Matching Constrained Monte Carlo at NLO

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Status of NLO matching of CMC algorithm for LHC

This work is supported by the EU grant mTkd-CT-2004-510126 In partnership with the CERN Physics Department and by the Polish Ministry of Scientific Research and Information Technology grant No 620/E-77/6.PRUE/DIE 188/2005-2008.

Overview

- Constrained Monte Carlo (CMC)
  - Combination of Two-Hemispheres
- NLO Calculation
- Subtraction Term for NLO calculation
- Subtraction Term vs. Parton Shower
- Matching Prescription

Constrained Monte Carlo

- Solve evolution equation

$$Q^2 \frac{dD(x,Q^2)}{dQ^2} = \int_x dz P(z) D(x/z, Q^2)$$

- Can have different arguments of running coupling and treatment of soft region
  - depart from DGLAP evolution
- Iterative solution can be written in terms of four-momentum of emissions

Constrained Monte Carlo

- Iterative Solution

$$
\begin{aligned}
D(x,Q^2) &= e^{-\Phi(Q^2,Q_0^2|x)} D(x,Q_0^2) \\
&\times \sum_{n=0}^{\infty} \int_0^1 dx_0 D(x_0,Q_0^2) \left[\prod_{i=1}^{n} \int_{t_{i-1}}^{t} \frac{dQ_i^2}{Q_i^2} \int_0^1 dz_i \right] \\
&\times e^{-\Phi(Q^2,Q_n^2|x_n)} \left[\prod_{i=1}^{n} \frac{K(Q_i^2, z_i|x_{i-1})}{1-z_i} e^{-\Phi(Q_i^2,Q_{i-1}^2|x_{i-1})} \right] \\
&\times \delta(x - x_0 \prod_j z_j) \\
&= \int dx_0 \, \mathcal{U}(x,Q^2;x_0,Q_0^2) D(x_0,Q_0^2)
\end{aligned}
$$

- Insert delta condition to fix final *x* value
  - This constraint is satisfied by CMC algorithm (see talk of S. Jadach)

Combining two hemispheres

- We have an algorithm to generate one shower
  - Want to construct algorithm for *HH* → *colourless object*
- To combine two we must fix the partonic *s*
  - Necessary for resonances, etc.
- Impose an additional delta function

$$\delta(s - (px_0 + \bar{p}\bar{x}_0 - \sum_F k_i - \sum_B \bar{k}_i)^2)$$

- Now s is given by the full momentum reconstructed *s*, rather than just $x_1 x_2 S$

Combining two hemispheres

- Result is

$$d\sigma = \int dx d\bar{x}\, D(x, Q^2) D(\bar{x}, Q^2) d\sigma(s) \delta_s$$

  - A factorization formula
  - If we "downgrade" our shower to produce partons with no k_T we retrieve standard collinear factorization formula
- The extra delta can be satisfied by rescaling momentum by common factor, Y

Combining two hemispheres

- CMC evolution variable is related to rapidity
- The maximum evolution time is defined for each hemisphere by some division line, η^*

$$\eta^* = \frac{1}{2} \ln \frac{x}{\bar{x}}$$

- Final result for cross section is

$$\begin{aligned} d\sigma &= \int dx d\bar{x} \int dx_0 d\bar{x}_0 \mathcal{U}(q^*, x; q_0, x_0) \mathcal{U}(\bar{q}^*, \bar{x}; \bar{q}_0, \bar{x}_0) \\ &\quad \times D(x_0, q_0) D(\bar{x}_0, \bar{q}_0) d\sigma(s) \delta_s \end{aligned}$$

Issues for matching

- In order to use the evolution operators, $\mathcal{U}$, and match the hard process at NLO we must address
  - IR regularization of ME
  - Double counting
  - Finite effects in collinear region
  - Negative weights
  - Connection to factorization theorems and schemes

NLO Matrix Element

- NLO calculation (from Feynman diagrams) is generally of the form (in 4-2ε dimensions)

$$d_{n+1}\sigma_V = \left[\left(\frac{A_V}{\varepsilon^2} + \frac{B_V}{\varepsilon}\right) d_n\sigma_B + d_n\sigma_{V,reg}\right] \delta(p^+)\delta(p^-)dp^+dp^-$$

$$d_{n+1}\sigma_R = d_{n+1}\sigma_F + \frac{\delta(p^+)\delta(p^-)}{\varepsilon^2} d_n\sigma_S dp^+dp^- + \frac{dp^+dp^-}{\varepsilon}\left(d_n\sigma_{c+}(p^+)\delta(p^-) + d_n\sigma_{c-}(p^-)\delta(p^+)\right)$$

  - Angular variable is implicit
  - p^+ and p^- are light-cone components of gluon emission

NLO Calculation

- Soft singularities from real cancel with virtual singularities
- Collinear singularities are treated by factorization scheme
- Using the factorization scheme and sense of distributions the NLO result is

$$d\sigma_{NLO} = d\hat{\sigma}_{SV} + d\hat{\sigma}_{c+} + d\hat{\sigma}_{c-} + d\sigma_F$$

Counterterm for NLO

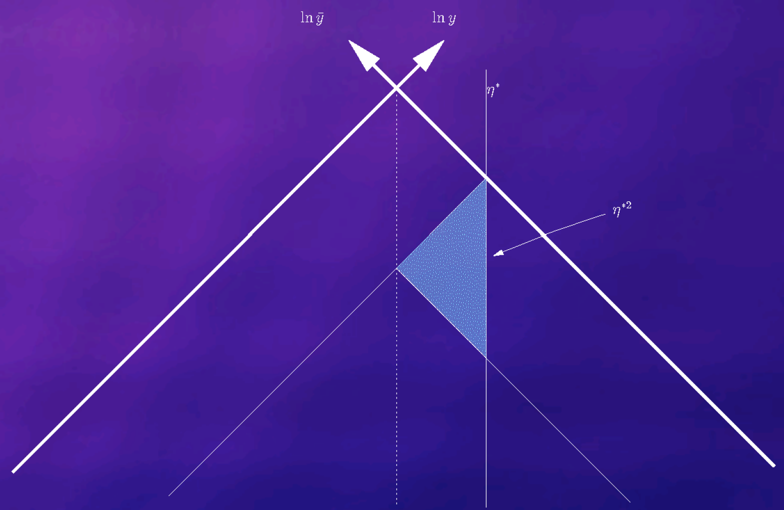
- To avoid double counting we use kernel of MC as a counterterm
- Virtual counterterm is minus the integral of the real one
- Subtraction terms phase space is divided in the same manner as the MC , η^*

$$d\sigma_{V,ct} = \int \frac{dp^+}{(p^+)^{1+\varepsilon}} \frac{dp^-}{(p^-)^{1+\varepsilon}} \left[K(p^+,\varepsilon)\theta_F(\eta^*) + K(p^-,\varepsilon)\theta_B(\eta^*) \right] d_n\sigma_B$$

- The virtual counterterm is then

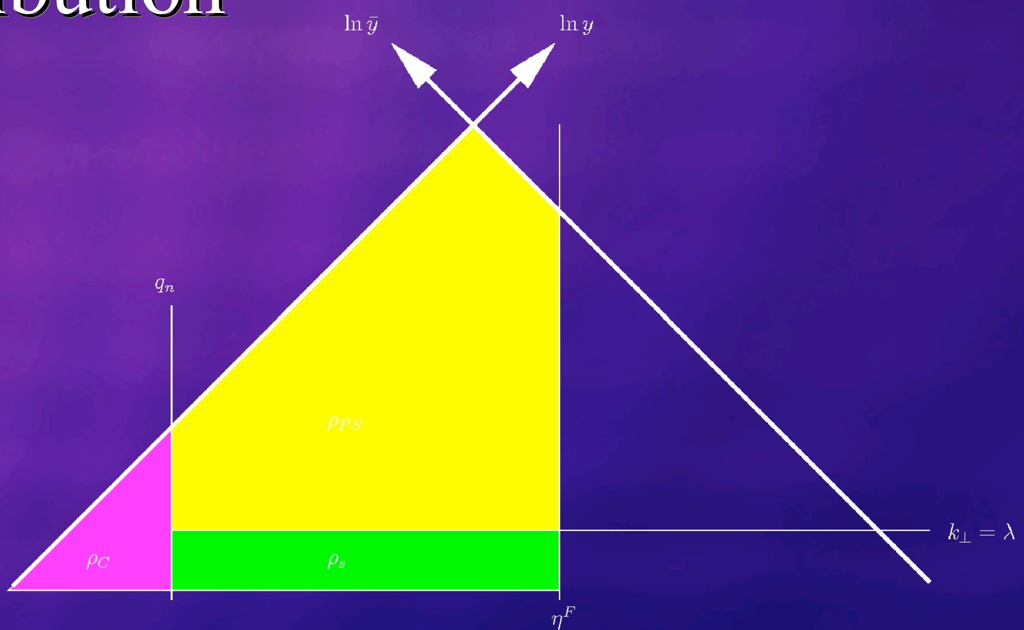
$$d\sigma_{V,ct} = 4\pi\alpha_S\mu_R^{2\varepsilon}s^{-\varepsilon}C_F N(\varepsilon)H(\varepsilon)d\sigma_B\left(\frac{4}{\varepsilon^2}+\frac{6}{\varepsilon}+4\eta^{*2}+8\right)$$

- The η^{*2} piece is residual effect of hemisphere matching. Disappears from all physical observables



Subtraction Term vs. Parton Shower

- Due to ordering and IR regulator the subtraction term differs from PS contribution



Subtraction Term vs. Parton Shower

- We have the regularized quantity

$$d\hat{\sigma}_{R/V} = d\sigma_{R/V} - d\sigma_{R/V,ct}$$

- Where the counterterm can be written as

$$d\sigma_{R/V,ct} = \rho_{PS}^{R/V} + \rho_{C}^{R/V} + \rho_{S}^{R/V}$$

- And the quantity

$$d\sigma_R - \rho_{PS}^{R} = d\hat{\sigma}_R + \rho_C^R + \rho_S^R$$

will be of interest later on…

Subtraction Term vs. Parton Shower

- The soft region can be ignored for IR safe observables
  - These are "unresolvable" emissions from PS
- The collinear region has the same structure as in collinear pdf
  - Can isolate poles order-by-order and use these to renormalize the bare proton distribution
- These collinear poles have the same form as in standard pdf, can use the same prescription as MS

Subtraction Term vs. Parton Shower

- After using a factorization scheme there is some finite scheme-dependent piece of ρ_C remaining

$$\hat{\rho}^R_{C,\overline{MS}}(z) = \rho^R_C - \frac{1}{2\pi\bar{\varepsilon}} P(z) d\sigma_B dz$$

  - This defines a collinear NLO correction
  - For those familiar with MC@NLO these are the same as the 2 tilde contributions
    - Treated by longitudinal boost
- Optimal treatment for CMC is still under investigation

Matching prescription

- We return to our factorization formula

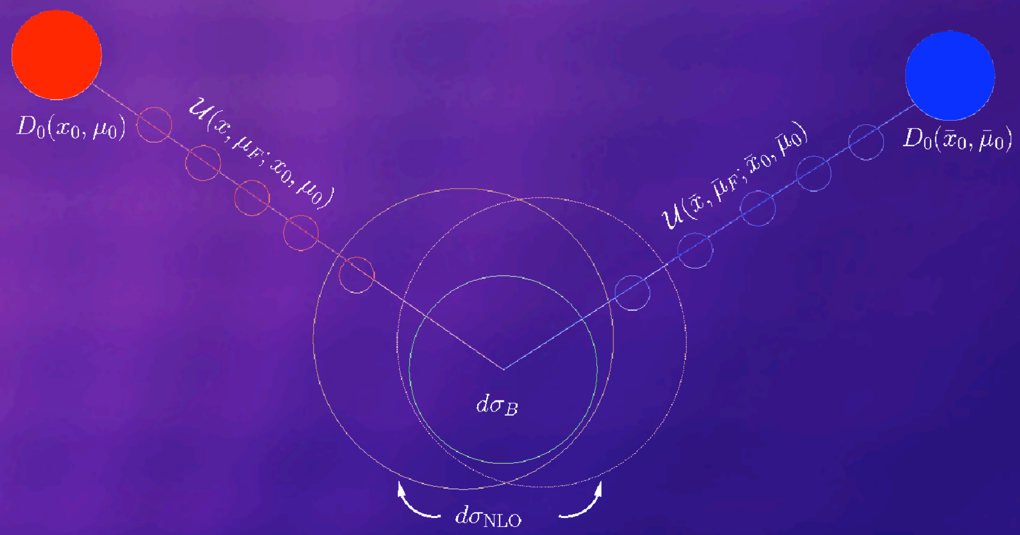
$$d\sigma = \int dx d\bar{x} \int dx_0 d\bar{x}_0 \mathcal{U}(q^*, x; q_0, x_0)\mathcal{U}(\bar{q}^*, \bar{x}; \bar{q}_0, \bar{x}_0) \times D(x_0, q_0)D(\bar{x}_0, \bar{q}_0)d\sigma(s)\delta_s$$

- We then separate each evolution operator at some rapidity, ξ $(x' = zx \;\; ; \;\; \bar{x}' = \bar{z}\bar{x})$

$$\int dx d\bar{x} dx_0 d\bar{x}_0 dx' d\bar{x}'$$
$$\times \Big[\mathcal{U}(q^*, x; \xi, x')\mathcal{U}(\xi, x'; q_0, x_0)\mathcal{U}(\bar{q}^*, \bar{x}; \bar{q}_0, \bar{x}_0)\delta(\bar{x}')\theta_F(\eta^*, \xi)$$
$$+\mathcal{U}(q^*, x; q_0, x_0)\mathcal{U}(\bar{q}^*, \bar{x}; \xi, \bar{x}')\mathcal{U}(\xi, \bar{x}'; \bar{q}_0, \bar{x}_0)\delta(x')\theta_B(\eta^*, \xi)\Big]$$
$$\times D(x_0, q_0)D(\bar{x}_0, \bar{q}_0)d\sigma(s)\delta_s$$

Matching prescription

- Logical choice of ξ last emission from either leg



- Choose emission with highest p_T

Matching prescription

- We define an additional weight factor, β such that the NLO cross section is

$$
\begin{aligned}
d\sigma^{NLO} &= \int dx\,d\bar{x}\,dx_0\,d\bar{x}_0\,dx'\,d\bar{x}' \\
&\times \Big[\mathcal{U}(q^*, x; \xi, x')\mathcal{U}(\xi, x'; q_0, x_0)\mathcal{U}(\bar{q}^*, \bar{x}; \bar{q}_0, \bar{x}_0)\delta(\bar{x}')\theta_F(\eta^*, \xi) \\
&+\mathcal{U}(q^*, x; q_0, x_0)\mathcal{U}(\bar{q}^*, \bar{x}; \xi, \bar{x}')\mathcal{U}(\xi, \bar{x}'; \bar{q}_0, \bar{x}_0)\delta(x')\theta_B(\eta^*, \xi)\Big] \\
&\times D(x_0, q_0)D(\bar{x}_0, \bar{q}_0)d\sigma(s)\delta_s\beta(\xi, x', \bar{x}')
\end{aligned}
$$

Matching prescription

- Expanding to $O(\alpha_S)$

$$d\sigma_{NLO} = \int dx d\bar{x} dx' d\bar{x}' \Big[D(x',\xi) D(\bar{x},\bar{q}^*) \left(1 + \rho^V_{PS} + \rho^R_{PS}\right) \theta_F(\eta^*,\xi) \delta(\bar{x}') + D(x,q^*) D(\bar{x}',\xi) \left(1 + \bar{\rho}^V_{PS} + \bar{\rho}^R_{PS}\right) \theta_B(\eta^*,\xi) \delta(x') \Big] \left[\left(\beta_0^{(0)} + \beta_0^{(1)}\right) \delta(x - x') \delta(\bar{x} - \bar{x}') + \beta_1^{(1)}(\xi, x', \bar{x}') \right] d\sigma_B(s)$$

- Comparing to standard factorization formula

$$d\sigma_{NLO} = \int dx d\bar{x} f(x,\mu_F^2) f(\bar{x},\bar{\mu}_F^2) d\hat{\sigma}_{NLO}(x\bar{x}S, \mu_F^2, \bar{\mu}_F^2)$$

Matching prescription

- Thus we find

$$\begin{aligned}
\beta_0^{(0)} &= 1 \\
\beta_0^{(1)} &= \frac{d\sigma_V - \rho_{PS}^V - \bar{\rho}_{PS}^V - PDF}{d\sigma_B} \\
&= \frac{d\hat{\sigma}_V}{d\sigma_B} + \frac{\hat{\rho}_C^V + \hat{\bar{\rho}}_C^V}{d\sigma_B} \\
&= \beta_0^1 + \beta_0^{c+} + \beta_0^{c-} \\
\beta_1^{(1)} &= \frac{d\sigma_R - \rho_{PS}^R - \bar{\rho}_{PS}^R - PDF}{d\sigma_B} \\
&= \frac{d\hat{\sigma}_R}{d\sigma_B} + \frac{\hat{\rho}_C^R(z)}{d\sigma_B} + \frac{\hat{\bar{\rho}}_C^R(\bar{z})}{d\sigma_B} \\
&= \beta_1(\xi, x', \bar{x}') + \beta_1^{c+}(z) + \beta_1^{c-}(\bar{z})
\end{aligned}$$

Conclusion

- We have a Monte Carlo which generates parton shower in both forward and backward hemisphere
  - Constructed in a non-Markovian way
  - Full coverage of phase space
- Prescription for regularizing NLO calculation
- Method for matching parton shower to NLO calculation
- Some connection factorization theorem
  - Scheme dependent contributions

Conclusion

- Currently implementing $W^+ W^-$ at NLO
- Remaining issues
  - Must implement quark-gluon transitions in parton shower for two hemispheres
  - Must fit initial parton distributions to DIS data
  - Would like to use matched NLO DIS for fits
- Once working for one process should be quick to implement additional processes