QED$\otimes$QCD Resummation with Shower/ME Matching and IR-Improved DGLAP Theory for LHC Physics

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Outline:

- **Introduction**
- **Review of YFS Theory and Its Extension to QCD**
- **Extension to QED$\otimes$QCD and QCED**
- **QED$\otimes$QCD Threshold Corrections, Shower/ME Matching, IR-Improved DGLAP Theory at the LHC**
- **Conclusions**

Papers by S. Jadach and B.F.L. Ward, S. Jadach, *et al.*, M. Phys. Lett. A **14** (1999) 491, hep-ph/0205062; *ibid.* **12** (1997) 2425; *ibid.***19** (2004) 2113; hep-ph/0509003,0508140

Motivation

- FNAL/RHIC $t\bar{t}$ PRODUCTION; POLARIZED pp PROCESSES; $b\bar{b}$ PRODUCTION; J/Ψ PRODUCTION: SOFT n(G) EFFECTS ALREADY NEEDED
  $\Delta m_t = 5.1$ GeV with SOFT n(G) UNCERTAINTY $\sim$ 2-3 GeV, ..., ETC.

- FOR THE LHC/TESLA/LC, THE REQUIREMENTS WILL BE EVEN MORE DEMANDING AND OUR QCD SOFT n(G) MC EXPONENTIATION RESULTS WILL BE AN IMPORTANT PART OF THE NECESSARY THEORY – YFS EXPONENTIATED $\mathcal{O}(\alpha_s^2)L$, IN THE PRESENCE OF SHOWERS, ON AN EVENT-BY-EVENT BASIS, WITHOUT DOUBLE COUNTING AND WITH EXACT PHASE SPACE.

- HOW RELEVANT ARE QED HIGHER ORDER CORRECTIONS WHEN QCD IS CONTROLLED AT $\sim 1\%$ PRECISION?

- CROSS CHECK OF QCD LITERATURE:
  1. PHASE SPACE – CATANI, CATANI-SEYMOUR, ALL INITIAL PARTONS MASSLESS
  2. RESUMMATION – CATANI ET AL., BERGER ET AL., ...
  3. NO-GO THEOREMS

- CROSS CHECK OF QED LITERATURE:

  1. ESTIMATES BY SPIESBERGER, STIRLING, ROTH and WEINZIERL – FEW PER MILLE EFFECTS FROM QED CORRECTIONS TO STR. FN. EVOLUTION.

  2. WELL-KNOWN POSSIBLE ENHANCEMENT OF QED CORRECTIONS AT THRESHOLD, ESPECIALLY IN RESONANCE PRODUCTION

  $\Rightarrow$ HOW BIG ARE THESE EFFECTS AT THE LHC?

- TREAT QED AND QCD SIMULTANEOUSLY IN THE YFS EXPONENTIATION TO ESTIMATE THE ROLE OF THE QED AND TO ILLUSTRATE AN APPROACH TO SHOWER/ME MATCHING.

PRELIMINARIES

- WE USE THE GPS CONVENTIONS OF JWW FOR SPINORS; PHOTON-GLUON POLARIZATION VECTORS FOLLOW THEREFROM:

$$(\epsilon^{\mu}_{\sigma}(\beta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}u_{\sigma}(\beta)}{\sqrt{2}\;\bar{u}_{-\sigma}(k)u_{\sigma}(\beta)}, \quad (\epsilon^{\mu}_{\sigma}(\zeta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}\mathfrak{u}_{\sigma}(\zeta)}{\sqrt{2}\;\bar{u}_{-\sigma}(k)\mathfrak{u}_{\sigma}(\zeta)}, \tag{1}$$

- REPRESENTATIVE PROCESSES

  $pp \rightarrow V + n(\gamma) + m(g) + X \rightarrow \bar{\ell}\ell' + n'(\gamma) + m(g) + X$,

  where $V = W^{\pm}, Z$,and $\ell = e, \mu,\ \ell' = \nu_e, \nu_\mu(e, \mu)$ respectively for $V = W^{+}(Z)$, and $\ell = \nu_e, \nu_\mu,\ \ell' = e, \mu$ respectively for $V = W^{-}$.

Review of YFS Theory and Its Extension to QCD

QED CASE – S. Jadach et al., YFS2, YFS3, BHLUMI, BHWIDE, KORALZ, KKMC, YFSWW3, YFSZZ, KoralW

For $e^+(p_1)e^-(q_1) \rightarrow \bar{f}(p_2)f(q_2) + n(\gamma)(k_1, \cdot, k_n)$**, renormalization group improved YFS theory (PRD36(1987)939) gives**

$$d\sigma_{exp} = e^{2\alpha\, Re\, B + 2\alpha\, \tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D}$$
$$\bar{\beta}_n(k_1, \ldots, k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}$$

where the YFS real infrared function $\tilde{B}$ and the virtual infrared function B are known and where we note the usual connections

$$2\alpha\, \tilde{B} = \int^{k \le K_{max}} \frac{d^3 k}{k_0} \tilde{S}(k)$$

$$D = \int d^3 k \frac{\tilde{S}(k)}{k^0} \left(e^{-iy\cdot k} - \theta(K_{max} - k)\right) \qquad (2)$$

for the standard YFS infrared emission factor

$$\tilde{S}(k) = \frac{\alpha}{4\pi^2}\left[Q_f Q_{(\bar{f})'}\left(\frac{p_1}{p_1\cdot k} - \frac{q_1}{q_1\cdot k}\right)^2 + (\ldots)\right] \quad (3)$$

if Q_f is the electric charge of f in units of the positron charge. For example, the YFS hard photon residuals $\bar{\beta}_i$ in (1), $i = 0, 1, 2$, are given in S. Jadach *et al.*,CPC102(1997)229 for BHLUMI 4.04 $\Rightarrow$ YFS exponentiated exact $\mathcal{O}(\alpha)$ and LL $\mathcal{O}(\alpha^2)$ cross section for Bhabha scattering via a corresponding Monte Carlo realization of (1).

In hep-ph/0210357(ICHEP02), Acta Phys.Polon.B33,1543-1558,2002, we have extended the YFS theory to QCD:

$$d\hat{\sigma}_{\rm exp} = \sum_n d\hat{\sigma}^n$$
$$= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}}$$
$$* \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3 P_2}{P_2^{\,0}} \frac{d^3 Q_2}{Q_2^{\,0}} \quad (4)$$

where now the hard gluon residuals $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ defined by

$$\tilde{\bar{\beta}}_n(k_1,\ldots,k_n) = \sum_{\ell=0}^{\infty} \tilde{\bar{\beta}}_n^{(\ell)}(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$.

- We stress that the arguments in the earlier papers (**DeLaney** *et al.* **PRD52(1995)108, PLB342(1995)239**) are not really sufficient to derive the respective analog of eq.(4); for, **they did not really expose the compensation between the left over genuine non-Abelian IR virtual and real singularities between** $\int dPh\bar{\beta}_n$ **and** $\int dPh\bar{\beta}_{n+1}$ **respectively that really allows us to isolate** $\tilde{\bar{\beta}}_j$ **and distinguishes QCD from QED, where no such compensation occurs.**
- **Our exponential factor corresponds to the** $N = 1$ **term in the exponent in Gatheral's formula (Phys. Lett.B133(1983)90) for the general exponentiation of the eikonal cross sections for non-Abelian gauge theory; his result is an approximate one in which everything that does not eikonalize and exponentiate is dropped whereas our result (4) is exact.**

Extension to QED⊗QCD and QCED

Simultaneous exponentiation of QED and QCD higher order effects,

hep-ph/0404087,

gives

$$
\begin{aligned}
B^{nls}_{QCD} &\rightarrow B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED},\\
\tilde{B}^{nls}_{QCD} &\rightarrow \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED},\\
\tilde{S}^{nls}_{QCD} &\rightarrow \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED} \equiv \tilde{S}^{nls}_{QCED}
\end{aligned} \tag{5}
$$

which leads to

$$
\begin{aligned}
d\hat{\sigma}_{\rm exp} &= e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \\
&\prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \\
&\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3 p_2}{p_2^0}\frac{d^3 q_2}{q_2^0},
\end{aligned} \tag{6}
$$

where the new YFS residuals

$\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$, **with n hard gluons and m hard photons,** represent the successive application of the YFS expansion first for QCD and subsequently for QED.

The infrared functions are now

$$\mathrm{SUM_{IR}(QCED)} = 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED}$$
$$D_{\mathrm{QCED}} = \int \frac{dk}{k^0}\left(e^{-iky} - \theta(K_{max}-k^0)\right)\tilde{S}^{nls}_{QCED} \quad (7)$$

where K_{max} is a dummy parameter – here the same for QCD and QED.

Infrared Algebra(QCED):

$x_{avg}(QED) \cong \gamma(QED)/(1+\gamma(QED))$

$x_{avg}(QCD) \cong \gamma(QCD)/(1+\gamma(QCD))$

$\gamma(A) = \frac{2\alpha_A \mathcal{C}_A}{\pi}(L_s - 1)$, $A = QED, QCD$

$\mathcal{C}_A = Q_f^2, C_F$, respectively, for $A = QED, QCD$

$\Rightarrow$ QCD dominant corrections happen an order of magnitude earlier than those for QED.

$\Rightarrow$ Leading $\tilde{\bar{\beta}}^{(0,0)}_{0,0}$ -level gives a good estimate of the size of the effects we study.

QED⊗QCD Threshold Corrections

,Shower/ME Matching & IRI-DGLAP Theory at the LHC

We shall apply the new simultaneous QED⊗QCD exponentiation calculus to the sinlge Z production with leptonic decay at the LHC (and at FNAL) to focus on the ISR alone, for defi niteness. See also the work of Baur *et al.*, Dittmaier and Kramer, Zykunov for exact $\mathcal{O}(\alpha)$ results and Hamberg *et al.*, van Neerven and Matsuura and Anastasiou *et al.* for exact $\mathcal{O}(\alpha_s^2)$ results.

For the basic formula

$$d\sigma_{exp}(pp \to V + X \to \bar{\ell}\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s), \tag{8}$$

we use the result in (6) here with semi-analytical methods and structure functions from Martin *et al.*.

A MC realization will appear elsewhere.

SHOWER/ME MATCHING

- **Note the following: In (8) WE DO NOT ATTEMPT *HERE* TO REPLACE HERWIG and/or PYTHIA –**
  WE INTEND *HERE* TO COMBINE OUR EXACT YFS CALCULUS, $d\hat{\sigma}_{exp}(x_i x_j s)$, WITH HERWIG and/or PYTHIA BY USING THEM/IT TO GENERATE A PARTON SHOWER STARTING FROM (x_1, x_2) AT FACTORIZATION SCALE μ AFTER THIS POINT IS PROVIDED BY $\{F_i\}$:
  THERE ARE TWO APPROACHES TO THE MATCHING UNDER STUDY, ONE BASED ON p_T-MATCHING AND ONE BASED ON SHOWER-SUBTRACTED RESIDUALS $\{\hat{\hat{\bar{\beta}}}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m)\}$– SEE hep-ph/0509003.
- **THIS COMBINATION OF THEORETICAL CONSTRUCTS CAN BE SYSTEMATICALLY IMPROVED WITH EXACT RESULTS ORDER-BY-ORDER IN α_s, α, WITH EXACT PHASE SPACE.**
- **THE RECENT ALTERNATIVE PARTON EVOLUTION ALGORITHM BY JADACH and SKRZYPEK, Acta. Phys. Pol.B35, 745 (2004), CAN ALSO BE USED.**
- **LACK OF COLOR COHERENCE $\Rightarrow$ ISAJET NOT CONSIDERED HERE.**

With this said, we compute , with and without QED, the ratio

$r_{exp} = \sigma_{exp}/\sigma_{Born}$

to get the results (We stress that we *do not* use the narrow resonance approximation here.)

$$r_{exp} = \begin{cases} 1.1901 & ,\ \text{QCED} \equiv \text{QCD+QED, LHC} \\ 1.1872 & ,\ \text{QCD, LHC} \\ 1.1911 & ,\ \text{QCED} \equiv \text{QCD+QED, Tevatron} \\ 1.1879 & ,\ \text{QCD, Tevatron} \end{cases} \quad (9)$$

$\Rightarrow$

$\ast$***QED IS AT .3% AT BOTH LHC and FNAL.***

$\ast$***THIS IS STABLE UNDER SCALE VARIATIONS.***

$\ast$***WE AGREE WITH BAUR ET AL., HAMBERG ET AL., van NEERVEN and ZIJLSTRA.***

$\ast$***QED EFFECT SIMILAR IN SIZE TO STR. FN. RESULTS.***

$\ast$***DGLAP SYNTHESIZATION HAS NOT COMPROMISED THE NORMALIZATION.***

IR-IMPROVED DGLAP THEORY

- WELL-KNOWN: ERROR ON PDF'S
  -- MUST CONTROL FOR PRECISION
  LHC PHYSICS
  -- ~ % LEVEL DESIRED
- EXPONENTIATED DGLAP EVOLUTION
  POSSIBLE AVENUE FOR BETTER PRECISION

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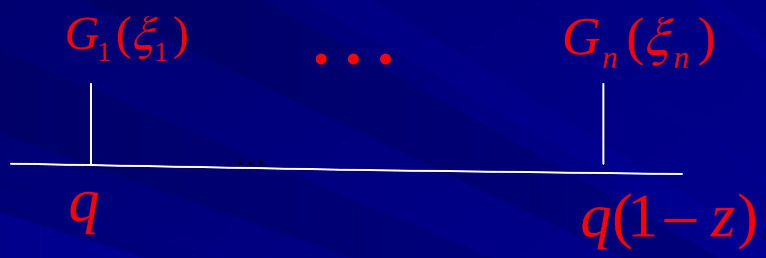
DGLAP THEORY SOFT LIMIT

$$z = \sum_j \xi_j$$



$$q \to q(1-z) + G_1(\xi_1) + \cdots + G_n(\xi_n)$$

$$\Rightarrow$$

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$$P_{BA}(z) = \frac{1}{2} z(1-z) \overline{\sum_{spins}} \frac{|V_{A\to B+C}|^2}{p_\perp^2}$$

$$\Rightarrow$$

$$P_{BA}(z) = \frac{1}{2} z(1-z) \overline{\sum_{spins}} \frac{|V_{A\to B+C}|^2}{p_\perp^2} z^{\gamma_q} F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q}$$

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IR-IMPROVEMENT

- EXPONENTIATED NS DGLAP KERNEL

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+z^2}{1-z}(1-z)^{\gamma_q}$$

- WHERE

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0}$$

$$\delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right)$$

$$F_{YFS}(\gamma_q) = \frac{e^{-C_E \gamma_q}}{\Gamma(1+\gamma_q)}$$

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- NOTE

$$\int_{k_0} dz/z = C_0 - \ln k_0$$

experimentally distinguishable from

$$\int_{k_0} dz/z^{1-\gamma} = C'_0 - k_0^{\gamma}/\gamma$$

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(ANTICIPATED BY GRIBOV & LIPATOV)

LEP DATA

OPAL SiW preliminary

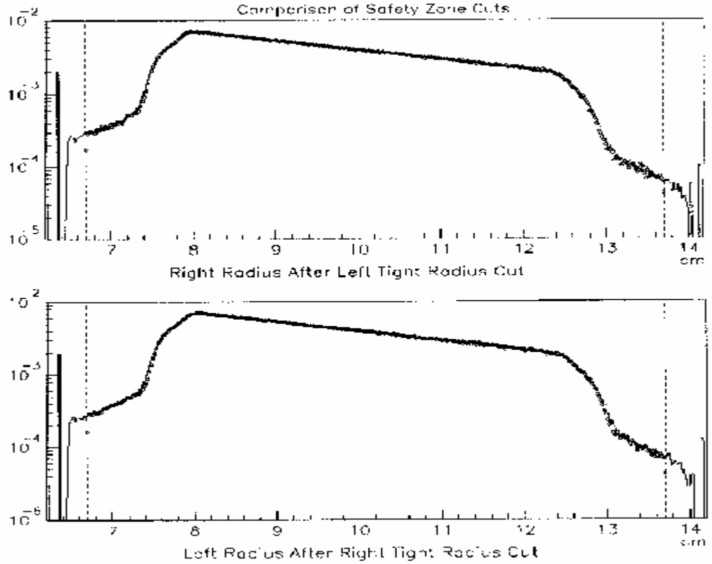

Figure 6: Radial distribution in the OPAL SiW detector compared to the Monte Carlo simulation with the BHLUMI generator.

due to better survey of the detector position as well as due to improvements of the detector simulation in the Monte Carlo. These improvements are well seen in the DELPHI SAT results where the lead mask was replaced by the tungsten mask, the geometrical survey was improved, and finally a lot of effort was put into the Monte Carlo simulation, particularly the simulation of interactions on the mask edge. Actual improvement with time was even bigger than what can be seen in Table 4 since some improvements, for example Monte Carlo simulation, could help in reducing the error of the data taken before the simulation was made.

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NORMALIZATION CONDITION

$$\int_0^1 dz P_{qq}(z) = 0$$

$$\Rightarrow$$

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q \delta(1-z) \right]$$

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IR-IMPROVED DGLAP KERNELS

$$P_{qq}^{\exp}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q \delta(1-z) \right]$$

$$P_{Gq}^{\exp}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}$$

$$P_{GG}^{\exp}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \Big\{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G}$$

$$+ \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G \delta(1-z) \Big\}$$

$$P_{qG}^{\exp}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \}$$

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HIGHER ORDER DGLAP KERNELS

$O(\alpha_s^2)$, $O(\alpha_s^3)$ KERNELS: Curci, Furmanski and Petronzio, Floratos et al., Moch et al., etc. - consider

$$P_{ns}^{+} = P_{qq}^{v} + P_{\bar{q}q}^{v} \equiv \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^{n+1} P_{ns}^{(n)+}$$

$\mathrm{O}(\alpha_s)$:

$$\mathrm{P}_{\mathrm{ns}}^{(0)+}(z) = 2C_F \left\{ \frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right\}$$

$\Rightarrow$ $\mathrm{P}_{\mathrm{ns}}^{(0)+}(z)$ agrees with unexponentiated $P_{qq}(z)$ except for overall factor of 2

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- Floratos et al., etc. :

  exact $P_{ns}^{(1)+}(z)$

- Moch et al. :

  exact $P_{ns}^{(2)+}(z)$

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Applying our QCD Exponentiation Master Formula to $q \rightarrow q + X$, $\bar{q} \rightarrow q + X'$ $\Rightarrow$

$$d\hat{\sigma}_{\exp} = \sum_n d\hat{\sigma}^n$$

$$= e^{SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{QCD}}$$

$$* \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3 P_2}{P_2^0} \frac{d^3 Q_2}{Q_2^0}$$

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- Hard Gluon Residuals:

$$\tilde{\bar{\beta}}_n(k_1,\ldots,k_n) = \sum_{\ell=0}^{\infty} \tilde{\bar{\beta}}_n^{(\ell)}(k_1,\ldots,k_n)$$

FREE OF IR DIV. TO ALL ORDERS IN α_s

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$$P_{ns}^{+,\exp}(z) = \left(\frac{\alpha_s}{4\pi}\right)\left(2P_{qq}^{\exp}(z)\right) + F_{YFS}(\gamma_q)e^{\frac{1}{2}\delta_q}\Big[\left(\frac{\alpha_s}{4\pi}\right)^2\left\{(1-z)^{\gamma_q}\,\overline{P}_{ns}^{(1)+}(z) + \overline{\mathrm{B}}_2\,\delta(1-z)\right\} + \left(\frac{\alpha_s}{4\pi}\right)^3\left\{(1-z)^{\gamma_q}\,\overline{P}_{ns}^{(2)+}(z) + \overline{\mathrm{B}}_3\,\delta(1-z)\right\}\Big]$$

Resummed Residuals :

$$\overline{P}_{\mathrm{ns}}^{(\mathrm{i})+}(z) = P_{ns}^{(i)+}(z) - B_{1+i}\,\delta(1-z) + \Delta_{ns}^{(i)+}(z),$$

where

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- where

$$\Delta_{ns}^{(1)+}(z) = -4C_F \pi\delta_1 \left\{ \frac{1+z^2}{1-z} - f_q \delta(1-z) \right\}$$

$$\Delta_{ns}^{(2)+}(z) = -4C_F (\pi\delta_1)^2 \left\{ \frac{1+z^2}{1-z} - f_q \delta(1-z) \right\} - 2\pi\delta_1 \overline{P}_{ns}^{(1)+}(z)$$

and

$$\overline{B}_2 = B_2 + 4C_F \pi\delta_1 f_q$$

$$\overline{B}_3 = B_3 + 4C_F (\pi\delta_1)^2 f_q - 2\pi\delta_1 \overline{B}_2$$

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- The constants are

$$B_2 = 4C_G C_F \left(\frac{17}{24} + \frac{11}{3}\varsigma_2 - 3\varsigma_3\right) - 4C_F n_f \left(\frac{1}{12} + \frac{2}{3}\varsigma_2\right)$$

$$+ 4C_F^2 \left(\frac{3}{8} - 3\varsigma_2 + 6\varsigma_3\right)$$

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$$
\begin{aligned}
B_3 = {} & 16 C_G C_F n_f \left(\frac{5}{4} - \frac{167}{54}\zeta_2 + \frac{1}{20}\zeta_2^2 + \frac{25}{18}\zeta_3\right) \\
& + 16 C_G C_F^2 \left(\frac{151}{64} + \zeta_2\zeta_3 - \frac{205}{24}\zeta_2 - \frac{247}{60}\zeta_2^2 + \frac{211}{12}\zeta_3 + \frac{15}{2}\zeta_5\right) \\
& + 16 C_G^2 C_F \left(-\frac{1657}{576} + \frac{281}{27}\zeta_2 - \frac{1}{8}\zeta_2^2 - \frac{97}{9}\zeta_3 + \frac{5}{2}\zeta_5\right) \\
& + 16 C_F n_f^2 \left(-\frac{17}{144} + \frac{5}{27}\zeta_2 - \frac{1}{9}\zeta_3\right) \\
& + 16 C_F^2 n_f \left(-\frac{23}{16} + \frac{5}{12}\zeta_2 + \frac{29}{30}\zeta_2^2 - \frac{17}{6}\zeta_3\right) \\
& + 16 C_F^3 \left(\frac{29}{32} - 2\zeta_2\zeta_3 + \frac{9}{8}\zeta_2 + \frac{18}{5}\zeta_2^2 + \frac{17}{4}\zeta_3 - 15\zeta_5\right)
\end{aligned}
$$

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Contact with Wilson Expansion

- N-th Moment of Forward Compton Amplitudes $T_{i,\ell}$, i=L,2,3, ℓ=q, G, (Gorishnii et al.)

$$P_N = \left[\frac{q^{\{\mu_1} \cdots q^{\mu_N\}}}{N!} \frac{\partial^N}{\partial p^{\mu_1} \cdots \partial p^{\mu_N}} \right] \Bigg|_{p=0},$$

projects coefficient of $1/(2x_{Bj})^N \Rightarrow$

we resum terms $\Leftrightarrow \gamma_q$ - dependent anomalous dimensions

NOT IN WILSON'S EXPANSION

LARGE λ NOT ALL ON TIP OF LIGHTCONE

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Effects on Parton Distributions

Moments of kernels ⇔ Logarithmic exponents for evolution

$$\frac{dM_n^{NS}(t)}{dt} = \frac{\alpha_s(t)}{2\pi} A_n^{NS} M_n^{NS}(t) \tag{43}$$

where

$$M_n^{NS}(t) = \int_0^1 dz z^{n-1} q^{NS}(z,t) \tag{44}$$

and the quantity A_n^{NS} is given by

$$\begin{aligned} A_n^{NS} &= \int_0^1 dz z^{n-1} P_{qq}(z), \\ &= C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)] \end{aligned} \tag{45}$$

where $B(x,y)$ is the beta function given by

$$B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$$

.

Compare the usual result

$$A_n^{NS^o} \equiv C_F\left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2\sum_{j=2}^{n}\frac{1}{j}\right]. \quad (46)$$

- ASYMPTOTIC BEHAVIOR: IR-improved goes to a multiple of $-f_q$, consistent with $\lim_{n\to\infty} z^{n-1} = 0$ for $0 \le z < 1$; usual result diverges as $-2C_F \ln n$.
- Different for finite n as well: for $n = 2$ we get, for example, for $\alpha_s \cong .118$,

$$A_2^{NS} = \begin{cases} C_F(-1.33) & , \text{ un-IR-improved} \\ C_F(-0.966) & , \text{ IR-improved} \end{cases} \quad (47)$$

- **For completeness we note**

$$M_n^{NS}(t) = M_n^{NS}(t_0) e^{\int_{t_0}^{t} dt' \frac{\alpha_s(t')}{2\pi} A_n^{NS}(t')}$$
$$= M_n^{NS}(t_0) e^{\bar{a}_n [Ei(\frac{1}{2}\delta_1 \alpha_s(t_0)) - Ei(\frac{1}{2}\delta_1 \alpha_s(t))]}$$
$$\underset{t,t_0 \text{ large with } t>>t_0}{\Longrightarrow} M_n^{NS}(t_0) \left(\frac{\alpha_s(t_0)}{\alpha_s(t)}\right)^{\bar{a'}_n} \quad (48)$$

where $Ei(x) = \int_{-\infty}^{x} dr e^r/r$ **is the exponential integral function,**

$$\bar{a}_n = \frac{2C_F}{\beta_0} F_{YFS}(\gamma_q) e^{\frac{\gamma_q}{4}} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)]$$
$$\bar{a'}_n = \bar{a}_n \left(1 + \frac{\delta_1}{2} \frac{(\alpha_s(t_0) - \alpha_s(t))}{\ln(\alpha_s(t_0)/\alpha_s(t))}\right) \quad (49)$$

with

$\delta_1 = \frac{C_F}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right)$.

Compare with un-IR-improved result where last line in eq.(48) holds exactly with $\bar{a'}_n = 2A_n^{NS^o}/\beta_0$.

- For $n = 2$, taking $Q_0 = 2$GeV and evolving to $Q = 100$GeV, with $\Lambda_{QCD} \cong .2GeV$ and $n_f = 5$ for illustration, (48,49) $\Rightarrow$ a shift of evolved NS moment by $\sim\ 5\%$, of some interest in view of the expected HERA precision (see for example, T. Carli et al., Proc. HERA-LHC Wkshp, 2005).

- Introduction of IR-Improved DGLAP kernels in HERWIG and PYTHIA in progress.

Conclusions

YFS THEORY (EEX AND CEEX) EXTENDS TO NON-ABELIAN GAUGE THEORY AND ALLOWS SIMULTANEOUS EXPN OF QED AND QCD WITH PROPER SHOWER/ME MATCHING BUILT-IN.

FOR QED$\otimes$QCD

- FULL MC EVENT GENERATOR REALIZATION IS POSSIBLE.
- SEMI-ANALYTICAL RESULTS FOR QED (AND QCD) THRESHOLD EFFECTS AGREE WITH LITERATURE ON Z PRODUCTION
- AS QED IS AT THE .3% LEVEL, IT IS NEEDED FOR 1% LHC THEORY PREDICTIONS; DITTO FOR IRI-DGLAP.
- A FIRM BASIS FOR THE COMPLETE $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ MC RESULTS NEEDED FOR THE FNAL/LHC/RHIC/TESLA/LC PHYSICS HAS BEEN DEMONSTRATED AND ALL THE LATTER IS IN PROGRESS.