Ratchet Model of Baryogenesis - Part 2

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May 9, 2011 @ Pheno 2011

Work in collaboration with

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Review of Part 1 (from last year)

- Many models of baryogenesis generate baryon number via the coherent evolution of a scalar field which carries baryon number:

  - S. Dimopoulos and L. Susskind, Phys. Rev. D 18, 4500 (1978)
  - I. Affleck and M. Dine, Nucl. Phys. B 249, 361 (1985)
  - A. G. Cohen and D. B. Kaplan, Phys. Lett. B 199 (1987) 251
  - A. Dolgov and K. Freese, Phys. Rev. D 51, 2693 (1995)
  - Etc.

E.g. Dimopoulos-Susskind Model:

- Phys. Rev. D18, 4500 (1978)
- Consider a scalar field which carries Baryon number:

$$J^{\mu} = i\phi^{*}\overleftrightarrow{\partial}^{\mu}\phi, \qquad B(t) = \int d\vec{x}\, J^{0}(\vec{x},t)$$

- Assume B, C, and CP violating potential:

$$V(\phi) = \lambda(\phi\phi^{*})^{n}(\phi + \phi^{*})(\alpha\phi^{3} + \alpha^{*}\phi^{*3})$$

$$B : \phi \longrightarrow e^{i\gamma}\phi$$

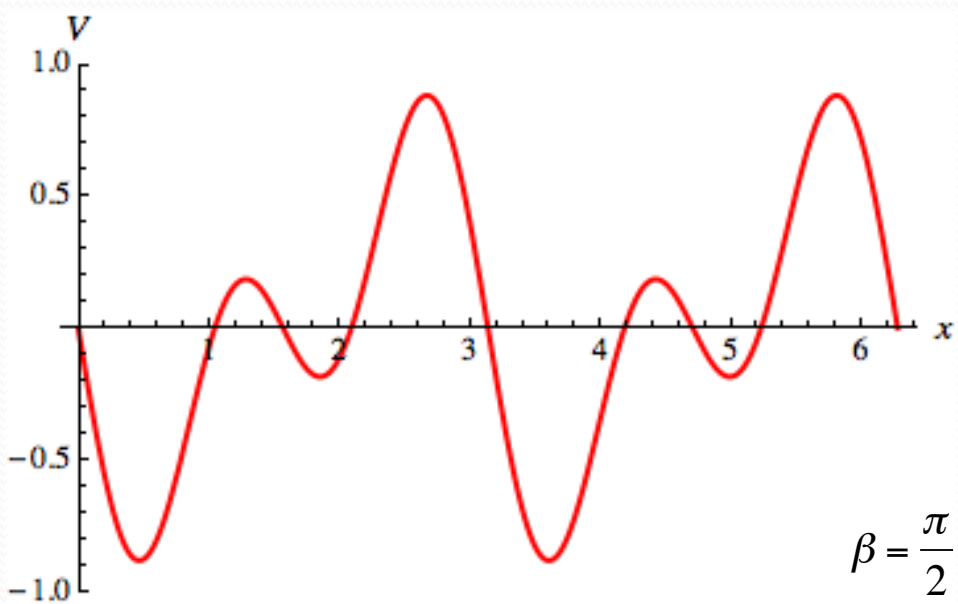
$$C : \phi \longrightarrow \phi^{*}$$

$$CP : \phi(t,x) \longrightarrow \pm\phi^{*}(t,-x)$$

- Baryon number depends only on the time derivative of the phase:

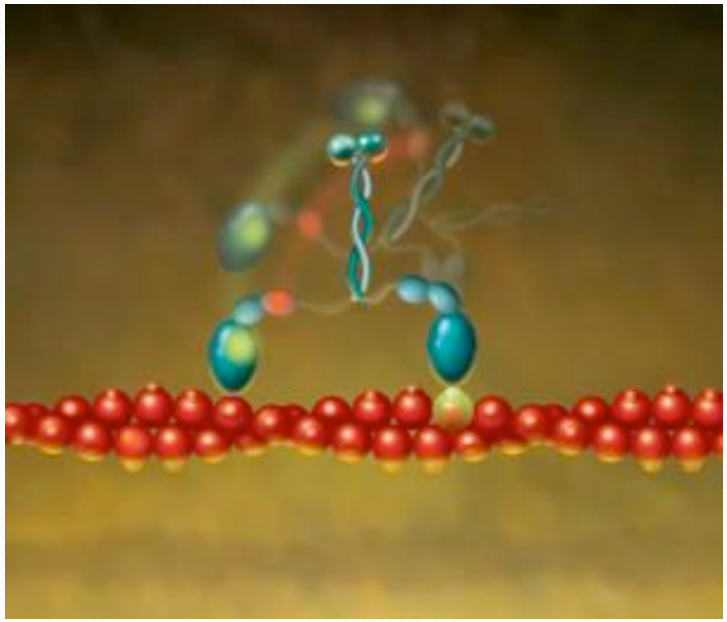
$$\hat{\phi} = \left|\hat{\phi}\right| e^{i\theta}, \qquad B = R^3(t)\, i\phi \overleftrightarrow{\partial}_t \phi^* = i\hat{\phi}\overleftrightarrow{\partial}_\tau \hat{\phi}^* = 2\frac{d\theta}{d\tau}$$

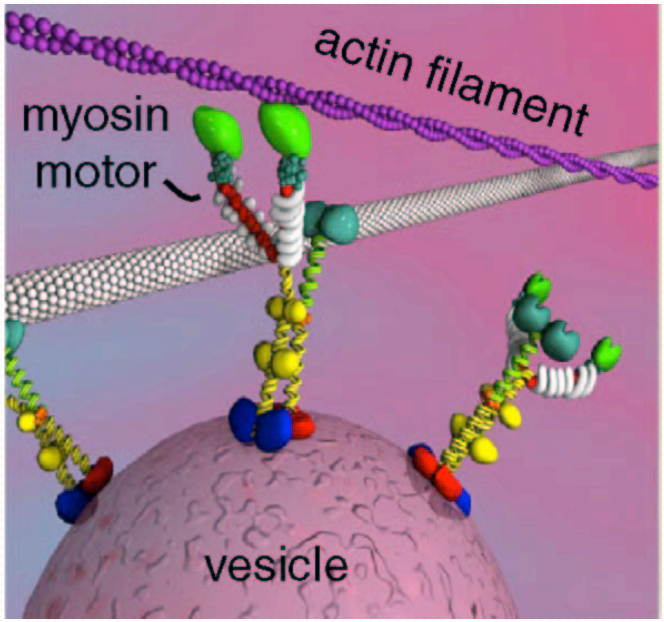
$$V(\theta) = 4\lambda \left|\hat{\phi}\right|^{4+2n} \cos\theta \cdot \cos(3\theta + \beta), \qquad \alpha = e^{i\beta}$$



Ratchet Model:

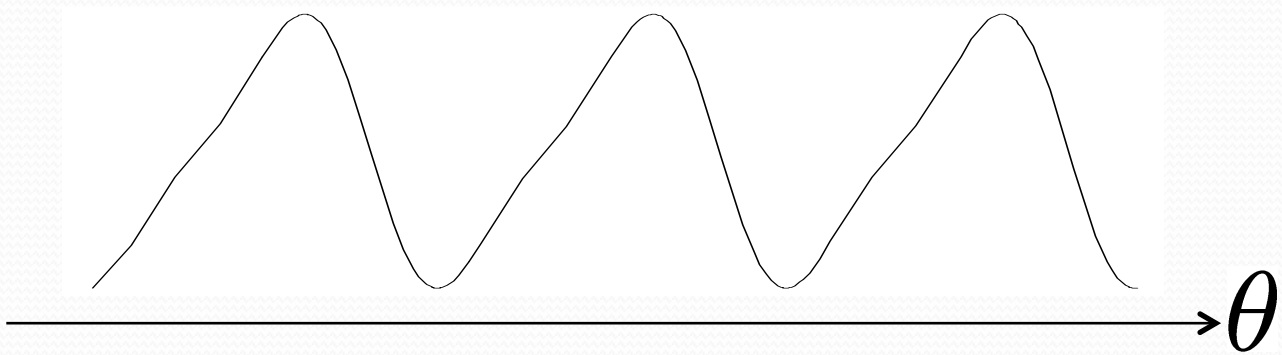
- Used in the theory of biological motors.



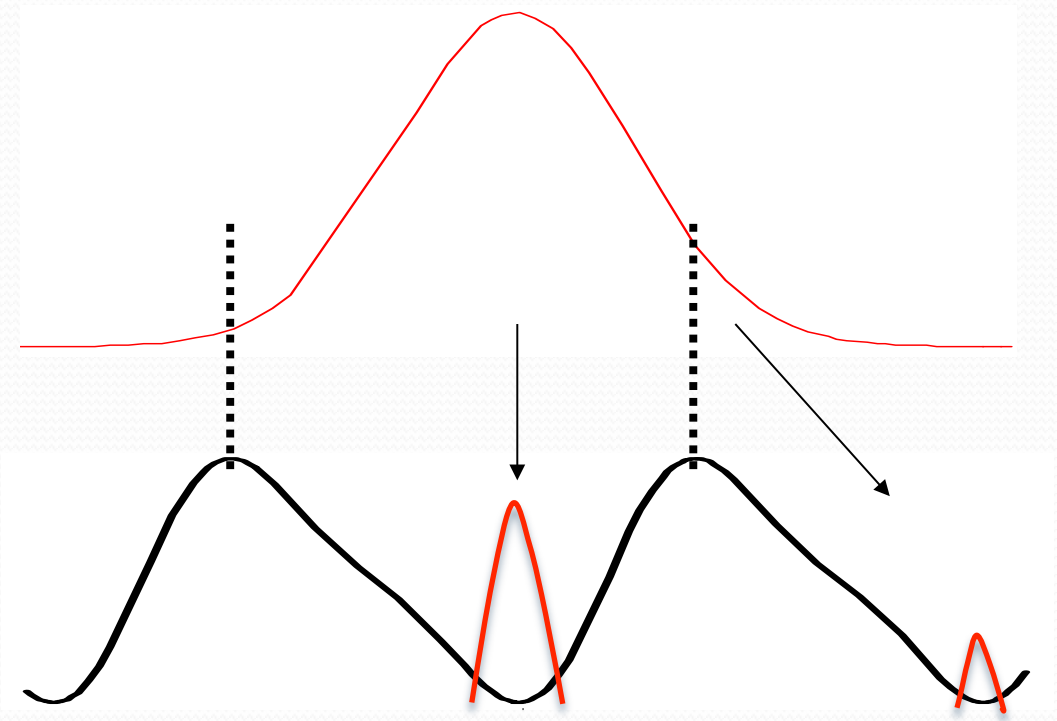


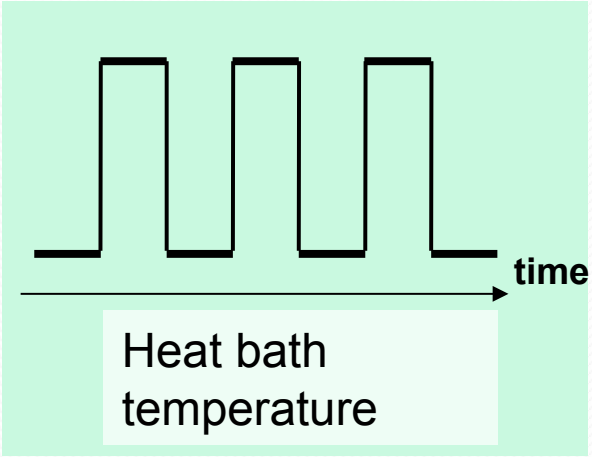
Ratchet Model:

- Generates directed motion from random thermal fluctuations without a biased external force.
  For the mechanism to work, it is known that:
  1. The potential must be spatially asymmetric.
  2. The heat bath must transition either periodically or randomly between two or more states.



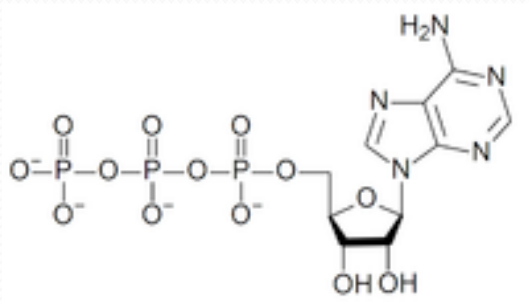
Ratchet Model:

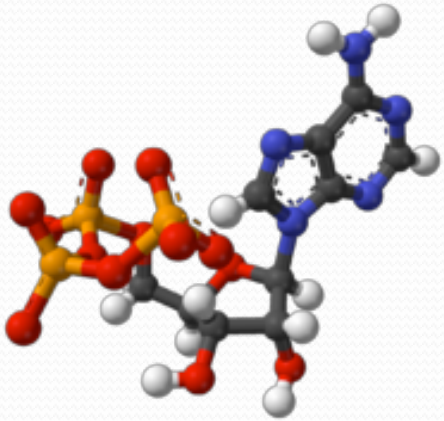




ATP:

- Biological motors are fueled by ATP (Adenosine Tri-Phosphate)





- Introduce interaction with ATP-like particle:

$$\phi + \Phi_{ATP} \leftrightarrow \phi + \Phi_{ADP} + Q$$

Assume Q ≈ potential barrier height.

Equation of Motion:

- Dimopoulos-Susskind with $n = 0$

$$\frac{d^2\theta}{d\tau^2} + \frac{\partial V}{\partial \theta} + \lambda^2 \frac{d\theta}{d\tau} = 0$$

$$V = \lambda(\phi + \phi^*)(\alpha\phi^3 + \alpha^*\phi^{*3})$$
$$= 4\lambda \left|\hat{\phi}\right|^4 \cos\theta \cdot \cos(3\theta + \beta)$$

- With fluctuating thermal bath

$$\frac{d^2\theta}{d\tau^2} + \frac{\partial V}{\partial \theta} + \lambda^2 \frac{d\theta}{d\tau} - \sqrt{2D(\tau)}\,\xi(\tau) = 0$$

$$\langle \xi(\tau) \rangle = 0, \quad \langle \xi(\tau)\xi(\sigma) \rangle = \delta(\tau - \sigma)$$

Fokker-Planck Equation:

- The equation of motion is equivalent to:

$$\frac{\partial}{\partial \tau} p(\theta,\tau) + \frac{\partial}{\partial \theta} j(\theta,\tau) = 0$$

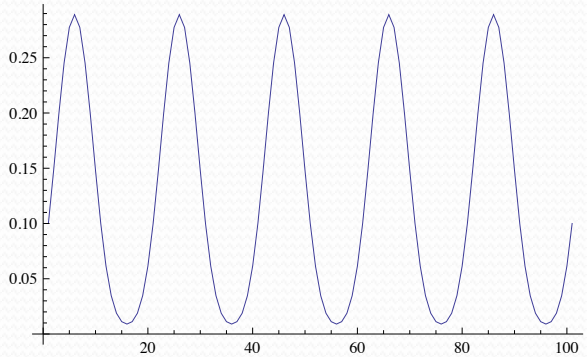
$$j(\theta,\tau) = -\left[V'(\theta) + D(t) \frac{\partial}{\partial \theta} \right] p(\theta,\tau)$$

- Assume:

$$D(\tau) = D_0 \left[1 + A \sin(\omega \tau) \right]^2$$

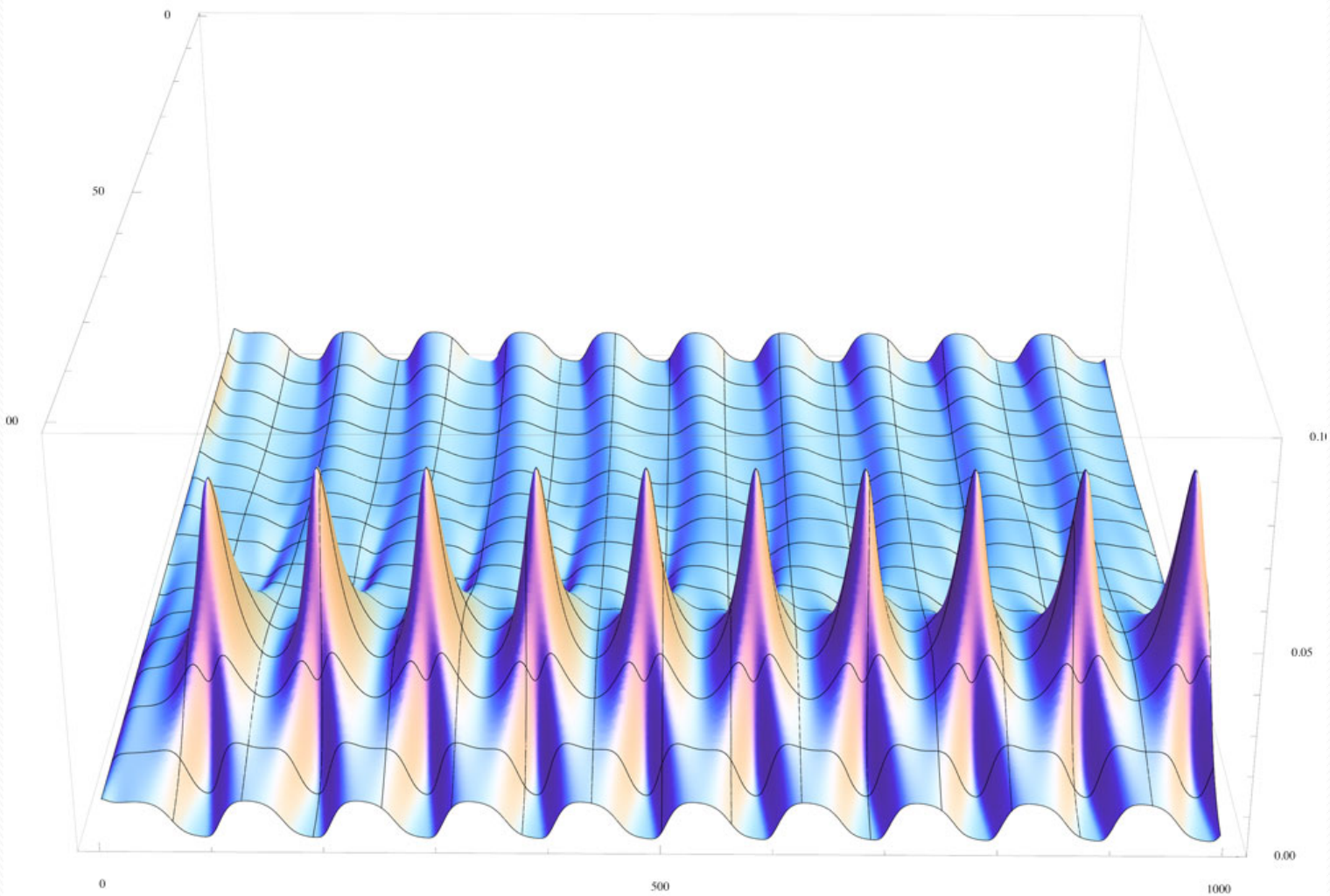
- Generate:

$$B \propto \frac{1}{T} \int_0^T j(\theta,\tau)\, d\tau$$

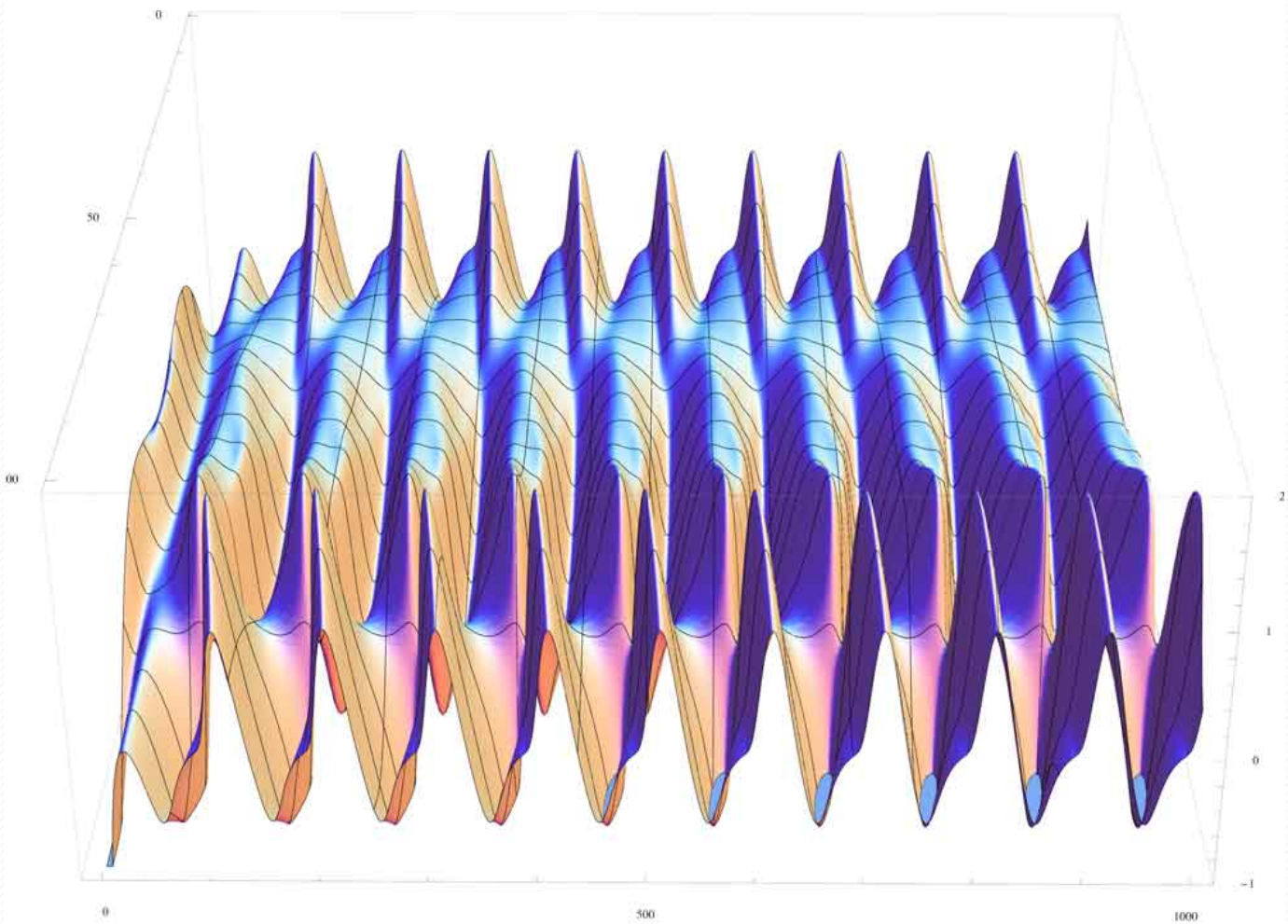


- P. Reimann, et al., Phys. Lett. A215 (1996) 26

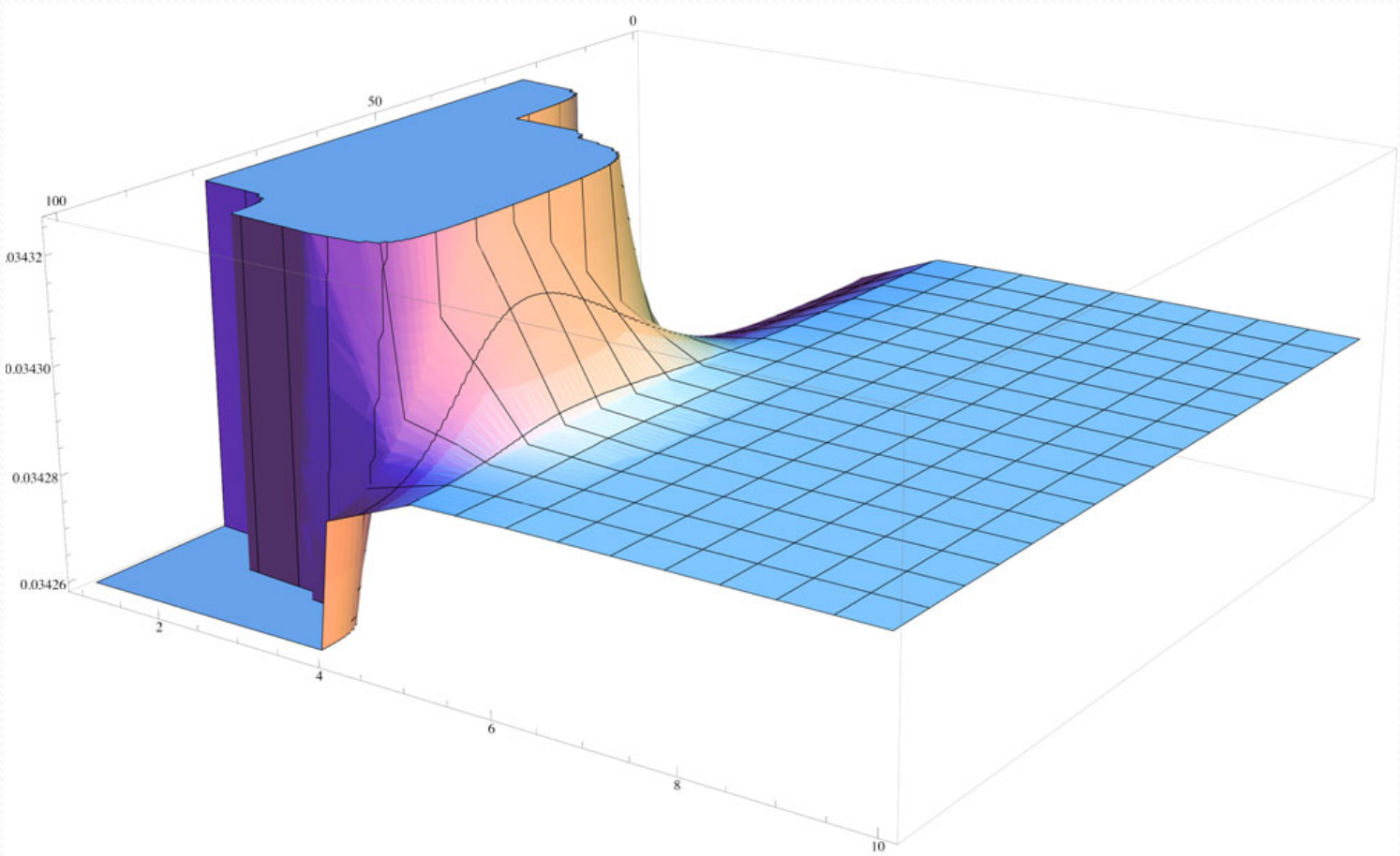
Sample Solution: Probability Density



Sample Solution: Current Density



Sample Solution: Average Current per Period



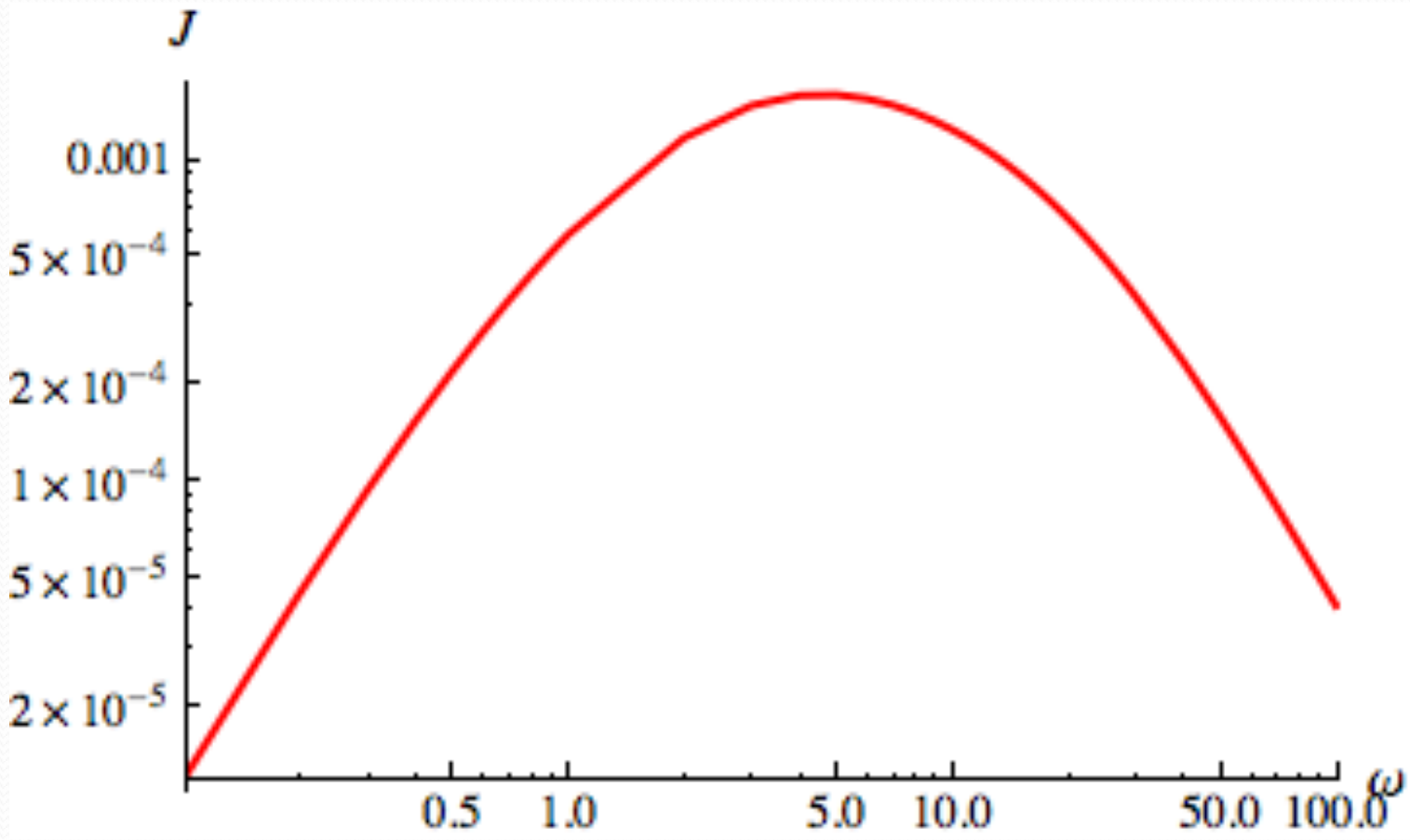
What could the ATP/ADP particles be?

$$\phi + \Phi_{ATP} \leftrightarrow \phi + \Phi_{ADP} + Q$$

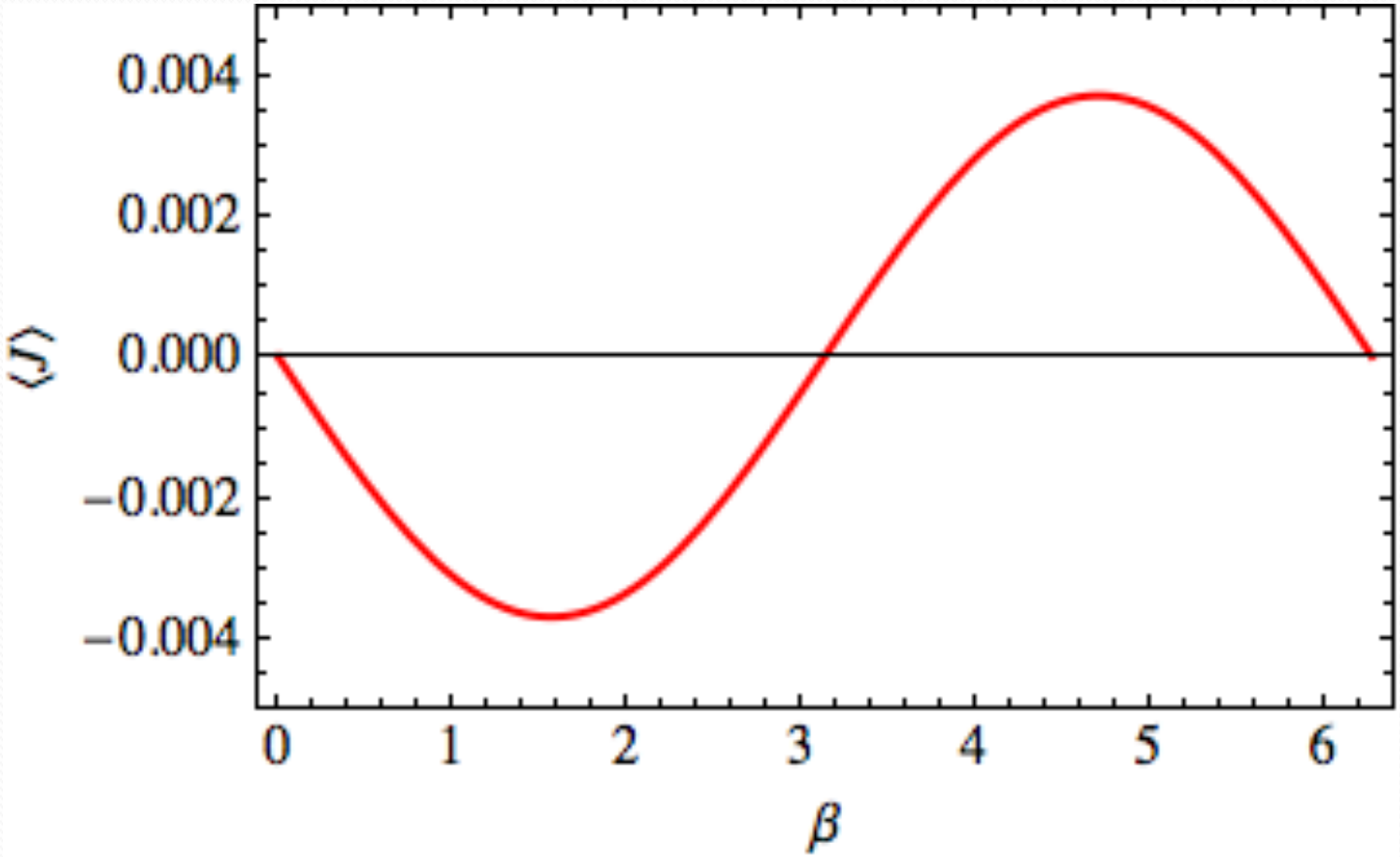
- Inflaton at reheating?
- KK modes?
- Technimesons?

End of Part 1

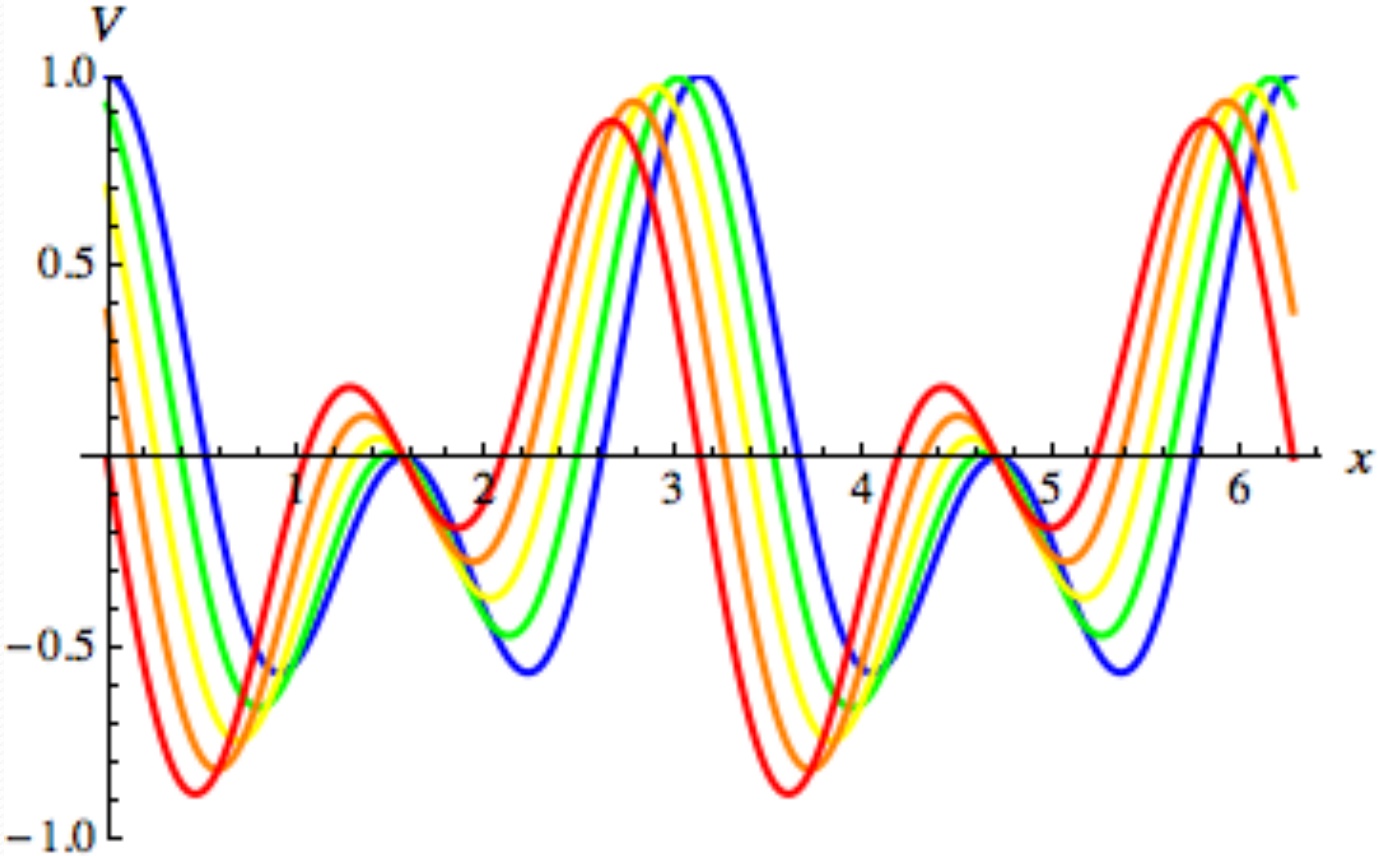
Omega dependence of the induced current:



Beta-dependence of the induced current:



Beta-dependence of the potential:



$$V(x) \propto \cos(x)\cos(3x + \beta)$$

Fast Oscillation Expansion:

P. Reimann, et al., Phys. Lett. A215 (1996) 26

Expand in inverse powers of the oscillational frequency of D(t):

$$\frac{\partial}{\partial \tau} p(\theta,\tau) + \frac{\partial}{\partial \theta} j(\theta,\tau) = 0$$

$$j(\theta,\tau) = -\left[V'(\theta) + D(t)\frac{\partial}{\partial \theta}\right] p(\theta,\tau)$$

$$p(x,\tau) = \sum_{n=0}^{\infty} \omega^{-n} p_n(x,\tau)$$

$$\leftarrow \quad j(x,\tau) = \sum_{n=0}^{\infty} \omega^{-n} j_n(x,\tau)$$

$$J = \sum_{n=0}^{\infty} \omega^{-n} J_n$$

Fast Oscillation Expansion:

$$\frac{\partial p_0(x,\tau)}{\partial \tau} = 0$$

$$\frac{\partial p_n(x,\tau)}{\partial \tau} = \frac{\partial}{\partial x}\left[V'(x) + D(\tau)\frac{\partial}{\partial x}\right]p_{n-1}(x,\tau)$$

$$j_n(x,\tau) = -\left[V'(x) + D(\tau)\frac{\partial}{\partial x}\right]p_n(x,\tau)$$

$$J_n = \frac{1}{2\pi}\int_0^{2\pi} d\tau\; j_n(x,\tau)$$

Can be solved order by order.

Results:

$$J_0 = 0$$

$$J_1 = 0$$

$$J_2 \propto \int_0^L dx\, V'(x)\left[V''(x)\right]^2$$

For V(x) = cos(x) cos(3x+β) we have:

$$\int_0^L dx\, V'(x)\left[V''(x)\right]^2 = -12\sin\beta$$

Next order:

After some tedious calculations, we find:

$$J_3 = A \int_0^L dx\, V'(x) \left[V'''(x)\right]^2$$
$$+ B \int_0^L dx\, V'''(x) \left[V'(x)\right]^4$$
$$+ C \int_0^L dx \left[V'(x)\right]^2 V''(x) V'''(x)$$

For V(x) = cos(x) cos(3x+β) the A and B integrals are proportional to sinβ, while the C integral vanishes.

Higher order contributions seem difficult to calculate.

Conclusions:

- For the potential $V(x) = \cos(x)\cos(3x+\beta)$, numerical calculations indicate that the generated current is proportional to $\sin\beta$.
- There must exist a very good reason for such a simple dependence on β.
- Using the fast oscillation expansion, we were able to show analytically that the current must be proportional to $\sin\beta$ up to order ω^{-3}
- We have not been able to prove that the current is proportional to $\sin\beta$ to all orders (yet).