PERTURBATIVELY STABLE EVOLUTION OF QUARKS AND GLUONS AT SMALL x

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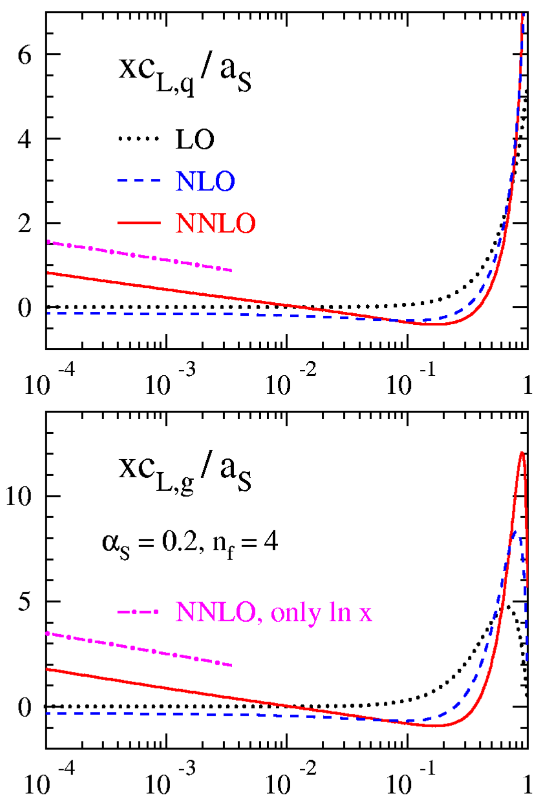
NNLO CORRECTIONS
PERTURBATIVE INSTABILITY AT SMALL x

THEORY

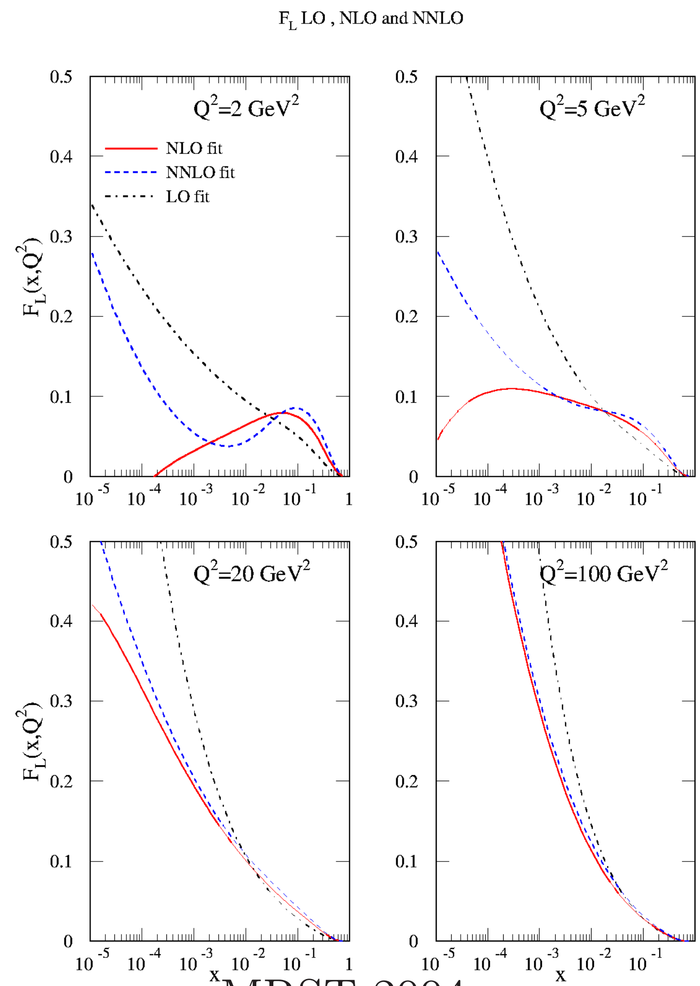
- Perturbation theory unstable
- leading log approx no good



the coefficient function C_L

Moch, Vermaseren, Vogt 2005

PHENOMENOLOGY
F_L FIT



MRST 2004

RESUMMATION

THE APPROACH OF ALTARELLI, BALL, S.F.

- PERTURBATIVE STABILIZATION ($n_f = 0$)
  - DUALITY (MOMENTUM CONSERVATION)
  - EXCHANGE SYMMETRY
  - RUNNING COUPLING
- EXTENSION TO THE QUARK SECTOR
  - $n_f \neq 0$ RESUMMATION
  - CHOICE OF FACTORIZATION SCHEME
- RESULTS
  - SPLITTING FUNCTIONS
  - QUARK AND GLUON EVOLUTION

THE FIRST INGREDIENT: DUALITY (fixed coupling)

THE ALTARELLI-PARISI EQN IS AN INTEGRO–DIFFERENTIAL EQUATION $\Rightarrow$ IT CAN BE EQUIVALENTLY VIEWED AS Q^2–EVOLUTION EQUATION FOR x–MOMENTS (usual RG eqn.), OR x–EVOLUTION EQUATION FOR Q^2–MOMENTS (BFKL eqn.)

EVOLUTION IN $t = \ln Q^2$

$$\frac{d}{dt} G(N,t) = \gamma(N,\alpha_s)\; G(N,t)$$

MELLIN x–MOMENTS

$$G(N,t) = \int_0^\infty d\xi\, e^{-N\xi}\; G(\xi,t)$$

EVOLUTION IN $\xi = \ln 1/x$

$$\frac{d}{d\xi} G(\xi,M) = \chi(M,\alpha_s)\; G(\xi,M)$$

MELLIN Q^2–MOMENTS

$$G(\xi,M) = \int_{-\infty}^\infty dt\, e^{-Mt}\; G(\xi,t)$$

THE TWO EQUATIONS HAVE THE SAME SOLUTIONS PROVIDED THE EVOLUTION KERNELS ARE RELATED BY

$$\chi(\gamma(N,\alpha_s),\alpha_s) = N$$

$$\gamma(\chi(M,\alpha_s),\alpha_s) = M$$

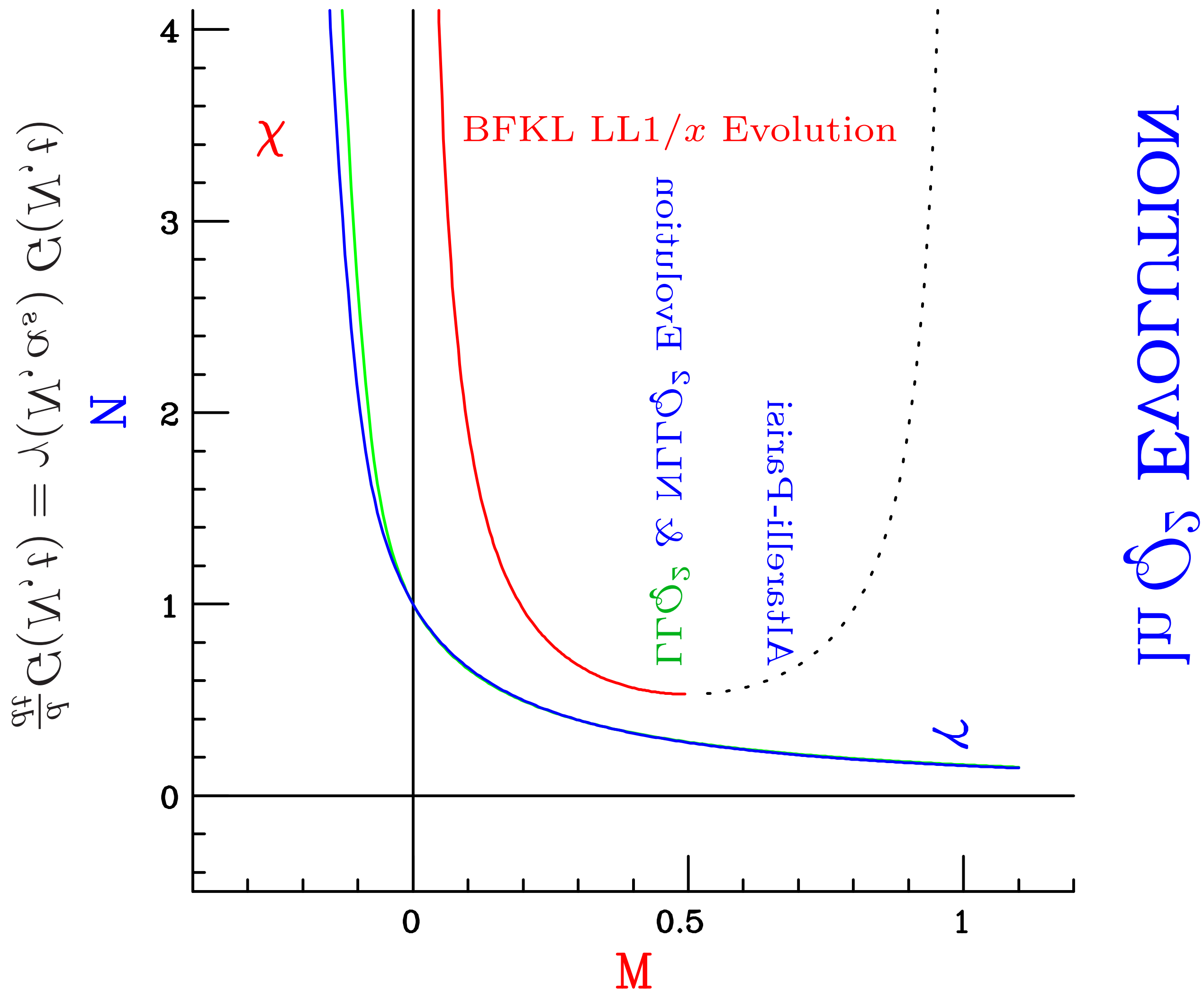
& BOUNDARY CONDITIONS RELATED BY

$$H_0[M] \to G_0(N) = H_0[\gamma(N,\alpha_s)]/\chi'(\gamma(N,\alpha_s))$$

. . . CAN SWITCH FROM LLQ^2 TO LL$1/x$

CHOOSING THE EVOLUTION KERNEL

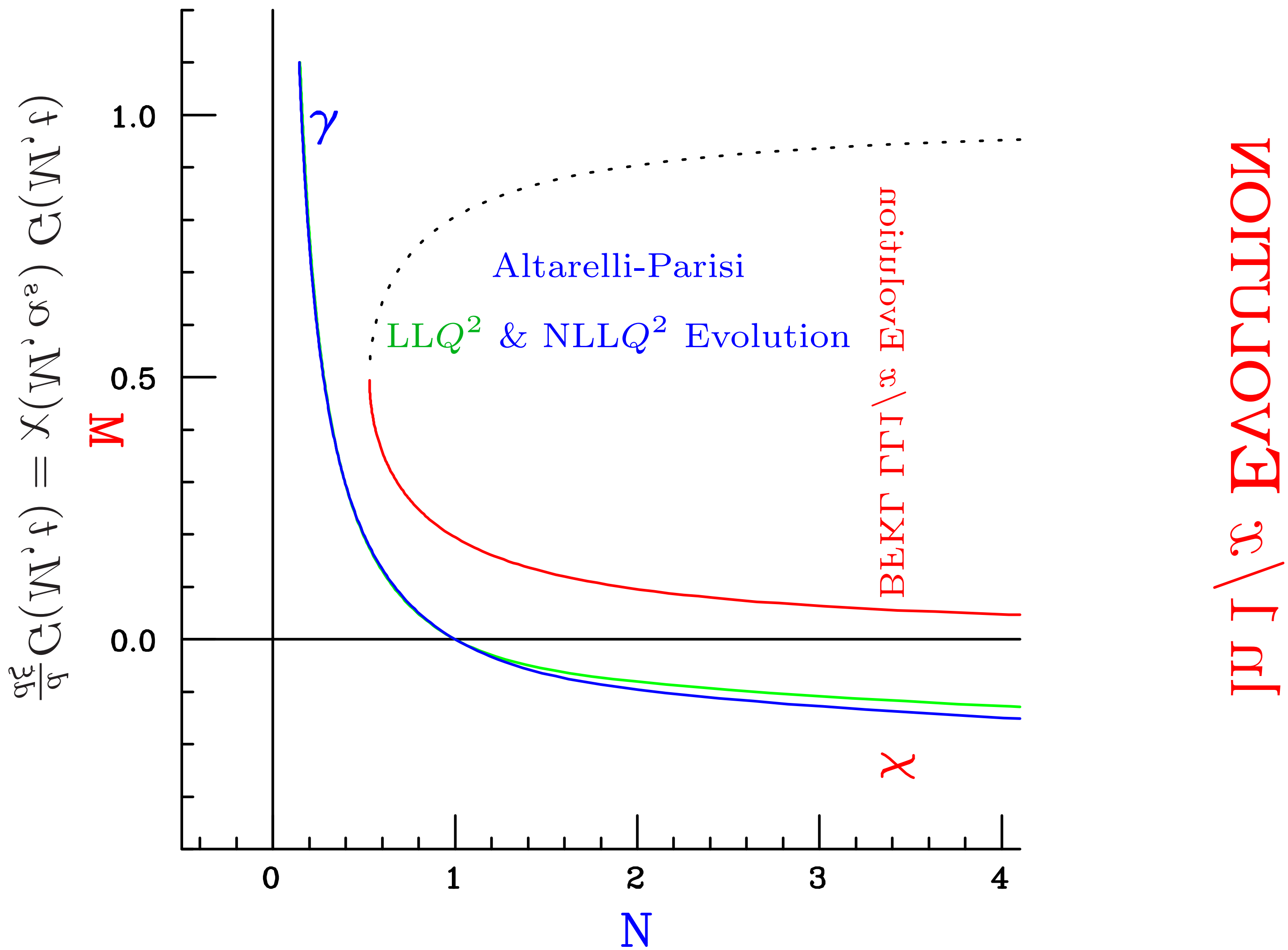
$\ln 1/x$ EVOLUTION



$$\frac{d}{d\xi}G(M,t) = \chi(M,\alpha_s)\ G(M,t)$$

. . . IN EITHER EQUATION!

$\ln Q^2$ Evolution

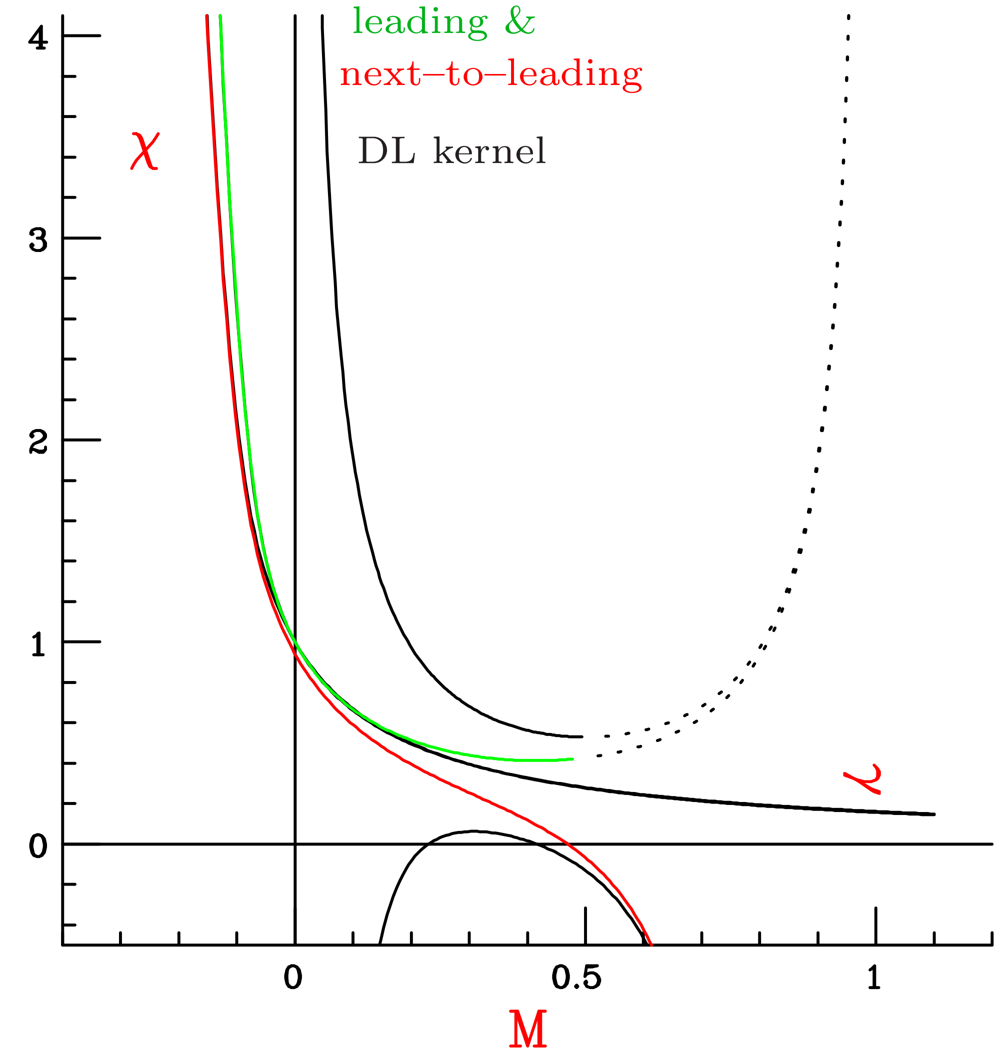


$$\frac{d}{dt}G(N,t) = \gamma(N,\alpha_s)\ G(N,t)$$

Double–Leading Evolution

$\chi_s \underset{M\to 0}{\sim} \frac{\alpha_s}{\alpha_s+M} = \frac{\alpha_s}{M} - \frac{\alpha_s^2}{M^2} + \frac{\alpha_s^3}{M^3} + \cdots$

$\chi(\gamma) = M \to \chi(0) = 1$



Momentum Conservation

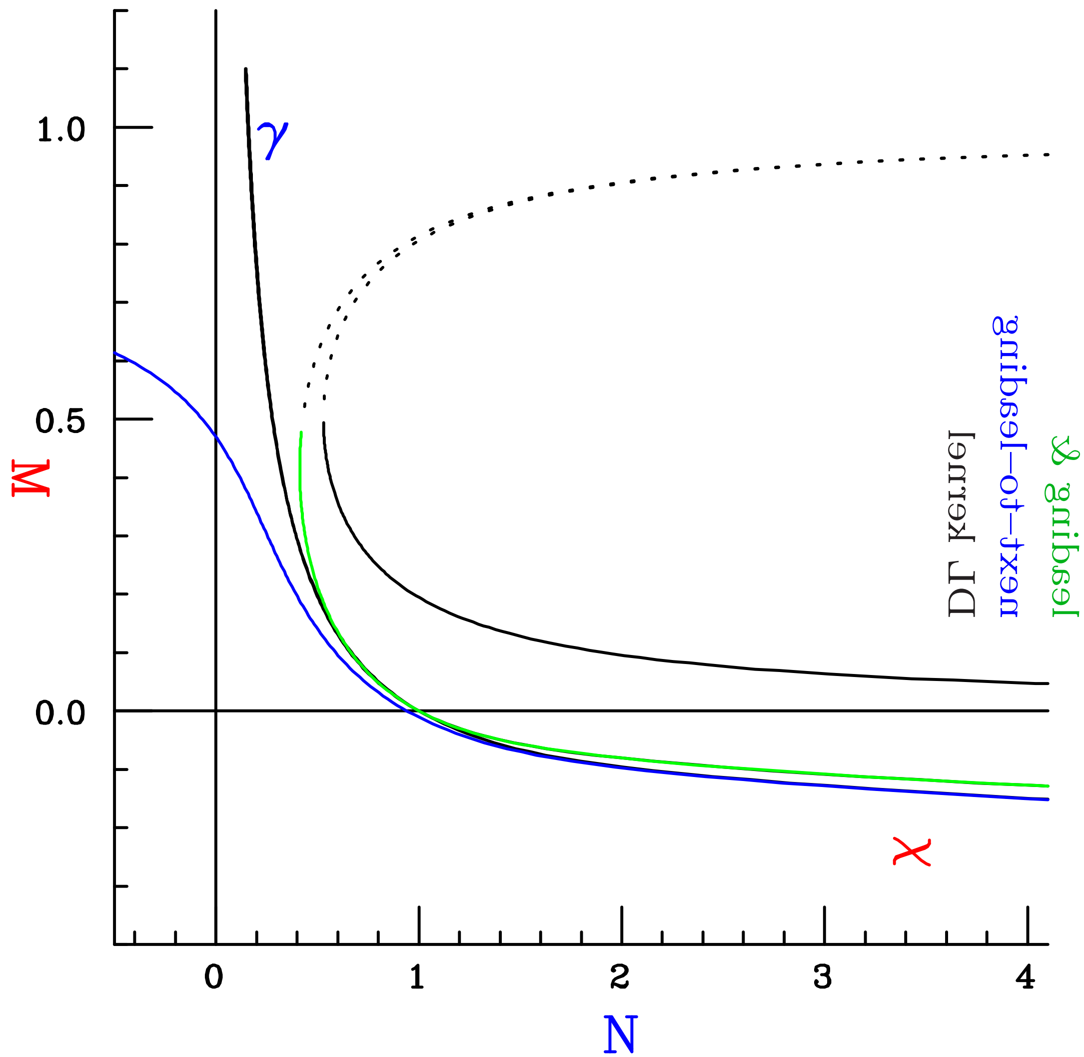
$\gamma(1) = 0$

- perturbative expansion stabilized in $M \sim 0$ region

Double–Leading Evolution

Momentum Conservation!

$$\gamma(1) = 0$$

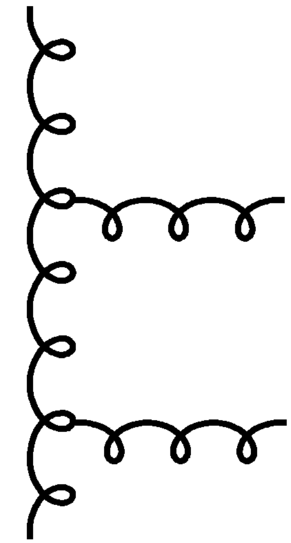


$$\chi(\gamma) = N \rightarrow \chi(0) = 1$$

$$\chi_s(M) \underset{M \to 0}{\sim} \frac{\alpha}{\alpha + M} = \frac{\alpha}{M} - \frac{\alpha^2}{M^2} + \frac{\alpha^3}{M^3} + \ldots$$

THE SECOND INGREDIENT: EXCHANGE SYMMETRY

DIAGRAMS FOR $\ln 1/x$ EVOLUTION KERNEL



$$\frac{d}{d\xi} G(\xi, M) = \chi(M, \alpha_s)\ G(\xi, M)$$

$$\chi(\xi, M) = \int_{-\infty}^{\infty} \frac{dQ^2}{Q^2} \left(\frac{Q^2}{k^2}\right)^{-M} \chi(\xi, \tfrac{Q^2}{k^2})$$

SYMMETRIC UPON INTERCHANGE

OF INITIAL AND FINAL PARTON VIRTUALITIES

$$Q^2 \leftrightarrow k^2 \Leftrightarrow M \leftrightarrow 1 - M$$

COLLINEAR RES. OF $\frac{1}{M}$ POLES $\leftrightarrow$ ANTICOLLINEAR RES. OF $\frac{1}{1-M}$ POLES

SYMMETRY BREAKING

- DIS KINEMATIC VARIABLES $s = \frac{Q^2}{x}$ (small x)
- RUNNING OF THE COUPLING $\alpha_s(Q^2)$

BOTH CAN BE DETERMINED EXACTLY

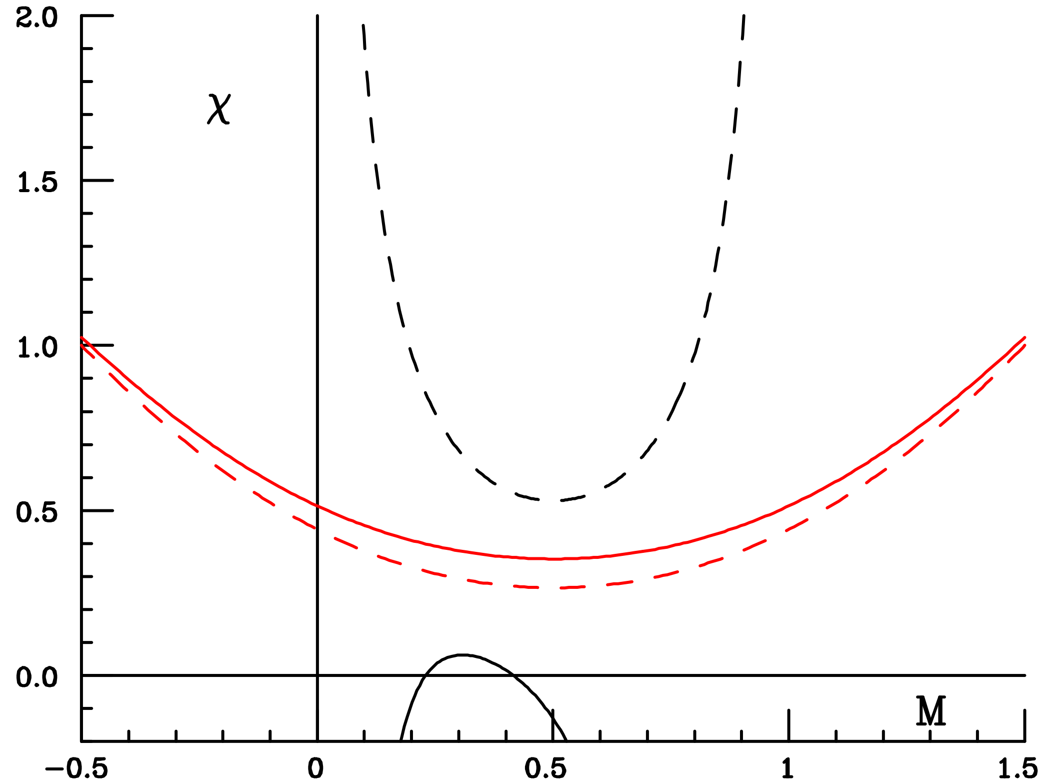
SYMMETRIZED EXPANSION

THE χ KERNEL

MOMENTUM CONSERVATION + SYMMETRY$\Rightarrow\chi$ ALWAYS HAS A MINIMUM

SYMMETRIC VARIABLES

- LO, NLO SYMMETRIC RESUMMED CLOSE TO EACH OTHER
- χ IS AN ENTIRE FCTN (QUADRATIC APPROX. IS EXCELLENT!)
- RESUMMED NLO HIGHER THAN LO



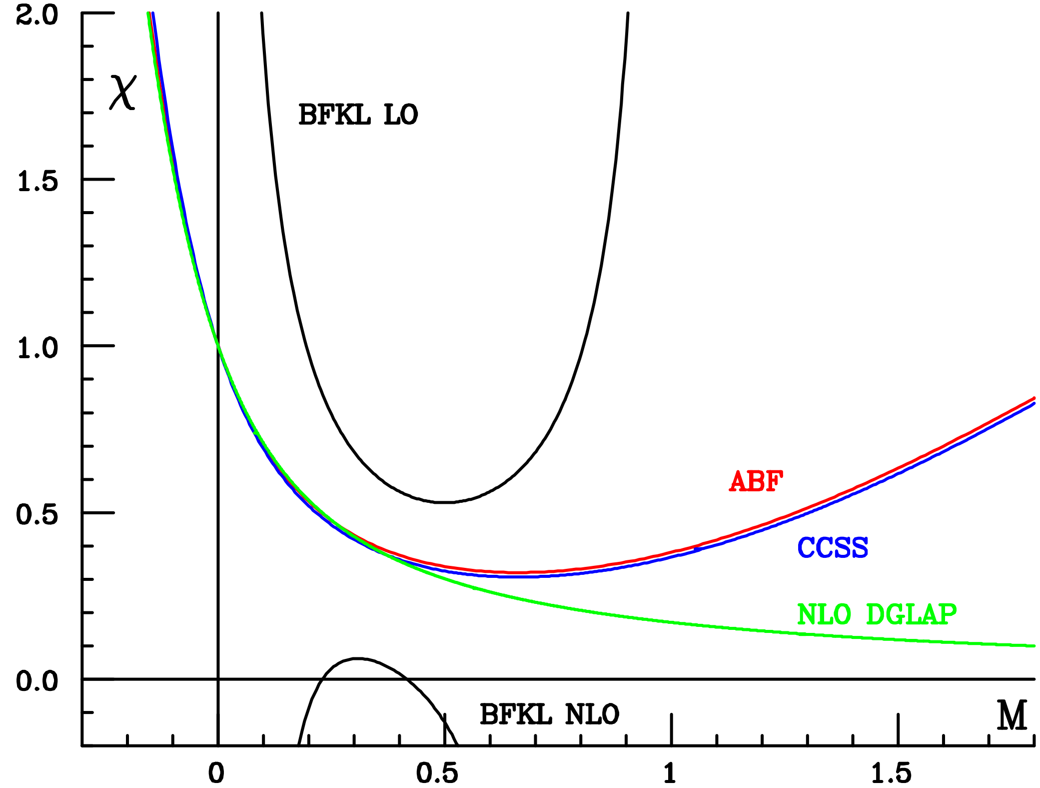
SYMMETRIZED EXPANSION

THE χ KERNEL

MOMENTUM CONSERVATION + SYMMETRY$\Rightarrow\chi$ ALWAYS HAS A MINIMUM

ASYMMETRIC VARIABLES

- LO, NLO SYM. CLOSE TO EACH OTHER
- LO, NLO SYM. CLOSE TO AP
- CURVATURE & INTERCEPT SAME IN SYM. & ASYM. VARIABLES
- RESULT DETERMINED BY MOM. CONS. + SYM. COMPARE CCSS



THE THIRD INGREDIENT: RUNNING COUPLING

- THE RUNNING OF THE COUPLING $\alpha(t) = \alpha_\mu[1 - \beta_0 \alpha_\mu t + \ldots]$ ($t \equiv \ln \frac{Q^2}{\mu^2}$) IS LEADING LOG Q^2, BUT NEXT–TO–LEADING LOG $\frac{1}{x}$
- UPON M-MELLIN TRANSFORMATION ($\ln x$ EVOLUTION) $\alpha_s(t)$ BECOMES AN OPERATOR:

$$\hat{\alpha}_s \equiv \frac{\alpha_{\mu^2}}{1 - \beta_0 \alpha_{\mu^2} \frac{d}{dM}} + O(\alpha_{\mu^2})$$

$\Rightarrow$ EVOLUTION EQUATION

for $G(N, M)$ with b.c. $H_0(M)$

$$NG(N, M) = \chi(\hat{\alpha}_s, M) G(N, M) + H_0(M)$$

- NOTE: OPERATOR ORDERING $\Leftrightarrow$ ARGUMENT OF THE COUPLING

$$\int_{-\infty}^{\infty} \frac{dk^2}{k^2} \sum_{p=1}^{\infty} \left[\alpha_s^p(Q^2) K_L^{(p)}(Q^2/k^2) + \alpha_s^p(k^2) K_R^{(p)}(Q^2/k^2) \right] G(\xi, k^2) \Leftrightarrow$$

$$\Leftrightarrow \sum_{p=1}^{\infty} \left[\hat{\alpha}_s^p \chi_L^{(p)}(M) + \chi_R^{(p)}(M) \hat{\alpha}_s^p \right] G(\xi, M)$$

RUNNING COUPLING DUALITY
THE OPERATOR APPROACH:

DUAL KERNEL INVERSION

$$\chi(\hat{\alpha}_s, \gamma(\hat{\alpha}_s, N)) = N$$
$$\gamma(\hat{\alpha}_s, \chi(\hat{\alpha}_s, M)) = M$$

ACTING ON $G(N, M)$

DUALITY STILL HOLDS TO ALL ORDERS!:

$\Rightarrow$ CAN DETERMINE γ (χ) AS A FUNCTIONAL OF FIXED-COUPLING DUAL γ_s (χ_s):

at LO, using $\chi_0 = N\hat{\alpha}^{-1}$, $\gamma \neq \gamma_s$ because $[N\hat{\alpha}^{-1}, \chi_0(M)] \neq 0$, so

$$\gamma\left(N\hat{\alpha}^{-1}\right) = \gamma_s\left(N\hat{\alpha}^{-1}\right) - \frac{1}{2}N\beta_0\gamma_s''\left(N\hat{\alpha}^{-1}\right)/\gamma_s'\left(N\hat{\alpha}^{-1}\right) + \ldots$$

RESULTS UP TO NLO FOR χ, NNNLO FOR γ (including β_1 terms)

EXACT ASYMPTOTIC SOLUTION

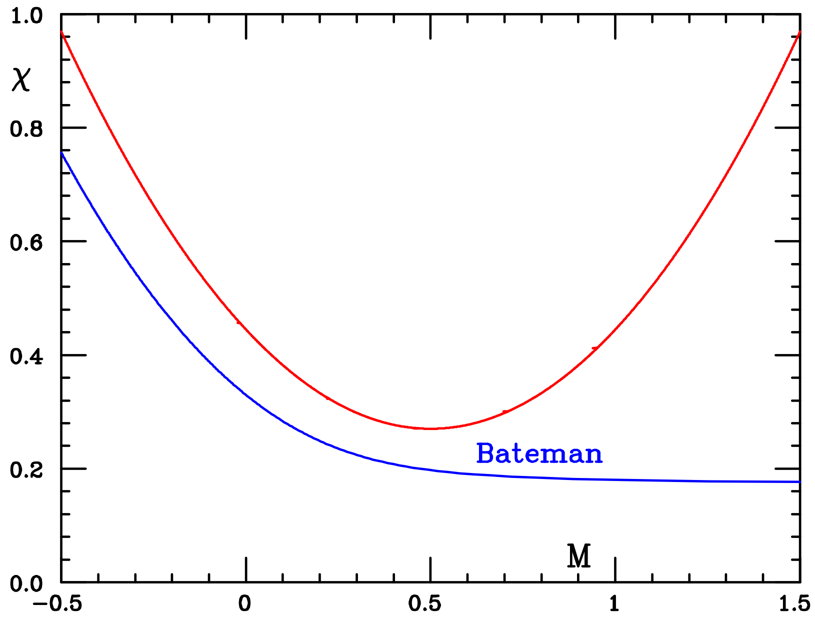
ASYMPTOTIC BEHAVIOUR CONTROLLED BY

MINIMUM OF $\chi(M) \Leftrightarrow$ RIGHTMOST SING. OF $\gamma(N)$

QUADRATIC KERNEL $\chi_q(\hat{\alpha}_s, M) = [c(\hat{\alpha}_s) + \frac{1}{2}\kappa(\hat{\alpha}_s)(M - M_s)^2]$

CAN SOLVE EXACTLY WITH LINEARIZED $c(\hat{\alpha}_s)$, $\kappa(\hat{\alpha}_s)$
IN TERMS OF BATEMAN FUNCTION $K_\nu(x)$:

THE EFFECTIVE RESUMMED KERNEL

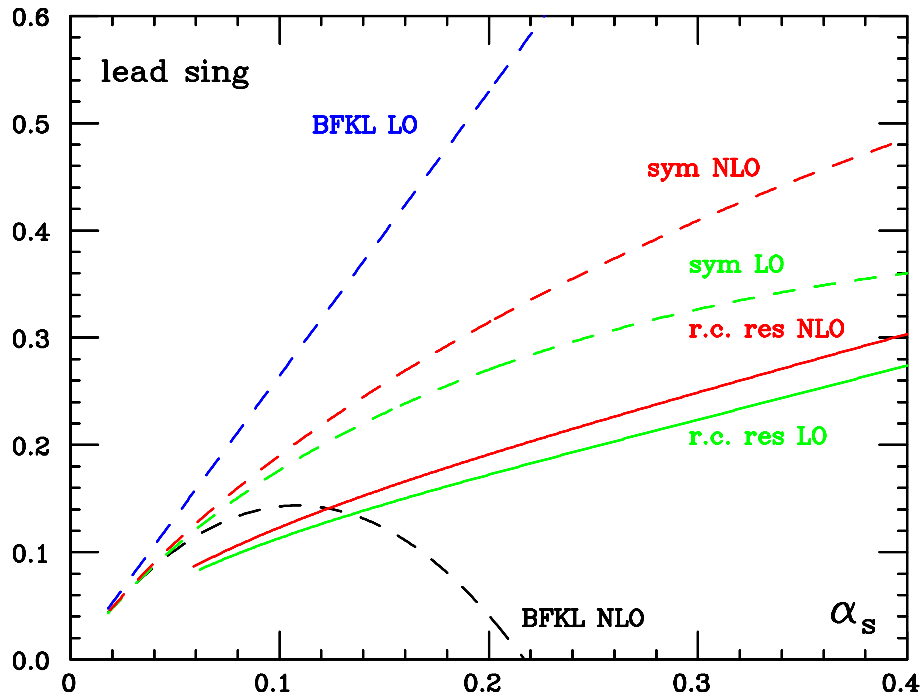


- $G(N,t) \propto K_{2B(\alpha_s,N)}\left[\frac{1}{\beta_0 \bar{\alpha}_s(t) A(\alpha_s,N)}\right]$
  A, B DEPEND ON α_s, N TRHOUGH c, κ
- ASYMPTOTIC LEADING LOG SMALL x SERIES RESUMMED
- BRANCH CUT IN γ REPLACED BY SIMPLE POLE

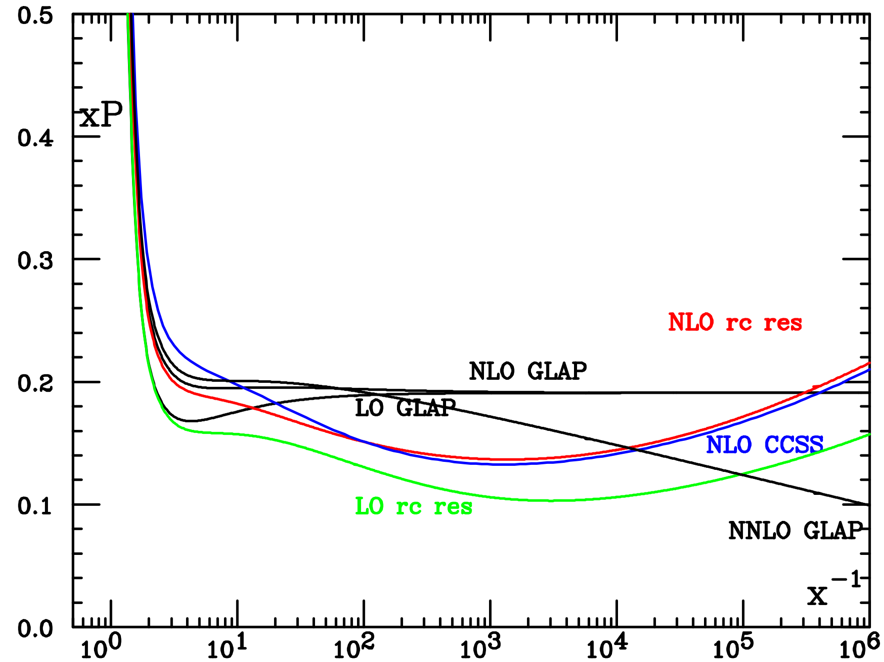
FULL RESUMMATION: QUALITATIVE FEATURES: $n_f = 0$

SINGULARITY IN ANOM. DIM. AT $N = \alpha \Rightarrow$ ASYMPT. SMALL-x POWER $G \sim x^{-\alpha}$

SMALL-x POWER VS. α_s



SPLITTING FUNCTION



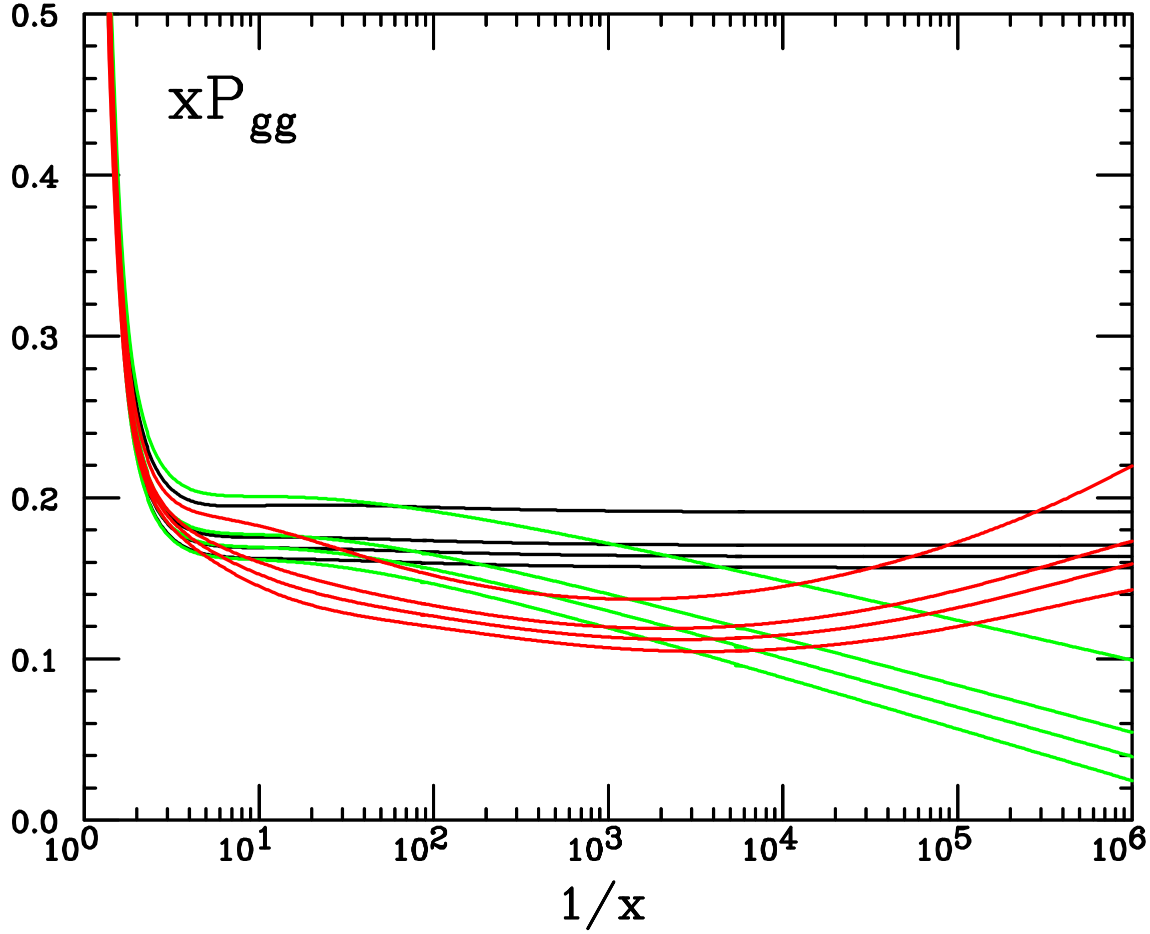
- RESUMMED EXPANSION CONVERGES RAPIDLY
- BELOW $x \lesssim 10^{-2}$ SPLITTING FUNCTION COINCIDES WITH SMALL x ASYMPTOTIC SOLUTION (G. Altarelli, R. Ball, S. Forte, C. Frugiuele, 2007)
- SMALL x INTERCEPT & CURVATURE DETERMINE RESUMMED BEHAVIOUR

$n_f \neq 0$: THE GLUON SECTOR

- MUST REMOVE CUT FROM AP DIAGONALIZATION (P_+ NOT WELL DEFINED)
- n_f DEPENDENCE NOT NEGLIGIBLE
- SMALL x RISE SOFTENED BY COUPLING TO QUARKS

P_{gg}, $n_f = 0, 3, 4, 5$ (top to bottom)

NLO, NNLO, RESUMMED

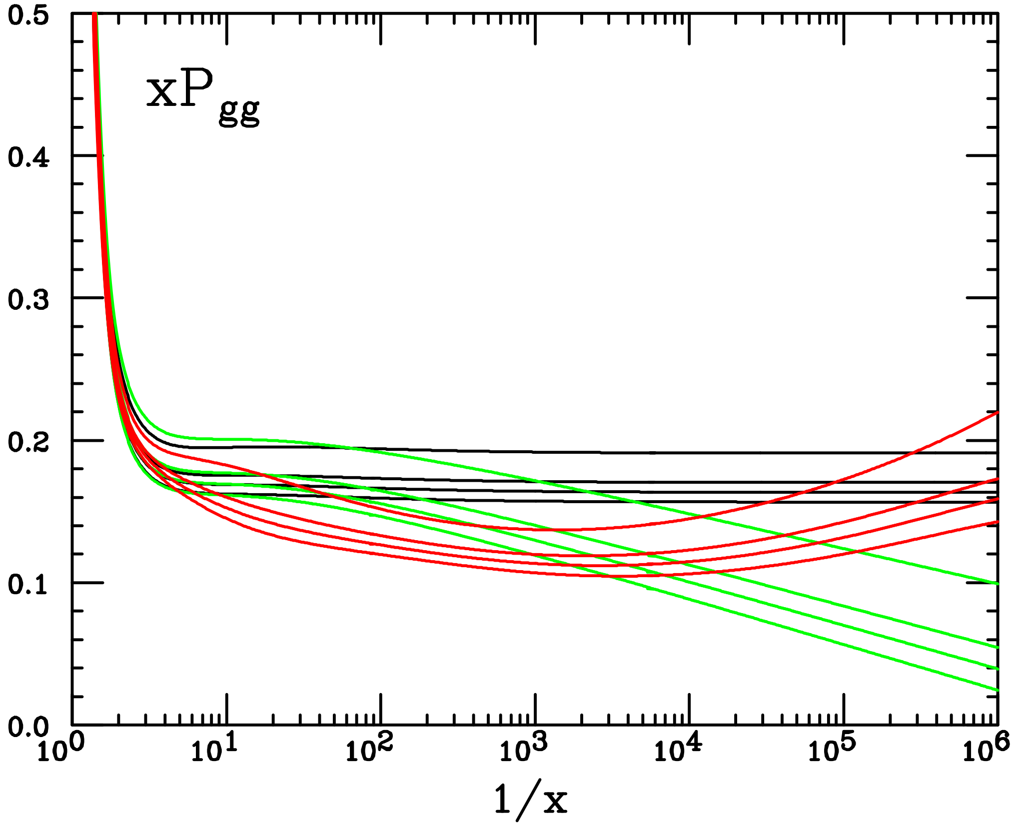


$n_f \neq 0$: THE GLUON SECTOR

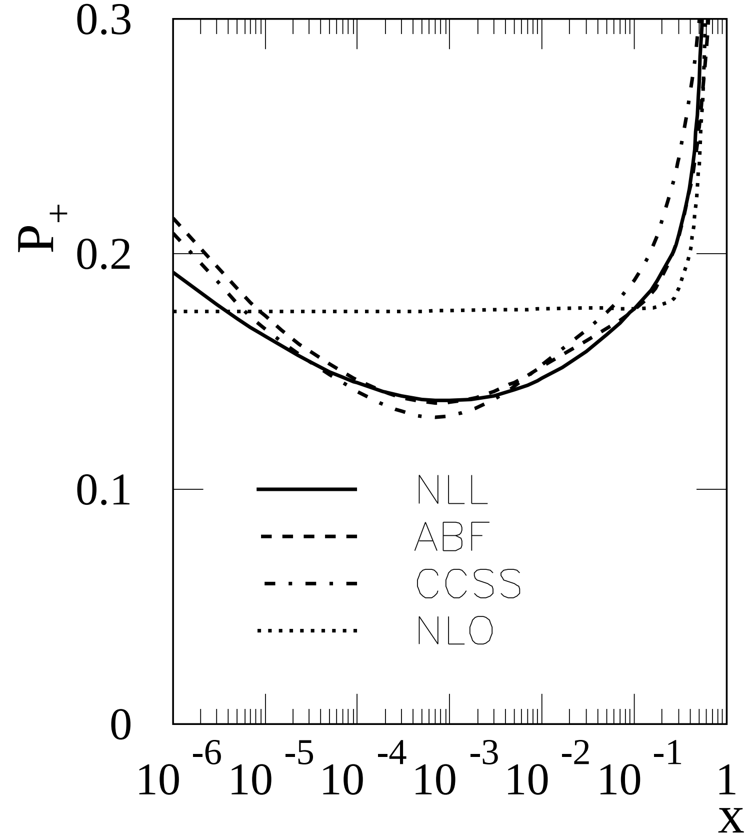
CAN COMPARE WITH THORNE & WHITE: AGREEMENT DETERIORATES AS $n_f \neq 0$

P_{gg}, $n_f = 0, 3, 4, 5$ (top to bottom)

NLO, NNLO, RESUMMED



THORNE & WHITE ($n_f = 4$)



$n_f \neq 0$: THE QUARK SECTOR

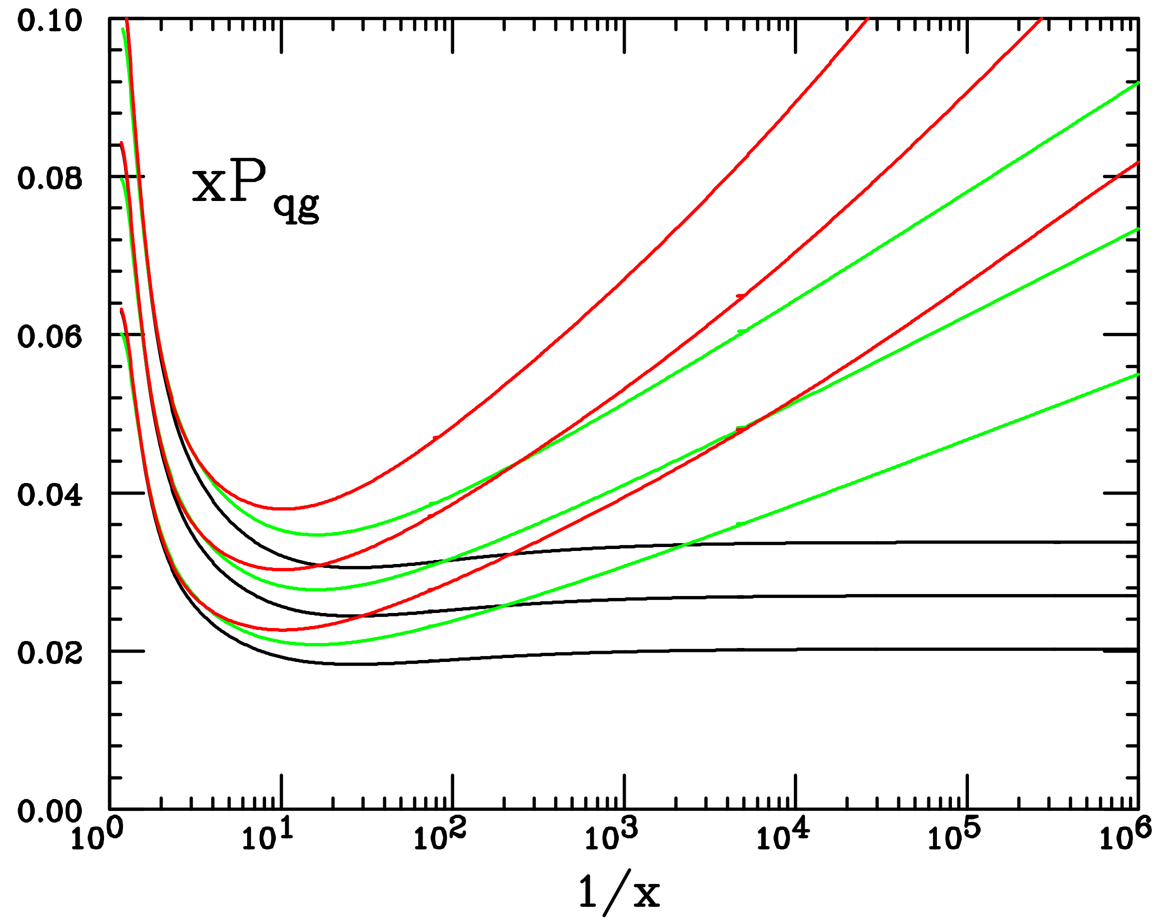
USE $Q_0\overline{\text{MS}}$ SCHEME: (Ball, Forte, 2005)

COINCIDES WITH $\overline{\text{MS}}$ AT LARGE x (NLO) BUT SMALL x R.C. SINGULARITIES IN GLUON SECTOR

- γ_{qg} SAME AS IN $\overline{\text{MS}}$
- γ_+ SAME AS IN Q_0
- CAN DETERMINE RESUMMED γ_{qg} (Catani & Hautmann)

P_{qg}, $n_f = 3, 4, 5$ (top to bottom)

NLO, NNLO, RESUMMED

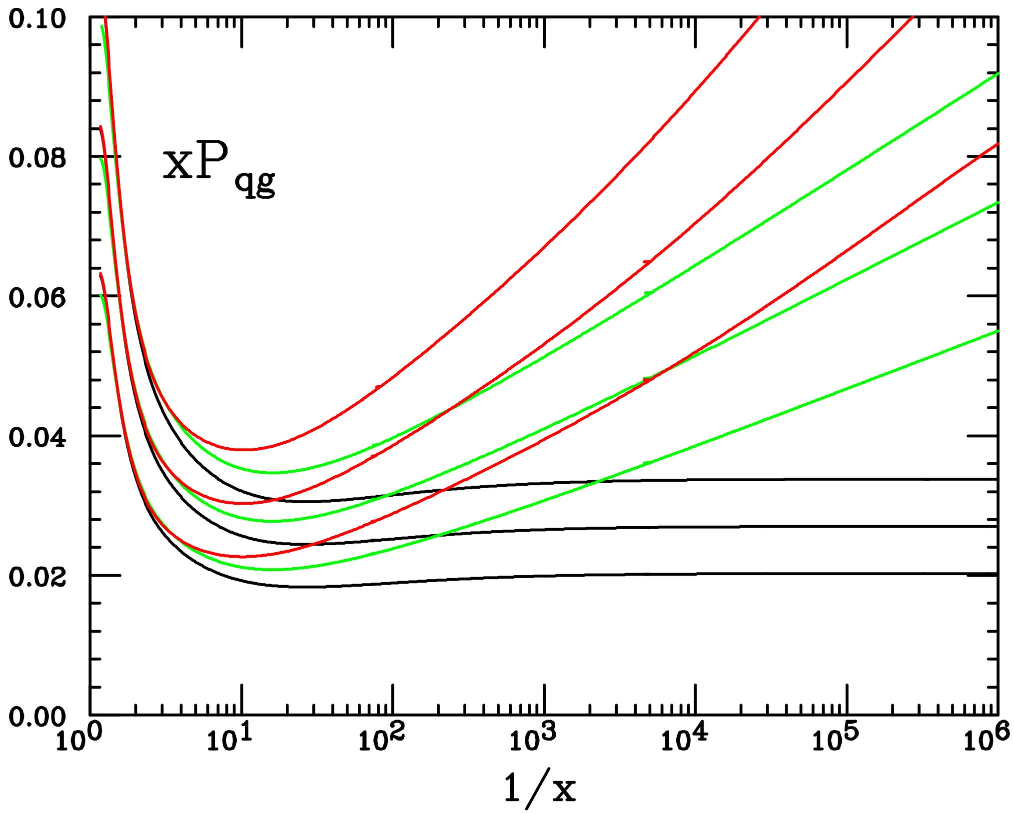


$n_f \neq 0$: THE QUARK SECTOR

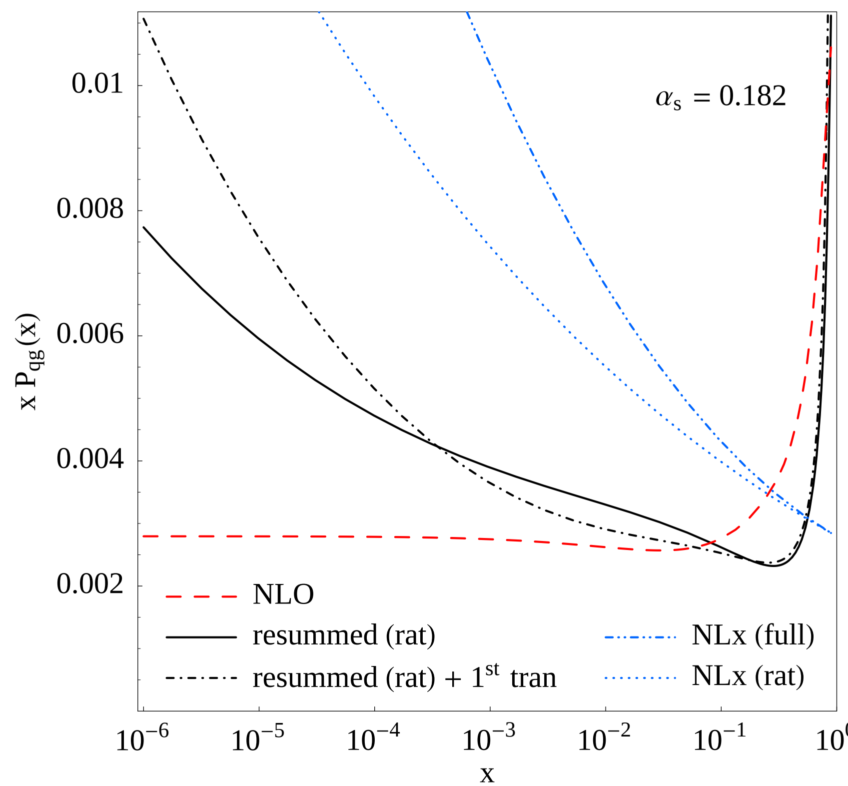
COMPARISON WITH CCSS: QUALITATIVELY SIMILAR (BUT CCSS RISE MILDER)

P_{qg}, $n_f = 0, 3, 4, 5$ (top to bottom)

NLO, NNLO, RESUMMED



CCSS

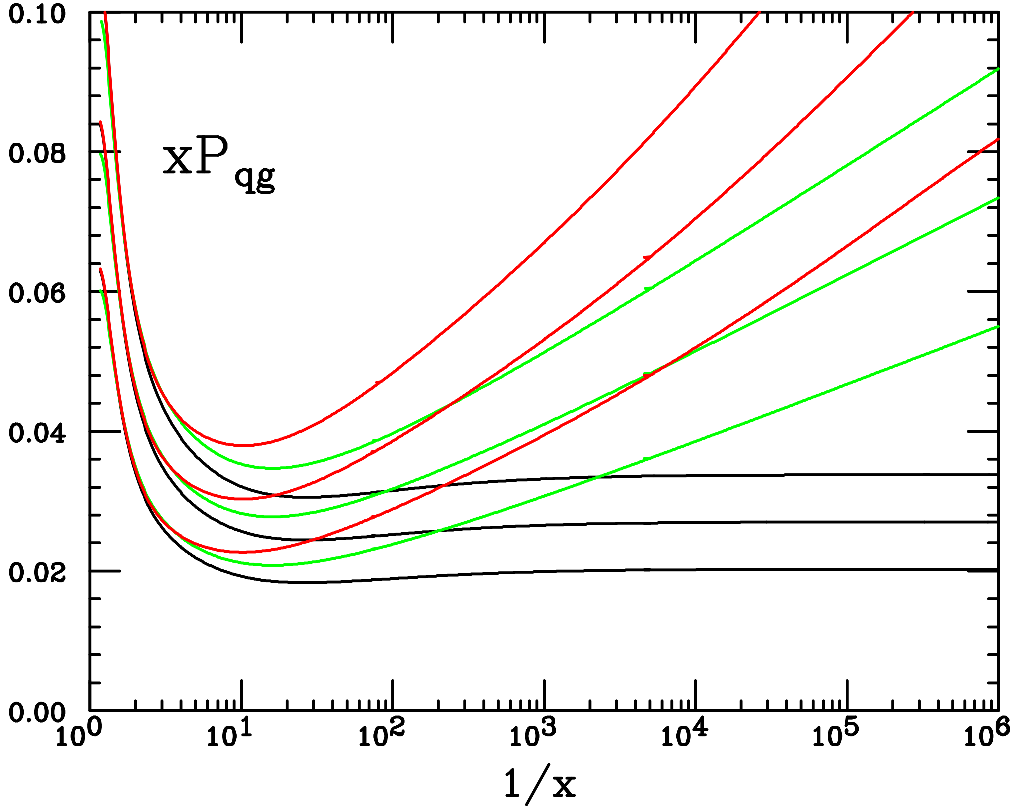


$n_f \neq 0$: THE QUARK SECTOR

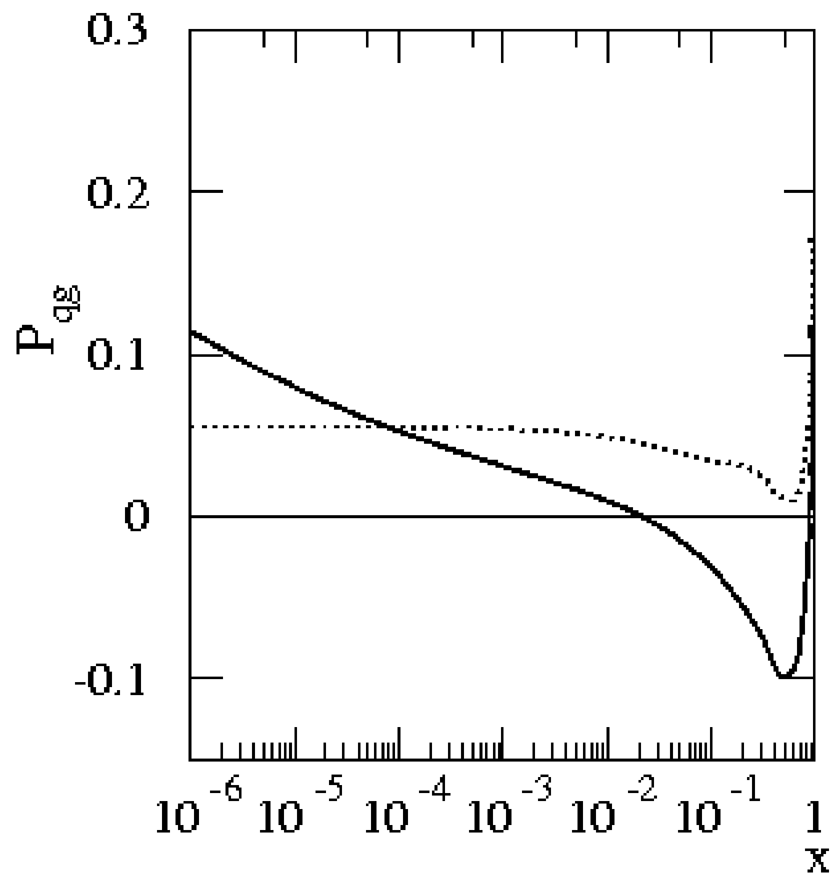
COMPARISON WITH THORNE & WHITE: QUALITATIVELY NOT SO SIMILAR (TW DIP)

P_{qg}, $n_f = 0, 3, 4, 5$ (top to bottom)

NLO, NNLO, RESUMMED

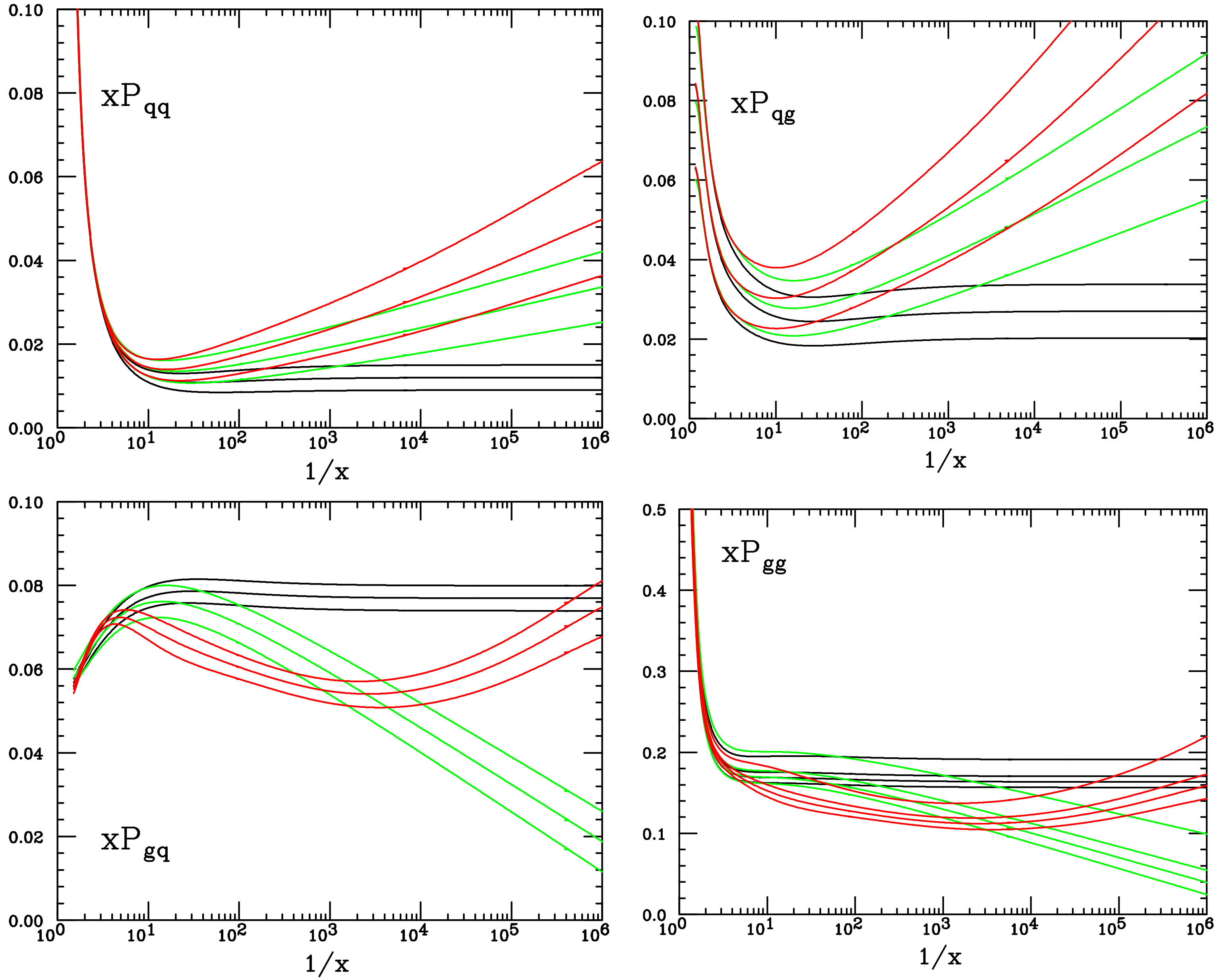


THORNE & WHITE



THE SPLITTING FUNCTION MATRIX

NLO, NNLO, RESUMMED

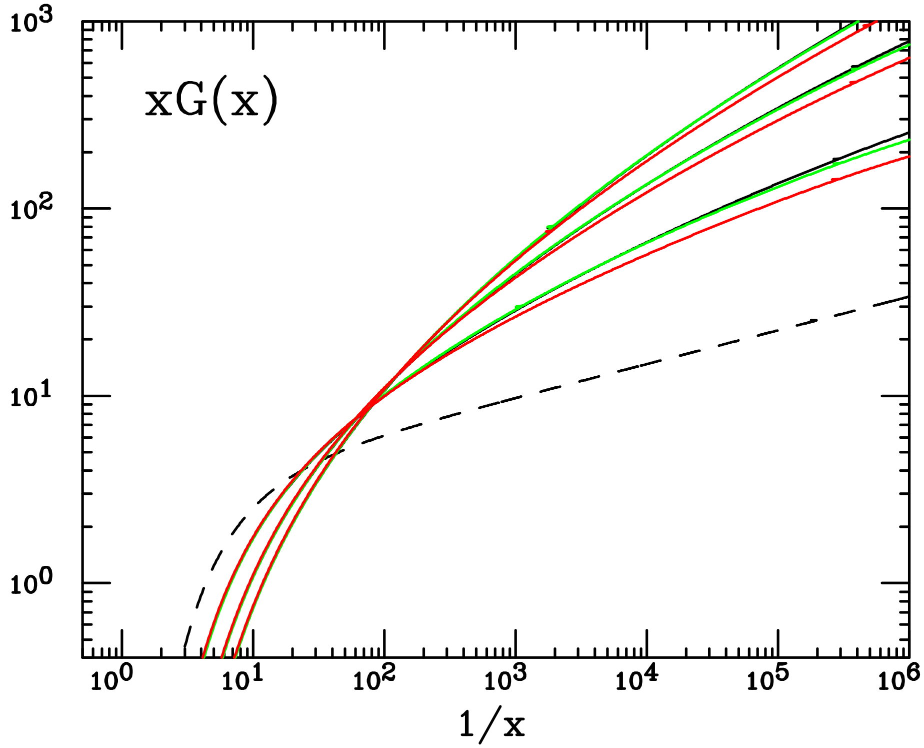


QUARK AND GLUON EVOLUTION

evolve toy $G = (x, Q_0) = x^{-0.18}(1-x)^5$, $Q(x, Q_0) = \frac{1}{3}G(x, Q_0)$, $Q_0 = 2$ GeV

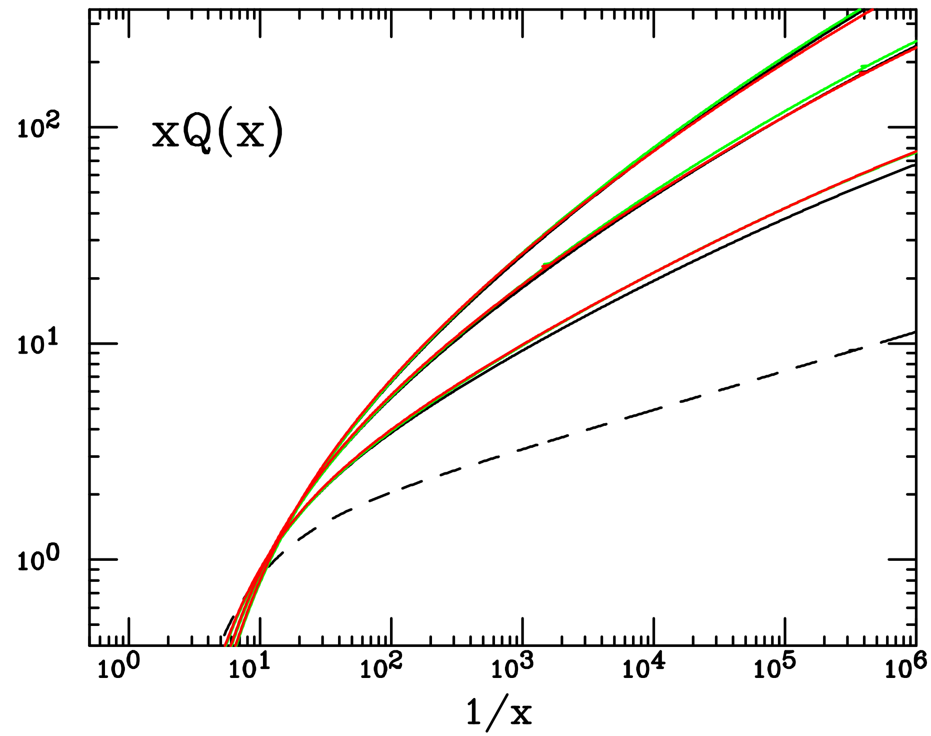
GLUON

$Q = 2,\ 10,\ 100,\ 1000$ GeV (bottom to top)



QUARK

NLO, NNLO, RESUMMED



- LO VS NLO DIFFERENCE LARGER THAN FIXED VS RESUMMED
- RESUMMED LIES BETWEEN NLO & NNLO
- RESUMMATION EFFECT SIZABLE AT MEDIUM-LARGE Q^2
- RESUMMED GLUON BELOW UNRESUMMED, QUARK JUST BELOW

CONCLUSIONS

- Matching of Altarelli-Parisi and BFKL fully understood at running coupling level
- Generic small x behaviour: (soft) simple pole
- resummed perturbative expansion stable
- Resummation softens gluons and quarks down to pretty low x

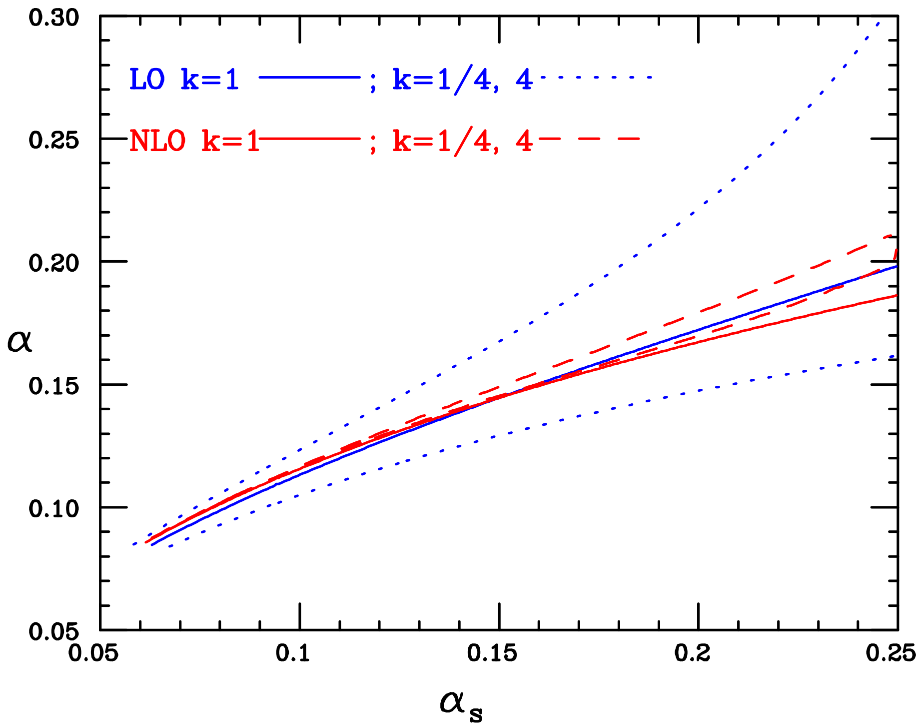
EXTRAS

PHENOMENOLOGY: PERTURBATIVE STABILITY

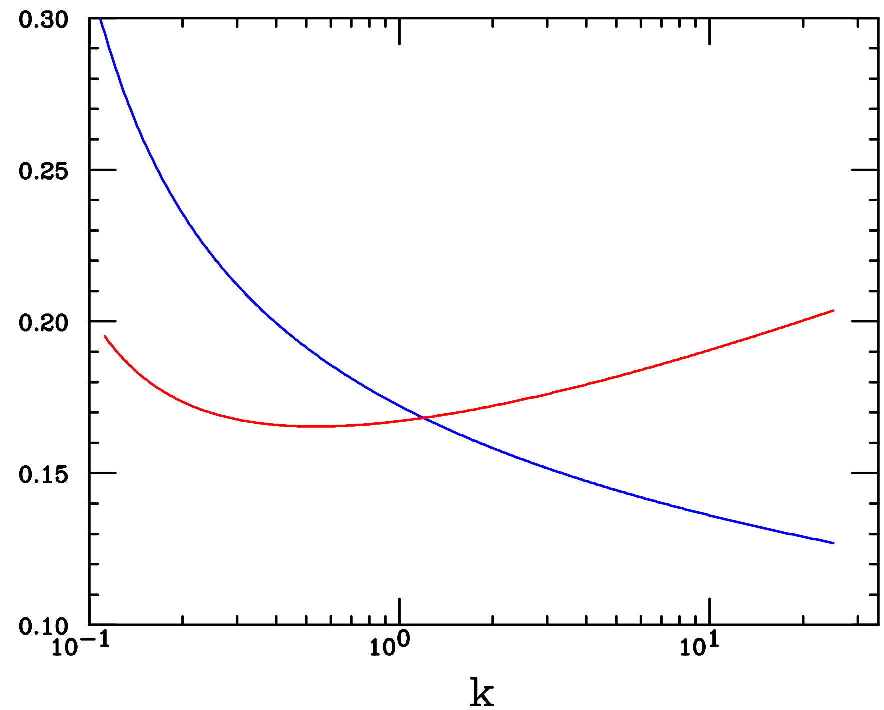
SIMPLE POLE IN ANOMALOUS DIMENSION $\Rightarrow$ ASYMPTOTIC SMALL-x POWER $G \sim x^{-\alpha}$

- LEADING AND NEXT-TO-LEADING POWER QUITE FLAT, CLOSE TO EACH OTHER
- DEP. ON RENORMALIZATION GETS WEAKER $\Rightarrow$ NLO STABILITY

SMALL-x POWER VS. α_s



SMALL-x POWER VS. REN. SCALE

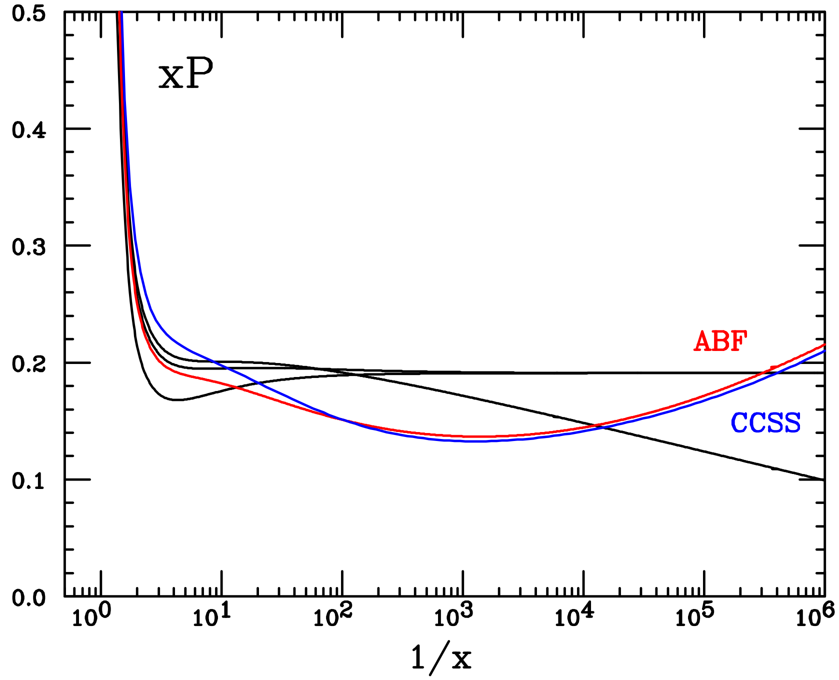


ABF VS. CCSS

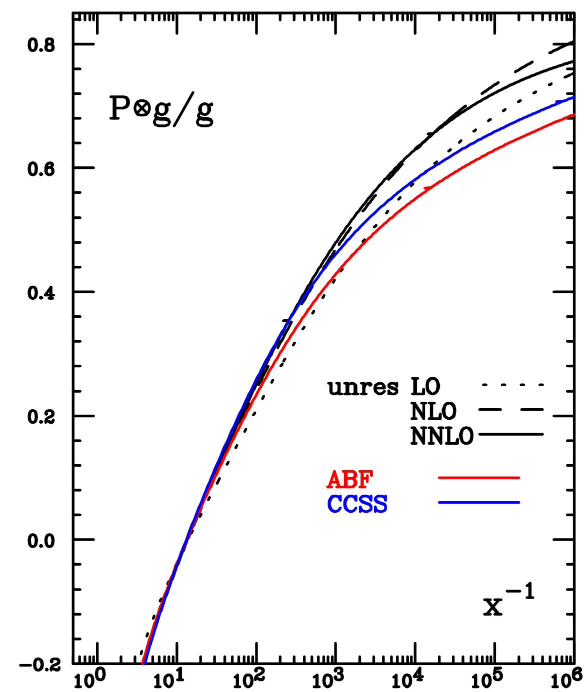
THE CCSS APPROACH:

- NUMERICAL BUT EXACT SOLUTION OF x-SPACE BFKL
- ABF & CCSS VERY CLOSE AT FIXED COUPLING LEVEL
- CCSS TREATS RUNNING BFKL EXACTLY, BUT GLAP AT FIXED COUPLING

SPLITTING FUNCTION



GLUON CONVOLUTION



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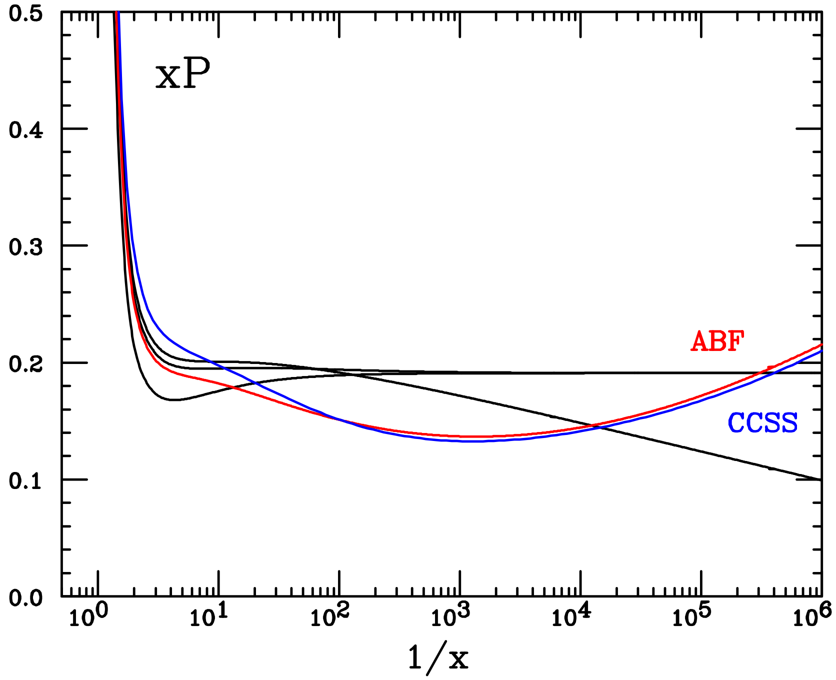

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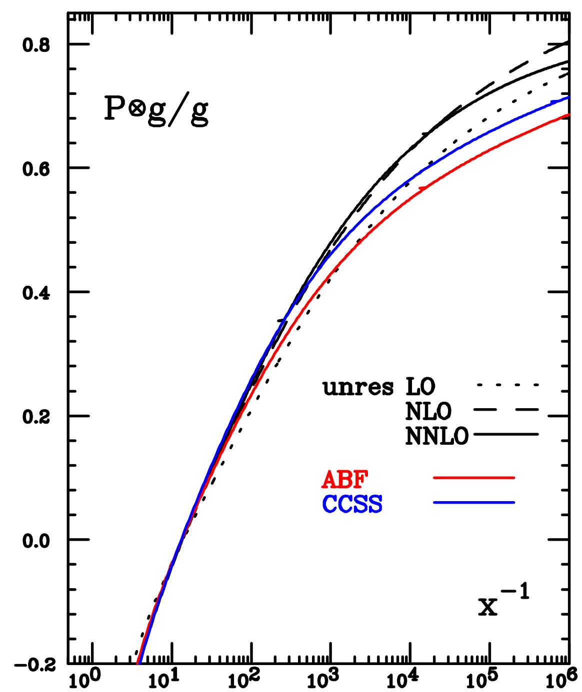
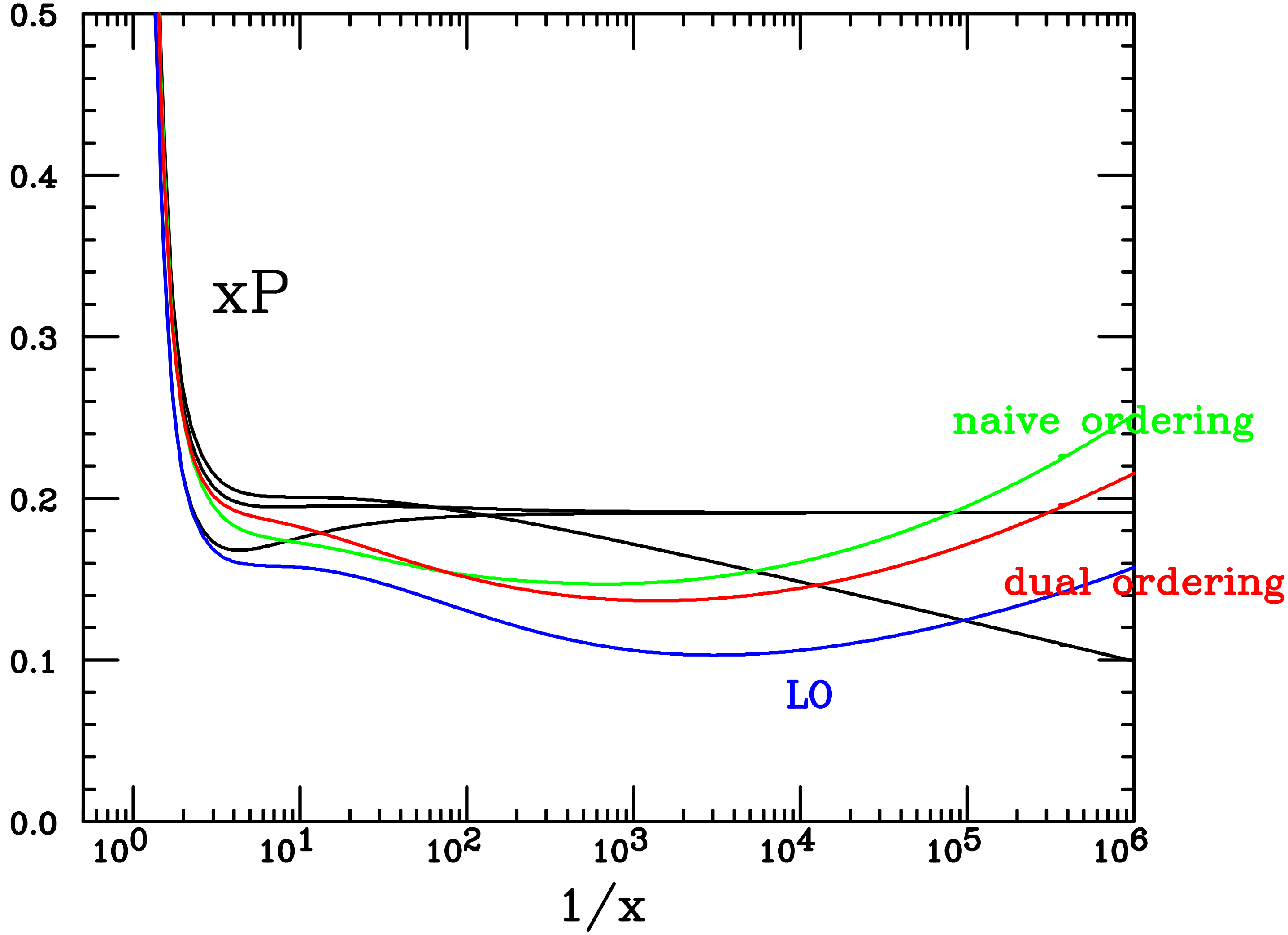


- SPLITTING FUNCTIONS CLOSER THAN TH. ERROR (!)
- DIFFERENCE IN GLUON DUE TO LARGE-x TERMS

EFFECTS OF OPERATOR ORDERING

$$\bar{\chi}_{\sigma\,LO}(\hat{\alpha}_s, M, N) = \chi_s\left(\hat{\alpha}_s\left(M+\frac{N}{2}\right)^{-1}\right)+\chi_s\left(\left(1-M+\frac{N}{2}\right)^{-1}\hat{\alpha}_s\right)+\tilde{\chi}_0(\hat{\alpha}_s, M, N)$$



RESUMMATION 2000

- DOUBLE-LEADING PERTURBATIVE EXPANSION STABLE
- STRONG DEPENDENCE ON NONPERTURBATIVE ALL-ORDER INTERCEPT λ (value of $\chi(M)$ at min.)
- GOOD AGREEMENT WITH DATA IF λ FITTED (FINE-TUNED)