Charged current deep-inelastic scattering at three loops

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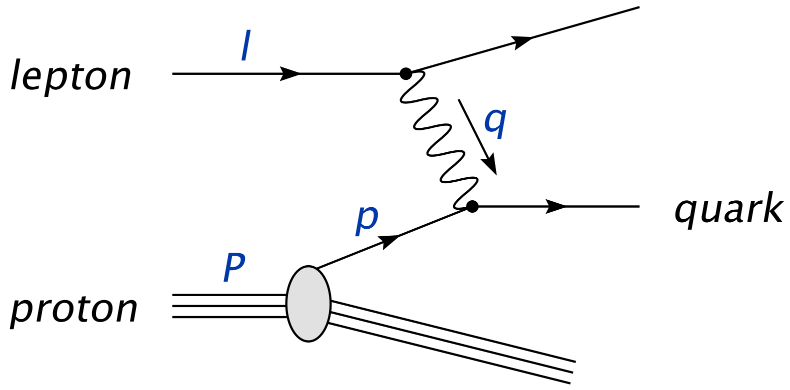
Plan

- Charged-current deep-inelastic scattering
- Review of experimental results
- Perturbative QCD corrections at three loops in QCD
- Work done in collaboration with M. Rogal and A. Vogt
  - *Charged current deep-inelastic scattering at three loops* S.M., M. Rogal
  - *Phenomenology of charged current DIS* S.M., M. Rogal, A. Vogt

Introduction

- Deep-inelastic lepton-hadron scattering ($e^{\pm}p$, $e^{\pm}n$, νp, $\bar{\nu}p$, ...)



- **Kinematic variables**
  - momentum transfer $Q^2 = -q^2$
  - Bjorken variable $x = Q^2/(2p \cdot q)$
- **Gauge boson exchange**
  - neutral current: γ, Z
  - charged current: $W^{\pm}$
- Cross section $\sigma \simeq L^{\mu\nu} W_{\mu\nu}$
  - leptonic tensor $L_{\mu\nu}$ for neutral/charged current
  - hadronic tensor parametrized through structure functions

$$W_{\mu\nu} = e_{\mu\nu} \frac{1}{2x} F_L(x, Q^2) + d_{\mu\nu} \frac{1}{2x} F_2(x, Q^2) + \mathrm{i}\epsilon_{\mu\nu\alpha\beta} \frac{p^\alpha q^\beta}{p \cdot q} F_3(x, Q^2)$$

NC (neutral-current)

- NC $e^{\pm}p$ cross section

$$\frac{d^2\sigma^{NC}(e^{\pm}p)}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}[(1+(1-y)^2)F_2 - y^2 F_L \mp (1-(1-y)^2)xF_3]$$

  - valence/sea parton distributions $q \pm \bar{q}$

$F_2 = \sum_q A_q\, x\, (q+\bar{q})$ with $A_q = e_q^2 + \mathcal{O}(\frac{Q^2}{Q^2+M_Z^2})$

$F_3 = \sum_q B_q\, x\, (q-\bar{q})$ with $B_q = \mathcal{O}(\frac{Q^2}{Q^2+M_Z^2})$

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CC (charged-current)

- CC $e^{\pm}p$ cross section $\longrightarrow$ flavour separation:

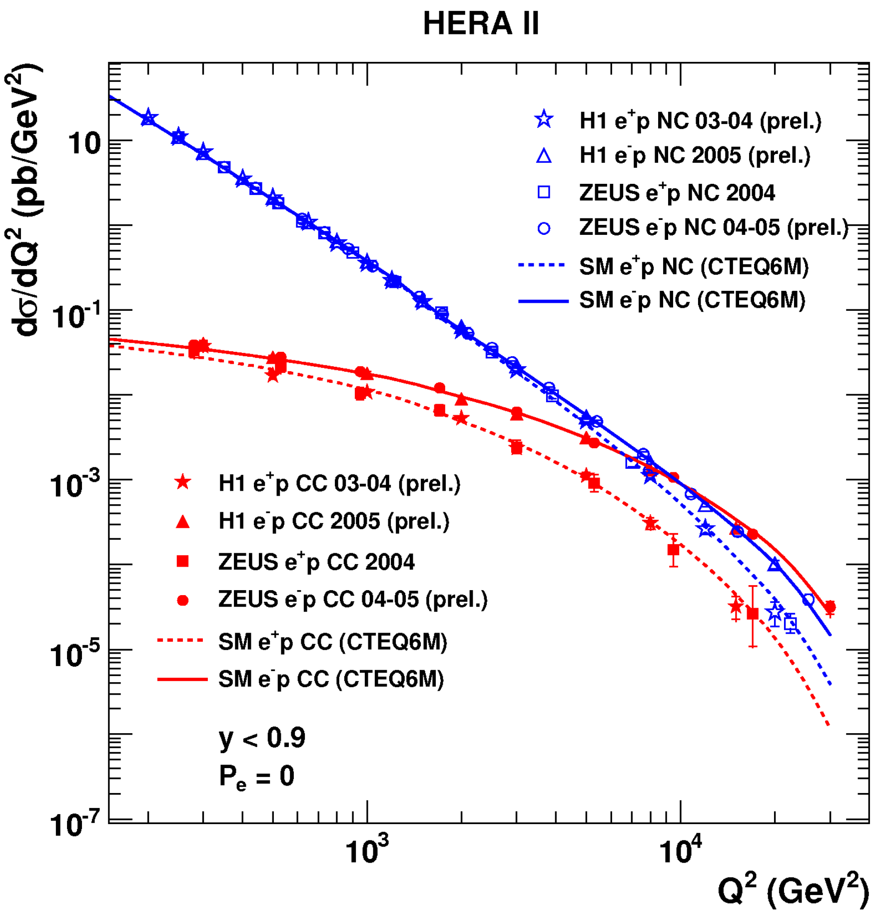
$$\frac{d^2\sigma^{CC}(e^{+}p)}{dxdQ^2} = \frac{G_F^2}{2\pi}(\frac{M_W^2}{M_W^2+Q^2})^2\left[\bar{u}+\bar{c}+(1-y)^2(d+s)\right]$$

$$\frac{d^2\sigma^{CC}(e^{-}p)}{dxdQ^2} = \frac{G_F^2}{2\pi}(\frac{M_W^2}{M_W^2+Q^2})^2\left[u+c+(1-y)^2(\bar{d}+\bar{s})\right]$$

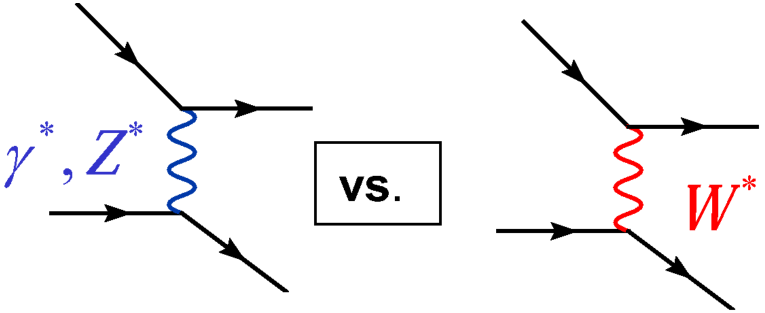
DIS experiments

EW unification at HERA

- NC vs. CC deep-inelastic scattering



- NC and CC DIS cross sections comparable in size for Q^2 at electroweak scale



Electron polarization at HERA

- CC DIS with polarized electrons
  - cross section is linearly proportional to polarization P_e

$$\sigma_{CC}^{e^{\pm}p}(P_e) = (1 \pm P_e) \cdot \sigma_{CC}^{e^{\pm}p}(P_e = 0) \qquad P_e = \frac{N_R - N_L}{N_R + N_L}$$

- Standard model prediction
  - vanishing cross section for $P_e = +1(-1)$ in $e^{-(+)}$ scattering

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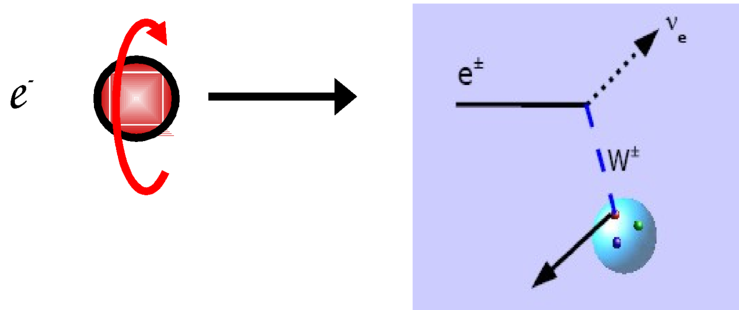
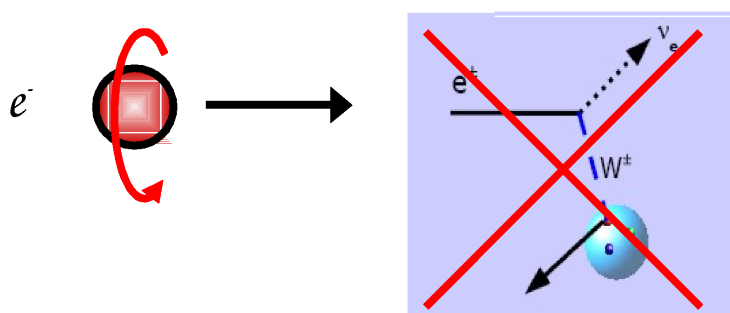
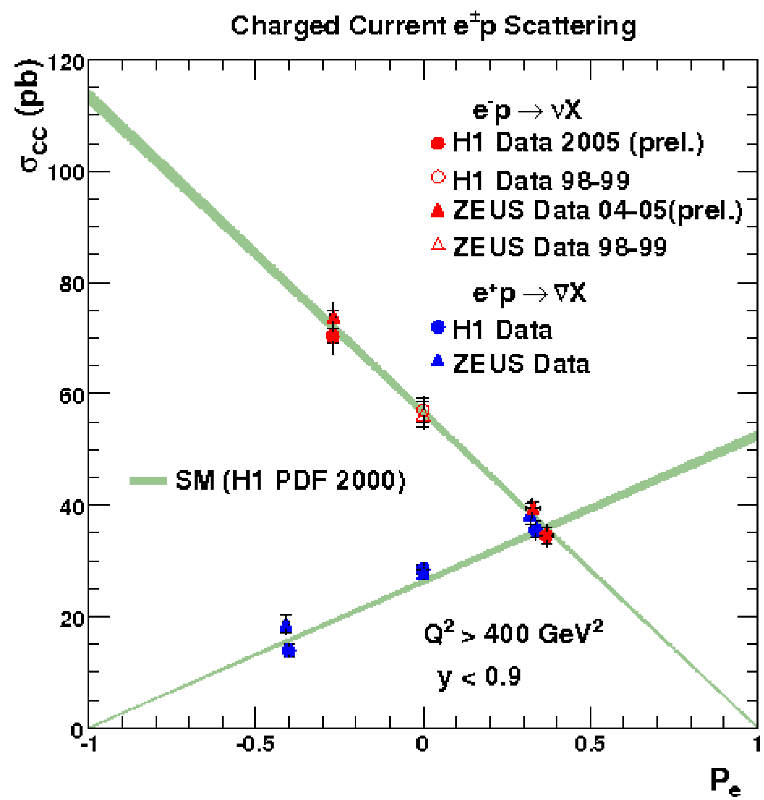
lefthanded electrons interact (CC)



righthanded electrons do not!





Paschos-Wolfenstein relation

- Exact relation for massless quarks and isospin zero target
  Paschos,Wolfenstein'73; Llewelin Smith'83

$$R^- = \frac{\sigma(\nu_\mu N \to \nu_\mu X) - \sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \to \mu^- X) - \sigma(\bar{\nu}_\mu N \to \mu^+ X)} = \frac{1}{2} - \sin^2\theta_W$$

  - measurement of $\sin^2\theta_W$ NuTeV '01 with large deviations from Standard model expectations
- QCD corrections to Paschos-Wolfenstein relation
  Davidson, Forte, Gambino, Rius, Strumia '01; Dobrescu, Ellis '03; S.M. McFarland '03
  - second moments of valence PDFs $q^- = \int dx\, x(q - \bar{q})$
  - expansion in isoscalar combination $u^- + d^-$

$$R^- = \frac{1}{2} - \sin^2\theta_W + \left[1 - \frac{7}{3}\sin^2\theta_W + \frac{8\alpha_s}{9\pi}\left(\frac{1}{2} - \sin^2\theta_W\right)\right] \times \left(\frac{u^- - d^-}{u^- + d^-} - \frac{s^-}{u^- + d^-} + \frac{c^-}{u^- + d^-}\right)$$

- main uncertainties in s^-
  Martin, Roberts, Stirling, Thorne '04; Lai, Nadolsky, Pumplin, Stump, Tung, Yuan '07
- QCD corrections to coefficient functions small S.M. McFarland '03

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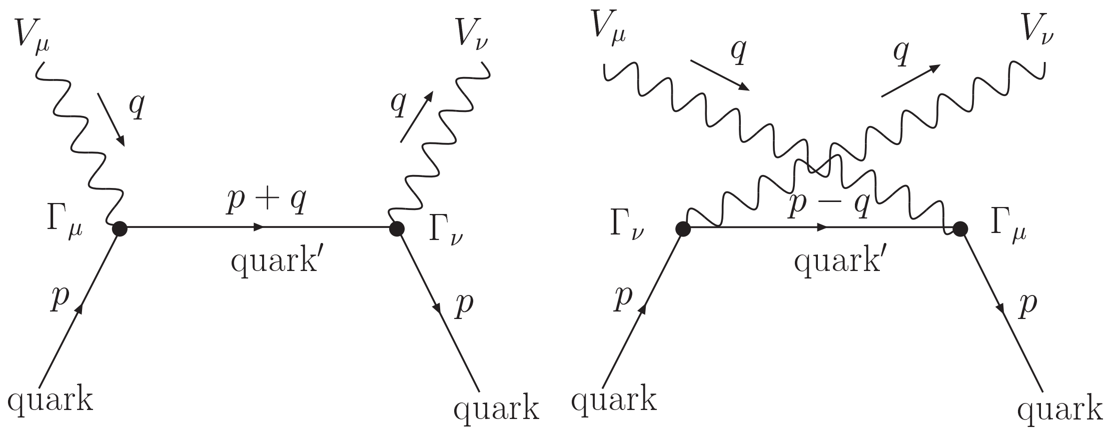
Calculation

- Leading order diagrams at parton level
  - Vector and axial-vector interaction $a\gamma^\mu + b\gamma^\mu\gamma^5$



- Mellin moments with definite symmetry properties
  - process dependent distinction even/odd N (from OPE)

$$F_i(N,Q^2) = \int_0^1 dx\, x^{N-2} F_i(x,Q^2)\,, \quad i = 2, L$$

$$F_3(N,Q^2) = \int_0^1 dx\, x^{N-1} F_3(x,Q^2)$$

Known knowns

[joeyspeak]

- NC (exchange via γ gauge boson) $\longrightarrow F_2^{eP}$
  CC (exchange via $W^\pm$ gauge boson) $\longrightarrow F_2^{\nu p+\bar{\nu}p}$, $F_3^{\nu p+\bar{\nu}p}$

  even N for F_2, odd N for F_3

  - NLO Bardeen, Buras, Duke, Muta ‘78
  - N^{2}LO Zijlstra, van Neerven ‘92
  - N^{3}LO S.M., Vermaseren, Vogt ‘05

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  - order N^{3}LO still missing this talk: S.M., Rogal '07

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Unknown unknowns

- ...

In practice

- Calculations of the third order QCD corrections
- Large number of diagrams (3466 in total)
  - diagram generation with QGRAF Nogueira '93

	$\mathcal{O}(\alpha_s^0)$	$\mathcal{O}(\alpha_s^1)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	sum
$F_2^{\nu P - \bar{\nu} P}$	1	4	55	1016	1076
$F_L^{\nu P - \bar{\nu} P}$	1	4	55	1016	1076
$F_3^{\nu P - \bar{\nu} P}$	1	4	63	1246	1314

- Calculation of diagrams
  - computer alsgebra system FORM Vermaseren '89-'07
  - package MINCER Larin, Tkachev, Vermaseren '91

Results

$$
\begin{aligned}
C^{\rm ns}_{3,10} \;=\; & \\
& 1 + a_s C_F \frac{1953379}{138600} + a_s^2 C_F n_f \left(-\frac{537659500957277}{15975002736000} \right) + a_s^2 {C_F}^2 \left(\frac{5973994463755245 89}{14760902528064000} \right. \\
& \left. + \frac{7202}{105} \zeta_3 \right) + a_s^2 C_A C_F \left(\frac{5832602058122267}{29045459520000} - \frac{99886}{1155} \zeta_3 \right) \\
& + a_s^3 C_F {n_f}^2 \left(\frac{513397566731946171 91}{996360920644320000} + \frac{48220}{18711} \zeta_3 \right) + a_s^3 {C_F}^2 n_f \left(-\frac{1254838179460551213513 53}{209235793335307200000} \right. \\
& \left. - \frac{59829376}{3274425} \zeta_3 + \frac{24110}{693} \zeta_4 \right) + a_s^3 {C_F}^3 \left(-\frac{7444742236066958785254013 07}{7088908678200207936000000} + \frac{28630985464358}{24960941775} \zeta_3 \right. \\
& \left. + \frac{151796299}{8004150} \zeta_4 - \frac{53708}{99} \zeta_5 \right) + a_s^3 C_A C_F n_f \left(-\frac{18522135004550748775 3}{2264456637828 00000} + \frac{8071097}{39690} \zeta_3 \right. \\
& \left. - \frac{24110}{693} \zeta_4 \right) + a_s^3 C_A {C_F}^2 \left(\frac{1977007872933860773207544 9}{83694317334122880000 00} - \frac{619383700181}{5546875950} \zeta_3 \right. \\
& \left. - \frac{151796299}{5336100} \zeta_4 - \frac{37322}{99} \zeta_5 \right) + a_s^3 {C_A}^2 C_F \left(\frac{9379871963905664812514 3}{36231306205248000000} - \frac{43202630363}{20582100} \zeta_3 \right. \\
& \left. + \frac{151796299}{16008300} \zeta_4 + \frac{195422}{231} \zeta_5 \right)
\end{aligned}
$$

Checks

- Known Mellin moments for $F_{2,L}^{\nu P+\bar{\nu}P}$ (even) and $F_3^{\nu P+\bar{\nu}P}$ (odd) recalculated
- All calculations with gauge parameter ξ for gluon propagator

$$\mathrm{i}\frac{-g^{\mu\nu} + (1-\xi)q^{\mu}q^{\nu}}{q^2}$$

- Adler sum rule for DIS structure functions $\longrightarrow C_{2,1}^{\mathrm{ns}} = 1$

$$\int_0^1 \frac{dx}{x}\left(F_2^{\nu P}(x,Q^2) - F_2^{\nu N}(x,Q^2)\right) = 2$$

  - measures isospin of the nucleon in the quark-parton model
  - neither perturbative nor non-perturbative corrections in QCD
- Gottfried sum rule (charged lepton(l)-proton(P) or neutron(N) DIS)

$$\int_0^1 \frac{dx}{x}\left(F_2^{lP}(x,Q^2) - F_2^{lN}(x,Q^2)\right)$$

  - Conjecture: difference of non-singlet coefficient functions for even and odd Mellin moments subleading in color $[C_F - C_A/2] \simeq 1/N_c$
    Broadhurst, Kataev, Maxwell '04

Checks (cont'd)

- $\delta C_{i,n}^{\rm ns} = C_{i,n}^{\nu P + \bar\nu P} - C_{i,n}^{\nu P - \bar\nu P}$ with color coefficient $[C_F - C_A/2]$
- e.g.

$$\delta C_{2,3}^{\rm ns} =$$

$$+a_s^2 C_F [C_F - C_A/2] \left(-\frac{4285}{96} - 122\zeta_3 + \frac{671}{9}\zeta_2 + \frac{128}{5}{\zeta_2}^2 \right)$$

$$+a_s^3 C_F [C_F - C_A/2]^2 \left(\frac{1805677051}{466560} - \frac{2648}{9}\zeta_5 + \frac{10093427}{810}\zeta_3 - \frac{1472}{3}{\zeta_3}^2 - \frac{7787113}{1944}\zeta_2 \right.$$

$$\left. + \frac{55336}{9}\zeta_2\zeta_3 - \frac{378838}{45}{\zeta_2}^2 - \frac{8992}{63}{\zeta_2}^3 \right)$$

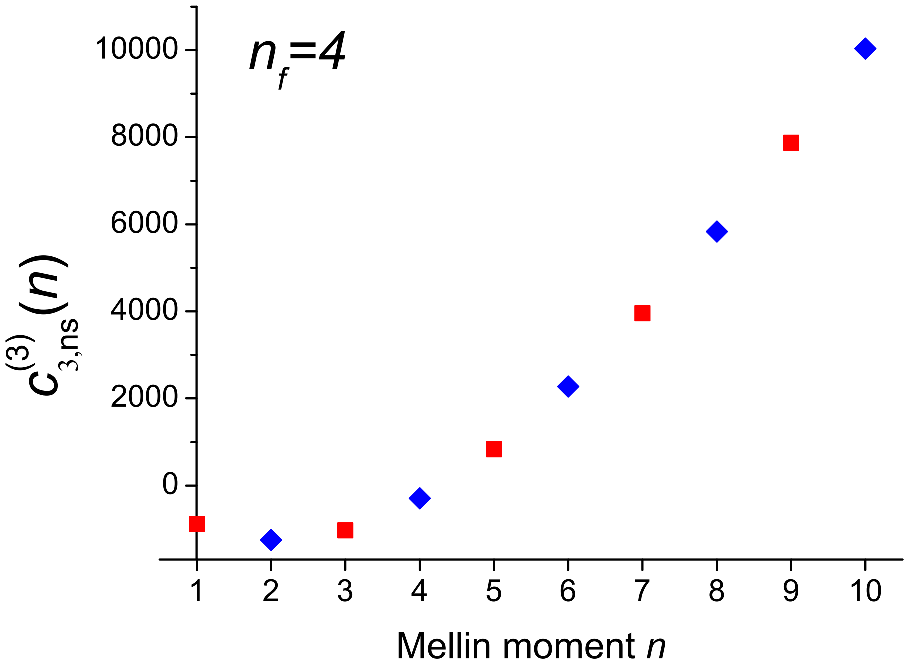
$$+a_s^3 {C_F}^2 [C_F - C_A/2] \left(-\frac{5165481803}{1399680} + \frac{40648}{9}\zeta_5 - \frac{9321697}{810}\zeta_3 + \frac{1456}{3}{\zeta_3}^2 + \frac{8046059}{1944}\zeta_2 \right.$$

$$\left. -4984\zeta_2\zeta_3 + \frac{798328}{135}{\zeta_2}^2 - \frac{56432}{315}{\zeta_2}^3 \right)$$

$$+a_s^3 n_f C_F [C_F - C_A/2] \left(\frac{20396669}{116640} - \frac{1792}{9}\zeta_5 + \frac{405586}{405}\zeta_3 - \frac{139573}{486}\zeta_2 \right.$$

$$\left. + \frac{1408}{9}\zeta_2\zeta_3 - \frac{50392}{135}{\zeta_2}^2 \right)$$

First ten integer Mellin moments



- First ten integer Mellin moments $c^{(3)}_{3,\rm ns}$ with $n_f = 4$ flavors

Parametrization (easy-to-use)

- $L_0 = \ln(x)$, $L_1 = \ln(1-x)$, +-distribution $\mathcal{D}_i = \ln(1-x)^i/(1-x)_+$

$c^{(3)}_{3,\,\nu+\bar{\nu}}(x) \cong$

$$
\begin{aligned}
& 512/27\,\mathcal{D}_5 - 5440/27\,\mathcal{D}_4 + 501.099\,\mathcal{D}_3 + 1171.54\,\mathcal{D}_2 - 7328.45\,\mathcal{D}_1 + 4442.76\,\mathcal{D}_0 \\
& \quad - 9172.68\,\delta(x_1) - 512/27\,L_1^5 + 8896/27\,L_1^4 - 1396\,L_1^3 + 3990\,L_1^2 + 14363\,L_1 \\
& \quad - 1853 - 5709\,x + x\,x_1(5600 - 1432\,x) - L_0 L_1(4007 + 1312\,L_0) - 0.463\,xL_0^6 \\
& \quad - 293.3\,L_0 - 1488\,L_0^2 - 496.95\,L_0^3 - 4036/81\,L_0^4 - 536/405\,L_0^5 \\
+ \quad & n_f\,\{\,640/81\,\mathcal{D}_4 - 6592/81\,\mathcal{D}_3 + 220.573\,\mathcal{D}_2 + 294.906\,\mathcal{D}_1 - 729.359\,\mathcal{D}_0 + 2575.46\,\delta(x_1) \\
& \quad - 640/81\,L_1^4 + 32576/243\,L_1^3 - 660.7\,L_1^2 + 959.1\,L_1 + 516.1 + x\,x_1(635.3 + 310.4\,x) \\
& \quad - 465.2\,x + 31.95\,x_1 L_1^4 + L_0 L_1\,(1496 + 270.1\,L_0 - 1191\,L_1) - 1.200\,xL_0^4 + 366.9\,L_0 \\
& \quad + 305.32\,L_0^2 + 48512/729\,L_0^3 + 304/81\,L_0^4\,\} \\
+ \quad & n_f^2\,\{\,64/81\,\mathcal{D}_3 - 464/81\,\mathcal{D}_2 + 7.67505\,\mathcal{D}_1 + 1.00830\,\mathcal{D}_0 - 103.2602\,\delta(x_1) - 64/81\,L_1^3 \\
& \quad + 992/81\,L_1^2 - 49.65\,L_1 + 11.32 + 51.94\,x - x\,x_1(44.52 + 11.05\,x) + 0.0647\,xL_0^4 \\
& \quad - L_0 L_1\,(39.99 + 5.103\,L_0 - 16.30\,L_1) - 16.00\,L_0 - 2848/243\,L_0^2 - 368/243\,L_0^3\,\} \\
+ \quad & fl_{02}\,n_f\,\{2.147\,L_1^2 - 24.57\,L_1 + 48.79 - x_1(242.4 - 150.7\,x) - L_0 L_1\,(81.70 + 9.412\,L_1) \\
& \quad + xL_0\,(218.1 + 82.27\,L_0^2) - 477.0\,L_0 - 113.4\,L_0^2 + 17.26\,L_0^3 - 16/27\,L_0^5\,\}\,x_1
\end{aligned}
$$

Approximate parametrizations

- $c^{(3)}_{2,\,\nu-\bar{\nu}}(x)$, $c^{(3)}_{3,\,\nu-\bar{\nu}}(x)$ and $c^{(3)}_{L,\,\nu-\bar{\nu}}(x)$
- approximation based on first five moments to come
- stay tuned . . .

Summary

- New results for fixed Mellin moments at order α_s^3

  $F_{2,L}^{\nu p-\bar{\nu}p}$ (odd) and $F_3^{\nu p+\bar{\nu}p}$ (even)

- Conjecture from Gottfried sum rule Broadhurst, Kataev, Maxwell '04
  - verified for first integer Mellin moments

Outlook

- Results for symbolic Mellin-N (complete Bjorken x) in the near future with chain of the calculation
  - generation of diagrams with QGRAF (topologies)
  - generation of database of Feynman graphs (depend on the process considered)
  - automatic evaluation of diagrams using FORM programs
- Use machinery in other applications