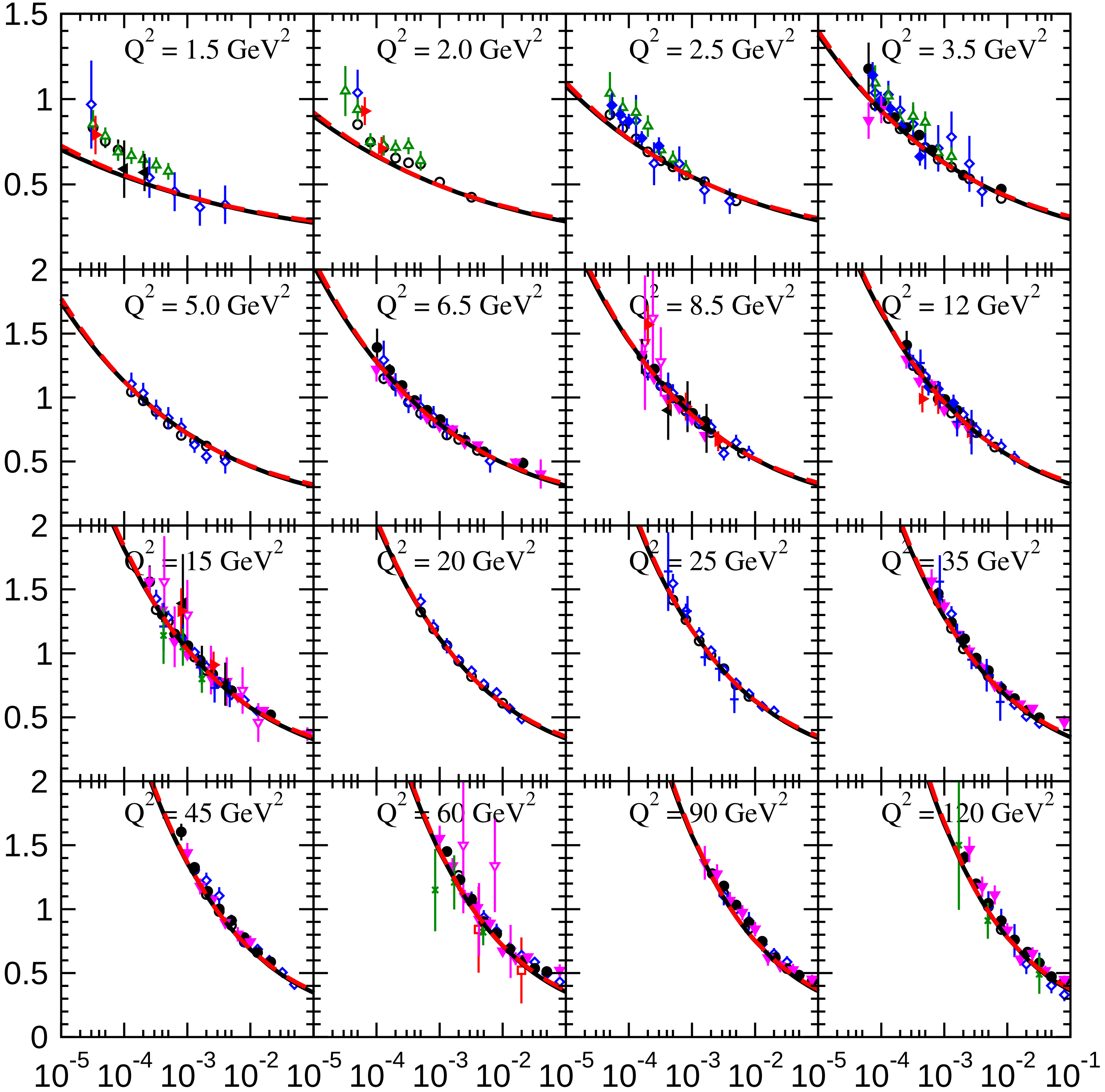
at LO

$$F_2(x, Q^2) = \frac{5}{18} f_S(x, Q^2)$$

at NLO

$$F_2(x, Q^2) = \frac{5}{18} \left[f_S(x, Q^2) + \frac{2f}{3} a_s(Q^2) f_G(x, Q^2) \right].$$

Fits of HERA experimental data of the structure function $F_2(x, Q^2)$ (A.Yu.Illarionov, A.V.K. and G.Parente, 2004)



Q^2 = 1.5 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 2.5 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 5.0 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 12 GeV^2
Q^2 = 15 GeV^2
Q^2 = 20 GeV^2
Q^2 = 25 GeV^2
Q^2 = 35 GeV^2
Q^2 = 45 GeV^2
Q^2 = 60 GeV^2
Q^2 = 90 GeV^2
Q^2 = 120 GeV^2
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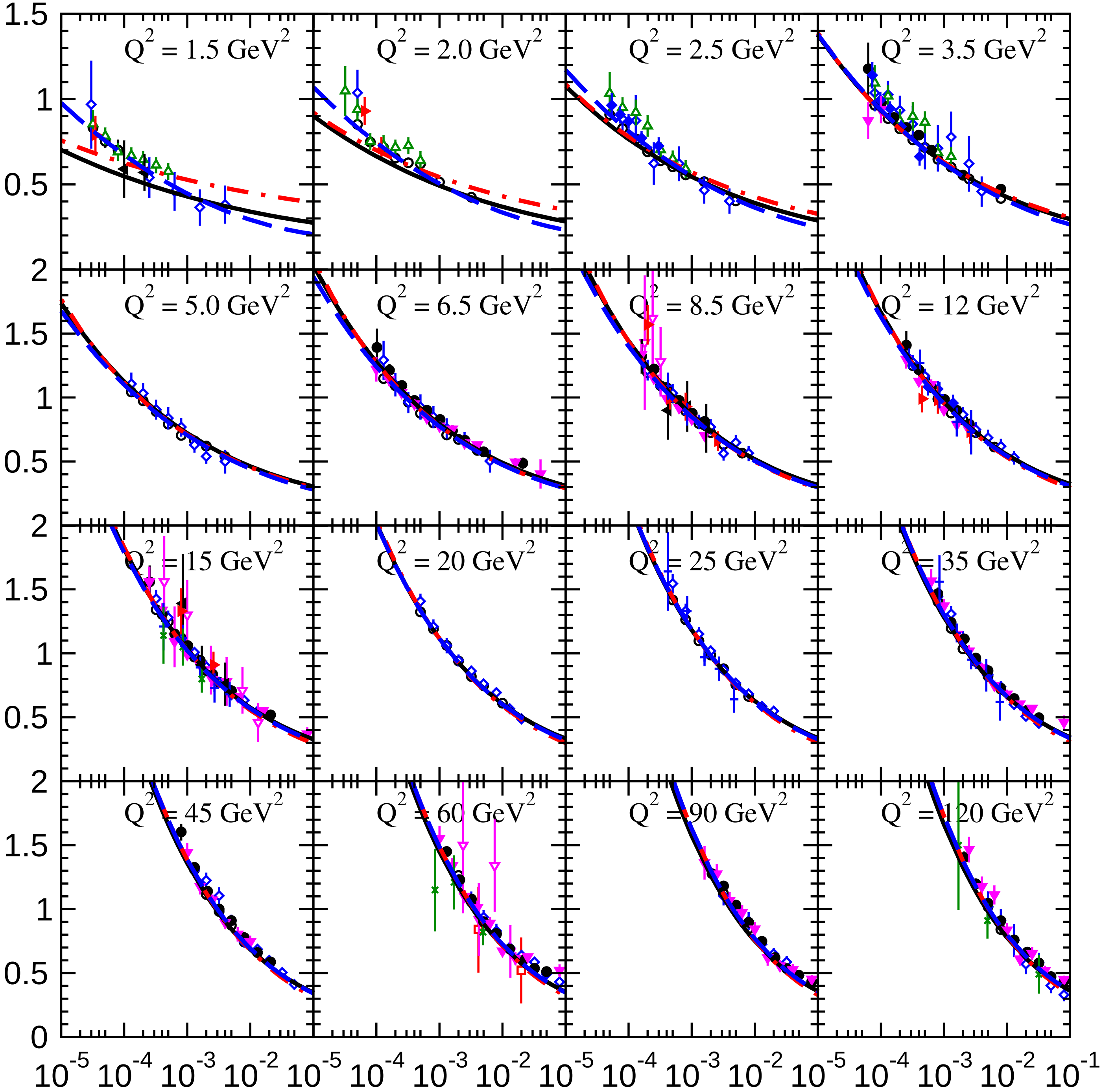

Q^2 = 1.5 GeV^2
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Q^2 = 8.5 GeV^2
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Q^2 = 35 GeV^2
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Q^2 = 90 GeV^2
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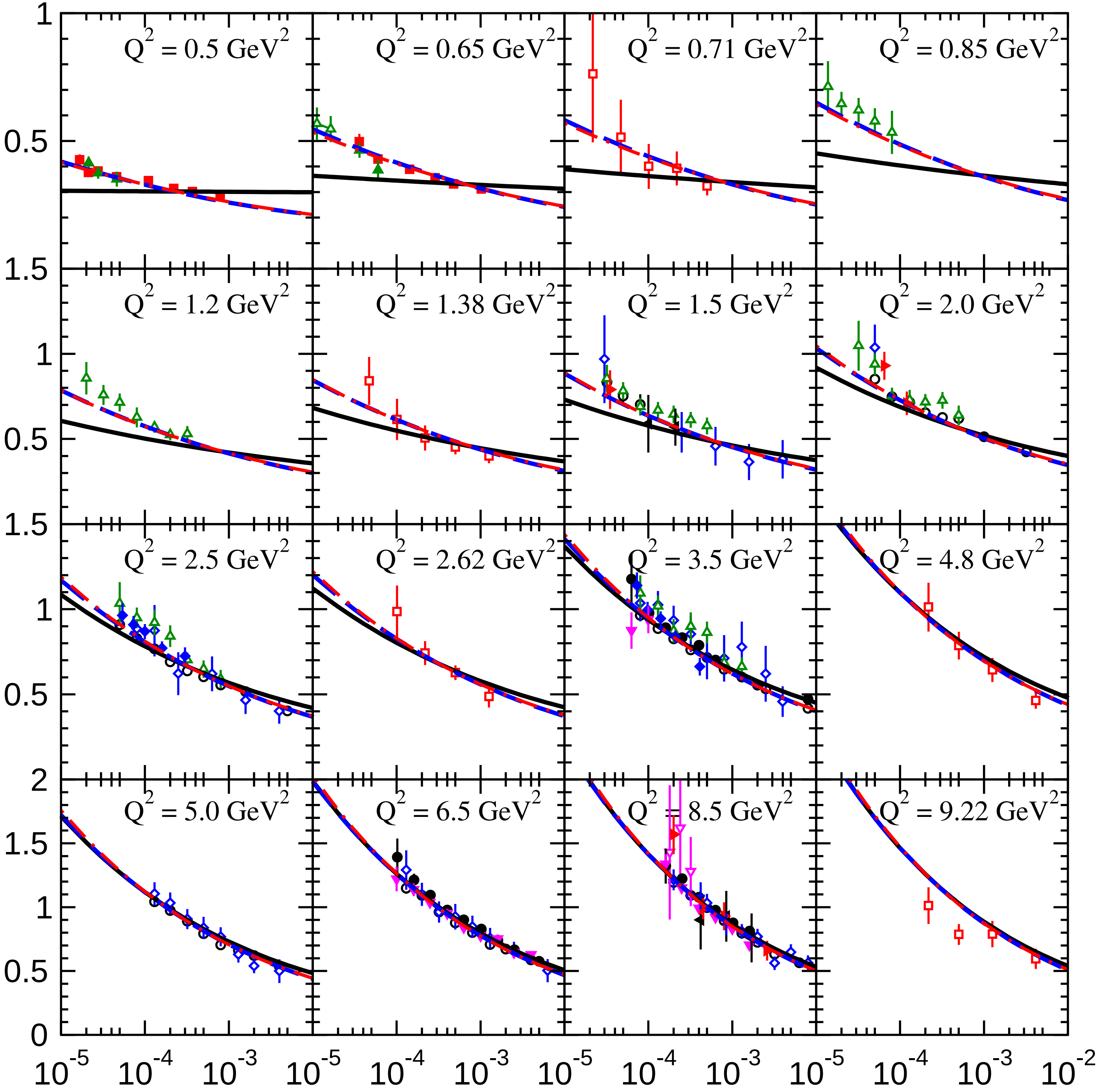

$Q^2 = 0.5$ GeV^2
$Q^2 = 0.65$ GeV^2
$Q^2 = 0.71$ GeV^2
$Q^2 = 0.85$ GeV^2
$Q^2 = 1.2$ GeV^2
$Q^2 = 1.38$ GeV^2
$Q^2 = 1.5$ GeV^2
$Q^2 = 2.0$ GeV^2
$Q^2 = 2.5$ GeV^2
$Q^2 = 2.62$ GeV^2
$Q^2 = 3.5$ GeV^2
$Q^2 = 4.8$ GeV^2
$Q^2 = 5.0$ GeV^2
$Q^2 = 6.5$ GeV^2
$Q^2 = 8.5$ GeV^2
$Q^2 = 9.22$ GeV^2
10^{-5} 10^{-4} 10^{-3} 10^{-2}

2. Results

A. **1. Valent part** $f_V(x,Q^2)$

$$f_V(x,Q^2) = \overline{f}_V(x,Q^2)\cdot\left(1+\sum_{k=1}^{N}\alpha_{k,V}x^k\right) \quad (N=1,2,3),$$

$$\overline{f}_V(x,Q^2) = \overline{A}_V(s)\,x^{\lambda_V}\,(1-x)^{\beta_V(s)}\left[\,1\;+\;\left(\frac{\overline{B}_V(s)}{\overline{A}_V(s)}-1\right)x\right],$$

where

$$\beta_V(s) = \beta_V(0)+\hat{d}_{NS}s, \quad \hat{d}_{NS}=\frac{32}{3}\frac{1}{2\beta_0},$$

$$\overline{B}_V(s) = B_V(0)\frac{e^{-p_Vs}}{\Gamma(1+\beta_V(s))}, \quad p_V=\hat{d}_{NS}\left(\gamma_E-\frac{3}{4}\right)s.$$

From quark counting rules: $\beta_V(0)\sim 3$.

The coefficients from low x asymptotics are

$$\overline{A}_V(s) = A_V(0)e^{-d_{NS}(1-\lambda_V)s},$$
$$d_{NS}(n) = \frac{32}{3}\frac{1}{2\beta_0}\left[\Psi(n+1) + \gamma_E - \frac{3}{4} - \frac{1}{2n(n+1)}\right],$$

where $\Psi(n+1)$ is Euler Ψ-function

and $\lambda_V \sim Const$ ($\neq$ function of s): $\lambda_V \sim 0.5 \div 0.7$

2. Sum rule

$$\int_0^1 \frac{dx}{x} f_V(x, Q^2) = Q_V, \quad Q_V = 3$$

Because porameterization, the sum rule can be applied only in one point: $s = s_C$. At other points, the sum rule can accepted only approximately.

So, (for $\alpha_{k,V} = 0$)

$$Q_V = \frac{\Gamma(\lambda_V)}{\Gamma(\lambda_V + 2 + \beta_V(s_C))} [\lambda_V B_V(0) e^{-p_V s_C} + \Gamma(2 + \beta_V(s_C)) A_V(0) e^{-d_{NS}(1-\lambda_V) s_C}]$$

It is possible to choose some "middle" value of s.

If $s_C = 0$, we have (for $\alpha_{k,V} = 0$)

$$Q_V = \frac{\Gamma(\lambda_V)}{\Gamma(\lambda_V + 2 + \beta_V(0))} \left[\lambda_V B_V(0) + \Gamma(2 + \beta_V(0)) A_V(0)\right]$$

B. 1. Nonsinglet part $\Delta(x,Q^2)$

It has Q^2 dependence with replacement

$$\beta_V(0) \to \beta_{NS}(0), \quad \lambda_V \to \lambda_{NS},$$
$$B_V(0) \to B_{NS}(0), \quad A_V(0) \to A_{NS}(0).$$

Moreover $\beta_i(0)$ and λ_i $(i = V, NS)$ should be close each other.

Thus

$$f_{NS}(x,Q^2) = \overline{f}_{NS}(x,Q^2) \cdot \left(1 + \sum_{k=1}^{N} \alpha_{k,NS} x^k\right) \quad (N = 1,2,3),$$
$$\overline{f}_{NS}(x) = \overline{A}_{NS}(s)\, x^{\lambda_{NS}} \,(1-x)^{\beta_{NS}(s)} \left[\, 1 \; + \; \left(\frac{\overline{B}_{NS}(s)}{\overline{A}_{NS}(s)} - 1\right) x \right],$$

where

$$\beta_{NS}(s) = \beta_{NS}(0) + \hat{d}_{NS}s, \quad \hat{d}_{NS} = \frac{32}{3}\frac{1}{2\beta_0},$$
$$\overline{B}_{NS}(s) = \overline{B}_{NS}(0) \frac{e^{-p_{NS}s}}{\Gamma(1+\beta_{NS}(s))}, \quad p_{NS} = \hat{d}_{NS}\left(\gamma_E - \frac{3}{4}\right)s,$$
$$\overline{A}_{NS}(s) = A_{NS}(0) e^{-d_{NS}(1-\lambda_{NS})s},$$

2. Sum rule

$$\int_0^1 \frac{dx}{x} f_{NS}(x,Q^2) = Q_{NS},$$

where the number Q_{NS} depends on considered process.

For example, for

$$\int_0^1 \frac{dx}{x}\left[F_2^{ep}(x,Q^2) - F_2^{en}(x,Q^2)\right] = Q_{NS}^{ep-en}, \quad Q_{NS}^{ep-en} = \frac{1}{3}$$

Note that at $f = 4$

$$F_2^{ep}(x,Q^2) = \frac{5}{18} f_{SI}(x,Q^2) + \frac{1}{6} f_{NS}^{ep}(x,Q^2),$$
$$F_2^{en}(x,Q^2) = \frac{5}{18} f_{SI}(x,Q^2) + \frac{1}{6} f_{NS}^{en}(x,Q^2),$$

where $f_{SI}(x,Q^2)$ is the singlet part

$$f_{SI}(x,Q^2) = f_V(x,Q^2) + f_{NS}(x,Q^2)$$

C. 1. Gluon density $f_G(x, Q^2)$ and sea part $f_S(x, Q^2)$

They are mixed during the DGLAP evolution.

After diagonalization we have

$$f_G(x,Q^2) = f_G^+(x,Q^2) + f_G^-(x,Q^2),$$
$$f_S(x,Q^2) = f_S^+(x,Q^2) + f_S^-(x,Q^2),$$

Moreover, at low x the $+$ component of sea quarks is diven by the $+$ component of gluons and, correspondly, the $-$ component of gluons is diven by the $-$ component of sea quarks:

$$f_S^+(x,Q^2) \sim \frac{f}{9}\frac{\rho I_1(\sigma)}{I_0(\sigma)} f_G^+(x,Q^2),$$
$$f_G^-(x,Q^2) \sim -\frac{4}{9} f_S^-(x,Q^2)$$

$$f^{\pm}_{(S,G)}(x,Q^2) = \overline{f}^{\pm}_{(S,G)}(x,Q^2)\cdot\left(1+\sum_{k=1}^{N}\alpha_{k,(S,G)}x^k\right) \quad (N=1,2,3),$$

$$\overline{f}^{-}_{S}(x,Q^2) = \overline{A}^{-}_{S}(s)\,(1-x)^{\beta_-(s)} \cdot\left[1 \;+\; \left(\frac{\overline{B}^{-}(s)}{\overline{A}^{-}(s)}-1\right)x\right],$$

$$\overline{f}^{+}_{G}(x,Q^2) = \left[\overline{A}^{+}_{G}(s)+\frac{4}{9}\overline{A}^{+}_{S}(s)\,(1-x)^{1+\beta_-(s)-\beta_+(s)}\right] \cdot(1-x)^{\beta_+(s)}\left[1 \;+\; \left(\frac{\overline{B}^{+}(s)}{\overline{A}^{+}(s)}-1\right)x\right],$$

where

$$\beta_\pm(s) = \beta_\pm(0)+\hat{d}_\pm s, \quad \hat{d}_- = \hat{d}_{NS}\frac{32}{3}\frac{1}{2\beta_0}, \quad \hat{d}_+ = \frac{24}{2\beta_0}$$

$$\overline{B}^{\pm}(s) = B^{\pm}(0)\frac{e^{-p_\pm s}}{\Gamma(1+\beta_\pm(s))},$$

$$p_- = \hat{d}_{NS}\left(\gamma_E-\frac{3}{4}\right)s, \quad p_+ = \hat{d}_+\left(\gamma_E-\frac{\beta_0}{12}\right)s$$

are coefficients from large x asymptotics. The constant $\beta_{\pm}(0)$ can be taken from quark counting rules:

$$\beta_{+}(0) \sim \beta_{NS}(0) + 1 \sim 4,$$
$$\beta_{-}(0) \sim \beta_{+}(0) + 1 \sim 5,$$

The coefficients from low x asymptotics are

$$\begin{aligned}
\overline{A}_S^{-}(s) &= A_S(0) e^{-\overline{d}_{-}(1)s}, \\
\overline{A}_S^{+}(s) &= A_S(0) e^{-\overline{d}_{+}(1)s}, \quad \overline{A}_G^{+}(s) = A_G(0) e^{-\overline{d}_{+}(1)s}, \\
\overline{d}_{-}(1) &= \frac{16f}{27\beta_0}, \quad \overline{d}_{+}(1) = 1 + \frac{20f}{27\beta_0},
\end{aligned}$$

where it is possible to put

$$\frac{B^{+}(0)}{A^{+}(0)} - 1 = \frac{B^{-}(0)}{A^{-}(0)} - 1 \equiv t$$

2. Sum rule. Full momentum

$$\int_0^1 dx[f_G(x,Q^2) + f_{SI}(x,Q^2)] = 1,$$

Because porameterization, the sum rule can be applied only in one point: $s = s_C$. At other points, the sum rule can accepted only approximately.

If $s_C = 0$, we have (for $\alpha_k = 0$)

$$1 = \frac{\Gamma(\lambda_V + 1)}{\Gamma(\lambda_V + 3 + \beta_V(0))} \left[(\lambda_V + 1)\overline{B}_V(0) + \Gamma(2 + \beta_V(0))\overline{A}_V(0)\right]$$
$$+A_S(0) \left[\frac{1}{\beta_-(0) + 1} + \frac{t}{\beta_-(0) + 2}\right] + A_G(0) \left[\frac{1}{\beta_+(0) + 1} + \frac{t}{\beta_+(0) + 2}\right]$$

Conclusion

- I have demonstrated the low x and large x asymptotics of parton densities and also their parameterizations.
- Low x asymptotics are in good agreement with data from HERA.

Next steps:

- To fix parameters of the parameterizations from DIS data.
- To analyse of some LHC processes and to compare the results with MRST, CTEQ and Alekhin predictions.