

Angles and Mixings

Marco Ciuchini

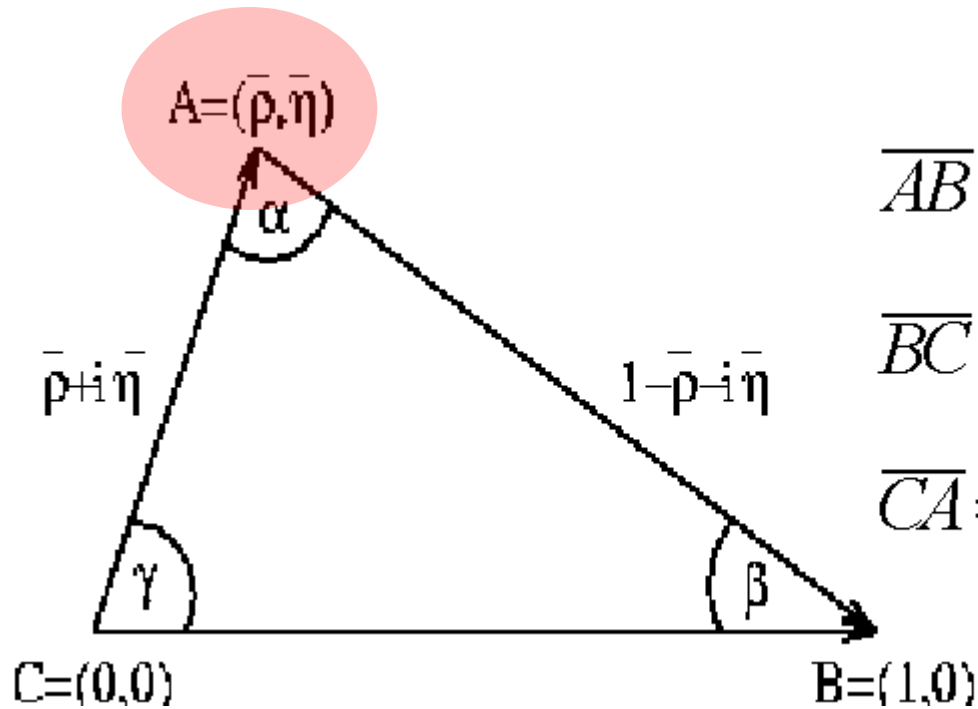
INFN Roma Tre

- UT angles from "tree" decays
 - γ from $B \rightarrow D$ K-like modes
 - α from $B \rightarrow \pi\pi / \rho\pi / \rho\rho$ modes
 - β from $B \rightarrow J/\psi$ K-like modes
- UT angles from penguin decays
 - β from $b \rightarrow s$ penguin transitions
- neutral meson mixings
 - ($B_d - \bar{B}_d$ mixing)
 - $B_s - \bar{B}_s$ mixing: Δm_s and ϕ_{B_s}
 - an "intruder": $D - \bar{D}$ mixing

Unitarity Triangle

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$



$$\overline{AB} = \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*}$$

$$\overline{BC} = 1$$

$$\overline{CA} = \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$$

Triangle

sides \leftrightarrow angles

$|VV^*| \leftrightarrow \alpha, \beta, \gamma$

$$\alpha \equiv \arg \left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right), \quad \beta \equiv \arg \left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right), \quad \gamma \equiv \arg \left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right)$$

γ from $B \rightarrow D^{(*)}K$

- no penguins

four different flavours

- many modes

$$X_{CPES} = K_s \pi, K_s \rho, \pi^+ \pi^-, \dots$$

$$X_{CPNES} = K^+ \pi^-, K^{*+} \pi^-, \dots$$

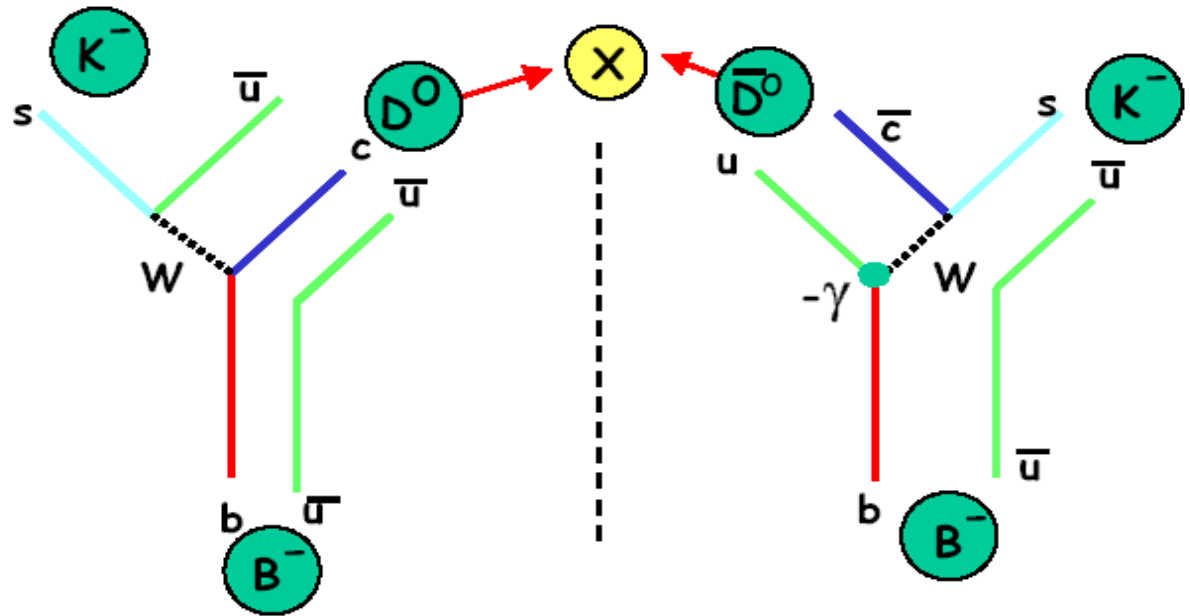
$$K^{*+} K^-, \pi^+ \rho^-, \dots$$

B^\pm : CP violation in the decay

$$\Rightarrow \gamma$$

B^0 : CP violation in the interference between mixing and decay

$$\Rightarrow \sin(2\beta + \gamma)$$

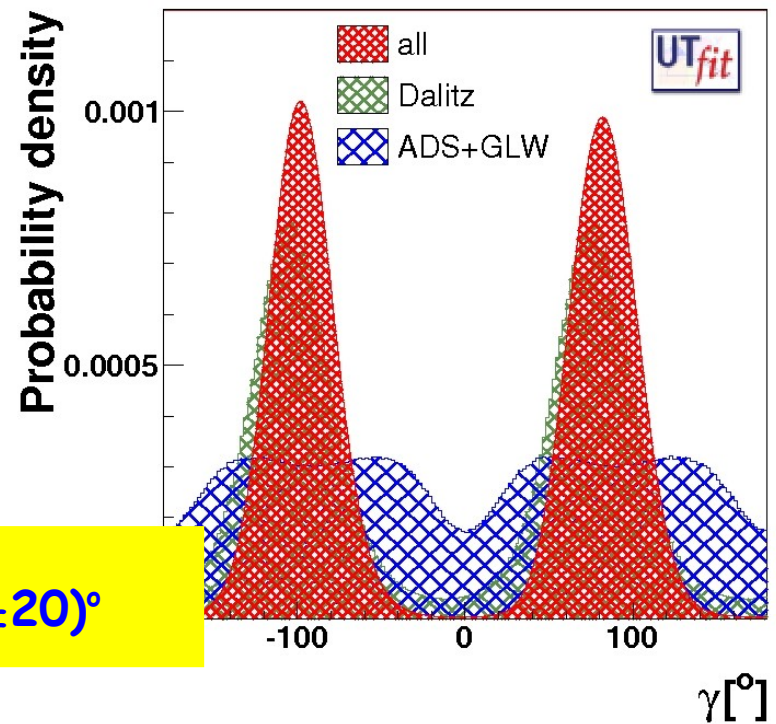


GLW: $X=CPES$

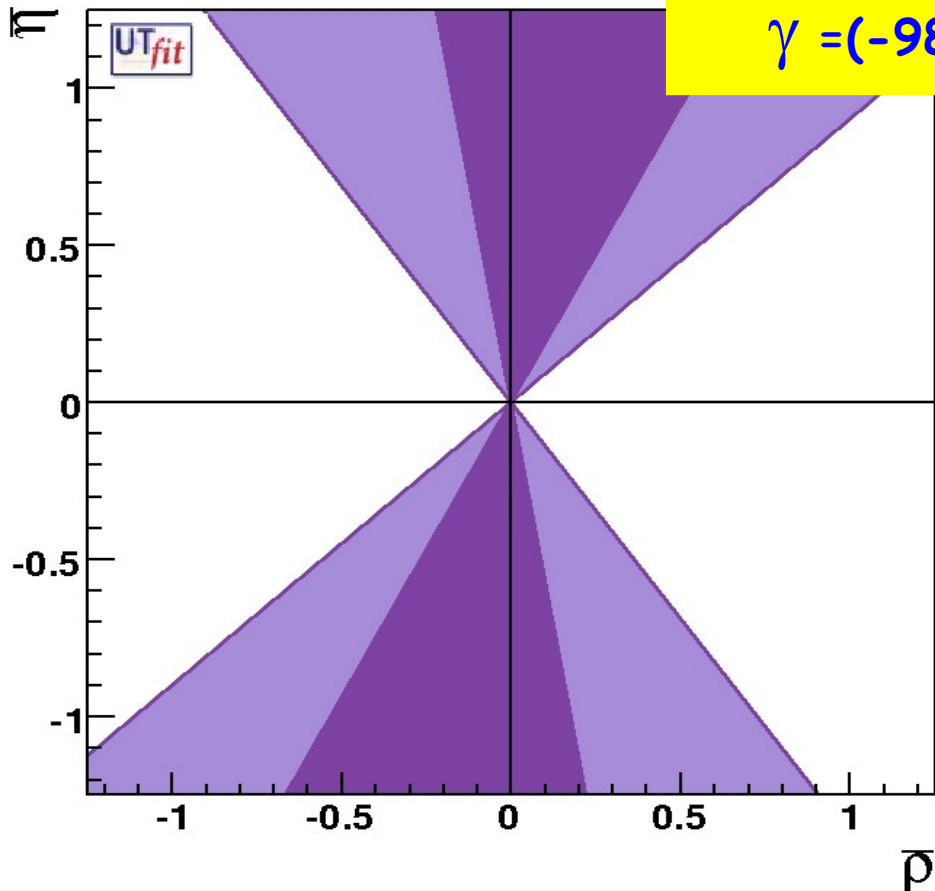
ADS: $X=K^+\pi^-$ etc.

GGSZ: Dalitz analysis of $D^0 \rightarrow 3$ -body modes, for ex. $K_s \pi^+ \pi^-$

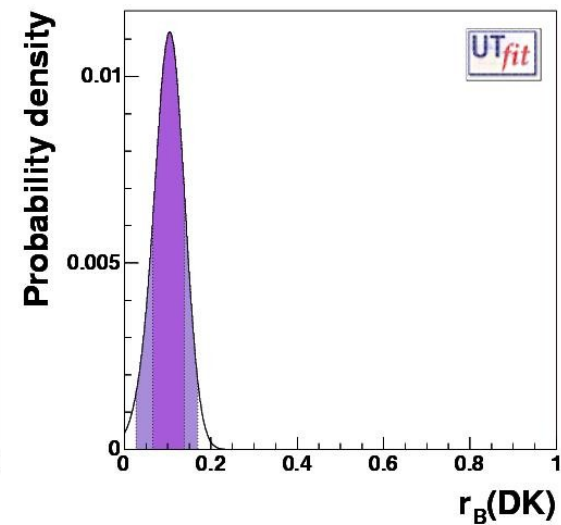
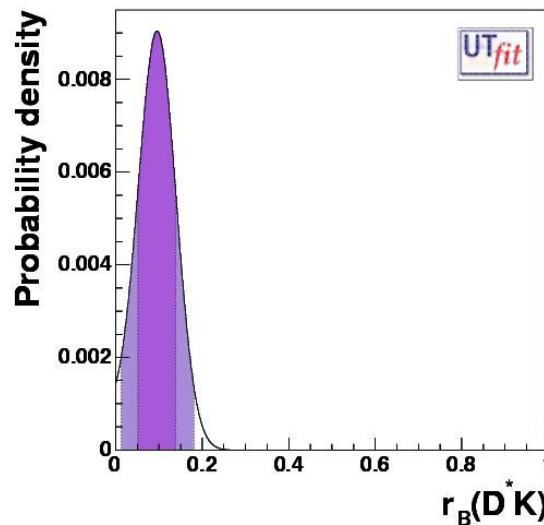
Present determination of γ



$$\gamma = (-98 \pm 20)^\circ \cup (82 \pm 20)^\circ$$



$$r_B(D^{(*)} K) = \left| \frac{A(B^- \rightarrow D^{(*)} K^-)}{A(B^- \rightarrow \bar{D}^{(*)} K^-)} \right|$$



Perspectives for γ

WG2, preliminary

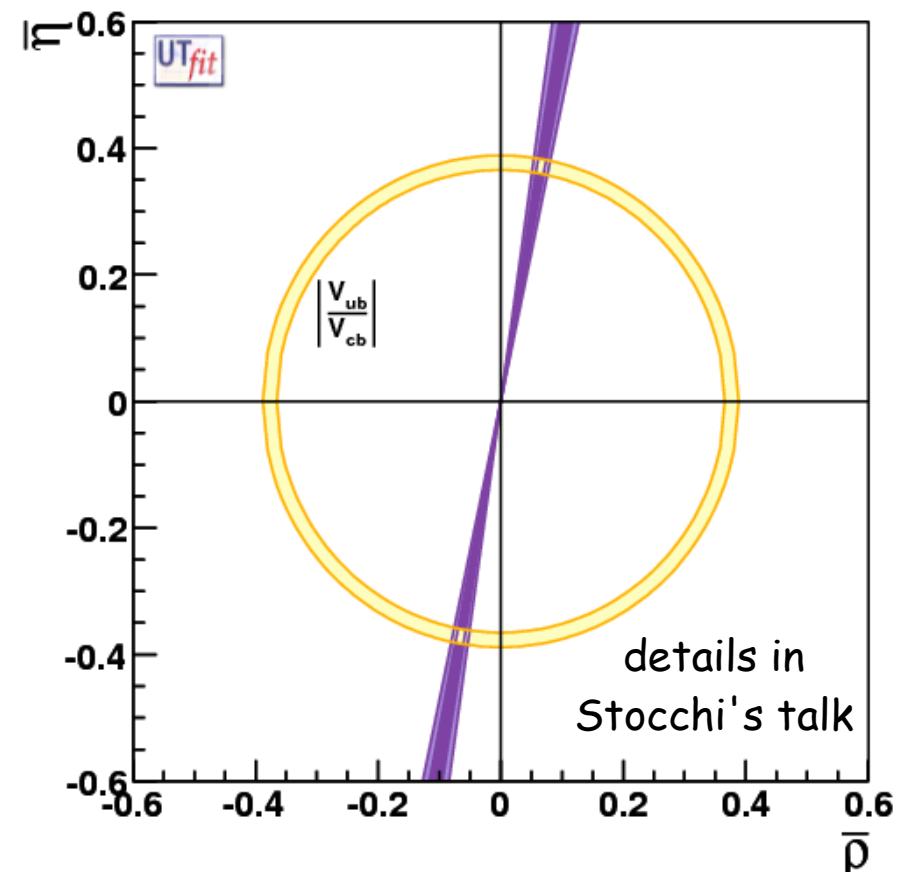
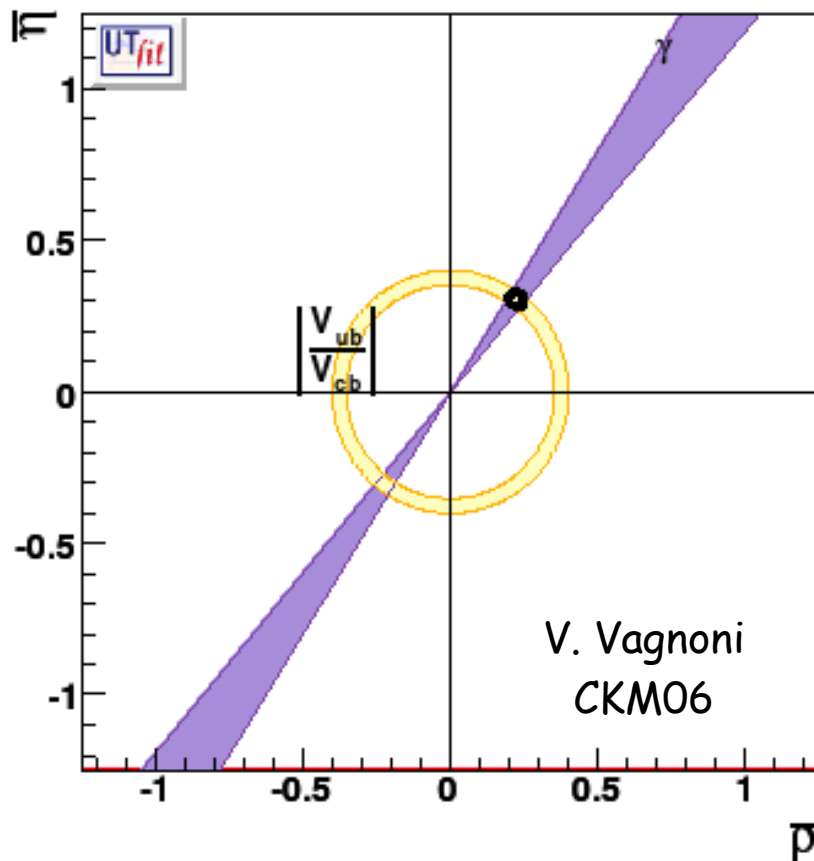
BF (Now)	BF(End '08)	LHCb	LHCb	SBF	ITE
$\sim 1 \text{ ab}^{-1}$	2 ab^{-1}	2 fb^{-1}	10 fb^{-1}	50 ab^{-1}	
30° (46%)	15° (23%)	4.6° (7%)	2.6° (4%)	2° (3%)	$O(0.1\%)$

LHCb (5 years)

- GLW, ADS, GGSZ
- $B_s \rightarrow D_s K$

SuperB (5 years)

- GLW, ADS, GGSZ



α from $B \rightarrow \rho\rho/\rho\pi/\pi\pi$ and isospin symmetry

Parametrization of the $B \rightarrow \rho\rho/\pi\pi$ amplitudes (neglecting EWP)

Gronau, London, PRL65 (1990) 3381

$$A^{+-} = -T e^{-i\alpha} + P e^{i\delta_P}$$

$$A^{+0} = -\frac{1}{\sqrt{2}} e^{-i\alpha} (T + T_c e^{i\delta_C})$$

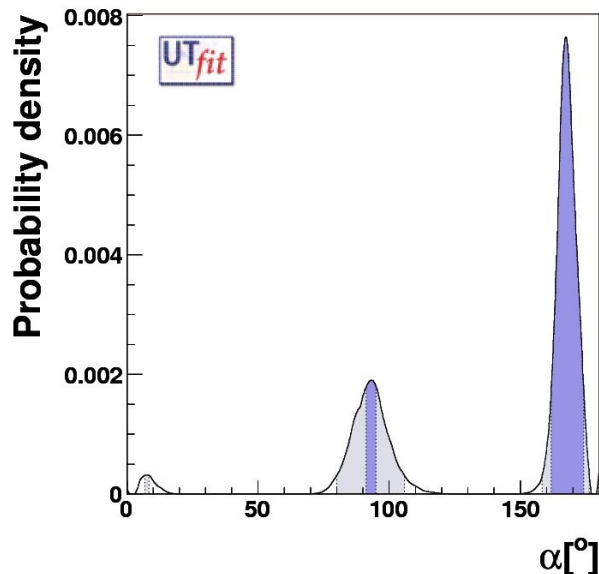
▶ 6 unknowns: $T, T_c, P, \delta_P, \delta_C, \alpha$

▶ 6 observables: $3 \times \text{BR}_{\text{ave}}, C_{+-}, S_{+-}, C_{00}$

$$A^{00} = A^{+0} - \frac{1}{\sqrt{2}} A^{+-}$$

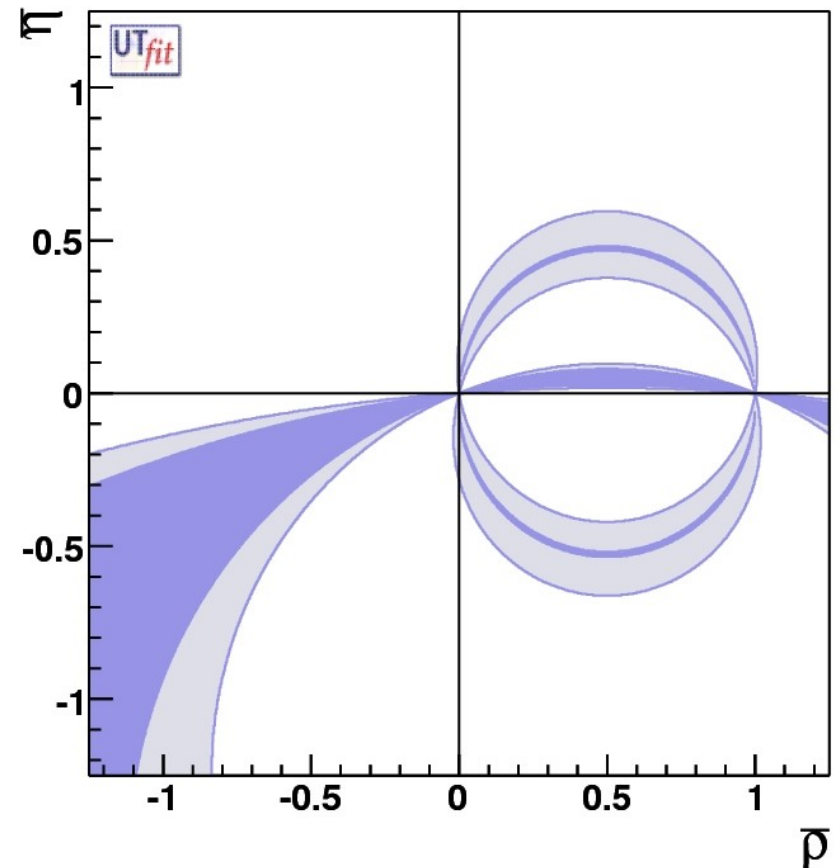
+ time-dependent Dalitz plot study
of $(\rho\pi)^0$ à la Snyder-Quinn

PRD48 (1993) 2139



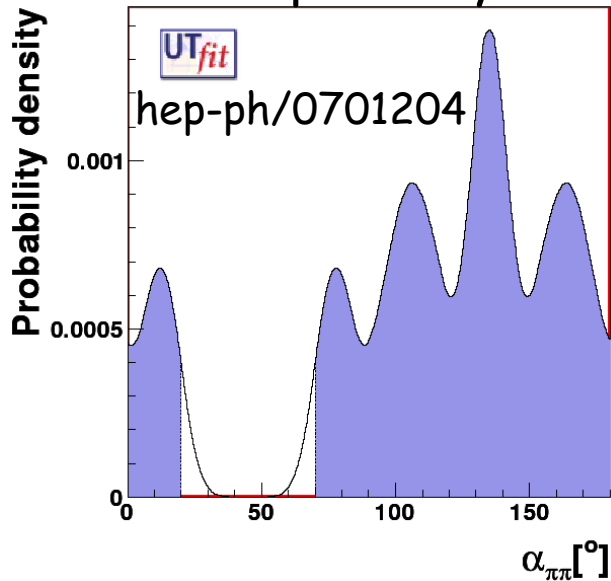
$\alpha = [7, 8]^\circ \cup$
 $[80, 106]^\circ \cup$
 $[158, 176]^\circ @95\%$

SM-like solution:
 $\alpha = (92 \pm 7)^\circ$

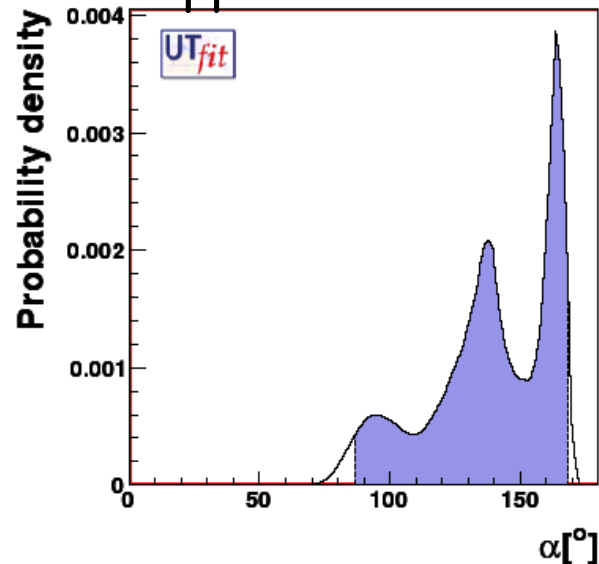


Bayesian probability

isospin analysis



isospin analysis + upper bound on P

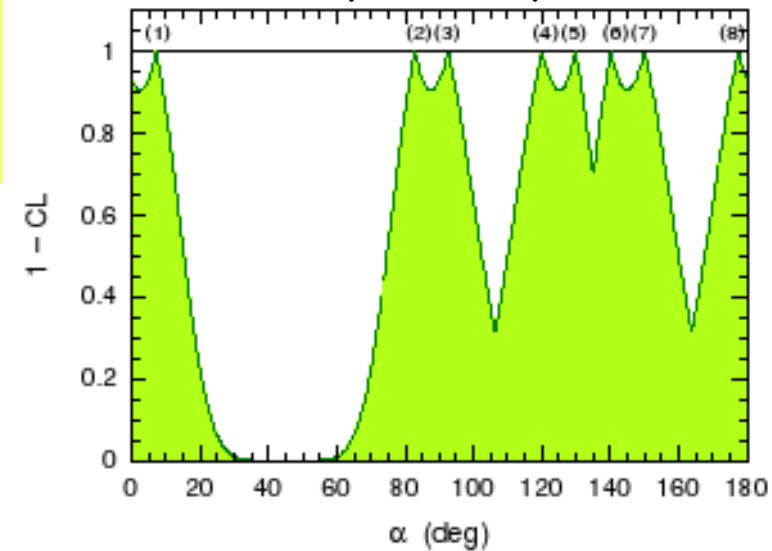


A popular mistake:
 α from $B^- \rightarrow \pi\pi$

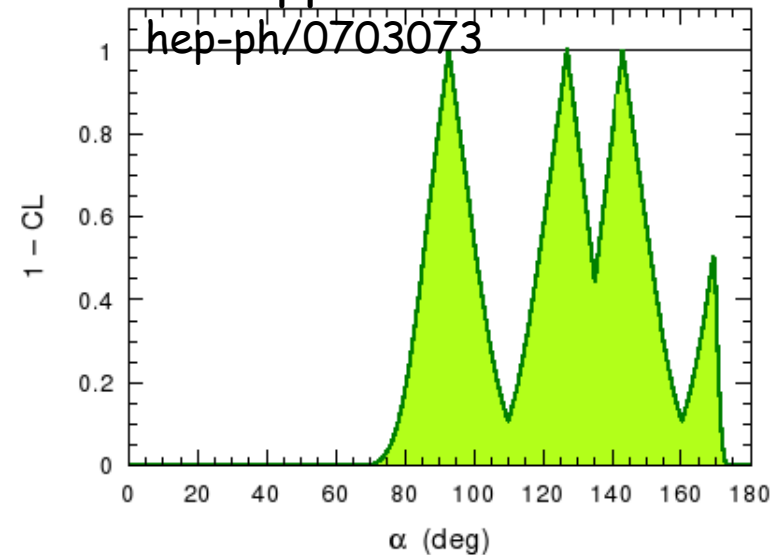
- can CP be violated with $\alpha = 0$? Remember that $C_{+-} \neq 0$ at $> 5\sigma$
- is the isospin analysis reliable if SU(3) violation is $> 3000\%$? $\alpha < 2$ implies $P > 30$
BR($B_s \rightarrow K^+K^-$) requires $P \sim 1$ using SU(3) + reasonable breaking

Frequentist C.L.

isospin analysis



isospin analysis + upper bound on P



Perspectives for α

WG2, preliminary

BF (Now)	BF(End '08)	LHCb	LHCb	SBF	ITE
$\sim 1 \text{ ab}^{-1}$	2 ab^{-1}	2 fb^{-1}	10 fb^{-1}	50 ab^{-1}	
10° (11%)	7° (8%)	8.1° (9%)	4.6° (5%)	1.5° (1.6%)	O(few %)

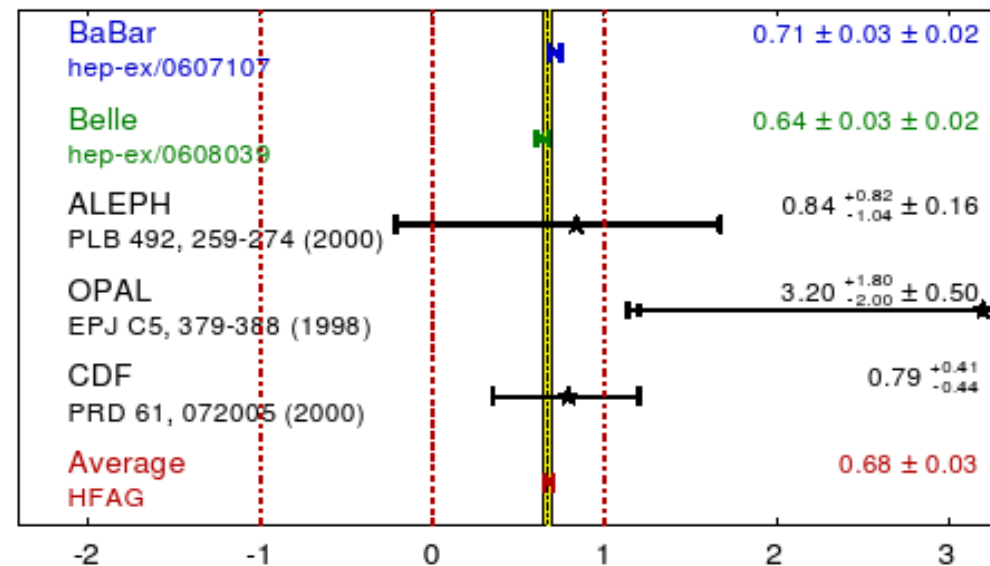
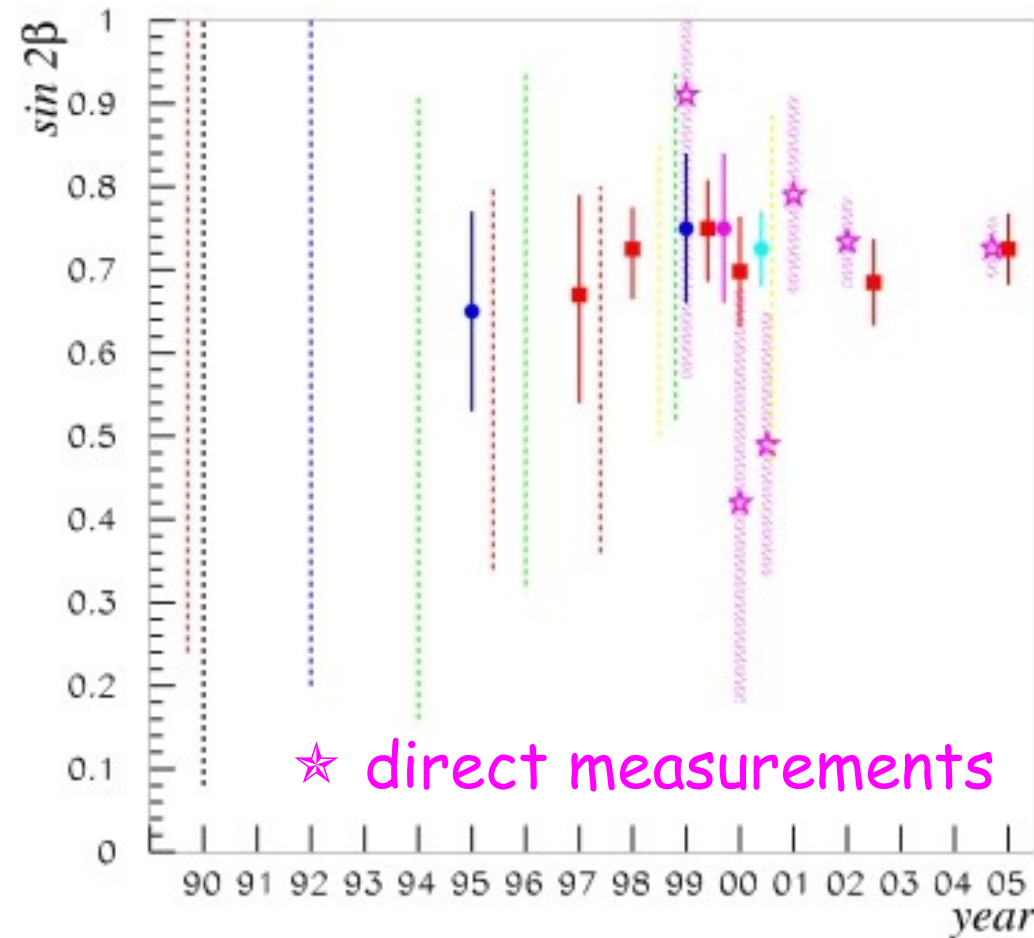
- Theoretical error from "isospin breaking"
 - ElectroWeak Penguins
 - dominant LxL EWP's calculable: $\Delta\alpha \sim$ few degrees
 - η - η' - π^0 mixing Gronau, Zupan, hep-ph/0502139
 - model-dependent estimate: $\Delta\alpha \sim 1$ degree
 - $(m_d - m_u)/\Lambda_{\text{QCD}}$ and α_e effects in matrix elements
 - order-of-magnitude estimate: $\Delta\alpha$ small
- Sensitivity to New Physics
 - potentially sensitive to NP in penguins
 - isospin analysis sensitive to EWP-like contributions only
 - QCD-penguin-like NP just redefine hadronic amplitudes

β from $B \rightarrow J/\psi K^{(*)}$

$$a_{CP}^{J/\psi K_s}(t) = -\text{Im} \left(\frac{q}{p} \frac{\bar{A}}{A} \right) \sin(\Delta m_B t) = \sin 2\beta \times \sin(\Delta m_B t)$$

$$\sin(2\beta) \equiv \sin(2\phi_1)$$

HFAG
ICHEP 2006
PRELIMINARY



Predictions existed
since early 90s

$$\sin 2\beta = 0.65 \pm 0.12$$

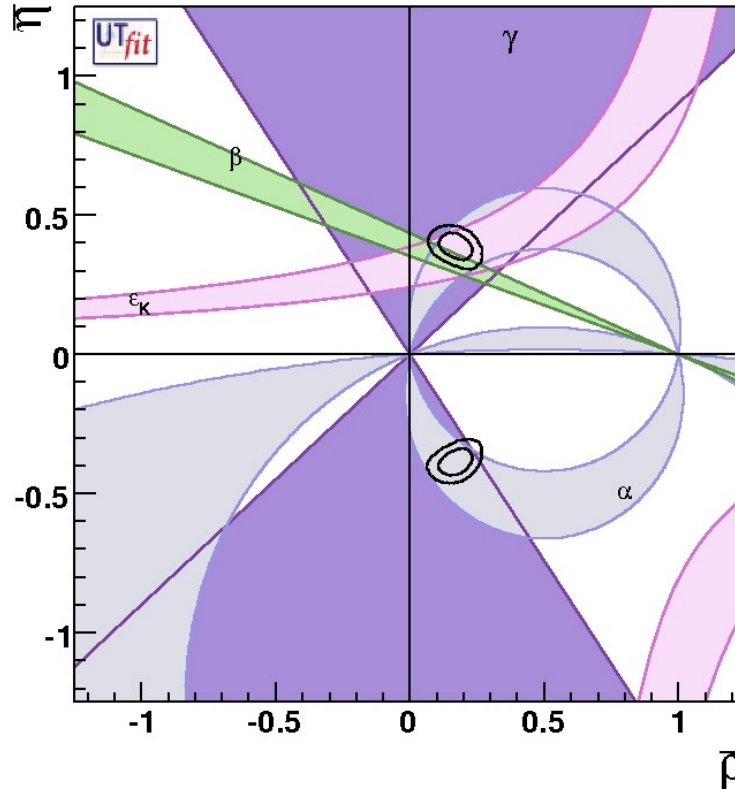
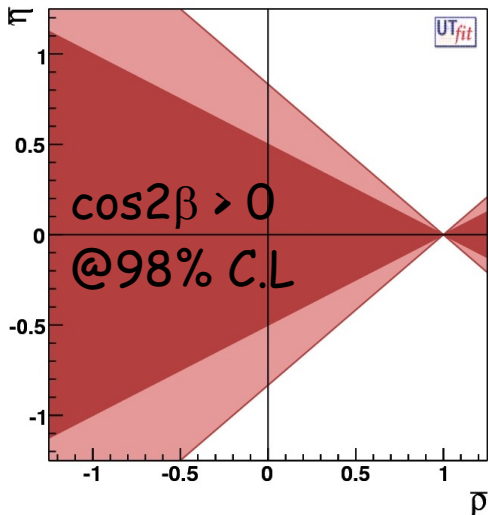
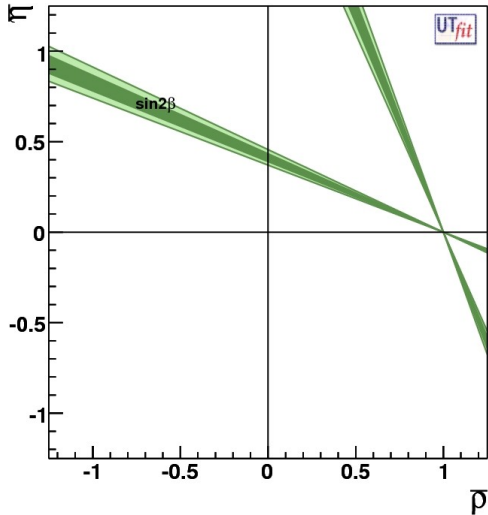
MC et al., hep-ph/9501265

current determination of 2β

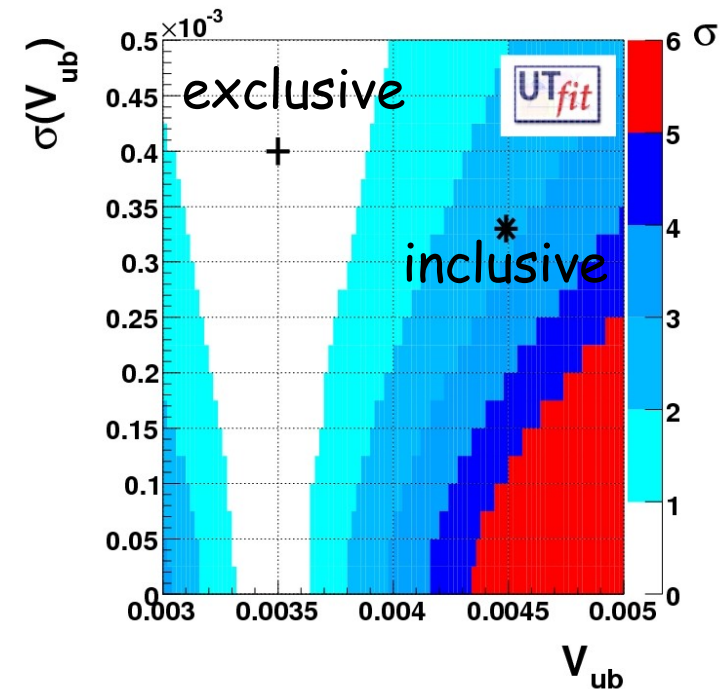
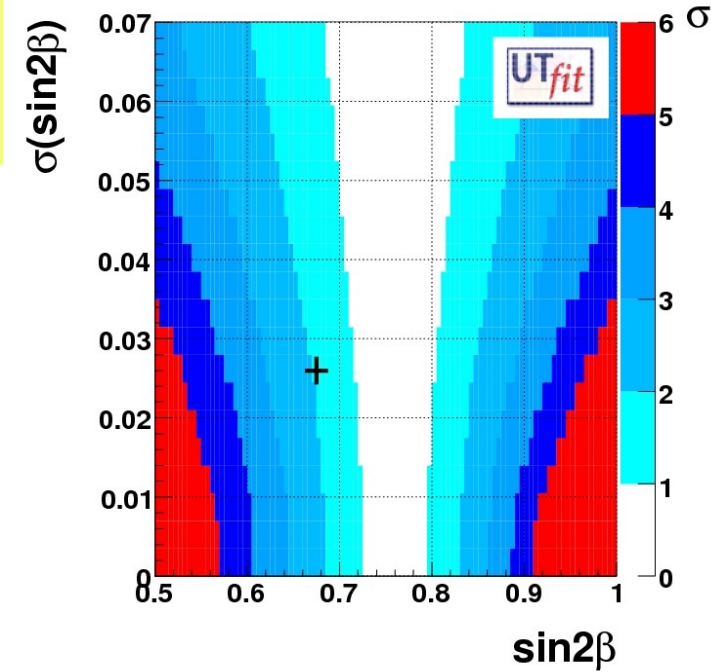
$$\sin 2\beta^{\text{exp}} = 0.68 \pm 0.03$$

$$\sin 2\beta^{\text{fit}} = 0.76 \pm 0.04$$

Tension!



likely due to
 V_{ub} inclusive:
no NP at work



How precious is the golden mode?

$$A(B \rightarrow J/\psi K^0) = V_{cb}^* V_{cs} \underbrace{(E_2 + P_2)}_{\lambda^2 T} - V_{ub}^* V_{us} \underbrace{(P_2^{GIM} - P_2)}_{\lambda^4 P}$$

- the "polluting" amplitude $P = P_2^{GIM} - P_2$ is a pure u-penguin

naive estimate: $V_{cb}^* V_{cs} P / V_{ub}^* V_{us} T \sim \lambda^2 \alpha(m_b) / 4\pi \sim 10^{-3}$

confirmed by factorization Boos, Mannel, Reuter, Li, Mishima, hep-ph/0403085 hep-ph/0610120

Data-driven estimate getting a little help from a friend:

$$A(B \rightarrow J/\psi \pi^0) = V_{cb}^* V_{cd} \underbrace{(E_2 + P_2)}_{\lambda^3 T} - V_{ub}^* V_{ud} \underbrace{(P_2^{GIM} - P_2 - \cancel{E_2})}_{\lambda^3 P}$$

Strategy: CPS, hep-ph/0507290

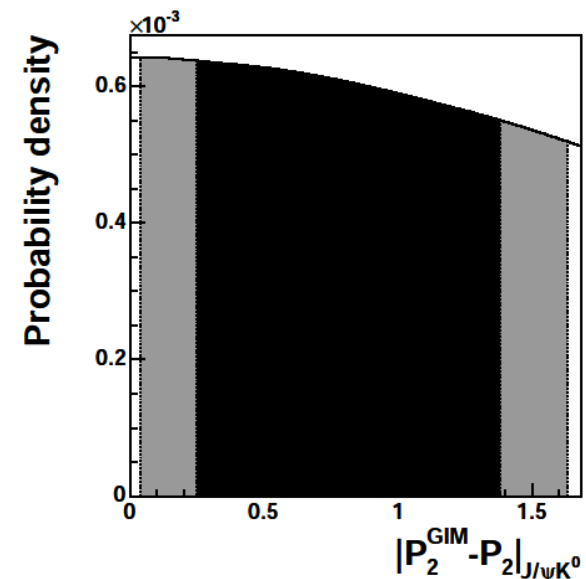
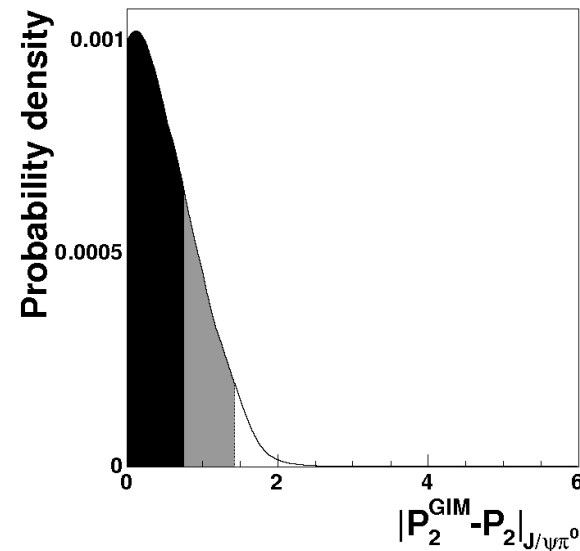
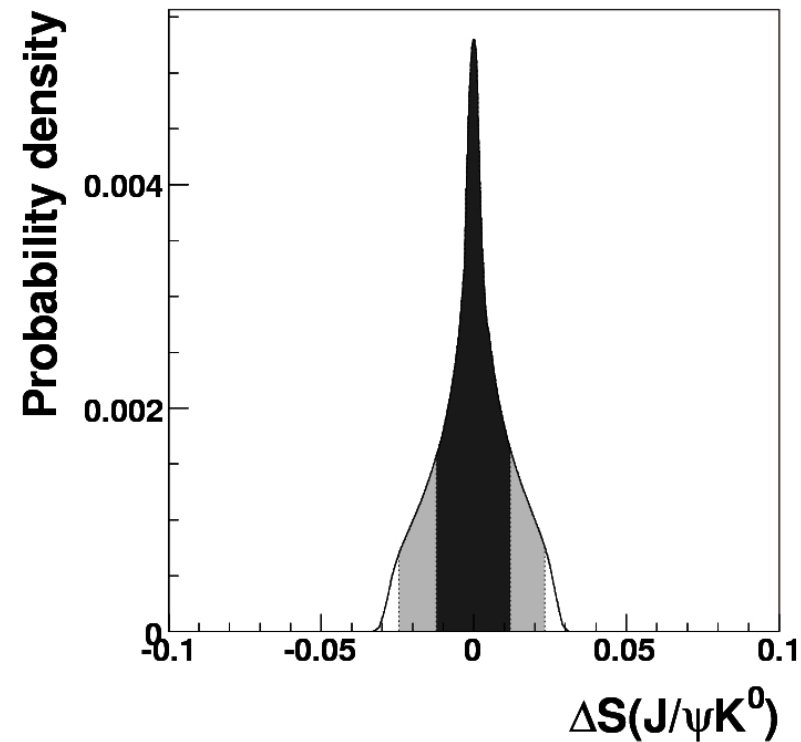
1. fix the *range* of $|P|$ from $J/\psi \pi^0$
2. use it to evaluate ΔS in $J/\psi K^0$

range instead of value: weaker use of SU(3)

Using this info in the $B^0 \rightarrow J/\psi K^0$ fit :

$BR^{tb} [10^{-4}]$	8.61 ± 0.35	$BR^{exp} [10^{-4}]$	8.63 ± 0.35
C_{CP}^{tb}	0.00 ± 0.015	C_{CP}^{exp}	0.001 ± 0.031
S_{CP}^{out}	0.759 ± 0.040	S_{CP}^{in}	0.759 ± 0.037
$ E_2 - P_2 $	1.465 ± 0.035		

The input from $B^0 \rightarrow J/\psi \pi^0$ i.e. $|P| < 1.7$ (99% probability) is crucial



$$\begin{aligned} \Delta S(J/\psi K^0) &= \sin 2\beta_{eff} - \sin 2\beta \\ &= 0.000 \pm 0.012 \end{aligned}$$

Perspectives for $\sin 2\beta$

WG2, preliminary

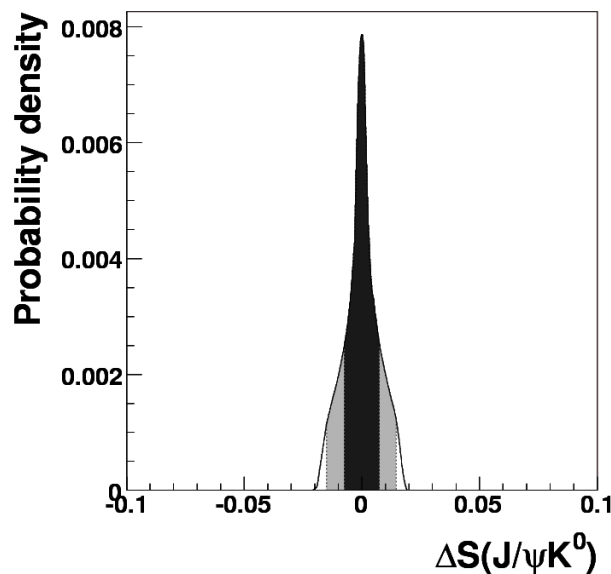
^a : statistical error only

BF (Now)	BF(End '08)	LHCb	LHCb	SBF	ITE
$\sim 1 \text{ ab}^{-1}$	2 ab^{-1}	2 fb^{-1}	10 fb^{-1}	50 ab^{-1}	
0.026 (4%)	0.023 (3.3%)	0.017 (2.4%) ^a	0.008 (1%) ^a	0.013 (2%)	$\lesssim 1\%$

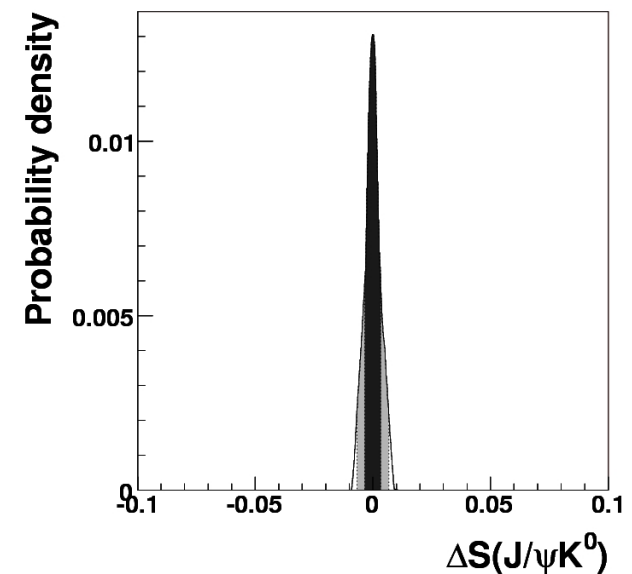
- the measurement is systematic-dominated at SBF
- theoretical error is likely not to be a problem

scaling the $B \rightarrow J/\psi \pi^0$ analysis:

2 ab^{-1} : $\Delta S = 0.000 \pm 0.007$



30 ab^{-1} : $\Delta S = 0.000 \pm 0.003$



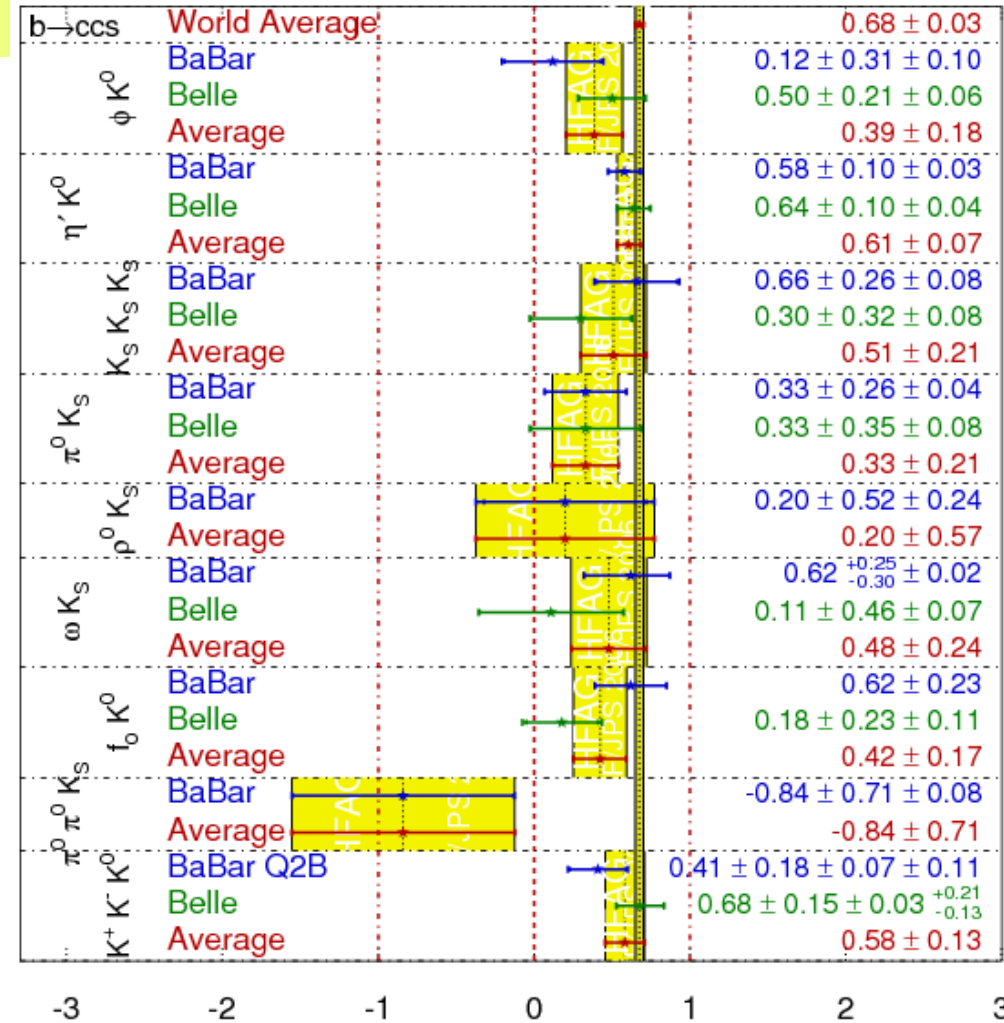
$\sigma^{\text{sys}}(S) > 0.005-0.01$
irreducible (and
largely correlated)
systematic error

β from $b \rightarrow q\bar{q}s$ and NP

- decays dominated by a single $b \rightarrow s$ penguin amplitude
 - $a_{CP}(t)$ measures $\sin 2\beta$ in the SM up to DCS corrections
 - NP corrections to penguins change the expectation $\sin 2\beta \rightarrow \sin 2\beta^{\text{eff}}$
- Best place to look for NP in 2-body non-leptonic decays

$$\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}})$$

HFAG
DPF/JPS 2006
PRELIMINARY

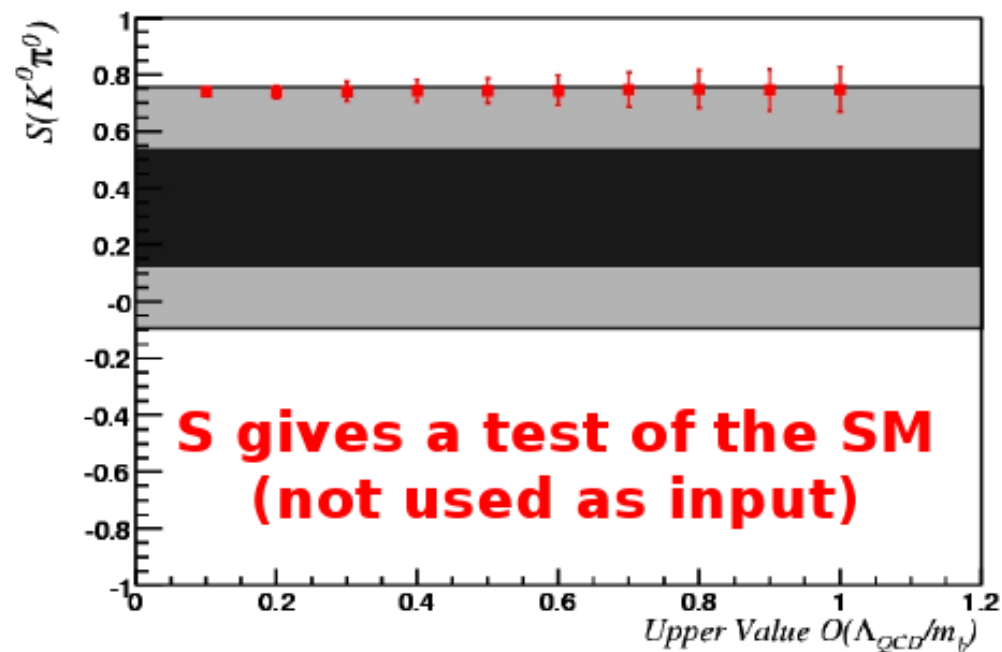


$$A(B^0 \rightarrow \phi K^0) = V_{cs} V_{cb}^* \times P_{1+2}(c) + V_{us} V_{ub}^* \times P_{1+2}(u)$$

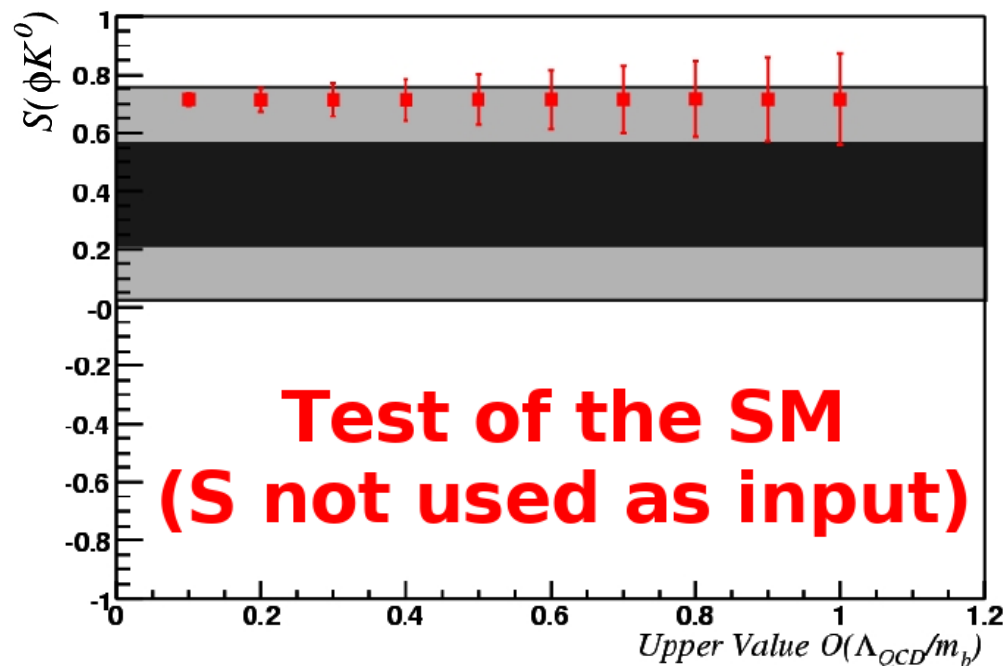
$$A(B^0 \rightarrow K^0 \pi^0) = V_{cs} V_{cb}^* \times \lambda^2 P_1(c) + V_{us} V_{ub}^* \times \{E_2 + \lambda^4 P_1(u)\}$$

Contributions from doubly-Cabibbo-suppressed SM amplitudes can fake a NP signal

- “Data-driven” method to put bounds on th. uncertainties: M.Pierini, CKM '06
- color-allowed emission diagram E_1 computed in factorization
 - leading P term fitted using BR_{ave} and direct CP asymmetries
 - other hadronic amplitudes varied between 0 and E_1



$$\Delta S(K^0\pi^0) < \sim 0.1$$



$$\Delta S(\phi K^0) < \sim 0.2$$

Using for instance QCD factorization to compute the amplitudes, ΔS can be evaluated

- the ratio P_u/P_c is roughly constant and universal: $\Delta S \sim 0.02$
- the ratio E_2/P_c is responsible for the different ΔS
- the size and sign of the corrections are dictated by the size of E_2/P_c and relative sign between the two ratios

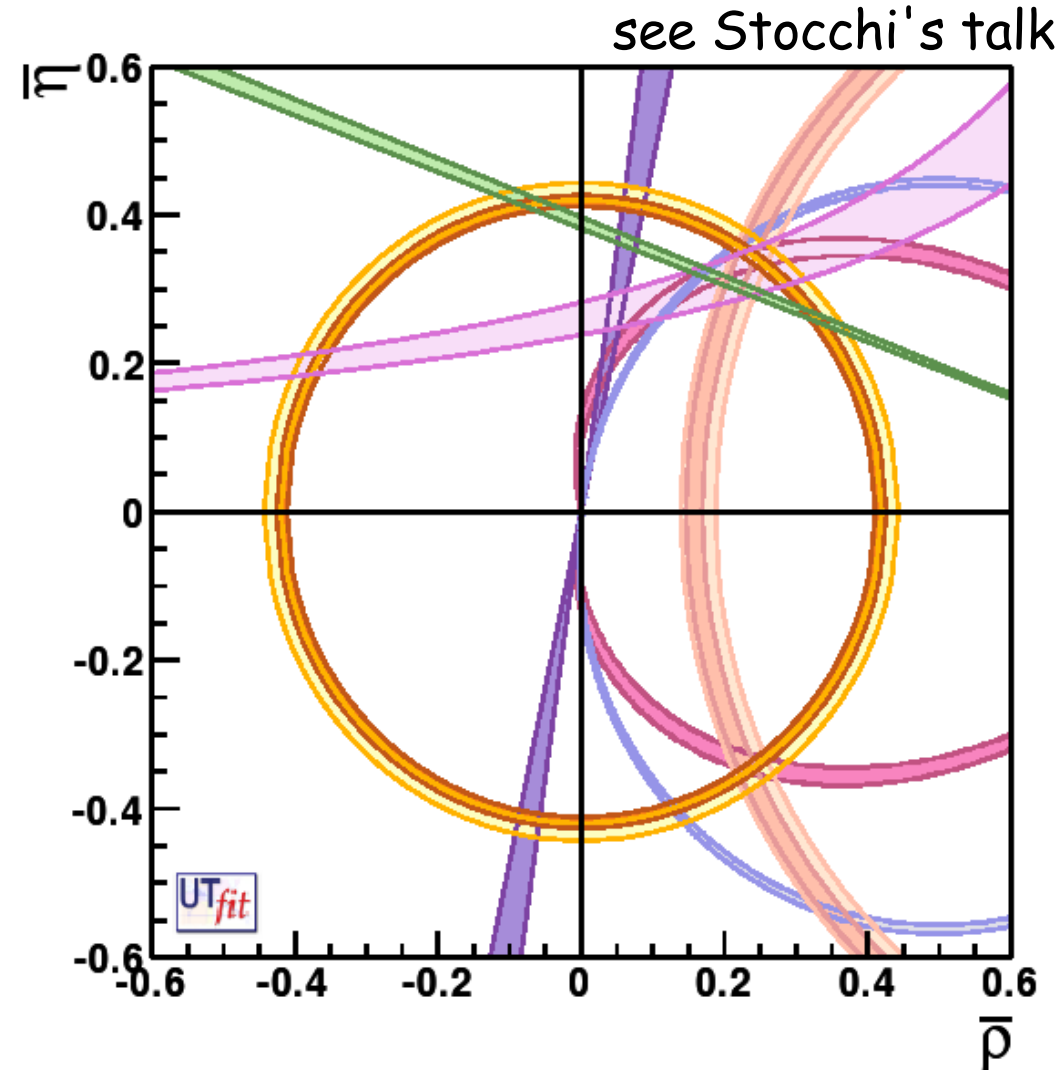
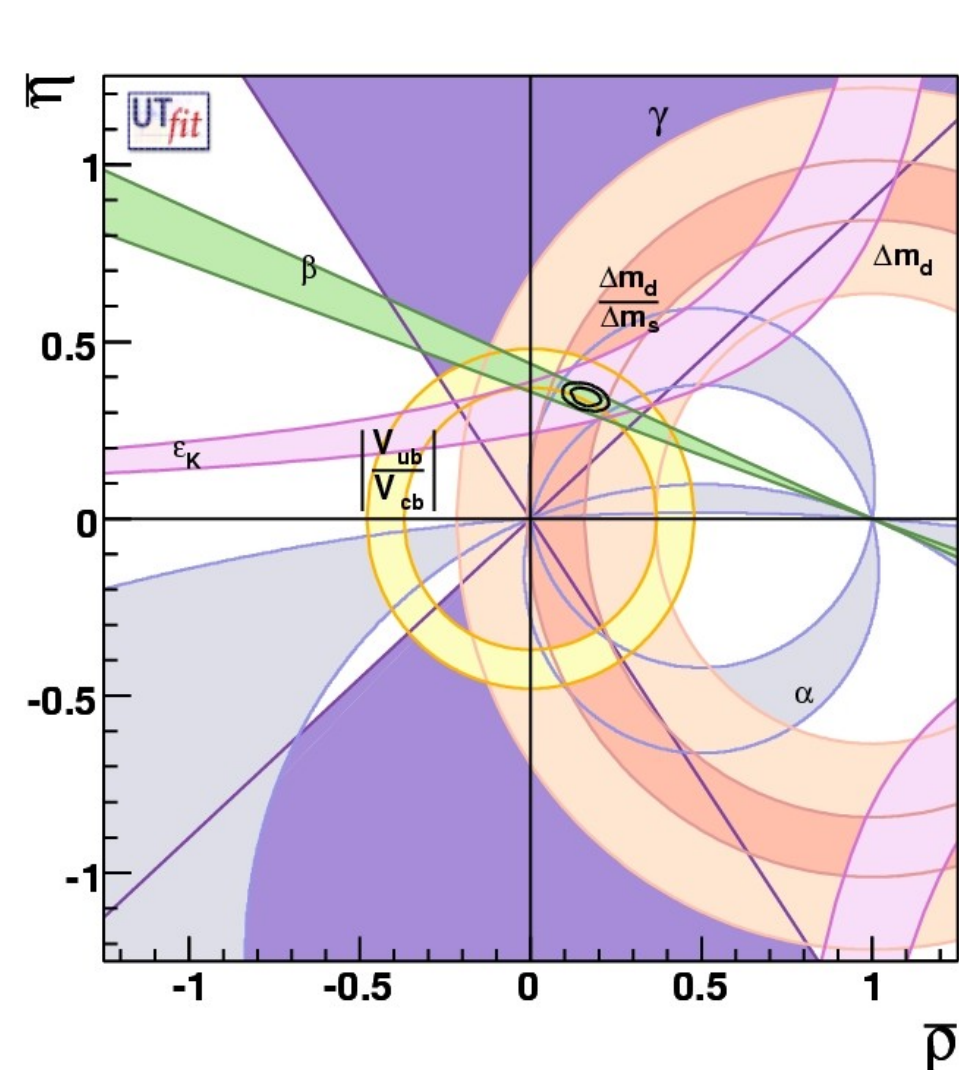
WG2, preliminary

Mode	ΔS_f (Theory)	ΔS_f [Range]	Mode	ΔS_f (Theory)	ΔS_f [Range]
$\pi^0 K_S$	$0.07^{+0.05}_{-0.04}$	[+0.03, 0.13]	$\rho^0 K_S$	$-0.08^{+0.08}_{-0.12}$	[-0.29, 0.01]
$\eta' K_S$	$0.01^{+0.01}_{-0.01}$	[+0.00, 0.03]	ϕK_S	$0.02^{+0.01}_{-0.01}$	[+0.01, 0.05]
ηK_S	$0.10^{+0.11}_{-0.07}$	[-0.76, 0.27]	ωK_S	$0.13^{+0.08}_{-0.08}$	[+0.02, 0.21]

but: factorization of c-penguins is still debated
power-suppressed terms can be important

Sides and angles consistent within present errors

It could be much different with SBF accuracy



Neutral meson mixing

Mixing amplitudes with generic NP contributions:

$$A_{B_q} = A_{B_q}^{SM} e^{2i\beta_q} + A_{B_q}^{NP} e^{2i(\beta_q + \phi_q^{NP})} = C_{B_q} e^{2i(\beta_q + \phi_{B_q})} A_{B_q}^{SM}$$

Observables:

$$\Delta m_q = 2 C_{B_q} |A_{B_q}^{SM}|$$

$$a_{SL}^q = \text{Im} \left(\Gamma_{12}^q / A_{B_q} \right)$$

$$\Delta \Gamma_q = -\Delta m_q \text{Re} \left(\Gamma_{12}^q / A_{B_q} \right)$$

$$S^{B_d \rightarrow J/\psi K_s} = \sin 2(\beta_d + \phi_{B_d})$$

$$S^{B_s \rightarrow \phi K_s} = \sin 2(\beta_s + \phi_{B_s})$$

$$a_{CH}^{\text{dimuon}} = f(a_{SL}^d, a_{SL}^s)$$

Standard Model: $C_{B_q}=1$, $\phi_{B_q}=0$

CKM fits determine ρ , η , C_{B_q} , ϕ_{B_q} simultaneously

UTfit, hep-ph/0509219; hep-ph/0605213

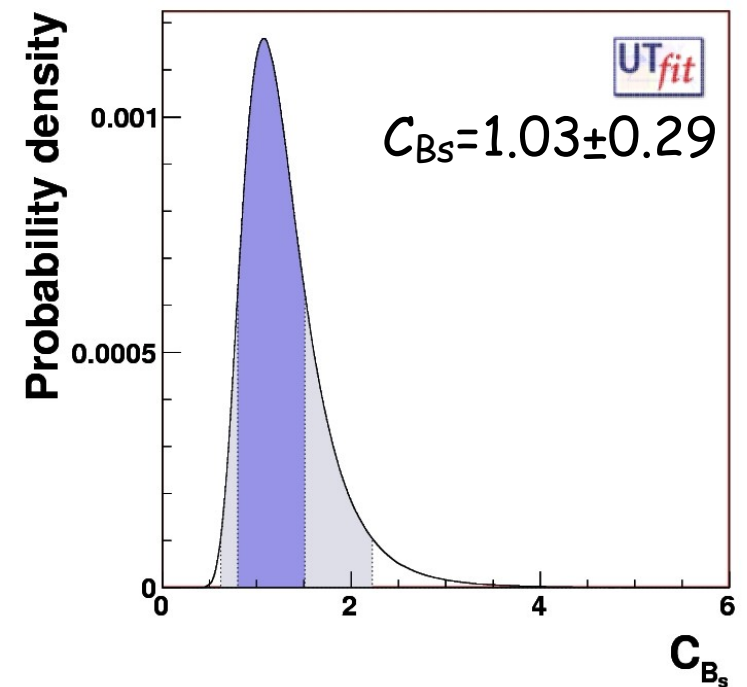
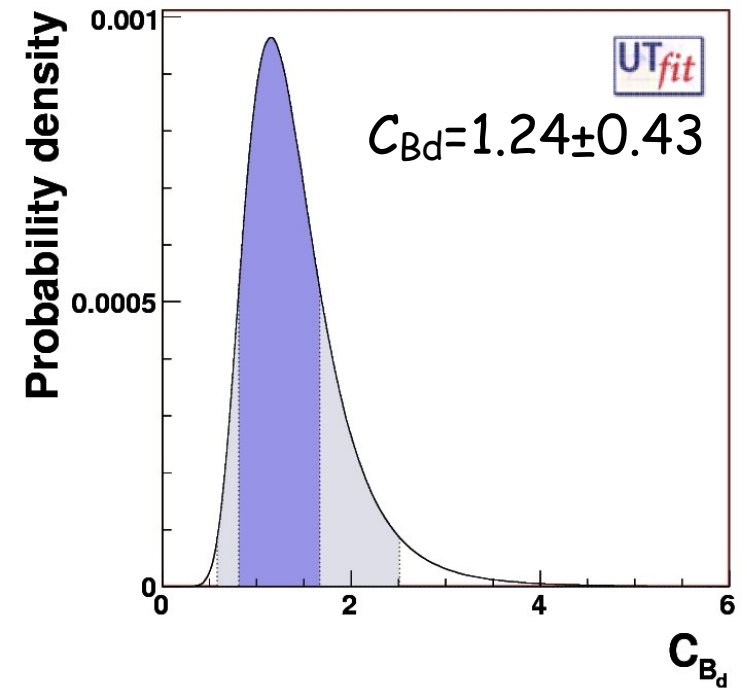
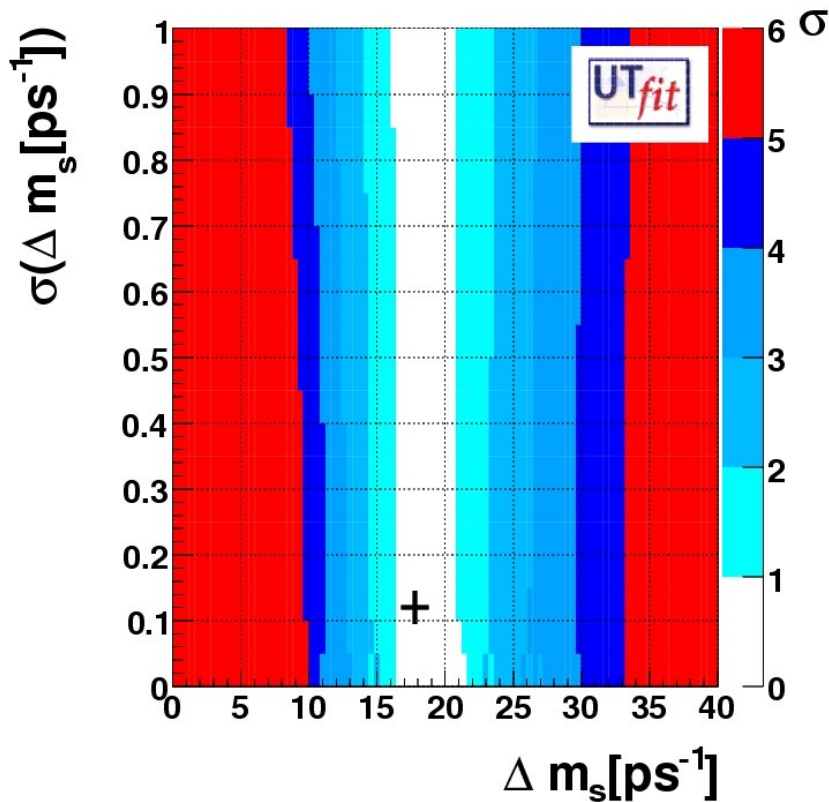
Information on the moduli

- B_d mixing:

$$\Delta m_d = 0.507 \pm 0.005 \text{ ps}^{-1}$$

- B_s mixing:

$$\Delta m_s = 17.77 \pm 0.01 \pm 0.07 \text{ ps}^{-1}$$



Accessing the B_s mixing phase

Recent ϕ_{B_s} -sensitive measurements from the Tevatron

$$a_{CH}^{dimuon} = (-9.2 \pm 4.4 \pm 3.2) \times 10^{-3} \quad \text{measured by D0}$$

$$\Delta \Gamma_s = (0.47_{-0.24}^{+0.19} \pm 0.01) \text{ ps}^{-1} \quad \text{measured by CDF}$$

$$A_{SL}^s = (24.5 \pm 19.3 \pm 3.5) \times 10^{-3} \quad \text{measured by D0}$$

+ the t -dependent angular analysis of $B_s \rightarrow J/\psi \phi$ by D0
resulting in a 3D likelihood for Γ_s , $\Delta \Gamma_s$ and ϕ_{B_s}

For comparison the SM expectations at NLO are:

MC et al, hep-ph/0308029; Lenz, Nierste, hep-ph/0612167

$$\Delta \Gamma_s / \Gamma_s = (11 \pm 4) \times 10^{-2}$$

$$A_{SL}^s = (2.3 \pm 0.5) \times 10^{-5}$$

Spectacular measurements of B_s observables
and particularly of ϕ_{B_s} expected from LHCb

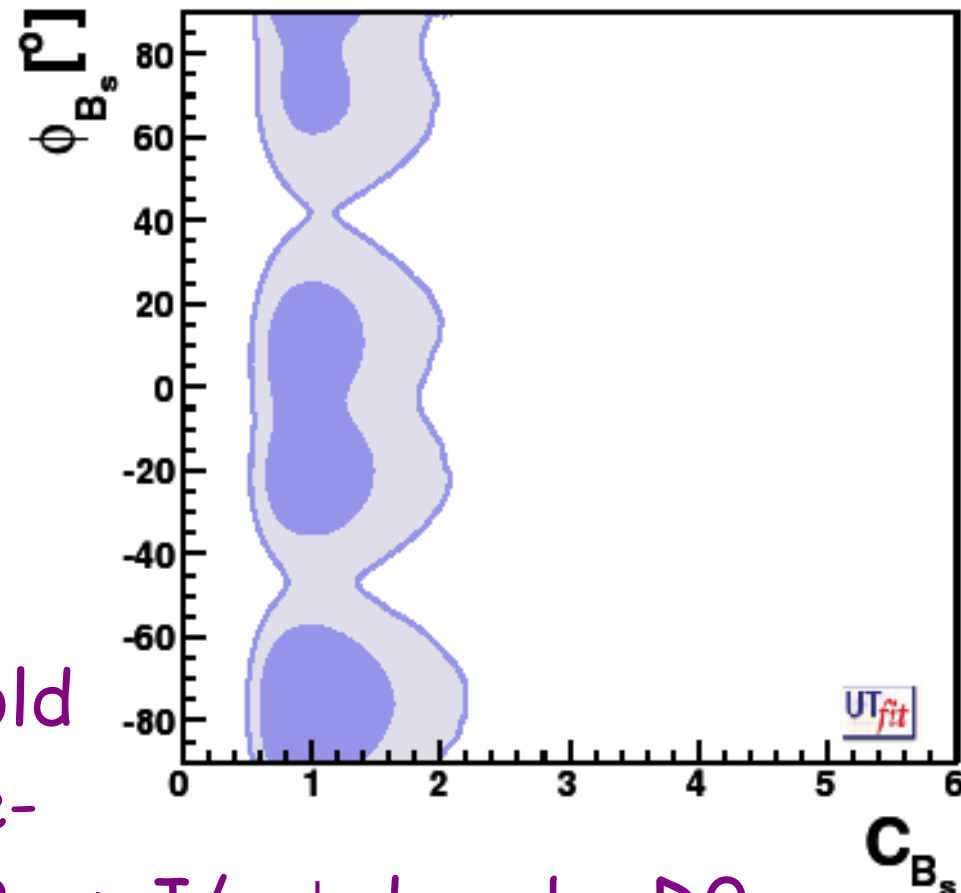
Information on the phases

- B_d mixing:

$$\phi_{B_d} = (-3 \pm 2)^\circ$$

- B_s mixing:

$$\phi_{B_s} = (-75 \pm 14)^\circ \cup (-19 \pm 11)^\circ \cup (9 \pm 10)^\circ \cup (102 \pm 16)^\circ$$



The result retains the the 4-fold ambiguity of the untagged time-dependent angular analysis of $B_s \rightarrow J/\psi \phi$ done by D0

$$(\phi_{B_s}, \cos\delta_{1,2}) \leftrightarrow (-\phi_{B_s}, -\cos\delta_{1,2}), (\phi_{B_s}, \Delta\Gamma_s) \leftrightarrow (\pi + \phi_{B_s}, -\Delta\Gamma_s)$$

only weakly lifted by A_{CH} and A_{SL}^s which prefer $\phi_{B_s} < 0$

Factorization and $SU(3)$ give opposite signs for $\cos\delta_{1,2}$

MSSM + generic soft SUSY-breaking terms

A useful tool: the mass-insertion approximation

SuperCKM basis + perturbative smass diagonalization

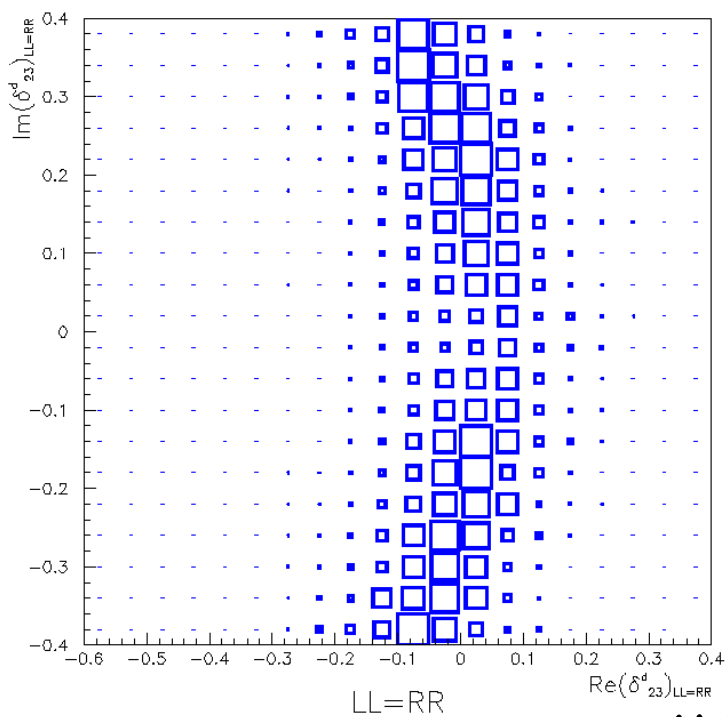
► expansion parameters: $(\delta_{ij}^q)_{AB} \equiv \frac{(M_{ij}^2)^q_{AB}}{\tilde{m}^2}$

All flavour-changing NP effects in the squark propagators

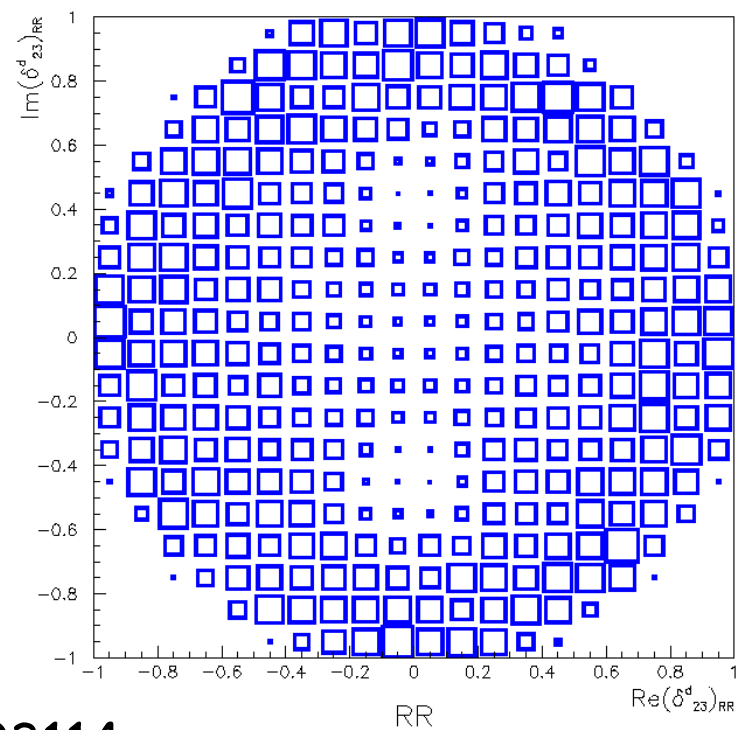
$$\begin{array}{ccc} & (\delta_{ij}^q)_{AB} & q = \{u, d\}, (A, B) = \{L, R\} \\ (\tilde{q}_i)_A & \text{---} \text{---} \text{---} \times \text{---} \text{---} \text{---} & (\tilde{q}_j)_B \quad (i, j) = \{1, 2, 3\} \end{array}$$

**FCNC and CP violation impose
model-independent bounds on the δ 's**

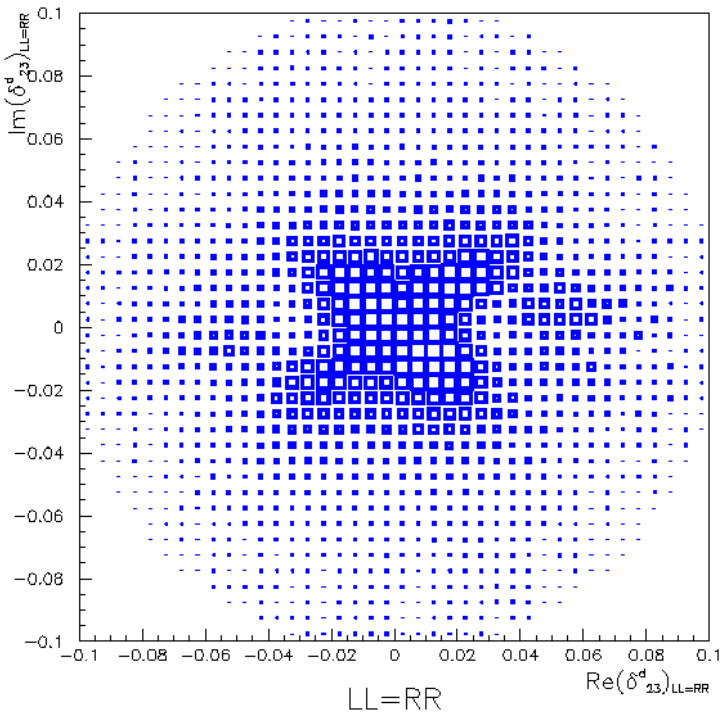
NB: only dominant gluino contributions are considered



← before →
 Tevatron B_s
 measurements
 $|\delta|$ unbounded
 $m_{gl} = m_{sq} = -\mu = 350 \text{ GeV}$
 $\tan \beta = 3$

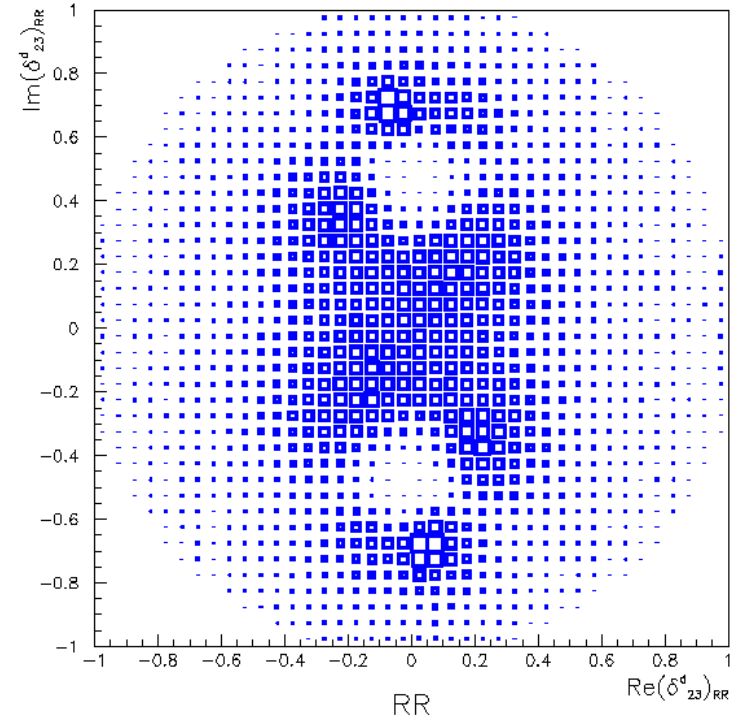


MC, Silvestrini, hep-ph/0603114



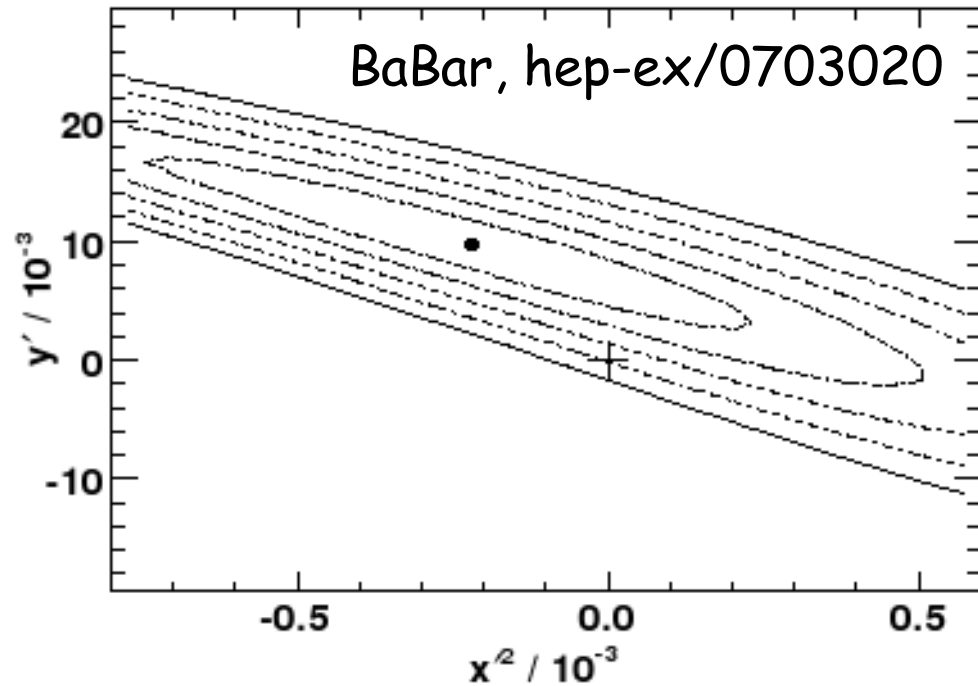
← after →
 Tevatron B_s
 measurements

$ (\delta_{23}^d)_{LL} $	$ (\delta_{23}^d)_{RR} $
$2 \cdot 10^{-1}$	$7 \cdot 10^{-1}$
$ (\delta_{23}^d)_{LL=RR} $	$(\delta_{23}^d)_{LR,RL}$
$5 \cdot 10^{-2}$	$5 \cdot 10^{-3}$



WG2, preliminary

Breaking news: evidence of $D-\bar{D}$ mixing



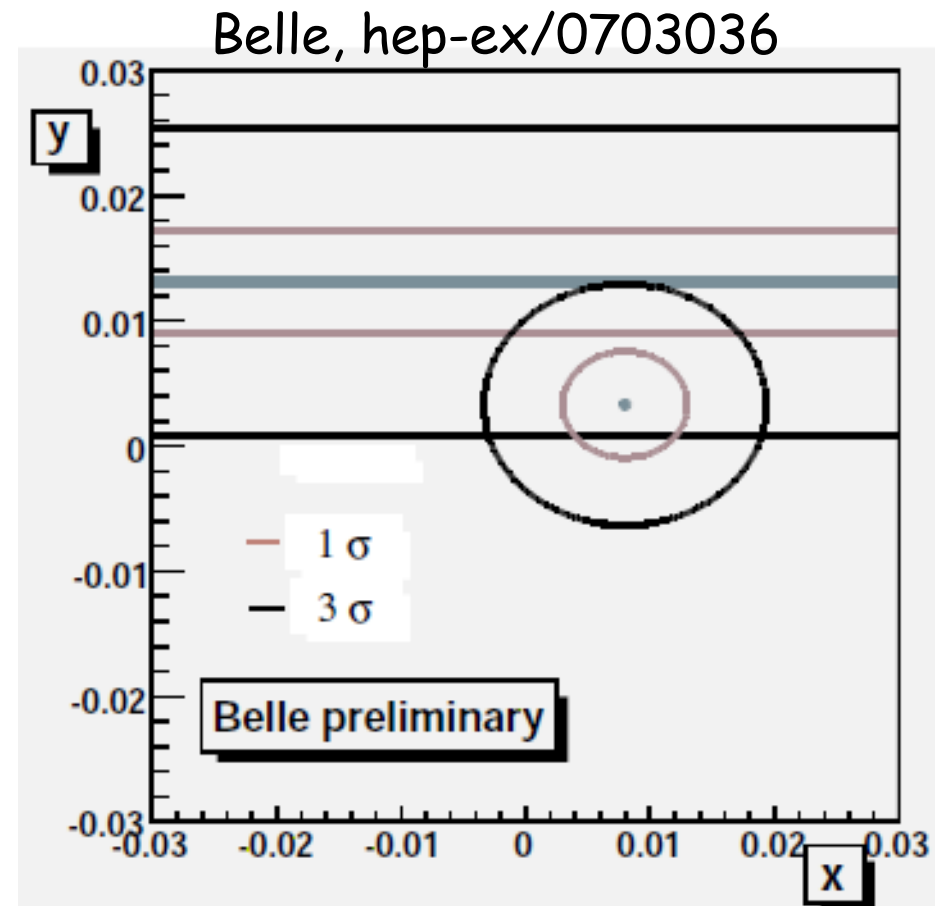
Parameter	Value
x_+^{f2}	$(-0.24 \pm 0.43 \pm 0.30) \cdot 10^{-3}$
x_-^{f2}	$(-0.20 \pm 0.41 \pm 0.29) \cdot 10^{-3}$
y_+^f	$(9.8 \pm 6.4 \pm 4.5) \cdot 10^{-3}$
y_-^f	$(9.6 \pm 6.1 \pm 4.3) \cdot 10^{-3}$
x	$(8.0 \pm 3.4) \cdot 10^{-3}$
y	$(3.3 \pm 2.8) \cdot 10^{-3}$
y_{CP}	$(13.1 \pm 4.1) \cdot 10^{-3}$
A_Γ	$(0.1 \pm 3.4) \cdot 10^{-3}$

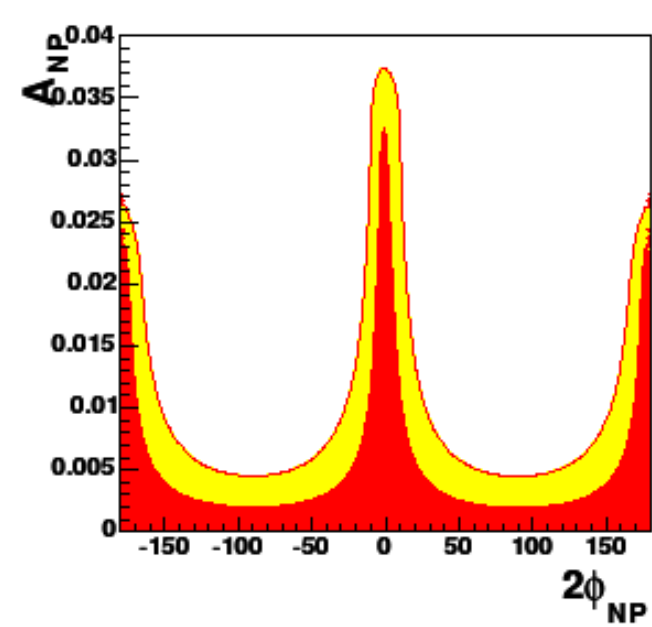
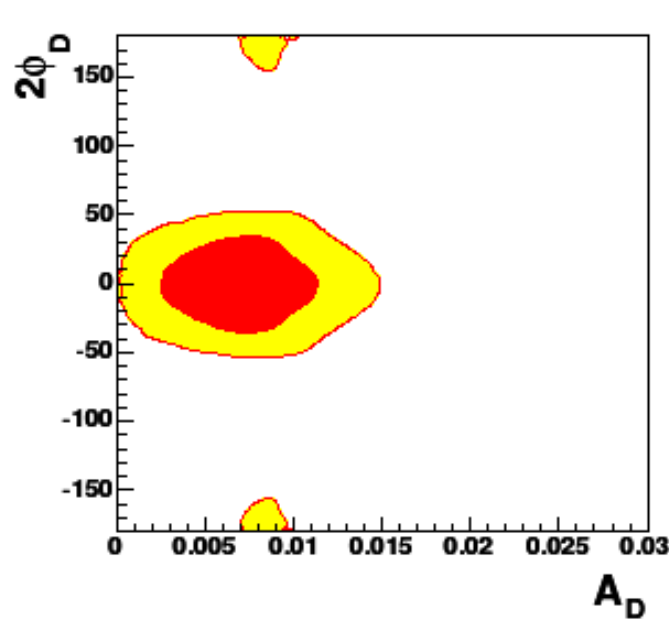
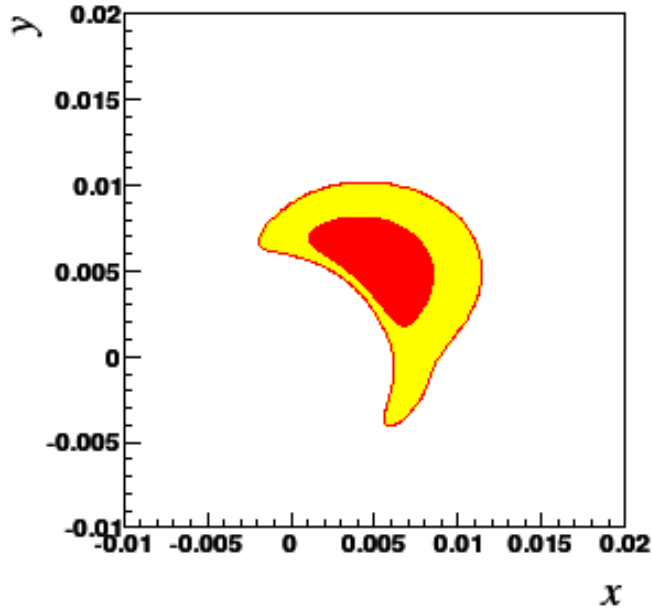
$$y'_\pm = (1 \pm A_m)(y' \cos 2\phi_D \mp x' \sin 2\phi_D),$$

$$x_\pm^{f2} = (1 \pm 2A_m)(x' \cos 2\phi_D \pm y' \sin 2\phi_D)^2,$$

$$y_{CP} = y \cos 2\phi_D - A_m x \sin 2\phi_D,$$

$$A_\Gamma = A_m y \cos 2\phi_D - x \sin 2\phi_D,$$



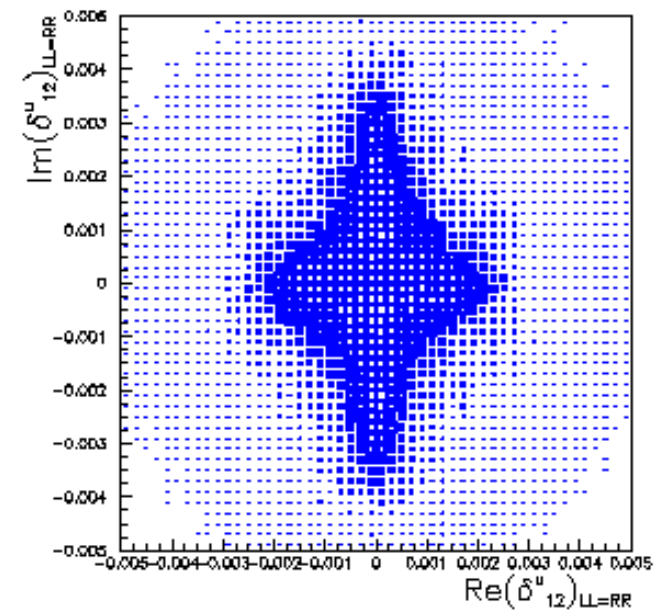
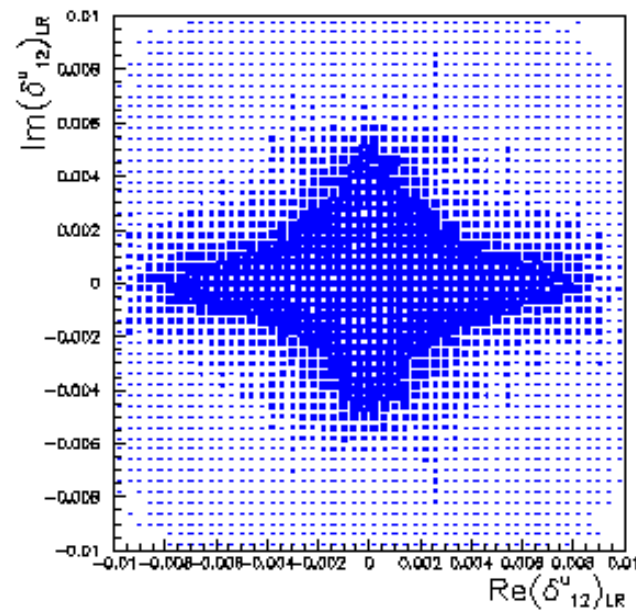
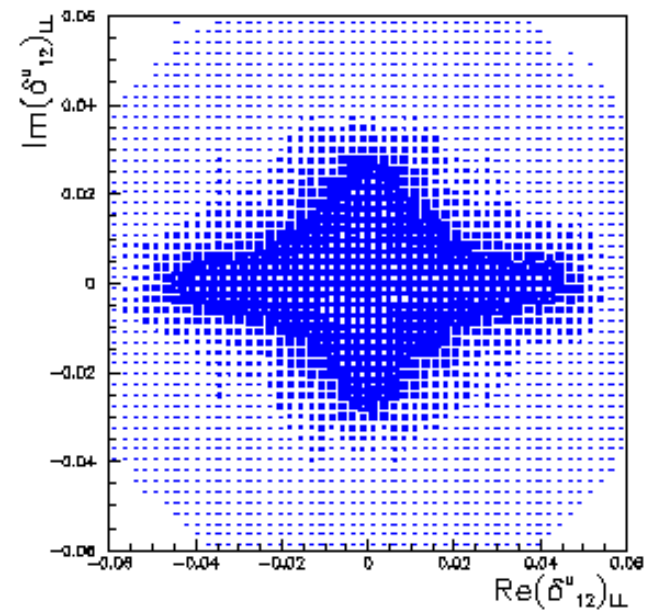


MC at al, hep-ph/0703204
 Nir, hep-ph/0703235

Bounds on MSSM MI's improved by an order of magnitude

$m_{\tilde{q}}$	$m_{\tilde{g}}$	$ (\delta_{12}^u)_{LL,RR} $	$ (\delta_{12}^u)_{LR,RL} $	$ (\delta_{12}^u)_{LL=RR} $
350	350	0.032	0.0056	0.0027

**Quark-squark alignment ruled out
 for squark masses within LHC reach**



Conclusions (i)

γ is poorly known (error $\sim 20\%$ + ambiguity) but will improve with statistics. No systematic and th. limitations up to SBF. Very well measured at LHCb

Error on α is 10% if ambiguities are eliminated with external information. Measurement eventually limited by isospin breaking effects.
Sensitive to EWP-like NP

β known at 5%. Systematically limited at $\sim 10 \text{ ab}^{-1}$. Th. error under control in $b \rightarrow c\bar{c}s$, limiting factor for NP searches in $b \rightarrow s$ penguin dominated decays

Conclusions (ii)

NP contributions to the B_d mixing amplitude are quite constrained from B-factory data

Missing information on B_s physics starts coming from the Tevatron. Modulus of the amplitude is already strongly constrained. First information on the phase start to put some non-trivial bound

Evidence of $D-\bar{D}$ mixing constrains NP in the up sector. SUSY models with quark-squark alignment are no longer viable unless $m_{sq} > 2 \text{ TeV}$

T H A N K S
TO

the WG2 subgroup conveners

angles: M. Bona, A. Soni,

K. Trabelsi, G. Wilkinson

b- \rightarrow s: F. Muheim

mixing: V. Lubicz, J. van Huten

and to all the contributors
(even the virtual ones...)