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This talk is based on a fresh  
Tel-Aviv preprint [hep-ph/7002053](#)  
(GLM + Kormilitzin).

We were provoked by a recent  
KMR preprint [hep-ph/0609312](#)  
which suggests a procedure to  
extract  $G_{3P}$ , the bare triple  
Pomeron coupling, from high mass  
SD data in the reaction

$$\gamma^* + P \rightarrow \bar{q}/q + M$$

In as much as we like the basic  
idea suggested by KMR, our results  
are different and may be relevant  
to the calculations of survival  
probabilities for Higgs and hard  
dijets.

## I) Introduction:

Survival probability is a damping factor necessitated by unitarity and QCD corrections to an initial LRG diffractive cross section.

$$S_D^2(s) = \frac{\sigma_D^{\text{out}}(s)}{\sigma_D^{\text{in}}(s)}$$

In general, the survival probability can be perceived as a product of 3 independent factors

$$S^2(s, \Delta Y) = S_{\text{soft}}^2(s) \cdot S_{g\text{-rad}}^2(\Delta Y) \cdot S_{\text{hard}}^2(s)$$

- 1)  $S_{\text{soft}}^2$  is due to the soft rescatterings of the spectator partons. It is a consequence of  $s$ -channel unitarity imposed on the soft scattering amplitude. Its calculation is coupled to a global fit of the soft scattering data.

2)  $S_{g\text{-rad}}^2$  is initiated by gluon radiation emitted by the partons taking part in the hard LRG diffractive sub process. This correction is strongly suppressed by the Sudakov factor which is included in the calculation of the hard sub process.

3)  $S_{\text{hard}}^2$  is a consequence of possible s-channel unitarity corrections in the hard pQCD sector. It is commonly ignored even though it may be relevant in DIS at small  $(x, \alpha^2)$ .

$\Rightarrow$  present estimates:  $S^2 = S_{\text{soft}}^2$ .

In the spirit of KMR we have examined two SD reactions

$$1) p+p \rightarrow p+M$$

$$2) \bar{r}^*+p \rightarrow \bar{J}/\psi + M$$

(4)

## II) Single diffraction in p-p scattering

The calculation of  $S_{SD}^2$  in the eikonal model is simple

$$A_{el}(s, b) = i(1 - e^{-\frac{1}{2}\Omega(s, b)})$$

$$G_{in}(s, b) = 1 - e^{-\Omega(s, b)}$$

Given a diffractive  $b$ -space amplitude  $M_D(s, b)$ , be it soft or hard, it is corrected to include initial state rescatterings  $|M_D|^2 \rightarrow e^{-\Omega} |M_D|^2$ .

We define the survival probability

$$S_D^2 = \frac{\int d^2b e^{-\Omega} |M_D|^2}{\int d^2b |M_D|^2} = \frac{\sigma_D^{out}(s)}{\sigma_D^{in}(s)}$$

The model does not fit the energy dependence of  $\sigma_{SD}(s)$ .

Reason: The model takes into account only elastic rescatterings. i.e. it assumes that  $\sigma_{diff} \ll \sigma_{el}$  which is not compatible with the data.

In a two channel model we include <sup>(4)</sup>  
 in the initial rescattering chain also  
 diffractive states. In the simplest  
 approximation diffractively produced  
 hadrons at a vertex are considered as  
 a single diffractive state  $\Psi_D$  which  
 is orthonormal to the hadronic (proton)  
 wave function  $\Psi_h$ :  $\langle \Psi_h | \Psi_D \rangle = 0$ .

The  $2 \times 2$  interaction operator  $T$  is  
 diagonalized by two base wave  
 functions  $\Psi_1$  and  $\Psi_2$ .

$$\Psi_h = \alpha \Psi_1 + \beta \Psi_2$$

$$\Psi_D = -\beta \Psi_1 + \alpha \Psi_2$$

$$\alpha^2 + \beta^2 = 1$$

The high energy amplitudes are

$$A_{i,k} = \langle \Psi_i \Psi_k | T | \Psi_i \Psi_k \rangle = A_{i,k} \delta_{i,i'} \delta_{k,k'}$$

The two channel model has, in (5)  
 general 4 amplitudes. For  $pp(p\bar{p})$   
 it reduces to 3 amplitudes since  
 $A_{12} = A_{21}$ . We get:

$$a_{ee}(s,b) = \alpha^4 A_{b1} + 2\alpha^2\beta^2 A_{b2} + \beta^4 A_{b22}$$

$$a_{sd}(s,b) = \alpha\beta \left[ -\alpha^2 A_{b1} + (\alpha^2 - \beta^2) A_{b2} + \beta^2 A_{b22} \right]$$

$$a_{dd}(s,b) = \alpha^2\beta^2 (A_{b1} - 2A_{b2} + A_{b22})$$

A further simplification is obtained if  
 we neglect  $\sigma_{DD}$ , assuming  $a_{dd} = 0$   
 and thus  $A_{b22} = 2A_{b2} - A_{b1}$ . We obtain:

$$a_{ee}(s,b) = A_{b1} - 2\beta^2 (A_{b1} - A_{b2})$$

$$a_{sd}(s,b) = -\alpha\beta (A_{b1} - A_{b2})$$

Two channel eikonalization implies

$$A_{i,\pm}(s,b) = i \left( 1 - e^{-\frac{\Re i,\pm}{2}} \right)$$

$$G_{i,\pm}^{in}(s,b) = e^{-\Re i,\pm}$$

(6)

In the GLM model we assume  
 Regge factorization of the couplings  
 and a Gaussian  $b$ -dependence

$$\Omega_{i,j,k}(S,b) = \frac{g_i g_k}{\pi R_{i,k}^2} \left(\frac{S}{S_0}\right)^\Delta e^{-\frac{b^2}{R_{i,k}^2}}$$

$$R_{i,k}^2(S) = R_{0,i,k}^2 + 4\alpha_p' \ln \frac{S}{S_0}$$

$$R_{0,j,2}^2 = \frac{1}{2} R_{0,j,1}^2$$

$$R_{0,2,2}^2 = 0$$

and we assume a linear  $\mathbb{P}$  trajectory

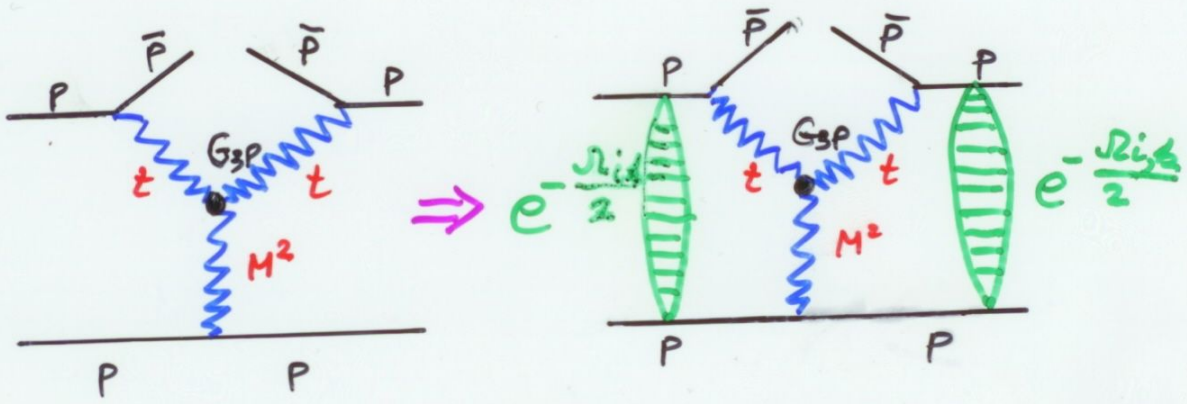
input:  $\alpha_{\mathbb{P}}(t) = 1 + \Delta + \alpha_p' t$

The survival probability of SD integrated  
 over its entire  $M^2$  range is

$$S_{SD}^2 = \frac{\int d^2b |a_{SD}|^2}{\int d^2b |a_{SD}^{in}|^2} = \frac{V_{SD}^{out}}{V_{SD}^{in}}$$

$$a_{SD}^{in} = \frac{i d\beta}{2} \left( -\alpha^2 \Omega_{1,1} + (\alpha^2 - \beta^2) \Omega_{1,2} + \beta^2 \Omega_{2,2} \right)$$

The reduction to a 2 amplitude model  
 is straight forward. Note that it breaks  
 Regge factorization on the input level !!!



For the calculation of high mass  $S_{3P}^2$  we use Mueller's triple Pomeron formalism.

$$M^2 \frac{dG_{3P}}{dM^2 dt} = G_{3P}(t) \frac{g_P^2(t) g_P(0)}{16\pi^2} \left(\frac{S}{M^2}\right)^{2\alpha_P(t)-2} \left(\frac{M^2}{S_0}\right)^\Delta$$

is transformed to  $b$ -space. Note that the radius has an additional  $r_0^2 = 0.5 \text{ GeV}^{-2}$  to account for the small, but non zero, radius of the 3P vertex.  $g_P(t)$  is exponential in  $t \Rightarrow b$ -Gaussian.

For a two channel calculation we write

$$T(S, M^2; b) = \sum_{i,k,l} \langle P|l\rangle^2 \langle P|k\rangle T_{\frac{1}{2}}^{l,i}(S, M^2; b) \cdot \langle P|k\rangle \langle P|i\rangle^2$$

where

$$\langle P|1\rangle = \alpha \quad \langle P|2\rangle = \beta$$



$$S_{3P}^2(S, M^2) = \frac{\int d^2b N(S, M^2; b)}{\int d^2b D(S, M^2; b)} \quad (8)$$

$$N = \left( \alpha^6 T_1^{1,1} e^{-\Omega_{1,1}} + 2\alpha^4 \beta^2 T_1^{1,2} e^{-\frac{\Omega_{1,1} + \Omega_{1,2}}{2}} \right. \\ \left. + \alpha^2 \beta^4 T_1^{2,2} e^{-\Omega_{1,2}} \right. \\ \left. + \alpha^4 \beta^2 T_2^{1,1} e^{-\Omega_{1,2}} + 2\alpha^2 \beta^4 T_2^{1,2} e^{-\frac{\Omega_{2,2} + \Omega_{1,2}}{2}} \right. \\ \left. + \beta^6 T_2^{2,2} e^{-\Omega_{2,2}} \right)$$

$$D = (\text{the same without } e^{-\dots})$$

Our original aim was to compare the  $S^2$  outputs of the 2 amplitude and the 3 amplitude models. The data base of Model A (2 amp.) does not contain the 5 reported DD data points. These are included in the data base of Model B (3 amp.). We found that Model B had two  $\chi^2$  comparable dips. Both are included denoted B(1) and B(2)

Model	$\Delta$	$\alpha_p^2 \text{ GeV}^{-2}$	$\beta$	$R_{0;1,1} \text{ GeV}^{-2}$	$g_1 \text{ GeV}^{-1}$	$g_2 \text{ GeV}^{-1}$	$\frac{\chi^2}{\text{d.o.f.}}$
A	0.126	0.25	0.464	16.34	3.6	12.1	1.50
B(1)	0.15	0.17	0.767	21.88	2.2	63.3	1.48
B(2)	0.15	0.20	0.783	20.8	2.0	35.2	1.57

Remarkably, all 3 models have very similar outputs with comparable  $\frac{\chi^2}{d.o.f.}$ . Even though the  $\chi^2$  seems a bit high its the best one can get due to the spread of the SD data and the different  $\sigma_{tot}$  values at  $\sqrt{s} = 1800$  GeV.

Model A and B Tevatron results are compatible but the two models develop a difference with growing energy.

At LHC :

	$\sigma_{tot}$	$\sigma_{ee}$	$\sigma_{sd}$	$\sigma_{dd}$	$S_{sd}^2$
2ch	103	24.5	12	—	0.04
3ch	111	26.4	12	3	0.0002

B(1)

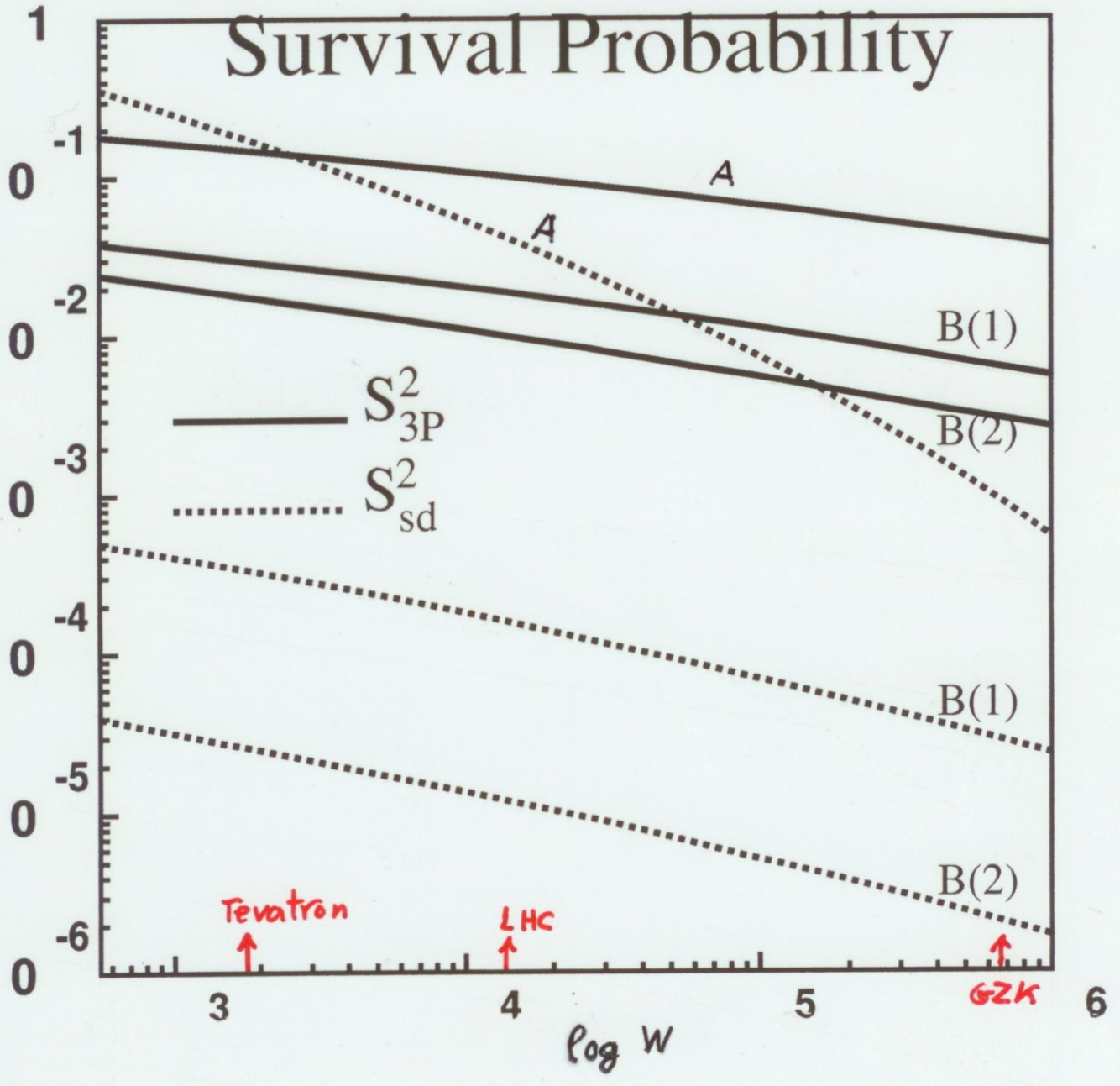
The data base does not constrain well the parameters of Model B, hence, the two solutions. To re-check we ran our Model B fit on the data base of Model A (no DD data) and obtained almost identical results.

Note though that the 5 DD fitted points are about 50% of the reported experimental values. As it stands, this is dictated by the SD relatively high number of data points.

$S_{3P}$  is weakly dependent on  $M^2$ . We present its  $M^2$  averaged values.

With the exception of low energies  $S_{3P}^2 \rightarrow S_{sd}^2$ .

Both  $S_{3P}^2$  and  $S_{sd}^2$  in Model B are 1-3 orders of magnitude smaller than the corresponding Model A  $S^2$  values.



Be reminded that all 3 models reproduce the data well with comparable  $\chi^2$ .

The interpretation of this observation is that Model B has very large input unscreened cross sections which are strongly reduced by the very small  $S^2$  values to reproduce the data well.

$$\frac{\sigma_{sd}^{in} (2 \text{ amp.})}{\sigma_{sd}^{in} (3 \text{ amp.})} \approx \frac{S_{sd}^2 (3 \text{ amp.})}{S_{sd}^2 (2 \text{ amp.})}$$

Are these results relevant to  $S^2$  calculations relating to Higgs and hard dijets ???

It is commonly believed that different models which reproduce the soft scattering data base well provide compatible  $S_{soft}^2$  values.

$W_{TeV}$	$S_{sd}^2 (GJJ)$		$S_{dd}^2 (JGJ)$		$S_{cd}^2 (GJJ(H)G)$ exclusive	
	GLM	KMR	GLM	KMR	GLM	KMR
1.8	0.12	0.10	0.17	0.15	0.044	0.045
14.0	0.08	0.06	0.11	0.10	0.027	0.026

Same compatible  $S^2$  values have been obtained by FS and others.

Even though GLM and KMR are eikonal two channel models they are dynamically different:

GLM two channels refers to the diversity of the intermediate rescattering options, i.e. elastic and diffractive.

KMR two channels relate to two different dynamical contributions to the input non screened diffractive cross sections.

There is, though, a significant difference between  $S_{diff}^2$  and  $S_H^2$  (or  $S_{2\sigma}^2$ ).

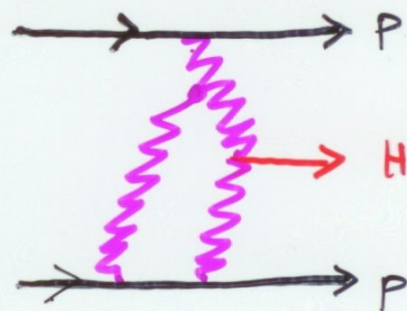
Soft diffraction data is part of the fitted data base. Consequently, the compensation between  $S_{diff}^{in}$  and  $S_{diff}^2$  is in built. Dijets or Higgs are produced in a hard QCD process where the input is decoupled from the soft sector which determines  $S^2$ . The problem is highly non linear and I hesitate to guess. Our calculations will be finished soon!

### III) Eikonalization in the hard sector:

We wish to examine if eikonalization is a viable procedure with which we can assess the role of unitarity in a strictly hard scattering.

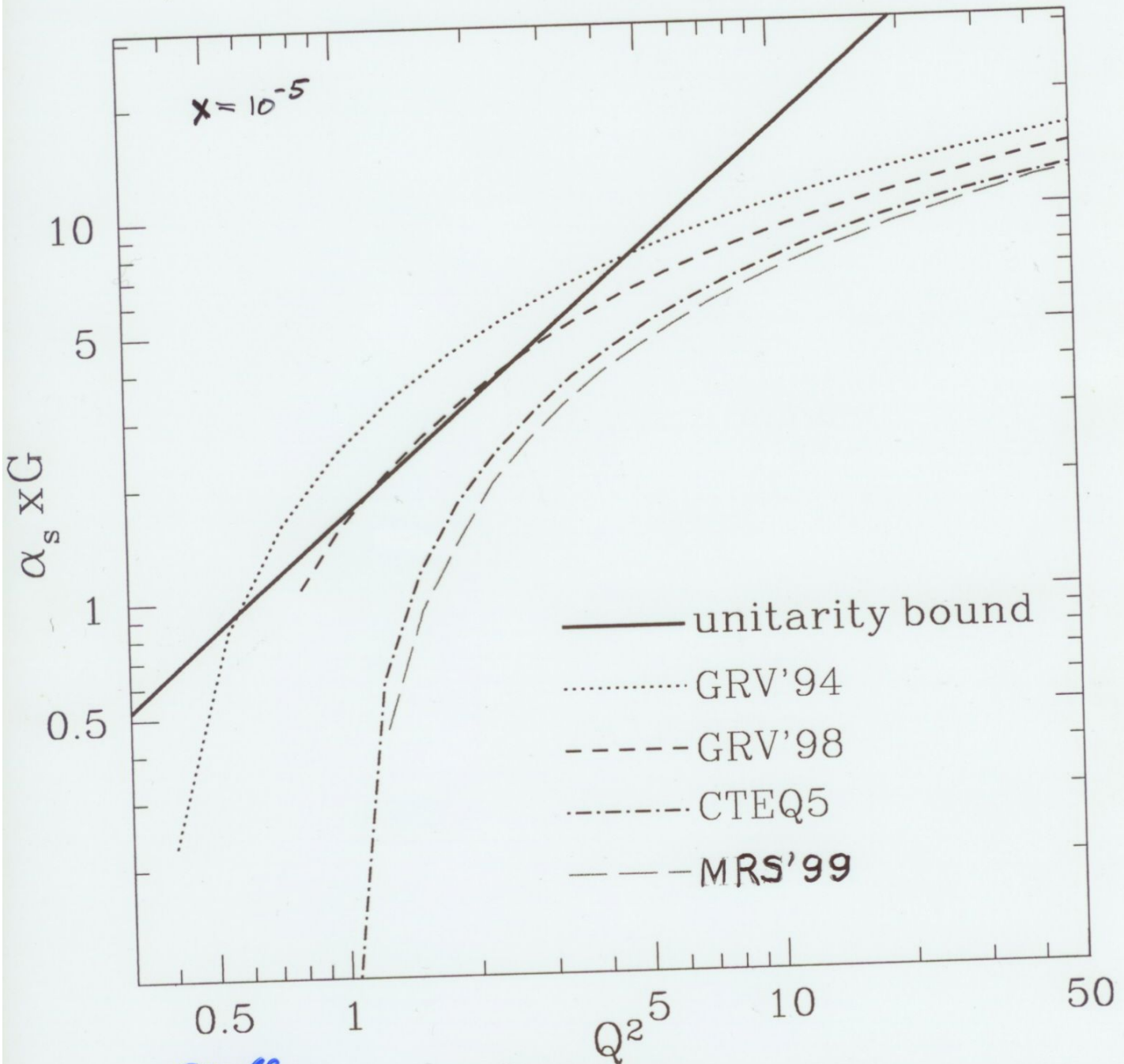
Note that the suppression due to gluon radiation from the partons participating in the hard subprocess is already included in the hard calculation through the Sudakov factor.

KMR (hep-ph/0602247) recently showed that the rescattering depicted in the diagram results in a very small correction.



One may ask, never the less, what is the consequence of  $xG(x, Q^2)$  getting too close to the black unitarity bound. If so, the SC due to the percolation of a  $q\bar{q}$  dipole can be calculated in the eikonal model. This is simply demonstrated in a dipole LLA DGLAP.

Ayala, Gay-Ducati, Levin:  $\frac{\partial^2 x G(x, Q^2)}{\partial y \partial \ln Q^2} < \frac{2}{\pi} R_H^2 Q^2$

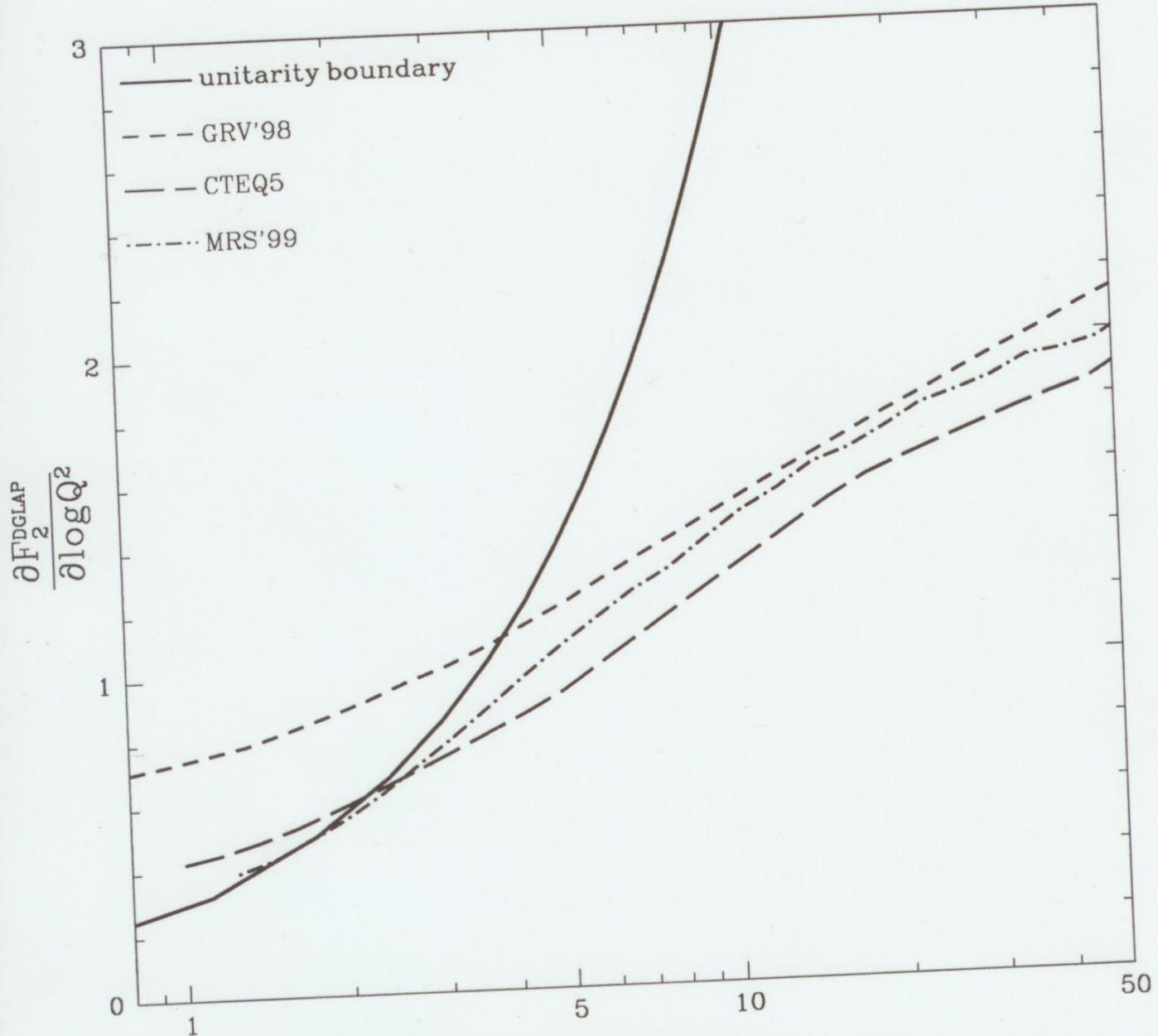


Small x  
LLA DGLAP:  $\frac{\partial^2 x G(x, Q^2)}{\partial y \partial \ln Q^2} = \frac{N_c}{\pi} \alpha_s(Q^2) x G(x, Q^2)$

$x G(x, Q^2) < \frac{2}{\pi N_c \alpha_s(Q^2)} R_H^2 Q^2$   
 $R_H^2 \approx 4 \text{ GeV}^{-2}$  obtained from HERA  $\gamma p \rightarrow \gamma/4 P$

In a correlated small x bound:

$x = 10^{-5}$



$$\frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} = \frac{\partial x G(x, Q^2)}{\partial \ln Q^2} = \frac{Q^2}{3\pi^2} \times G(x, Q^2)$$

$$\frac{\partial x G(x, Q^2)}{\partial \ln Q^2} = \frac{Q^2}{3\pi^2} \int d^2 b_{\perp} \text{Im} a_{ee}^H(x, r_{\perp}^2)$$

$$Q^2 = \frac{4}{r_{\perp}^2}$$

Setting  $a_{ee}^H \equiv 1$  provides the bound which was tested by H1 and Zeus.



Hard eikonalization was applied by GLM to reproduce the HERA experimental data of  $\frac{\partial F_2}{\partial \ln Q^2}$  and  $\sigma p \rightarrow \gamma/\psi p$ .

Despite its phenomenological success the GLM program was unable to suggest definite unitarity signatures derived from the small  $(x, Q^2)$  data.

The trigger for our analysis was the poor reproduction of the small  $(x, Q^2)$   $\frac{\partial F_2}{\partial \ln Q^2}$  data by the early p.d.f. editions.

The over estimation was corrected by our eikonalization, but then similar proper results were obtained by newer p.d.f. editions. Similar problems were detected at even smaller  $(x, Q^2)$  values. The new data was fitted by our model and then reproduced soon after by yet newer p.d.f. editions, etc. In 2001 we got tired and retired from this game.

Saturated (or high density) p.d.f.'s reproduce the data (explicitly  $\frac{\partial F_2}{\partial \ln Q^2}$  and  $\sigma_p \rightarrow \sigma/\psi p$ ) but the problem is that these p.d.f.'s offer an excellent prediction while their predictive power, at exceedingly small  $(x, Q^2)$ , may be limited.

Phenomenological models in the spirit of Golec-Biernat, Wüsthoff, Bartels, Kovalsxi, ... are doing well but their translation to p-p scattering is not simple. As far as I know there is no decisive pQCD unitarity signature observed in p-p scattering as yet!

#### IV) Single diffraction in $\mathbb{P}/4$ photoproduction: (18)

The high mass single diffraction  $\gamma p \rightarrow \mathbb{P}/4 M$  is a relatively clean channel to obtain the value of  $G_{3p}$ , the bare triple  $\mathbb{P}$  coupling. This observation is at the core of the KMR preprint.

The argument: this reaction is initiated by the charm component of the photon which has a small absorptive cross section since its interaction stems from short distances  $r^2 \propto \frac{1}{m_c^2}$ , hence its  $S_{3p}^2$  should be close to 1.

We aimed to quantify this assessment in a dedicated  $F_2$  model (Kormilitzin M.Sc. Thesis). The model reproduces  $F_2$  extremely well for  $Q^2$  getting as small as  $Q^2 = 0.25 \text{ GeV}^2$ .

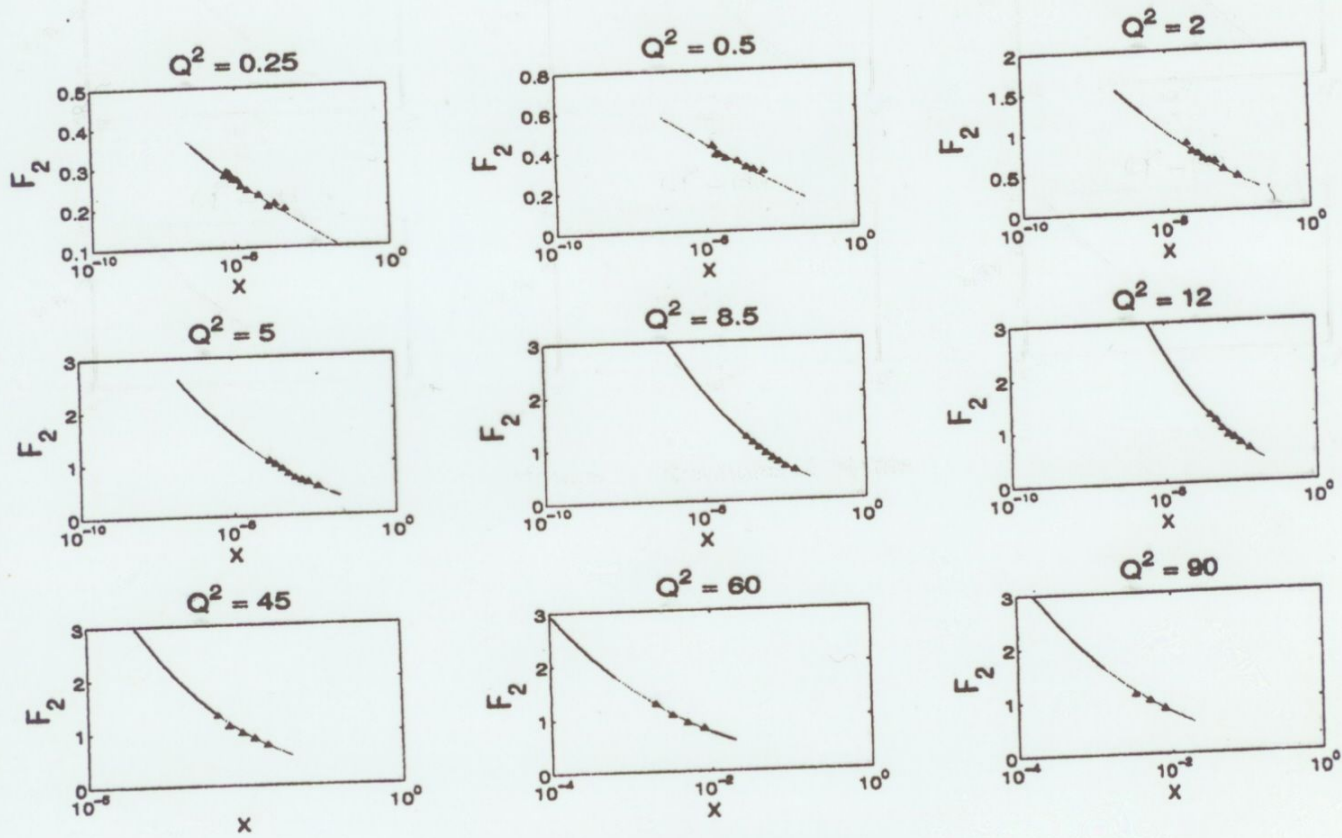
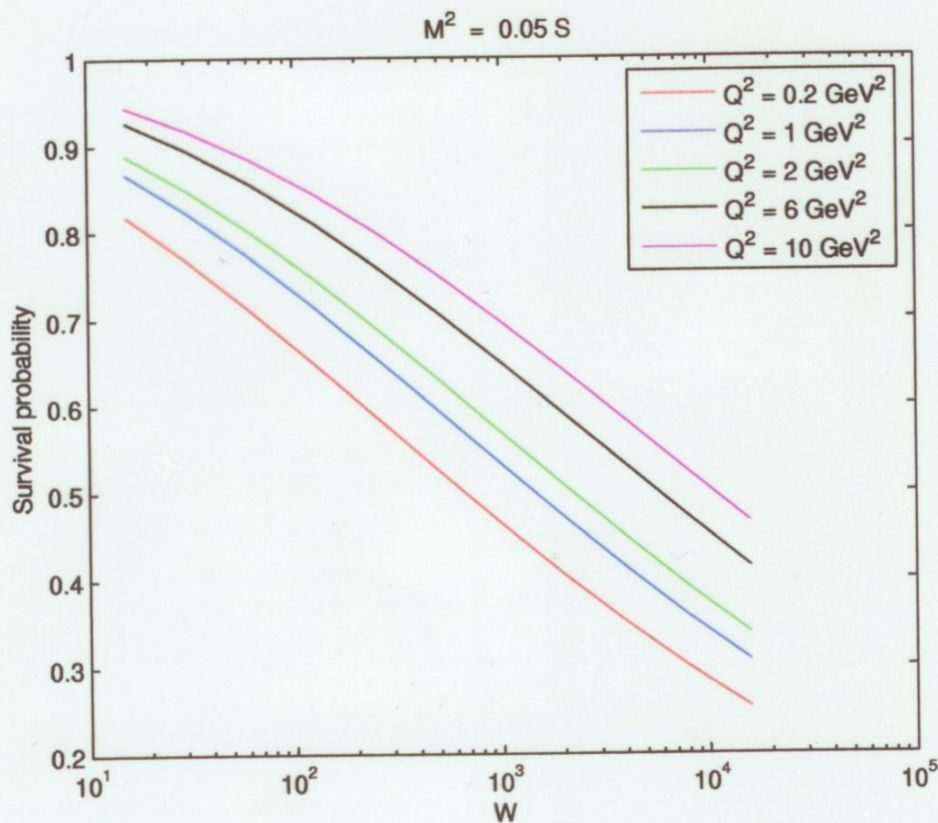


Figure 1: Examples of  $F_2$  fits.

The interesting observation is that when convoluted with  $e^{-\nu_2 s}$  (single channel  $S_{\text{soft}}^2$  calculation), we obtain survival probabilities which are quite lower than 1.



We suggest to modify the LMR proposal and compare

$$R_{\frac{3}{4}} = \frac{\sigma(\gamma^* p \rightarrow \frac{3}{4} M_H)}{\sigma(\gamma^* p \rightarrow \frac{3}{4} P)} \quad \text{and} \quad R_P = \frac{\sigma(pp \rightarrow P M_H)}{\sigma(pp \rightarrow PP)}$$

The above ratios are proportional to  $S_{3P}^2(\frac{3}{4}) G_{3P}^{(HHS)}$  and  $S_{3P}^2(pp) G_{3P}^{(SSS)}$ . The two couplings are presumed to be equal, but this should be experimentally checked!