General Constrained Monte Carlo algorithm for single hadron

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- **People involved:**

  K. Golec-Biernat*, A. van Hameren, S. Jadach*, M. Jeżabek, G. Nanava, W. Płaczek*, E. Richter-Wąs, M. Skrzypek*, P. Sawicki, P. Stephens, Z. Wąs*

- **Papers* on QCD MC evolution and new parton shower MCs:**

  hep-ph/0312355, NP Proc. 135:138(04), hep-ph/0504205, hep-ph/0509178, hep-ph/0504263, hep-ph/0603031, NP Proc. 157:241(06), hep-ph/0701174 and more in preparation.

- **The aim is dedicated MC (parton shower) for W/Z production at LHC, with NLO QCD and 1-st order EW/QED corrections from SANC group (Dubna)**

- **Other LHC related MC projects at IFJ-PAN:**

  TAUOLA, PHOTOS, WINHAC.

QCD evolution in Constrained Monte Carlo, CMC

- Evolution equation describes response of the PDF(x, q) to a change of scale parameter $Q = q = \mu_F$ controlling size of the real/virtual emission phase space.
- MC can solve precisely $\sim 0.1\%$ wide range of the QCD evolution equations, from DGLAP LL, NLL, to CCFM, BFKL and more.
- Markovian MC is (MMC) the natural way to do it.
- Constrained MC (CMC) does the same but allows to predefine x and parton type $f = G, q_i, \bar{q}_i$ resulting from the evolution.
- CMC is an alternative for "backward evolution". Technically difficult.
- CMC can be used as a building block for a new parton shower MC, with better phase space coverage.
- 1-st examples of CMC for full LL DGLAP by S.J. and M. Skrzypek, hep-ph/0504263.
- This work done in collaboration with M. Skrzypek and W. Placzek. Help of Z. Was and P. Stephens warmly acknowledged.

Evolution equation

Generic evolution equation covering several types of evolution reads

$$\partial_t D_f(t,x) = \sum_{f'} \int_x^1 du\, \mathcal{K}_{ff'}(t,x,u) D_{f'}(t,u).$$

Standard LL DGLAP case:

$$\mathcal{K}_{ff'}(t,x,u) = \frac{1}{x}\mathcal{P}_{ff'}\left(t,\frac{x}{u}\right) = \frac{\alpha(t)}{\pi}\frac{1}{x}P_{ff'}\left(t,\frac{x}{u}\right),$$

where $P_{ff'}(z)$ is the standard LL kernel.

The same in compact matrix notation:

$$\partial_t \mathbf{D}(t) = \mathbf{K}(t)\,\mathbf{D}(t).$$

Solution of the evolution equation

Given initial $\mathbf{D}(t_0)$, at any later time $t \geq t_0$ solution is:

$$\mathbf{D}(t) = \exp\left(\int_{t_0}^{t} \mathbf{K}(t')dt'\right)_T \mathbf{D}(t_0) = \mathbf{G}_{\mathbf{K}}(t, t_0)\mathbf{D}(t_0).$$

The usual time-ordered exponential evolution operator used:

$$\mathbf{G}_{\mathbf{K}}(t, t_0) = \mathbf{G}(\mathbf{K}; t, t_0) = \exp\left(\int_{t_0}^{t} \mathbf{K}(t')dt'\right)_T = \mathbf{I} + \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int_{t_0}^{t} dt_i \theta_{t_i > t_{i-1}} \mathbf{K}(t_i),$$

Multiplication rule:

$$\left(\mathbf{K}(t_2)\mathbf{K}(t_1)\right)_{f_2, f_1}(x_2, x_1) = \sum_{f'} \int_{x_2}^{x_1} dx' \; \mathcal{K}_{f_2 f'}(t_2, x_2, x')\mathcal{K}_{f' f_1}(t_1, x', x_1).$$

$$\mathbf{K} = \mathbf{K}^A + \mathbf{K}^B$$

Instrumental identity for $\mathbf{K}(t) = \mathbf{K}^A(t) + \mathbf{K}^B(t)$:

$$\mathbf{D}(t) = \mathbf{G}_{\mathbf{K}^B}(t, t_0)\, \mathbf{D}(t_0)$$
$$+ \sum_{n=1}^{\infty} \left[\prod_{i=1}^{n} \int_{t_0}^{t} dt_i\ \theta_{t_i > t_{i-1}} \mathbf{G}_{\mathbf{K}^B}(t_{i+1}, t_i) \mathbf{K}^A(t_i) \right] \mathbf{G}_{\mathbf{K}^B}(t_1, t_0)\, \mathbf{D}(t_0)$$

where $t_{n+1} \equiv t$ and $\mathbf{G}_{\mathbf{K}^B}$ is the evolution operator with the kernel $\mathbf{K}^B$.

Formal proof and applications:
hep-ph/0701174 by S.J., M. Skrzypek, Z. Was.

Application (1): $\mathbf{K} = \mathbf{K}^A + \mathbf{K}^B$

Isolating bremsstrahlung and flavour-changing parts of the evolution.

Kernel splits:

$$\mathcal{K}(t)_{ff'} = \mathcal{K}^A(t)_{ff'} + \mathcal{K}^B(t)_{ff'} = (1-\delta_{ff'})\mathcal{K}(t)_{ff'} + \delta_{ff'}\mathcal{K}(t)_{ff}.$$

Solution in the standard integro-tensorial notation for CMC:

$$D_f(t,x) = \int_x^1 dx_0\; G_{ff}(K^B;t,t_0;x,x_0)\; D_f(t_0,x_0) + \sum_{n=1}^{\infty} \sum_{f_{n-1},\ldots,f_1,f_0}$$
$$\times \int_x^1 dx_0 \left[\prod_{i=1}^{n} \int_{t_{i-1}}^{t} dt_i \int_0^1 dx_i \int_0^1 dx_i'\; \theta_{x_i<x_i'<x_{i-1}}\right] G_{ff}(K^B;t,t_n,x,x_n)$$
$$\times \left[\prod_{i=1}^{n} \mathcal{K}^A_{f_i f_{i-1}}(t_i,x_i,x_i')\; G_{f_{i-1}f_{i-1}}(K^B;t_i,t_{i-1},x_i',x_{i-1})\right] D_{f_0}(t_0,x_0),$$

where $f_n \equiv f$ and $G_{ff}(K^B...)$ represents pure bremsstrahlung evolution operator $\mathbf{G}_{\mathbf{K}^B}$. Only flavour-changing kernels $\mathcal{K}^A$ seen.

Application (2): $\mathbf{K^B} = \mathbf{K}^V + \mathbf{K}^R$

Resumming virtual IR-divergent part $\mathcal{K}^V$ of kernel

$$\mathcal{K}^B_{ff}(t,x,u) = \mathcal{K}^V_{ff}(t,x,u) + \mathcal{K}^R_{ff}(t,x,u),$$
$$\mathcal{K}^V_{ff}(t,x,u) = -\delta_{x=v}\mathcal{K}^v_{ff}(t,x),$$
$$\mathcal{K}^R_{ff}(t,x,u) = \theta_{x<u-\Delta(x,u)}\mathcal{K}_{ff}(t,x,u),$$

where $\Delta(x,u)$ is finite IR cut-off, not necessarily infinitesimal.

$$\mathbf{G}_{\mathbf{K}^V+\mathbf{K}^R}(t,t_0) = \mathbf{G}_{\mathbf{K}^V}(t,t_0)\,\mathbf{D}(t_0) +$$
$$+\sum_{n=1}^{\infty}\left[\prod_{i=1}^{n}\int_{t_0}^{t} dt_i\;\theta_{t_i>t_{i-1}}\mathbf{G}_{\mathbf{K}^V}(t_{i+1},t_i)\mathbf{K}^R(t_i)\right]\mathbf{G}_V(t_1,t_0).$$

$\mathbf{K}^V(t_i)$ is diagonal in the flavour index, hence:

$$\{G_{\mathbf{K}^V}(t_{i+1},t_i)\}_{ff}(x,u) = \delta_{x=u}\;e^{-\Phi_f(t_{i+1},t_i|x)},$$
$$\Phi_f(t_{i+1},t_i|x) = \int_{t_{i-1}}^{t_i} dt\;\mathcal{K}^v_{ff}(t,x),$$

Basic formula for CMC for pure bremsstrahlung

The same in the integro-tensorial notation:

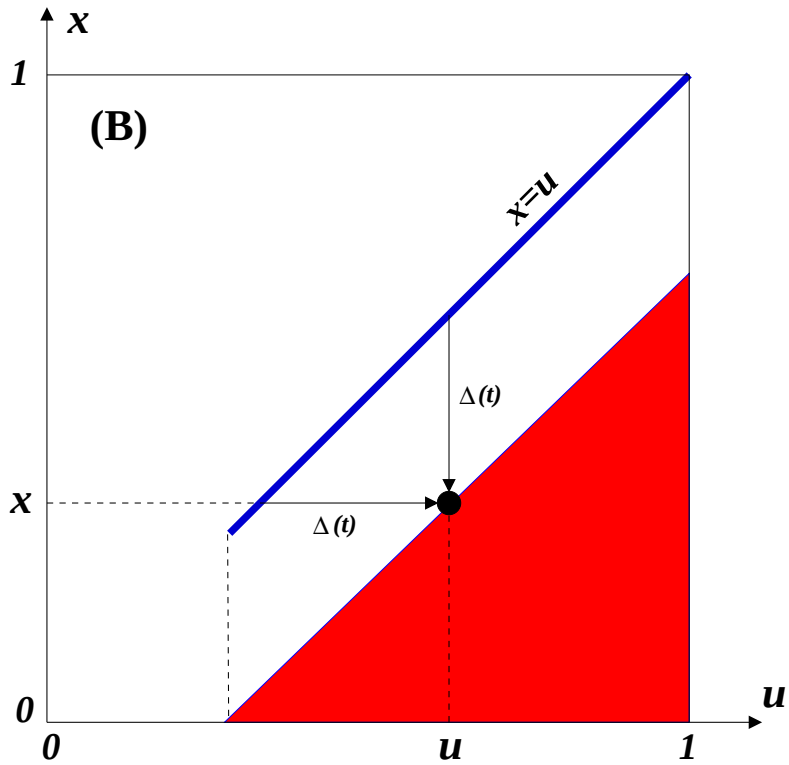
$$G_{ff}(K^B; t_b, t_a, x, u) \equiv \{\mathbf{G}_{\mathbf{K}^B}(t_b, t_a)\}_{ff}(x, u) =$$

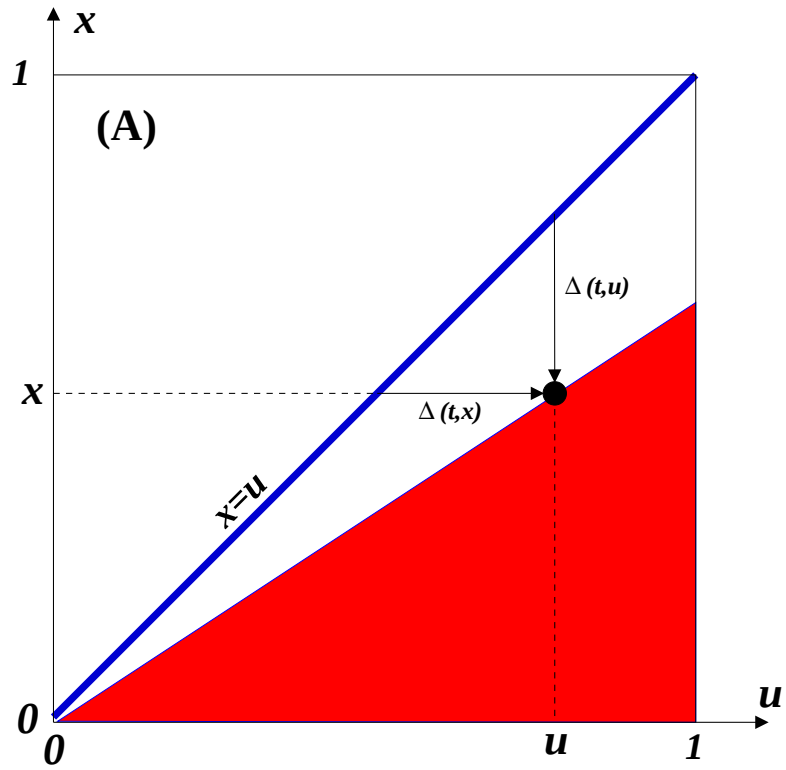
$$= e^{-\Phi_f(t_b, t_a|x)}\delta_{x=u} + \sum_{n=1}^{\infty}\left[\prod_{i=1}^{n}\int_{t_a}^{t_b} dt_i\ \theta_{t_i > t_{i-1}}\int_{x}^{u} dx_i\right]$$

$$\times\ e^{-\Phi_f(t_b, t_n|x)}\left[\prod_{i=1}^{n}\mathcal{K}^R_{ff}(t_i, x_i, x_{i-1})e^{-\Phi_f(t_i, t_{i-1}|x_{i-1})}\right]\delta_{x=x_n},$$

where $t_{n+1} \equiv t_b$, $t_0 \equiv t_a$ and $x_0 \equiv u$.

Two types of infrared (IR) cut-off





Virtual part of the kernel

For all cases considered, the momentum sum rule

$$0 = \partial_t \sum_f \int dx\, x D_f(t,x)$$

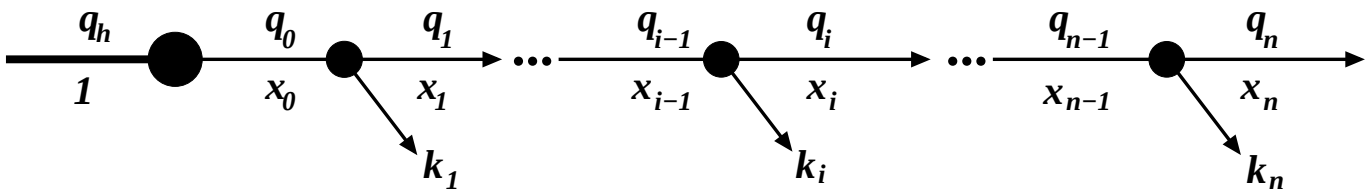
$$= \sum_f \int_0^1 du \left\{ -u\mathcal{K}^v_{ff}(t,u) + \sum_{f'} \int_0^u dx\, x\mathcal{K}^\theta_{ff'}(t,x,u) \right\} D_{f'}(t,u)$$

is imposed. The same as in DGLAP.

The sum rule determines unambiguously the virtual part of the kernel

$$\mathcal{K}^v_{ff}(t,u) = \sum_{f'} \int_0^1 \frac{dx}{u} x\, \mathcal{K}^\theta_{f'f}(t,x,u).$$

Relating evolution variables to 4-momenta



Lightcone variables $q^{\pm} = q^0 \pm q^3$.
Normalize parton momenta with respect to energy E_h of the initial hadron

$$q_h^+ = 2E_h, \quad \text{and} \quad q_i^+ = x_i 2E_h, \quad \text{where} \quad 0 \le x_i \le 1.$$

Parton initiating cascade $q_0^+ = x_0 2E_h$. For emitted parton:

$$k_i^+ = (x_{i-1} - x_i)2E_h,$$

$$k_i^T = (x_{i-1} - x_i)e^{t_i} = (x_{i-1} - x_i)2E_h e^{-\eta_i}$$

$$k_i^- = (k_i^T)^2/k_i^+,$$

Convention $k_i^T \equiv e^{t_i}(x_{i-1} - x_i)$ relates rapidity and evolution time:

$$e^{t_i} = e^{\ln(2E_h)-\eta_i} \quad \Rightarrow \quad \eta_i = \ln(2E_h) - t_i.$$

Minimum rapitity $\eta_{\min} = 0$ implies $t_{\max} = \ln(2E_h)$
and minimum $k_{\min}^T = \lambda$ implites $t_{\min} = \ln\lambda - \ln x_0$.

Three types of kernels

Bremsstrahlung kernels of (A), (B) and (C) type:

$$\mathcal{K}_{ff}^{(A)\theta}(t,x,u) = \frac{\alpha_S(e^t)}{\pi}\frac{1}{u}P_{ff}(x/u)\ \theta_{u-x\geq u\epsilon},$$

$$\mathcal{K}_{ff}^{(B)\theta}(t,x,u) = \frac{\alpha_S((1-x/u)e^t)}{\pi}\frac{1}{u}P_{ff}(x/u)\ \theta_{u-x\geq u\lambda e^{-t}},$$

$$\mathcal{K}_{ff}^{(C)\theta}(t,x,u) = \frac{\alpha_S((u-x)e^t)}{\pi}\frac{1}{u}P_{ff}(x/u)\theta_{u-x\geq \lambda e^{-t}}.$$

Inclusion of quark-gluon transitions, $X = A, B, C$:

$$x\mathcal{K}_{f'f}^{\theta(X)}(t,x,u) = \delta_{f'f}\ x\mathcal{K}_{f'f}^{\theta(X)}(t,x,u) + (1-\delta_{f'f})\frac{\alpha_S(e^t)}{\pi}F_{f'f}(z)\theta_{1-z>\epsilon}\ ,$$

Case (A) is LL DGLAP,

case (C) is CCFM without non-Sudakov formfactor.

Formfactor in most complicated case (C)

$$\Phi_f(t_1,t_0|u) = \mathbf{\Phi}_f(t_1,t_0|u) + \Phi_f^b(t_1,t_0|u) + \Phi_f^c(t_1,t_0|u),$$

where

$$\mathbf{\Phi}_f(t_1,t_0|u) = A_{ff}\frac{2}{\beta_0}\rho_2(t_1+\ln u, t_0+\ln u; t_\lambda),$$

is IR divergent (ρ_2 consists of logs) and

$$\Phi_f^b(t_1,t_0|u) = \int_{t_0}^{t_1} dt \int_0^1 dz\, \frac{\alpha_S((1-z)ue^t)}{\pi} F_{ff}(z)\, \theta_{(1-z)u>\lambda e^{-t}}\ ,$$

$$\Phi_f^c(t_1,t_0|u) = \int_{t_0}^{t_1} dt\, \frac{\alpha_S(e^t)}{\pi} \sum_{f'\neq f} \int_0^1 dz\, F_{f'f}(z)\, \theta_{(1-z)u>\lambda e^{-t}}\ .$$

One integral done analytically and one numerically.

Generic CMC class I for pure bremsstrahlung

$$D(v) = \Psi'(v) e^{-\int_{v_0}^{v_\mu} K(v')dv'} \left\{ \delta_{\Psi(v)=\Psi(v_0)} + \right.$$

$$\left. + \sum_{n=1}^{\infty} \frac{1}{n!} \left[\int_{v_0}^{v} \prod_{i=1}^{n} K(v_i) dv_i \right] \delta_{\Psi(v)=\sum_{j=1}^{n} \Psi(v_j)} \right.$$

- $\Psi(v)$ must be monotonously growing, $\Psi'(v) \geq 0$,
- $\delta(v - v_0)$ represent "no-emission events", v_0 anywhere outside (v_0, v_μ),
- here, v_0 associated to IR boundary
- true upper integration limit is $\leq v \leq v_\mu$ determined by the δ-function
- parameter v_μ (factorisation scale) adjusted later on,
- $1/n!$ from integration over t, see below for refi nements.
- typical $v_i = \ln(x_{i-1} - x_i), \ln(1 - x_i/x_{i-1})$,
- typical $\Psi(v) = v$ or $\Psi(v) = \exp(v)$

Preparatory step

Mapping (and its inverse) which removes $K(v)$ from the integrand

$$r_i = R(v_i) = \int_{v_0}^{v_i} K(v')dv', \qquad v_i = V(r_i) = R^{-1}(r_i).$$

Our basic formula transforms into:

$$D(v) = \Psi'(v)e^{-R(v_\mu)}\left\{\delta_{\Psi(v)=\Psi(v_0)} + \sum_{n=1}^{\infty}\frac{1}{n!}\left[\int_0^{R(v)}\prod_{i=1}^{n} dr_i\right]\delta_{\Psi(v)=\sum_{j=1}^{n}\Psi(R^{-1}(r_j))}\right\}.$$

If $f(r) = \Psi(R^{-1}(r))$ is very steeply growing function of r, then the constraint is effectively resolved by single $r_j \simeq R(v)$, the biggest one!

Class CMC algorithm exploits the above, see next slides

Transforming constraint in 3 steps

STEP ONE: Introduce new auxiliary integration variable X countered by δ-function:

$$D(v) = e^{-R(v_\mu)}\delta_{v=v_0} + \Psi'(v)e^{-R(v_\mu)}$$
$$\times \sum_{n=1}^{\infty} \frac{1}{n!} \int dX \left[\int_0^{R(v)} \prod_{i=1}^{n} dr_i \right] \delta_{\Psi(v)=\sum_{j=1}^{n} \Psi(R^{-1}(r_j))} \delta_{R(v)=X+\max_j r_j} \cdot$$

STEP TWO: Change variables $r_i = r_i' - X$:

$$D(v) = e^{-R(v_\mu)}\delta_{v=v_0} + \Psi'(v)e^{-R(v_\mu)}$$
$$\times \sum_{n=1}^{\infty} \frac{1}{n!} \int dX \left[\int_0^{R(v)} \prod_{i=1}^{n} dr_i' \, \theta_{r_i>0} \right] \delta_{\Psi(v)=\sum_{j=1}^{n} \Psi(R^{-1}(r_j'-X))} \delta_{R(v)=\max_j r_j'} \cdot$$

From now on we have to watch out for $r_i = r_i' - X > 0$ explicitly.

Transforming constraint in 3 steps

STEP THREE: Eliminate old constraint by integrating over X

$$D(v) = e^{-R(v_\mu)}\delta_{v=v_0} + \sum_{n=1}^{\infty}\frac{1}{n!}\left[\int_0^{R(v)}\prod_{i=1}^{n} dr'_i\ \theta_{r_i>0}\right]\delta_{R(v)=\max_j r'_j}\ \mathfrak{J},$$

where $X_0(r'_1, r'_2, ..., r'_n)$ found by solving numerically (iterative method) the original constraint for X.

Jacobian factor $\mathfrak{J}$ enters well behaved MC weight:

$$w^{\#} = \frac{\Psi'(v)}{R'(v)\sum_{j=1}^{n}\Psi'(v_j)(R'(v_j))^{-1}}\prod_{i=1}^{n}\theta_{r_i>0}.$$

and we obtain elegant formula for the MC

$$D(v) = e^{-R(v_\mu)}\delta_{v=v_0} + R'(v)e^{-R(v_\mu)}\sum_{n=1}^{\infty}\frac{1}{n!}\left[\int_0^{R(v)}\prod_{i=1}^{n} dr'_i\right]\delta_{\max_j r'_j=R(v)}\ w^{\#},$$

Standardized/Normalized formula for CMC

Using $\xi_i = r'_i/R(v)$ and Poisson distribution $P_n(\lambda) = \exp(-\lambda)\lambda^n/n!$:

$$D(v) = e^{-R(v_\mu)}\delta_{v=v_0} + R'(v)e^{R(v)-R(v_\mu)} \sum_{n=1}^{\infty} P_{n-1}(R(v)) \left[\prod_{i=1}^{n} \int_0^1 d\xi_i \right] \frac{\delta_{\max \xi_j = 1}}{n} \, w^{\#}(\xi),$$

Neglecting (temporarily) MC weight one gets

$$D(v)|_{w^{\#}=1} = e^{-R(v_\mu)}\delta_{v=v_0} + \theta_{v>v_0} R'(v) e^{R(v)-R(v_\mu)},$$

which is analytical and is normalized nicely to one

$$\int_{v_0}^{v_\mu} D(v)|_{w^{\#}=1} dv = e^{-R(v_\mu)} + \int_{\exp(-R(v_\mu))}^{1} d\left(e^{R(v)-R(v_\mu)}\right) = 1.$$

In the implementation of quark-gluon transitions with FOAM using variable $U = U(v) = e^{R(v)-R(v_\mu)} \in (0,1)$ helps

$$\int_0^{\exp(-R(v_\mu))} dU + \int_{\exp(-R(v_\mu))}^{1} dU = \int_0^1 dU.$$

General CMC algorithm for gluonstrahlung

- Generate v according to $D|_{w^{\#}=1}(v)$ times hard process xsect. using variable $U = e^{R(v)-R(v_\mu)} \in (0,1)$ (using FOAM)
- If $U \leq e^{-R(v_\mu)}$ then set $v = v_0$ and $n = 0$ (no-emission event).
- Otherwise U is translated into v ($v > v_0$) and $n > 0$ is generated according to Poisson $P(n-1|R(v))$.
- Generate $\xi_i \in (0,1)$ $i = 1, 2, ..., n$, except one of them $\xi_j = 1$, where $j = 1, 2, ...n$ is chosen randomly with equal chance.
- Variables ξ_i translate into r'_i; the transcendental equation defining X_0 shift is solve numerically.
- Knowing X_0, all $v_i(r_i(r'(i(\xi_i)))$ are calculated.
- MC weight $w^{\#}$ is evaluated. The check on $r_i > 0$ done earlier.

Dealing with t-ordering, generic case

All cases (A-C), bremsstrahlung, and any similar ones with t-ordering:

$$D(t,t_0|v) = \Psi'(v) e^{-\int_{t_0}^{t} dt' \int_{v_0(t')}^{v_\mu} \bar{\mathbb{P}}(v',t') dv'} \Bigg\{ \delta_{\Psi(v)=\Psi(v_0)} +$$

$$+ \sum_{n=1}^{\infty} \left[\prod_{i=1}^{n} \int_{t_{i-1}}^{t} dt_i \int_{v_0(t_i)}^{v} dv_i \, \bar{\mathbb{P}}(t_i, v_i) \right] w^{\mathbb{P}}(\mathbf{t}, \mathbf{v}) \, \delta_{\Psi(v)=\sum_{j=1}^{n} \Psi(v_j)} \Bigg\},$$

where $t_n \equiv t$. Boldface $\mathbf{t}$ denotes vector $(t_1, t_2, ..., t_n)$ and $\mathbf{v}$ similarly.

- $\bar{\mathbb{P}}$ typically represents IR part of kernel
- For $w^{\mathbb{P}} = 1$ integrand features pairwise symmetry with respect to simultaneous interchange $(t_i, v_i) \leftrightarrow (t_k, v_k)$
- $w^{\mathbb{P}} = 1$ (including formfactor) may violate pairwise symmetry

Dealing with t-ordering in the basic CMC algorithm

- Symmetrize pairwise. Sum over permutations $\mathfrak{P}$ of the pairs (v_i, t_i), with $1/n!$ factor:

$$\frac{1}{n!}\sum_{\mathfrak{P}}\left[\prod_{i=1}^{n}\int_{t_0}^{t} dt_i^{\mathfrak{P}} \int_{v_0(t_i^{\mathfrak{P}})}^{v} dv_i^{\mathfrak{P}}\ \bar{\mathbb{P}}(t_i^{\mathfrak{P}}, v_i^{\mathfrak{P}})\right]\theta_{t_n^{\mathfrak{P}}>t_{n-1}^{\mathfrak{P}}>\ldots>t_1^{\mathfrak{P}}}\ w^{\mathbb{P}}(\mathbf{t}^{\mathfrak{P}}, \mathbf{v}^{\mathfrak{P}}).$$

- Undo permutation $\mathfrak{P}$ in pairwise symmetric part of integrand:

$$\frac{1}{n!}\left[\prod_{i=1}^{n}\int_{t_0}^{t} dt_i \int_{v_0(t_i)}^{v} dv_i\ \bar{\mathbb{P}}(t_i, v_i)\right]\sum_{\mathfrak{P}}\theta_{t_n^{\mathfrak{P}}>t_{n-1}^{\mathfrak{P}}>\ldots>t_1^{\mathfrak{P}}}\ w^{\mathbb{P}}(\mathbf{t}^{\mathfrak{P}}, \mathbf{v}^{\mathfrak{P}}).$$

- Only one permutation $\mathfrak{P}_{\mathbf{t}}$ contributes for a given $\mathbf{t} = (t_1, t_2, ..., t_n)$:

$$D(t, t_0|v) = \Psi'(v)e^{-\int_{t_0}^{t} dt' \int_{v_0(t')}^{v_\mu} \bar{\mathbb{P}}(v',t')dv'}\Bigg\{\delta_{\Psi(v)=\Psi(v_0)}+$$

$$+\sum_{n=1}^{\infty}\frac{1}{n!}\left[\prod_{i=1}^{n}\int_{t_{i-1}}^{t} dt_i \int_{v_0(t_i)}^{v} dv_i\ \bar{\mathbb{P}}(t_i, v_i)\right]\delta_{\Psi(v)=\sum_{j=1}^{n}\Psi(v_j)}\ w^{\mathbb{P}}(\mathbf{t}^{\mathfrak{P}_{\mathbf{t}}}, \mathbf{v}^{\mathfrak{P}_{\mathbf{t}}})\Bigg\}.$$

- For $w^{\mathbb{P}}$ temporarily neglected, t_i generated unordered. $\mathfrak{P}_{\mathbf{t}}$ is read from the ordering in $\mathbf{t}$. This $\mathfrak{P}_{\mathbf{t}}$ then used to construct x_i out of v_i and to calculate $w^{\mathbb{P}}$.

Final formula for generalized CMC (bremsstrahlung)

Interchange of the integration order

$$\int_{t_{i-1}}^{t} dt_i \int_{v_0(t_i)}^{v} dv_i \; \bar{\mathbb{P}}(t_i, v_i) = \int_{v_0}^{v} dv_i \int_{t_{i-1}(v_i)}^{t(v_i)} dt_i \; \bar{\mathbb{P}}(t_i, v_i) = \int_{v_0}^{v} dv_i \; K(v_i) \int_0^1 d\sigma_i,$$

plus the additional mapping

$$\sigma(t, v) = \frac{\int_{t_{\min}(v)}^{t} dt' \; \bar{\mathbb{P}}(t', v)}{\int_{t_{\min}(v)}^{t_{\max}(v)} dt' \; \bar{\mathbb{P}}(t', v)} = \frac{\int_{t_{\min}(v)}^{t} dt' \; \bar{\mathbb{P}}(t', v)}{K(v)},$$

leads to:

Final generic formula for CMC bremsstrahlung

$$D(t, t_0|v) = e^{-R(v_\mu)} \delta_{v=v_0} + \theta_{v>v_0} R'(v) e^{R(v)-R(v_\mu)}$$
$$\times \sum_{n=1}^{\infty} P_{n-1}(R(v)) \left[\prod_{i=1}^{n} \int_0^1 d\xi_i \int_0^1 d\sigma_i \right] \frac{\delta_{\max_j \xi_j} = 1}{n} \; w^{\#}(\xi) \; w^{\mathbb{P}}(\mathbf{t}^{\mathfrak{P}_\mathbf{t}}, \mathbf{v}^{\mathfrak{P}_\mathbf{t}}).$$

APPLICATION: CMC bremsstrahlung kernel (C), CCFM-like

Starting point:

$$\frac{x}{u} G_{ff}(K^B; t_b, t_a, x, u) = e^{-\Phi_f(t_b, t_a|x)} \delta_{x=u}$$
$$+ \sum_{n=1}^{\infty} \left[\prod_{i=1}^{n} \int_{t_a}^{t_b} dt_i \; \theta_{t_i > t_{i-1}} \int_x^u dx_i \frac{x_i}{x_{i-1}} \mathcal{K}^R_{ff}(t_i, x_i, x_{i-1}) \right]$$
$$\times \, e^{-\sum_{j=0}^{n} \Phi_f(t_j, t_{j-1}|x_{j-1})} \delta_{x=x_n},$$

where $x = x_n$, $u = x_0$, $t_0 = t_a$ and $t_n = t_b$.

The evolution kernel:

$$x_i \mathcal{K}^R_{ff}(t_i, x_i, x_{i-1}) = \alpha_S(e^t(1-z_i)/x_{i-1}) \; z_i P(z_i), \quad z_i = x_i/x_{i-1}.$$

APPLICATION: CMC bremsstrahlung kernel (C)

Intermediate formula is:

$$\frac{x}{u}G_{ff}(K^B;t_b,t_a,x,u) = e^{-\mathbf{\Phi}_f(t_b,t_a|1-x)}h(x,u)\Bigg\{\delta_{\exp(v)=1}+$$

$$+\sum_{n=1}^{\infty}\left[\prod_{i=1}^{n}\int_{t_{i-1}}^{t_b}dt_i\int_{v_0(t_i)}^{v}dv_i\;\bar{\mathbb{P}}(t_i,v_i)\right]\delta_{\exp(v)=\sum_{j=1}^{n}\exp(v_j)}\;w^{\bar{\mathbb{P}}}\Bigg\},$$

with the following list of (C)-specific components :

$$\bar{\mathbb{P}}(t_i,v_i) = \alpha_S(e^{t_i+v_i})A_{ff}, \quad v_i = \ln(y_i), \quad v_0(t) = \ln\lambda - t,$$

$$v = \ln(u-x), \quad \Psi(v) = e^v, \quad \Psi'(v) = e^v = u-x,$$

$$w^{\bar{\mathbb{P}}} = e^{\mathbf{\Phi}_f(t_b,t_a|u)-\sum_{j=0}^{n}\Phi_f(t_j,t_{j-1}|x_{j-1})}\prod_{i=1}^{n}\frac{\alpha_S(e^{t_i+v_i})z_i(1-z_i)P_{ff}(z_i)}{\bar{\mathbb{P}}(t_i,v_i)},$$

$$h(x,u) = e^{\mathbf{\Phi}_f(t_b,t_a|1-x)-\mathbf{\Phi}_f(t_b,t_a|u)}.$$

$$R(v;t_b,t_a) = A_{ff}\frac{2}{b_0}\rho_2(t_b+v,t_a+v;\ln\lambda), \quad K(v) = \partial_v R(v;t_b,t_a),$$

APPLICATION: CMC bremsstrahlung kernel (C)

The final standardized formula used in the MC reads:

$$\int_x^1 du\, G_{ff}(K^B; t_b, t_a, x, u) =$$

$$= \int_0^1 dU\, \frac{u}{x} h(x,u) \Big\{ \theta_{U<\exp(-R(v_\mu))}\big|_{v=v_0(t_b)}$$

$$+ \theta_{U>\exp(-R(v_\mu))}$$

$$\times \sum_{n=1}^{\infty} P_{n-1} R(v)) \Big[\prod_{i=1}^{n} \int_0^1 d\xi_i \int_0^1 d\sigma_i \Big] \frac{\delta_{\max_j \xi_j} = 1}{n}$$

$$\times\, w^{\#}(\xi)\, w^{\mathbb{P}}(\mathbf{t}^{\mathfrak{P}_\mathbf{t}}, \mathbf{v}^{\mathfrak{P}_\mathbf{t}}) \Big\},$$

where $U = U(x,u) = e^{R(v)-R(v_\mu)} = e^{R(\ln(u-x))-R(\ln(1-x))}$.

Complete CMC with quark-gluon transitions

Integral for FOAM, in terms of $3n+1$ variables $U_0, U_k, \alpha_k, \beta_k \in (0,1), k = 1, 2, ...n$:

$$D_f(t,x) = \int_0^1 dU_0(x,x_0)\; H^{(X)}(x_0,x) \sum\!\!\!\!\!\!\int dG_0\; W_0^G D_f(t_0,x_0) +$$

$$+ \sum_{n=1}^{N} \sum_{f_{n-1},\ldots,f_1,f_0} \left[\prod_{k=1}^{n} \int_0^1 (t - t_{k-1}) d\alpha_k \right] \int_0^1 dU_n(x,x_n)\; H^{(X)}(x_n,x) \sum\!\!\!\!\!\!\int dG^{(n)}\; W_n^G$$

$$\times \left[\prod_{k=1}^{n} \int_0^1 d\beta_k\; (1-x_k) \mathcal{P}^A_{f_k f_{k-1}}(t_k, x_k, x'_k(\beta_k)) \right.$$

$$\left. \times \int_0^1 dU_{k-1}(x'_k, x_{k-1})\; H^{(X)}(x_{k-1}, x'_k) \sum\!\!\!\!\!\!\int dG^{(k-1)}\; W_{k-1}^G \right] D_{f_0}(t_0,x_0),$$

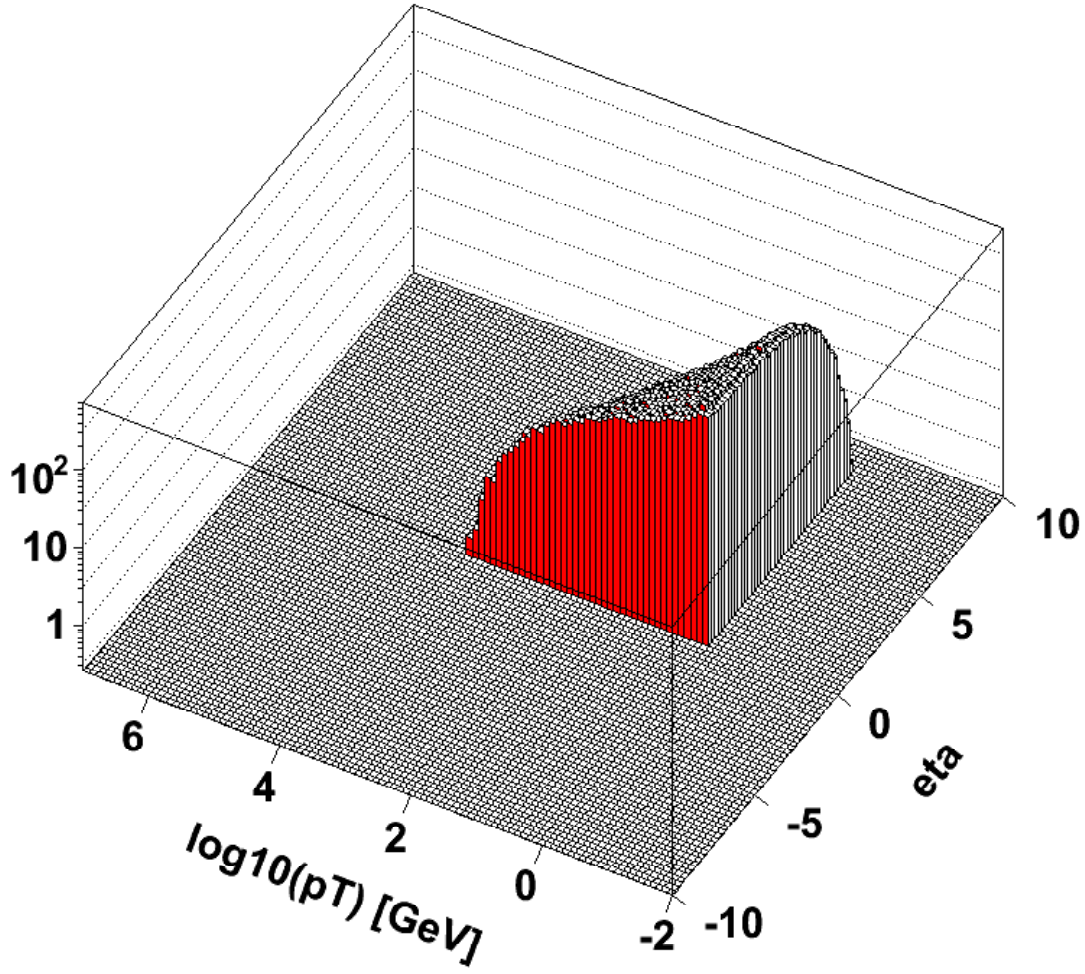
where for all three cases $X = A, B, C$, bremsstrahlung multi-differentials dG are

$$\sum\!\!\!\!\!\!\int dG = \theta_{U<\exp(-R(v_\mu))}|_{v=v_0(t_b)} +$$

$$+ \theta_{U>\exp(-R(v_\mu))} \sum_{n=1}^{\infty} P(n-1|R(v)) \left[\prod_{i=1}^{n} \int_0^1 d\xi_i \int_0^1 d\sigma_i \right] \frac{\delta_{\max_j \xi_j} = 1}{n},$$

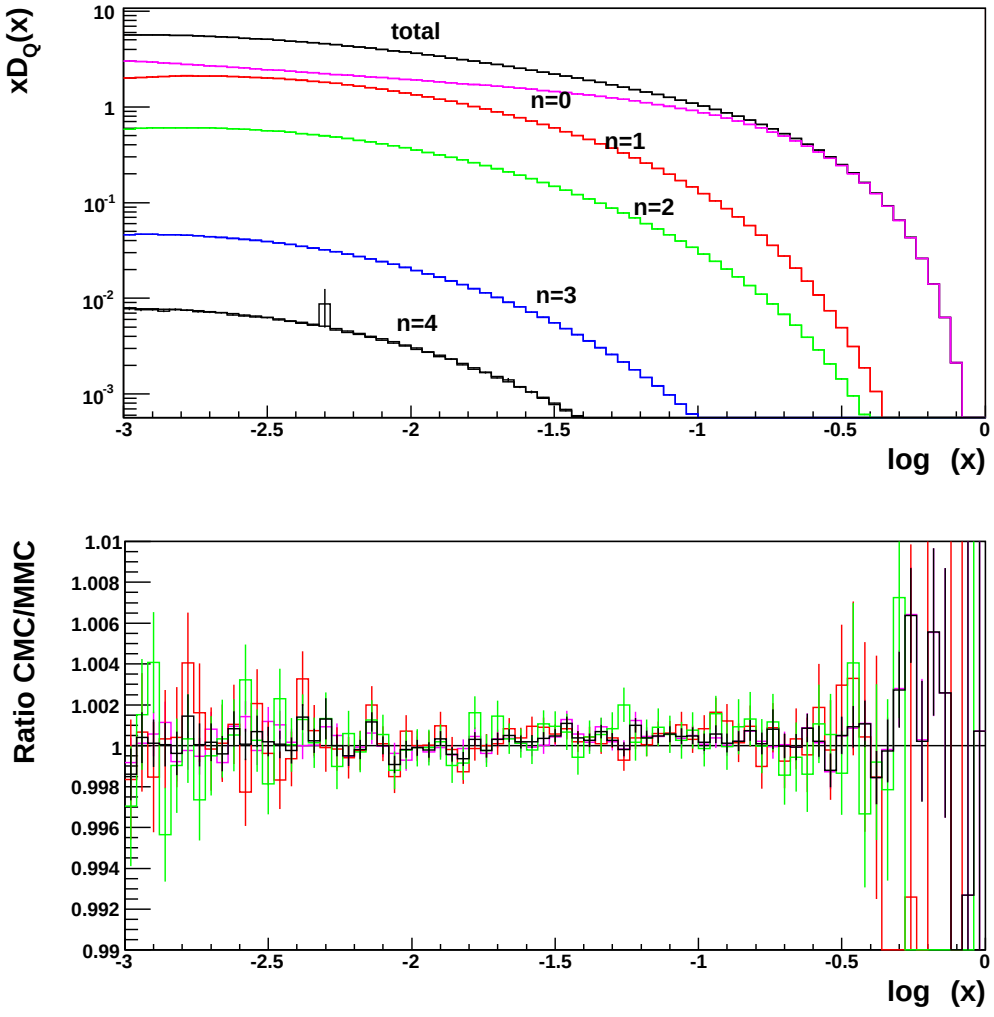
$$\int_0^1 dU \sum\!\!\!\!\!\!\int dG \equiv 1, \quad W^G = w^{\#}(\xi)\; w^{\mathbb{P}}(\mathbf{t}^{\mathfrak{P}_\mathbf{t}}, \mathbf{v}^{\mathfrak{P}_\mathbf{t}}).$$

CMC kernel (C), numerical results



Distribution of rapidity and log of k^T from CMC for $2E_h = 1000$GeV and $\lambda = 1$GeV, for kernel type (C), pure bremsstrahlung.

CMC kernel (C); Example numerical results



NEW!!! Quark distribution from CMC and MMC for evolution with kernel (C'), $\lambda = 1$GeV and $e^{t_{\max}} = 2E_h = 1$TeV. Contributions from fi xed number of the quark-gluon transitions $n = 0, 1, 2, 3, 4$ are also shown. The ratios CMC/MMC in the lower plot are separately for the total result and for the number of quark-gluon transition $n = 0, 1, 2$. MC statistics is 10^{10} weighted events for both CMC and MMC.

Recent developments and plans

Recent activity:

- In progress: 2 single evolutions into one MC for W/Z production at LHC, more and more testing!
- Getting more realistic distributions of W/Z rapidity and kT
- Quark-gluon transitions in double CMC
- Non-Sudakov formfactor for full CCFM compatibility
- QCD NLO in the hard process (easy?) and evolution (difficult?). See talk by Phil Stephens.

Plans:

- Better EW +QED FSR matrix element, from WINHAC/SANC
- xchecks with uPDFs of CASCADE/SMALLX?
- CMC/MMC for DIS process, fitting F_2
- Establishing relation to k^T-ordering and CSS in b-space