Bounds on DIS Observables from the Colour Dipole Picture

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in collaboration with

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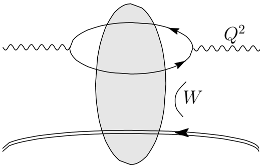
The Colour Dipole Picture

Dipole Picture *(Nikolaev, Zakharov (1991,1992), Mueller (1994,1995))*:

- successful description of HERA data down to very low Q^2
- key idea:
  - $\gamma \rightarrow q\bar{q}$
  - + interaction of $q\bar{q}$ "colour dipole" with p
- saturation at low x ?
  - means low Q^2 at HERA, pQCD problematic
  - implement saturation via dipole picture



GBW model

Golec-Biernat,Wüsthoff (1998) model:

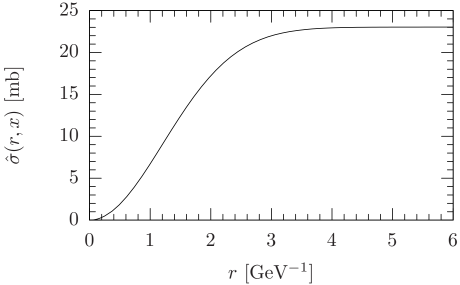
dipole cross section

$$\hat{\sigma}_{\text{GBW}}(r, x) = \hat{\sigma}_0 \left(1 - e^{-\frac{1}{4}(r/r_0(x))^2}\right)$$

with dipole size r and saturation scale

$$r_0(x) = \left(\frac{x}{x_0}\right)^{\lambda/2}$$

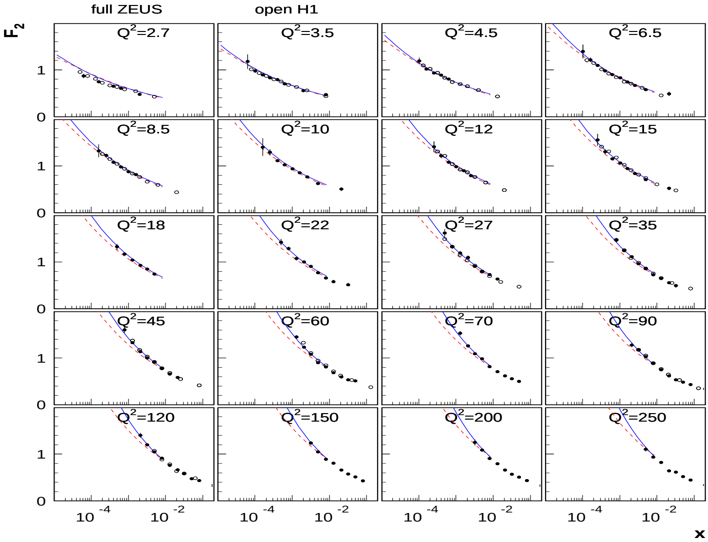
($\hat{\sigma}_0 \approx 23$ mb, $\lambda \approx 0.3$, $x_0 \approx 3 \cdot 10^{-4}$)



($Q^2 = 1$ GeV^2, $W = 100$ GeV)

Success of the Dipole Picture

Medium - High Q^2



Bartels, Golec-Biernat, Kowalski (2002):
GBW+DGLAP (solid), GBW (dashed), ZEUS (full), H1 (open)

Success of the Dipole Picture

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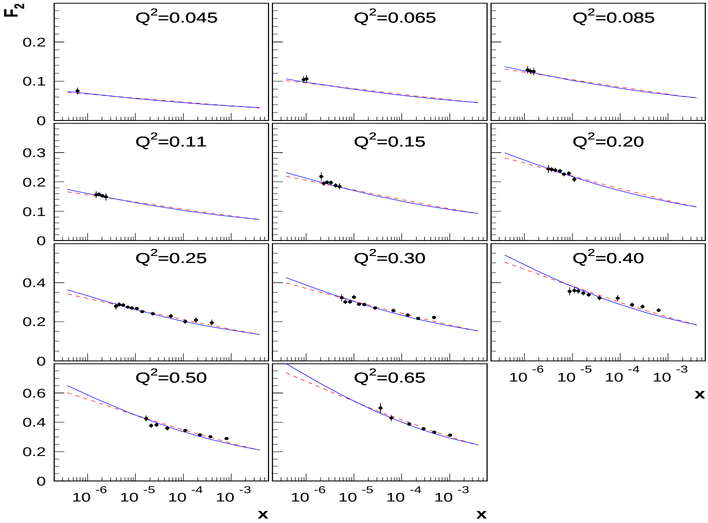

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GBW+DGLAP (solid), GBW (dashed), ZEUS BPT97.

Outline

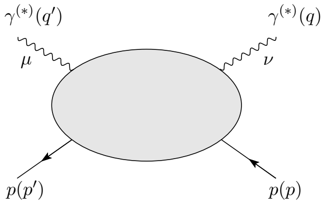
Layout

Compton Scattering

$\gamma^{(*)} p \to \gamma^{(*)} p$ with nonperturbative techniques:
C. Ewerz, O. Nachtmann (2004, 2006)

- matrix element

$$\mathcal{M}^{\mu\nu}_{s's}(p', p, q) = \frac{i}{2\pi m_p} \int d^4x \; e^{-iqx} \langle p(p', s') | \mathrm{T}^* J^\mu(0) J^\nu(x) | p(p, s) \rangle$$



- electromagnetic current

$$J^\lambda(x) = \sum_{q=u,d,\ldots} \bar{q}(x) Q_q \gamma^\lambda q(x)$$

- interpolating field operator for proton

$$\psi_p(x) = \Gamma_{\alpha\beta\gamma} u_\alpha(x) u_\beta(x) d_\gamma(x)$$

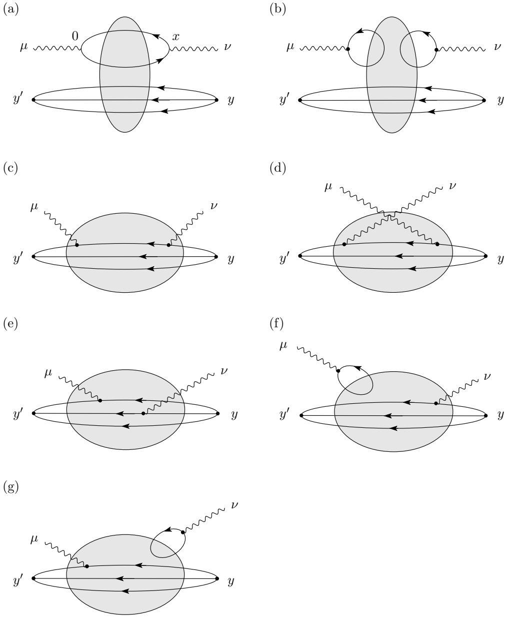
- LSZ reduction, Green's functions via functional integrals
- perform $q, \bar{q}$ functional integrations

Skeleton Decomposition

- classify by quark line skeleton

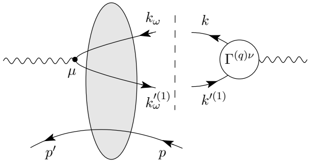
$$\mathcal{M} = \mathcal{M}^{(a)} + \ldots + \mathcal{M}^{(g)}$$

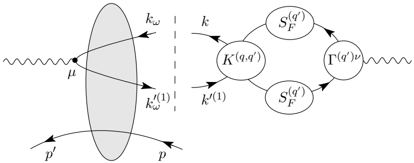
- leading at high energy:
  - class (a) and (b)
- (b) higher order in α_s than (a), suppressed at high Q^2
- consider class (a) only
  - dipole picture



Towards the Dipole Picture

- transition from currents to $q\bar{q}$:
  - insert factors of 1 with free quark prop.
- renormalisation of $\gamma q\bar{q}$ vertex:
  - Dyson-Schwinger-equation
- pinch in ω integration for $\Delta E \to 0$, at high $\boldsymbol{q}$:
  - for quark long. momenta in $\boldsymbol{q}$ direction
  - singles out leading term in $\mathcal{M}^{(a)}$





proceed with perturbative treatment:

- $\gamma q\bar{q}$ vertex: leading order
- define bound dipole states

and finally arrive at ...

The Dipole Picture

Compton Scattering in the Dipole Picture

$\gamma p \to \gamma p$ cross section

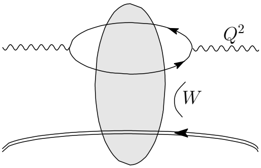
$$\sigma^{\gamma p}_{T,L}(W, Q^2) = \sum_q \int \mathrm{d}^2 r \; \hat{\sigma}^{(q)}(r, W) w^{(q)}_{T,L}(r, Q^2)$$

with photon wave function density

$$w^{(q)}_{T,L}(r, Q^2) = \int_0^1 \mathrm{d}\alpha \left| \psi^{(q)}_{T,L}(\alpha, \boldsymbol{r}, Q^2) \right|^2$$

and dipole cross section

$$\hat{\sigma}^{(q)}(r, W)$$



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Bounds on $R = \sigma_L/\sigma_T$

Applicability of dipole picture:
Predictions by the framework alone, valid for any dipole cross section $\hat{\sigma}$?

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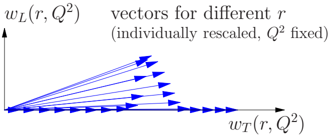

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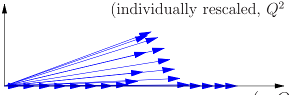

- $\Rightarrow$ for any dipole picture based $R = \sigma_L^{\gamma p}/\sigma_T^{\gamma p}$:

$$\inf_{q,r} \frac{w_L^{(q)}(r, Q^2)}{w_T^{(q)}(r, Q^2)} \leq R \leq \sup_{q,r} \frac{w_L^{(q)}(r, Q^2)}{w_T^{(q)}(r, Q^2)}$$

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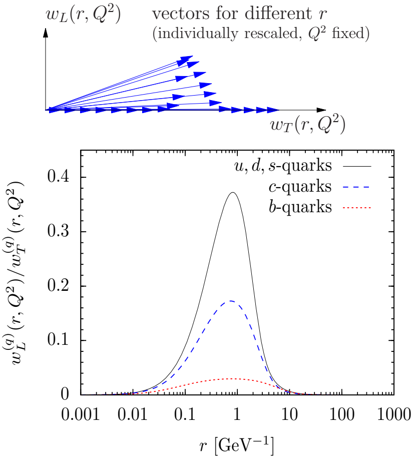
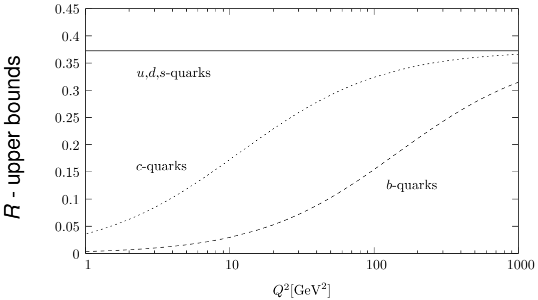
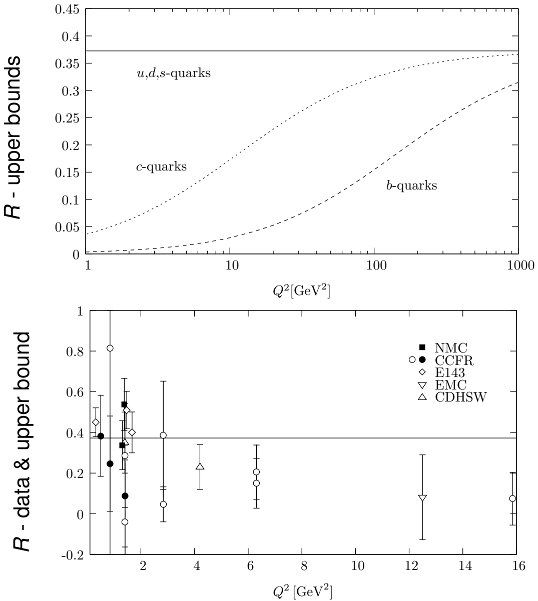


C. Ewerz, O. Nachtmann (2006)



Ratio $R = \sigma_L/\sigma_R$: upper bounds from dipole picture and data.



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Layout

Bounds on F_2 ratios

Consider structure function
$F_2(W, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{\text{em}}} (\sigma_L^{\gamma p} + \sigma_T^{\gamma p})$ at two different Q^2

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$$\begin{pmatrix} F_2(W, Q_1^2) \\ F_2(W, Q_2^2) \end{pmatrix} = \sum_q \int \mathrm{d}^2 r \, \frac{\hat{\sigma}^{(q)}(r, W)}{4\pi^2\alpha_{\text{em}}} \cdot$$

$$\cdot \begin{pmatrix} Q_1^2 \left(w_T^{(q)}(r, Q_1^2) + w_L^{(q)}(r, Q_1^2) \right) \\ Q_2^2 \left(w_T^{(q)}(r, Q_2^2) + w_L^{(q)}(r, Q_2^2) \right) \end{pmatrix}$$

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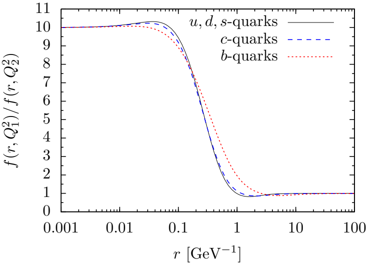
- $\Rightarrow$ for any dipole cross section $\hat{\sigma}^{(q)}(r, W)$:

$$\inf_{q,r} \frac{f^{(q)}(r, Q_1^2)}{f^{(q)}(r, Q_2^2)} \leq \frac{F_2(W, Q_1^2)}{F_2(W, Q_2^2)} \leq \sup_{q,r} \frac{f^{(q)}(r, Q_1^2)}{f^{(q)}(r, Q_2^2)}$$

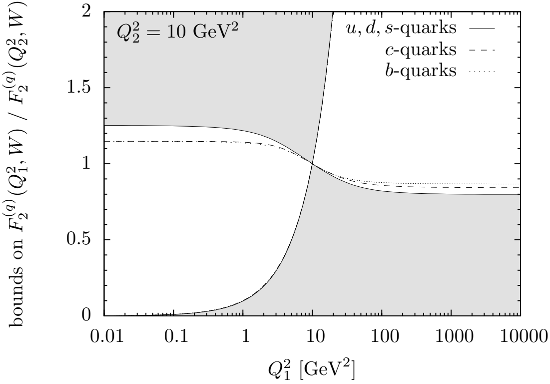
  with

$$f^{(q)}(r, Q^2) \equiv Q^2 \left(w_T^{(q)}(r, Q^2) + w_L^{(q)}(r, Q^2) \right)$$



Bounds on F_2 ratios
Results for different flavours

C. Ewerz, O. Nachtmann (2006)



Improved Bounds on Q^2 dependence of σ_r

Consider reduced cross section

$$\sigma_r(W, Q^2) = \frac{Q^2}{4\pi^2\alpha_{\text{em}}}\left(\sigma_T^{\gamma p} + \frac{2(1-y)}{1+(1-y)^2}\sigma_L^{\gamma p}\right)$$

with $y = \frac{W^2+Q^2}{s_{ep}}$ at three different Q^2

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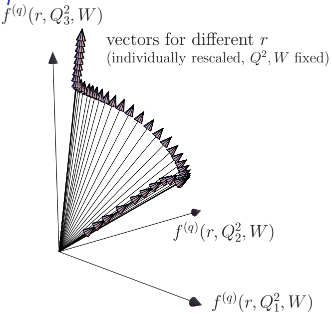

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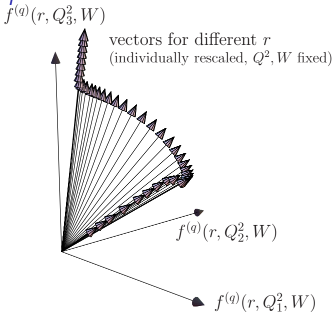

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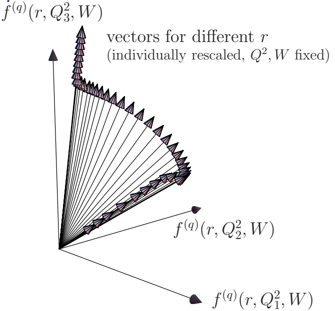
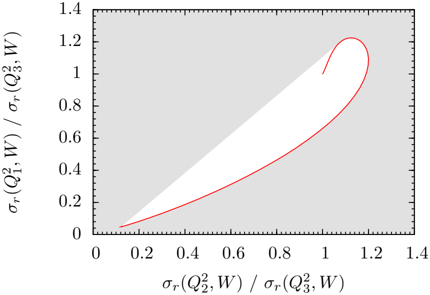
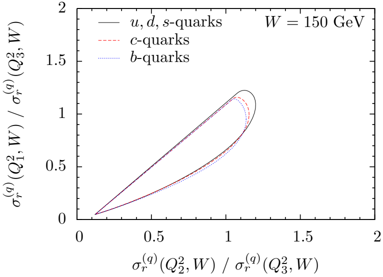




Correlated ratio bounds on σ_r - flavour and W dependence

Bounds for $(Q_1^2, Q_2^2, Q_3^2) = (4, 10, 80)$ GeV^2:



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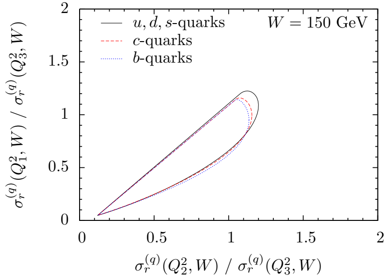
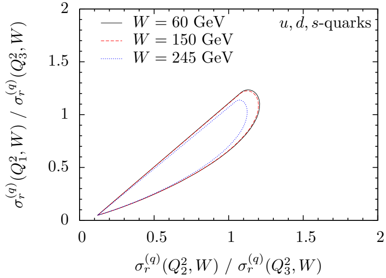
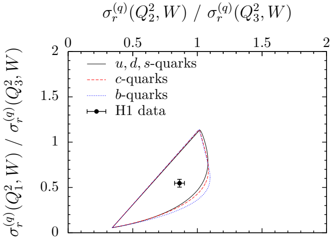




Correlated ratio bounds on σ_r confronted with data

high W, moderate Q^2 $\longrightarrow$

$W = 247$ GeV,
$(Q_1^2, Q_2^2, Q_3^2) = (2, 12, 35)$ GeV^2



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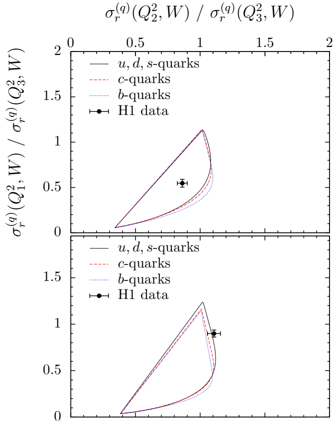

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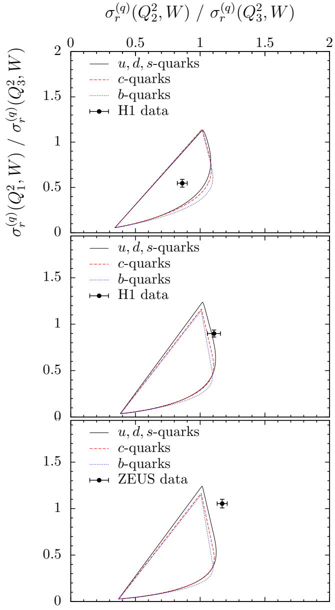
lower W, incl. high Q^2 $\longrightarrow$

$W = 75$ GeV,
$(Q_1^2, Q_2^2, Q_3^2) = (4, 45, 120)$ GeV^2



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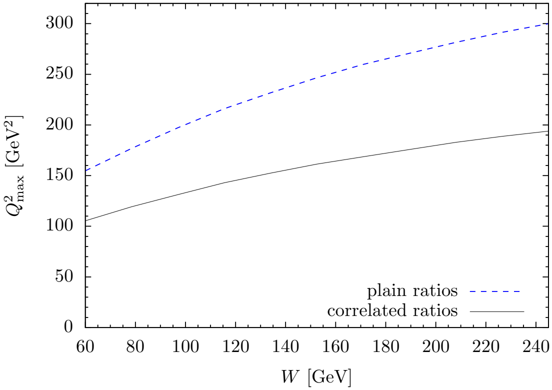

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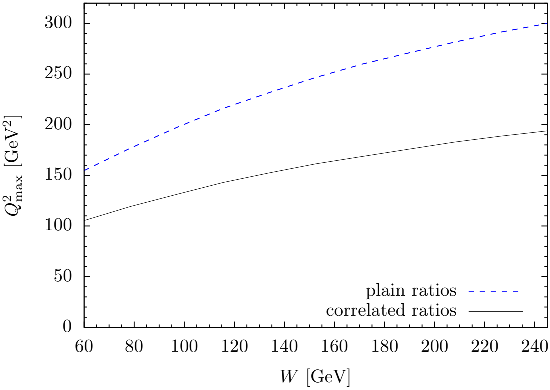

- note: $\hat{\sigma} = \hat{\sigma}(r, W)$ assumed, limits do not apply for $\hat{\sigma} = \hat{\sigma}(r, x)$

Summary

Dipole Picture:

- nonperturbative foundations
- general bounds
  - for $R = \sigma_L/\sigma_T$ and "any" $\hat{\sigma}$, relevant at low Q^2 ?
  - for $F_2(W, Q_1^2)/F_2(W, Q_2^2)$ and $\hat{\sigma}(r, W) \Rightarrow$ application limited to max. Q^2

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Outlook:

- photon wave function beyond leading order
- F_L measurement at HERA

Supplementary Slides

- Energy dependence of the Dipole Cross Section
- Bounds on F_2 ratios vs. ALLM97

The correct energy dependence of $\hat{\sigma}$

Which energy variable should be used for $\hat{\sigma}$?

- limit $W \Rightarrow \infty$, Q^2 fixed:
  dipoles similar to hadrons with $\hat{\sigma}^{(q)} = \hat{\sigma}^{(q)}(r, W)$
  - independent of Q^2
  - $\Leftrightarrow$ indep. of $x = Q^2/(W^2 + Q^2)$ for given W
- γp scattering in perturbative limit $W, Q^2 \Rightarrow \infty$, Q^2/W^2 fixed:
  $\hat{\sigma} \propto$ gluon density $g(x, Q^2)$:
  - dependent on Q^2

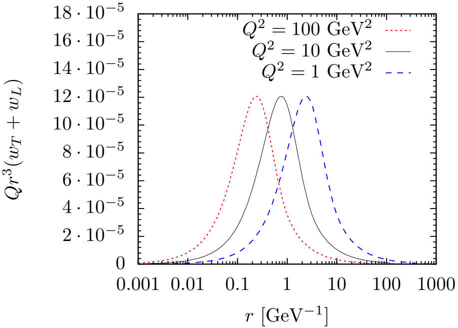
Way out through photon wave function ?

- photon wave funct. carries Q^2 info $\Rightarrow$ favoured r of prod. dipoles dep. on Q^2
- consider $\hat{\sigma} = const \cdot r^2$:

$$\sigma^{\gamma p} \propto \int \mathrm{d}r \, r \cdot r^2 \cdot (w_T + w_L)$$

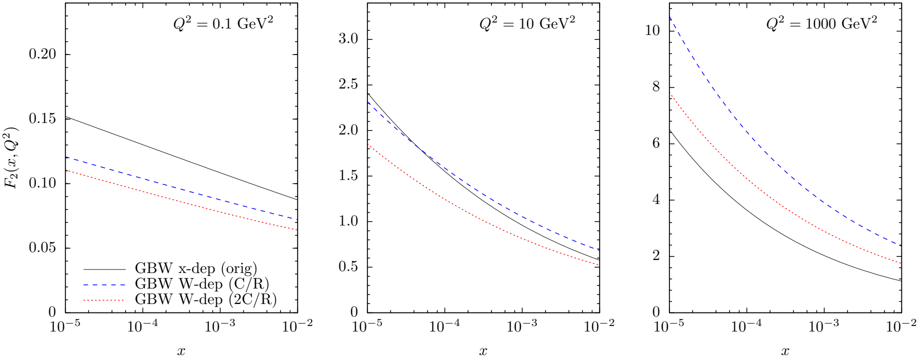
- peak at

$$r_{\max}(Q^2) = \frac{C}{Q}, \qquad \text{with } C = 2.40$$



Can we rewrite energy dependences of $\hat{\sigma}$ based on r_{max} ?

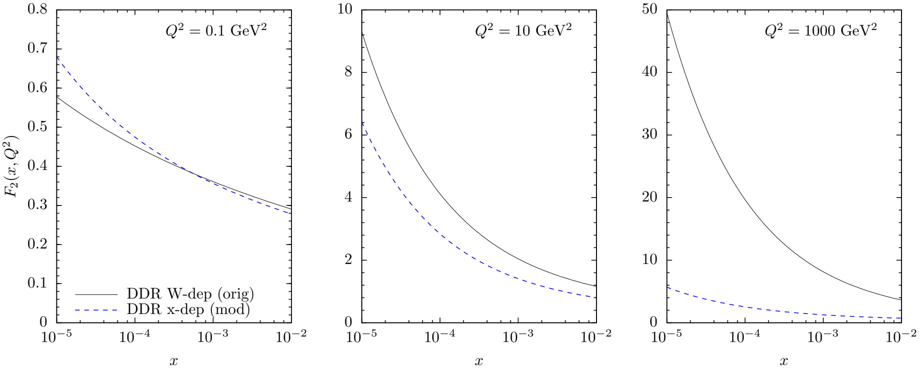
F_2 for GBW using peak size $r_{\max}(Q^2)$



Elimination of Q^2-dependence: F_2 vs. x for Golec-Biernat,Wüsthoff (1999) model.

solid black: original Q^2-dependent $\hat{\sigma}$,
blue dashed: Q^2-dependence in $\hat{\sigma}$ eliminated by $Q \to C/r$,
red dotted: $Q \to 2\,C/r$.

F_2 for DDR using peak size $r_{max}(Q^2)$

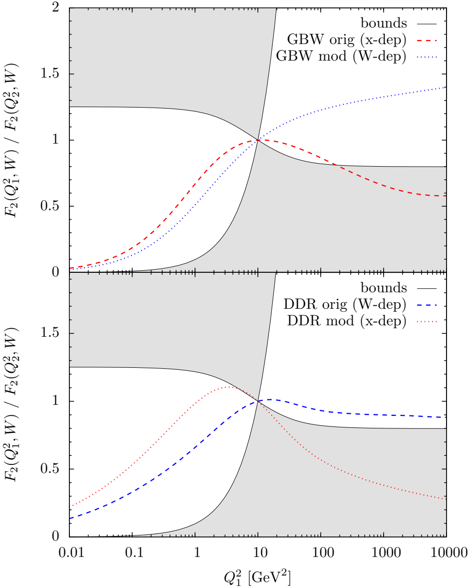


Introduction of Q^2-dependence: F_2 vs. x for Donnachie,Dosch,Rueter (1999-2001) model.

solid black: original Q^2-independent $\hat{\sigma}$,
blue dashed: Q^2-dependence introduced by $r \rightarrow C/Q$ in $\hat{\sigma}/r^2$.

Energy dependencies and general bounds on F_2 ratios

Simple substitution leads to substantial change:



Bounds on F_2 ratios
Results and ALLM97 fit

C. Ewerz, O. Nachtmann (2006)