Central jet vertex in k_T-factorisation at NLO

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Motivation

- We want to understand the strong force (QCD)
  - per se as one of the fundamental forces in nature
  - as background at collider experiments
- soft energy scale $\rightsquigarrow$ confinement $\rightsquigarrow$ no free quarks/ gluons observable, but **jets** of hadronized particles
- hard energy scale $\rightsquigarrow$ asymptotic freedom $\rightsquigarrow$ access via perturbative QCD
- factorization to disentangle soft from hard physics

Motivation - need for BFKL resummation

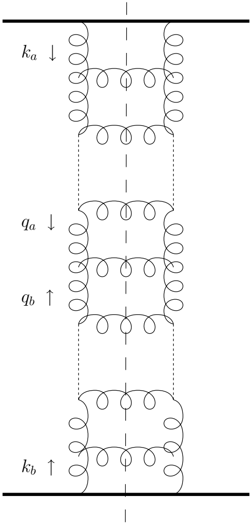
perturbative QCD $=$ expansion in coupling α_s

- large but ordered scales (e.g. $s \gg |t| \gg \Lambda_{\mathrm{QCD}}$) $\rightsquigarrow$ large logs ($\log s/t$) for each additional emission in multi Regge kinematics $\rightsquigarrow$ compensating smallness of α_s
- need to resum terms $\sim (\alpha_s \log s/t)^n$
  $\rightsquigarrow$ LO **B**alitsky-**F**adin-**K**uraev-**L**ipatov equation ['75-'78]
- resummation of terms $\sim \alpha_s(\alpha_s \log s/t)^n$
  $\rightsquigarrow$ NLO BFKL equation ['98]

Outline

1. Motivation and Introduction ✓
2. Jet production vertex at central rapidity
   - Jet production at LO
     $\gamma^*\gamma^*$, *pp*, unintegrated gluon density
   - Jet production at NLO
     $\gamma^*\gamma^*$, *pp*, unintegrated gluon density
3. Summary

Total cross section at LO BFKL



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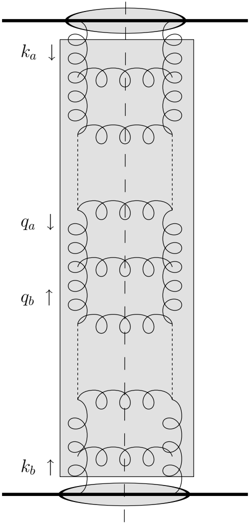
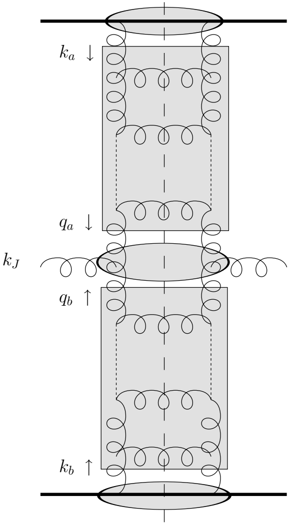


$$\sigma(s) = \int \frac{d^2\mathbf{k}_a}{2\pi\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b}{2\pi\mathbf{k}_b^2} \Phi_A(\mathbf{k}_a)\, \Phi_B(\mathbf{k}_b) \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} f_\omega(\mathbf{k}_a, \mathbf{k}_b).$$

- with impact factors Φ
- Green's function f_ω obeys BFKL equation

$$\omega f_\omega(\mathbf{k}_a, \mathbf{k}_b) = \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) + \int d^2\mathbf{k}\; \mathcal{K}(\mathbf{k}_a, \mathbf{k}) f_\omega(\mathbf{k}, \mathbf{k}_b)$$

Jet production at LO BFKL



$$\frac{d\sigma}{d^2\mathbf{k}_{Jet}\, dy_{Jet}} = \int \frac{d^2\mathbf{k}_a}{2\pi\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b}{2\pi\mathbf{k}_b^2}\, \Phi_A(\mathbf{k}_a)\, \Phi_B(\mathbf{k}_b)$$

$$\times \int d^2\mathbf{q}_a \int d^2\mathbf{q}_b \int \frac{d\omega}{2\pi i} \left(\frac{s_{AJ}}{s_0}\right)^{\omega} f_\omega(\mathbf{k}_a, \mathbf{q}_a)$$

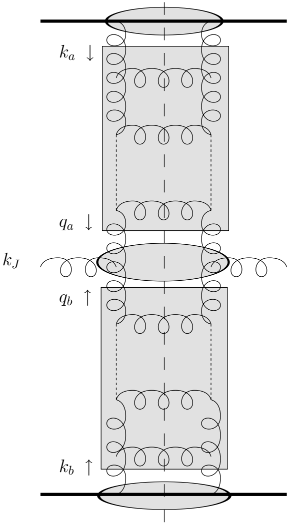
$$\times\, \mathcal{V}(\mathbf{q}_a, \mathbf{q}_b; \mathbf{k}_{Jet}, y_{Jet})$$

$$\times \int \frac{d\omega'}{2\pi i} \left(\frac{s_{BJ}}{s_0'}\right)^{\omega'} f_{\omega'}(-\mathbf{q}_b, -\mathbf{k}_b)$$

with the LO emission vertex

$$\mathcal{V} = \mathcal{K}^{(LO)}_{\mathrm{real}}(\mathbf{q}_a, -\mathbf{q}_b)\, \delta^{(2)}(\mathbf{q}_a + \mathbf{q}_b - \mathbf{k}_{Jet})\,.$$

$\gamma^*\gamma^*$ scattering (LO)



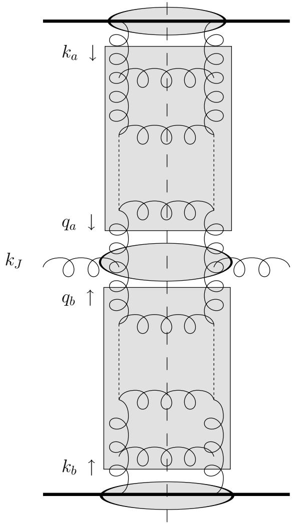
- impact factors and jet provide hard scale as well
- symmetric situation, choose s_0 as

$$s_0 = |\mathbf{k}_a|\,|\mathbf{k}_{Jet}|, \quad s_0' = |\mathbf{k}_{Jet}|\,|\mathbf{k}_b|$$

- natural language of rapidities:

$$\left(\frac{s_{AJ}}{s_0}\right)^{\omega} = e^{\omega(y_A - y_{Jet})}$$

pp scattering (LO)



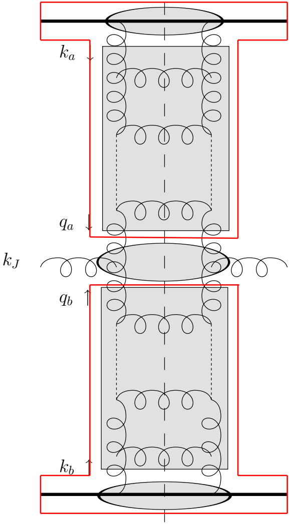
- only jet provides hard scale
- asymmetric situation, choose s_0 as

$$s_0 = \mathbf{k}_{Jet}^2, \quad s_0' = \mathbf{k}_{Jet}^2$$

- natural language of longitudinal momentum fractions

$$\left(\frac{s_{AJ}}{s_0}\right)^\omega = \left(\frac{1}{x_1}\right)^\omega$$

pp scattering (LO)



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Unintegrated gluon density

define the unintegrated gluon density

$$g(x, \mathbf{k}) = \int \frac{d^2\mathbf{q}}{2\pi\mathbf{q}^2}\,\Phi_P(\mathbf{q}) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i}\, x^{-\omega} f_\omega(\mathbf{q}, \mathbf{k})$$

which obeys the evolution equation

$$\frac{\partial g(x, \mathbf{q}_a)}{\partial \ln 1/x} = \int d^2\mathbf{q}\, \mathcal{K}(\mathbf{q}_a, \mathbf{q})\, g(x, \mathbf{q})$$

Then cross section can be written in k_T *factorization*

$$\frac{d\sigma}{d^2\mathbf{k}_{Jet}\, dy_{Jet}} = \int d^2\mathbf{q}_a \int d^2\mathbf{q}_b\, g(x_1, \mathbf{q}_a) g(x_2, \mathbf{q}_b) \mathcal{V}(\mathbf{q}_a, \mathbf{q}_b; \mathbf{k}_{Jet}, y_{Jet})$$

Changes at NLO BFKL

Q: Can we just replace the LO expressions for impact factors, kernel and Green's function by their NLO counterparts?

Changes at NLO BFKL

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A: No!

- real Kernel $\mathcal{K}_{\mathrm{real}}$ contains at NLO two particle production
  - jet algorithm
  - separation MRK $\leftrightarrow$ QMRK $\rightsquigarrow$ scale s_Λ
- energy scale s_0 is now a relevant parameter

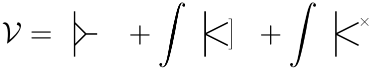
Jet definition

- remember: at LO $\mathcal{K}_{\text{real}} \sim$  $\sim \mathcal{V}$
- at NLO $\mathcal{K}_{\text{real}} \sim$  $+ \int$ 
- for  two possibilities:
  - both together form a jet
  - one forms the jet, other one unresolved
- define distance in rapidity-azimuthal angle space
  $R_{12} = \sqrt{(y_1 - y_2)^2 + (\phi_1 - \phi_2)^2}$
  - $\theta(R_0 - R_{12})$: 
  - $\theta(R_{12} - R_0)$: 
- open integration to extract jet

$$\mathcal{V} \sim$$  $+ \int$  $+ \int$ 

Subtraction term

- real and virtual parts with different $x_{1,2}$ configurations $\rightsquigarrow$ different $g(x_1, q_a)g(x_2, q_b)$ $\rightsquigarrow$ cancellation of divergences?



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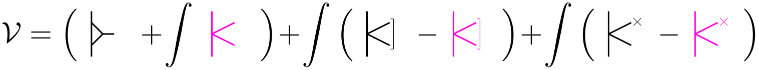

- add singular part of 2 particle production (in x configuration of virtual part) times $0 = 1 - \theta(R_0 - R_{12}) - \theta(R_{12} - R_0)$
- first bracket: analytical cancellation of divergences
- second and third bracket: numerical cancellation of divergences

$\gamma^*\gamma^*$ scattering (NLO)

- NLO calculation of the kernel was performed in framework with hard scale impact factors
- $\rightsquigarrow$ can keep (in principle) LO formula
  with NLO impact factors, Green's functions, jet vertex

pp scattering (NLO)

proton: soft scale   jet: hard scale

- in asymmetric situation: necessity of scale change $s_0 = |\mathbf{k}_a|\,|\mathbf{k}_{Jet}| \longrightarrow s_0 = \mathbf{k}_{Jet}^2$
- symmetric change $s_0 = |\mathbf{k}_a|\,|\mathbf{k}_{Jet}| \longrightarrow s_0 = f_1(|\mathbf{k}_a|)f_2(|\mathbf{k}_{Jet}|)$ could be compensated by change in only the impact factors and the vertex
- asymmetric change effects complete evolution; now from a soft scale to a hard scale

pp scattering - consequences of scale change

- modified Kernel for evolution of Green's function

$$\widetilde{\mathcal{K}}(\mathbf{q}_1,\mathbf{q}_2) = \mathcal{K}(\mathbf{q}_1,\mathbf{q}_2) - \frac{1}{2}\int d^2\mathbf{q}\,\mathcal{K}^{(LO)}(\mathbf{q}_1,\mathbf{q})\,\mathcal{K}^{(LO)}(\mathbf{q},\mathbf{q}_2)\ln\frac{\mathbf{q}^2}{\mathbf{q}_2^2}$$

$$\omega\tilde{f}_\omega(\mathbf{k}_a,\mathbf{q}_a) = \delta^{(2)}(\mathbf{k}_a-\mathbf{q}_a) + \int d^2\mathbf{q}\,\widetilde{\mathcal{K}}(\mathbf{k}_a,\mathbf{q})\,\tilde{f}_\omega(\mathbf{q},\mathbf{q}_a)$$

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- modified proton impact factor

$$\widetilde{\Phi}(\mathbf{k}_a) = \Phi(\mathbf{k}_a) - \frac{1}{2}\mathbf{k}_a^2\int d^2\mathbf{q}\,\frac{\Phi^{(LO)}(\mathbf{q})}{\mathbf{q}^2}\mathcal{K}^{(LO)}(\mathbf{q},\mathbf{k}_a)\ln\frac{\mathbf{q}^2}{\mathbf{k}_a^2}.$$

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- $\Rightarrow$ new NLO unintegrated gluon distribution

$$g(x,\mathbf{k}) = \int d^2\mathbf{q}\frac{\widetilde{\Phi}_P(\mathbf{q})}{2\pi\mathbf{q}^2}\int\frac{d\omega}{2\pi i}\tilde{f}_\omega(\mathbf{k},\mathbf{q})\,x^{-\omega}$$

$$\frac{\partial g(x,\mathbf{q}_a)}{\partial\ln 1/x} = \int d^2\mathbf{q}\,\widetilde{\mathcal{K}}(\mathbf{q}_a,\mathbf{q})\,g(x,\mathbf{q}).$$

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- modified vertex

$$\begin{aligned}\widetilde{\mathcal{V}}(\mathbf{q}_a,\mathbf{q}_b) &= \mathcal{V}(\mathbf{q}_a,\mathbf{q}_b)\\ &\quad -\frac{1}{2}\int d^2\mathbf{q}\, \mathcal{K}^{(LO)}(\mathbf{q}_a,\mathbf{q})\mathcal{V}^{(LO)}(\mathbf{q},\mathbf{q}_b)\ln\frac{\mathbf{q}^2}{(\mathbf{q}-\mathbf{q}_b)^2}\\ &\qquad -\frac{1}{2}\int d^2\mathbf{q}\, \mathcal{V}^{(LO)}(\mathbf{q}_a,\mathbf{q})\, \mathcal{K}^{(LO)}(\mathbf{q},\mathbf{q}_b)\ln\frac{\mathbf{q}^2}{(\mathbf{q}_a-\mathbf{q})^2}.\end{aligned}$$

Summary

We constructed a jet vertex

- in NLO k_T factorization
- explicitly free of divergences
- implications for definition of uPDFs at NLO
- kept track of dependence on all scales involved