A Matrix Formulation for Small-x Singlet Evolution

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- Aim of the work
  - To provide a reliable description of parton densities at small-x while keeping the well known behaviour at larger-x;
  - To devise a small-x resummation in matrix form:
    quarks and gluons are treated on the same ground
    (in a collinear factorization scheme as close as possible to $\overline{\mathrm{MS}}$)
- Outline
  - Review of 1-channel (gluon) improved BFKL equation
  - Limits of all resummation approaches developed so far
  - Formulation with 2 channels (sea-quarks and gluons):
    2×2 matrix kernel merging NLO + NLLx
  - Numerical results
  - Conclusions

Recalling 1-Channel Resummation...

At fixed coupling

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  Singular terms in one place correspond to less-singular terms in the other formulation
- From the knowledge of LO $A(\omega)$, NLO $B(\omega)$ ... one can predict higher order terms in BFKL hierarchy (presumably very large in the collinear limit $\gamma \to 0$)

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The RG analysis we developed (`hep-ph/0507106, 0601200`) accomplishes this task: by defining an improved kernel

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predicts and resums the most singular poles

$$\chi_0^\omega(\gamma) \sim \frac{1}{\gamma + \frac{\omega}{2}} + \frac{1}{1 + \frac{\omega}{2} - \gamma} \sim \frac{1}{\gamma} - \omega \frac{1}{2\gamma^2} \sim \frac{1}{\gamma} - \frac{\bar{\alpha}_s}{2\gamma^3} \qquad [\omega = \bar{\alpha}_s \chi(\gamma) \sim \frac{\bar{\alpha}_s}{\gamma}]$$

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From the *improved equation* $\mathcal{F}_g = \mathcal{K}\mathcal{F}_g$ we can compute

- hard Pomeron exponent $\omega_s(\bar{\alpha}_s)$
- resummed anom. dim. $\gamma_+(\bar{\alpha}_s,\omega)$ and effective split. funct. $P_{\rm eff}(\bar{\alpha}_s,x)$

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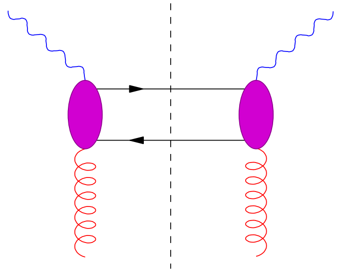
- These approaches limit themselves to a consistent resummation scheme for the gluon only
  deriving quark-sea by $\boldsymbol{k}$-factorization in DIS scheme



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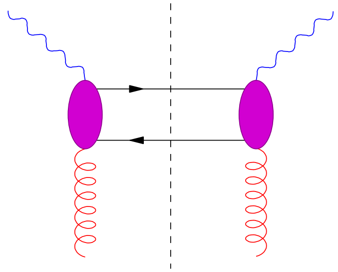

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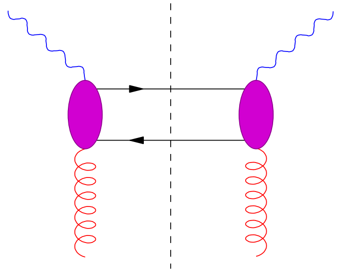

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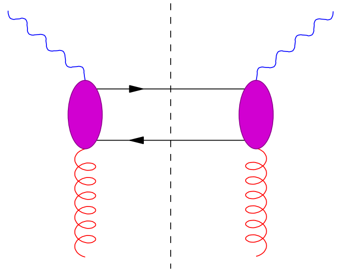

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- Our choice: incorporate known anomalous dimension up to NLO;
  at high-energy ($\omega \sim 0$) we incorporate NLLx BFKL kernel for the gluon channel only

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- Solution: rearrange colour factors and high-energy propagators $1/\omega$ between vertices of ladder diagrams;

  mathematically: ∃ diagonal similarity transform. $S^{-1}\Gamma_0^T S = \Gamma_0$

$$\Rightarrow \qquad \mathcal{K}_0 \sim \Gamma_0(\omega) \left(\frac{1}{\gamma + \frac{\omega}{2}} + \frac{1}{1 + \frac{\omega}{2} - \gamma} \right) \equiv \Gamma_0(\omega) \chi_c^{\omega}(\gamma)$$

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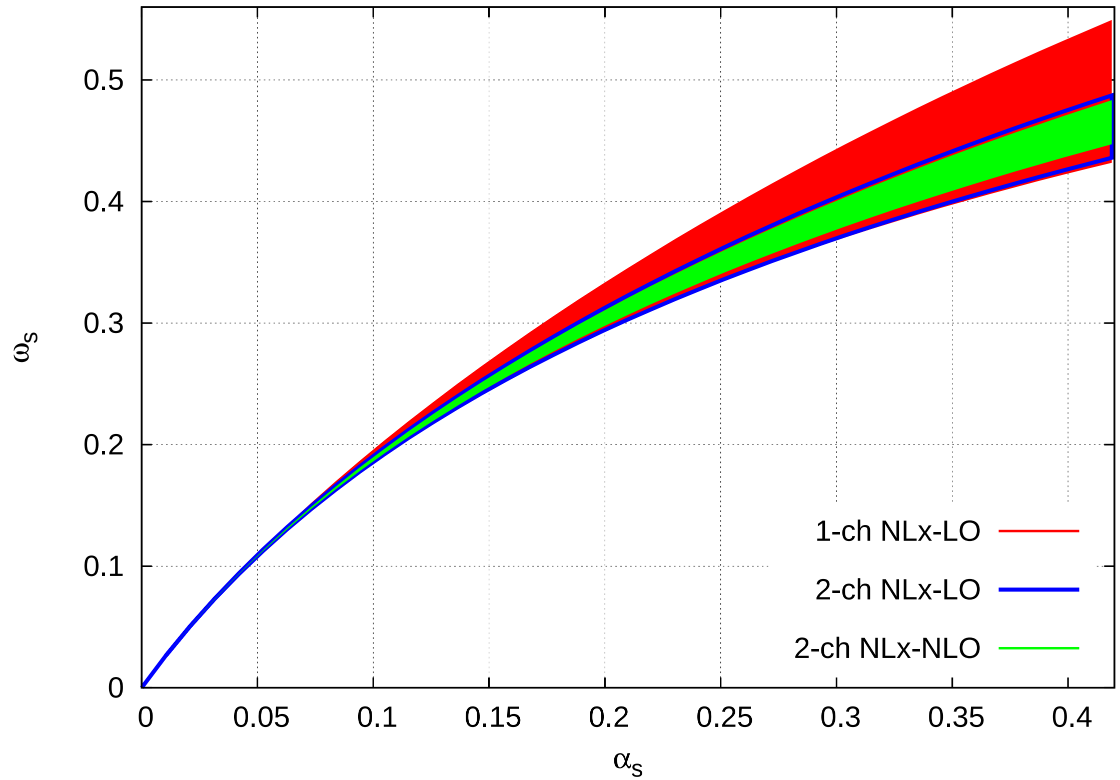
- Running coupling: introduced in $(\boldsymbol{k}, x)$ space (analytic double inverse Mellin transf.)

$$\mathcal{K}(\boldsymbol{k},\boldsymbol{k}';x) = \bar{\alpha}_s(\boldsymbol{k}^2_>)\mathcal{K}_0(\boldsymbol{k},\boldsymbol{k}';x) + \bar{\alpha}_s^2(\boldsymbol{k}^2_>)\mathcal{K}_1(\boldsymbol{k},\boldsymbol{k}';x)$$

($\mathcal{K}_1$ depends on the choice of run.coupl. scale $\boldsymbol{k}_> \equiv \max(\boldsymbol{k},\boldsymbol{k}')$)

Frozen Coupling Features

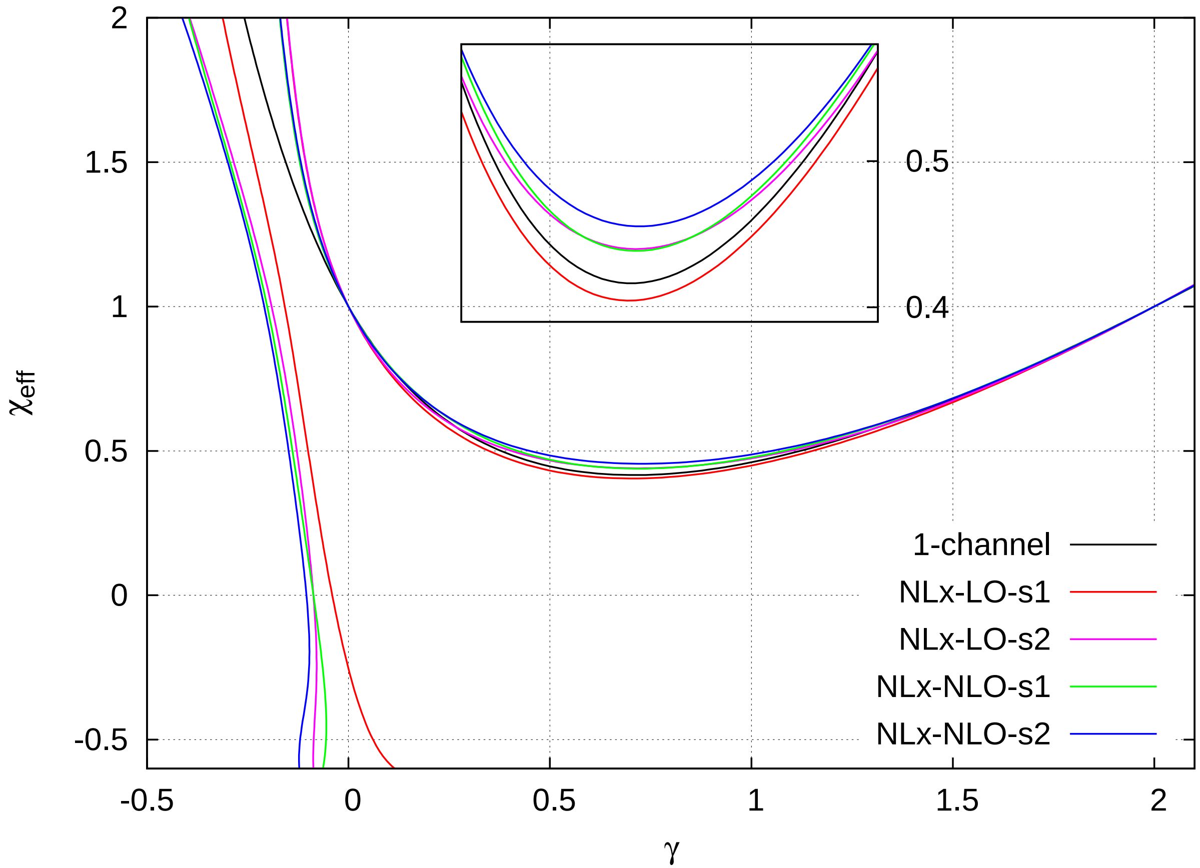
Hard Pomeron exponent $\omega_s(\alpha_s)$ and resummation scheme uncertainty



- various estimates are stable and compatible with each other
- resummation scheme uncertainty is reduced in the 2-channel formulation, in particular when NLO corrections are included

Frozen Coupling Features

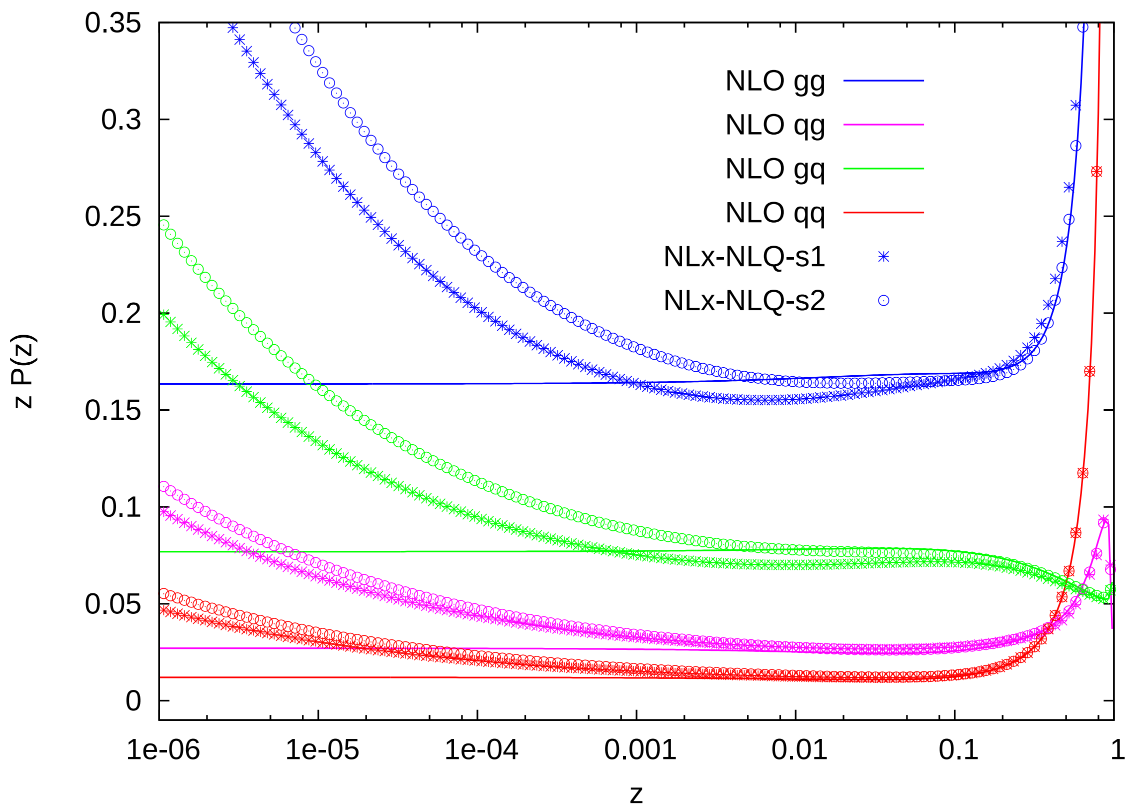
Effective eigenvalue functions: $\omega = \chi_{\pm}(\alpha_s, \gamma_{\pm})$ (here $\alpha_s = 0.4$)



fixed points at $\gamma = 0, 2$ and $\omega = 1$ $\Rightarrow$ momentum conservation in both collinear and anti-collinear limits

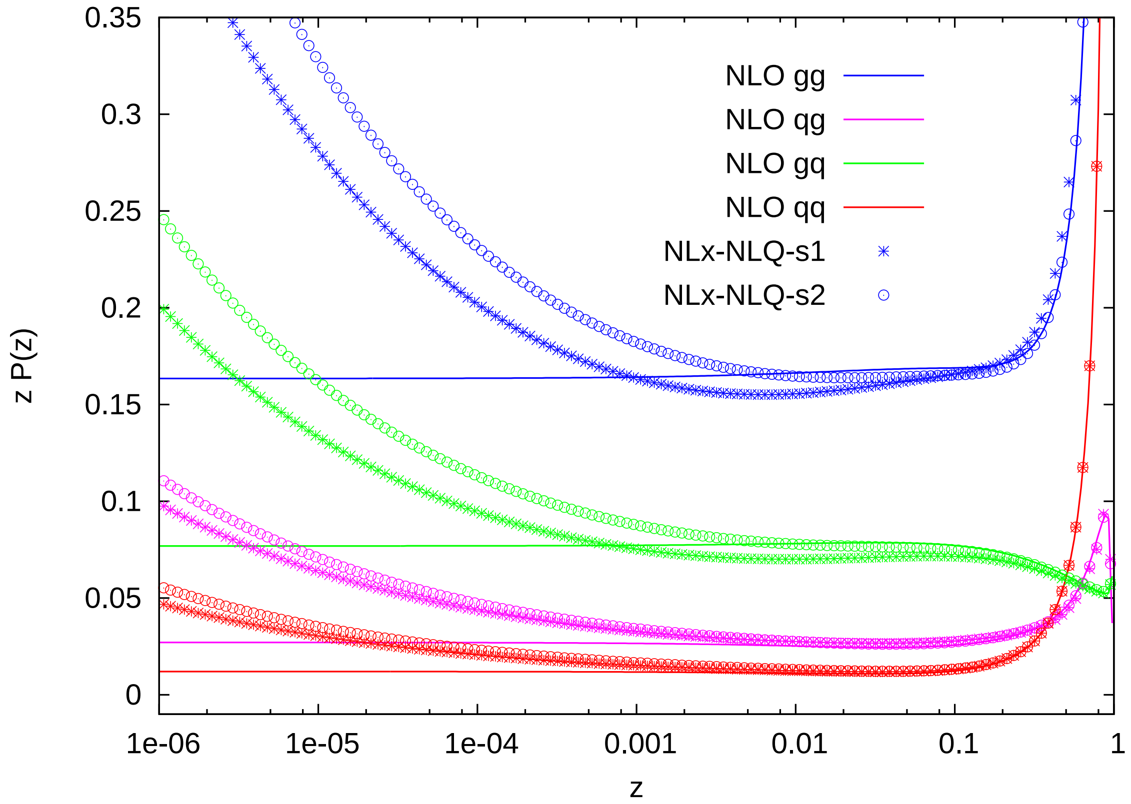
Frozen Coupling Features

Resummed $\overline{\mathrm{MS}}$ splitting functions $z\,P(\alpha_s, z)$ for $\alpha_s = 0.2$



Frozen Coupling Features

Resummed $\overline{\mathrm{MS}}$ splitting functions $z\,P(\alpha_s, z)$ for $\alpha_s = 0.2$



- at large-x fixed order and resummed splitting functions overlap
- at moderate-x resummed splitting functions show a small dip
- final rise sets in at very small-x
- resummation scheme uncertainty is small

Summary

- We propose a small-x evolution scheme in matrix form
  - quarks and gluons treated on the same ground
  - no need of $\boldsymbol{k}$-factorization to derive anomalous dimensions in DIS scheme
  - splitting functions already in $\overline{\mathrm{MS}}$ scheme

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- Running coupling can be straightforwardly introduced; features are under investigation
- A fully resummed fit needs resummed coefficient functions, but one could try first with LO impact factors with exact kinematics