SU(3) flavor symmetry in D Decays

Bhubanjyoti Bhattacharya

Dept. of Physics and Enrico Fermi Institute
University of Chicago
Talk at Pheno 2011

May 9, 2011

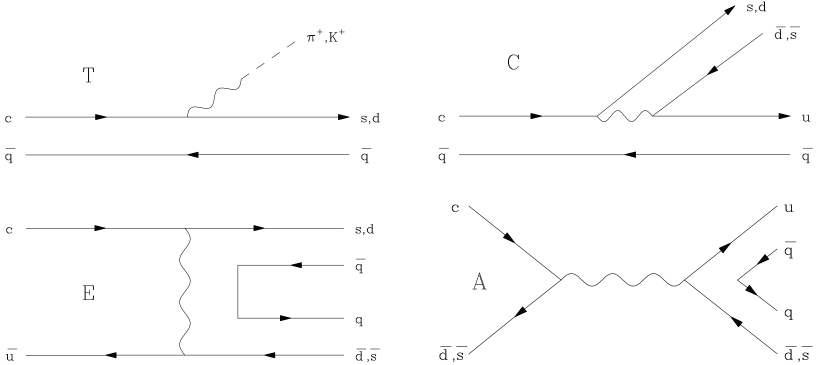
Work done in collaboration with J. L. Rosner. and C-W. Chiang
Thanks for useful discussions: M. Dubrovin, B. Meadows,
K. Mishra, R. Andreassen, BABAR Collaboration.

- Motivation
- SU(3) flavor symmetry
  - D decays to a pair of pseudoscalars
  - D decays to a pseudoscalar and a vector
- D decays to three bodies: Dalitz plots
  - $D^0 \to 3\pi$ Dalitz plot
  - Cross ratios in different Dalitz plots
- Summary

Motivation

- CP violation in the Standard Model: CKM matrix
- $\gamma = Arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$
- $B^\pm \to D\,K^\pm$ + Multibody D decays : Large strong phases from resonances (PRD **68**, 054018 (2003), Giri et. al.)
  We are interested in :
- Relative phases in 3–body D decays
- 3–body Dalitz plots through effective 2–body decays: intermediate resonance
- Study effective 2–body decays using Flavor SU(3). PRD **60**, 114026 (1999), Rosner

Diagrams for $D \to PP$ $\quad P = \pi^{\pm}, \pi^0; K^{\pm}, K^0, \overline{K}^0; \eta, \eta'$



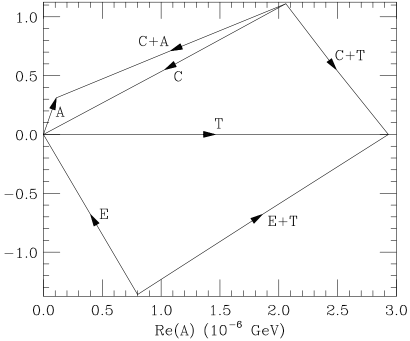
Flavor topology representation of "T", "C", "E" and "A" amplitudes. Example: $D^0 \to K^- \pi^+ = T + E$

T, C, E, A from $D \to PP$ decays

PRD **81**, 014026 (2010), BB and J. L. Rosner

$T = 2.93 \pm 0.02$ $\qquad$ $C = (2.34 \pm 0.03)\, e^{i\,(-152 \pm 1)^\circ}$

$E = (1.57 \pm 0.03)\, e^{i\,(121 \pm 1)^\circ}$ $\qquad$ $A = (0.33 \pm 0.14)\, e^{i\,(70 \pm 11)^\circ}$



"T", "C", "E" and "A" amplitudes on the complex plane. ($\chi^2 = 1.79$) Large phases of "C", "E" relative to "T" suggest importance of rescattering contributions

$D \to PV$ decays $V = \rho^{\pm}, \rho^0; K^{*\pm}, K^{*0}, \overline{K}^{*0}; \omega, \phi$

PRD **79**, 034016 (2009), BB and J. L. Rosner

- For $D \to PV$ keep track of spectator quark
- Label "T", "C", "E" and "A" with the suffix "P" or "V"
- Example: $D^0 \to K^{*-}\pi^+ = T_V + E_P$, $D^0 \to K^-\rho^+ = T_P + E_V$
- Larger parameter space: 8 (instead of 4) complex quantities
- The inputs (as before) are amplitudes of known decays
- Amplitudes for 5 CF known processes depend only on T_V, C_P and E_P. Set T_V real to obtain an exact solution (up to some discrete ambiguity)
- Amplitudes for 5 other CF processes also depend on T_P, C_V and E_V. Set T_P real to obtain an exact solution
- Multiple solutions: best fit to SCS decay rates chosen

Chosen solution for $D \to PV$

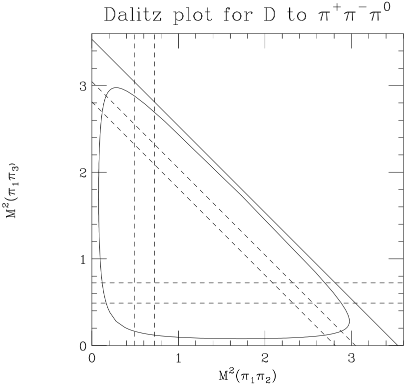
Fit parameters ($\chi^2 \sim 62$ for 12 degrees of freedom: SCS decays with known rates) :

$$T_V = 3.95 \pm 0.07 \qquad T_P = 7.46 \pm 0.21$$
$$C_P = (4.88 \pm 0.07)\, e^{i\,(-162 \pm 1)^\circ} \qquad C_V = (3.46 \pm 0.18)\, e^{\,i\,(172 \pm 3)^\circ}$$
$$E_P = (2.94 \pm 0.09)\, e^{i\,(-93 \pm 3)^\circ} \qquad E_V = (2.37 \pm 0.19)\, e^{i\,(-110 \pm 4)^\circ}$$

These results will be used to study three-body D decays.

Dalitz plot boundary for $D^0 \to \pi^0\pi^+\pi^-$

PRD **81**, 096008 (2010), BB, C-W. Chiang and J. L. Rosner



Kinematic Boundaries of $D \to 3\pi$ Dalitz plot. Dashed lines show positions of resonance bands.

Success of SU(3) flavor symmetry

- BABAR [arXiv:1001.3317] noticed isospin 0 dominance in the final state for $D^0 \to \pi^0\pi^+\pi^-$; Strong depopulation along the symmetry axes of the Dalitz plot.
- Table comparing flavor SU(3) predictions with BABAR data: (PRD **81**, 096008 (2010) for theory)

Channel	Fraction(%)	vs BABAR(%)
$I = 0$	92.9±6.7	94.24±0.40
$I = 1$	4.8±0.3	2.17±0.17
$I = 2$	2.3±0.8	3.58±0.29

Definition of isospin eigenstates:

$$
\begin{aligned}
I_0 &= [\rho^+\pi^- - \rho^0\pi^0 + \rho^-\pi^+]/\sqrt{3} \\
I_1 &= [\rho^+\pi^- - \rho^-\pi^+]/\sqrt{3} \\
I_2 &= [\rho^+\pi^- + 2\,\rho^0\pi^0 + \rho^-\pi^+]/\sqrt{6}
\end{aligned}
$$

Flavor SU(3) representations of relevant amplitudes:

Decay Mode	Rep.	$\mathcal{B}(\%)$
$\rho^+\pi^-$	$-(T'_P + E'_V)$	0.980±0.040
$\rho^0\pi^0$	$(E'_P + E'_V - C'_P - C'_V)/2$	0.373±0.022
$\rho^-\pi^+$	$-(T'_V + E'_P)$	0.497±0.023

Cancellations in appropriate places leads to isospin 0 dominance in theory. Flavor SU(3) must be giving correct relative strong phases for this cancellation.

Idea of amplitude ratios

PRD **82**, 074025 (2010) BB and J. L. Rosner:
Dalitz Plots for $D^0 \to \pi^0 K^+ K^-$, $D^0 \to K_S \pi^+ \pi^-$

- SU(3) flavor symmetry provides a relationship:

$$\frac{A(D^0 \to K^{*-}K^+)}{A(D^0 \to K^{*+}K^-)} = -\lambda^2 \frac{A(D^0 \to K^{*-}\pi^+)}{A(D^0 \to K^{*+}\pi^-)}; \qquad \lambda \sim 0.23$$

- Expt. phases obey the above relationship. Flavor SU(3) works!
- Comparison of data with theory:

Ratio	Predicted		Measured	
	Mag.	Ph.($^\circ$)	Mag.	Ph.($^\circ$)
$A(K^{*-}K^+)/A(K^{*+}K^-)$	0.685	− 19.2	0.601	− 37
$-\lambda^2 A(K^{*-}\pi^+)/A(K^{*+}\pi^-)$	0.685	− 19.2	0.562	− 2.5

- Ratios in $D^0 \to \pi^0\pi^+\pi^-$, $D^0 \to \pi^0 K^+ K^-$, $D^0 \to K^-\pi^+\pi^0$:

$$\frac{A(D^0 \to \rho^+\pi^-)}{A(D^0 \to \rho^-\pi^+)} = \frac{A(D^0 \to K^{*+}K^-)}{A(D^0 \to K^{*-}K^+)} = \frac{A(D^0 \to \rho^+ K^-)}{A(D^0 \to K^{*-}\pi^+)}$$

Ratio	Predicted		Measured	
	Mag.	Phase($^\circ$)	Mag.	Phase($^\circ$)
$A(\rho^+\pi^-)/A(\rho^-\pi^+)$	1.461	19.2	1.400	2
$A(K^{*+}K^-)/A(K^{*-}K^-)$	1.461	19.2	1.664	37
$A(\rho^+ K^-)/A(K^{*-}\pi^+)$	1.461	19.2	1.585	13

- Amplitude ratios from data agree well with theory.
- Discrepancies in phases are smaller than 19°. (Sign conventions have been taken care of.)
- Flavor SU(3) breaking may be the source of remaining discrepancies. (PRD **82**, 114032 (2010), BB, J. L. Rosner)

Summary:

- $D \to PP$ decays : framework with flavor SU(3) symmetry among "u", "d" and "s" quarks
- $D \to PV$ decays : exact fit to CF processes
- $D \to PV$ amplitudes and phases used to study features of $D \to PPP$ Dalitz plots
- Isospin 0 dominance in $D^0 \to \pi^0\pi^+\pi^-$ amplitude is supported by Flavor SU(3) framework
- Cross ratios between several Dalitz plot amplitudes: relative phases indicate flavor SU(3) works

Decays : $D \to PP$ or $D \to PV$: 2 body phase space is easy!

- Charmed (D) mesons: $D^0 = -c\bar{u}$, $D^+ = c\bar{d}$, $D_s^+ = c\bar{s}$
- Pseudoscalar (P) mesons:
  - $\pi^+ = u\bar{d}$, $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\pi^- = -d\bar{u}$
  - $K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\overline{K}^0 = s\bar{d}$, $K^- = -s\bar{u}$
  - $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$, $\eta' = (2\,s\bar{s} + u\bar{u} + d\bar{d})/\sqrt{6}$
- Vector (V) mesons:
  - $\rho^+ = u\bar{d}$, $\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\rho^- = -d\bar{u}$
  - $K^{*+} = u\bar{s}$, $K^{*0} = d\bar{s}$, $\overline{K}^{*0} = s\bar{d}$, $K^{*-} = -s\bar{u}$
  - $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$, $\phi = s\bar{s}$
- Measured decay rates from experiments and/or PDG database
- $\mathcal{B} = \frac{p^{(2j+1)}}{8\pi M_D^2 \Gamma_D} |\mathcal{A}|^2$, j = 0(1) for $PP(V)$
- $|\mathcal{A}|$ in terms of $\mathcal{B}$, p, M_D, Γ_D

- Assume SU(3) flavor symmetry between "u", "d" and "s"
- Leading order in weak expansion: "Tree" diagram (T)
- Non-leading order "Color Suppressed Tree" (C), "Exchange" (E) and "Annihilation" (A)
- Cabibbo-favored (CF) decay amplitudes $\sim V_{cs} V_{ud}^*$
- Singly-Cabibbo-suppressed (SCS) decay amplitudes $\sim V_{cs} V_{us}^*$ or $V_{cd} V_{ud}^*$
- Doubly-Cabibbo-suppressed (WS) decay amplitudes $\sim V_{cd} V_{us}^*$
- Ratio of amplitudes ($\lambda \simeq 0.23$)
  $CF : SCS(V_{cs} V_{us}^*) : SCS(V_{cd} V_{ud}^*) : WS = 1 : \lambda : -\lambda : -\lambda^2$

Input CF data

PRD **81**, 014026 (2010), BB and J. L. Rosner

Meson	Decay mode	$\mathcal{B}$ (%)	Rep.	Th. $\mathcal{B}$ (%)
D^0	$K^-\pi^+$	3.89±0.08	$T+E$	3.91
	$\overline{K}^0\pi^0$	2.38±0.09	$(C-E)/\sqrt{2}$	2.35
	$\overline{K}^0\eta$	0.96±0.06	$C/\sqrt{3}$	1.00
	$\overline{K}^0\eta'$	1.90±0.11	$-(C+3E)/\sqrt{6}$	1.92
D^+	$\overline{K}^0\pi^+$	3.07±0.10	$C+T$	3.09
D_s^+	$\overline{K}^0K^+$	2.98±0.17	$C+A$	2.94
	$\pi^+\eta$	1.84±0.15	$(T-2A)/\sqrt{3}$	1.81
	$\pi^+\eta'$	3.95±0.34	$2(T+A)/\sqrt{6}$	3.60

Predictions

Singly-Cabibbo-suppressed D decays to π and K ($|T| > |C|$) :

Meson	Decay mode	$\mathcal{B}$ (10^{-3})	Rep.	Th. $\mathcal{B}$ (10^{-3})
D^0	$\pi^+\pi^-$	1.45±0.05	$-(T' + E')$	2.24
	$\pi^0\pi^0$	0.81±0.05	$-(C' - E')/\sqrt{2}$	1.35
	K^+K^-	4.07±0.10	$(T' + E')$	1.93
	$K^0\overline{K}^0$	0.32±0.02	0	0
D^+	$\pi^+\pi^0$	1.18±0.06	$-(T' + C')/\sqrt{2}$	0.89
	$K^+\overline{K}^0$	6.12±0.22	$(T' - A')$	6.15
D_s^+	π^+K^0	2.52±0.27	$-(T' - A')$	3.08
	π^0K^+	0.62±0.23	$-(C' + A')/\sqrt{2}$	0.85

Singly-Cabibbo-suppressed D decays to π and K ($|T| < |C|$) :

Meson	Decay mode	$\mathcal{B}$ (10^{-3})	Rep.	Th. $\mathcal{B}$ (10^{-3})
D^0	$\pi^+\pi^-$	1.45±0.05	$-(T' + E')$	2.24
	$\pi^0\pi^0$	0.81±0.05	$-(C' - E')/\sqrt{2}$	1.36
	K^+K^-	4.07±0.10	$(T' + E')$	1.92
	$K^0\overline{K}^0$	0.32±0.02	0	0
D^+	$\pi^+\pi^0$	1.18±0.06	$-(T' + C')/\sqrt{2}$	0.88
	$K^+\overline{K}^0$	6.12±0.22	$(T' - A')$	0.73
D_s^+	π^+K^0	2.52±0.27	$-(T' - A')$	0.37
	π^0K^+	0.62±0.23	$-(C' + A')/\sqrt{2}$	0.86

Observations

- Deviations from Flavor SU(3):
  - $D^0 \to (\pi^+\pi^-, K^+K^-)$: explained by decay constants ($f_K/f_\pi = 1.2$) and form factors ($f_+(D \to K)/f_+(D \to \pi) > 1.$)
  - $D \to K^0\overline{K}^0$: quantifies the amount of Flavor SU(3) breaking
- Decays involving the singlets (η, η') also need SE' (small), SA' (large)
- Flavor SU(3) gives sum rule between rates: $2.6\,\sigma$ deviation from experiments

- $D^0 \to (\pi^0\pi^0, \pi^0\eta, \pi^0\eta', \eta\eta)$
- Sum rule relating $|\mathcal{A}|^2$ for these processes:

$$\begin{aligned} & 8\,|\mathcal{A}(D^0 \to \pi^0\eta')|^2 \quad + \quad 16\,|\mathcal{A}(D^0 \to \pi^0\pi^0)|^2 \\ = & \, 16\,|\mathcal{A}(D^0 \to \pi^0\eta)|^2 \quad + \quad 9\,|\mathcal{A}(D^0 \to \eta\eta)|^2 \end{aligned}$$

- Experimental numbers:

$$\begin{aligned} 8\,|\mathcal{A}(D^0 \to \pi^0\eta')|^2 + 16\,|\mathcal{A}(D^0 \to \pi^0\pi^0)|^2 &= 348 \pm 24 \\ 16\,|\mathcal{A}(D^0 \to \pi^0\eta)|^2 + 9\,|\mathcal{A}(D^0 \to \eta\eta)|^2 &= 460 \pm 35 \end{aligned}$$

- $2.6\,\sigma$ discrepancy with flavor SU(3) prediction

Correcting for Vector decay

In order to compare data from different Dalitz plots we need to take into account the factors arising from the vector meson decay. Relative phases (from Clebsch factors) for the $K^{*\pm}$ decays are important:

Dalitz Plot	Resonance Process	Rep.	Clebsch factor
	$\rho^0\overline{K}^0$	$(C_V - E_V)/\sqrt{2}$	1
$D^0 \to K_S\pi^+\pi^-$	$K^{*-}\pi^+$	$T_V + E_P$	$\sqrt{2/3}$
	$K^{*+}\pi^-$	$-\lambda^2(T_P + E_V)$	$-\sqrt{2/3}$
	$\phi\pi^0$	$\lambda C_P/\sqrt{2}$	$1/\sqrt{2}$
$D^0 \to \pi^0 K^+ K^-$	$K^{*-}K^+$	$\lambda(T_V + E_P)$	$-1/\sqrt{3}$
	$K^{*+}K^-$	$\lambda(T_P + E_V)$	$-1/\sqrt{3}$