Antenna subtraction with hadrons in the initial state

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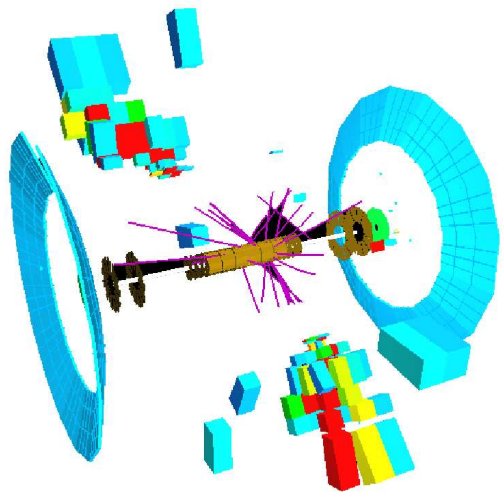
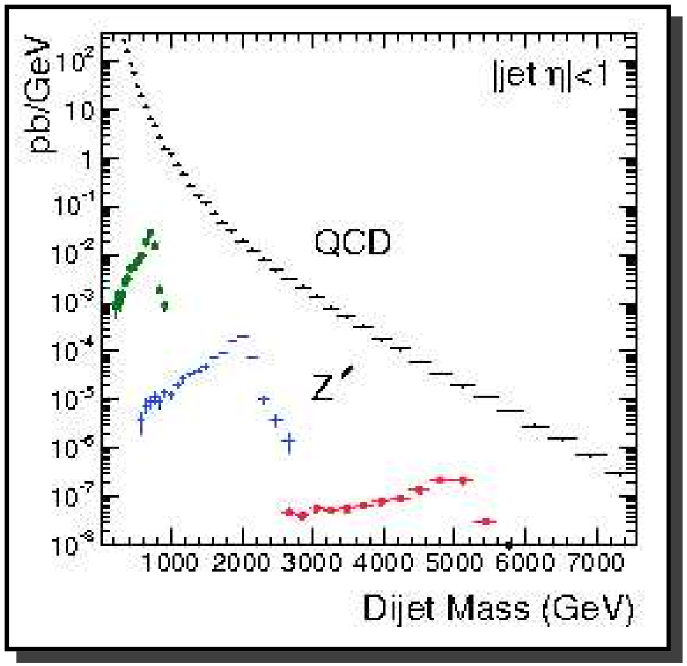
in collaboration with T. Gehrmann and D. Maître *do we need NNLO calculations?*

Jet production

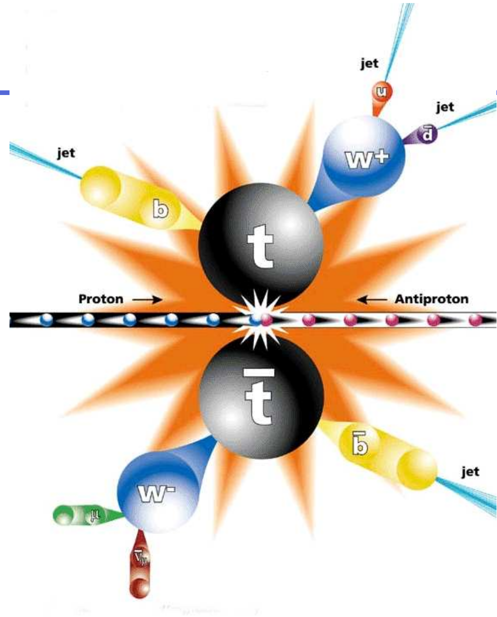
- dominat hard scattering process at LHC
- important input to constrain gluon PDFs and α_s
- rich in potential signals of new physics:
  - composite quarks
  - SUSY
  - extra gauge bosons, Z' and W'
  - Randall-Sundrum models (extra dimensions)





$t\bar{t}$ production

- third generation is the least known sector of the SM -except for the Higgs!-
- $t\bar{t}$ is key to measure top quark properties
- LHC will produce almost 1 $t\bar{t}$ per second at low luminosity!!
- $t\bar{t}$ is an important background for many searches of New Physics



$$\sigma(t\bar{t})_{\text{NLO}} \simeq 830 \text{ pb} \pm 15\% \text{ (scale+PDFs)}$$

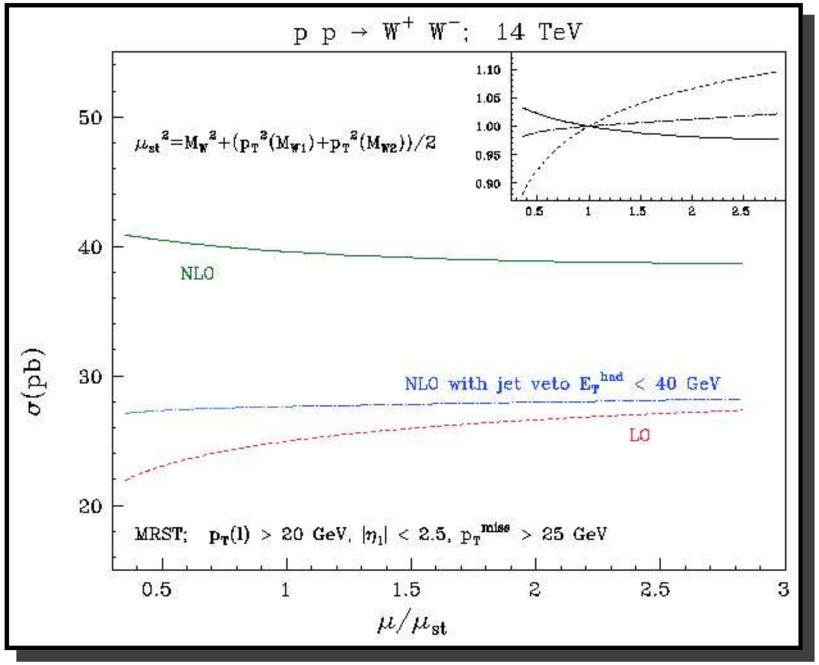
[Bonciani *et al.*;Cacciari]

	$\Delta\hat{\sigma}_{t\bar{t}(\mu)}/\hat{\sigma}_{t\bar{t}(\mu)}$		
	$1\,\text{fb}^{-1}$	$5\,\text{fb}^{-1}$	$10\,\text{fb}^{-1}$
Simulation samples (ϵ_{sim})	0.6%		
Simulation samples (F_{sim})	0.2%		
Pile-Up (30% On-Off)	3.2%		
Underlying Event	0.8%		
Jet Energy Scale (light quarks) (2%)	1.6%		
Jet Energy Scale (heavy quarks) (2%)	1.6%		
Radiation (Λ_{QCD}, Q_0^2)	2.6%		
Fragmentation (Lund b, σ_q)	1.0%		
b-tagging (5%)	7.0%		
Parton Density Functions	3.4%		
Background level	0.9%		
Integrated luminosity	10%	5%	3%
Statistical Uncertainty	1.2%	0.6%	0.4%
Total Systematic Uncertainty	13.6%	10.5%	9.7%
Total Uncertainty	13.7%	10.5%	9.7%

[CMS Physics TDR]

Vector boson pair production

- unique opportunity to probe the non-Abelian gauge symmetry of the Standard Model
- test the presence of anomalous couplings $\longrightarrow$ New Physics
- important backgrounds for Higgs and SUSY searches
- mild NLO corrections with jet veto
- important contributions from high p_T region



[Dixon, Kunszt, Signer]

Motivation

- high precision (NNLO) calculations needed for LHC:
  - processes used to measure fundamental parameters
  - important backgrounds in the searches of new physics
- necessity to develop general methods for these calculations, capable of dealing with differential distributions, arbitrary cuts, etc

$\Rightarrow$ antenna subtraction method for e^+e^- can be extended to hadronic collisions

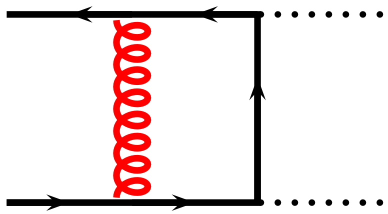
$\Rightarrow$ first step: NLO antenna subtraction with hadronic initial states

QCD corrections

amplitudes are singular due to soft and collinear radiation

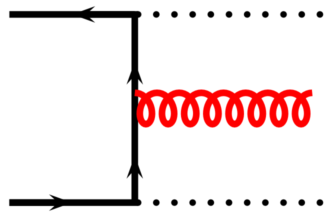
Virtual corrections

explicit singularites (loop integration)



Real corrections

“potential” singularites (phase space)



Singularites are guaranteed to cancel between real and virtual contributions

... but only after phase space integration...

and phase space integration is either not possible -e.g. jets- or not appropriate -e.g. differential cross sections-

how to extract the singularities from the real contributions?

Different approaches

- phase space slicing [Giele, Glover; Giele, Glover, Kosower]
  - split the phase space volume into singular and non-singular regions
  - in singular regions, matrix elements are approximated by their soft/collinear limits
  - these pieces are integrated analytically
  - and they cancel the explicit singularities of virtual components
  - in the non-singular regions it is safe to integrate numerically
- sector decomposition and expansion in plus distributions [Anastasiou, Melnikov, Petriello]
  - use sector decomposition of phase space integrals to isolate singularities
  - explicit poles in ϵ are extracted before integration
  - the finite coefficients are integrated numerically
  - cancellations of poles take place after numerical integration
  - delivers results at NNLO!!
- methods based on subtraction

Subtraction methods

$$\int R\, d\Phi_R + \int V\, d\Phi_V = \int (R - R_c)\, d\Phi_R + \int \left(V + R_c \frac{d\Phi_R}{d\Phi_V} \right) d\Phi_V$$

- R_c coincides with R in ALL singular regions
- it usually require one or several phase space mappings
- these mappings should allow phase space factorization: $d\Phi_R = d\Phi'_R\, d\Phi_V$
- R_c must be simple enough to be integrated analytically over $d\Phi'_R$

Several implementations, differing in the construction of R_c and the phase space mappings:

$\Rightarrow$ Ellis-Kunszt-Soper method NLO

$\Rightarrow$ Catani, Seymour dipoles NLO

$\Rightarrow$ Grazzini, Frixione NNLO (e^+e^-)

$\Rightarrow$ Weinzierl NNLO

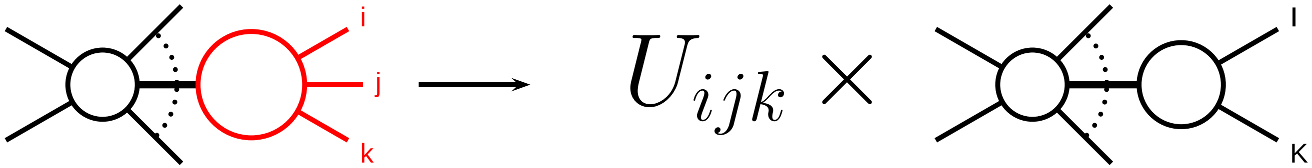
$\Rightarrow$ Somogyi, Trocsanyi, del Duca NNLO (e^+e^-) $\Leftarrow$ Gabor's talk starting in 15'!!

$\Rightarrow$ antennae: Kosower; Campbell, Cullen, Glover; Gehrmann-De Ridder, Gehrmann, Glover NNLO (e^+e^-)

$\Rightarrow$ Catani, Grazzini NNLO (pp, colourless final state)

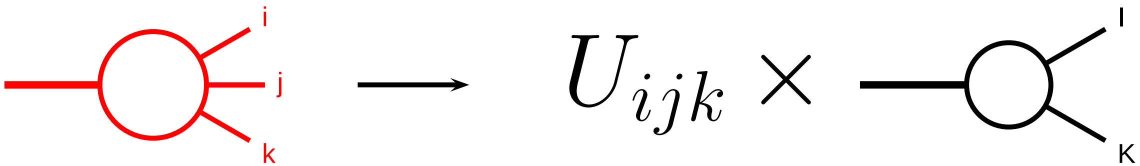
Antenna subtraction

⇒ matrix elements factorize in singular limits

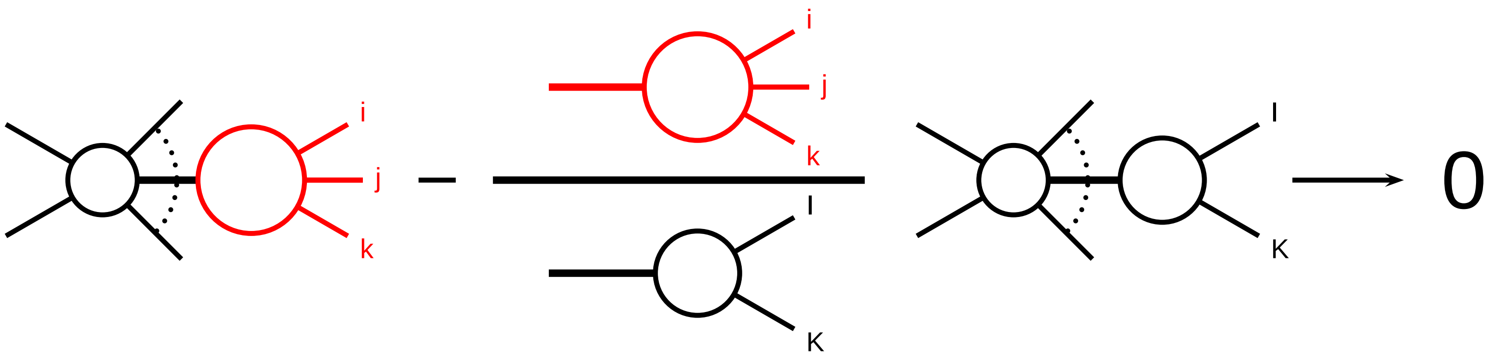


p_I and p_K are combinations of p_i, p_j and p_k only meaningful in the limit
i.e. if $j \parallel k$, $p_I = p_i$, $p_K = p_j + p_k$

⇒ infrared factorization is universal

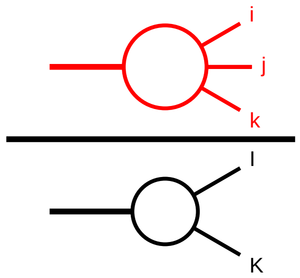


⇒ we can exploit universality to build subtraction terms



Antenna subtraction

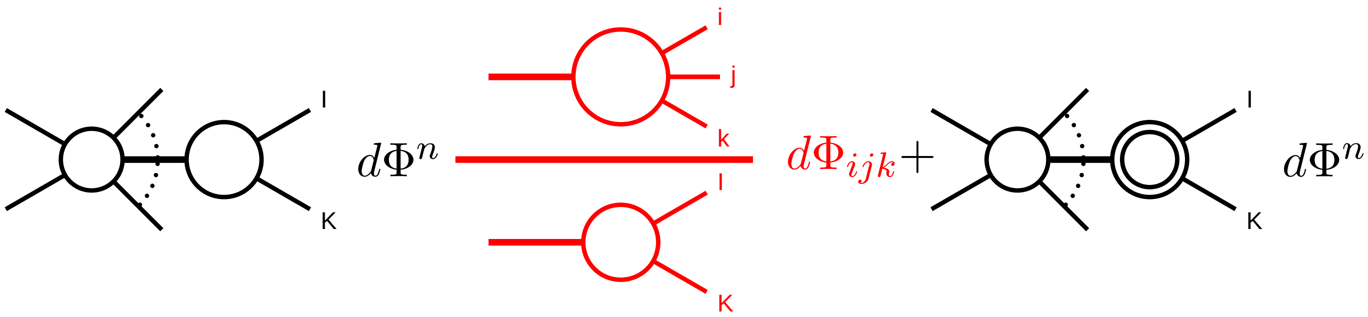
⇒ counterterms are built in terms of antenna functions

$X^0_{ijk} =$ 

⇒ each antenna interpolates between several singular limits

⇒ they have to be combined with mappings $(i, j, k) \to (I, K)$ also interpolating between these limits

⇒ phase space factorizes and the subtraction terms are combined with the virtual corrections to cancel poles in ϵ



Some details

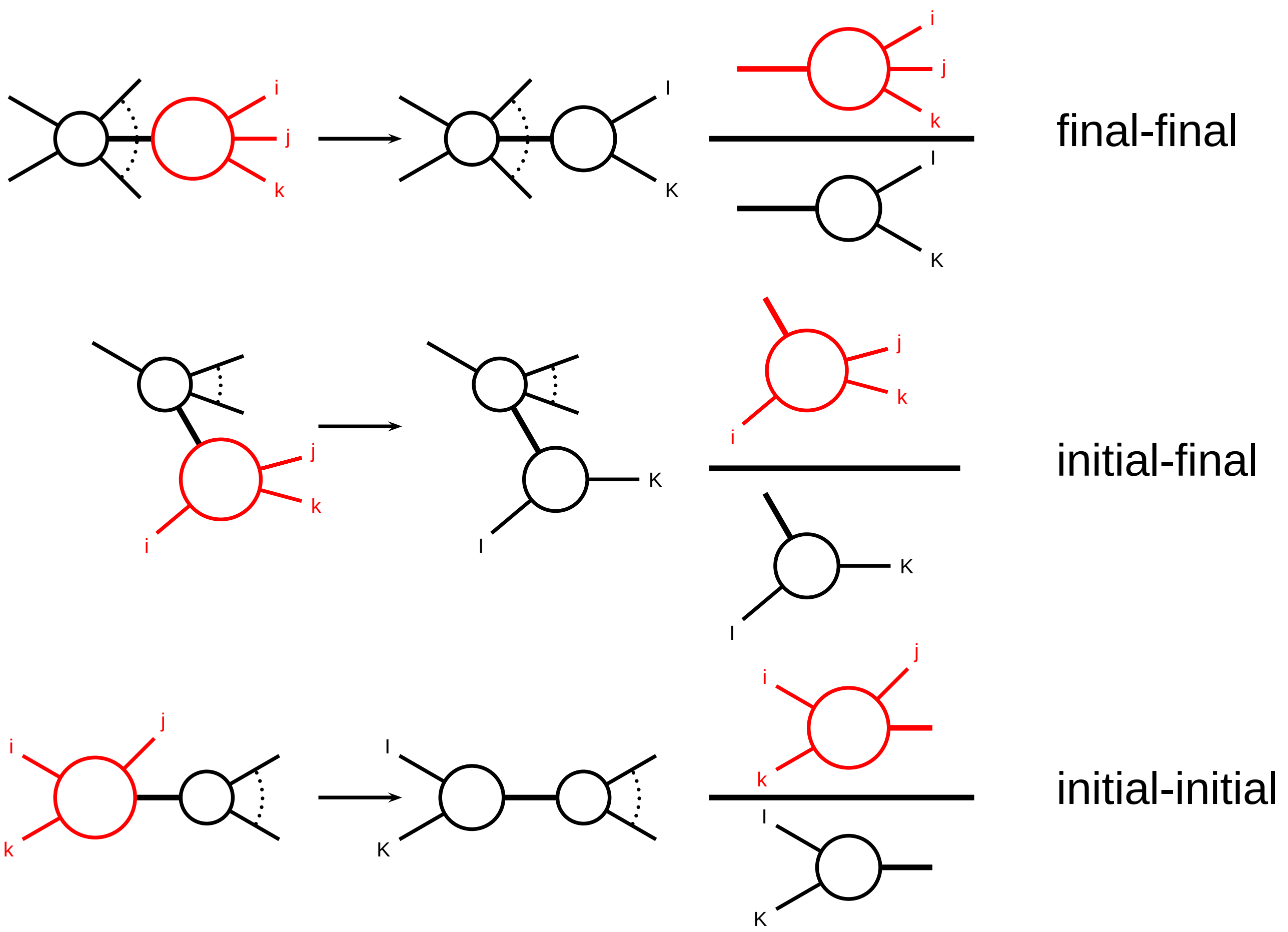
- $\Uparrow$ use of colour ordered amplitudes to simplify the singular structure
- $\Uparrow$ antennae interpolate between several limits, fewer terms needed compared to other subtraction approaches
- $\Uparrow$ antennae are just colour ordered matrix elements of physical processes
- $\Uparrow$ antennae can be integrated using well known techniques

last two points make NNLO extension relatively simpler than in other approaches

- $\Rightarrow$ missing angular correlations, subtraction are not fully local, can be dealt with by slicing the phase space
- $\Rightarrow$ in practice, some antenna functions might need to be decomposed in subantennae -with different associated mappings- to deal with with non-ordered emissions and degenerate antennae

Subtractions with hadrons in the initial states

We distinguish three configurations:



each requires a set of antennae and a phase space mapping

Subtraction terms (m jet production)

$$\mathrm{d}\hat{\sigma}^{R} - \mathrm{d}\hat{\sigma}^{S,(ff)} - \mathrm{d}\hat{\sigma}^{S,(if)} - \mathrm{d}\hat{\sigma}^{S,(ii)} = \text{finite}$$

$$\mathrm{d}\hat{\sigma}^{R} = \mathrm{d}\Phi_{m+1}(k_1,\ldots,k_{m+1};p_1,p_2)\,|\mathcal{M}_{m+1}(k_1,\ldots,k_{m+1};p_1,p_2)|^2\,J_m^{(m+1)}(k_1,\ldots,k_{m+1})$$

$$\mathrm{d}\hat{\sigma}^{S,(ff)} = \mathrm{d}\Phi_{m+1}(k_1,\ldots,k_{m+1};p_1,p_2) \times \sum_j X^0_{ijk}\,|\mathcal{M}_m(k_1,\ldots,K_I,K_K,\ldots,k_{m+1};p_1,p_2)|^2\,J_m^{(m)}(k_1,\ldots,K_I,K_K,\ldots,k_{m+1})$$

$$\mathrm{d}\hat{\sigma}^{S,(if)} = \mathrm{d}\Phi_{m+1}(k_1,\ldots,k_{m+1};p_1,p_2) \times \sum_j X^0_{i,jk}\,|\mathcal{M}_m(k_1,\ldots,K_K,\ldots,k_{m+1};xp_1,p_2)|^2\,J_m^{(m)}(k_1,\ldots,K_K,\ldots,k_{m+1})$$

$$d\hat{\sigma}^{S,(ii)} = \mathrm{d}\Phi_{m+1}(k_1,\ldots,k_{j-1},k_j,k_{j+1},\ldots,k_{m+1};p_1,p_2) \sum_j X^0_{ik,j}\left|\mathcal{M}_m(\tilde{k}_1,\ldots,\tilde{k}_{j-1},\tilde{k}_{j+1},\ldots,\tilde{k}_{m+1};x_1p_1,x_2p_2)\right|^2 J_m^{(m)}(\tilde{k}_1,\ldots,\tilde{k}_{j-1},\tilde{k}_{j+1},\ldots,\tilde{k}_{m+1})$$

NLO antenna functions

⇒ at NLO there are only 5 basic antenna functions:

- $qg\bar{q}$ $\qquad \gamma \to qg\bar{q}$
- qgg $\qquad \tilde{\chi} \to \tilde{g}gg$
- $qq'\bar{q}'$ $\qquad \tilde{\chi} \to \tilde{g}q'\bar{q}'$
- ggg $\qquad h \to ggg$
- $q\bar{q}g$ $\qquad h \to q\bar{q}g$

⇒ for initial-final and initial-initial configurations, one or two partons are crossed to the initial state, i.e.

$$A_{qg\bar{q}} \longrightarrow A_{q,gq}, \;\; A_{g,q\bar{q}}$$

⇒ the case qgg when a gluon is rotated to the initial state needs special consideration ($g \to qg$ looks like $\bar{q} \to g$ or $g \to g$ depending on the limit)

$$D_{g,qg} = D'_{g,qg} + D_{g,gq}$$

NLO mappings

⇒ phase space mappings $(i, j, k) \to (I, K)$ must:

1. interpolate between all the singular limits: j soft or collinear to either i or k
2. satisfy momentum conservation and keep all particles in their mass-shells
3. allow for phase space factorization

(4). NNLO extension: factorize in strongly ordered limits

⇒ final-final configurations are the easiest, 2 inmediately provides 3:

$$d\Phi_{n+1}(k_1, \ldots, k_i, k_j, k_k, \ldots, k_{n+1}; q) = d\Phi_n(k_1, \ldots, K_I, K_K, \ldots, k_{n+1}; q) \, \frac{1}{P_2} \, d\Phi_3(k_i, k_j, k_k; K_I + K_K)$$

key point is that mapped momenta are integrated over

⇒ several possibilities, straightforward extensions to NNLO

⇒ integrated antenna functions

$$\mathcal{X}_{ijk}(\epsilon) = \frac{1}{P_2} \int d\Phi_3 \, X_{ijk} \sim \mathbf{I}_{(ij)k}(s_{ijk}, \epsilon) + \mathbf{I}_{i(jk)}(s_{ijk}, \epsilon) + \mathcal{O}(\epsilon^0)$$

NLO mappings

initial-final and initial-initial configurations are more involved:

- ⇒ one or the two hard radiators are in the initial state
- ⇒ only longitudinal component of initial state momenta can be mapped, otherwise phase space doesn't factorize
- ⇒ phase space mapping is totally defined by this constraint and momentum conservation
- ⇒ phase space factorization now involves convolutions, i.e. for one initial state radiator:

$$d\Phi_{m+1}(k_1,\ldots,k_{m+1};p,r) \quad = \quad d\Phi_m(k_1,\ldots,K_K,\ldots,k_{m+1};xp,r)\frac{Q^2}{2\pi}d\Phi_2(k_j,k_k;p,q)\frac{dx}{x}$$

- ⇒ integrated antenna functions

$$\mathcal{X}_{i,jk}(x,\epsilon) = \frac{Q^2}{2\pi}\int d\Phi_2\, X_{i,jk} \sim \left(\mathbf{I}_{(ij)k}(s_{ijk},\epsilon) + \mathbf{I}_{i(jk)}(s_{ijk},\epsilon)\right)\delta(1-x) + \frac{1}{\epsilon}P^{(0)}_{(ij)i}(x) + \mathcal{O}(\epsilon^0)$$

- ⇒ initial-initial configurations require a mapping of ALL final state momenta -including non QCD particles- still some freedom in the construction of the mapping

Cancellation of singularities (m jet example)

$$\hat{\sigma}^V + \mathrm{d}\hat{\sigma}^{MF} + \mathrm{d}\hat{\sigma}^{S,(ff)} + \mathrm{d}\hat{\sigma}^{S,(if)} + \mathrm{d}\hat{\sigma}^{S,(ii)} = \text{finite}$$

$$\mathrm{d}\hat{\sigma}^V = \mathrm{d}\Phi_m(k_1,\ldots,k_m;p_1,p_2)\sum_{i,j}\mathbf{I_{ij}}(\epsilon)\,|\mathcal{M}_m(k_1,\ldots,k_{m+1};p_1,p_2)|^2\,J_m^{(m)}(k_1,\ldots,k_m)+\mathcal{O}(\epsilon^0)$$

$$\mathrm{d}\hat{\sigma}^{MF} = \int\frac{dx}{x}\sum_{i,j}\frac{1}{\epsilon}P_{ij}(x)\,\mathrm{d}\Phi_m(k_1,\ldots,k_m;x\,p_1,p_2)\,|\mathcal{M}_m(k_1,\ldots,k_m;x\,p_1,p_2)|^2\,J_m^{(m)}(k_1,\ldots,k_m)$$

$$\mathrm{d}\hat{\sigma}^{S,(ff)} = \mathrm{d}\Phi_m(k_1,\ldots,k_m;p_1,p_2)\,\mathcal{X}^0_{ijk}(\epsilon)\,|\mathcal{M}_m(k_1,\ldots,k_{m+1};p_1,p_2)|^2\,J_m^{(m)}(k_1,\ldots,k_m)$$

$$\mathrm{d}\hat{\sigma}^{S,(if)} = \int\frac{dx}{x}\,\mathcal{X}^0_{i,jk}(x,\epsilon)\,\mathrm{d}\Phi_m(k_1,\ldots,k_m;x\,p_1,p_2)\,|\mathcal{M}_m(k_1,\ldots,k_m;x\,p_1,p_2)|^2\,J_m^{(m)}(k_1,\ldots,k_m)$$

$$\mathrm{d}\hat{\sigma}^{S,(ii)} = \int\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\,\mathcal{X}^0_{ik,j}(x_1,x_2,\epsilon)\mathrm{d}\Phi_m(k_1,\ldots,k_m;x_1p_1,x_2p_2)\,|\mathcal{M}_m(k_1,\ldots,k_m;x_1p1,x_2p_2)|^2\,J_m^{(m)}(k_1,\ldots,k_m)$$

Constructing the counterterms (jet + vector boson production)

$$g + q_i \longrightarrow g + q_j + V$$

$$\left|M^0_{g_1 q_i, g_2 q_j}\right|^2 \;=\; |C_{ij}|^2\,(v_i^2 + a_i^2)\,N_{2,qg}\left[N\,A_4^0(j_q, \hat{1}_g, 2_g, \hat{i}_{\bar{q}}) + N\,A_4^0(j_q, 2_g, \hat{1}_g, \hat{i}_{\bar{q}})\right] + \mathcal{O}(1/N)$$

(1): $\hat{1}_g$ and $\hat{i}_{\bar{q}}$ in the first term; (2): j_q and $\hat{1}_g$ in the second term; (3): j_q and 2_g in the first term.

singular limits

- $2\times$ gluon 2 soft
- $2\times$ gluon 2 $\|$ gluon 1
- $1\times$ gluon 2 $\|$ quark j
- $1\times$ quark j $\|$ gluon 1

(2) and (3) actually share D^0_{g,gq_j} and $D^0_{g,q_j g}$

$$\mathrm{d}\hat{\sigma}^S_{q_i g, q_j g} = \sum_j |C_{ij}|^2\,(v_i^2 + a_i^2)\,N_{2,qg}\,\mathrm{d}\Phi_3(k_q, k_g, q; p_q, p_g)$$

$$\times\Big\{N\left[\underbrace{D^0_{q_i g,g}}_{(1)}\,A^0_{Q_i G, Q_j}\,J_1^{(1)}(K_{Q_j}) + \underbrace{D^0_{g,gq_j}}_{(2)}\,A^0_{q_i G, Q_j}\,J_1^{(1)}(K_{Q_j}) + \underbrace{D^0_{g,q_j g}}_{(3)}\,A^0_{q_i \bar{Q}_j, G}\,J_1^{(1)}(K_G)\right] + \mathcal{O}(1/N)$$

Extending the framework to NNLO

⇒ all the pieces are knwon for e^+e^-

⇒ partial results for 3 jets production, final results are imminent
[Gehrman-De Ridder, Gehrmann, Glover]

For hadronic collisions:

- antenna functions:
  – simply obtained by crossing, some issues with the splittings of some of them
  – integrated versions must be calculated for each configuration: working on them
- phase space mappings:
  – both initial-final and initial-initial NLO mappings have simple NNLO extensions
  – proper factorization in single soft or collinear limits

Summary

- first step towards completing a antenna subtraction formalism for hadron colliders at NNLO
- complete NLO framework
- phase space mappings and antenna functions at NNLO for all configurations
- possible application of NLO antennae and mappings to shower Monte Carlo's [W. Giele, HP^2 Workshop]
- NNLO integrated antennae with one or two partons in the initial state are on the way

...only missing piece besides ... a real, complete, calculation!