

An Update of the **MRST (MSTW)** Parton Distributions

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Major update in people involved.

[Dick Roberts](#) completely retired from project.

[Graeme Watt](#) started as responsive RA on parton distributions from [April 1st 2006](#). Now making major contribution to project – responsible for many of these new results.

Major changes in theory.

Implementation of new heavy flavour [VFNS](#) (see talk), particularly at [NNLO](#).

Inclusion of [NNLO](#) corrections to [Drell-Yan](#) data.

Some important changes as [NLO](#) → [NNLO](#).

Most important change compared to previous [NNLO](#) – new [VFNS](#). → significant change in partons.

Implementation of [fastNLO](#) – fast perturbative [QCD](#) calculations [Kluge](#), [Rabbertz](#), [Wobisch](#). Allows easy inclusion of new data.

New data included.

NuTeV and Chorus data on $F_2^{\nu,\bar{\nu}}(x, Q^2)$ and $F_3^{\nu,\bar{\nu}}(x, Q^2)$ replacing CCFR.

NuTeV and CCFR dimuon data included directly. Leads to a direct constraint on $s(x, Q^2) + \bar{s}(x, Q^2)$ and on $s(x, Q^2) - \bar{s}(x, Q^2)$. Affects other partons.

CDFII lepton asymmetry data in two different E_T bins – $25\text{GeV} < E_T < 35\text{GeV}$ and $35\text{GeV} < E_T < 45\text{GeV}$.

HERA inclusive jet data (in DIS).

New CDFII high- E_T jet data.

Direct high- x data on $F_L(x, Q^2)$.

Update to include all recent charm structure function data.

Obtain **NNLO** partons with uncertainties due to experimental errors for the first time.

Same procedure as before – 15 eigenvector sets of partons and $\Delta\chi^2 = 50$ for 90% confidence limit.

First time we have full **NNLO** with no major approximations. (Heavy flavours a major issue.)

In general size of uncertainties similar to at **NLO**.

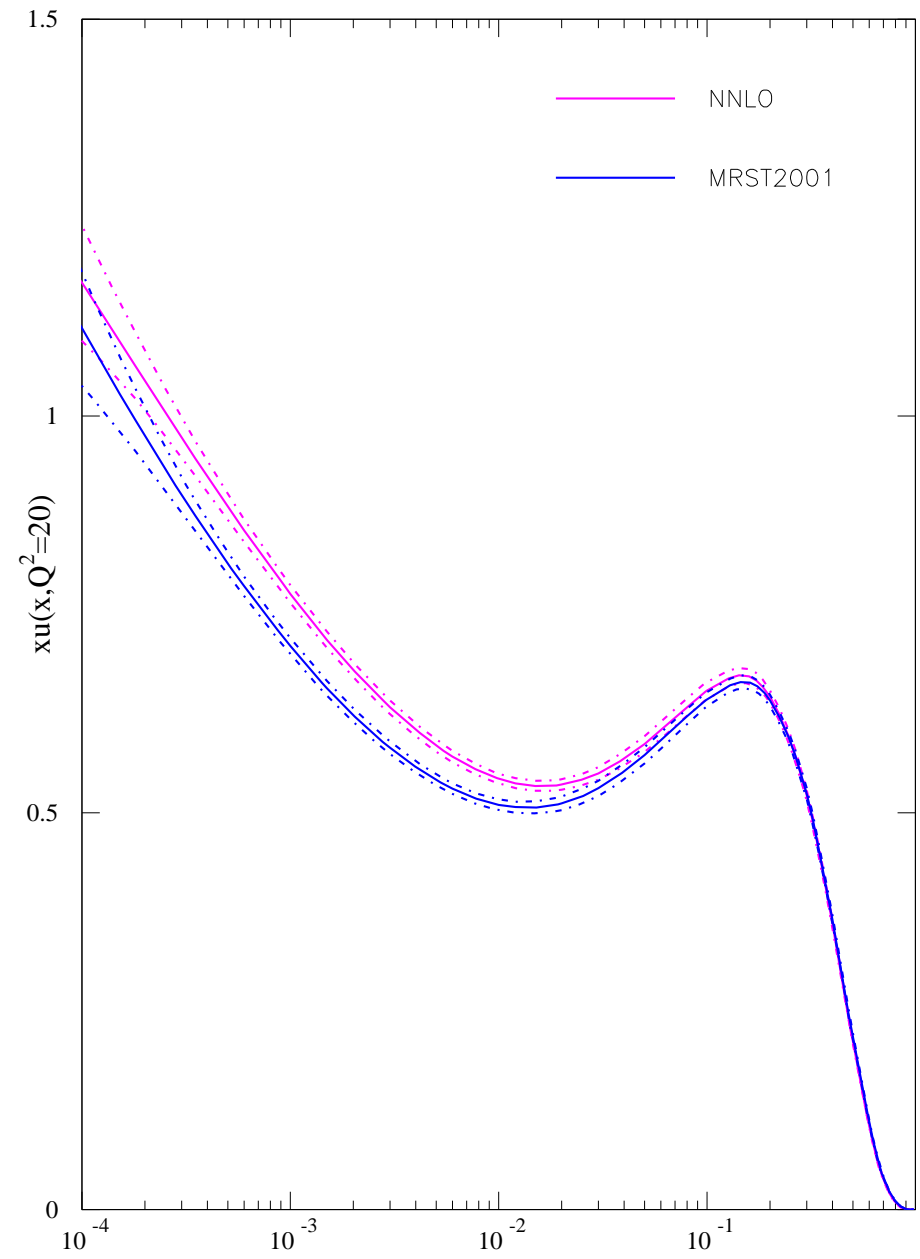
Constraint of partons comes mainly from , HERA neutral current, BCDMS, NMC, E866 Drell-Yan ratio, Tevatron jets, SLAC, E866 Drell-Yan, NuTeV, CDF W -asymmetry, *etc.* in (very) rough order of degree of constraint.

More work to do to estimate theoretical uncertainty. Certainly important in some regions.

Changes in Partons

At small x effect of coefficient functions, particularly $C_{2,g}(x, Q^2)$, important.

Change from **NLO** to **NNLO** greater than uncertainty in each.

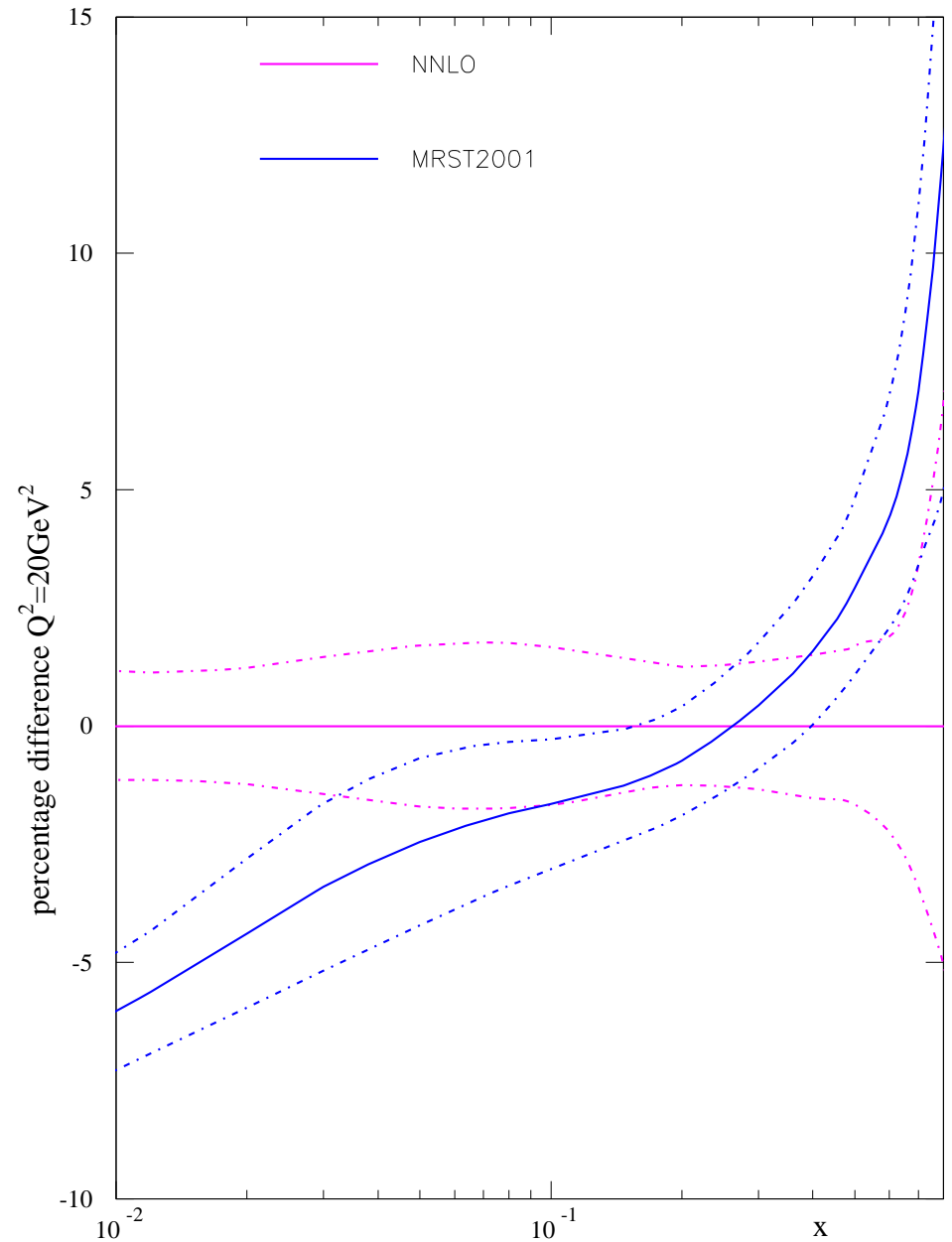


At large x coefficient functions important again,

$$C_{2,q}^2(x) \sim \left(\frac{\ln^3(1-x)}{1-x} \right)_+$$

Change from **NLO** to **NNLO** again larger than uncertainty in each.

No real change from **MRST2004NNLO** partons.



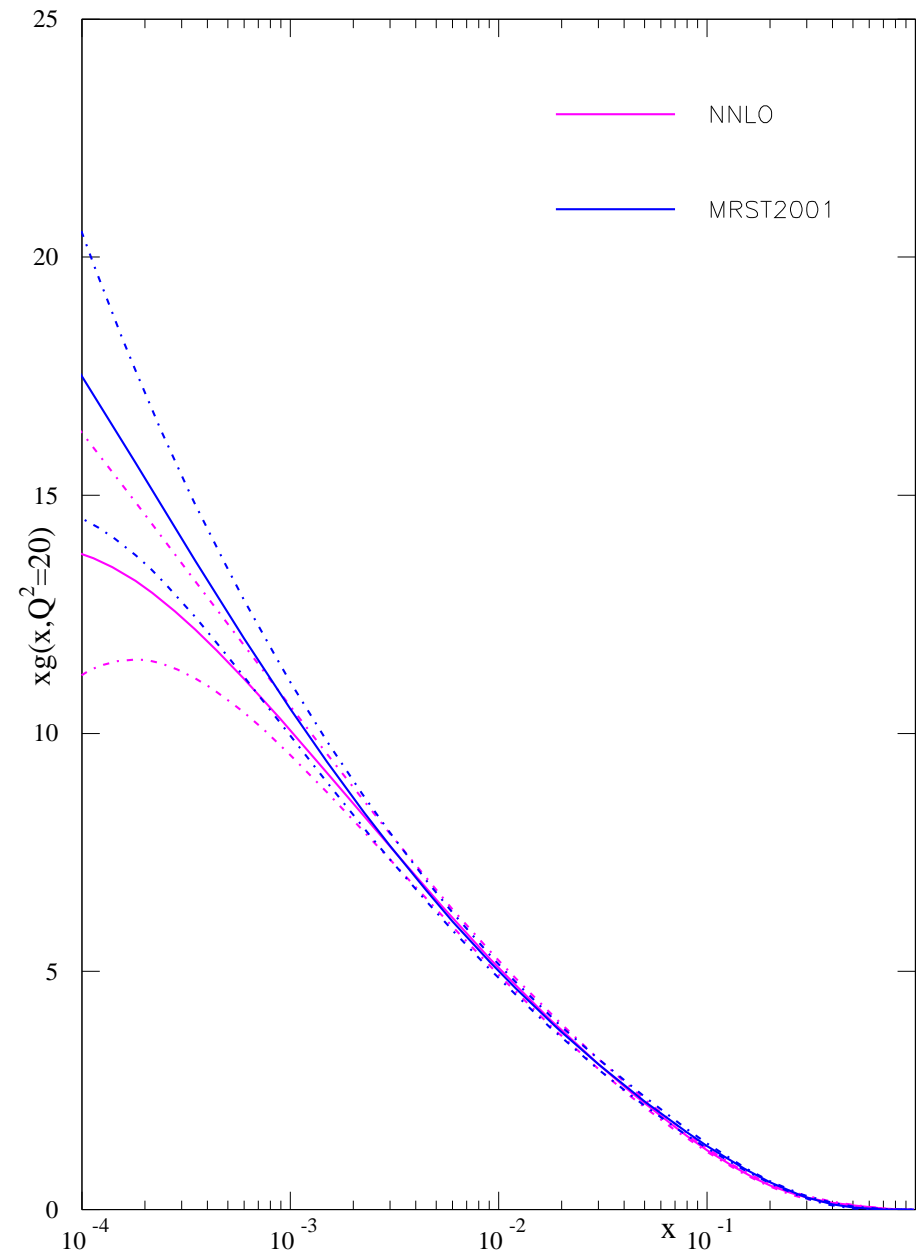
At small x effect of splitting functions particularly $P_{qg}^2(x, Q^2)$ important.

Positive $\ln(1/x)/x$ contribution at low x .

Affects gluon by fitting $dF_2(x, Q^2)/d \ln Q^2$.

Smaller at very low x .

NNLO coefficient functions very important for $F_L(x, Q^2)$.



Comparison to other (Alekhin) NNLO gluon.

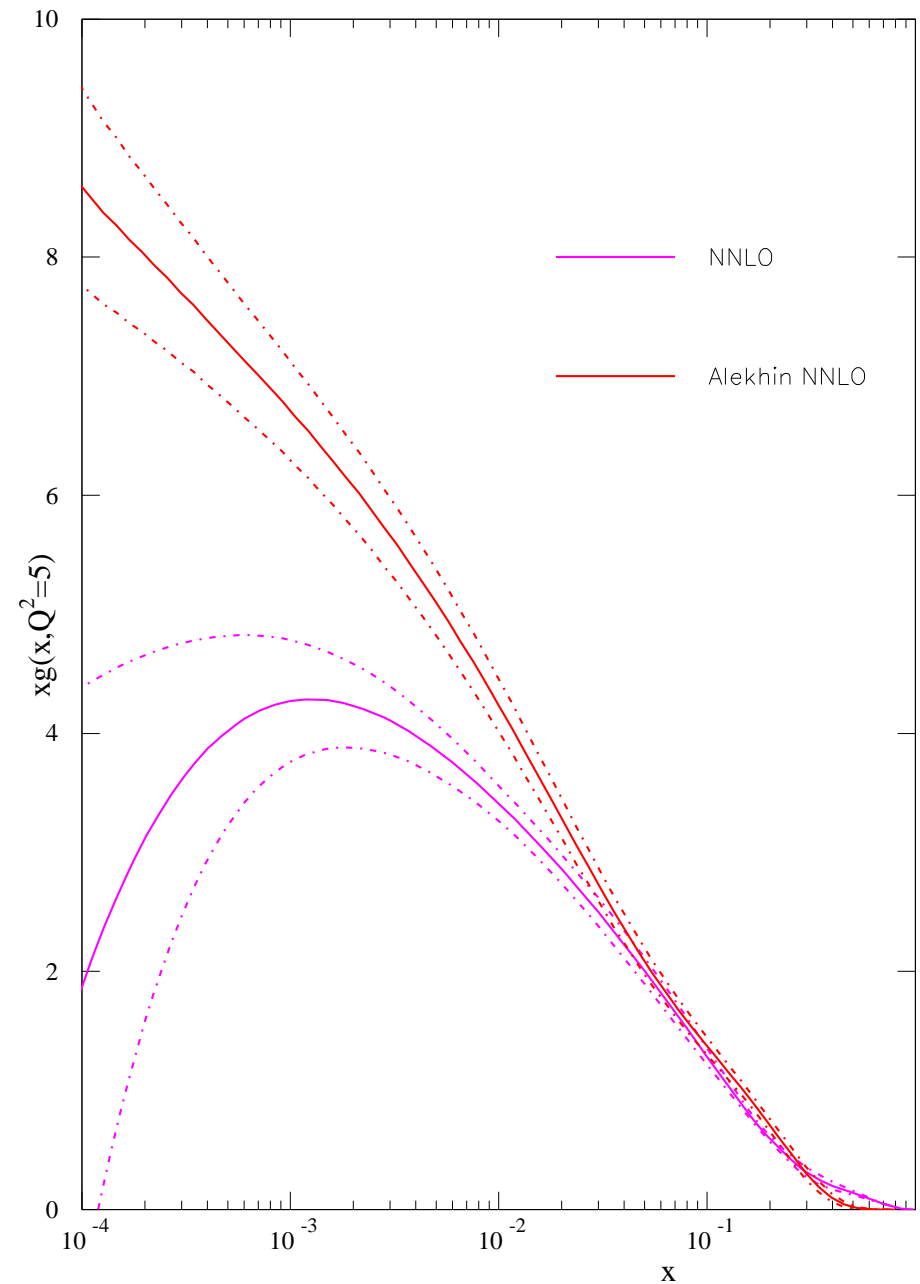
Hugely different at small x .

Differences much bigger than uncertainties.

Differences in heavy flavour treatments – already seen this is important.

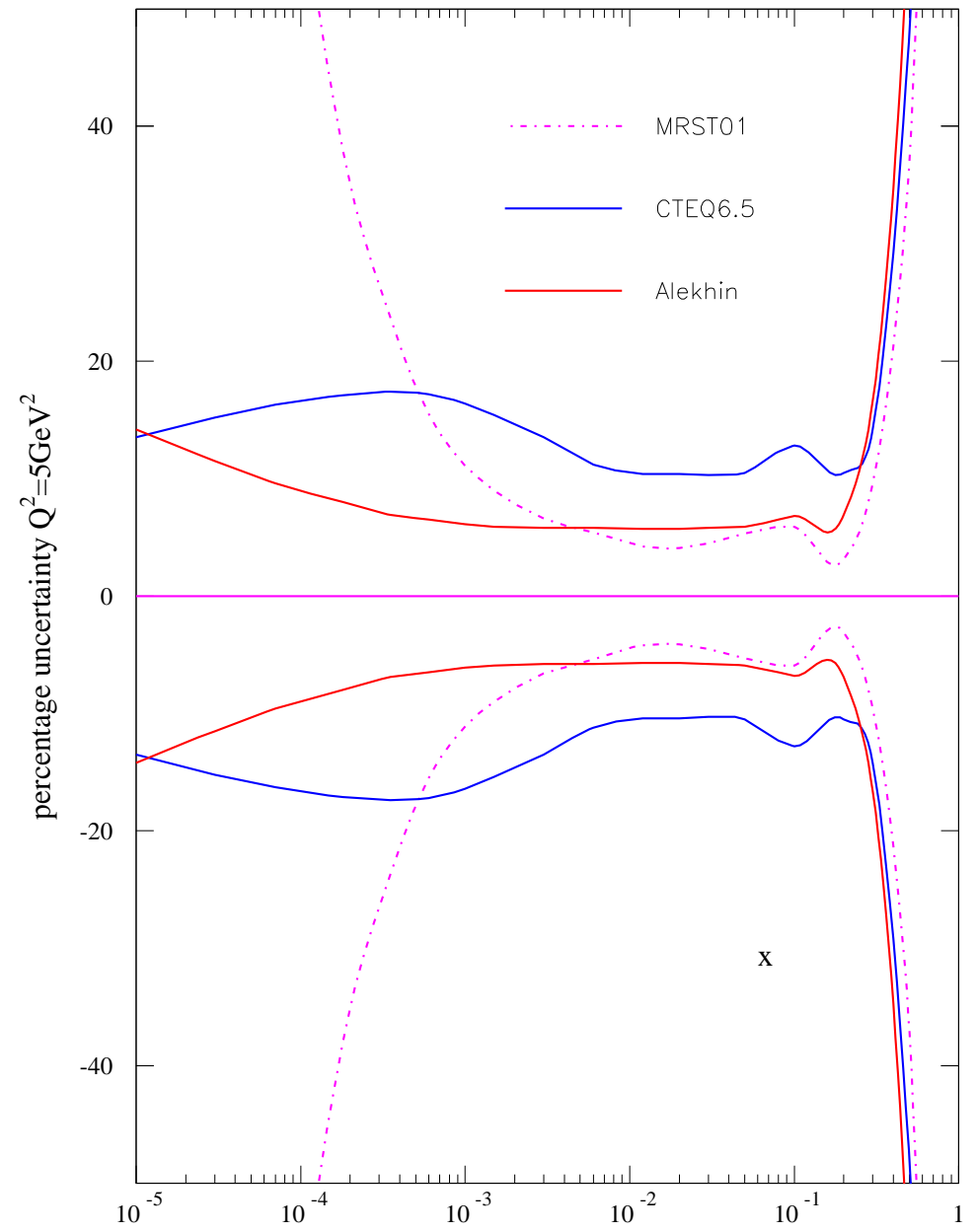
Differences in data fit and also in $\alpha_S(M_Z^2)$.

Note difference in uncertainty at low x not just in shape.



MRST uncertainty blows up for very small x , whereas Alekhin (and ZEUS and H1) gets slowly bigger, and CTEQ saturates (or even decreases).

Related to input forms and scales.



MRST (MSTW) parameterise at $Q_0^2 = 1\text{GeV}^2$ but allow negative and positive small x contributions. Very flexible. Represent true uncertainty at low x ?

Alekhin and ZEUS gluons input at higher scale – behave like $x^{-\lambda}$ at small x . Uncertainty due to uncertainty in one parameter.

CTEQ gluons input at $Q_0^2 = 1.69\text{GeV}^2$. Behave like x^λ at small x where λ large and positive. Input gluon valence-like.

Requires fine tuning. Evolving backwards from steep gluon at higher scale valence-like gluon only exists for very narrow range of Q^2 (if at all).

Small x input gluon tiny – very small absolute error. At higher Q^2 all uncertainty due to evolution driven by higher x , well-determined gluon. Very small x gluon no more uncertain than at $x = 0.01 - 0.001$.

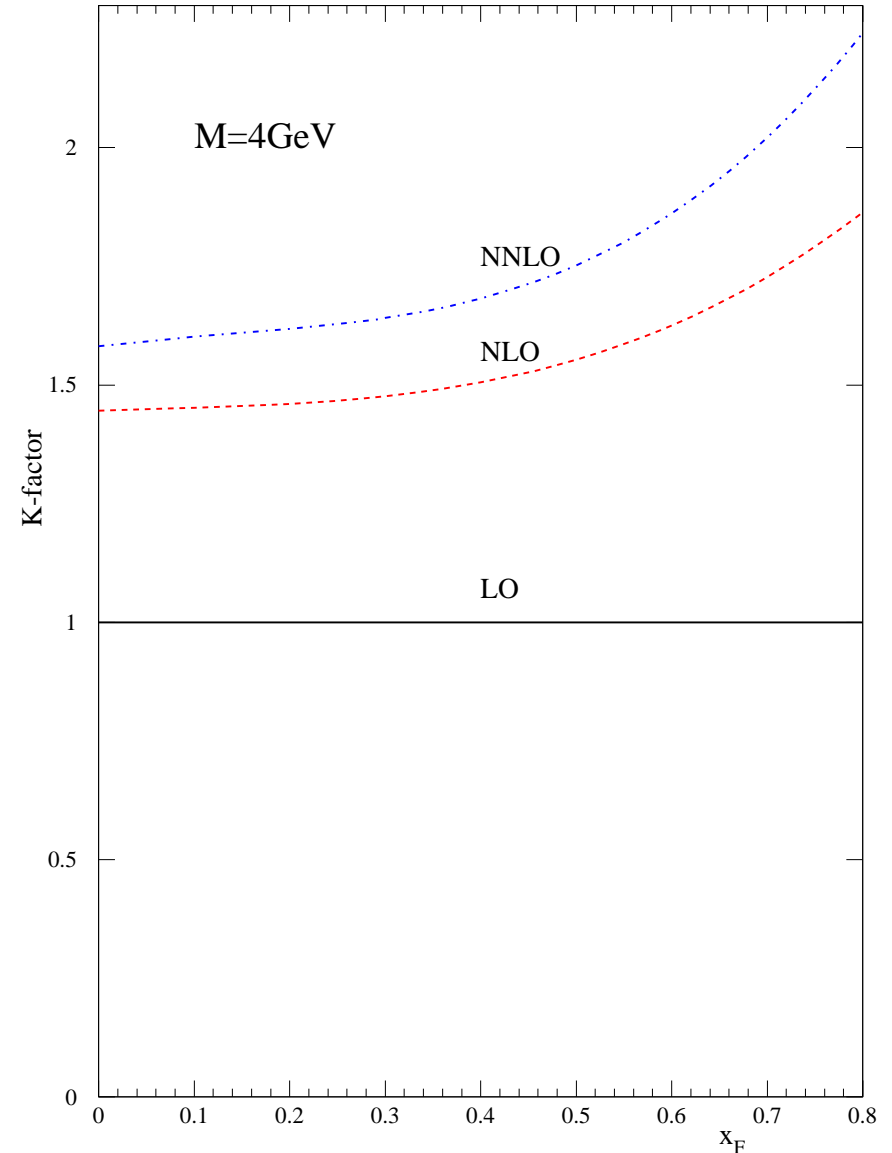
Drell-Yan corrections

The K -factors for Drell-Yan production at E866 – $\sqrt{s} = 38.8\text{GeV}$.

Enhancement at higher $x_F = x_1 - x_2$ due to logarithms. Similar to $\ln(1 - x)$ enhancement in structure functions.

NLO corrections large, NNLO corrections significant – 10% or more.

Higher order corrections to Drell-Yan Cross-section



Quality of fit to E866 Drell-Yan production at E866 in proton-proton collisions.

Now corrected for radiative corrections. Very roughly data increases by $\sim 4\%$.

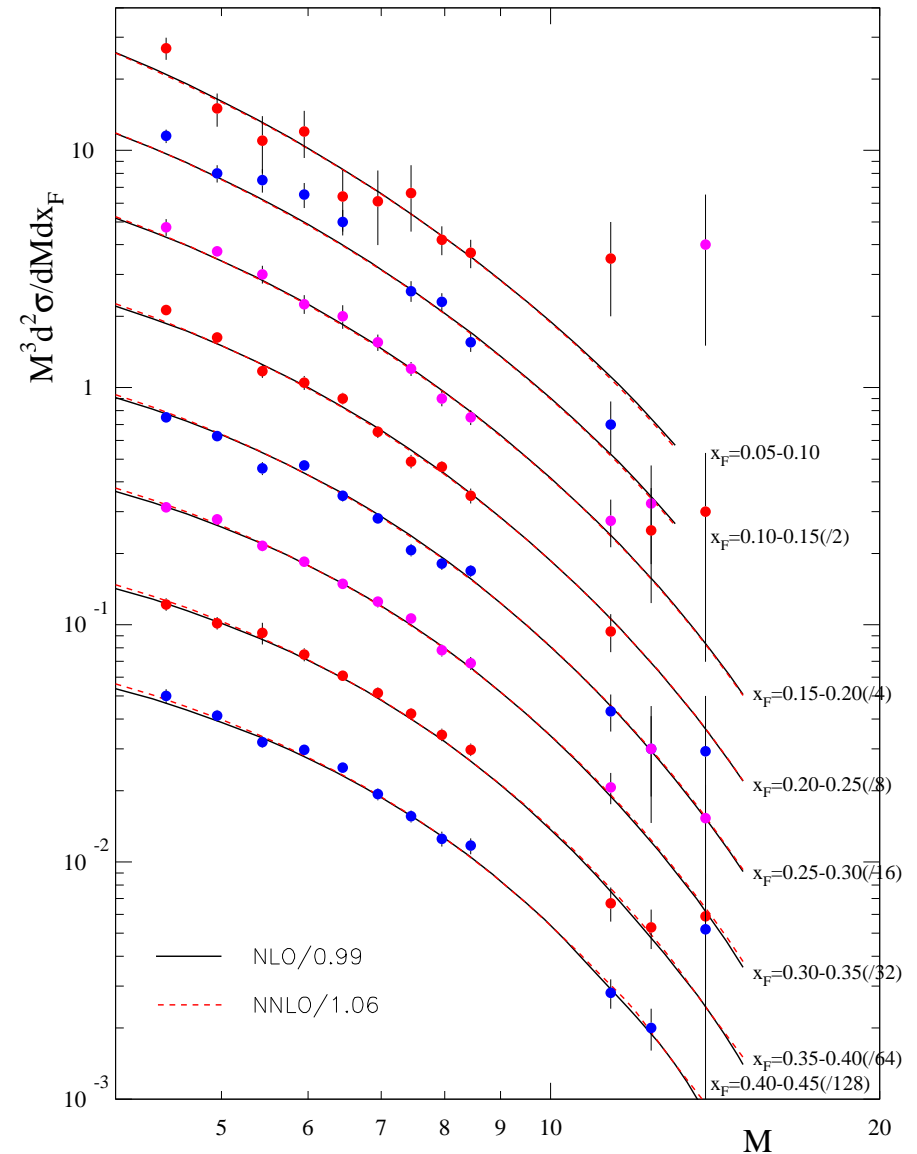
$\chi^2 = 225/174$ at NLO.

$\chi^2 = 240/174$ at NNLO.

Random scatter of points large – $\chi^2 \sim 220$ about best possible.

→ fit good.

E866 pp data and MRST fits ($x_F < 0.45$)



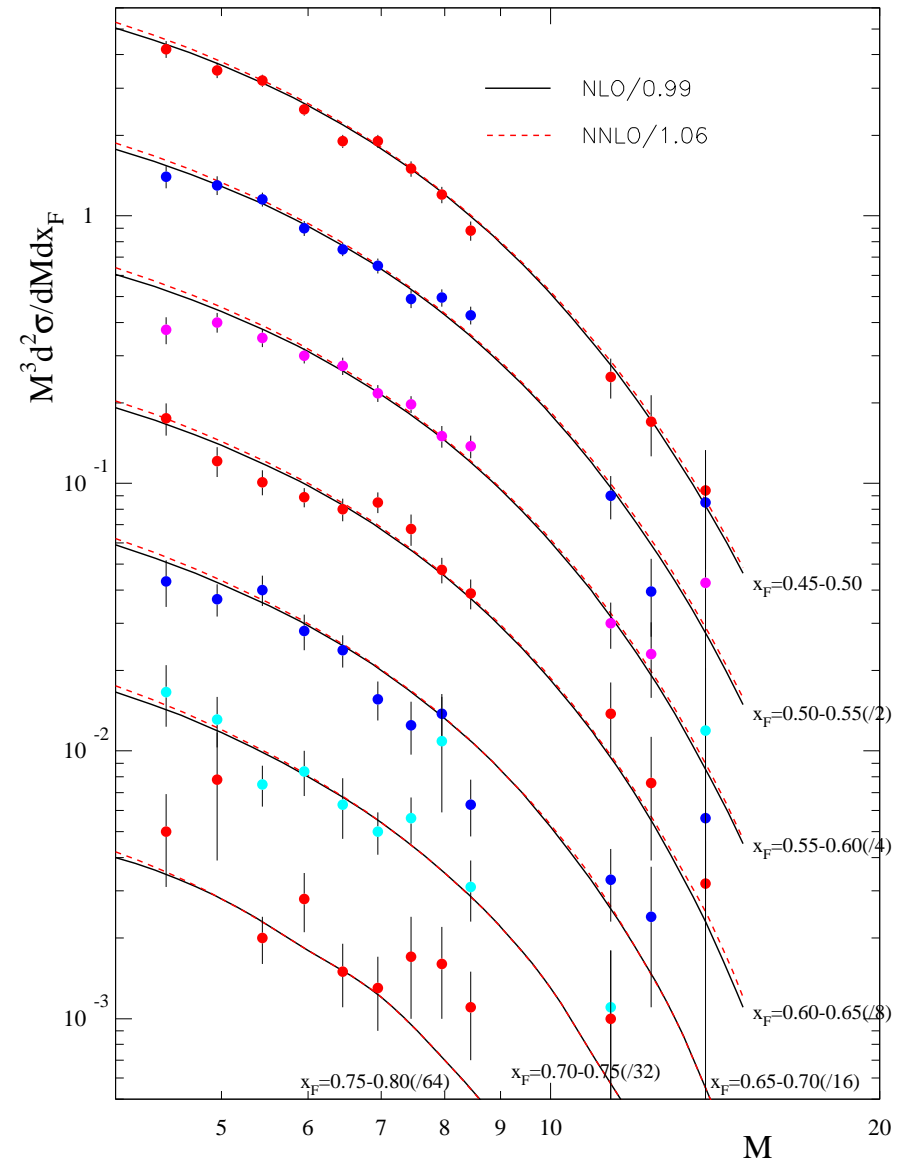
Consistent positive correction at NNLO requires data normalization equal 106%.

Sea for $x \leq 0.1$ and valence quarks already well-determined by structure function data.

Normalization uncertainty 6.5% — change of 6% ok.

Before corrections needed 10%.

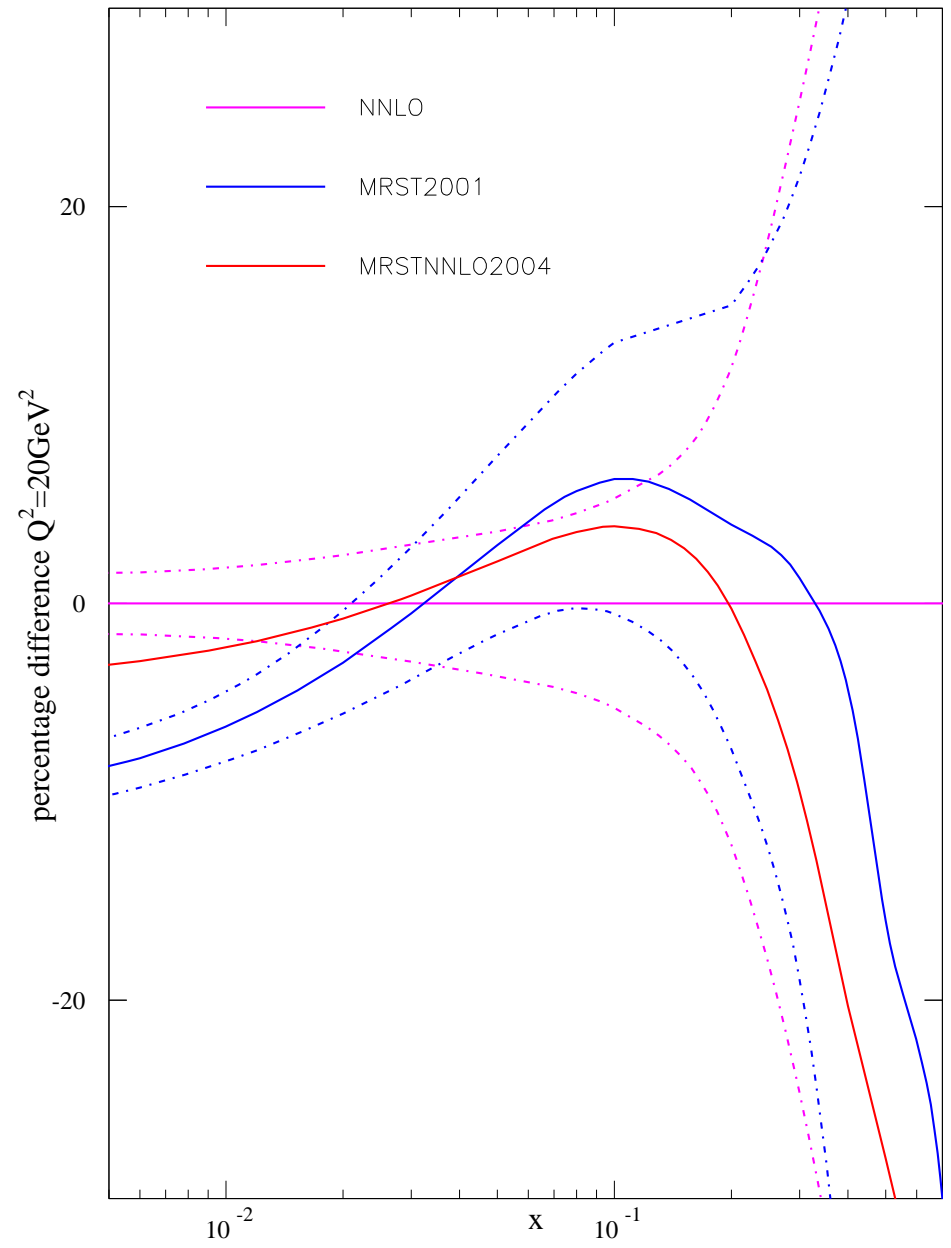
E866 pp data and MRST fits ($x_F > 0.45$)



Drell-Yan corrections affect sea quarks.

Even with increased normalization of data new NNLO partons smaller than MRST2004NNLO and NLO in region where constrained by data.

Must be bigger than these at smaller x as already seen.



Summary

Quality of full fit at **NLO** and at **NNLO**.

NNLO fairly consistently better than **NLO**.

Definite tendency for $\alpha_S(M_Z^2)$ to go up with all changes.

At **NLO** $\alpha_S(M_Z^2) = 0.121$.

At **NNLO** $\alpha_S(M_Z^2) = 0.119$.

Pull for high $\alpha_S(M_Z^2)$ at **NLO** from **NMC** data, **SLAC** data, **Tevatron** jets (indirectly) and $F_L(x, Q^2)$ data (against from **BCDMS** data).

Generally naturally improved by **NNLO** fit.

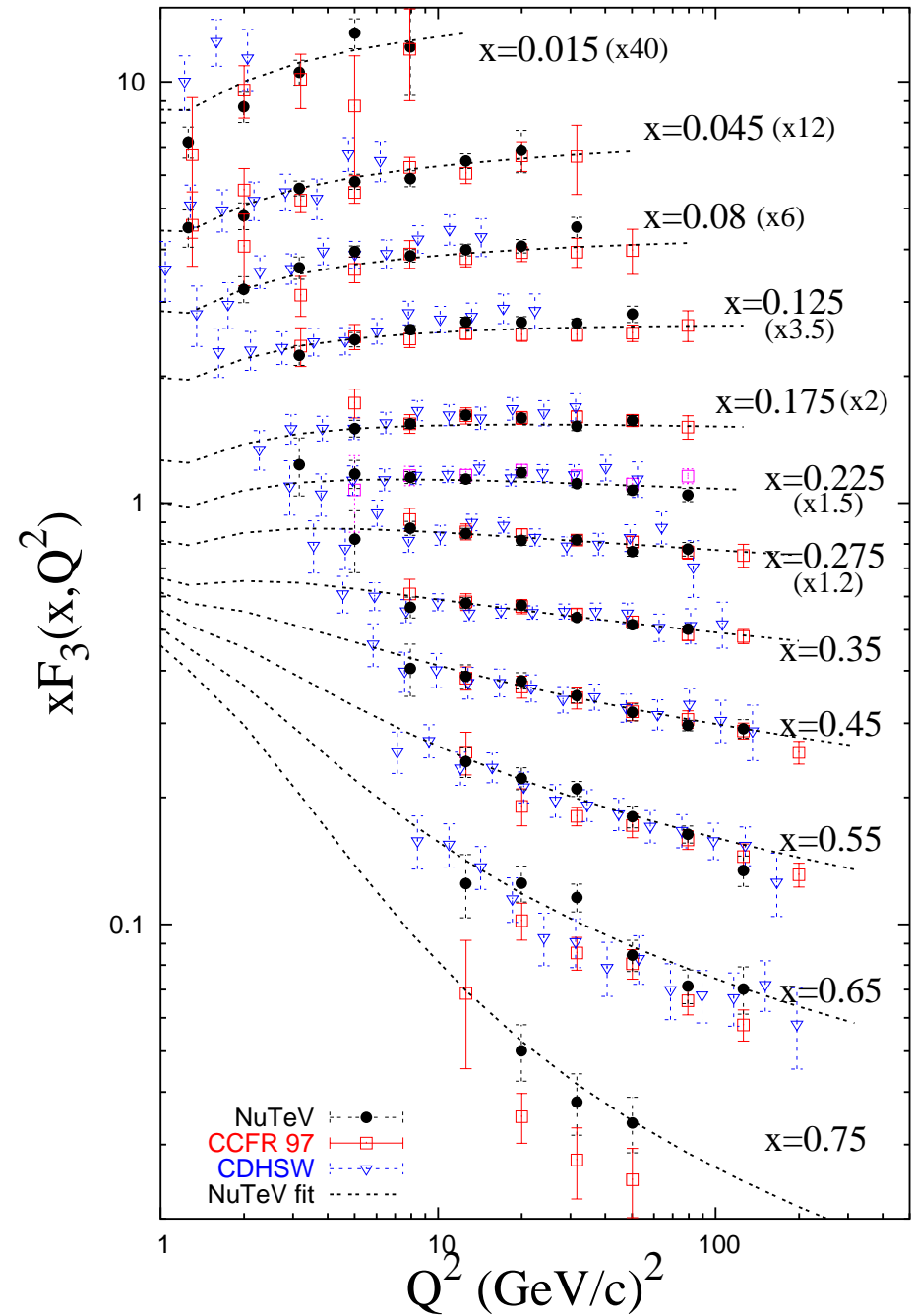
Some room for improvement.

New Data

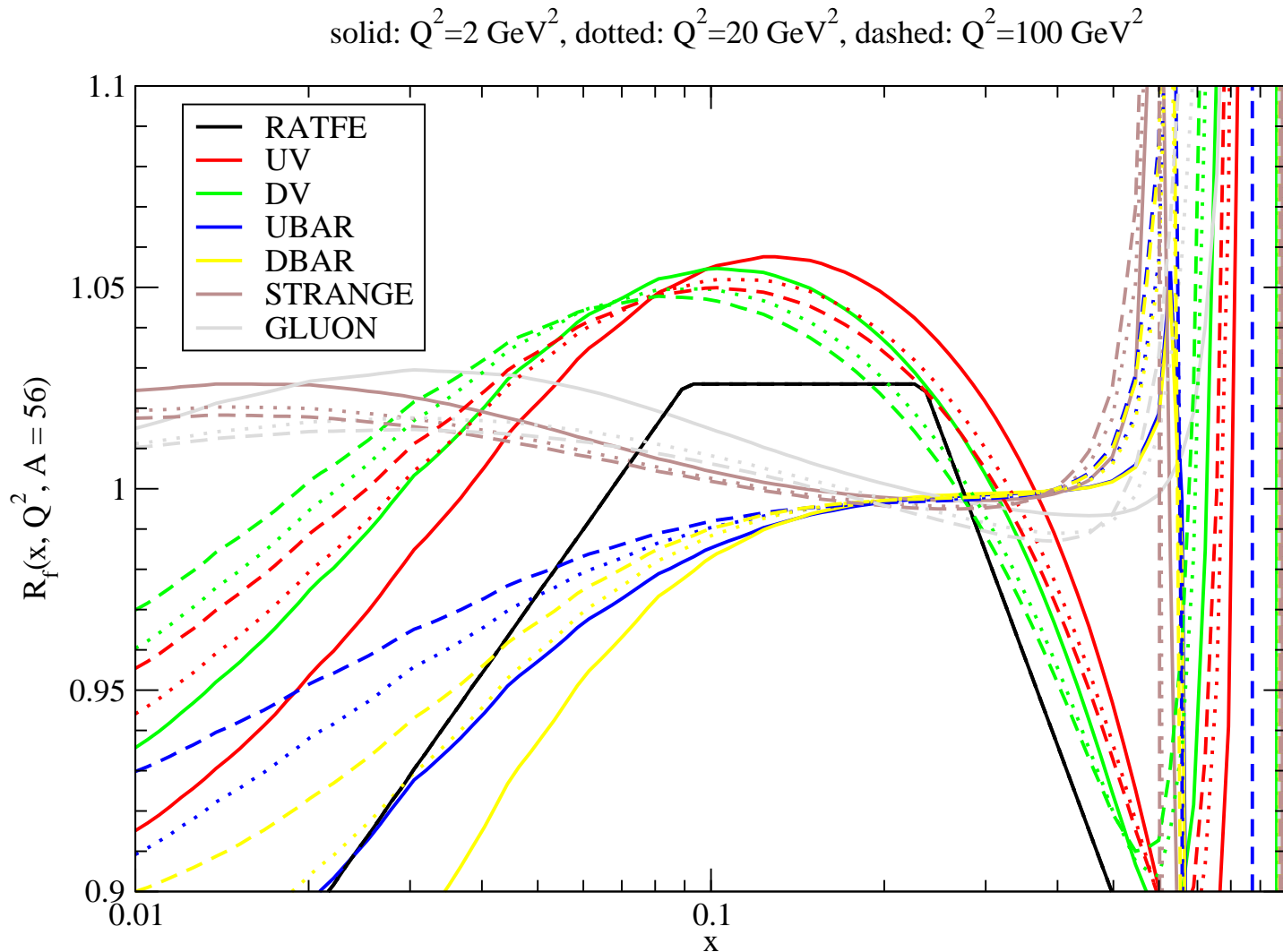
New **NuTeV** data not completely compatible with the older **CCFR** data.

Main source of discrepancy's calibration of magnetic field map of muon spectrometer \rightarrow muon energy scale.

However, previous parton distribution fits were perfectly compatible with **CCFR** data using **EMC** inspired Q^2 independent nuclear correction



Now implement far more sophisticated nuclear correction De Florian, Sassot.



Same general shape as before. Allow $\sim 3\%$ uncertainty on corrections. Cannot match high NuTeV data.

Chorus data also consistent with CCFR (lead not iron).

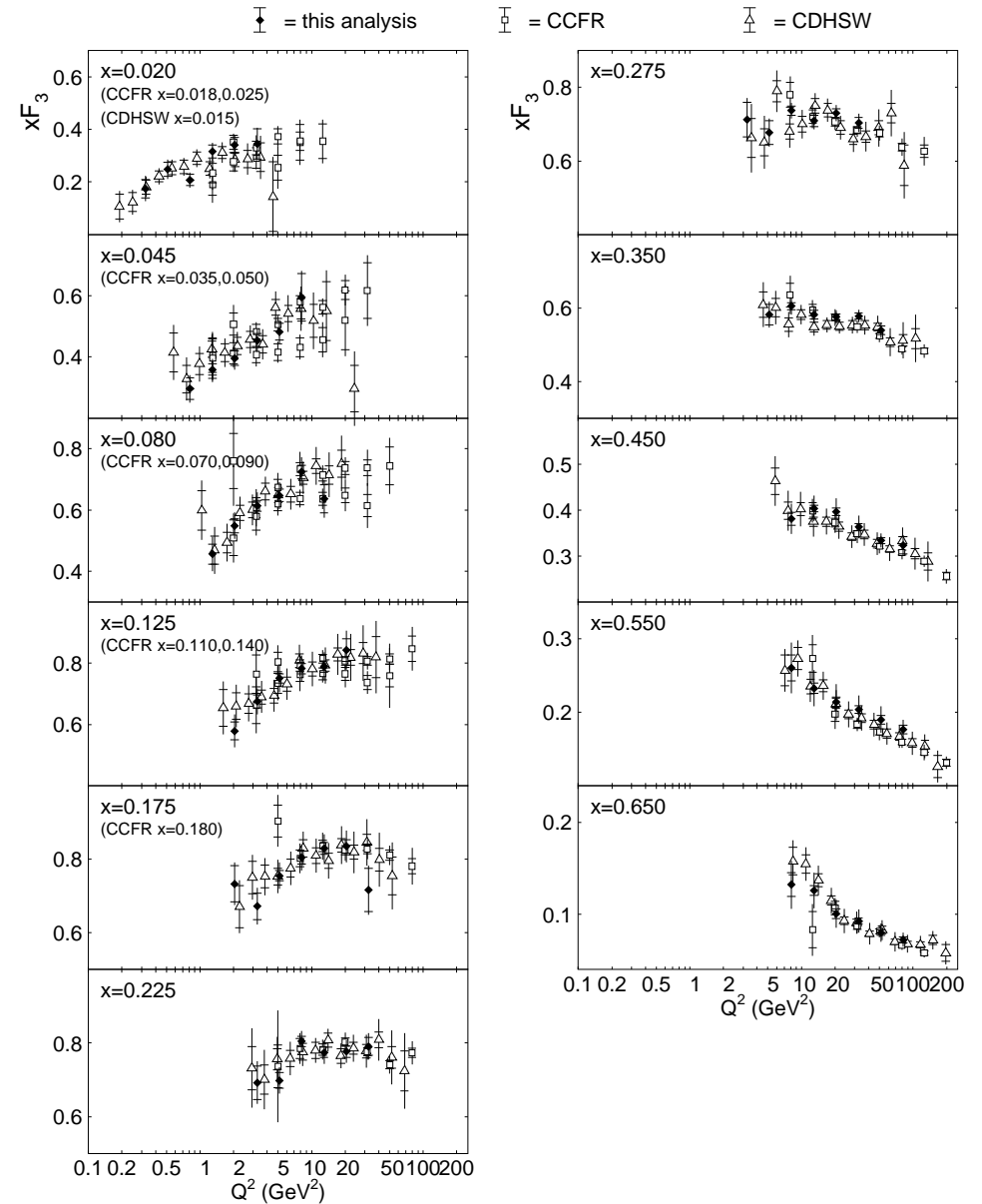
Inconsistencies at high x .

Partons in region of high x already well-determined from charged lepton structure functions.

Important information in the region $x < 0.3$, e.g. low x valence quarks - general consistency here.

Choose to cut neutrino structure function data for $x \geq 0.5$.

Also Chorus data at lower W^2 . $F_3(x, Q^2)$ expected to have larger higher twist corrections than $F_2(x, Q^2)$. Cut for $W^2 \leq 20\text{GeV}^2$.



CCFR/NuTeV dimuon cross-sections and strange quarks

$$\frac{d\sigma}{dx dy}(\nu_\mu(\bar{\nu}_\mu)N \rightarrow \mu^+\mu^-X) = B_c \mathcal{N} \mathcal{A} \frac{d\sigma}{dx dy}(\nu_\mu s(\bar{\nu}_\mu \bar{s}) \rightarrow c\mu^-(\bar{c}\mu^+)X),$$

B_c = semileptonic branching fraction

\mathcal{N} = nuclear correction

\mathcal{A} = acceptance correction.

ν_μ and $\bar{\nu}_\mu$ cross-sections probe s and \bar{s} (small mixing with d and \bar{d}).

Have previously indirectly used CCFR data to parameterise strange according to

$$s(x, Q_0^2) = \bar{s}(x, Q_0^2) = \frac{\kappa}{2}[\bar{u}(x, Q_0^2) + \bar{d}(x, Q_0^2)] \quad \kappa \approx 0.5$$

Now fit strange directly rather than assuming same shape as average of $\bar{u} + \bar{d}$ at input and some **fixed** fraction.

Also allow possibility of $s(x, Q_0^2) \neq \bar{s}(x, Q_0^2)$.

Make definitions at input

$$s^+(x, Q_0^2) \equiv s(x, Q_0^2) + \bar{s}(x, Q_0^2) = A_+(1-x)^{\eta_+} S(x, Q_0^2)$$

$$s^-(x, Q_0^2) \equiv s(x, Q_0^2) - \bar{s}(x, Q_0^2) = A_-(1-x)^{\eta_-} x^{-1+\delta_-} (1-x/x_0)$$

where $S(x, Q_0^2)$ is the total sea distribution and x_0 is determined by zero strangeness of proton, i.e.

$$\int_0^1 dx s^-(x, Q_0^2) = 0.$$

Extra freedom in both s^+ and s^- confirmed by fit.

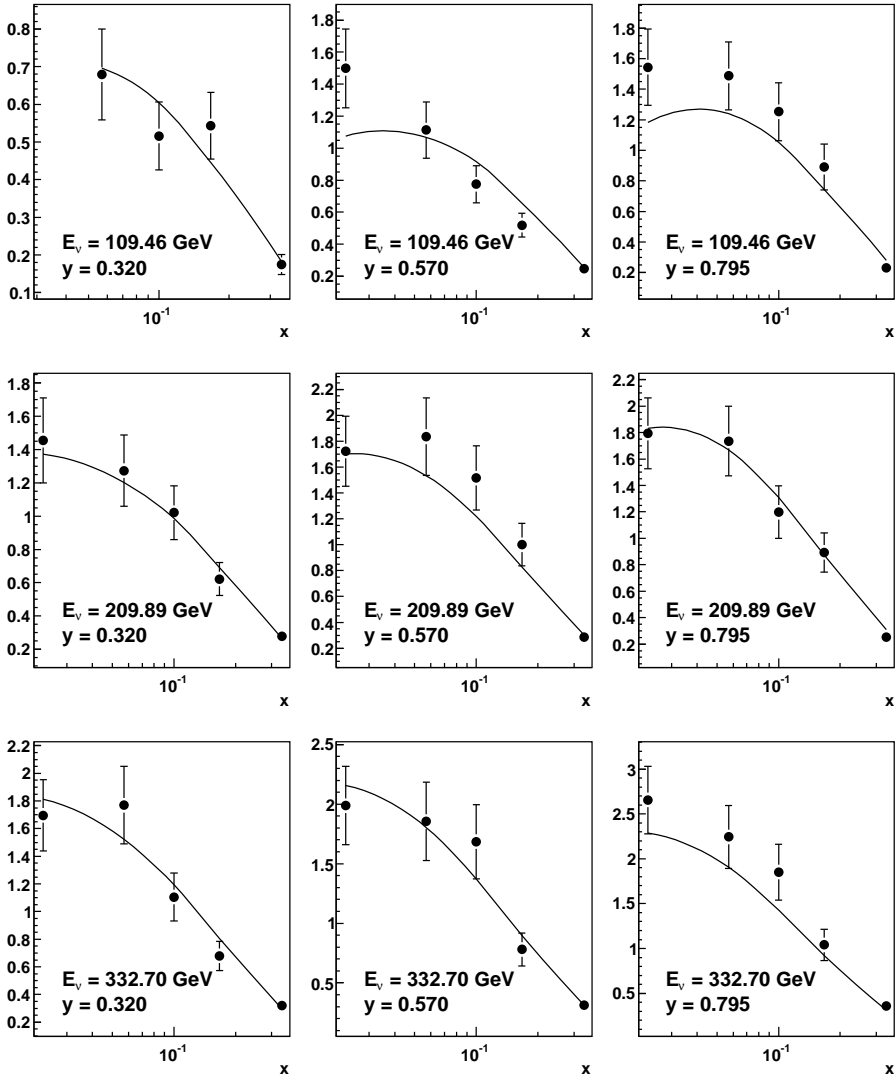
	χ_{CCFR}^2 86 pts	χ_{NuTeV}^2 84 pts	χ_{global}^2 2606 pts
$s = \bar{s} = (\bar{u} + \bar{d})/4$	68	66	2647
s^+ free, $s^- = 0$	63	54	2617
s^+ free, s^- free	64	40	2606

No improvement with further parameters.

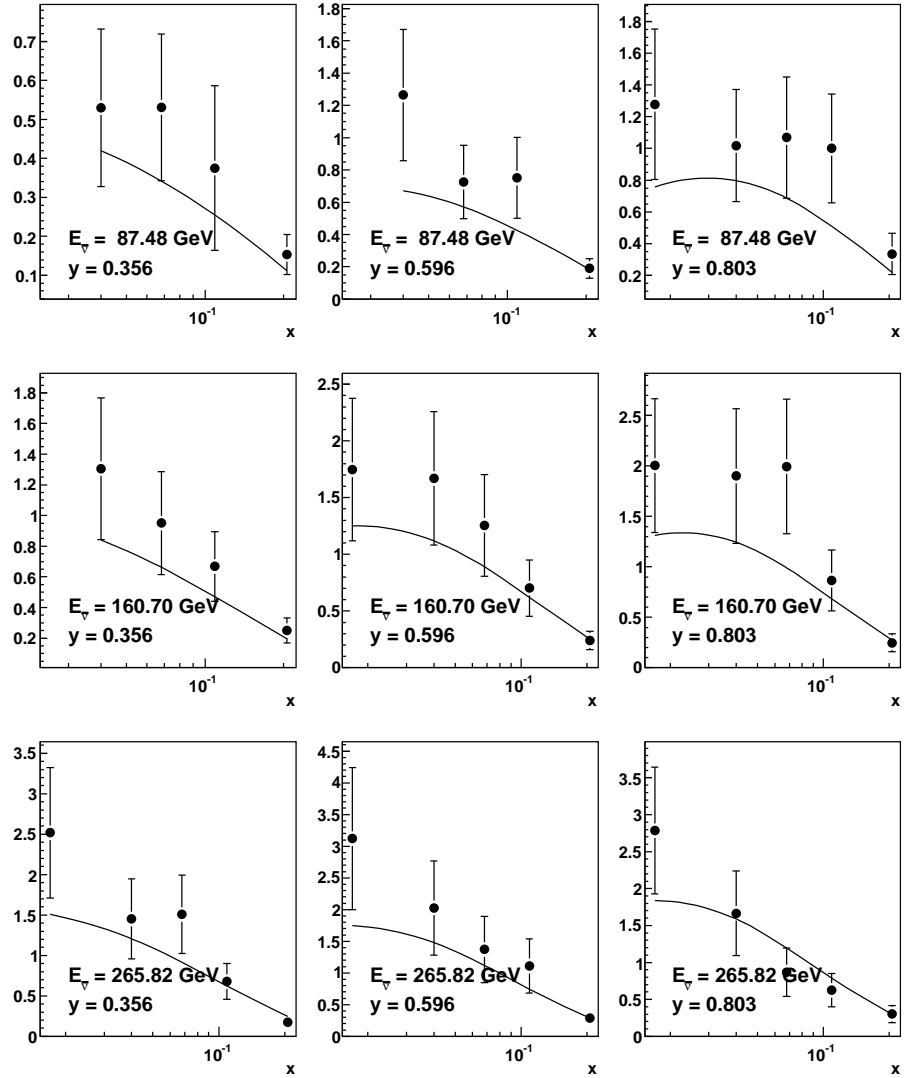
Data generally prefer s^+ free. Dimuon data only affected by s^- . Decoupled from other parameters to good approximation. $\delta_- = 0.2$ fixed, i.e. valence-like value.

Fit to data clearly very good.

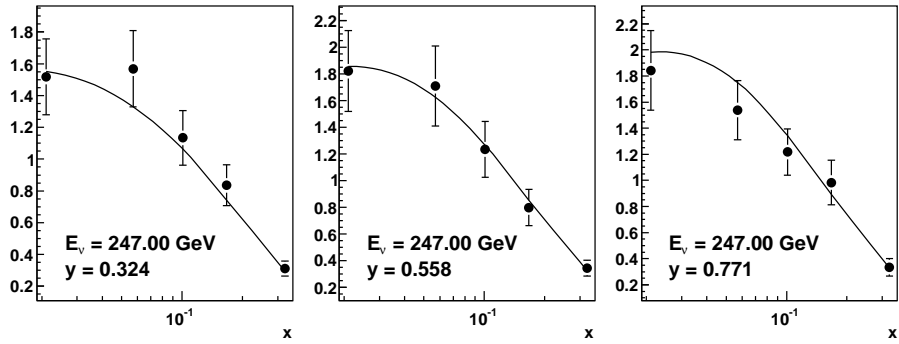
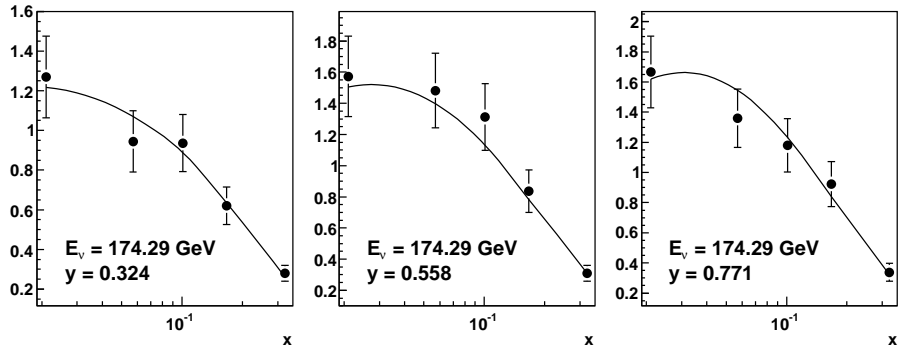
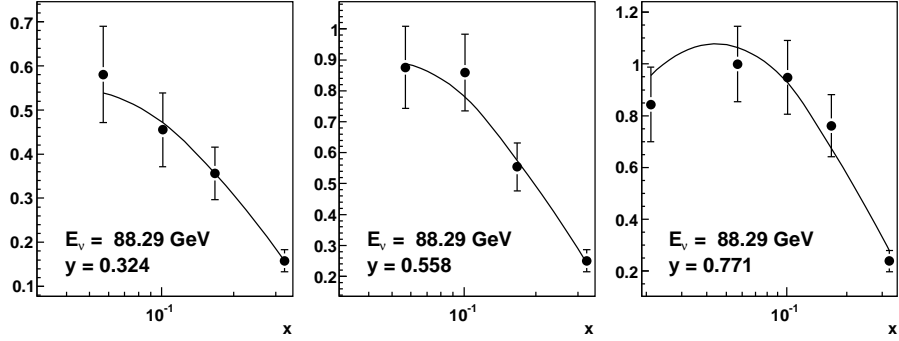
$$\text{CCFR } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\nu_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}, \chi^2 = 33/44 \text{ pts.}$$



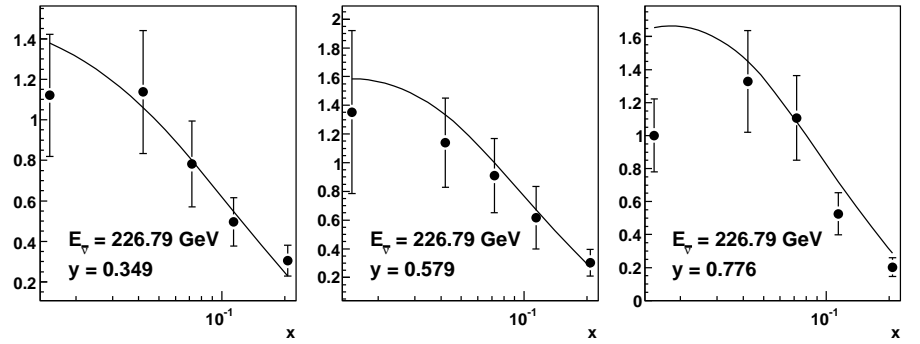
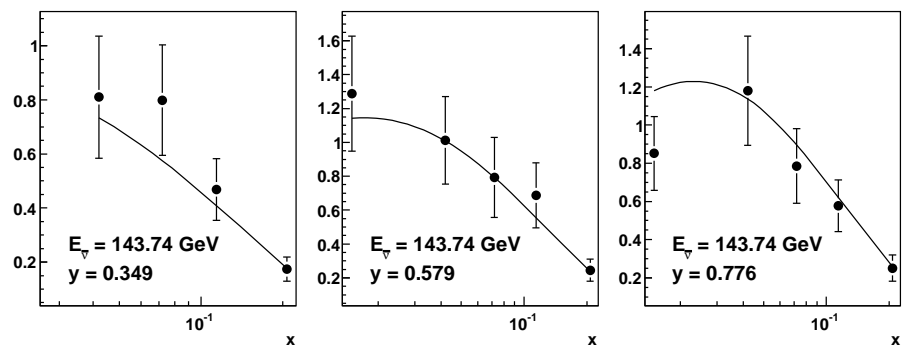
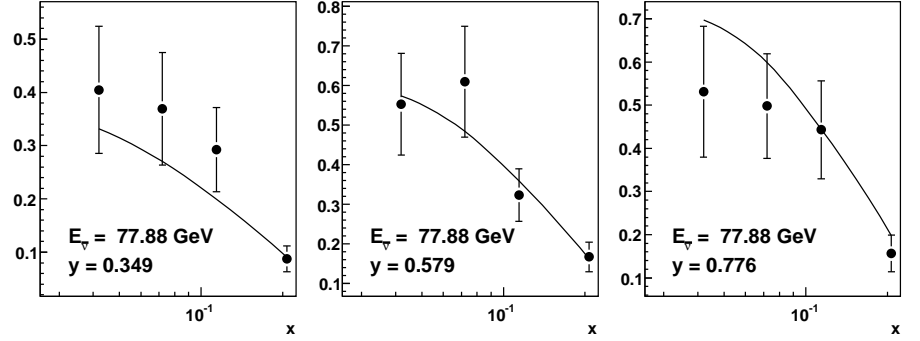
$$\text{CCFR } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\bar{\nu}_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}, \chi^2 = 32/42 \text{ pts.}$$



$$\text{NuTeV } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\nu_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}, \chi^2 = 10/21 \text{ DOF}$$

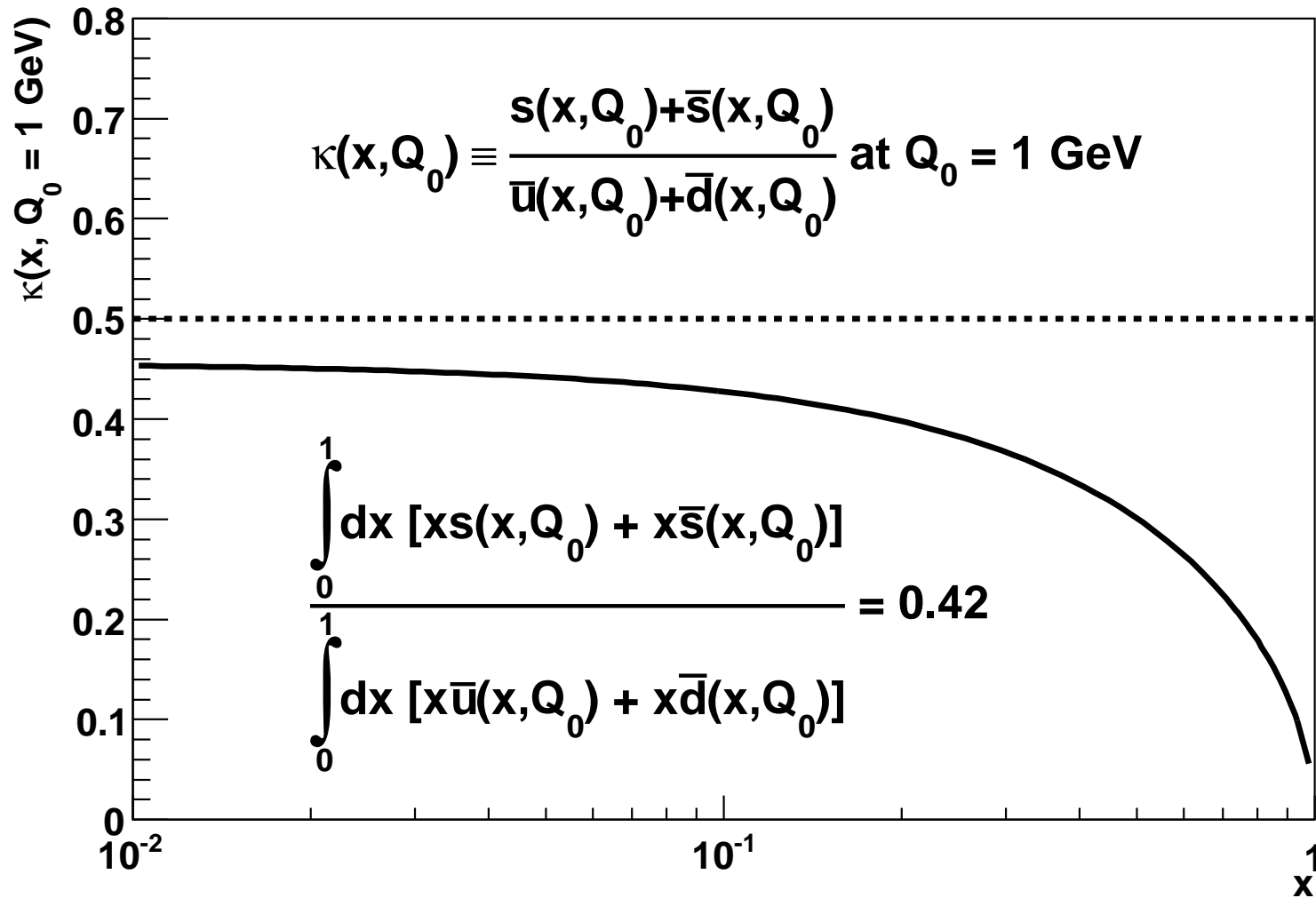


$$\text{NuTeV } \frac{100\pi}{G_F^2 M_N E_{\bar{\nu}}} \frac{d\sigma}{dx dy} (\bar{\nu}_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}, \chi^2 = 28/18 \text{ DOF}$$



Find reduced ratio of strange to non-strange sea compared to previous default $\kappa = 0.5$.

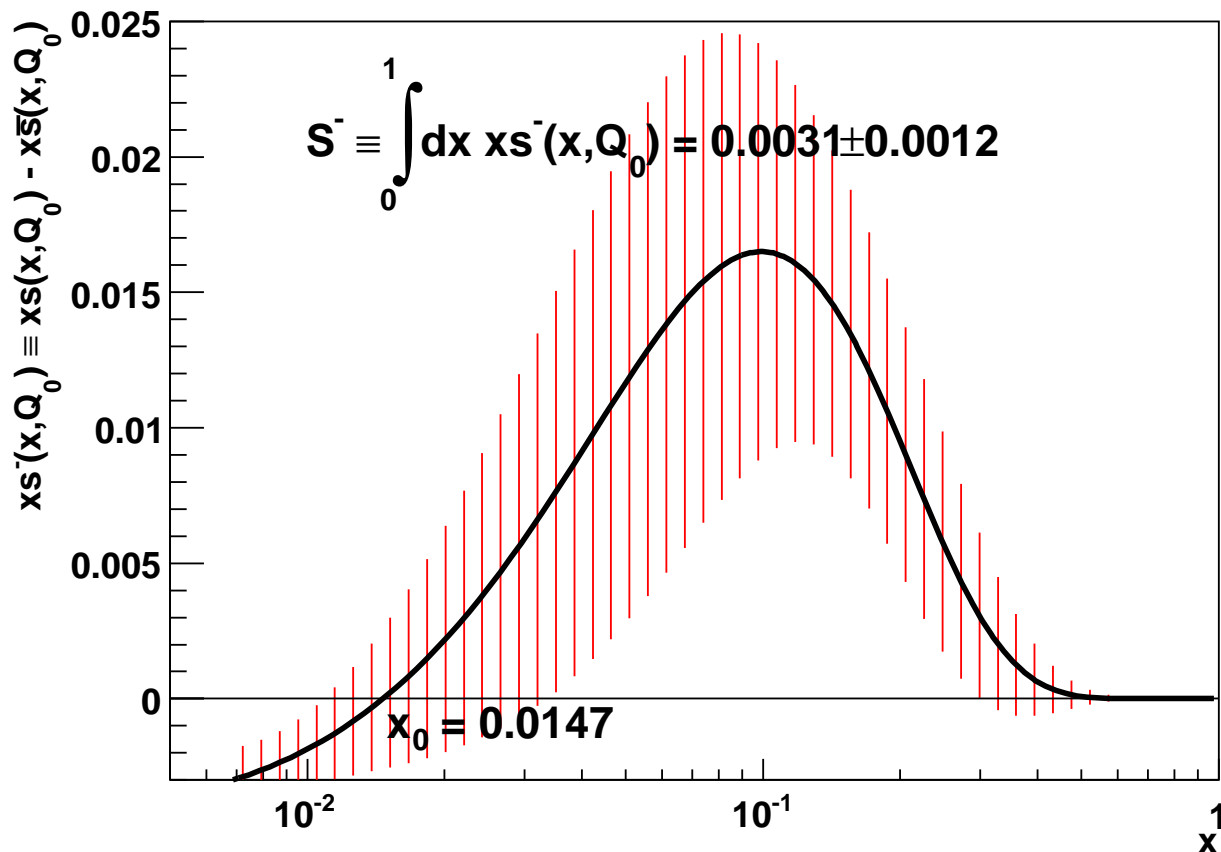
Suppression at high x , i.e. low W^2 . Effect of m_s ?



Strange sea asymmetry $xs(x, Q_0^2) - x\bar{s}(x, Q_0^2)$ constrained by dimuon data for $0.01 \geq x \geq 0.2$.

Positive, with central value 0.0031 ± 0.0012 (1σ). Nonzero value significantly greater than 1σ significance. At $Q^2 = 10\text{GeV}^2$ asymmetry of 0.0023 ± 0.0009 .

Need $S^- = 0.0068$ to bring NuTeV $\sin^2 \theta_W$ in line with world average.



Fitting to strange from **NUTEV** dimuon data affects uncertainties on partons other than strange.

Previously for us (and everyone else) strange a fixed proportion of total sea in global fit.

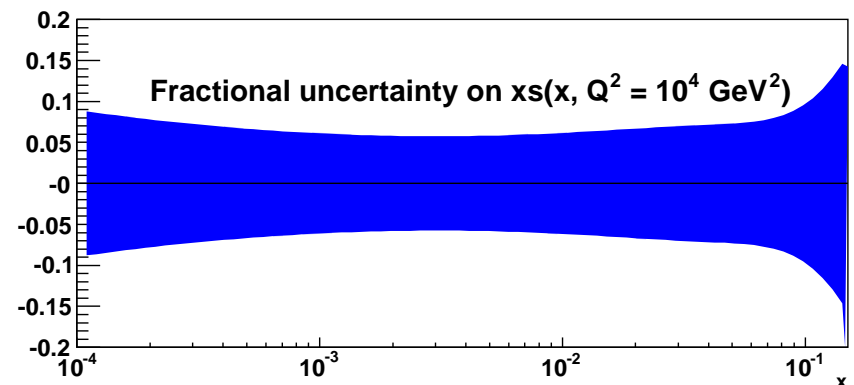
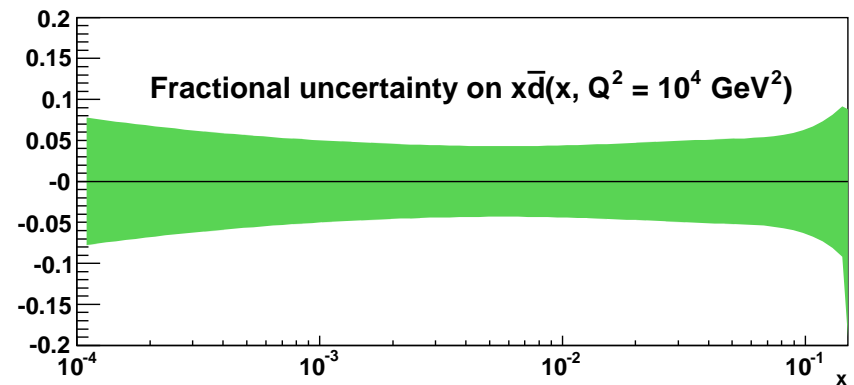
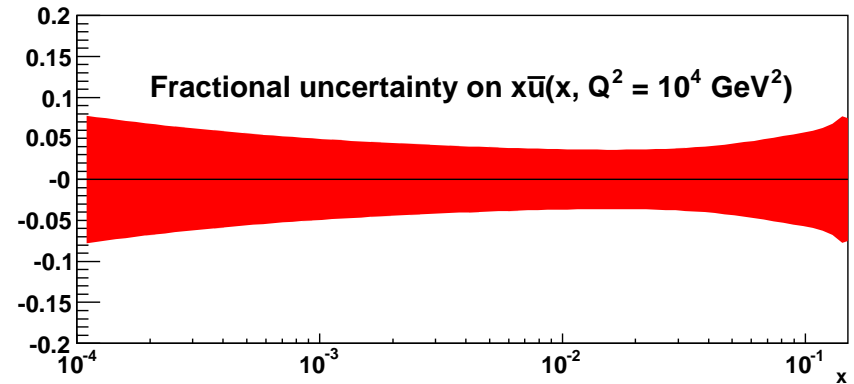
Genuine *larger* uncertainty on $s(x)$ — feeds into that on \bar{u} and \bar{d} quarks.

Low x data on $F_2(x, Q^2)$ constrains sum $4/9(u + \bar{u}) + 1/9(d + \bar{d} + s + \bar{s})$.

Changes in fraction of $s + \bar{s}$ affects size of \bar{u} and \bar{d} at input.

The size of the uncertainty on the small x anti-quarks roughly doubles — $\sim 1.5\% \rightarrow \sim 3\%$. (Remember uncertainties quoted as 90% confidence limits.)

MSTW 2007 NLO PDFs (preliminary)



W-asymmetry

The W -asymmetry at the Tevatron is defined by

$$A_W(y) = \frac{d\sigma(W^+)/dy - d\sigma(W^-)/dy}{d\sigma(W^+)/dy + d\sigma(W^-)/dy} \approx \frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)},$$

where $x_{1,2} = x_0 \exp(\pm y)$, $x_0 = \frac{M_W}{\sqrt{s}}$.

In practice it is the final state leptons that are detected, so it is really the **lepton asymmetry**

$$A(y_l) = \frac{\sigma(l^+) - \sigma(l^-)}{\sigma(l^+) + \sigma(l^-)}$$

which is measured. Defining angle of lepton in W rest frame

$$\cos^2 \theta^* = 1 - 4E_T^2/m_W^2 \quad \rightarrow \quad y_{lep} = y_W \pm 1/2 \log((1 + \cos \theta^*)/(1 - \cos \theta^*))$$

In practice at highish y_{lep}

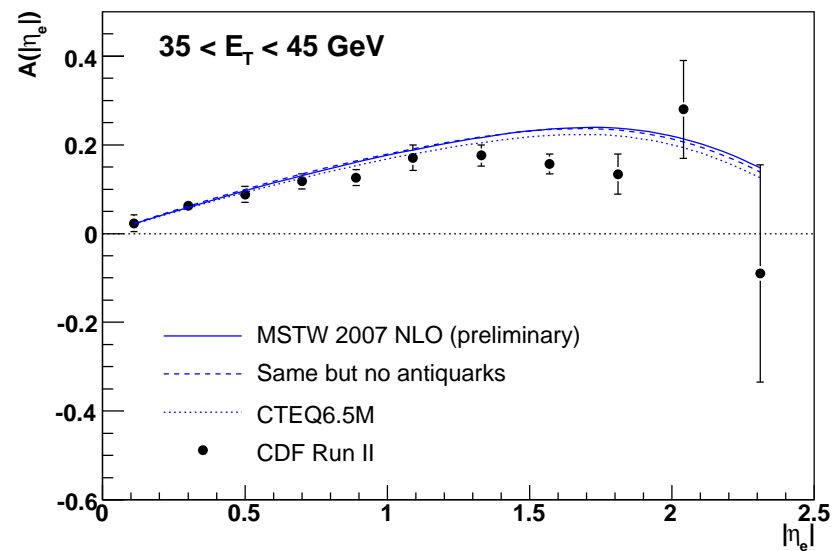
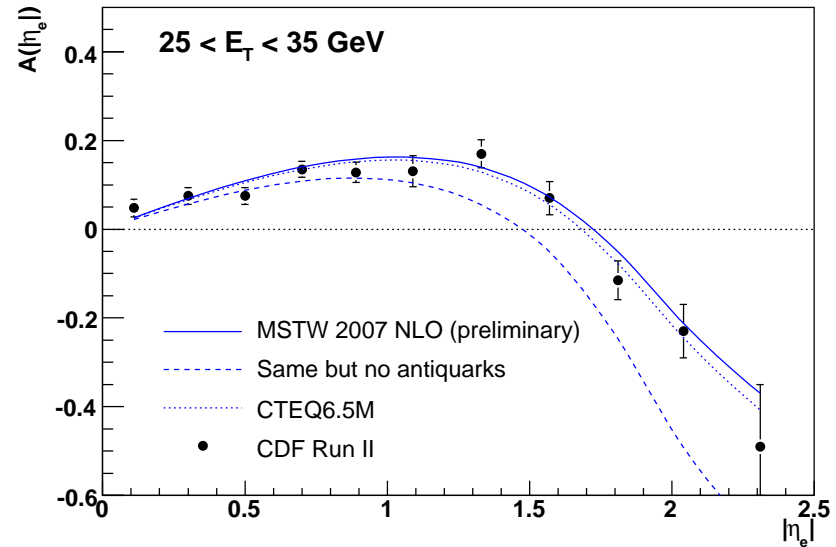
$$\sigma(l^+) - \sigma(l^-) \propto u(x_1)d(x_2)(1 - \cos \theta^*)^2 + \bar{u}(x_1)\bar{d}(x_2)(1 + \cos \theta^*)^2 - u(x_2)d(x_1)(1 + \cos \theta^*)^2$$

so fairly sensitive to anti-quarks at lower E_T .

Comparison of fits with various partons.
Some tension with neutrino structure function data.

CTEQ seems to be better shape for some reason.

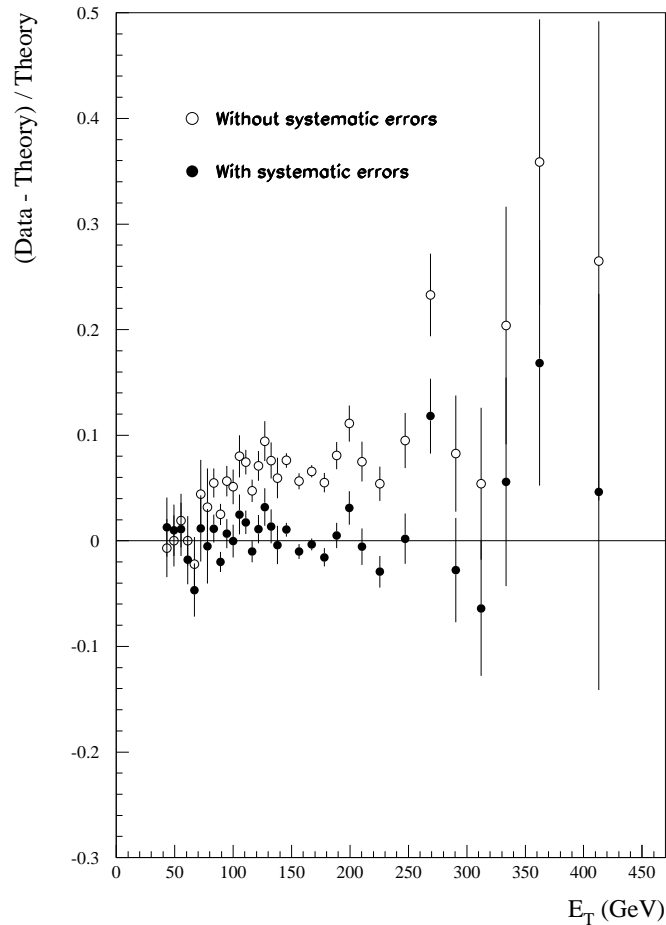
New CDF data does influence $d(x, Q^2)$ in MSTW fit.



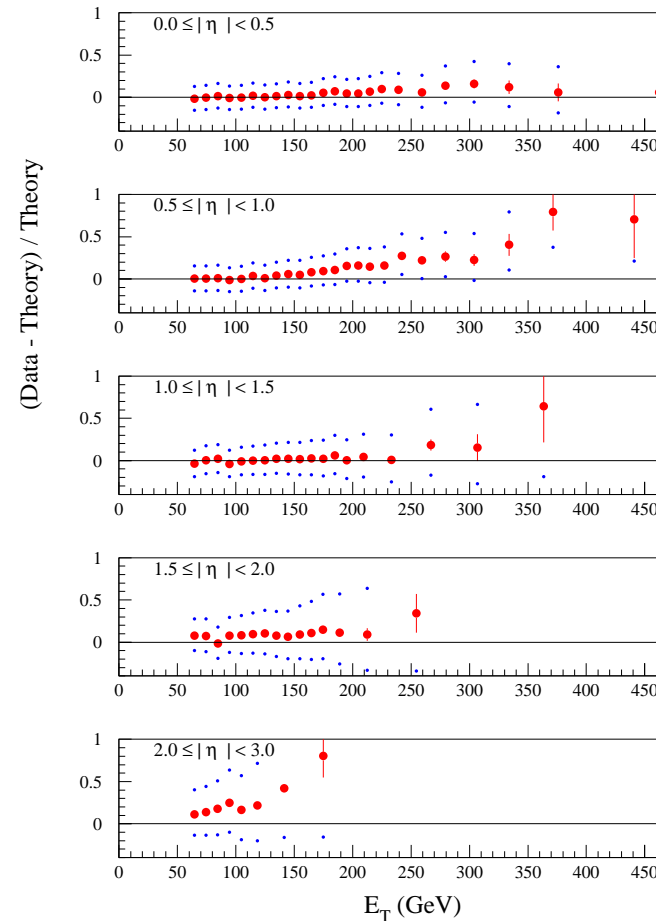
Now use **fast NLO** to implement **NLO** hard cross-section corrections to both **Tevatron** and **HERA** jets. Replaces previous “K-factors” and “pseudo-gluon data”.

No major effect on speed of fitting program. Slight influence on shape of gluon even using just **Tevatron** Run I data. (Hadronization corrections now included).

CDF Run I inclusive jet data, $\chi^2 = 50/33$ pts.



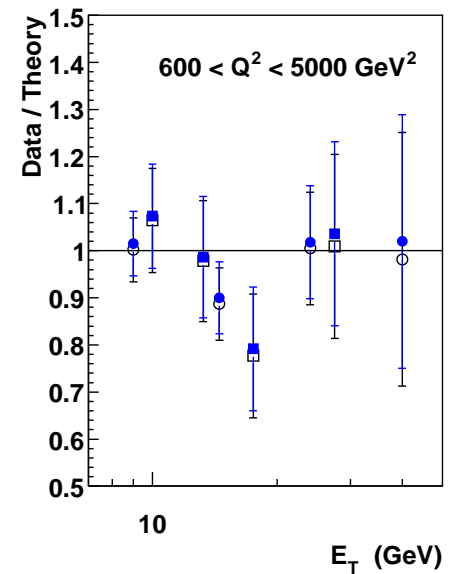
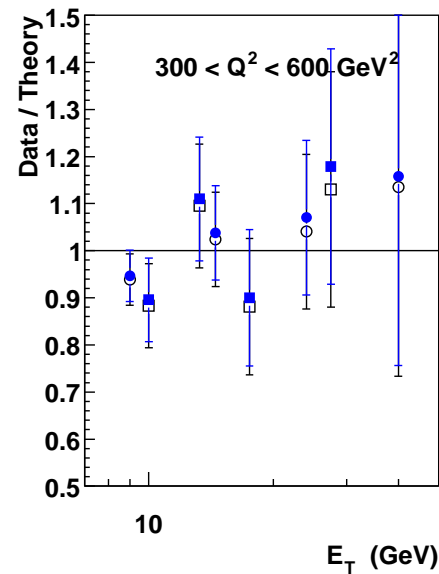
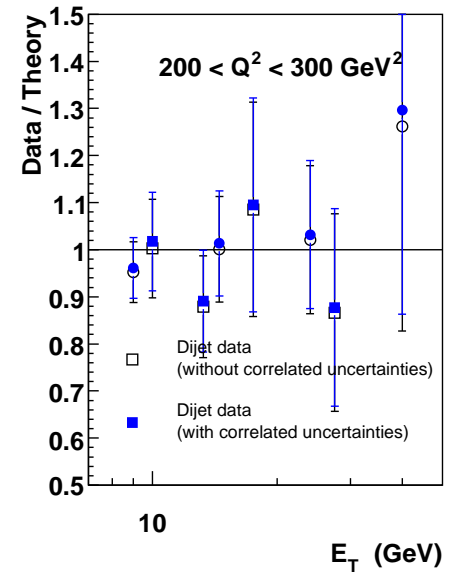
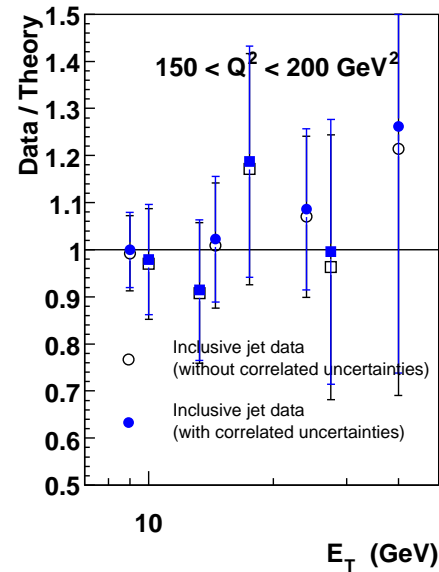
DØ Run I inclusive jet data, $\chi^2 = 58/90$ pts.



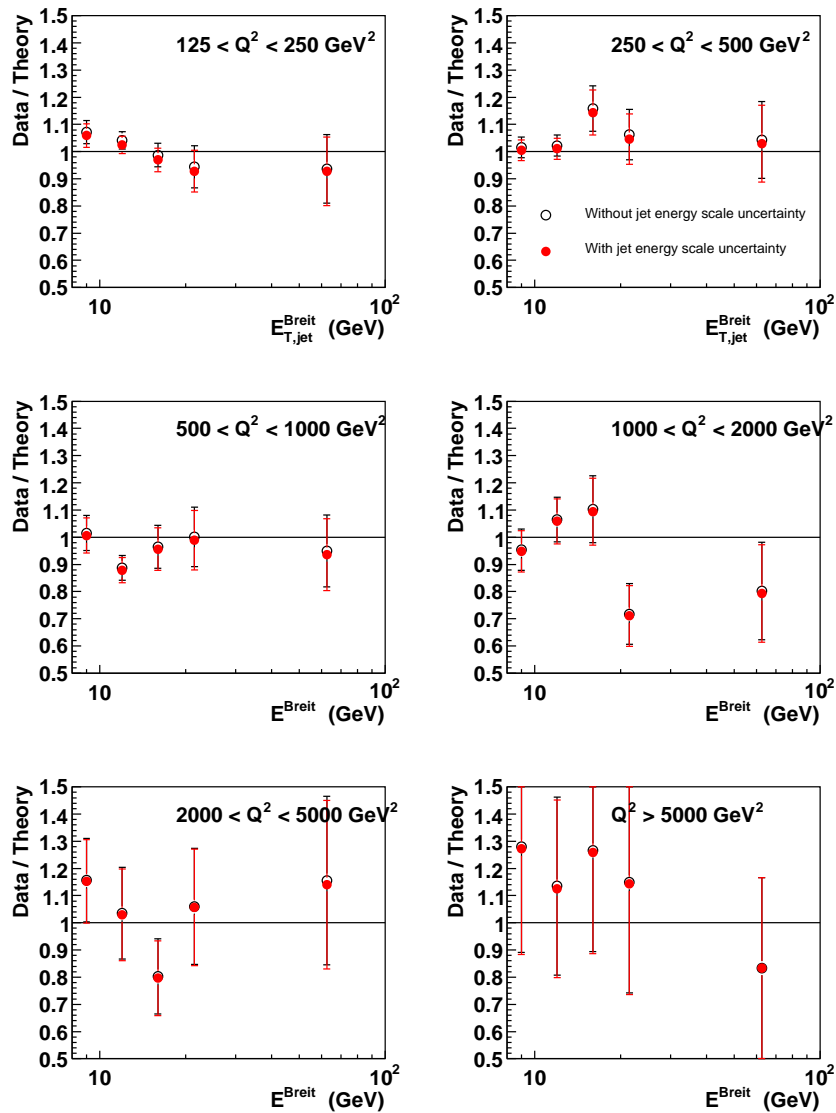
Also now include HERA inclusive and dijet DIS data using fastNLO.

Fit generally excellent. Correlated systematic uncertainties have little effect in this case.

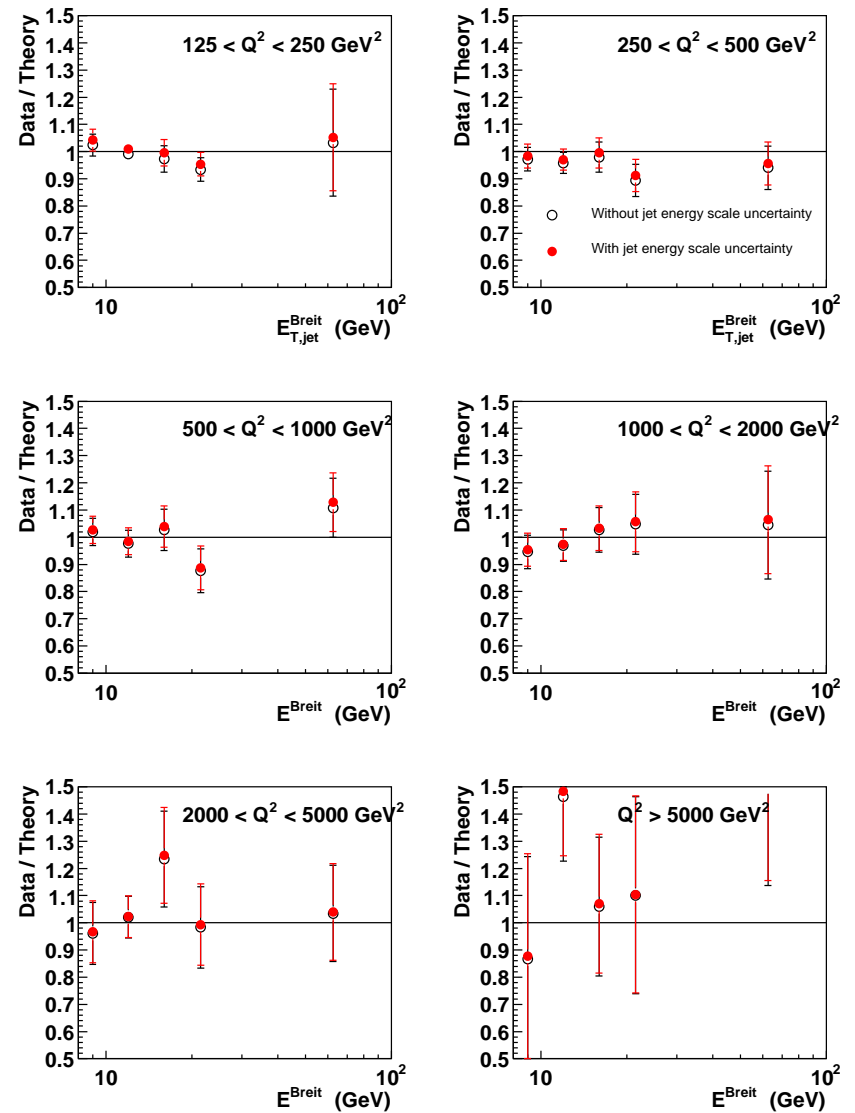
H1 95-97 incl. jet and dijet data, $\chi^2 = 13/32$ pts.



ZEUS 96-97 inclusive jet data, $\chi^2 = 30/30$ pts.

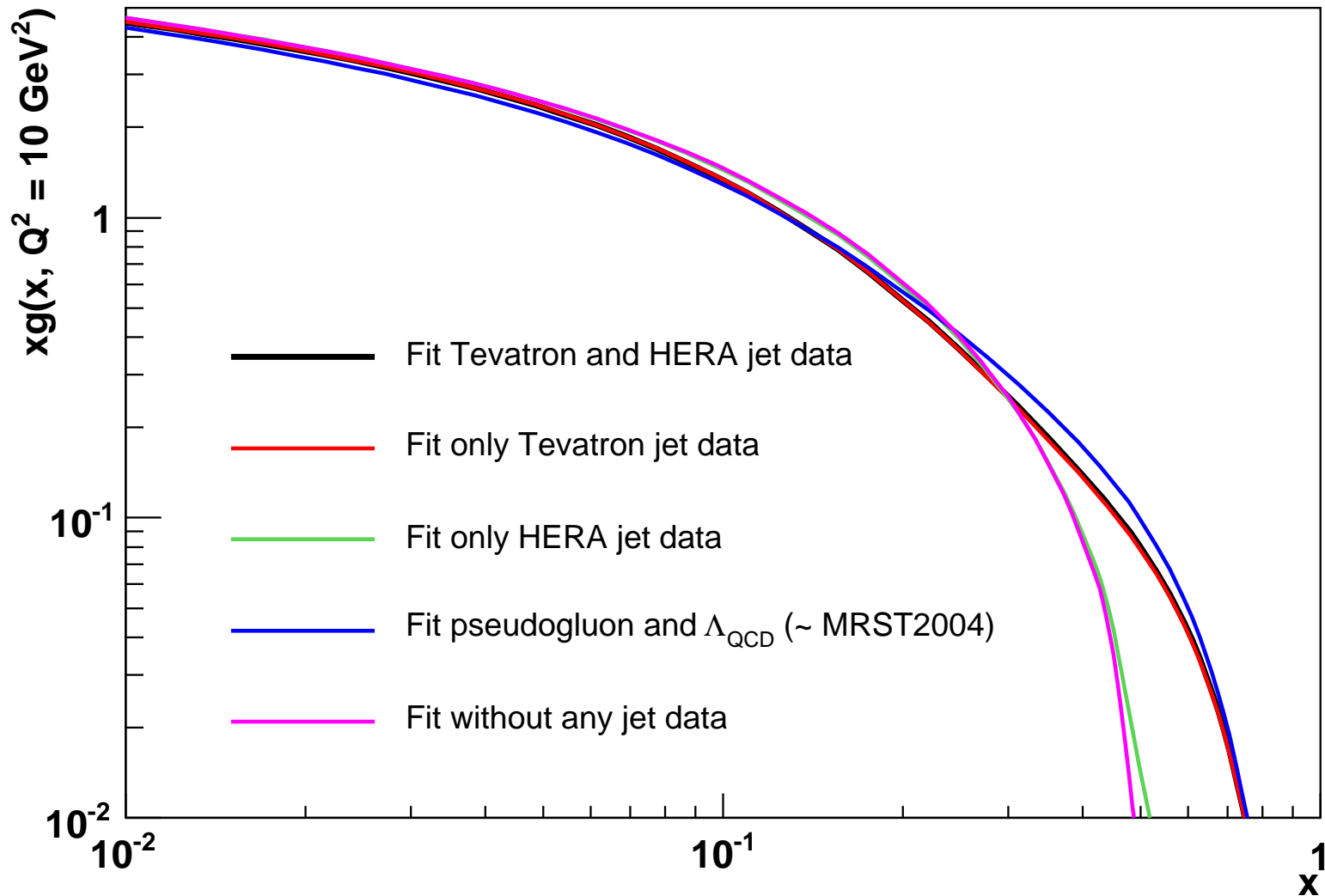


ZEUS 98-00 inclusive jet data, $\chi^2 = 19/30$ pts.



Perhaps more constraint from photoproduction data, but requires (rather uncertain) photon distributions.

Tevatron jet data are essential for constraining high x gluon – HERA jet data not sensitive to these x values and have much less pull.

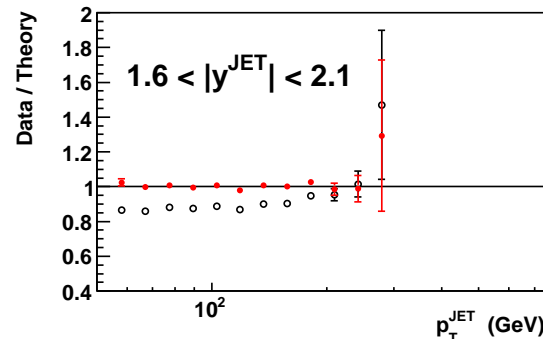
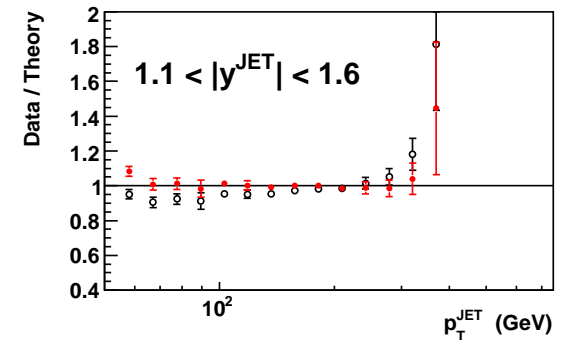
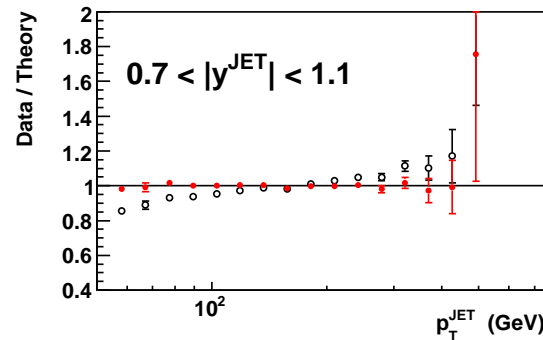
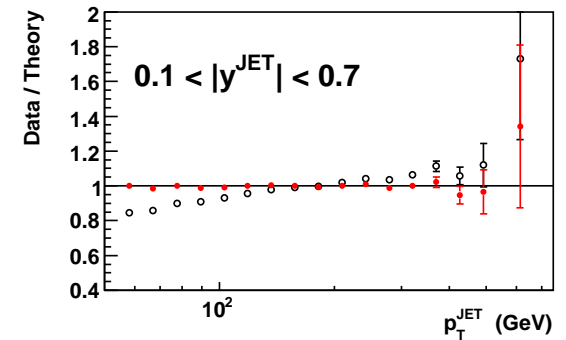
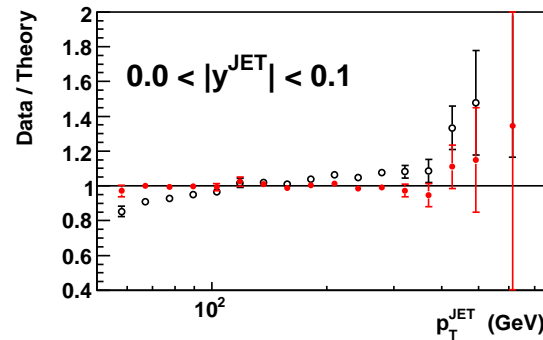


Now also include CDF Run II inclusive jet data in different rapidity bins using k_T jet algorithm (mid-point cone algorithm data seems very similar, but numbers not yet available).

Very good fit – $\chi^2 = 68/76$.

Full use of correlated systematic errors required for any sensible result.

CDF Run II inclusive jet data, $\chi^2 = 68/76$ pts.

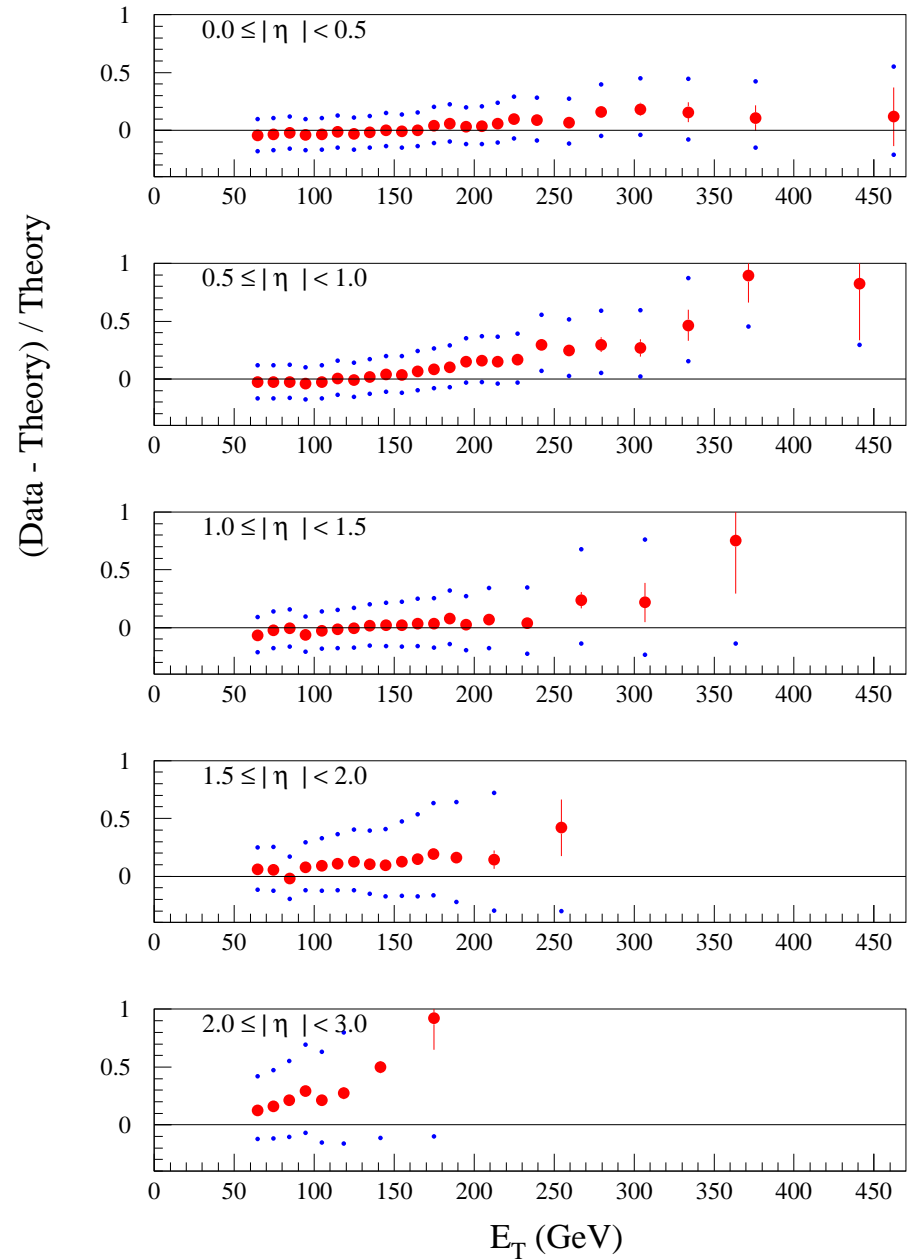


k_T algorithm with $D = 0.7$
MSTW 2007 NLO PDF fit (preliminary)

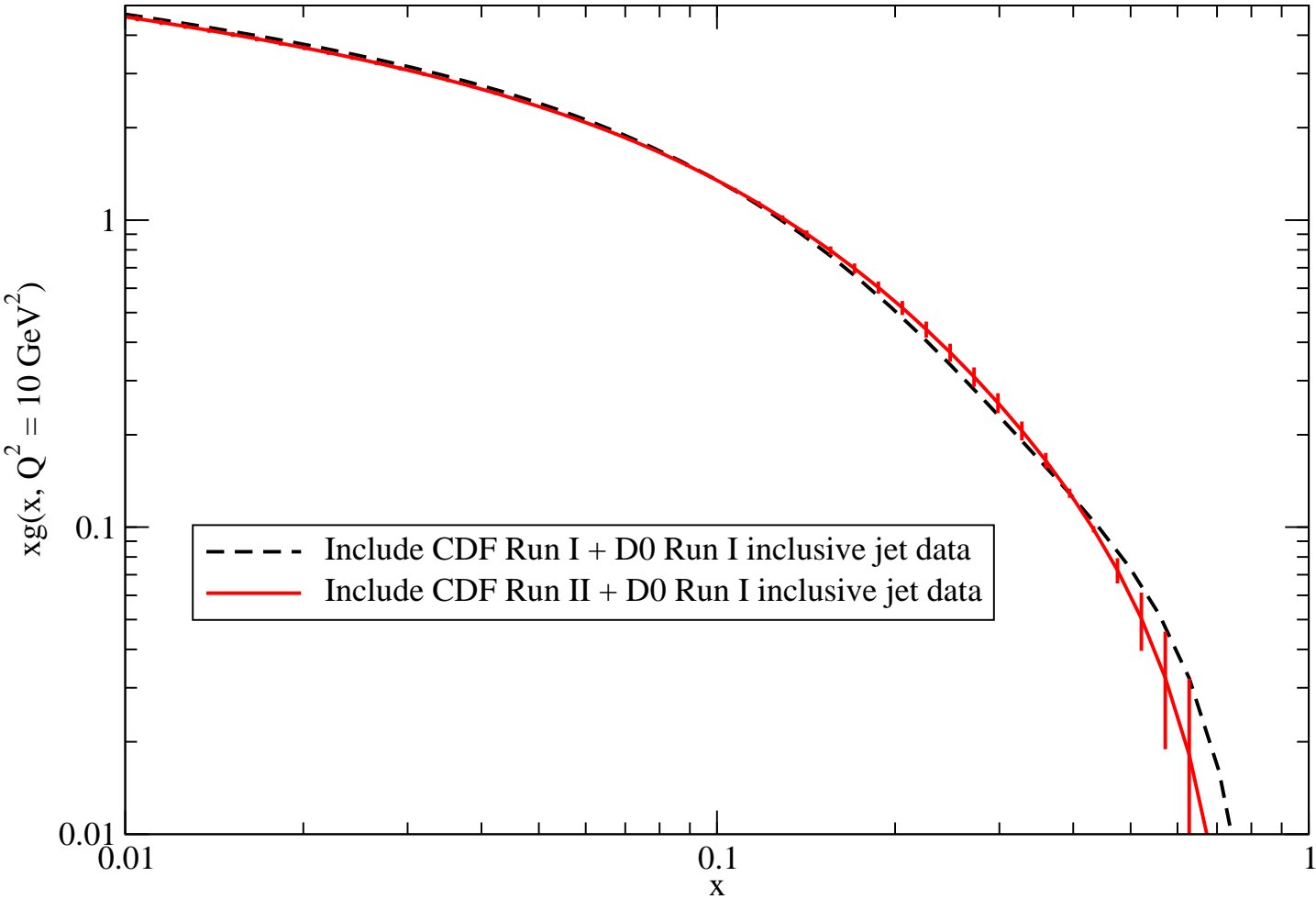
- Without systematic uncertainties
- With systematic uncertainties

Slight deterioration in fit to **D0** run I data in different rapidity bins.

$D\bar{D}$ Run I inclusive jet data, $\chi^2 = 71/90$ pts.



CDF run II data prefers a slightly smaller very high x gluon distribution compared to run I data. Just within uncertainties at our 1σ level.



Conclusions

NNLO essentially complete. **NNLO Drell-Yan** corrections quite large. Fit data well but **NLO** better. Provisional update of partons. Main difference due to heavy flavour prescription. This is important. In overall fit $\alpha_S(M_Z^2)$ creeping upwards.

Inclusion of new data. Neutrino structure function data inconsistent at high x . Cut at $x = 0.5$. Important effect at lower x . Dimuon data fitted directly. Important constraint on strange, and weak evidence for strangeness momentum asymmetry. New uncertainties on $s + \bar{s}$ feed into other partons.

New **CDF W -asymmetry** data more constraining for d_V and to some extent \bar{d} .

HERA jets, and **Tevatron** high- E_T jets, now fit using **fastNLO**. Works well and fit good. New run II **CDF** jet data included in fit. Small, but significant change.

Still require some theoretical fine-tuning and checking, mainly at **NNLO**. Some new data still to be included at **NNLO**.

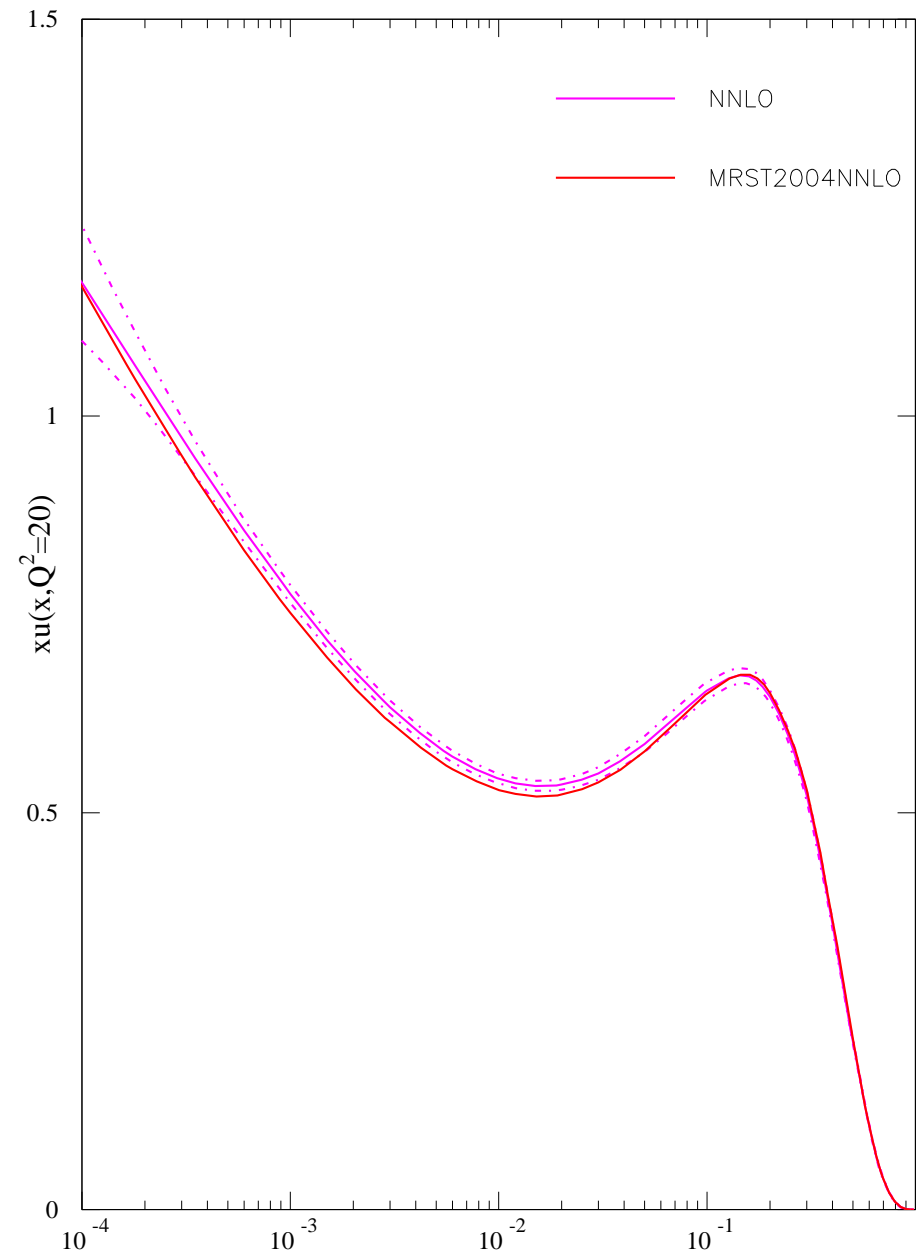
Will have full updated **NLO** and **NNLO** partons for **LHC** complete with uncertainties – experimental and theoretical.

Not much change in light quarks due to these theoretical updates.

Minor change – bit bigger than **MRST2004** at small x .

Slightly lower $s(x, Q^2) \rightarrow$ more $u(x, Q^2)$.

Also slightly higher $\alpha_S(M_Z^2)$. Negative **NNLO** correction bigger \rightarrow more $u(x, Q^2)$.



Previously used correction applied to theoretical prediction.

$$x < 0.0903 \quad R = 1.238 + 0.203 \log_{10} x$$

$$x > 0.2340 \quad R = 0.783 - 0.385 \log_{10} x$$

$$0.234 > x > 0.0903 \quad R = 1.026$$

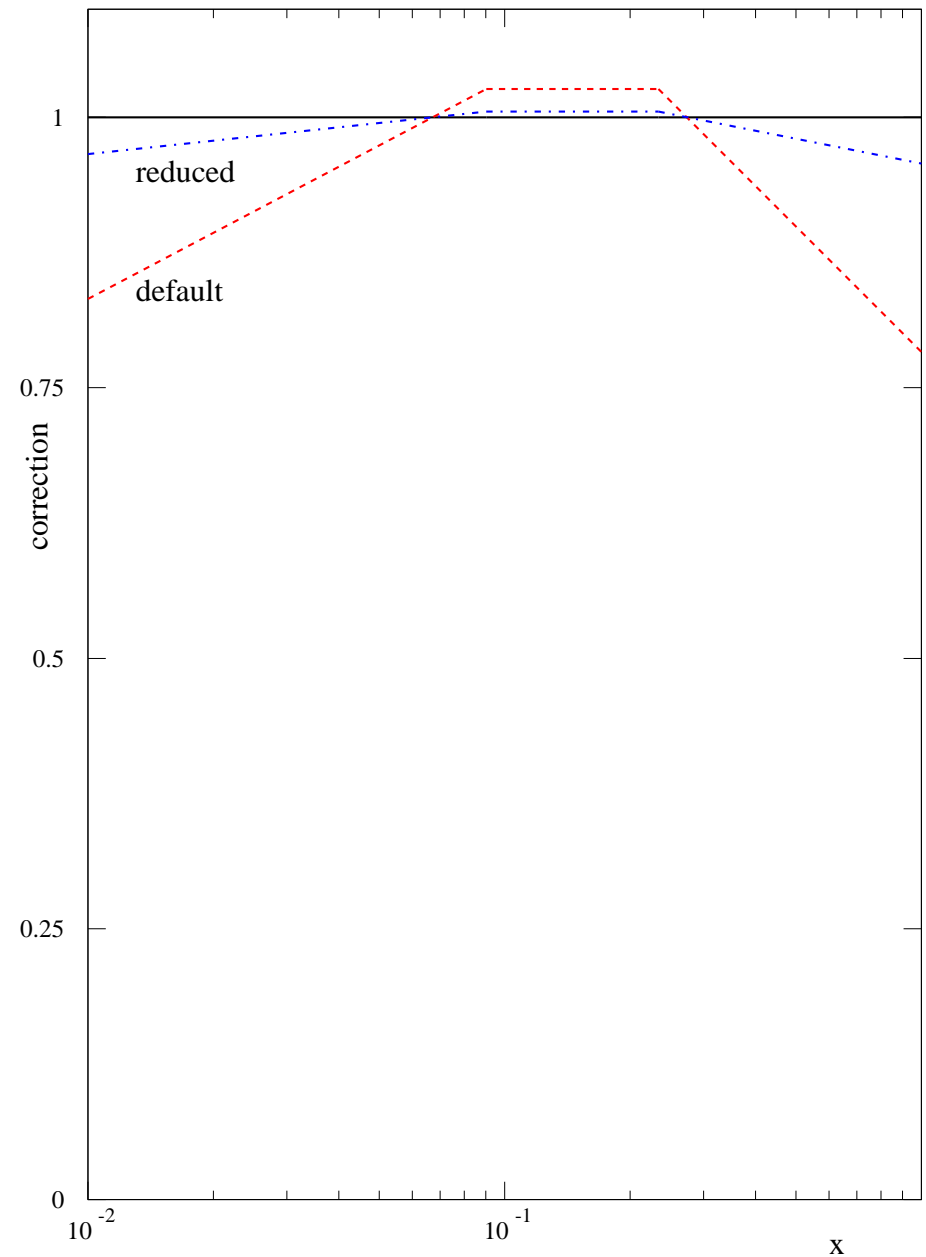
Far too large for new NuTeV data. High- x completely determined by valence quarks for both $F_2^{\nu, \bar{\nu}}(x, Q^2)$ and $F_3^{\nu, \bar{\nu}}(x, Q^2)$.

These well known from fixed target $F_2^p(x, Q^2)$ and $F^d(x, Q^2)$.

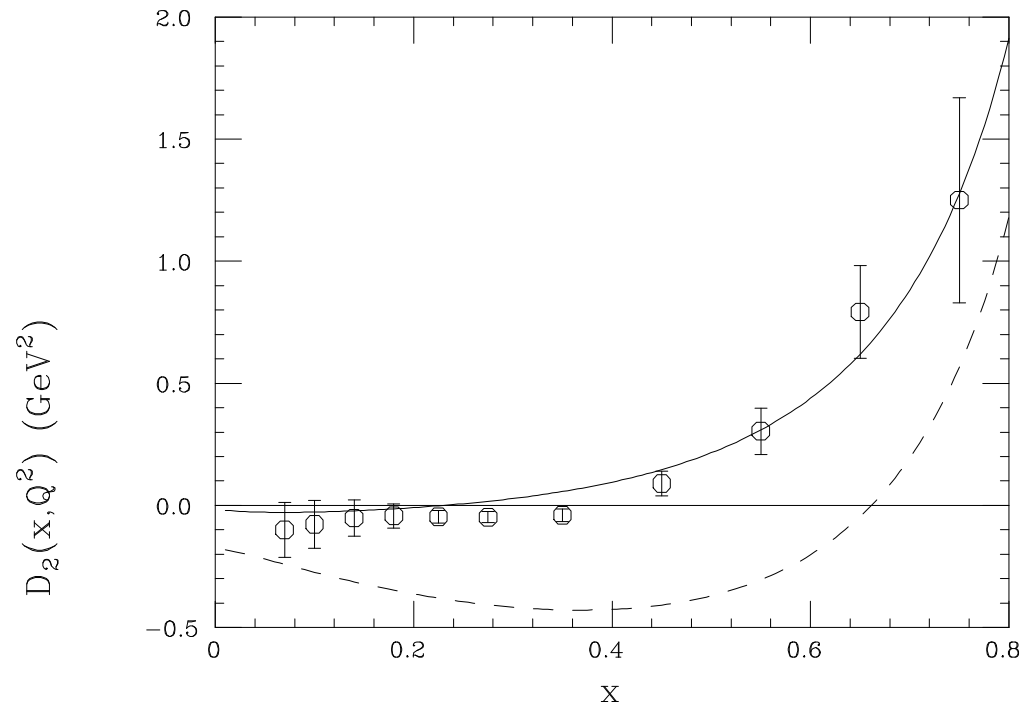
Try form $R^{eff} = 1 + A * (R - 1)$.

Best fit $A = 0.2$.

Nuclear corrections for NuTeV data



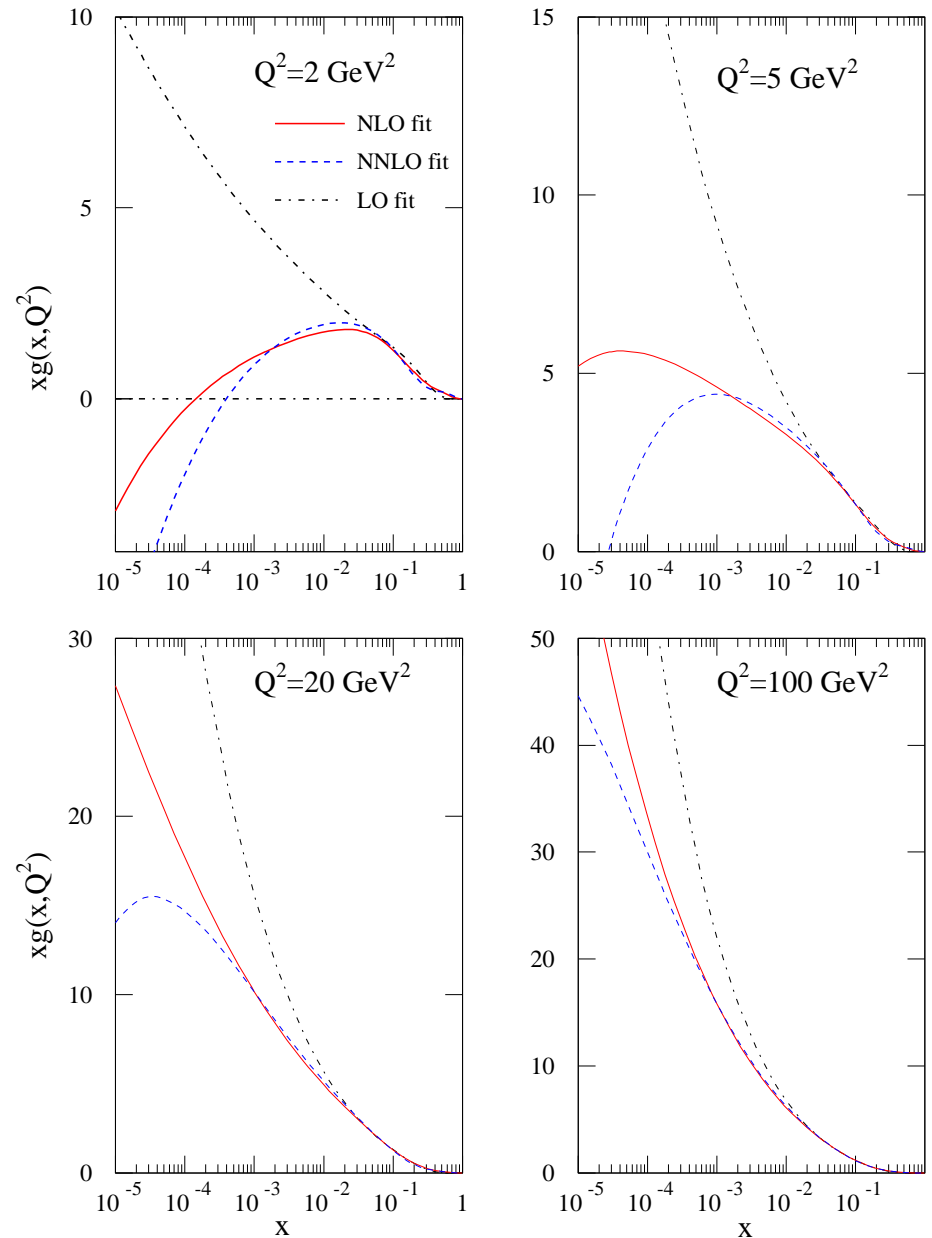
Renormalon prediction for $1/Q^2$ corrections for $F_2(x, Q^2)$ (solid line) and $xF_3(x, Q^2)$ (dashed line)
Dasgupta and Webber.



The gluon extracted from the global fit at **LO**, **NLO** and **NNLO**.

Additional and positive small- x contributions in P_{qg} at each order lead to smaller small- x gluon at each order.

Note - this conclusion relied on correct application of flavour thresholds in a General Variable Flavour Number Scheme at **NLO** not present in earlier approximate **NNLO MRST** fits. Correct treatment of flavour particularly important at **NNLO** because discontinuities in unphysical quantities appear at this order.

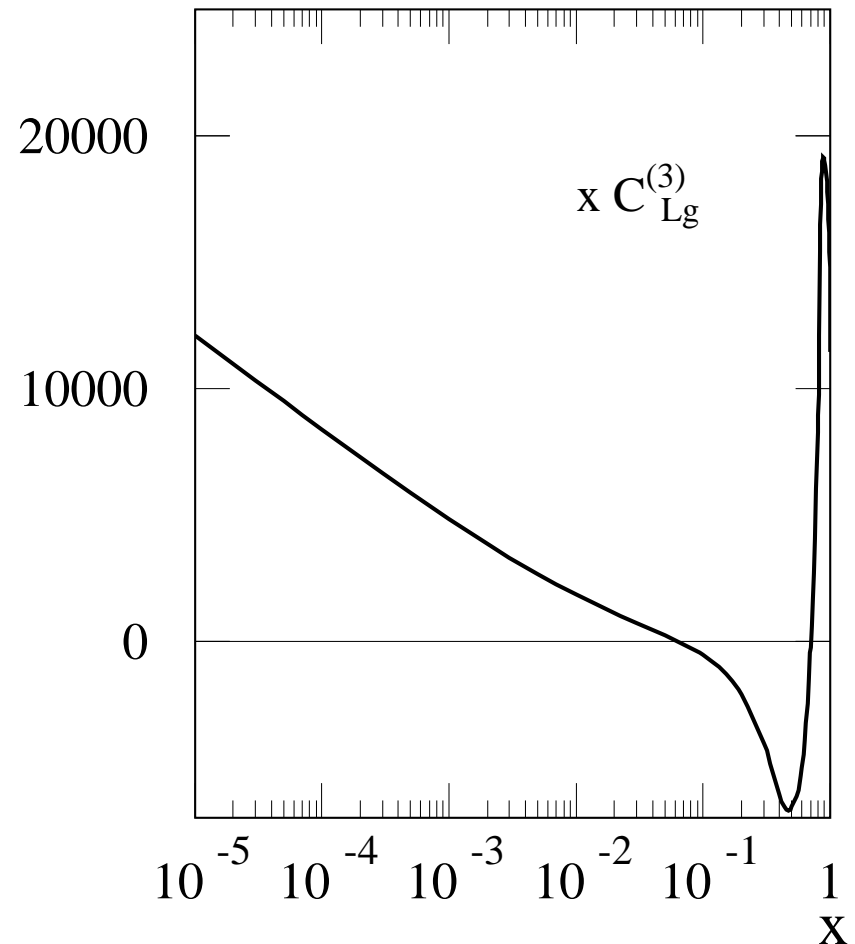


The NNLO $\mathcal{O}(\alpha_s^3)$ longitudinal coefficient function $C_{Lg}^3(x)$ given by

$$C_{Lg}^3(x) = n_f \left(\frac{\alpha_s}{4\pi} \right)^3 \left(\frac{409.5 \ln(1/x)}{x} - \frac{2044.7}{x} - \dots \right)$$

Clearly a significant positive contribution at small x .

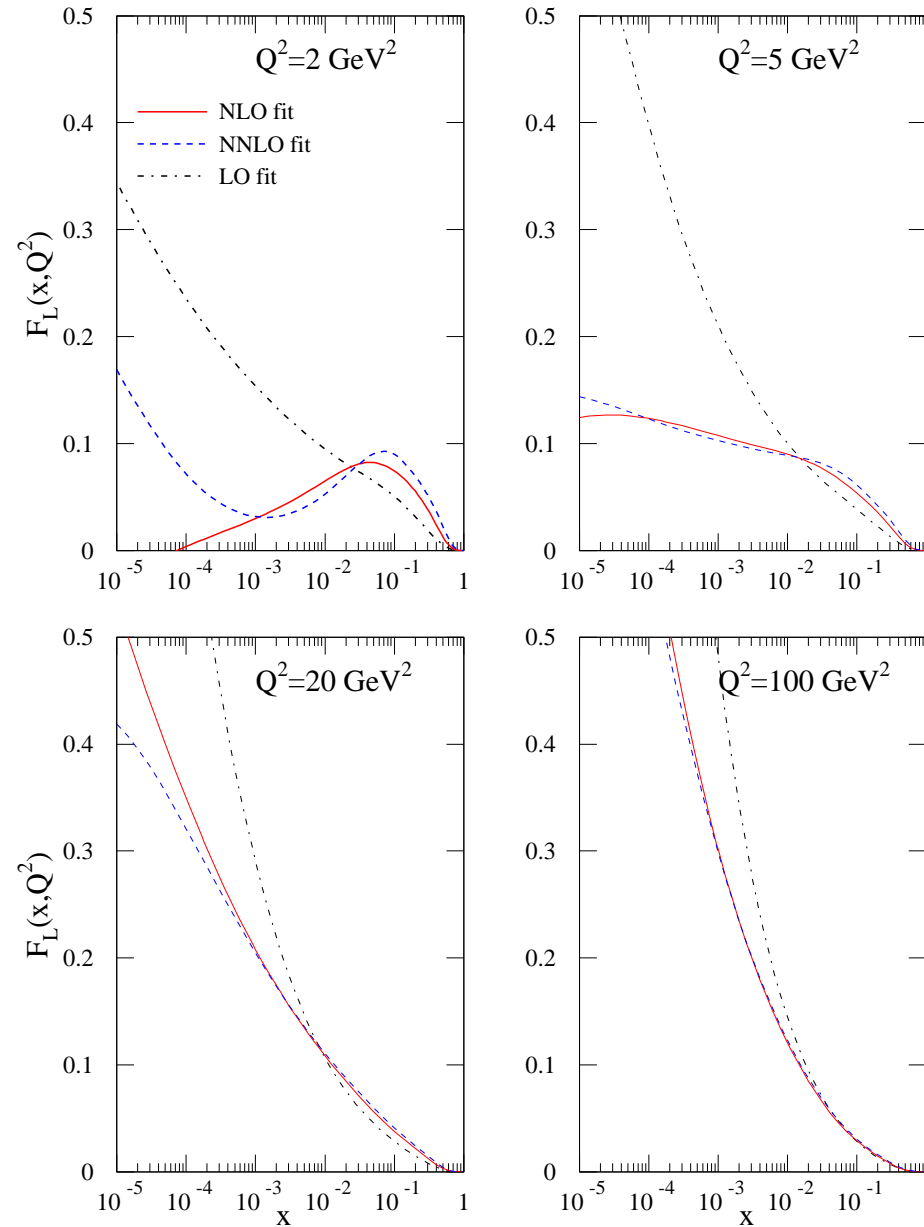
Counters decrease in small- x gluon.



$F_L(x, Q^2)$ predicted from the global fit at LO, NLO and NNLO.

NNLO coefficient function more than compensates decrease in NNLO gluon.

F_L LO, NLO and NNLO



Comparisons

Compare with only other NNLO partons on market – Alekhin2002.

Nothing from CTEQ?

Much larger $\alpha_S(M_Z^2)$ in this fit than that of Alekhin ($\alpha_S(M_Z^2) = 0.119$ compared to 0.114).

Not much difference in high- x valence quarks, except than explained by difference in $\alpha_S(M_Z^2)$. Very well-constrained.

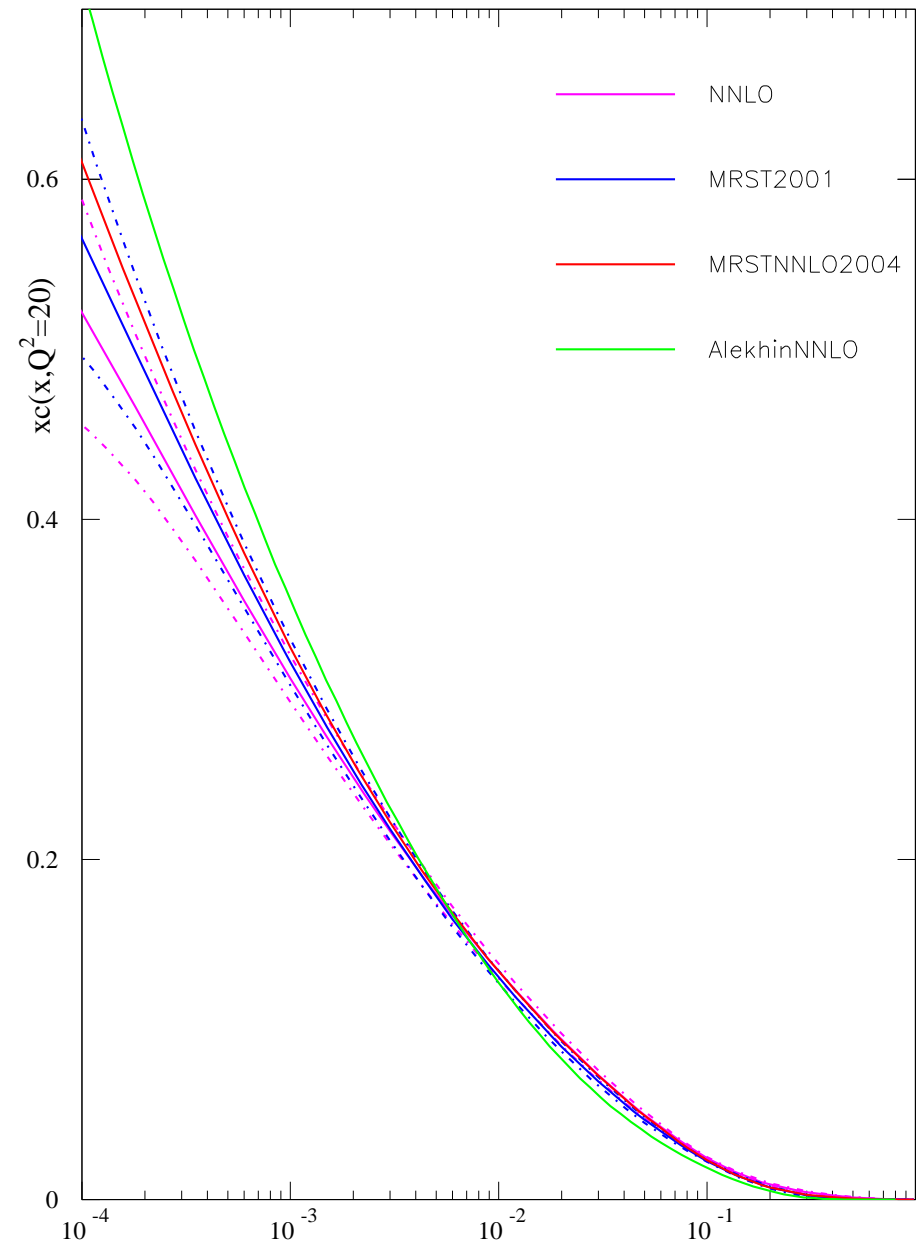
Differences in low- x sea quarks. Swamped by differences in flavour treatments – $\bar{u} - \bar{d}$ and $s(x, Q^2)$.

Main difference in gluon distribution.

Difference in gluon feeds through to charm.

Alekhin2002 much bigger at small x .

Starts from zero as with MRST2004NNLO.

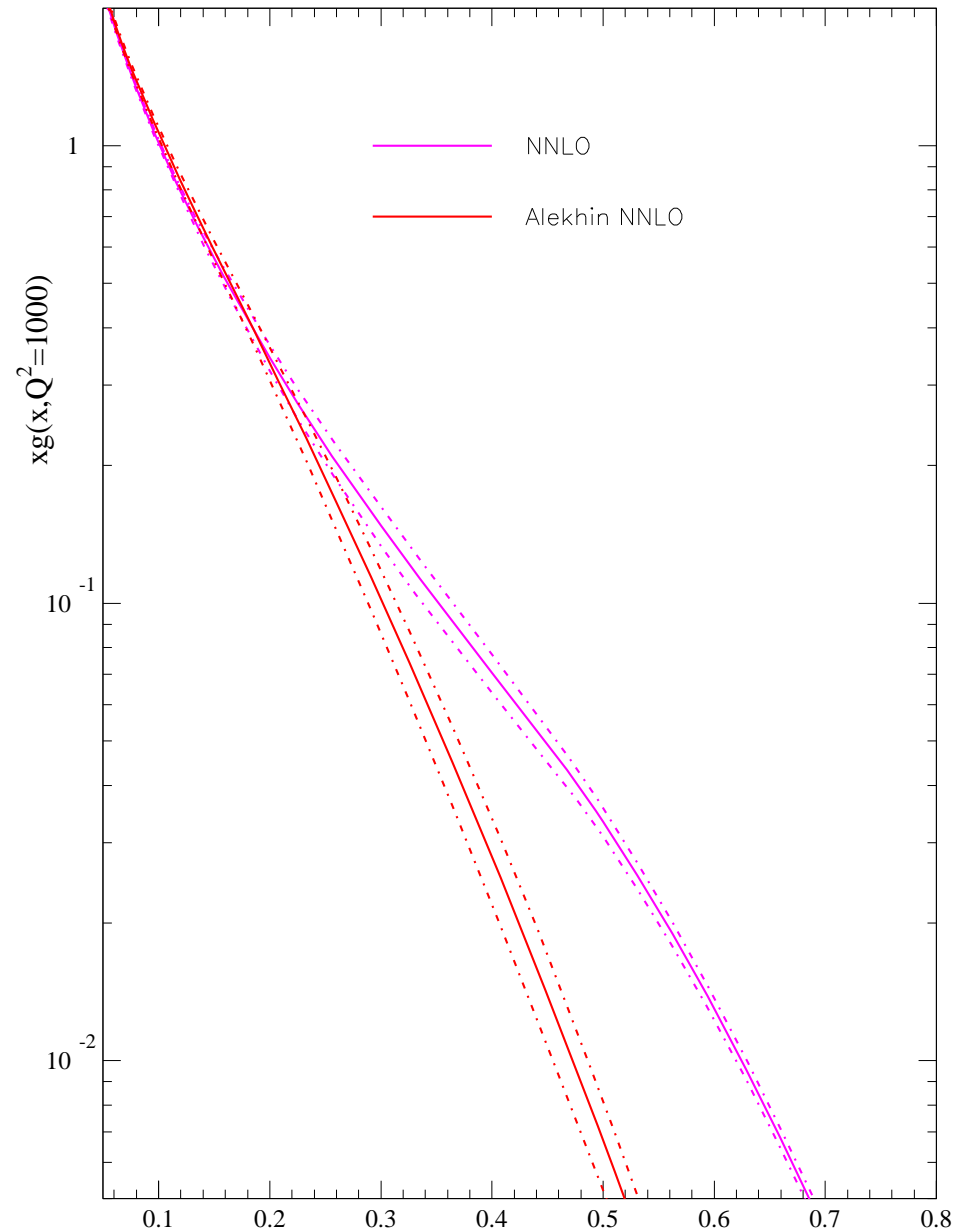


Big difference at high x and Q^2 .

Determined by Tevatron jet data for MRST. Fit now excellent.

Divergences at $x = 0.25$ corresponds to $E_T \sim 225\text{GeV}$.

In \overline{MS} scheme gluon more important for jets at high x at NNLO because high- x quarks smaller.



x