

The electroweak contribution to the top forward-backward asymmetry at Tevatron



MAX-PLANCK-GESELLSCHAFT



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$$p\bar{p} \rightarrow t\bar{t} + X$$

$$A_{FB}^{t\bar{t}} = \frac{\sigma(\Delta y > 0) - \sigma(\Delta y < 0)}{\sigma(\Delta y > 0) + \sigma(\Delta y < 0)}$$

$$\Delta y = y_t - y_{\bar{t}}$$

Definitions of A_{FB} :

$$A_{FB}^{pp\bar{p}} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)}$$

Why is A_{FB} interesting?

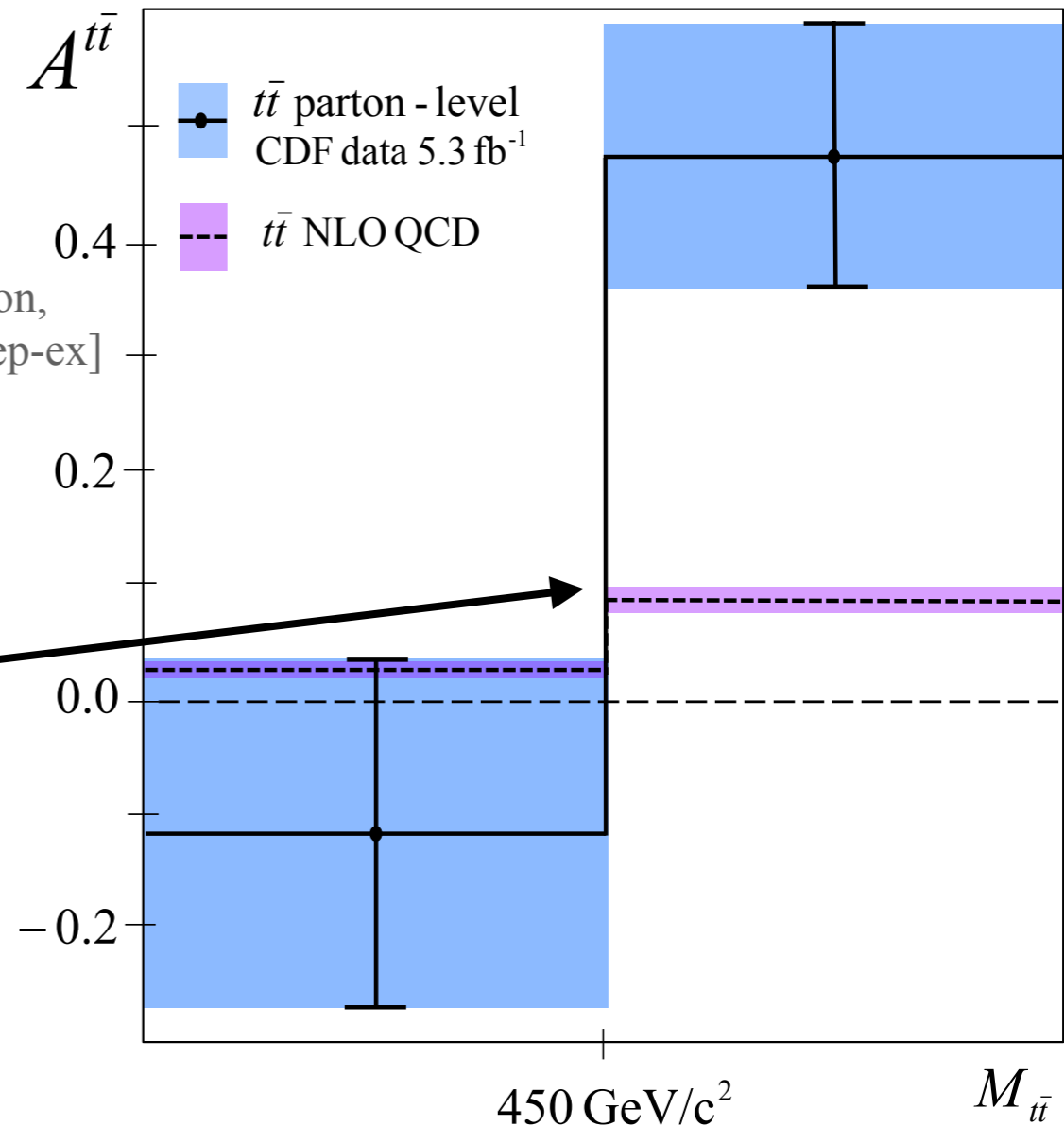
Theory

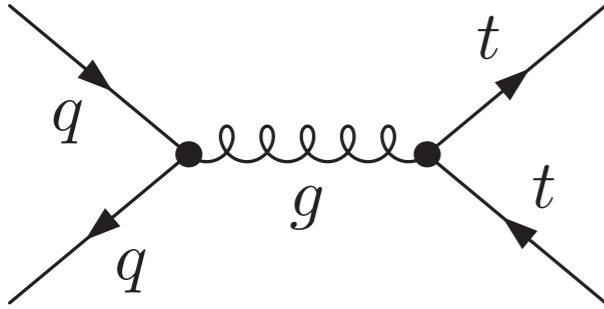
Experiment

$A_{FB}(\%)$	$A_{FB}^{t\bar{t}}$	$A_{FB}^{pp\bar{p}}$
data	15.8 ± 7.4	15.0 ± 5.5
MCFM	5.8 ± 0.9	3.8 ± 0.6

Only NLO QCD,
let's see SM
prediction!

CDF collaboration,
arXiv:1101.0034[hep-ex]





At LO partonic processes are not asymmetric.
 QCD produces the asymmetry only at NLO!
NLO in the cross-section, LO in A_{FB}

ONLY LO IS AVAILABLE!

NOT AVAILABLE, NOT TRIVIAL

$$A_{FB} = \frac{N}{D} = \frac{\alpha_s^3 N_1 + \alpha_s^4 N_2 + \dots}{\alpha_s^2 D_0 + \alpha_s^3 D_1 + \dots} = \frac{\alpha_s}{D_0} (N_1 + \alpha_s (N_2 - N_1 D_1 / D_0)) + \dots$$

QCD only at LO, but there is also electroweak theory.

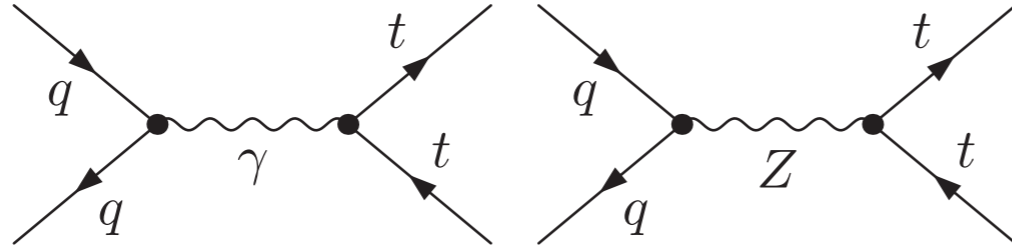
$$\mathcal{O}(\alpha_s \alpha) = 0$$

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \dots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \dots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2}{\alpha_s^2} \frac{\tilde{N}_0}{D_0}$$

$\alpha_s^2 D_0$ is the LO cross section, now we see the terms in N

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \dots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \dots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2 \tilde{N}_0}{\alpha_s^2 D_0}$$

$$\frac{\alpha^2 \tilde{N}_0}{\alpha_s^2 D_0}$$

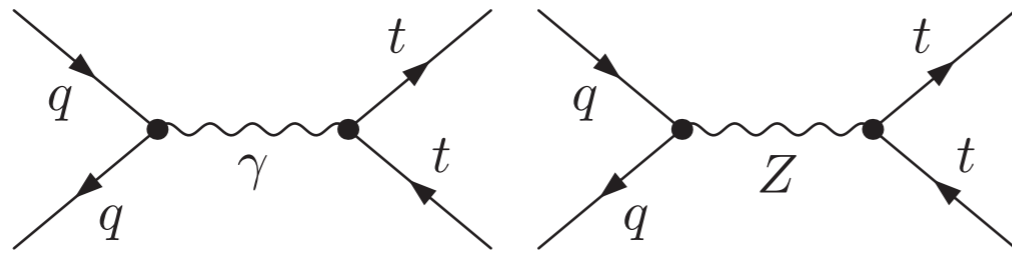


Different couplings for different chiralities produce asymmetric terms in the cross-section

$$\frac{d\sigma_{asym}}{d\cos\theta} = 2\pi\alpha^2 \underline{\cos\theta} \left(1 - \frac{4m_t^2}{s}\right) \left[\kappa \frac{Q_q Q_t A_q A_t}{(s - M_Z^2)} + 2\kappa^2 \frac{A_q A_t V_q V_t}{(s - M_Z^2)^2} \frac{s}{(s - M_Z^2)^2} \right]$$

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \dots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \dots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2 \tilde{N}_0}{\alpha_s^2 D_0}$$

$$\frac{\alpha^2 \tilde{N}_0}{\alpha_s^2 D_0}$$

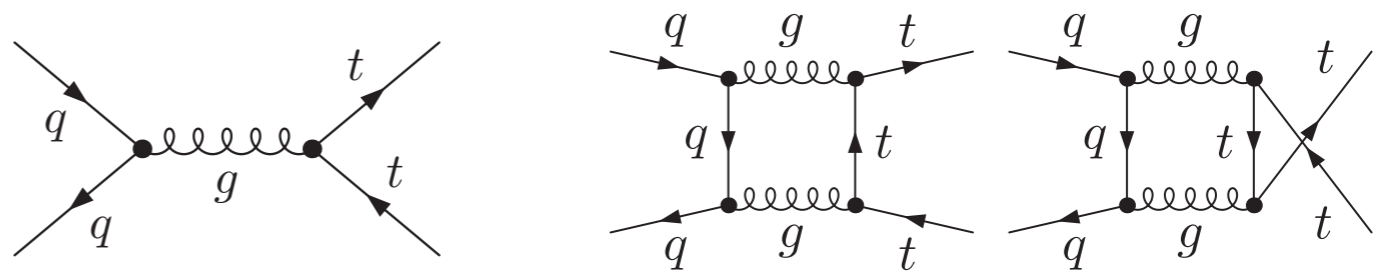


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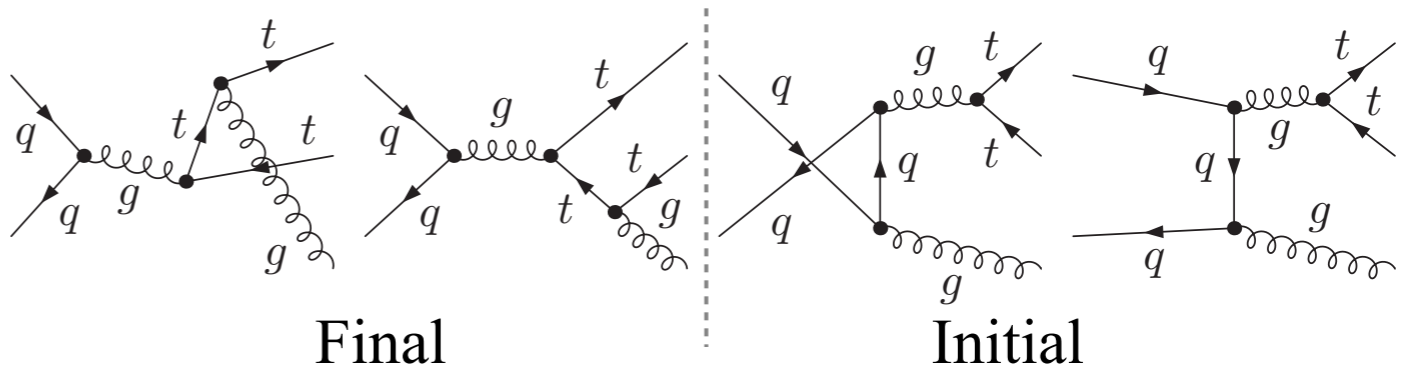
$$\frac{d\sigma_{asym}}{d\cos\theta} = 2\pi\alpha^2 \underline{\cos\theta} \left(1 - \frac{4m_t^2}{s}\right) \left[\kappa \frac{Q_q Q_t A_q A_t}{(s - M_Z^2)} + 2\kappa^2 \frac{A_q A_t V_q V_t}{(s - M_Z^2)^2} s \right]$$

$$\alpha_s \frac{N_1}{D_0}$$

VIRTUAL (Only Boxes)
NO UV, NO Coll. Div.
Only IR



REAL
Only interference of initial and final gluon emission is asymmetric.



Kuhn, Rodrigo - Phys.Rev. D59 (1999) 054017

REAL+VIRTUAL= IR finite

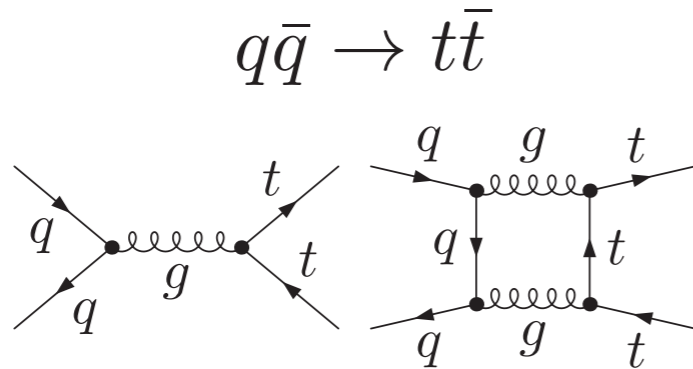
$$\alpha \frac{\tilde{N}_1}{D_0}$$

It's useful to divide electroweak contribution into QED (photon) and weak (Z) part.

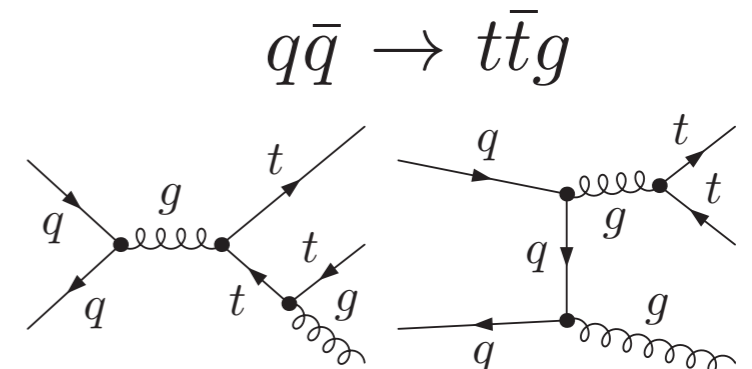
QED

QED can be easily obtained from QCD calculation and the substitution of one gluon into one photon in the squared amplitudes.

$$|\overline{\mathcal{M}^{t\bar{t}}}|^2_{\mathcal{O}(\alpha_s^3)}$$



$$|\overline{\mathcal{M}^{t\bar{t}g}}|^2_{\mathcal{O}(\alpha_s^3)}$$



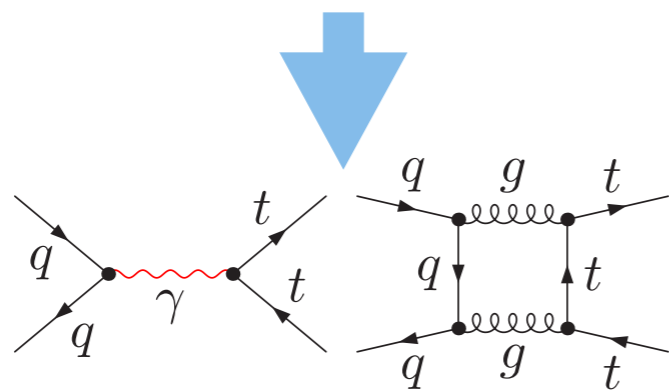
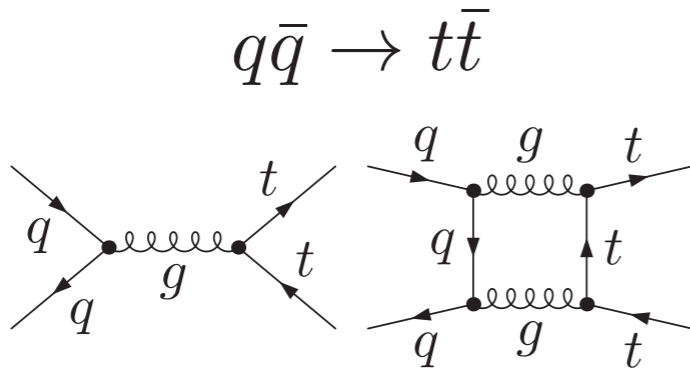
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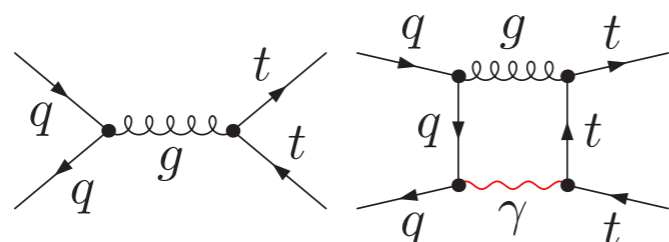
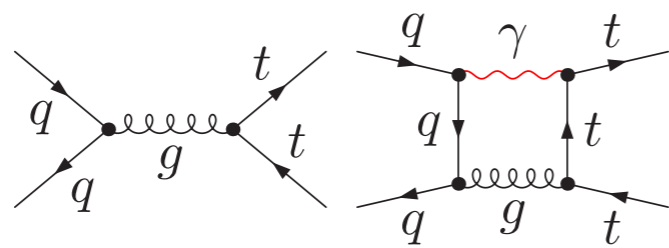
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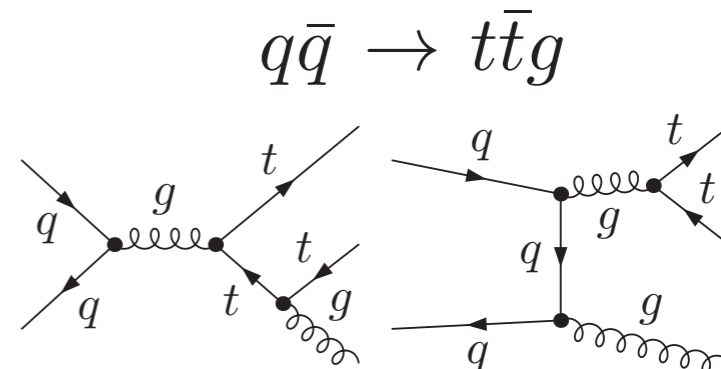
$$|\overline{\mathcal{M}^{t\bar{t}}}|^2_{\mathcal{O}(\alpha_s^3)}$$



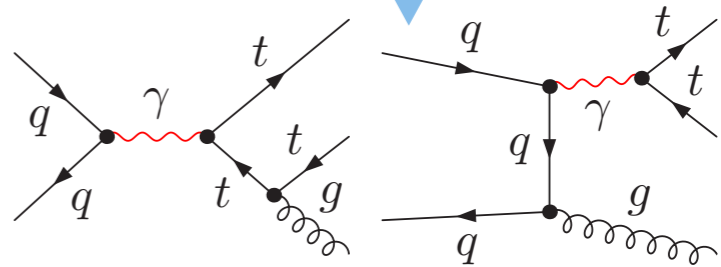
$$|\overline{\mathcal{M}^{t\bar{t}}}|^2_{\mathcal{O}(\alpha_s^2 \alpha)}$$



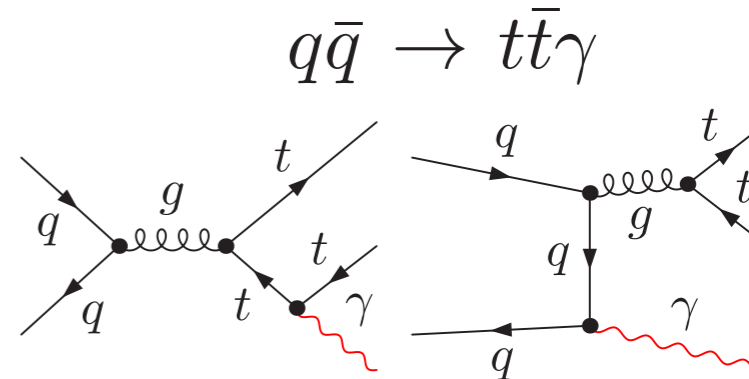
$$|\overline{\mathcal{M}^{t\bar{t}g}}|^2_{\mathcal{O}(\alpha_s^3)}$$



$$|\overline{\mathcal{M}^{t\bar{t}g}}|^2_{\mathcal{O}(\alpha_s^2 \alpha)}$$



$$|\overline{\mathcal{M}^{t\bar{t}\gamma}}|^2_{\mathcal{O}(\alpha_s^2 \alpha)}$$



DIFFERENCES:
Only couplings and color factor!

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \dots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 \tilde{D}_1 + \alpha_s^2 \alpha \tilde{D}_1 + \dots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2}{\alpha_s^2} \frac{\tilde{N}_0}{D_0}$$

$$R_{QED}(Q_q) = \frac{\alpha \tilde{N}_1^{QED}}{\alpha_s N_1} = Q_q Q_t \frac{36}{5} \frac{\alpha}{\alpha_s}$$

QED correction can be obtained from $\text{QCD} \times R_{QED}$

Weak

The same diagrams as QED part, but $\gamma \rightarrow Z$.

Z is not massless \rightarrow If we write $\text{Weak} = \text{QCD} \times R_{\text{Weak}}$.

R_{Weak} does not depend only on couplings and color factor

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \dots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \dots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2 \tilde{N}_0}{\alpha_s^2 D_0}$$

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Z is not massless \rightarrow If we write Weak = $QCD \times R_{Weak}$.

R_{Weak} does not depend only on couplings and color factor

EXISTING ESTIMATE



$$A_{FB}^{EW} = 0.09 \times A_{FB}^{QCD}$$

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \dots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 \tilde{D}_1 + \alpha_s^2 \alpha \tilde{D}_1 + \dots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2 \tilde{N}_0}{\alpha_s^2 D_0}$$

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QED correction can be obtained from $QCD \times R_{QED}$

Weak

The same diagrams as QED part, but $\gamma \rightarrow Z$.

Z is not massless \rightarrow If we write Weak = $QCD \times R_{Weak}$.

R_{Weak} does not depend only on couplings and color factor

EXISTING ESTIMATE



$$A_{FB}^{EW} = 0.09 \times A_{FB}^{QCD}$$

WE CALCULATED THE COMPLETE EW CONTRIBUTION



$$A_{FB}^{EW} \sim 0.25 \times A_{FB}^{QCD}$$

THIS NUMBER DEPENDS ON THE RENORMALIZATION SCALE OF α_s

NUMERICAL RESULTS

$\alpha^{-1} = 137.035$

$m_t = 172.0 \text{ GeV}$

$m_Z = 91.1875 \text{ GeV}$

$m_W = 80.399 \text{ GeV}$

Input

$\alpha_s?$

$$A_{FB} = \frac{N}{D} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \dots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \dots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2}{\alpha_s^2} \frac{\tilde{N}_0}{D_0}$$

α_s for LO \neq
 α_s for NLO

Expansion makes sense if α_s in N and D is the same.
 α_s from MRST2004QED \rightarrow α_s for NLO

Factorization scale μ_f
renormalization scale μ_r

$$\mu_f = \mu_r = (m_t/2, m_t, 2m_t)$$

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Factorization scale μ_f
renormalization scale μ_r

$$\mu_f = \mu_r = (m_t/2, m_t, 2m_t)$$

Output

$$A_{FB}^{t\bar{t}} = (9.7, 8.9, 8.3)\%$$

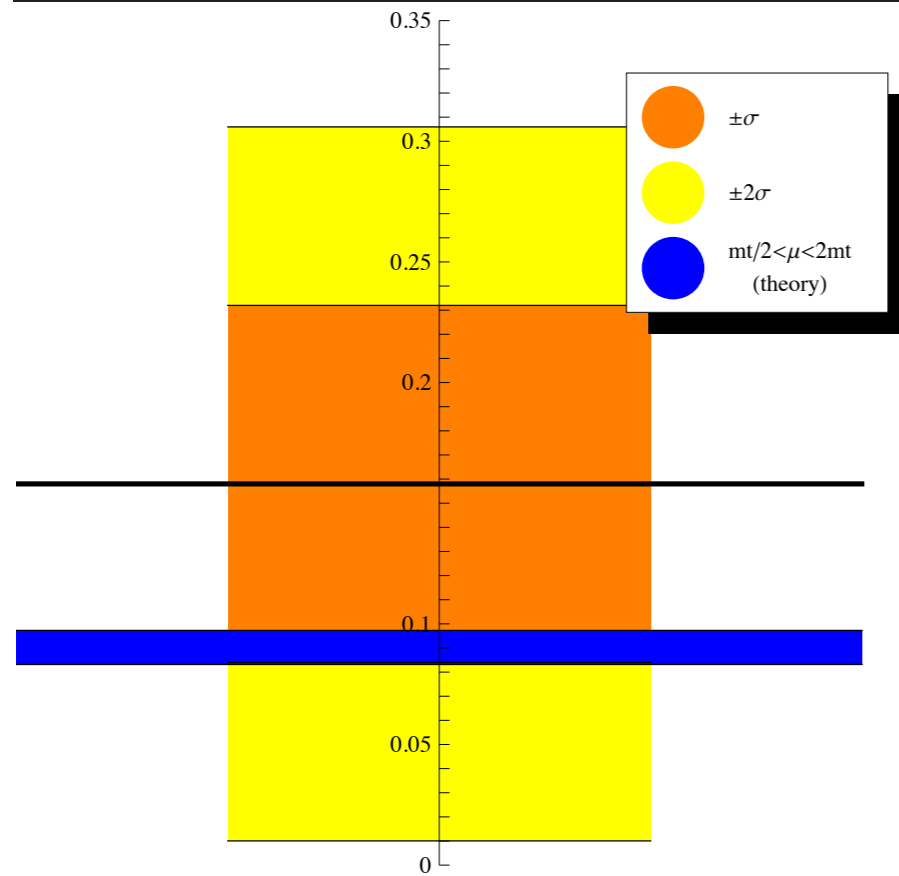
$$A_{FB}^{p\bar{p}} = (6.4, 5.9, 5.4)\%$$

Compare with

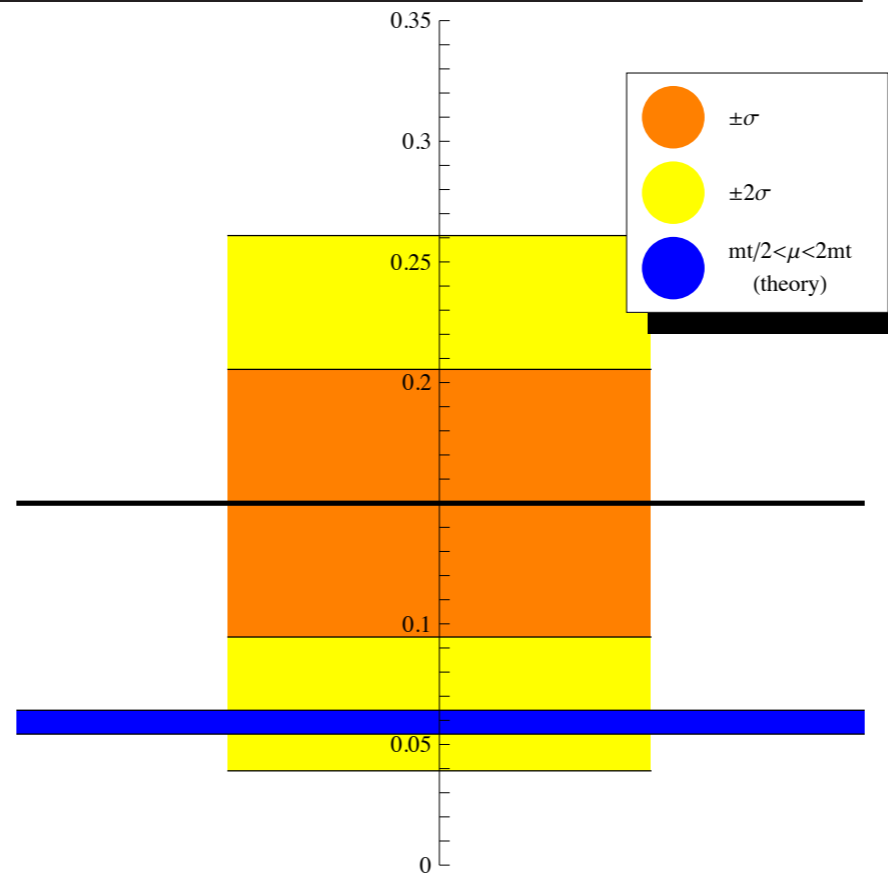
$A_{FB}(\%)$	$A_{FB}^{t\bar{t}}$	$A_{FB}^{p\bar{p}}$
data	15.8 ± 7.4	15.0 ± 5.5
MCFM	5.8 ± 0.9	3.8 ± 0.6

(a) $A_{FB}^{t\bar{t}}$

$A_{FB}^{t\bar{t}}$	$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$\mathcal{O}(\alpha_s) u\bar{u}$	7.01%	6.29%	5.71%
$\mathcal{O}(\alpha_s) d\bar{d}$	1.16%	1.03%	0.92%
$\mathcal{O}(\alpha)_{QED} u\bar{u}$	1.35%	1.35%	1.35%
$\mathcal{O}(\alpha)_{QED} d\bar{d}$	-0.11%	-0.11%	-0.11%
$\mathcal{O}(\alpha)_{weak} u\bar{u}$	0.16%	0.16%	0.16%
$\mathcal{O}(\alpha)_{weak} d\bar{d}$	-0.04%	-0.04%	-0.04%
$\mathcal{O}(\alpha^2/\alpha_s^2) u\bar{u}$	0.18%	0.23%	0.28%
$\mathcal{O}(\alpha^2/\alpha_s^2) d\bar{d}$	0.02%	0.03%	0.03%
tot $p\bar{p}$	9.72%	8.93%	8.31%

(a) $A_{FB}^{t\bar{t}}$ (b) $A_{FB}^{p\bar{p}}$

$A_{FB}^{p\bar{p}}$	$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$\mathcal{O}(\alpha_s) u\bar{u}$	4.66%	4.19%	3.78%
$\mathcal{O}(\alpha_s) d\bar{d}$	0.75%	0.66%	0.59%
$\mathcal{O}(\alpha)_{QED} u\bar{u}$	0.90%	0.90%	0.90%
$\mathcal{O}(\alpha)_{QED} d\bar{d}$	-0.07%	-0.07%	-0.07%
$\mathcal{O}(\alpha)_{weak} u\bar{u}$	0.10%	0.10%	0.10%
$\mathcal{O}(\alpha)_{weak} d\bar{d}$	-0.03%	-0.03%	-0.03%
$\mathcal{O}(\alpha^2/\alpha_s^2) u\bar{u}$	0.11%	0.14%	0.17%
$\mathcal{O}(\alpha^2/\alpha_s^2) d\bar{d}$	0.01%	0.02%	0.02%
tot $p\bar{p}$	6.42%	5.92%	5.43%

(b) $A_{FB}^{p\bar{p}}$

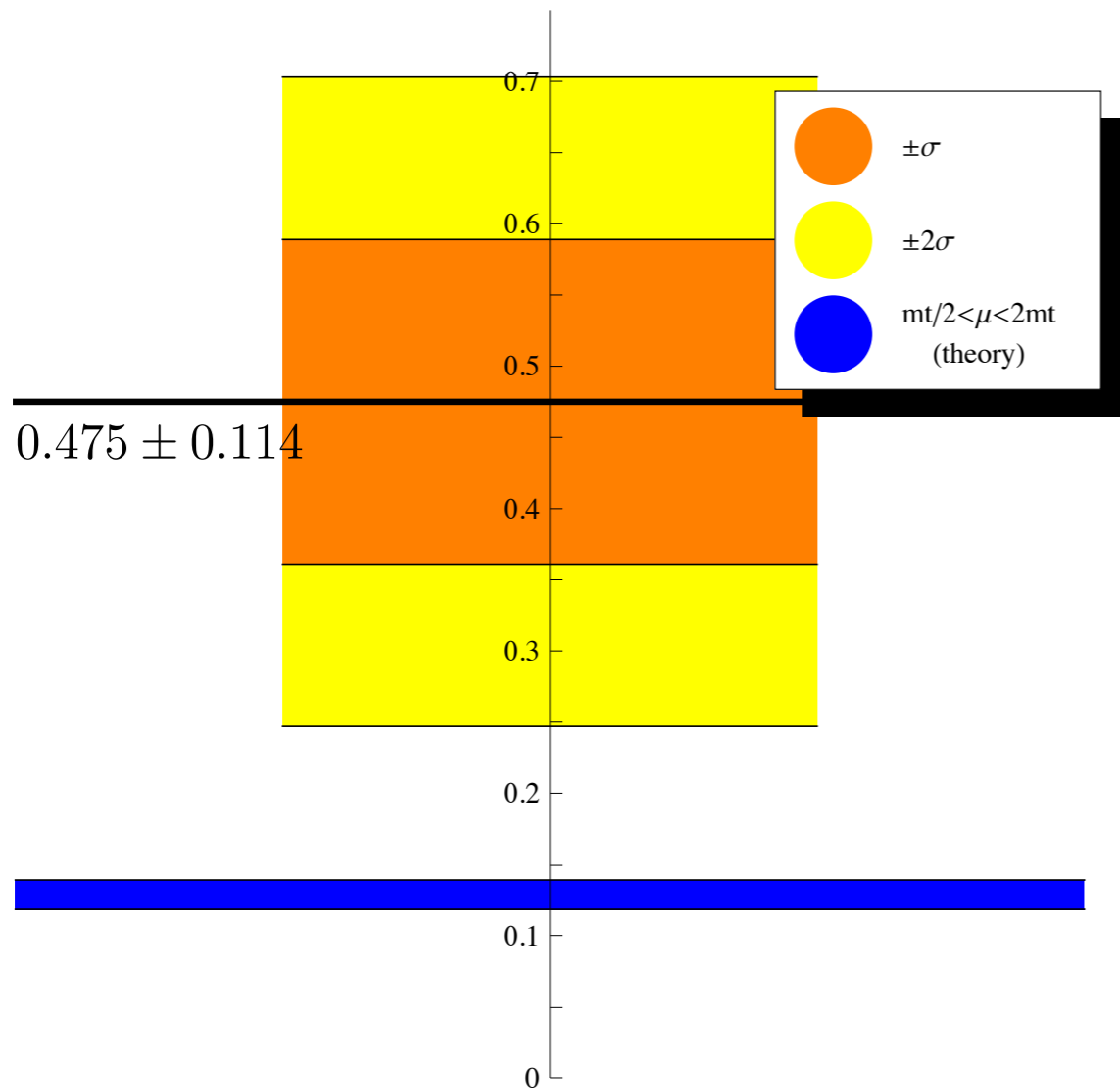
$$R_{QED}^{u\bar{u}} = (0.192, 0.214, 0.237)$$

$$R_{QED}^{d\bar{d}} = (-0.096, -0.107, -0.119)$$

a) at 1σ
b) inside 2σ

$$\frac{(A_{FB}^{t\bar{t}})^{EW}}{(A_{FB}^{t\bar{t}})^{QCD}} = (0.190, 0.220, 0.254)$$

$$\frac{(A_{FB}^{p\bar{p}})^{EW}}{(A_{FB}^{p\bar{p}})^{QCD}} = (0.186, 0.218, 0.243)$$



$$A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) = (13.9, 12.8, 11.9)\%$$

(a) $A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV})$

Also with cuts the gap between theory and experiment is decreased by EW terms. Anyway, if invariant mass $> 450 \text{ GeV}$, theory at 3σ .

CONCLUSION

A very important term is still missing: NNLO QCD differential cross section.

Total electroweak contribution is not negligible and increases QCD asymmetry by a factor ~ 1.2

The QED part can be simply calculated from QCD contribution of the subprocesses

EW cannot explain $A_{FB}(M_{INV} > 450 \text{ GeV})$, but new models cannot forget its contribution when they try to fill the gap between theory (SM) and experiment. Moreover we have to wait the NLO of QCD also for this region.

THANK YOU FOR THE ATTENTION!

EXTRA SLIDES

Hadronic process = partonic process \otimes PDF

$$H_1 H_2 \rightarrow t\bar{t} + X$$

$$p_1 p_2 \rightarrow t\bar{t} + X$$

$$f_{p_1, H_1}(x_1) f_{p_2, H_2}(x_2)$$

$$f_{p_1, H_2}(x_1) f_{p_2, H_1}(x_2)$$

Partonic process can be produced in two different directions with the same momenta

$$A_{FB} \neq 0$$



$$f_{p_1, H_1}(x_1) f_{p_2, H_2}(x_2) \neq f_{p_1, H_2}(x_1) f_{p_2, H_1}(x_2)$$

At LHC $H_1=H_2 \rightarrow A_{FB}=0$

At Tevatron only processes with p_1 or $p_2 = (\text{up, antiup, down, antidown})$ can produce asymmetric terms!

LO Cross-Section

$\sigma(\text{pb})$	$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$p\bar{p}$ (No cuts)	7.990	5.621	4.187
$p\bar{p}(M_{t\bar{t}} > 450 \text{ GeV})$	3.113	2.148	1.573
$p\bar{p}(\Delta y > 1)$	1.846	1.276	0.937

$$\kappa = \frac{1}{4 \sin^2(\theta_W) \cos^2(\theta_W)}$$

$$V_q = T_q^3 - 2Q_q \sin^2(\theta_W)$$

$$A_q = T_q^3$$

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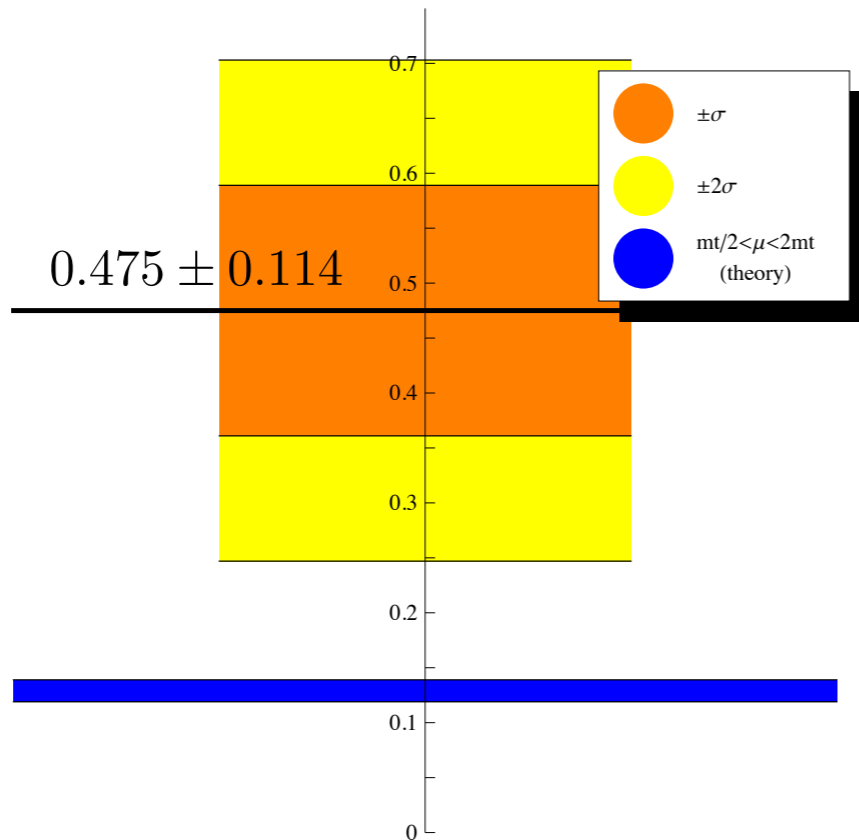
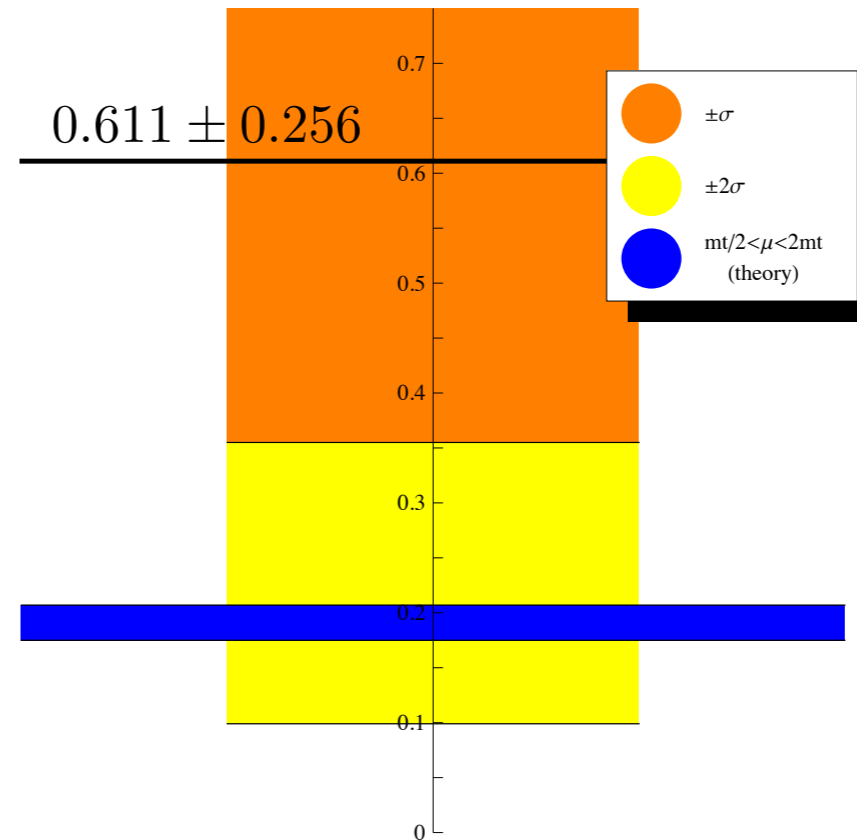
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(a) $A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV})$

$A_{FB}^{t\bar{t}}$		$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$\mathcal{O}(\alpha_s^3)$	$u\bar{u}$	10.13%	9.10%	8.27%
$\mathcal{O}(\alpha_s^3)$	$d\bar{d}$	1.44%	1.27%	1.14%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$u\bar{u}$	1.94%	1.95%	1.96%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$d\bar{d}$	-0.14%	-0.14%	-0.14%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$u\bar{u}$	0.28%	0.28%	0.28%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$d\bar{d}$	-0.05%	-0.05%	-0.05%
$\mathcal{O}(\alpha^2)$	$u\bar{u}$	0.26%	0.33%	0.41%
$\mathcal{O}(\alpha^2)$	$d\bar{d}$	0.03%	0.03%	0.04%
tot	$p\bar{p}$	13.90%	12.77%	11.91%

(b) $A_{FB}^{t\bar{t}}(|\Delta y| > 1)$

$A_{FB}^{t\bar{t}}$		$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
$\mathcal{O}(\alpha_s^3)$	$u\bar{u}$	15.11%	13.72%	12.41%
$\mathcal{O}(\alpha_s^3)$	$d\bar{d}$	2.28%	2.02%	1.84%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$u\bar{u}$	2.90%	2.94%	2.94%
$\mathcal{O}(\alpha_s^2\alpha)_{QED}$	$d\bar{d}$	-0.22%	-0.22%	-0.22%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$u\bar{u}$	0.25%	0.25%	0.26%
$\mathcal{O}(\alpha_s^2\alpha)_{weak}$	$d\bar{d}$	-0.09%	-0.09%	-0.08%
$\mathcal{O}(\alpha^2)$	$u\bar{u}$	0.35%	0.45%	0.55%
$\mathcal{O}(\alpha^2)$	$d\bar{d}$	0.04%	0.05%	0.06%
tot	$p\bar{p}$	20.70%	19.12%	17.75%

(a) $A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV})$ (b) $A_{FB}^{t\bar{t}}(|\Delta y| > 1)$