Modelling non-perturbative corrections to bottom-quark fragmentation

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The aim of the work presented in this talk is to describe heavy flavour fragmentation using a model based on NNLL threshold resummation and on an effective QCD coupling which do not exhibit the Landau pole.

Through this model we can reproduce with good accuracy the experimental data of B-hadron production in e^+e^- annihilation at Z^0 pole, without introducing any had-hoc non-perturbative component.

We have in plane to use this model to describe HERA and Tevatron B-hadron production data and to make predictions for LHC.

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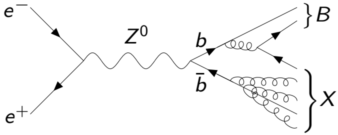
Outline

b-quark fragmentation

$$e^+e^- \rightarrow Z^0 \rightarrow B + X,$$



- Energy scales: $m_Z \geq E_b \geq m_b$, $Q = m_Z \sim 90 \; GeV$
- Threshold region: $x_b \equiv \frac{2\,E_b}{m_Z} \rightarrow 1$
- At α_S order $x_b + x_{\bar{b}} + \omega = 2$, we obtain

$$1 - x_b = \frac{1}{2} x_{\bar{b}}\, \omega \,(1 - \cos\theta_{g\bar{b}})$$

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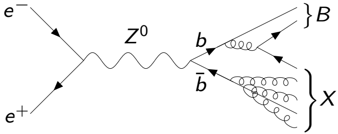

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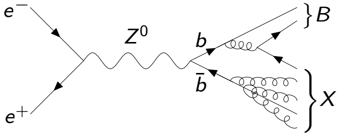

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- In the threshold region ($x_b \to 1$) the spectrum is affected by large logarithms of the type

$$\alpha^n \left[\frac{\ln^k(1-x_b)}{1-x_b}\right]_+ \qquad (n = 1, 2\cdots,\infty,\ k = 0, 1, 2, \cdots, 2n{-}1),$$

which are enhanced for soft or collinear emissions.

- The b-quark energy differential cross section contains also large logarithms of the form

$$\alpha_S^n \ln^k \frac{m_Z^2}{m_b^2} \qquad (n = 1, 2, \cdots, \infty \quad k = 1, 2, \cdots, n),$$

related to collinear emission of the partons with transverse momenta between $m_b^2 \le k_\perp^2 \le m_Z^2$, which are resummed using the Altarelli–Parisi (DGLAP) formalism.

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- Up to power corrections, the b-quark energy distribution factorizes as the following convolution [Mele & Nason ('91), Cacciari & Catani ('01)]:

$$\frac{1}{\sigma}\frac{d\sigma}{dx_b}(x_b;m_Z,m_b)=C(x_b;m_Z,\mu_F)\otimes E(x_b;\mu_F,\mu_{0F})\otimes D^{ini}(x_b;\mu_{0F},m_b)$$

  - $C(x_b;\, m_Z, \mu_F)$ is a coefficient function, describing the emission off a light parton. It contains the large-x logarithms which are process-dependent;
  - $D^{ini}(x_b;\, \mu_{0F}, m_b)$ is the initial condition of the perturbative fragmentation function at the scale $\mu_{0F} \simeq m_b$. It contains the large-x logarithms which are also process-independent;
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- In the Mellin space the threshold resummed form factor reads [Sterman ('87), Catani & Trentadue ('89)]:

$$\ln C_N = \int_0^1 dz \frac{z^{N-1}-1}{1-z}\left\{\int_{\mu_F^2}^{m_Z^2(1-z)} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] + B[\alpha_S(m_Z^2(1-z))]\right\}$$

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- When $z \to 1$ the integration in $k_\perp^2$ involves α_S at the Landau pole: it is necessary a prescription, e. g. the Minimal Prescription [Catani, Mangano, Nason & Trentadue ('96)].
- Our approach is different, we treat the unphysical Landau pole from the very beginning using the analytic QCD coupling [Aglietti & Ricciardi ('04)].

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Analytic QCD coupling

- Standard QCD coupling: physical cut at $\mu^2 < 0$ and unphysical pole at $\mu^2 = \Lambda^2_{QCD}$:

$$\alpha_S^{lo}(\mu^2) = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda^2_{QCD}}} .$$

- Analytic QCD coupling: same discontinuity along the cut but analytic elsewhere in the complex plane [Shirkov & Solovtsov ('97)]:

$$\bar{\alpha}_S^{lo}(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{\ln Q^2/\Lambda^2_{QCD}} - \frac{\Lambda^2_{QCD}}{Q^2-\Lambda^2_{QCD}}\right] , \quad LO \quad space-like$$

$$\lim_{Q^2\to 0} \bar{\alpha}_S(Q^2) = \frac{1}{\beta_0} , \qquad \lim_{Q^2\to\infty} \bar{\alpha}_S(Q^2) = \lim_{Q^2\to\infty} \alpha_S(Q^2) .$$

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$$\tilde{\alpha}_S(k_\perp^2) = \frac{i}{2\pi} \int_0^{k_\perp^2} ds \, Disc_s \, \frac{\bar{\alpha}_S(-s)}{s}.$$

- At leading order we have:

$$\tilde{\alpha}_S^{lo}(k_\perp^2) = \frac{1}{\beta_0} \left(\frac{1}{2} - \frac{1}{\pi} \arctan \frac{\ln \frac{k_\perp^2}{\Lambda_{QCD}^2}}{\pi} \right) ,$$

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- On the other hand the standard approach assumption is

$$\ln \frac{|s|}{\Lambda_{QCD}^2} \gg \pi \quad \Rightarrow \quad \frac{i}{2\pi} \int_0^{k_\perp^2} ds Disc_s \frac{\alpha_S(-s)}{s} \simeq \alpha_S(k_\perp^2).$$

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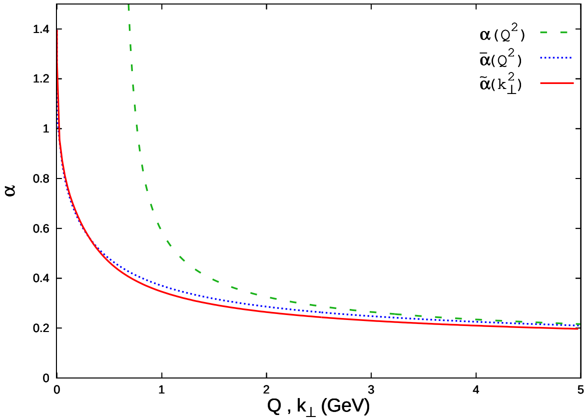

Figure 1: Time-like and space-like analytic couplings compared with the standard one.

Improved threshold resummation

- The improved threshold resummation formula therefore reads

$$\ln D_N^{ini} = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left\{ \int_{m_b^2(1-z)^2}^{\mu_{0F}^2} \frac{dk_\perp^2}{k_\perp^2} \tilde{A} \left[\tilde{\alpha}_S(k_\perp^2) \right] + \tilde{D} \left[\tilde{\alpha}_S(m_b^2(1-z)^2) \right] \right\}$$

analogous formulas hold for C_N and E_N.

- The coefficients for the time-like coupling are obtained by imposing the equality:

$$A(\alpha_S) = \tilde{A}(\tilde{\alpha}_S),$$

$$\text{where} \quad \tilde{A}(\tilde{\alpha}_S) = \sum_{n=1}^{\infty} \tilde{A}_n \, \tilde{\alpha}_S^n = \tilde{A}_1 \, \tilde{\alpha}_S + \tilde{A}_2 \, \tilde{\alpha}_S^2 + \tilde{A}_3 \, \tilde{\alpha}_S^3 + \cdots .$$

- Expressing the time-like coupling in terms of the standard one, we obtain:

$$\tilde{A}_1 = A_1; \quad \tilde{A}_2 = A_2; \quad \tilde{A}_3 = A_3 + \frac{(\pi\beta_0)^2}{3} A_1 \simeq 0.31 + 0.72 \simeq 1;$$

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Inverse Mellin transform

- The Mellin integration is performed exactly in numerical way; this is possible because the time-like coupling does not have the Landau pole and is regular for any value of N.
- The inverse transform from N-space to x-space is also made exactly in numerical way by the formula

$$f_N(\tilde{\alpha}_S) \equiv \int_0^1 z^{N-1} f(z, \tilde{\alpha}_S)\, dz,$$

$$f(z; \tilde{\alpha}_S) = \int_{C-i\infty}^{C+i\infty} \frac{dN}{2\pi i} z^{-N} f_N(\tilde{\alpha}_S),$$

where the constant C is chosen so that the integration contour in the N-plane lies to the right of all the singularities of $f_N(\tilde{\alpha}_S)$: no Minimal Prescription is needed.

Inverse Mellin transform

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Phenomenological Analysis

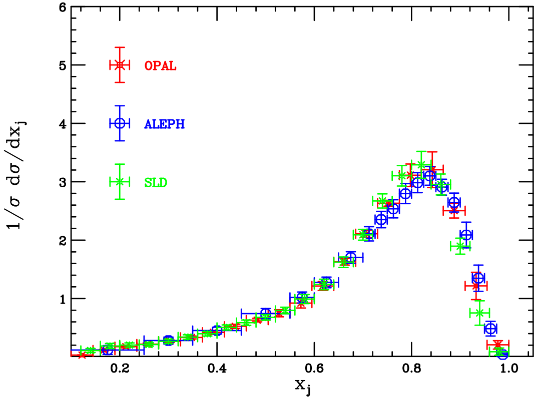

Figure 2: B-hadron spectrum in e^+e^- annihilation at Z^0 peak: prevision of the model compared with experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

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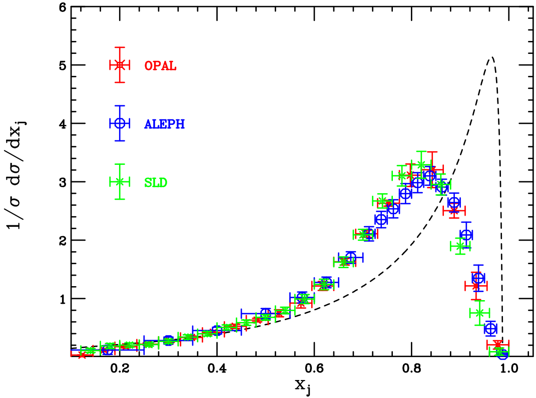

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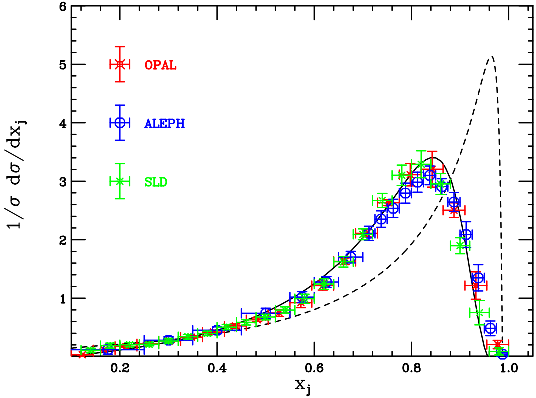

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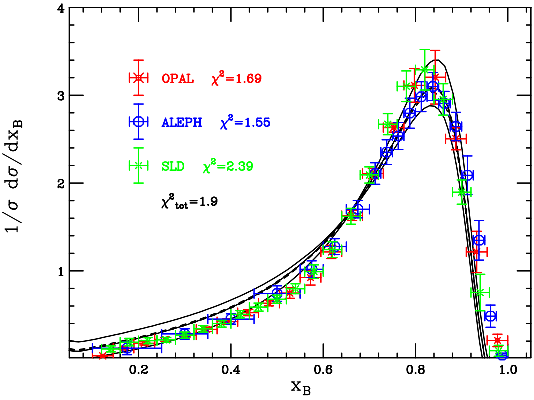

Figure 3: Model dependence on the factorizations scales.
Solid lines: $\mu_{0F} = m_b/2,\ m_b,\ 2m_b$;
dashed lines: $\mu_F = m_Z/2,\ m_Z,\ 2m_Z$.

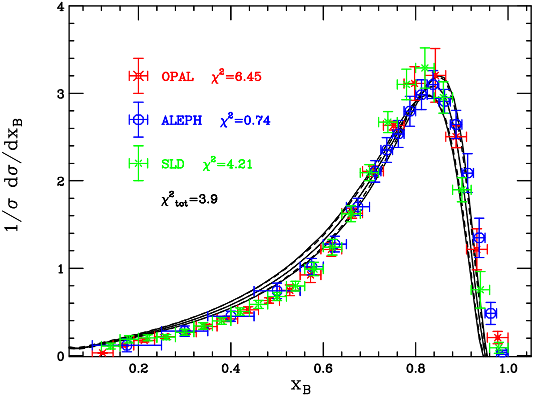

Figure 4: Model dependence on $\alpha_S(m_Z)$ and on m_b.
Solid lines: $\alpha_S(m_Z) = 0.117,\ 0.119,\ 0.121$;
dashed lines: $m_b = 4.7,\ 5.0,\ 5.3$ GeV.

	$\langle x\rangle$	$\langle x^2\rangle$	$\langle x^3\rangle$	$\langle x^4\rangle$
e^+e^- data σ_N^B	**0.7153 ± 0.0052**	**0.5401 ± 0.0064**	**0.4236 ± 0.0065**	**0.3406 ± 0.0064**
$[\sigma_N^B]_{\rm th}$	**0.6867 ± 0.0403**	**0.5019 ± 0.0472**	**0.3815 ± 0.0465**	**0.2976 ± 0.0462**
$\delta\sigma_N^B(\mu_R)$	0.0014	0.0011	0.0009	0.0007
$\delta\sigma_N^B(\mu_F)$	0.0066	0.0067	0.0059	0.0051
$\delta\sigma_N^B(\mu_{0R})$	0.0022	0.0028	0.0031	0.0033
$\delta\sigma_N^B(\mu_{0F})$	0.0364	0.0414	0.0398	0.0364
$\delta\sigma_N^B(m_b)$	0.0111	0.0145	0.0153	0.0150
$\delta\sigma_N^B(\bar{m}_b)$	0.0004	0.0005	0.0006	0.0006
$\delta\sigma_N^B(\bar{m}_c)$	0.0003	0.0005	0.0006	0.0006
$\delta\sigma_N^B(\bar{m}_s)$	0.0004	0.0007	0.0008	0.0008
$\delta\sigma_N^B(\alpha_S(m_Z^2))$	0.0113	0.0158	0.0173	0.0176
σ_N^b	0.7734 ± 0.0232	0.6333 ± 0.0311	0.5354 ± 0.0345	0.4617 ± 0.0346

Table 1: Moments σ_N^B from [DELPHI ('02)] and moments $[\sigma_N^B]_{\rm th}$ yielded by our calculation. We quote the uncertainties due to the parameters which enter in the perturbative calculations and compute the theoretical total error. We also present the moments σ_N^b of the standard NLL parton level result.

Semi-inclusive decay phenomenology [U.Aglietti, G.Ricciardi & G.F. ('06)]

Radiative B decays: $B \to X_s + \gamma \quad Q = m_b \simeq 5\ GeV$

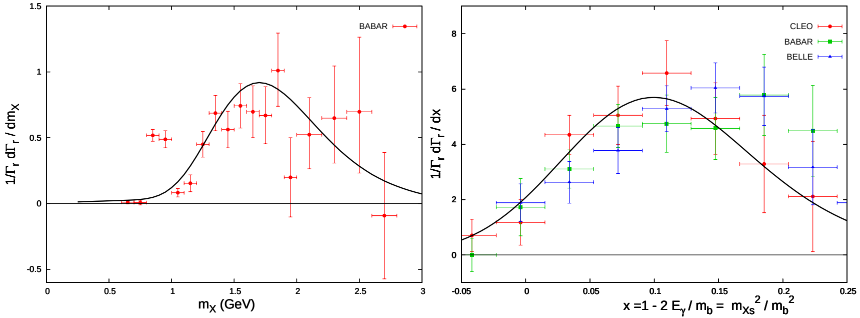

Figure 5: Invariant hadron mass distribution and photon energy distribution in the radiative B decays: prevision of the model compared with data [CLEO ('01), BaBar ('05), Belle ('05)]. Note the K^* peak at small hadron masses.

Semi-leptonic B decays: $B \to X_u + l + \nu_l \quad Q = 2E_X \lesssim 5\ GeV$

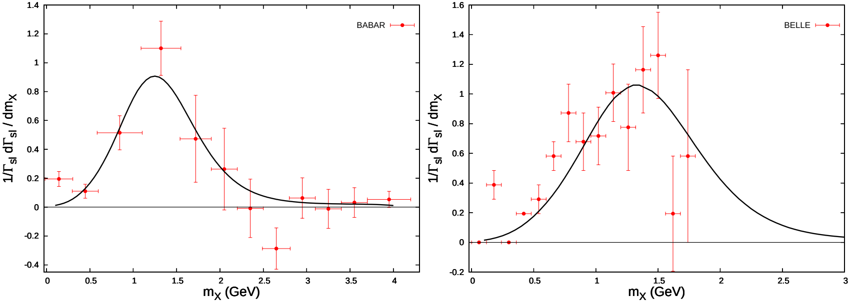

Figure 6: Invariant hadron mass distribution in the semileptonic B decays: prevision of the model compared with the experimental [Belle ('04), BaBar ('05)]. Note the π and the ρ peaks at small hadron masses.

Conclusions

- Heavy flavour physics in the threshold region is plagued by large logarithm in the perturbative expansion: an all order resummation is necessary. We have analyzed b fragmentation and semi-inclusive B decays.
- Threshold resummation involves the QCD coupling evaluated up to the Landau pole: a prescription is necessary.
- Through the analytic QCD coupling we develop a model that includes the large non-perturbative effects.
- Despite decays and fragmentation are quite different process, our model describes with good accuracy all measured spectra without introducing any ad-hoc non-perturbative component.

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Perspective

The work presented in this talk permit a wide range of improvements and developes.

- Using fixed-order calculation results [Melnikov & Mitov ('04)] it is possible to perform a complete NNLO+NNLL resummation. After this we are confident that the reduced theoretical uncertaintes will permit us to exctract $\alpha_S(m_Z)$ from experimental data.
- Use our model to study HERA and Tevatron hadron production data and to make prediction for LHC.
- Other possible extensions is to apply our formalism to other process and observables as the B production in top and Higgs decays or charm production in e^+e^- annihilations at the Z^0 peak ($m_Z \sim 90\ GeV$), at hadron colliders or even much below at $\Upsilon(4s)$ peak ($m_\Upsilon \sim 10\ GeV$).

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Back-up Slides

Comparison with DMW model

- Since the time-like coupling is regular for any value of $k_\perp$, we can compute the average of the coupling

$$\alpha_0 = \frac{1}{\mu_I} \int_0^{\mu_I} \tilde{\alpha}_S(k_\perp^2)\, dk_\perp ,$$

which is a free parameter to be determined with a fit to experimental data [Dokshitzer, Marchesini & Webber ('95)].

- Assuming $\alpha_S(m_b) = 0.22$, $n_f = 3$ and $\mu_I = 2\ GeV$, we obtain at leading order with the time-like coupling:

$$\alpha_0 \simeq 0.44.$$

- The fitted value from shape variable in e^+e^- data is around 0.45, not distant from our estimate.

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