

The S-parameter with Many Fermions on the Lattice

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on behalf of the LSD Collaboration

arXiv:1009.5967 (PRL, in press)

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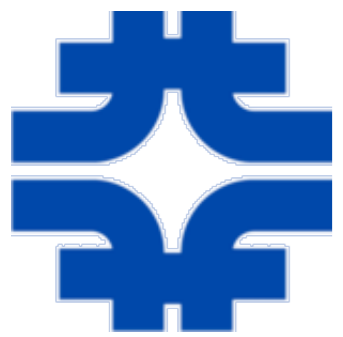


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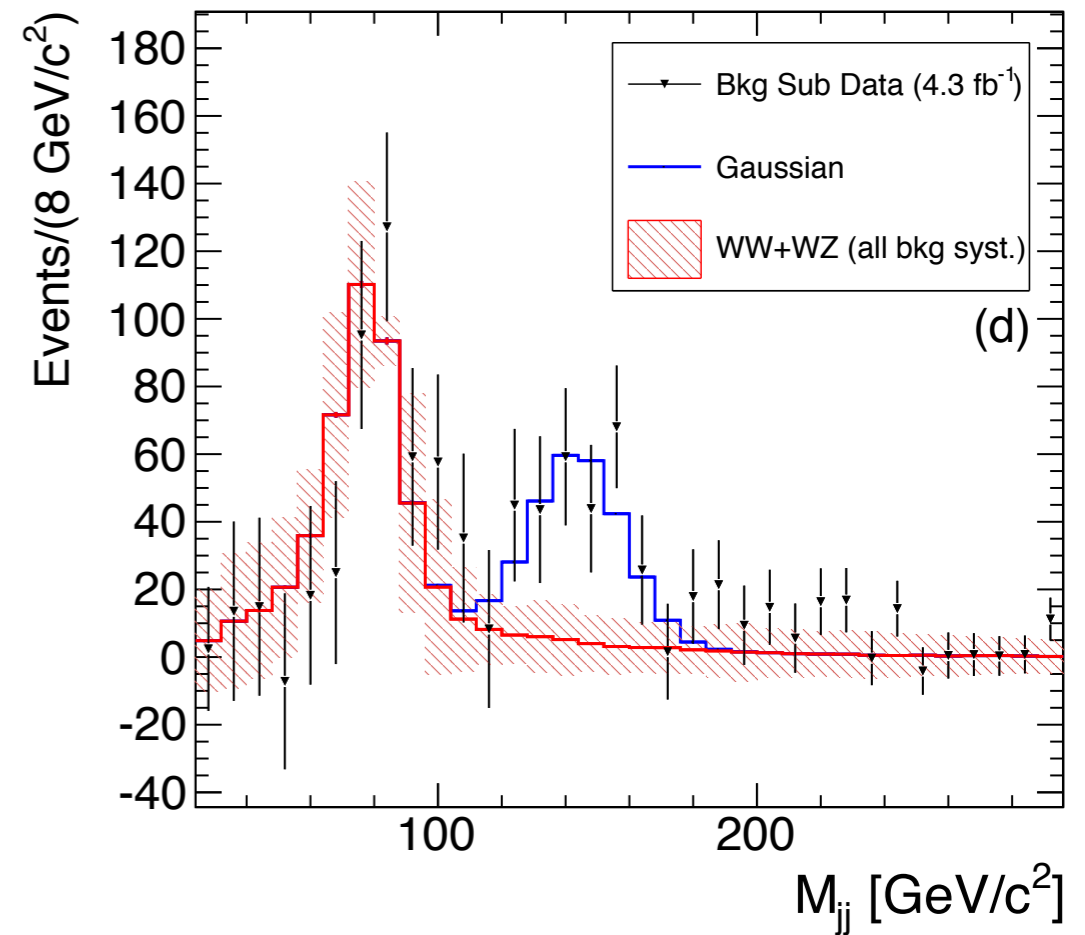


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Motivation

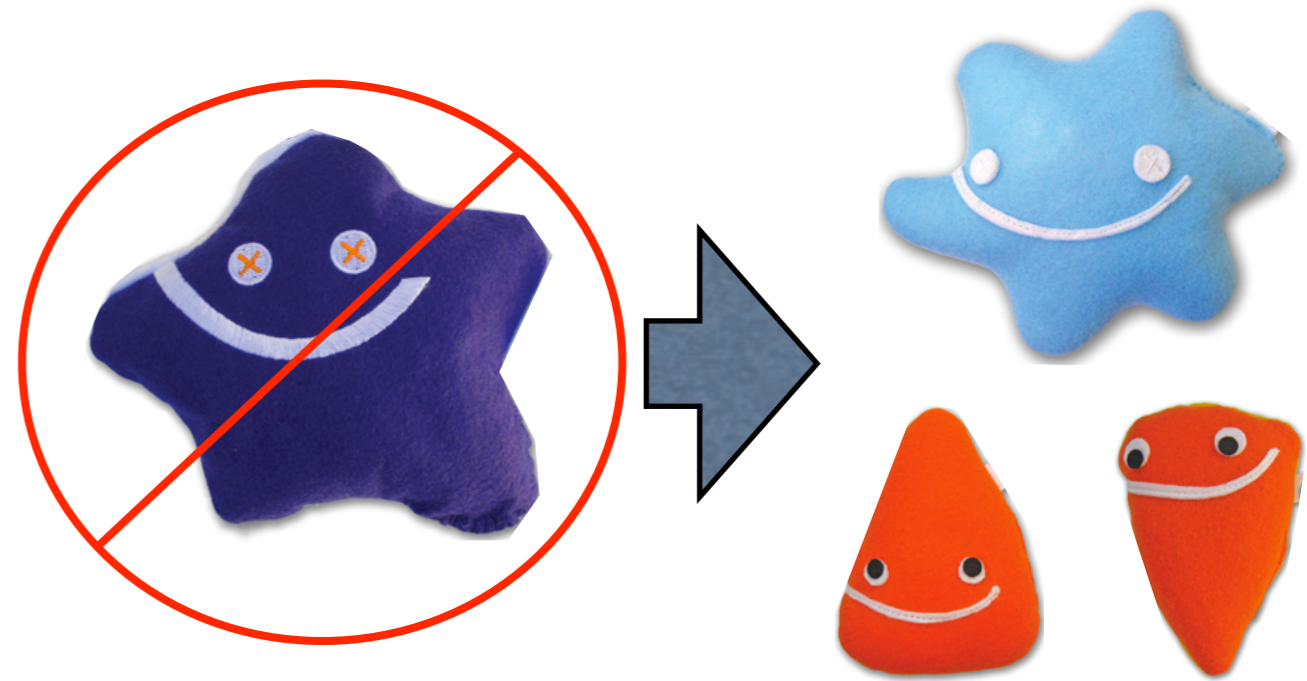
- With the LHC fully operational, we may be on the verge of discovering BSM physics (and there are already some possible hints!)
- New physics could easily be **strongly** coupled, in which case lattice will likely have a large role to play - generalized toolset for exploration of non-perturbative dynamics.



(arXiv:1104.0699)

One example: technicolor

- **Technicolor** theories replace the Higgs scalar field with new strong dynamics. Chiral symmetry breaking also breaks electroweak symmetry.



<http://particlezoo.net>

- **Minimal** or **one-doublet** technicolor is QCD, rescaled:
 $\Lambda_{QCD} \sim 1 \text{ GeV} \rightarrow \Lambda_{TC} \sim 1 \text{ TeV}.$

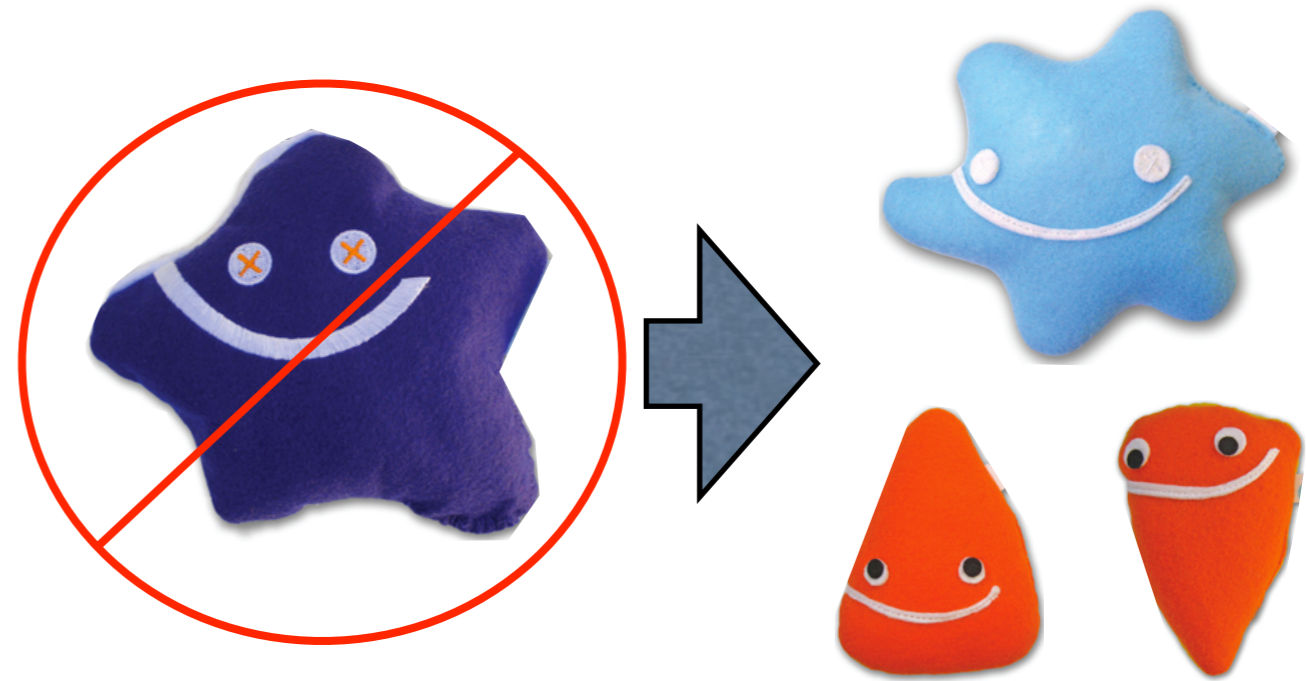
- Generically, technicolor models are in tension with precision EW, especially the **S-parameter**:

$$S \simeq 0.25 \frac{N_{TF}}{2} \frac{N_{TC}}{3} + \frac{1}{12\pi} \left(\frac{N_{TF}^2}{4} - 1 \right) \log \left(\frac{m_{\rho_T}^2}{m_{\pi_T}^2} \right)$$

(experiment: $S=0$ or slightly negative)

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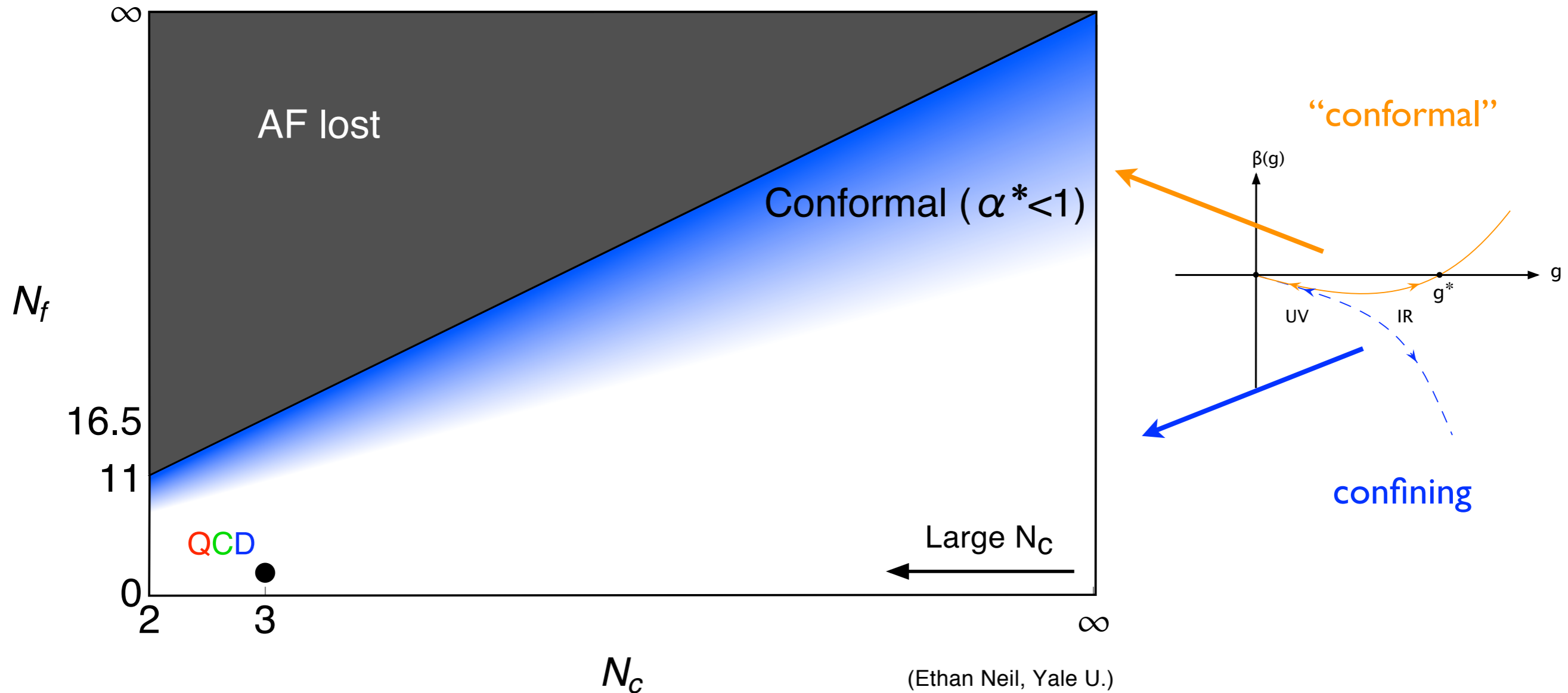
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relies on QCD pheno!

(experiment: $S=0$ or slightly negative)

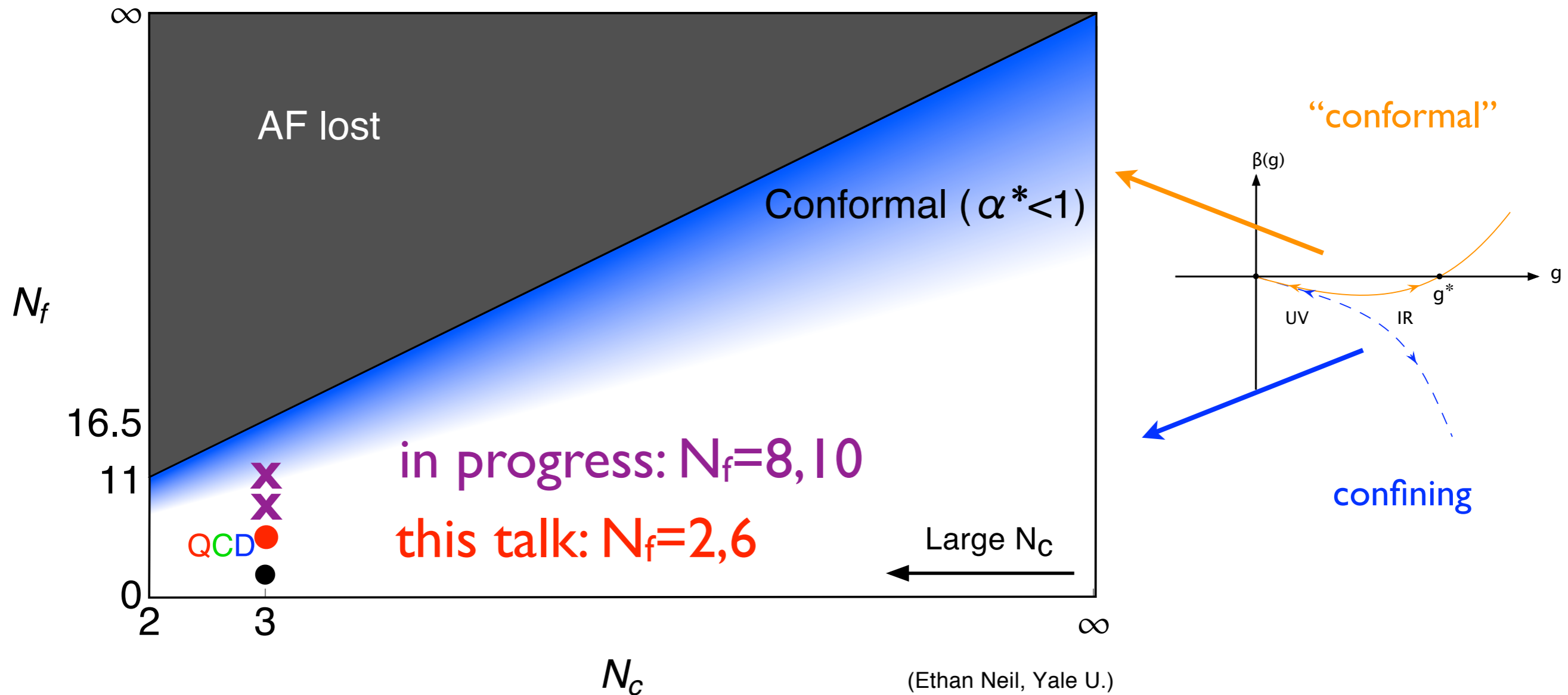
A roadmap in N_c and N_f



Large N_c expansion works well for **QCD**, but for large N_f , things change drastically (IR fixed point.)

Lattice can be applied anywhere with AF!

A roadmap in N_c and N_f



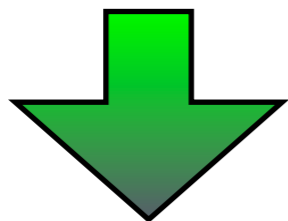
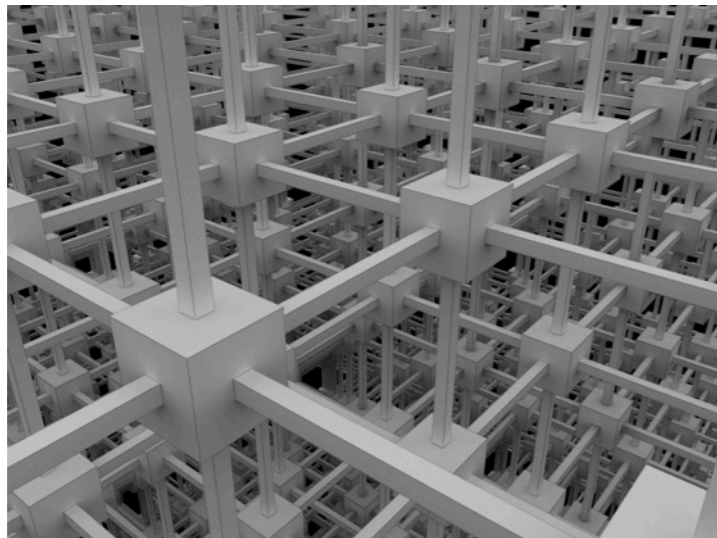
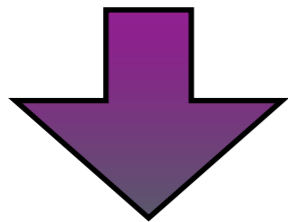
(Ethan Neil, Yale U.)

Large N_c expansion works well for QCD, but for large N_f , things change drastically (IR fixed point.)

Lattice can be applied anywhere with AF!

Going to the Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{O}(U, \bar{\psi}, \psi) \exp(-S[U, \bar{\psi}, \psi])$$



$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{U \in \mathcal{U}} \langle \mathcal{O} \rangle_U$$

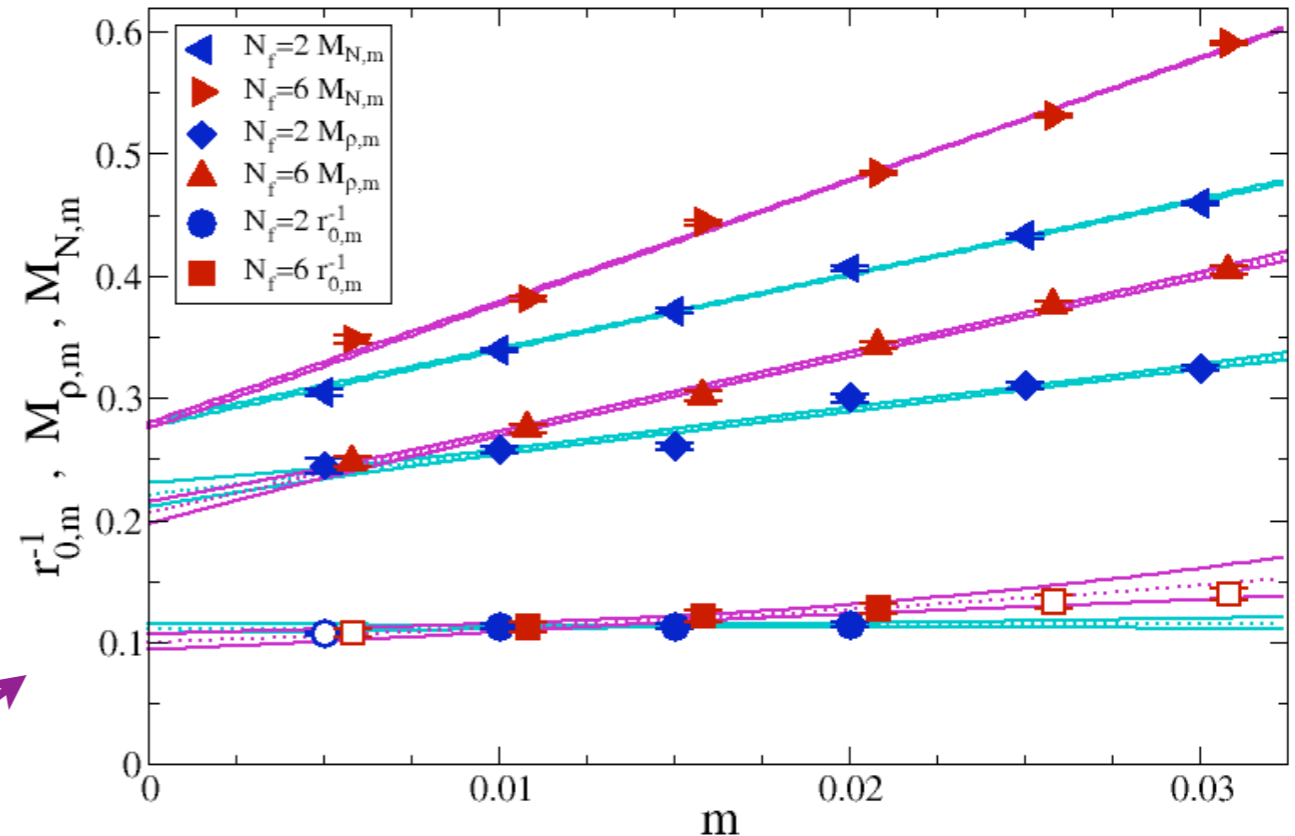
Discretize to make the path integral finite-dimensional (but sharply peaked!)

Importance sampling and Monte Carlo techniques give us an ensemble of field configurations, weighted by $\exp(-S)$

Most of the computational cost is in ensemble generation, so measure many different $\langle \mathcal{O} \rangle$

Lattice Ensembles

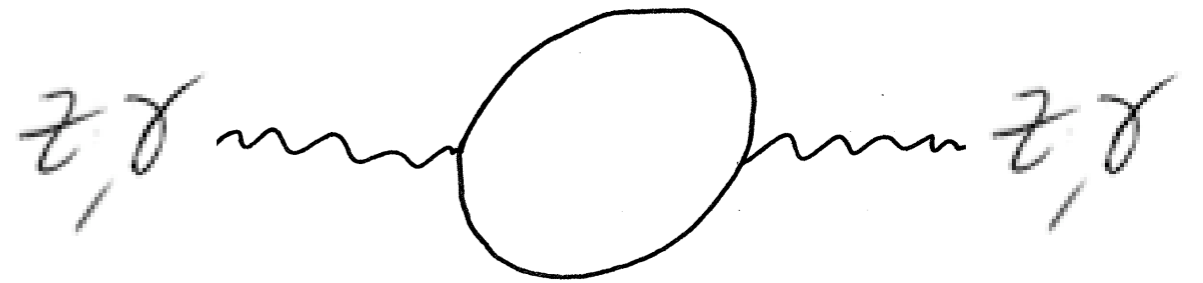
- This talk: two sets of ensembles with $N_f=2,6$ fermions (all with mass m)
- Ensembles are tuned to hold IR scale(s) fixed in chiral limit
- Goldstone mass kept small compared to box size



| am_f | $N_f = 2$ | | $N_f = 6$ | |
|--------|---------------|-----------|---------------|-----------|
| | " M_π " L | N_{cfg} | " M_π " L | N_{cfg} |
| 0.005 | 3.5 | 1430 | 4.7 | 1350 |
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| 0.020 | 6.5 | 720 | 7.8 | 400 |
| 0.025 | 7.0 | 600 | 8.8 | 420 |
| 0.030 | 7.8 | 400 | 9.8 | 360 |

S-parameter

S is sensitive to electroweak “**oblique corrections**”, i.e. vacuum polarization of EW gauge bosons. Can express using V/A currents:



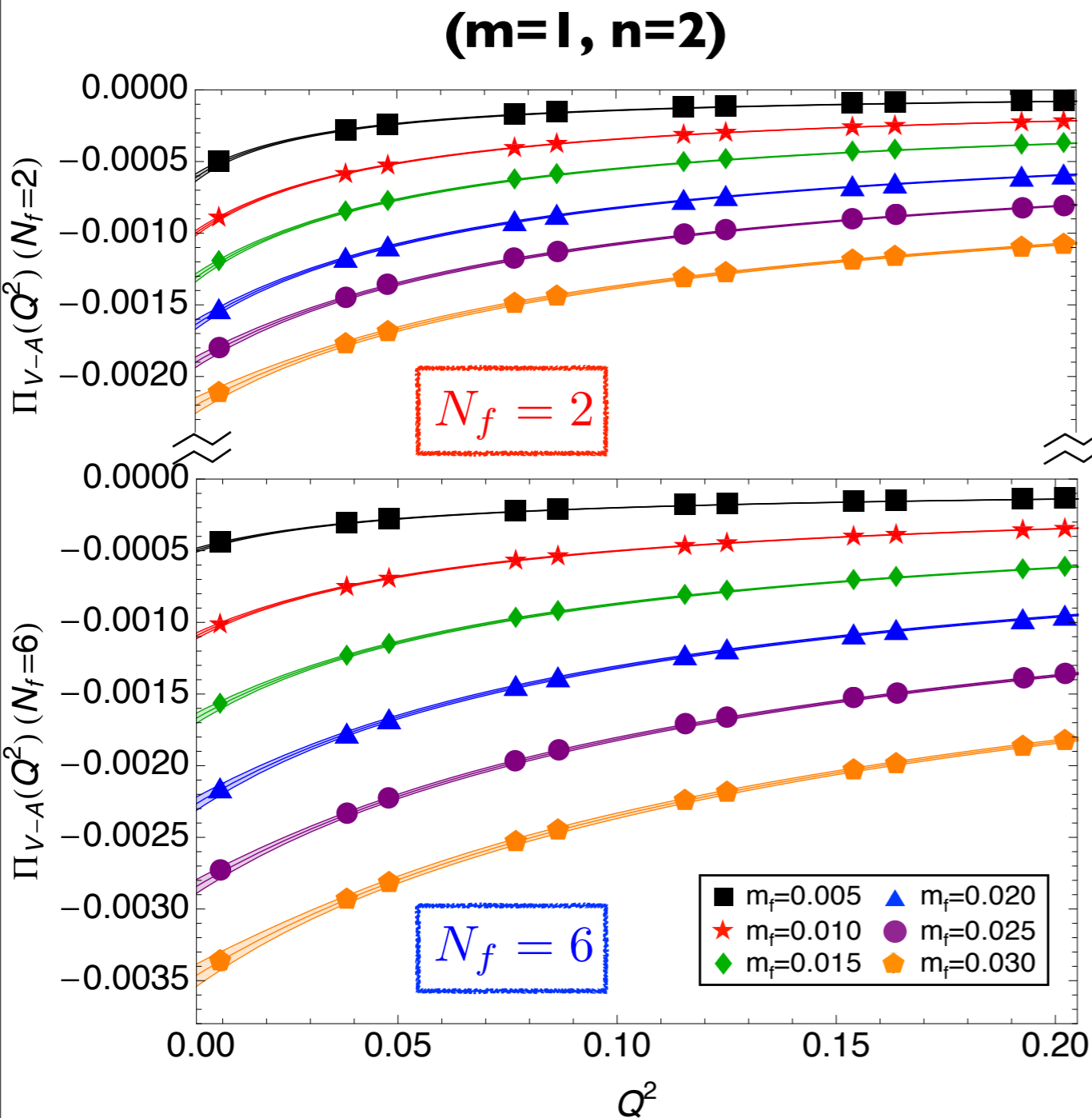
$$S = -4\pi(\Pi'_{VV}(0) - \Pi'_{AA}(0))$$

Overview of lattice measurement:

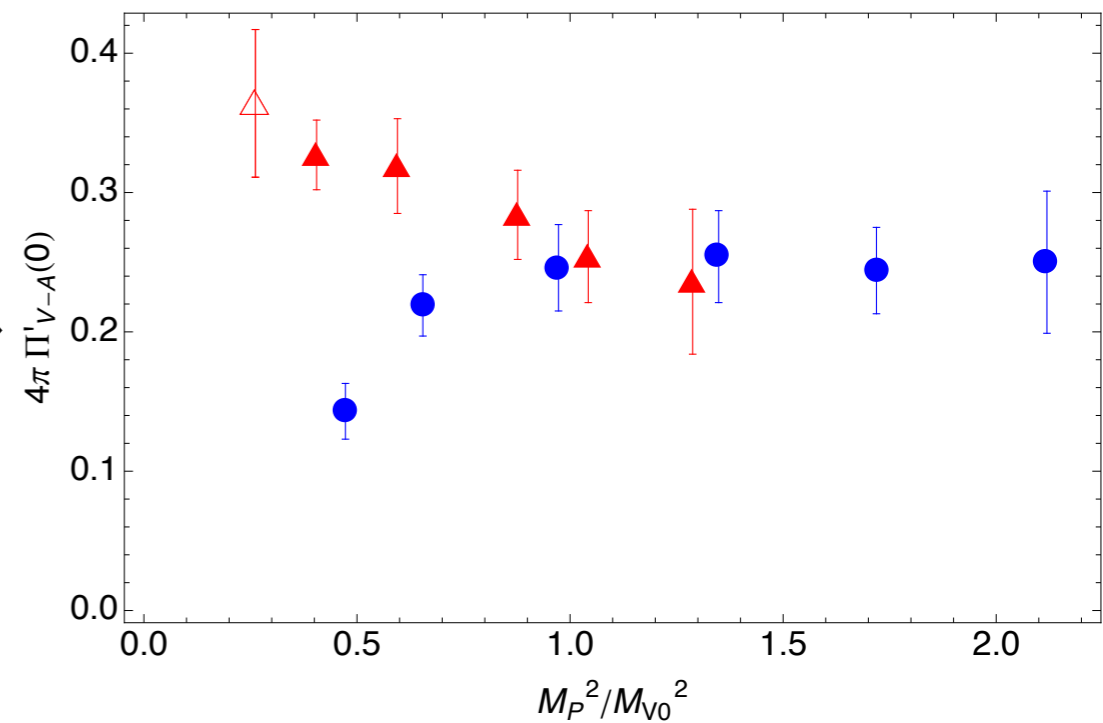
- Measure VV, AA correlators on chosen ensembles
- Fit to **Pade-(m,n) approximants**:
- Extract slope of (VV-AA) at zero q^2 , convert to S-parameter

$$\Pi_{V-A}(q^2) = \frac{\sum_m a_m q^{2m}}{\sum_n b_n q^{2n}}$$

Correlator fits



$$\Pi_{V-A}(q^2) = \frac{\sum_m a_m q^{2m}}{\sum_n b_n q^{2n}}$$



Expect agreement in the quenched limit $M_P^2 \rightarrow \infty$

From slope to S

$$S = \frac{1}{3\pi} \int_0^\infty \frac{ds}{s} \left\{ (N_f/2) [R_V(s) - R_A(s)] - \frac{1}{4} \left[1 - \left(1 - \frac{m_h^2}{s} \right)^3 \Theta(s - m_h^2) \right] \right\}$$

$\sim 4\pi\Pi'_{V-A}(0)$

ref. Higgs mass;
 we take $m_h \equiv M_{V0}$
 (=1 TeV, roughly)

Standard model subtraction:

- Removes the contribution of standard model Higgs doublet to S
- IR divergent - cancels precisely with divergence in the spectrum!

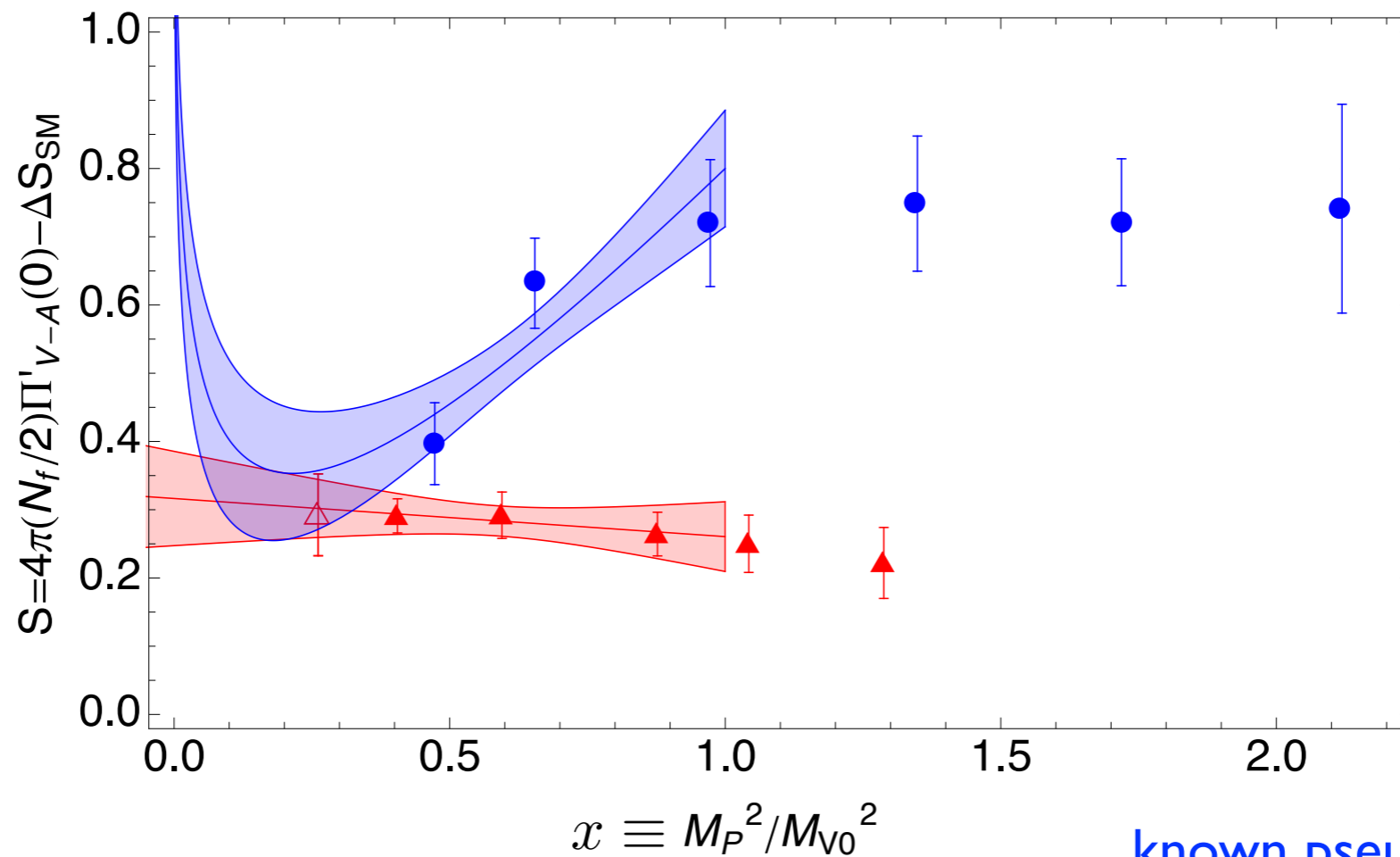
$$(H, \phi \rightarrow \pi_T)$$

Integrate:

$$\Delta S_{SM} = \frac{1}{12\pi} \left[\frac{11}{6} + \log \left(\frac{M_{V0}^2}{4M_P^2} \right) \right]^*$$

$$* \left(\frac{M_{V0}^2}{M_P^2} < 1/4 \right)$$

From slope to S

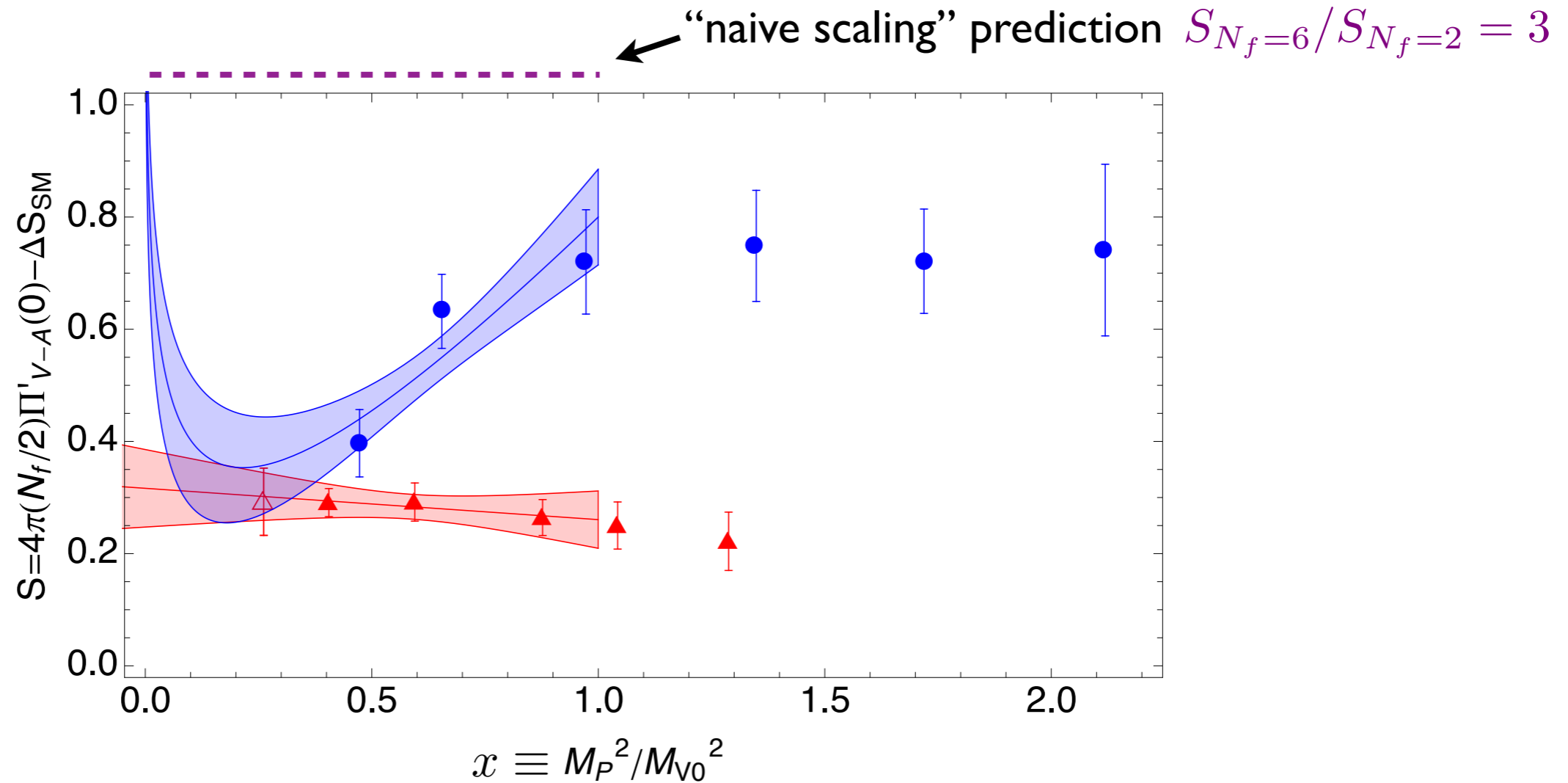


known pseudo-NGB contribution

Simple linear fit:
$$S(x) = A + Bx + \frac{1}{12\pi} \left(\frac{N_f^2}{4} - 1 \right) \log(1/x)$$

At two flavors, $S(m=0) = 0.35(6)$ - consistent with other results

From slope to S



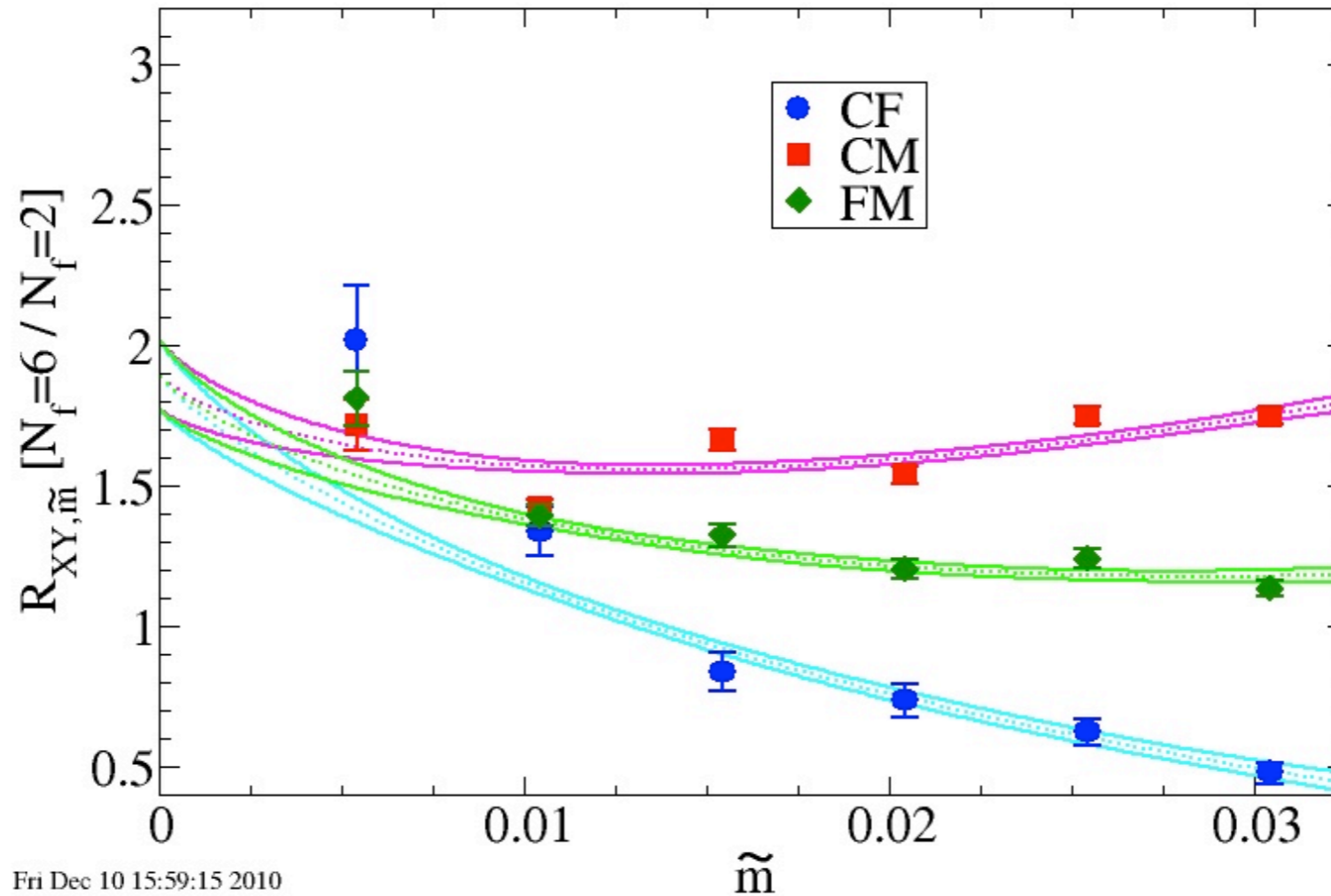
- $N_f=6$ result is much smaller than naive scaling predicts!
(But still much too large compared to experiment.)
- This is a “worst-case” S , assuming a model with all techni-doublets EW charged.

Conclusion

- Smaller S-parameter than expected for $N_f=6$ theory, compared to $N_f=2$
- Hints of dynamics unlike QCD, effects in the right direction to reduce tension with precision experiment in EWSB models
- Simulations underway at $N_f=8, 10$; initial setup for $SU(2)$ gauge group started

Backup Slides

Condensate Enhancement



Lattice scheme:

$$R^{(6)} = 1.95(12)$$

Renormalized:

$$R_{\overline{\text{MS}}}^{(6)} = 1.60(10)$$

Pert. theory est:

$$R_{\overline{\text{MS}},pt}^{(6)} \lesssim 1.15$$

(at 3.85 GeV!)

Simulation Details

- We use **domain wall** fermions to preserve as much chiral, flavor symmetry as possible. Residual χ SB is small:

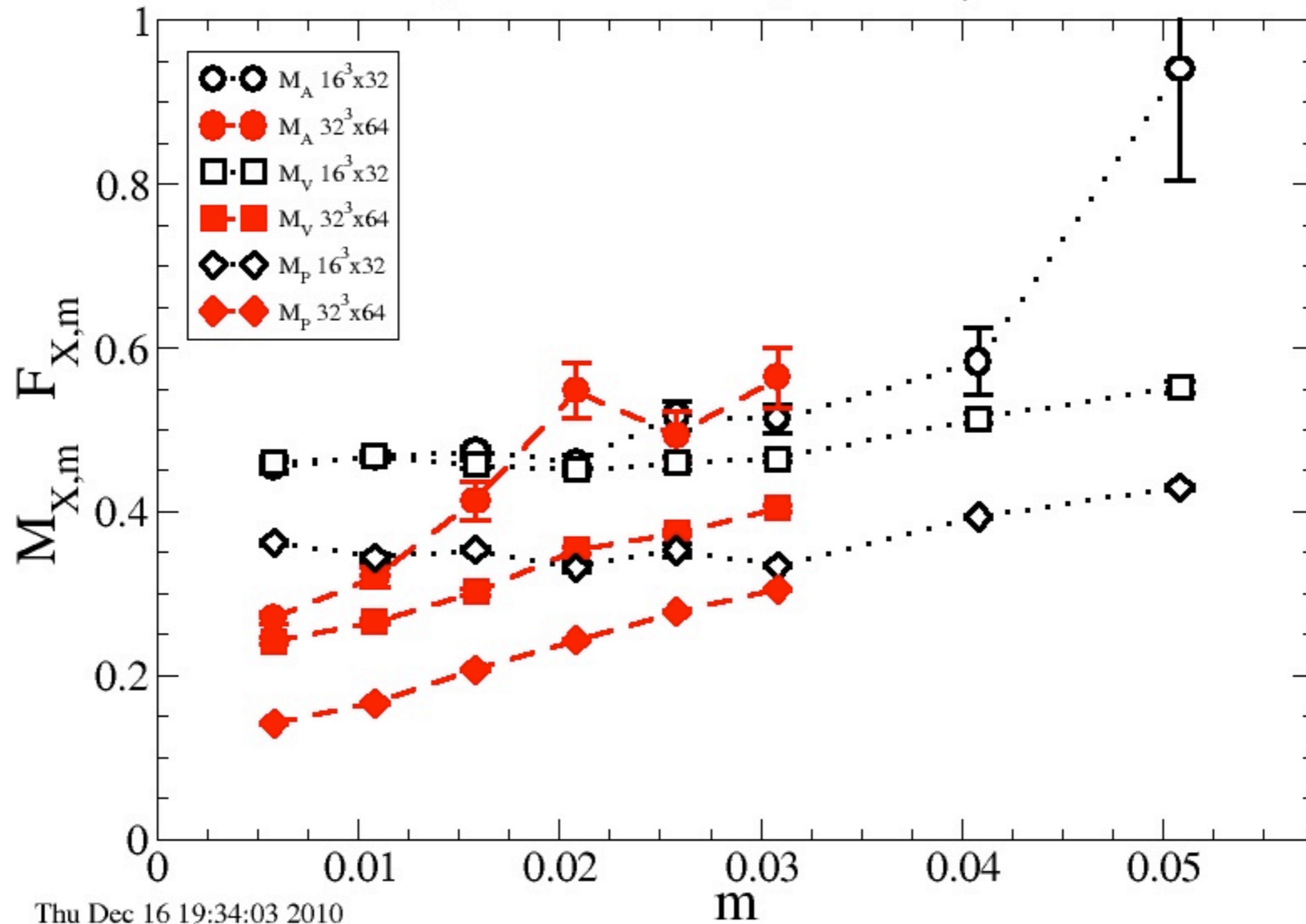
$$m_{res} = \begin{cases} 2.6 \times 10^{-5}, & N_f = 2 \\ 8.2 \times 10^{-4}, & N_f = 6 \end{cases}$$

- All volumes are $32^3 \times 64$, lattice spacing tuned to $a \sim 5m_\rho$.
At 2-flavors, this gives $a \sim 0.06 \text{ fm} = 3.6 \text{ GeV}^{-1}$, $L \sim 1.8 \text{ fm}$.

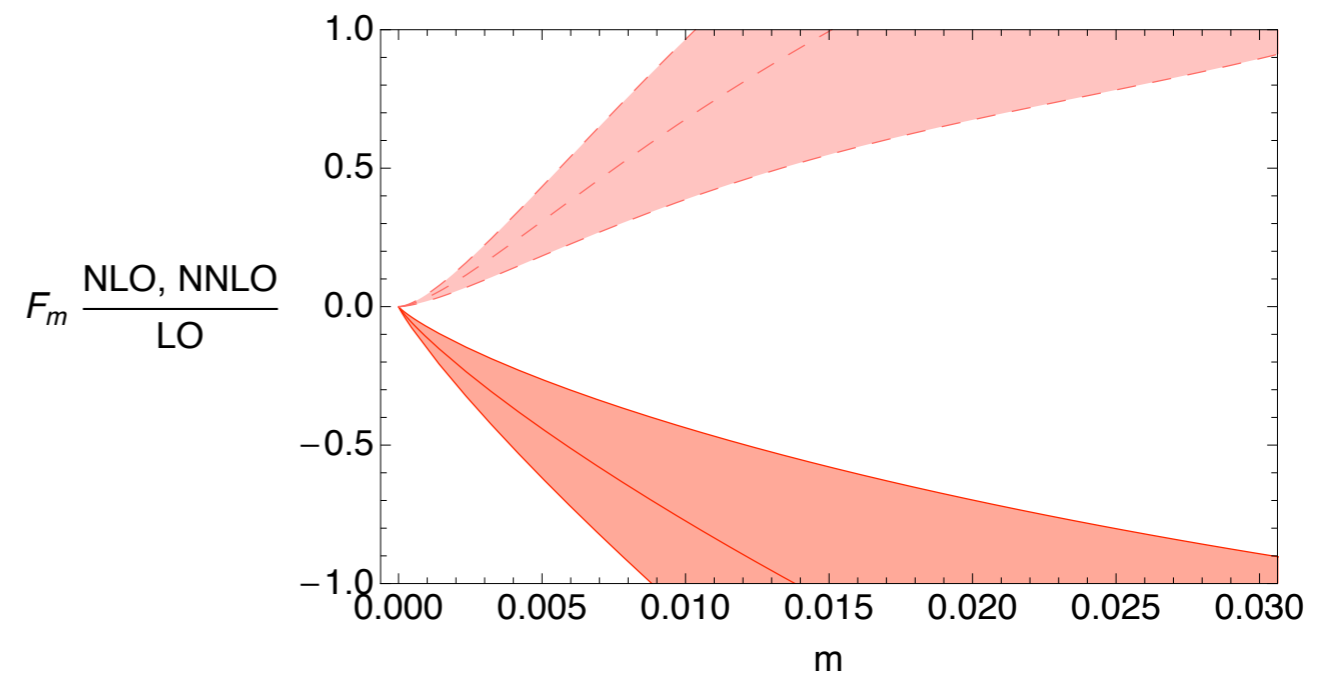
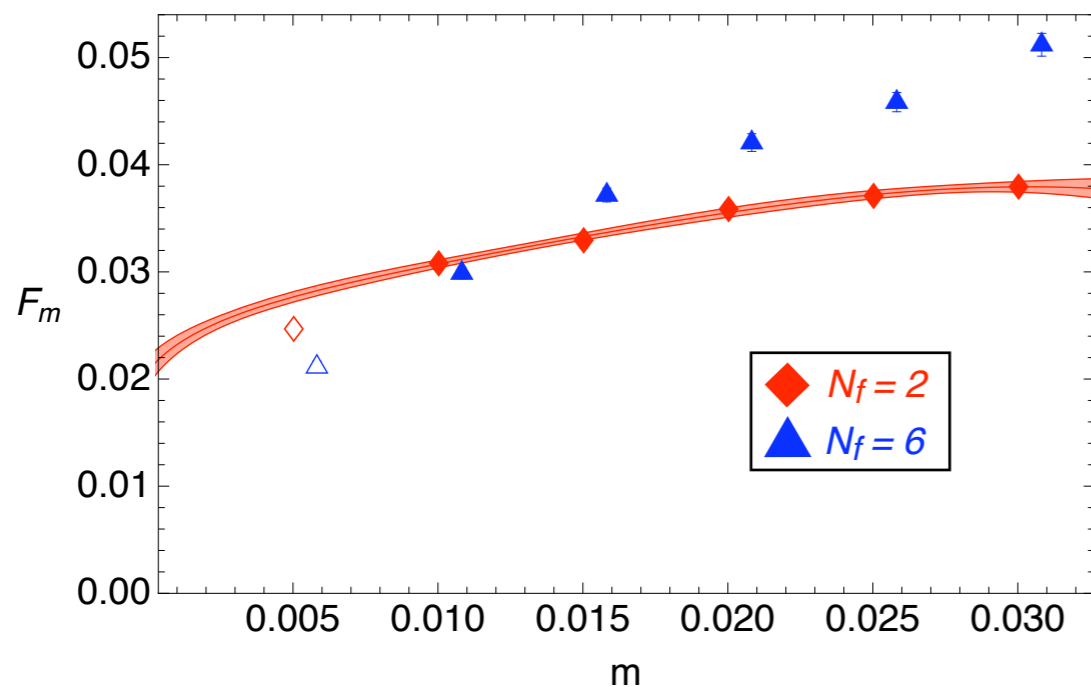
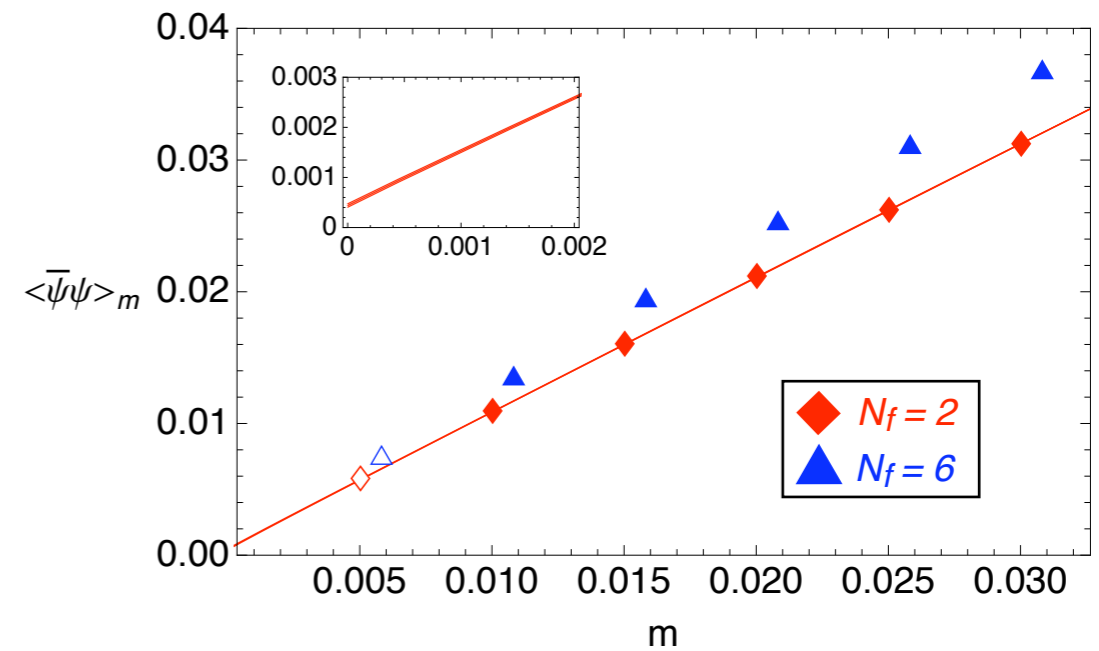
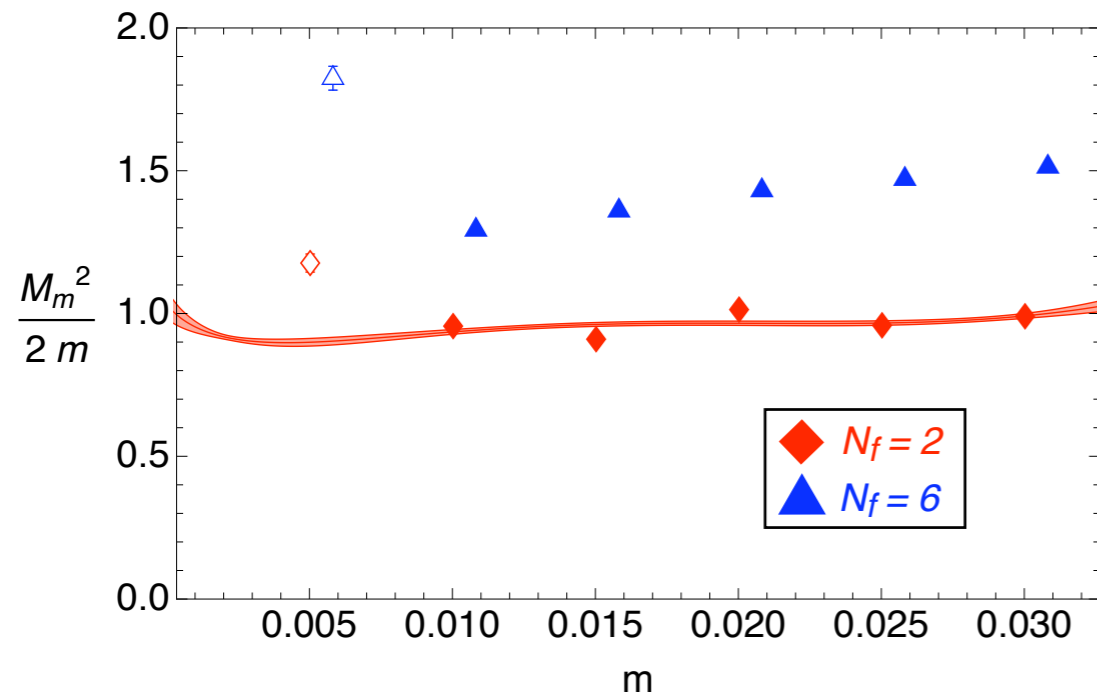
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Finite-Volume Effects?

$N_f=6, \beta=2.10, L_s=16, m_0=1.8$

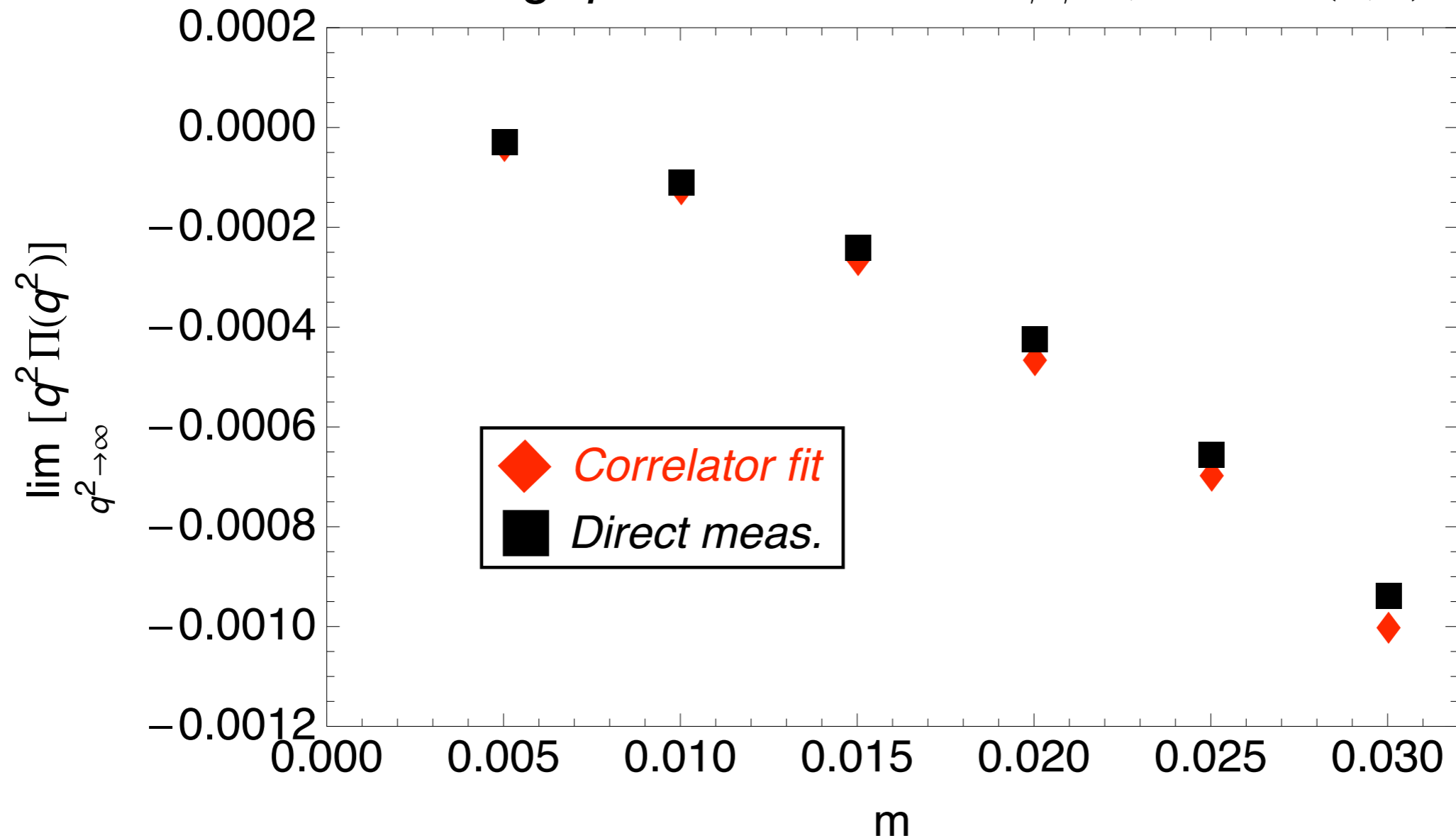


NNLO chiral fits



Momentum dependence

Leading $q^2 \rightarrow \infty$ coeff. and $\langle \bar{\psi}\psi \rangle$, Pade-(1,2)



Excellent agreement between direct measurement and OPE