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A Global Fit to Scattering Data with NLL BFKL Resummation

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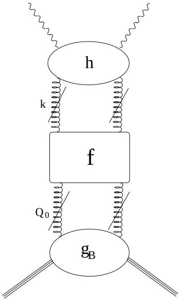
Overview

- Solution of the BFKL equation at NLL order with running coupling.
- Comparison of gluon splitting function with other resummation approaches (ABF, CCSS).
- Treatment of Heavy Flavours.
- Global parton fit using NLL resummed coefficient and splitting functions!

The High Energy Problem



- Coefficient and splitting functions for the proton structure functions unstable at low x due to terms $\sim x^{-1}\bar{\alpha}_S^n \log^m(1/x)$, $m \leq n-1$.
- Divergence due to t-channel gluon exchange at LL order, with some quark mixing at NLL order.
- Must resum the gluon 4-point function by solving the BFKL equation.
- Relate gluon to structure functions using the k_T factorisation formula (Collins & Ellis; Catani, Ciafaloni & Hautmann).

Running coupling solution of BFKL equation

- Mellin moments:

$$f(\gamma, N) = \int_0^\infty (k^2)^{-\gamma-1} \int_0^1 dx x^N f(x, k^2)$$

- Substitute LO running coupling into BFKL equation (Collins & Kwiecinski):

$$\frac{d^2 f(\gamma, N)}{d\gamma^2} = \frac{d^2 f_I(\gamma, Q_0^2)}{d\gamma^2} - \frac{1}{\bar{\beta}_0 N} \frac{d(\chi_0(\gamma) f(\gamma, N))}{d\gamma} + \frac{\pi}{3\bar{\beta}_0^2 N} \chi_1(\gamma) f(\gamma, N),$$

  with $\bar{\beta}_0 = 3/(\pi\beta_0)$.

- Solve with ansatz:

$$f(N, \gamma) = \exp\left(-\frac{X_1(\gamma)}{\bar{\beta}_0 N}\right) \int_\gamma^\infty A(\tilde{\gamma}) \exp\left(\frac{X_1(\tilde{\gamma})}{\bar{\beta}_0 N}\right) d\tilde{\gamma}$$

  (Ciafaloni & Colferai).

- Can shift lower limit $\gamma \to 0$ up to power-suppressed corrections (Thorne).
- Gluon factorises:

$$\mathcal{G}(N,t) = \mathcal{G}_E(N,t)\mathcal{G}_I(Q_0^2,N)$$

($t = \log Q^2/\Lambda^2$).

- Perturbative piece:

$$\mathcal{G}_E^1(N,t) = \frac{1}{2\pi\imath}\int_{1/2-\imath\infty}^{1/2+\imath\infty} \frac{f^{\beta_0}}{\gamma} \exp\left[\gamma t - X_1(\gamma,N)/(\bar{\beta}_0 N)\right] d\gamma$$

with:

$$X_1(\gamma,N) = \int_{\frac{1}{2}}^{\gamma} \left[\chi_0(\tilde{\gamma}) + N\frac{\chi_1(\tilde{\gamma})}{\chi_0(\tilde{\gamma})}\right] d\tilde{\gamma}.$$

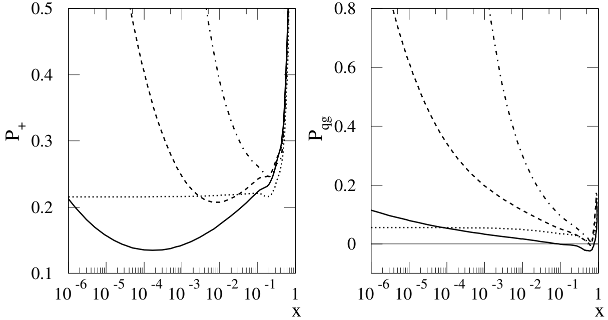
- Similarly, get structure functions:

$$\mathcal{F}_E^1(N,t) = \frac{1}{2\pi\imath}\int_{1/2-\imath\infty}^{1/2+\imath\infty} \frac{h(\gamma,N) f^{\beta_0}}{\gamma} \exp\left[\gamma t - X_1(\gamma,N)/(\bar{\beta}_0 N)\right] d\gamma$$

- If impact factors known, can disentangle all resummed coefficient and splitting functions (within a particular factorisation scheme).
- However, NLL impact factors $h(\gamma, N)$ not known. Work in progress (Bartels, Colferai, Gieseke & Kyrieleis).
- Instead LL factors with exact gluon kinematics have been calculated (Bialas, Navelet & Peschanski; White, Peschanski & Thorne).
- These provide a very good estimate to the full NLL impact factors (White & Thorne).
- Can use these to calculate all the NLL resummed coefficient and splitting functions in the DIS scheme.
- Finally, combine resummed results with NLO DGLAP:

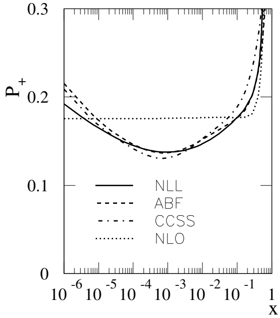
$$P^{tot.} = P^{NLL} + P^{NLO} - \left[P^{NLL(0)} + P^{NLL(1)}\right]$$

Results for Splitting Functions



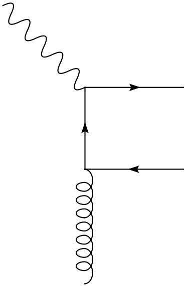
- Results shown at $n_f = 4,\ t = 6$.
- Running coupling suppresses low x divergence.
- NLL kernel and impact factor effects lead to even more suppression.
- Main feature is a dip below the NLO DGLAP result.

Comparison with Alternative Approaches



- The ABF and CCSS groups calculate P_+ $(= P_{gg} + \frac{4}{9} P_{qg})$ with $n_f = 0$.
- Results agree closely...
- ...even without resummation of BFKL kernel.
- Now need to consider heavy flavour coefficients...

Variable Flavour Number Schemes



- In DIS, can produce final state heavy quarks by boson gluon fusion (Witten).
- Diagrams diverge as $Q^2 \to \infty$ due to terms $\sim \alpha_S^n \log^n M^2/Q^2$.

- Get round this by defining parton distributions for the heavy species above a suitable matching scale e.g. $Q^2 = M^2$.
- Matching conditions exist at this scale (Buza, Matiounine, Smith & van Neerven) between the high Q^2 and low Q^2 partons.

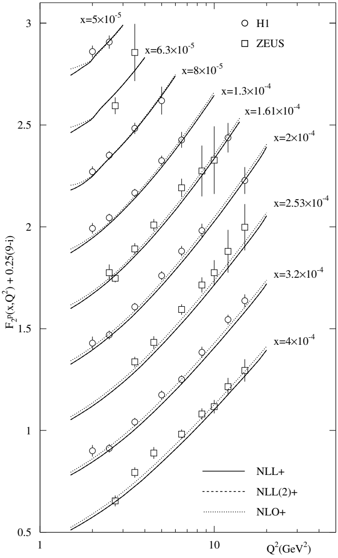
The DIS(χ) Scheme

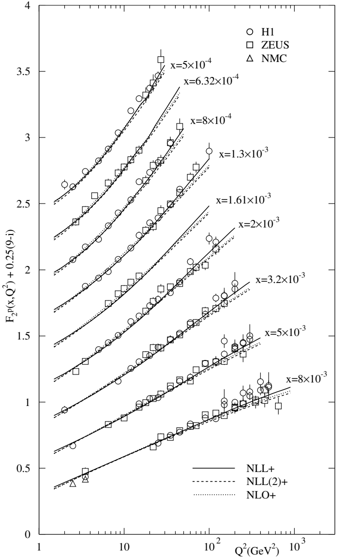
- Both sets of partons are ambiguous - one must fix a particular variable flavour scheme according to two types of choice.
- Have developed a scheme that allows one to disentangle the meaning of the impact factors in terms of heavy flavour coefficient and matching functions.
- Called the DIS(χ) scheme by analogy with the DIS scheme for massless quarks.
- Allows the consistent implementation of small x resummations in the heavy flavour sector.
- Approximate results obtained for massive resummed quantities at NLL+NLO order.
- Thus have everything necessary for a global parton fit!

Quality of Fit

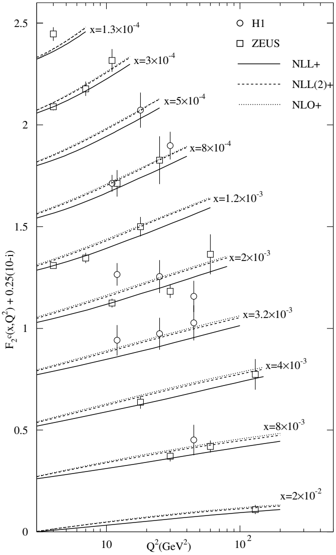
- Neutral and charged current data for F_2^p (including F_2^c) from H1, ZEUS.
- Data for F_2^p, F_2^n from BCDMS, NMC, SLAC and E665.
- $F_{2,3}^{\nu(\bar{\nu})N}$ from CCFR. F_2^D/F_2^p from NMC.
- Non-DIS: DY data from E866 / NuSea; DY asymmetry from NA51; $\sigma_{DY}^{PD}/\sigma_{DY}^{pp}$ from E866; W asymmetry from CDF.
- NLL resummed fit gives an overall fit quality $\chi^2 = 2249$ for 2181 data points.
- Compare NLO DIS scheme $\chi^2 = 2352$ and $\overline{\text{MS}}$ scheme $\chi^2 = 2307$.
- A previous LL resummed fit gave $\chi^2 = 2336$, with significant momentum conservation violation.
- Main improvement in the HERA data, as expected.
- Description of F_2^c benefits from DIS(χ) scheme.
- Resummation seems to decrease tension between data sets.

Results - F_2


x=5×10-5
x=6.3×10-5
x=8×10-5
x=1.3×10-4
x=1.61×10-4
x=2×10-4
x=2.53×10-4
x=3.2×10-4
x=4×10-4
H1
ZEUS
NLL+
NLL(2)+
NLO+
F2p(x,Q2) + 0.25(9-i)
Q2(GeV2)
3
2.5
2
1.5
1
0.5
1
10

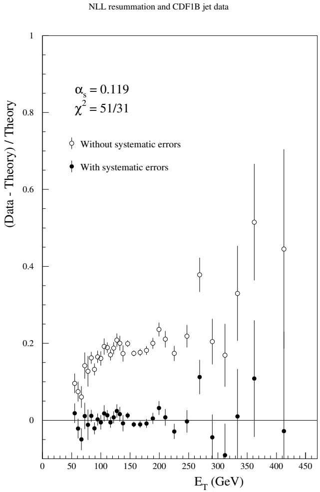

x=5×10-4
x=6.32×10-4
x=8×10-4
x=1.3×10-3
x=1.61×10-3
x=2×10-3
x=3.2×10-3
x=5×10-3
x=8×10-3
H1
ZEUS
NMC
NLL+
NLL(2)+
NLO+
F2p(x,Q2) + 0.25(9-i)
Q2(GeV2)
4
3.5
3
2.5
2
1.5
1
0.5
0
1
10
10^2
10^3

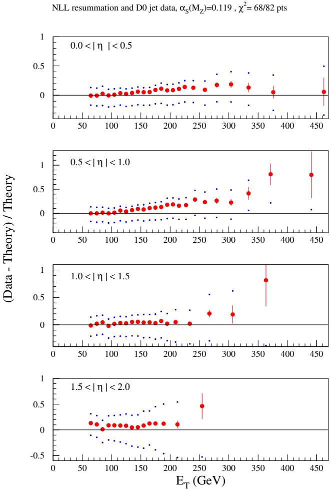
Results - F_2^c



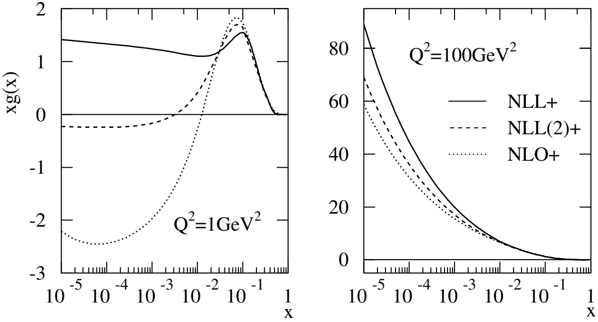
- Resummed fit performs better for small x data - note slope as Q^2 increases.
- Fit is also improved over the whole range of x.
- Resummed F_2^c at lower end of range allowed by data.

Comparison with Tevatron jet data



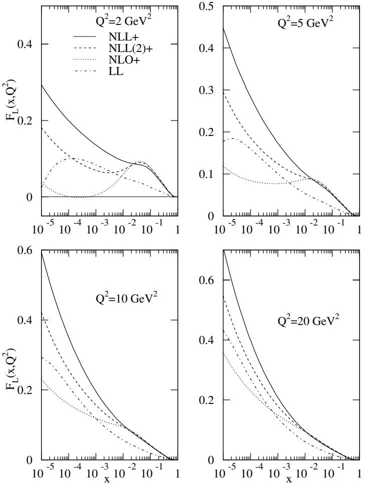


Gluon Distribution



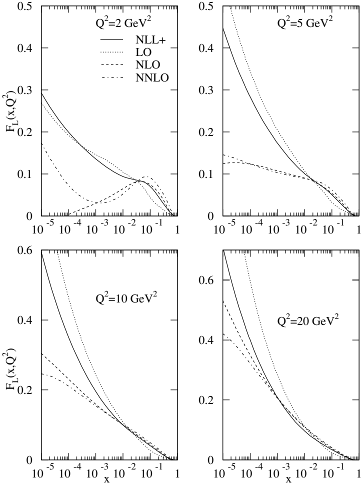
- Gluons differ for $x \lesssim 10^{-2}$.
- NLL resummed gluon positive and growing at small x!
- Not true at fixed order.
- Positive gluon avoids negative structure functions.
- See this in e.g. F_L...

Longitudinal Structure Function



- NLO result goes negative.
- LL prediction more stable but turns over due to negative gluon. Also inconsistent with NLO at high x.
- NLL result is much more sensible!

Longitudinal Structure Function



- Clearly see perturbative instability in fixed order results.
- This is cured by the resummation.

Conclusions

- Have combined NLL resummations with a NLO fixed order QCD expansion in massive and massless sectors, including running coupling effects.
- Results for P_+ comparable to alternative approaches, even without kernel resummation.
- Running coupling and NLL kernel corrections each lead to large suppressions in the small x divergence. NLL impact factors give further suppression.
- Global fit with improved splitting and coefficient functions gives an excellent description of data.
- Gluon from resummed fit positive at low x and Q^2.
- Prediction for F_L is stable in this regime.

$\Rightarrow$ Very compelling evidence for BFKL effects!