Collective Scattering of Light From Cold and Ultracold Atomic Gases

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Outline

1. Introduction

2. Collective scattering/instabilities involving **cold atoms**
   – Collective Atomic Recoil Lasing (CARL)
   2.1 outline of CARL model & experiments
   2.2 Optomechanical nonlinear optics
   2.3 Links with other physical phenomena

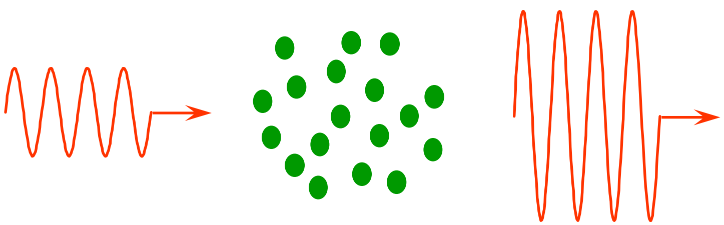
3. Collective scattering/instabilities involving **ultracold atoms**
   3.1 "Quantum CARL" model
   3.2 Quantum dynamics – LENS & MIT experiments
   3.3 Links with other physical phenomena

4. Conclusions & Acknowledgements

1. Introduction

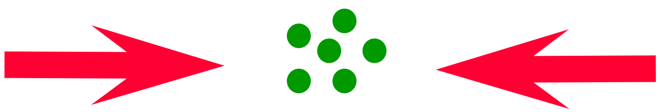
Most studies of atom-light interactions involve <u>either</u>

EM Field Evolution
(e.g. laser physics, spectroscopy)

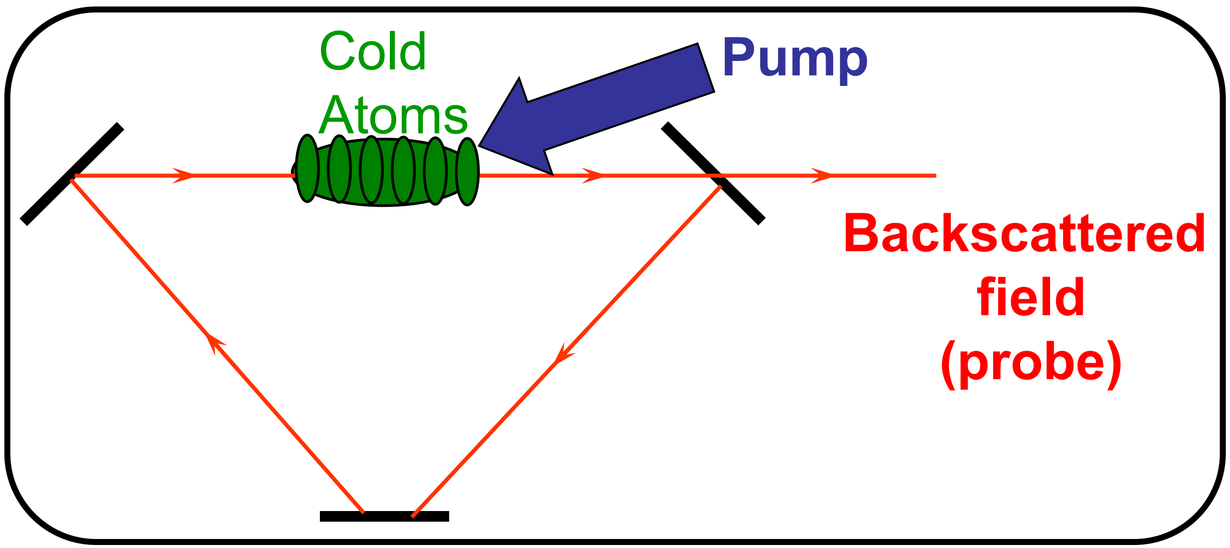


OR

Atomic centre-of-mass dynamics
(e.g. optical cooling)



This talk : Interactions involving self-consistent evolution of electromagnetic fields **AND** atomic centre of mass dynamics e.g. Collective Atomic Recoil Lasing (CARL)



CARL = instability involving simultaneous light amplification & spatial self-organisation/bunching of atoms

Related phenomena/terminology :

- Recoil-induced resonance
- Superradiant Rayleigh scattering
- Kapiza-Dirac scattering

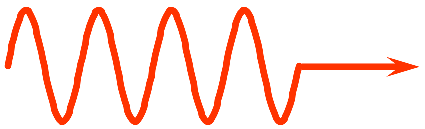
2. Collective Scattering/Instabilities Involving Cold Atoms - Collective Atomic Recoil Lasing (CARL) Model

•R.Bonifacio & L. DeSalvo, Nucl. Inst. Meth. **A 341**, 360 (1994)
•R.Bonifacio, L. DeSalvo, L.M. Narducci & E.J. D'Angelo PRA **50**, 1716 (1994).

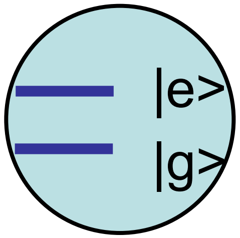
Model is <u>one-dimensional</u> and <u>semiclassical</u>.

There are three parts to the description of the atom+field system :

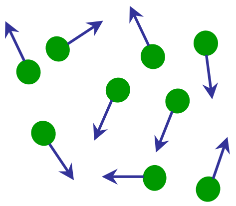
1. Optical field evolution



2. Internal atomic degrees of freedom (dipole moment, population difference)

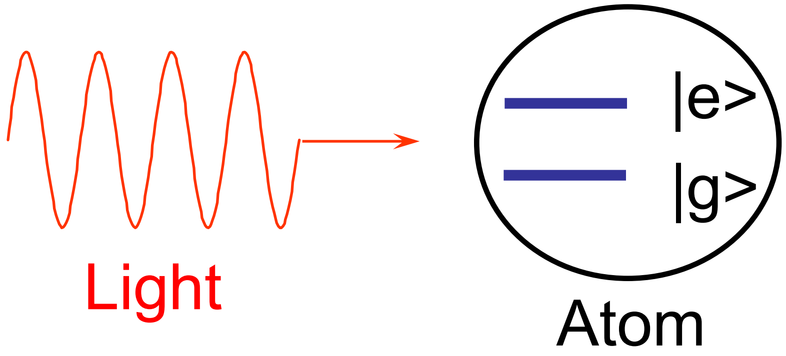


3. External atomic degrees of freedom (position, momentum)



2.1 Model

1. Internal atomic evolution



Proper treatment needs quantum description (Bloch equations)

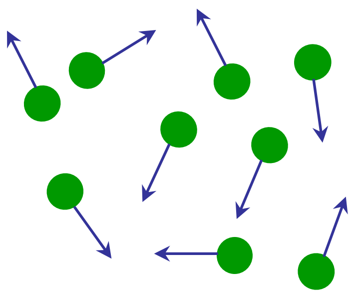
Here we consider only simplest case where light is far detuned from any resonance - atom behaves as a linear dipole.

i.e. induced dipole moment is proportional to electric field

$$\vec{d} = \alpha \vec{E}$$

α=polarisability

2. External atomic evolution



We consider atoms as classical point particles (for now)
i.e. point dipoles

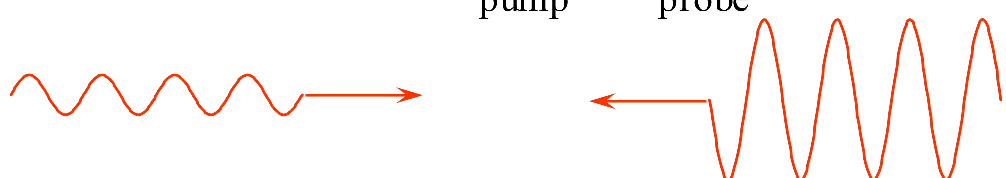
Force on each atom is the dipole force :

$$F_z = \vec{d} \cdot \frac{\partial \vec{E}}{\partial z}$$

d = dipole moment of atom
E = electric field

If E is the sum of two counterpropagating fields (as in CARL)

$$\vec{E} = \vec{E}_{pump} + \vec{E}_{probe}$$

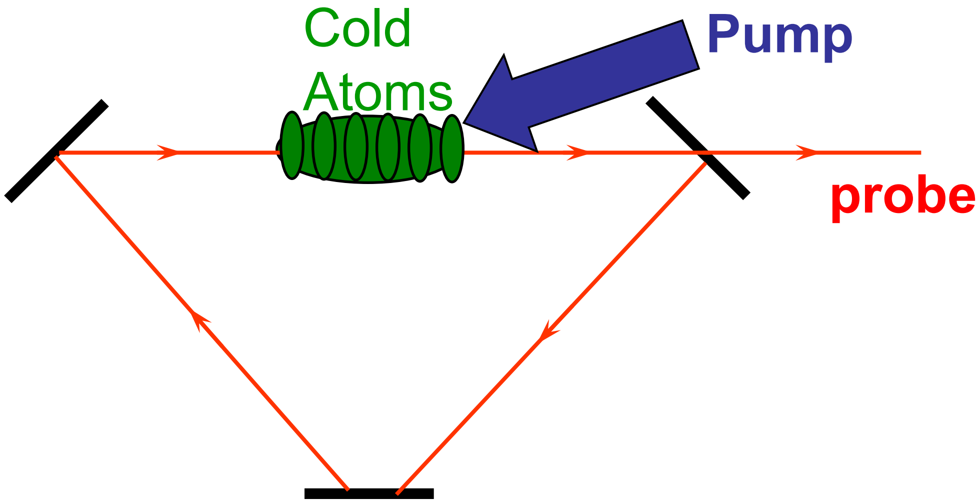


$$\vec{E}_{probe} = \left(E_{probe} e^{i(kz-\omega t)} + c.c.\right)\hat{x} \qquad \vec{E}_{pump} = \left(E_{pump} e^{-i(kz+\omega t)} + c.c.\right)\hat{x}$$

then $$F_z = \vec{d} \cdot \frac{\partial \vec{E}}{\partial z} \propto \left(E_{pump} E_{probe} e^{2ikz} + c.c.\right)$$

Spatially periodic by $\lambda/2$

3 Field evolution



Maxwell's wave equation

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\underline{E}(z,t) = \mu_0 \frac{\partial^2 \underline{P}(z,t)}{\partial t^2}$$

- Pump is assumed strong and undepleted.
- High-finesse cavity (usually) - light interacts with atoms for long times
  - modes are narrow and well separated in frequency
- Only one mode in each direction interacts with atoms
- Field source is a collection of point dipoles $\underline{P}(t) = \sum_j \underline{d}_j(t)\,\delta\left(\underline{x} - \underline{x}_j(t)\right)$

The equations for position, momentum and probe amplitude can be written as :

$$\frac{d\theta_j}{d\tau} = \bar{p}_j$$

$$\frac{d\bar{p}_j}{d\tau} = -(Ae^{i\theta_j} + c.c.)$$

$$\frac{dA}{d\tau} = \left\langle e^{-i\theta_j} \right\rangle - \kappa A$$

$$\theta_j = \frac{4\pi z_j}{\lambda}$$ Atom position in optical potential

$$\bar{p}_j = \frac{m\mathrm{v}_j}{\hbar k \rho}$$ Scaled momentum

$$A = \sqrt{\frac{2\varepsilon_0 \left|E_{\mathrm{probe}}\right|^2}{\hbar\omega\rho}}$$ Scaled scattered EM field amplitude

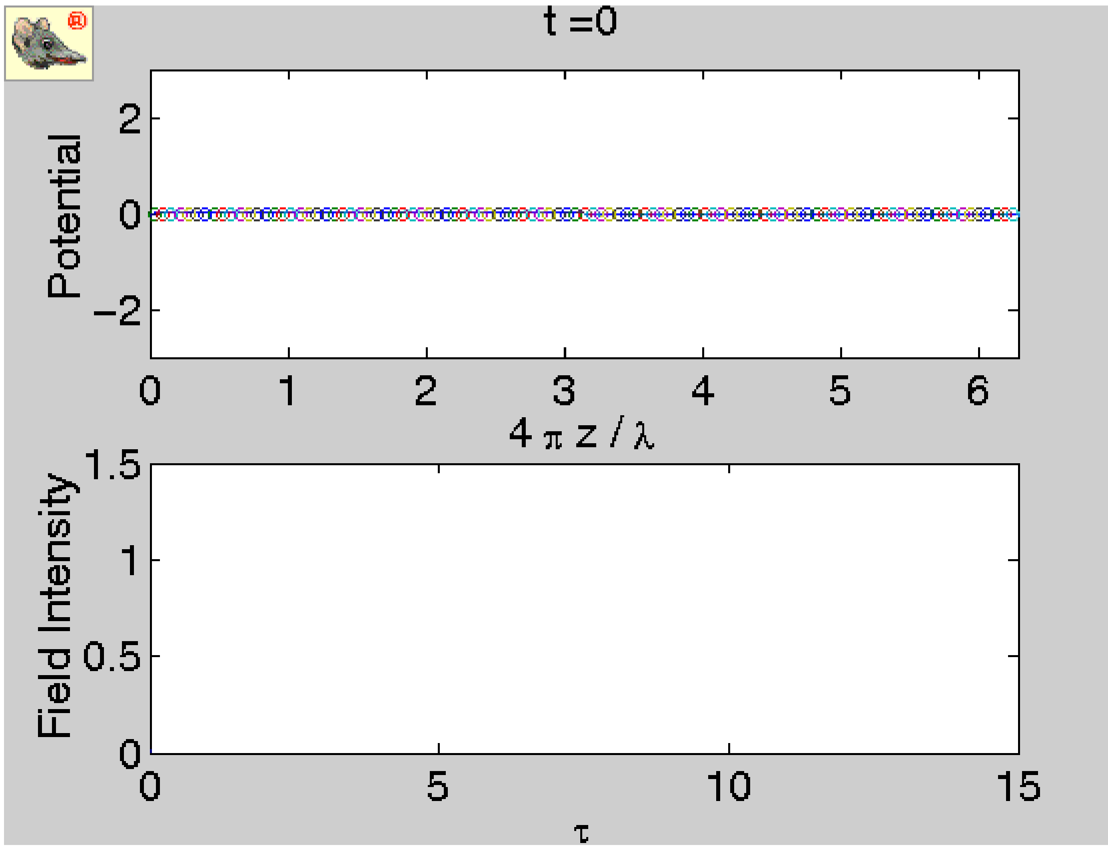
$$\tau = \omega_r \rho t$$ Scaled time variable

$$\kappa = \frac{K}{\omega_r \rho}$$ Scaled field decay rate

CARL parameter : $\rho \propto \frac{\left(\mathrm{I}_{\mathrm{pump}}\mathrm{n}\right)^{1/3}}{\Delta_a^{2/3}}$

CARL Instability animation

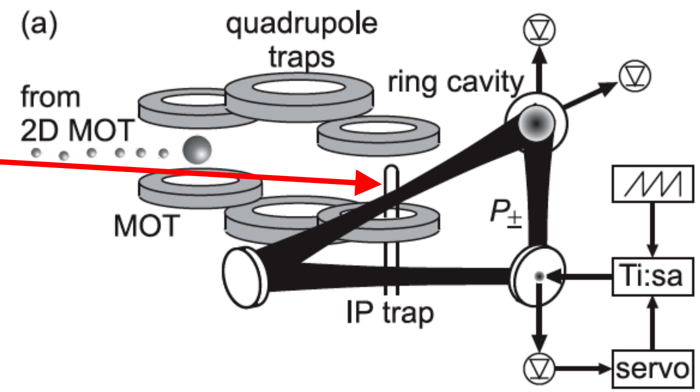
Animation shows evolution of atomic positions in the dynamic optical potential together with the scaled probe field intensity.



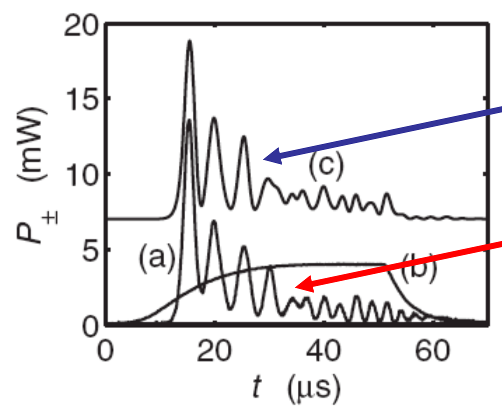
Results from CARL model agree well with experimental results :

*S. Slama, S. Bux, G. Krenz, C. Zimmermann, and Ph.W. Courteille, PRL **98** 053603 (2007).*

10^7 Rb atoms
T~1μK



Probe intensity evolution



Theory

Expt.

Probe intensity $\propto N^{4/3}$ in good cavity limit (κ<<1)

$\propto N^2$ in superradiant limit (κ>1)

Agreement with experiment is encouraging : what now?

Collective “optomechanical” instabilities e.g. CARL are of :

- practical interest in optical physics as

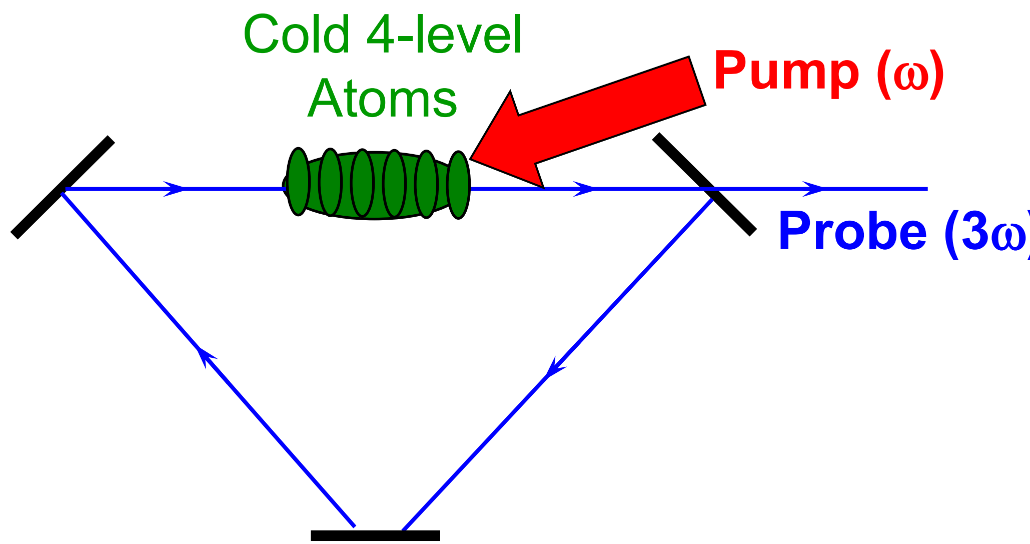
  e.g. source of new nonlinear optical phenomena

- fundamental interest as analogue/testing ground for various global coupling/mean-field models used in other fields

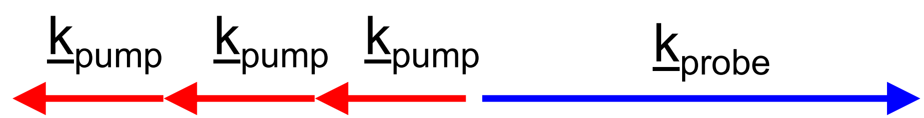
  e.g. plasma physics, condensed matter,
  mathematical biology & neuroscience

2.2 Optical application : optomechanical nonlinear optics

Using a similar model, it can be shown that 3^{rd} harmonic generation may be possible via an optomechanical instability related to CARL.

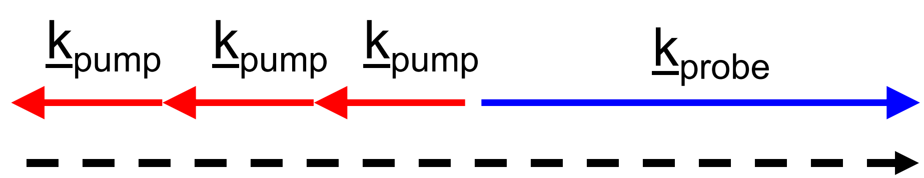


Direction of harmonic generation is opposite to that usually expected as phase mismatch for counterpropagating harmonic field is very large



$$|\Delta \underline{k}| = |\underline{k}_{probe} - 3\underline{k}_{pump}| = 6|\underline{k}_{pump}|$$

Generation of density modulation with period $\lambda/6$ causes "self-phase matching".



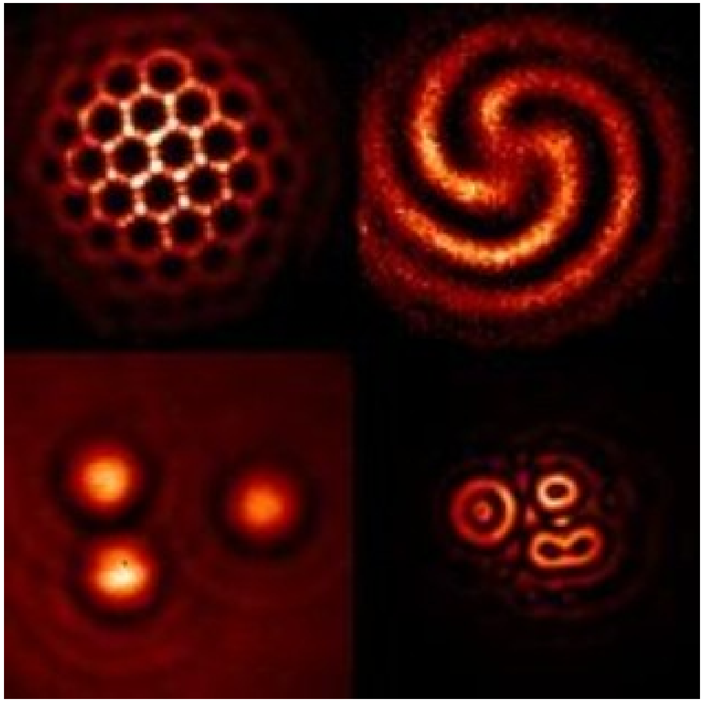
$$|\underline{k}_{grating}| = 6|\underline{k}_{pump}|$$

G.R.M. Robb & B.W.J. McNeil, PRL **94**, 023901 (2005).

2.2 Optical application : optomechanical nonlinear optics

Current project (Leverhulme Trust – Strathclyde) :

Optical patterns in cold atomic gases (theory/experiment)

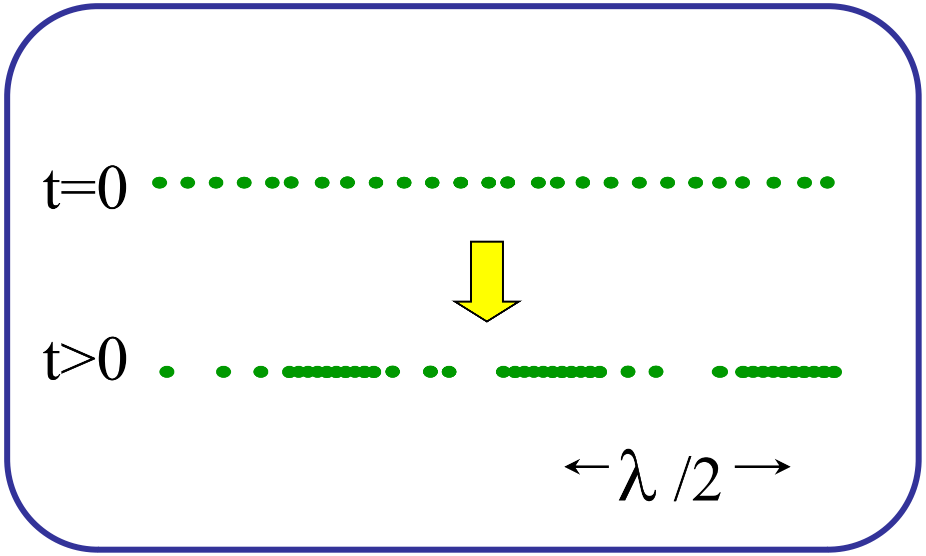


Nonlinear optical patterns have been produced in e.g.warm sodium gas

Nonlinearity here is due to internal atomic dynamics only.

How does nonlinear optical pattern formation differ in cold gases where internal **and** external atomic dynamics will be important.

2.3 Links with/analogues of other phenomena

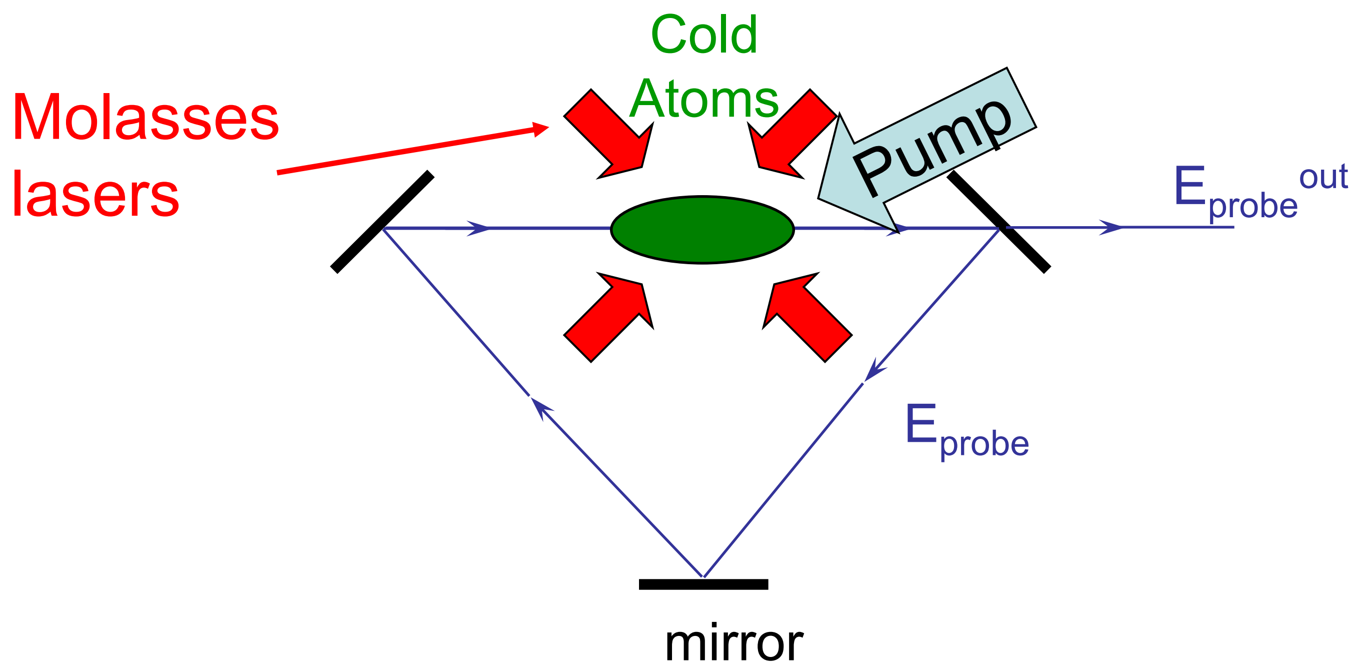


CARL can be interpreted as spontaneous ordering due to global coupling by light.

Versatility and controllability of cold-atom experiments make optomechanical instabilities a potentially useful analogue or testing ground for various processes involving self-organisation or synchronisation.

2.3 Links with/analogues of other phenomena

Example : "Viscous CARL" & the Kuramoto Model



Same setup as for CARL, but with additional "molasses" lasers to damp atomic momentum

For details see :

- G.R.M. Robb, N. Piovella, A. Ferraro , R. Bonifacio, Ph. W. Courteille and C. Zimmermann, Phys. Rev. A 69, 041403(R) (2004)
- J. Javaloyes, M. Perrin , G. L. Lippi, and A. Politi, Phys. Rev. A **70**, 023405 (2004)
- C. Von Cube, S. Slama, Ph. W. Courteille, C. Zimmermann, G.R.M. Robb, N. Piovella & R. Bonifacio **PRL 93**, 083601 (2004).
- Y. Kuramoto, Prog. Theor. Phys. Suppl. 79, 223 (1984).

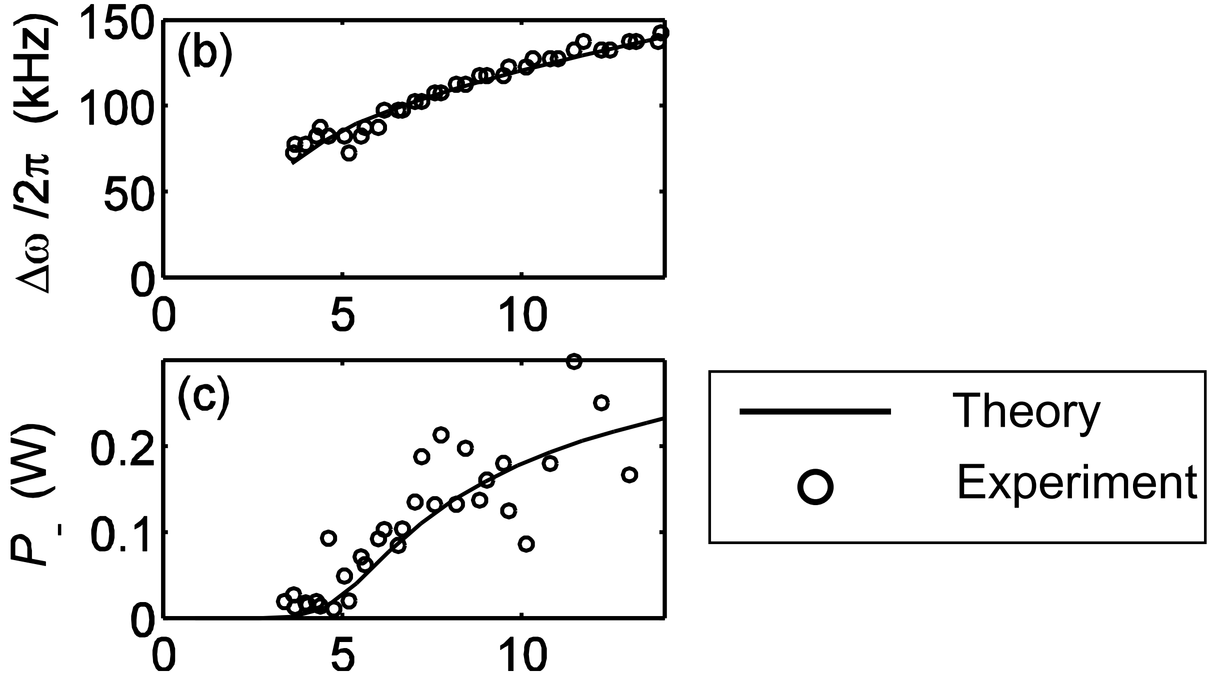
Addition of molasses/damping produces instability threshold:

Good agreement with experiment

-predicts threshold for pump power at ~4W

Beat frequency vs pump power

Backscattered power vs pump power



Intracavity pump power (W)

- G.R.M. Robb, et al. **PRA 69**, 041403 (2004)
- J. Javaloyes et al., **PRA 70**, 023405 (2004).
- C. Von Cube et al. **PRL 93**, 083601 (2004).

The threshold behaviour in the viscous CARL experiments is similar to that in the **Kuramoto model of collective synchronization in large systems of globally coupled oscillators.**

$$\frac{d\theta_j}{dt} = \omega_j + \frac{K}{N}\sum_{j=1}^{N}\sin\left(\theta_i - \theta_j\right) \qquad j = 1..N$$

θ_j is the phase of oscillator j **= atomic position**
ω_j is its (random) natural frequency **= thermal velocity**
Coupling constant K $\propto$ **pump power**

The Kuramoto model has been used to model a wide range of synchronisation phenomena in physics and mathematical biology.

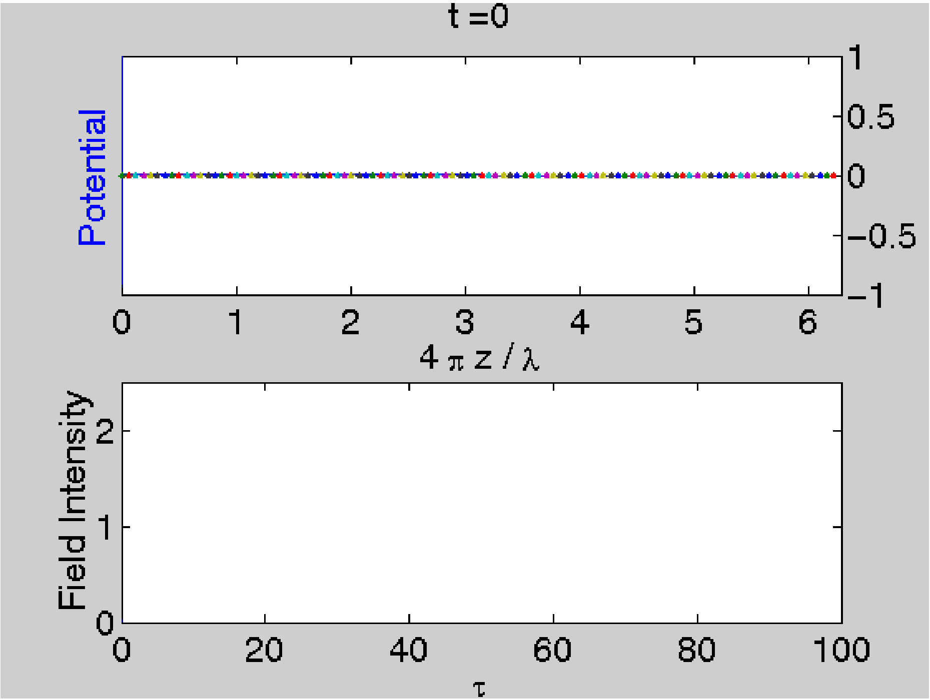
Similar equations describe synchronization of cold atoms in coupled by light, flashing fireflies, pacemaker cells in the heart and rhythmic applause!

SH Strogatz, Nature **410**, 268 (2001)

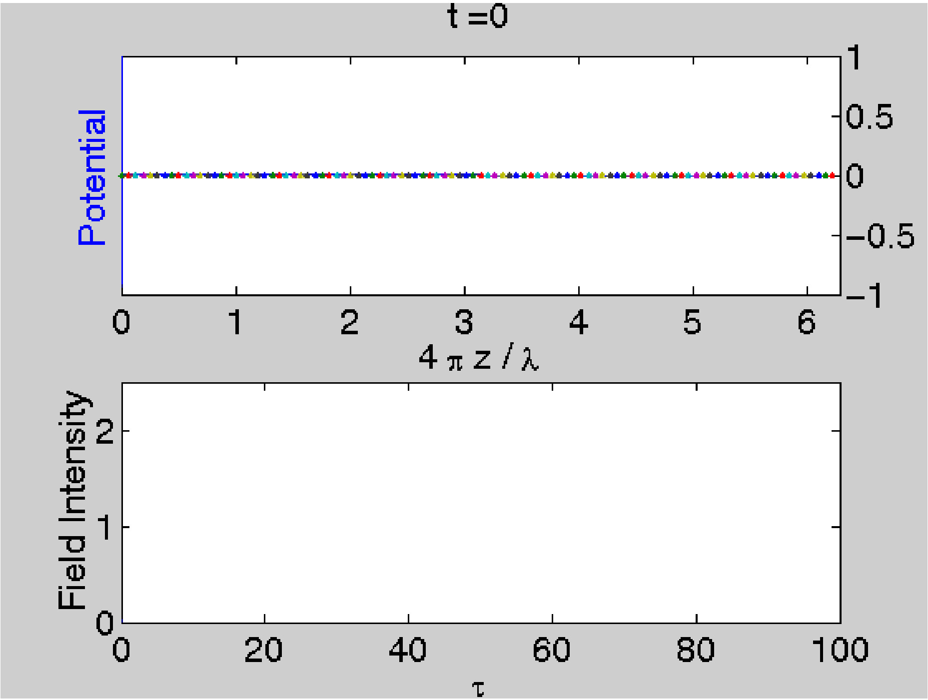
Coupling constant K $\propto$ **pump power**

Synchronisation transition occurs when K exceeds a threshold, K_c

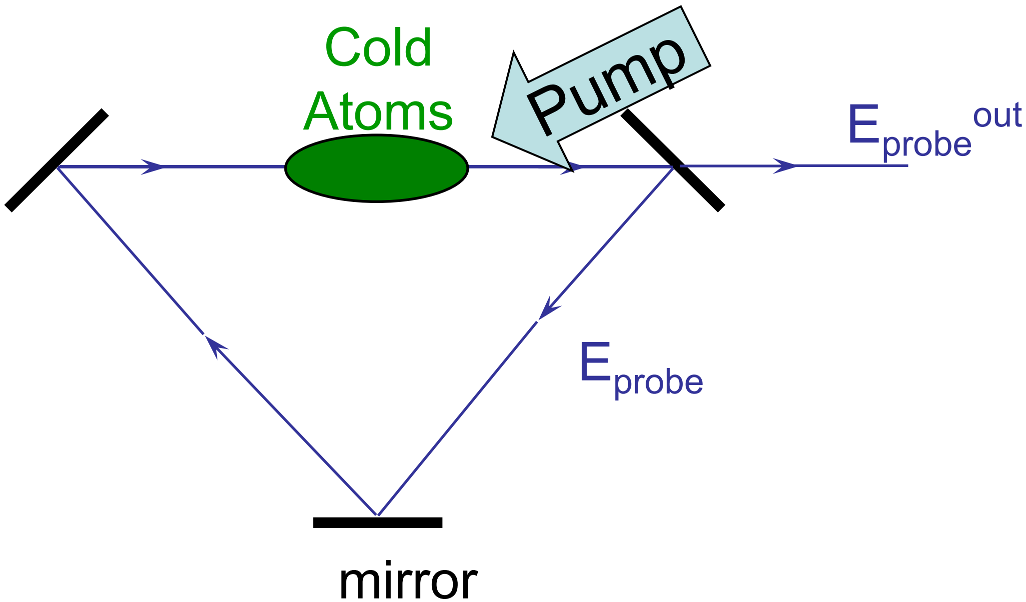
$K<K_c$ (weak pump)



$K>K_c$ (strong pump)



Another example : CARL and chaos



Now we consider a pump field which is **phase modulated**.

Pump field is of the form

$$e^{-i\left(k_{pump}z+\omega_{pump}t+\alpha_m \sin\omega_m t\right)}$$

where α_m = modulation amplitude
ω_m=modulation frequency

Incorporating a phase-modulated pump field into the CARL model, we obtain :

$$\frac{d\theta_j}{d\tau} = p_j$$

$$\frac{dp_j}{d\tau} = -(Ae^{i\left(\theta_j-\alpha_m \sin\Omega_m\tau\right)} + c.c.) \quad \text{(j=1..N)}$$

$$\frac{dA}{d\tau} = \left\langle e^{-i(\theta-\alpha_m \sin\Omega_m\tau)}\right\rangle + i\delta A$$

where $\Omega_m = \dfrac{\omega_m}{\omega_r\rho}$

is the scaled

modulation frequency

Robb & Firth, PRA 78, 041804 (2008)

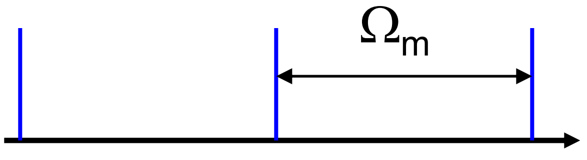
Using the identity $\exp\left(i\alpha_m \sin \Omega_m \tau\right) = \sum_{-\infty}^{\infty} J_n\left(\alpha_m\right) e^{in\Omega_m \tau}$

atom-light interaction now involves **many potentials/resonances** with

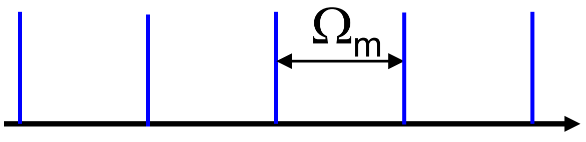
- phase velocities separated by Ω_m
- momentum width $\propto \sqrt{|A| J_n\left(\alpha_m\right)}$

Three different regimes :

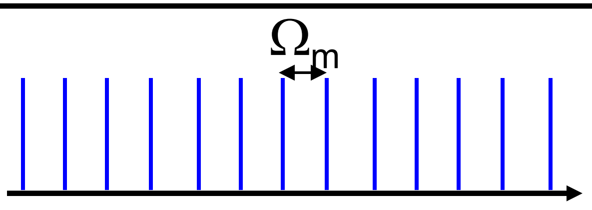
High frequency modulation (Ω_m >>1) :



Intermediate frequency modulation ($\Omega_m \approx 1$) :

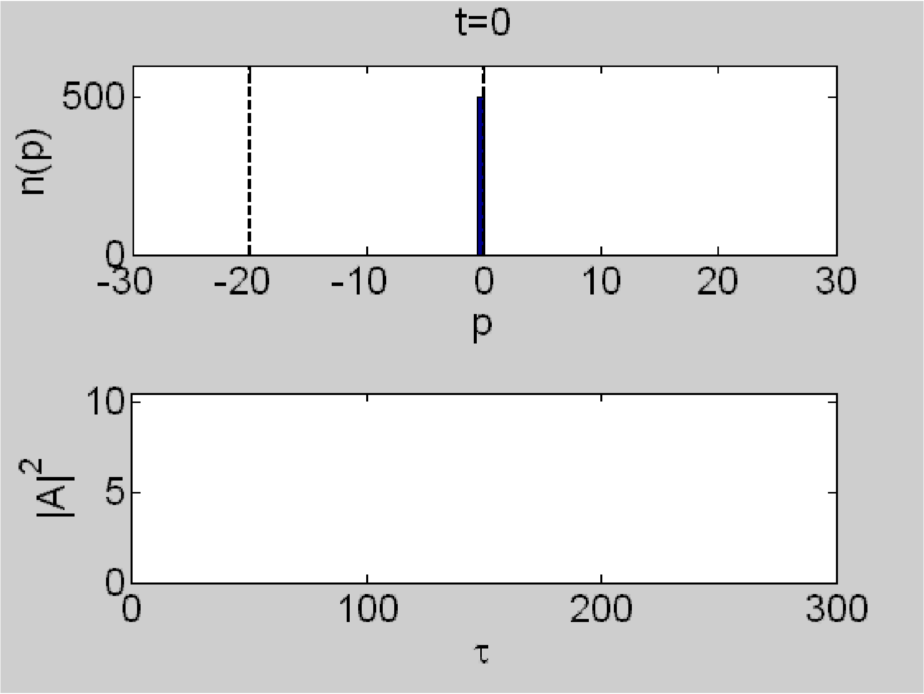


Low frequency modulation (Ω_m << 1) :



interaction involves many resonances with phase velocities separated by Ω_m and width $\propto \sqrt{|A| J_n(\alpha_m)}$

As scattered field amplitude (probe) is amplified, resonance overlap can occur, causing chaotic diffusion of atomic momentum.



Intermediate/low frequency phase modulation may be able to produce CARL intensities which greatly exceed those with a coherent pump.

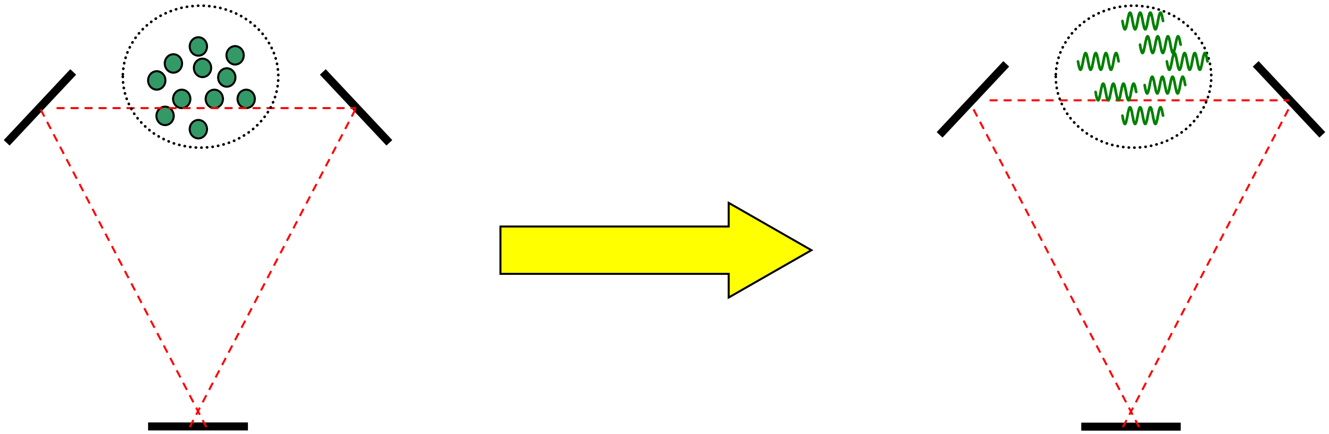
Perhaps possible to test quasilinear theories of e.g. plasma turbulence in this (non-plasma) system?
e.g. measurement of plasma diffusion coefficients?

3. Collective Scattering/Instabilities Involving Ultracold Atoms

So far we have treated the atomic gas as
a collection of classical point particles

For gases with sub-recoil temperatures (<~10μK) such as BECs this description fails

- we must then describe the atoms quantum mechanically



How does the transition from classical gas to BEC affect instabilities such as CARL?

3.1 Quantum CARL model

(i) Newtonian atomic motion equations are replaced with a Schrodinger equation for the single particle wavefunction $\Psi(\theta,\tau)$

$$\frac{d\theta_j}{d\tau} = \bar{p}_j$$

$$\frac{d\bar{p}_j}{d\tau} = -(Ae^{i\theta_j} + c.c.)$$

$$\Rightarrow \quad \frac{\partial \Psi(\theta,\tau)}{\partial \tau} = \frac{i}{\rho}\frac{\partial^2 \Psi}{\partial \theta^2} - \frac{\rho}{2}\left[Ae^{i\theta} - c.c.\right]\Psi$$

(ii) Average in wave equation becomes QM average

$$\frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j} \quad \Rightarrow \quad \int_0^{2\pi} d\theta \, |\Psi|^2 e^{-i\theta}$$

Maxwell-Schrodinger Equations

$$\frac{\partial \Psi(\theta,\tau)}{\partial \tau} = \frac{i}{\rho}\frac{\partial^2 \Psi}{\partial \theta^2} - \frac{\rho}{2}\left[Ae^{i\theta} - c.c.\right]\Psi$$

$$\frac{dA(\tau)}{d\tau} = \int_0^{2\pi} d\theta \, |\Psi|^2 e^{-i\theta} - \kappa A$$

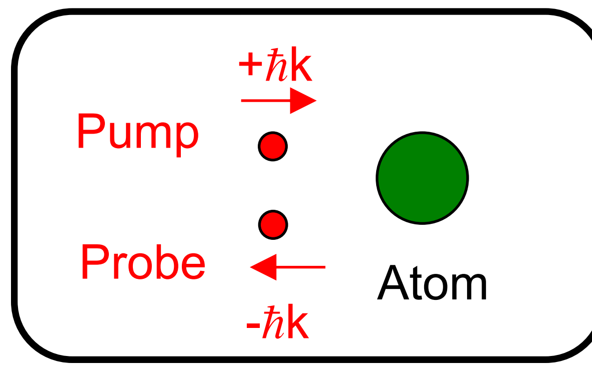
See : G. Preparata, PRA (1988)
N. Piovella et al. Optics Comm, **194**, 167 (2001)

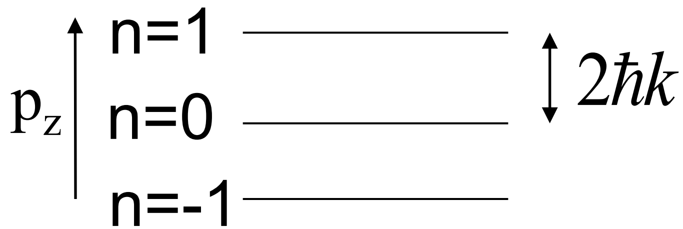
We assume uniform BEC density with L >> $\lambda/2$, so Ψ is periodic with period $\lambda/2$

$$\Psi(\theta,\tau) \propto \sum_{n=-\infty}^{\infty} c_n(\bar{t})\, e^{in\theta}$$

Momentum exchange no longer continuous.
Only <u>discrete</u> values of momentum exchange are possible :

$$p_z = n\,(2\hbar k)\ , \quad n = 0, \pm 1, ..$$





N. Piovella et al. Optics Comm, **194**, 167 (2001)
N. Piovella et al., Laser Physics **12**, 188 (2002)

Dynamical regime is determined by the CARL parameter, ρ

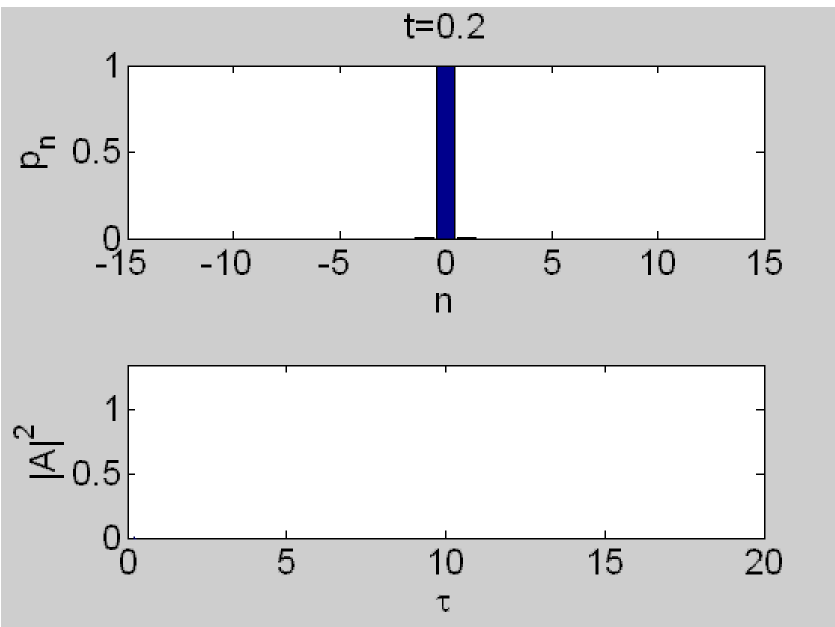
$$\frac{\partial \Psi(\theta,\tau)}{\partial \tau} = \frac{i}{\rho}\frac{\partial^2 \Psi}{\partial \theta^2} - \frac{\rho}{2}\left[A e^{i\theta} - c.c.\right]\Psi$$

$$\frac{dA(\tau)}{d\tau} = \int_0^{2\pi} d\theta \, |\Psi|^2 e^{-i\theta} - \kappa A$$

$$\rho \propto \frac{\left(I_{pump} n\right)^{1/3}}{\Delta_a^{2/3}}$$

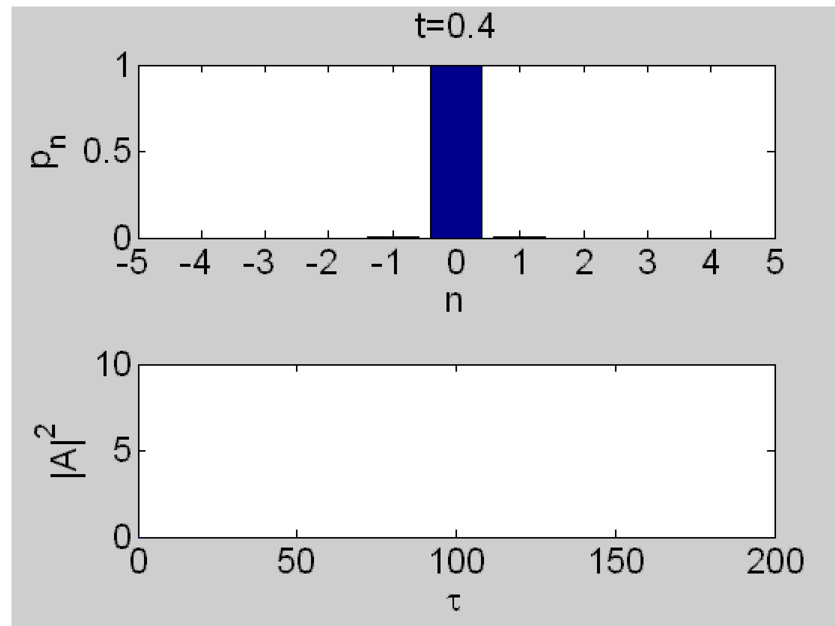
ρ can be interpreted as ~ number of photons scattered per atom

Classical CARL (ρ>>1)

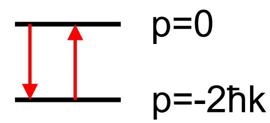


Many momentum states occupied
Field evolves as in particle model

Quantum CARL (ρ<1)

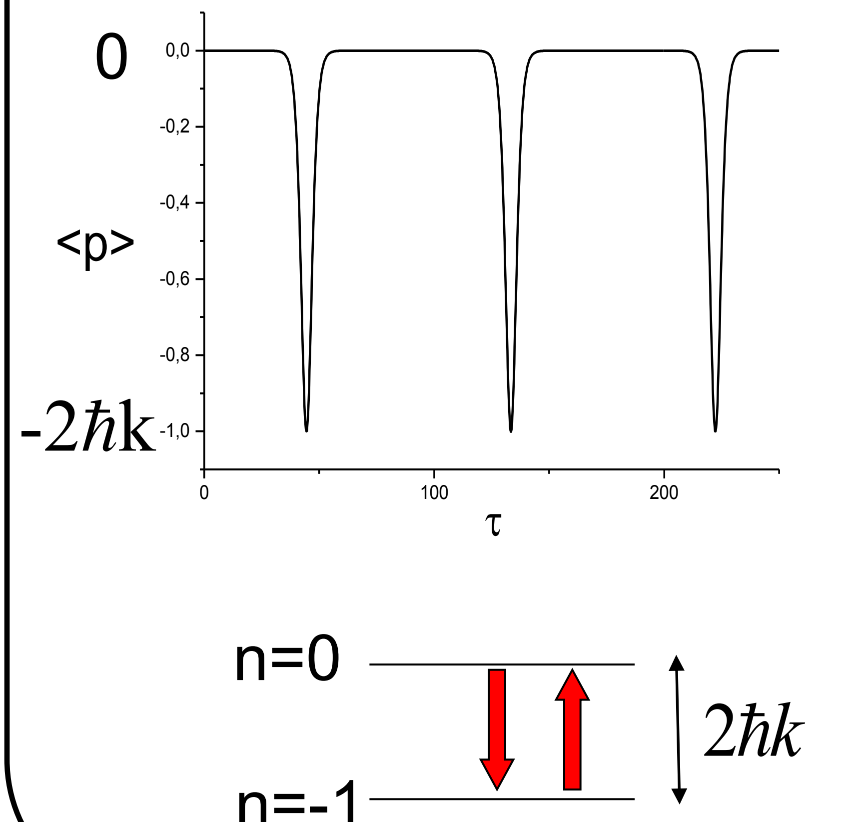


Only 2 momentum states occupied

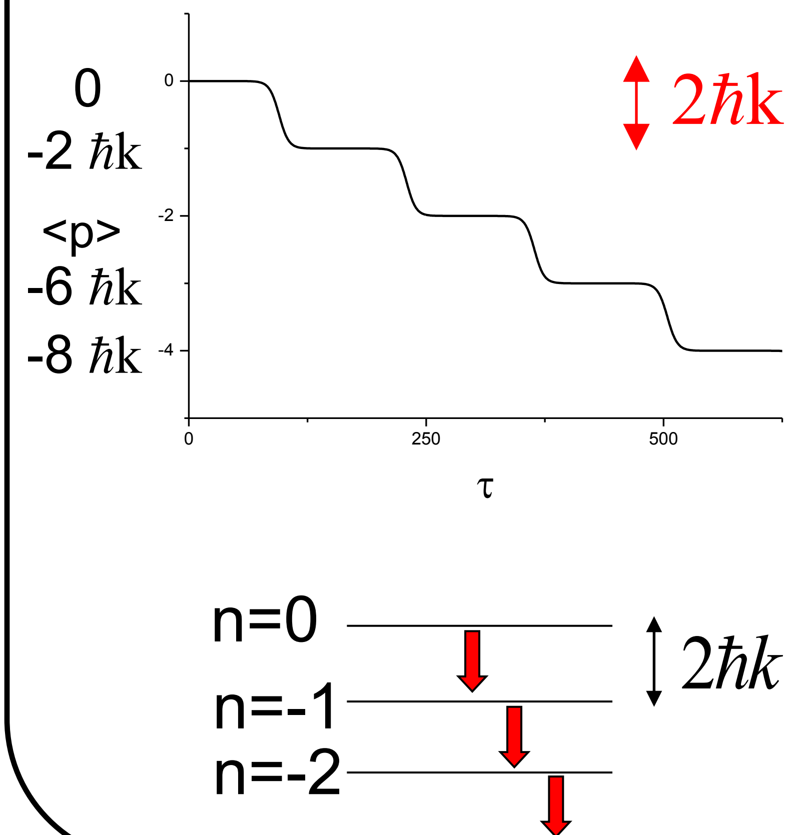


When BEC is in free space, light escapes rapidly
- Simplest model uses large K (~c/L)
-we see sequential superradiant scattering

No radiation losses (K=0)



Rapid radiation loss (large K)



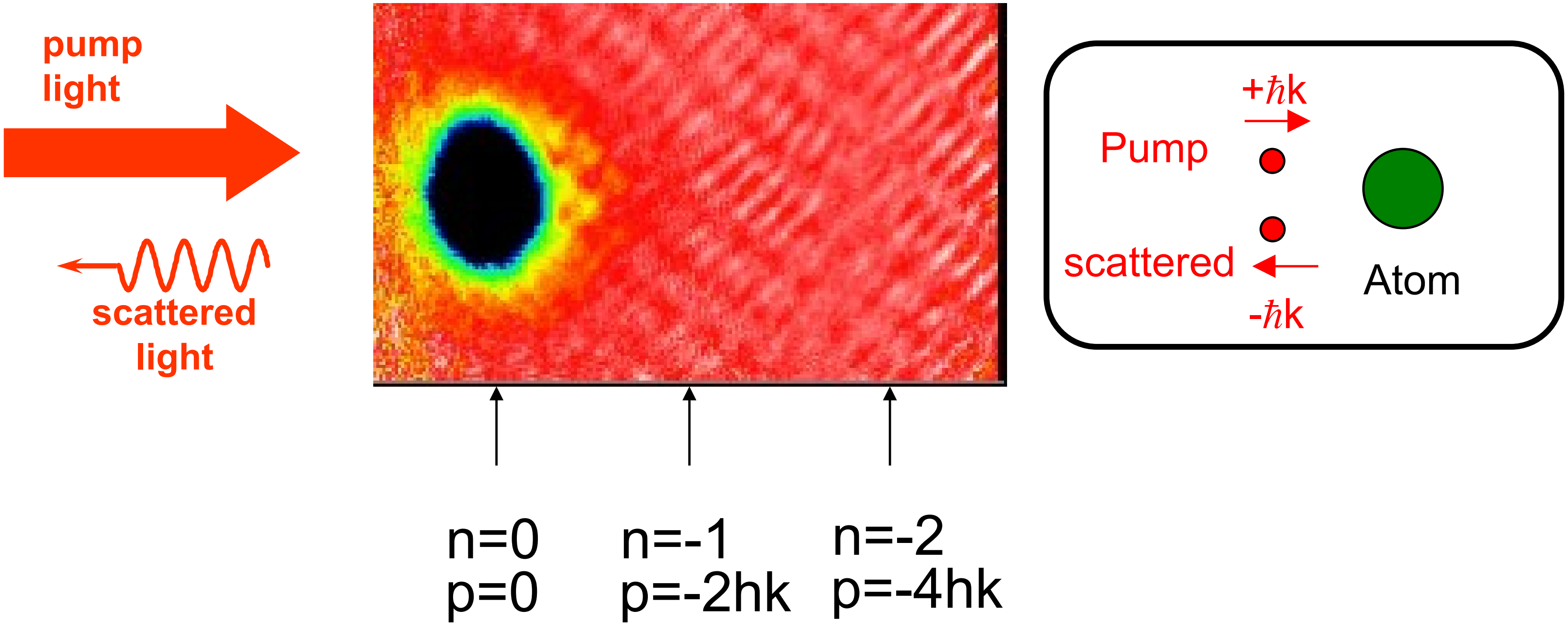
3.2 Quantum CARL dynamics – The LENS & MIT experiments

LENS experiments (Florence)

- ^{87}Rb BEC illuminated by pump laser

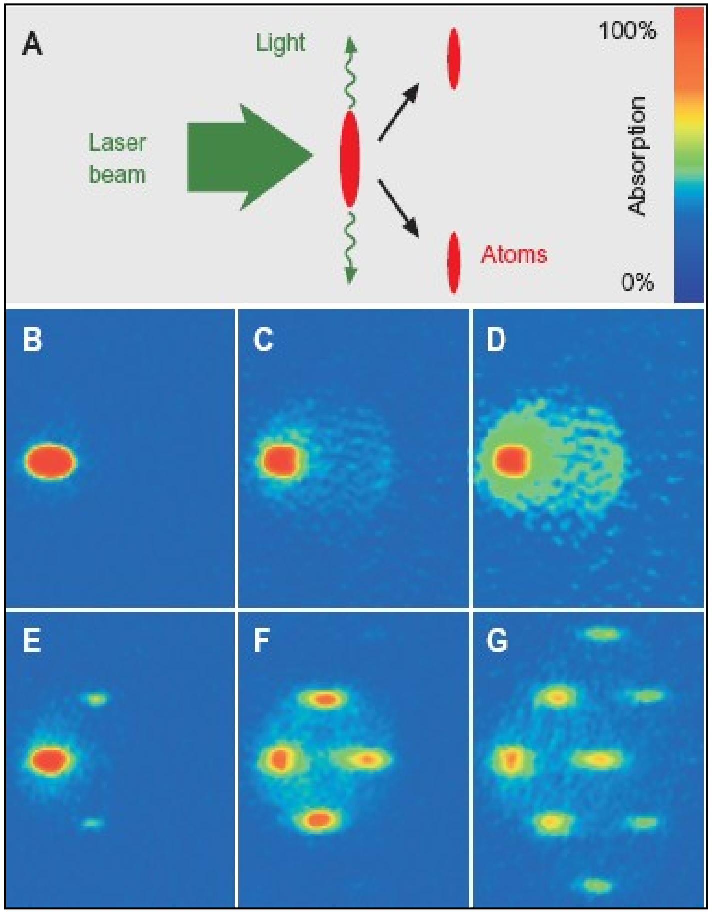
- Temporal evolution of the population in the first three atomic momentum states during the application of the light pulse

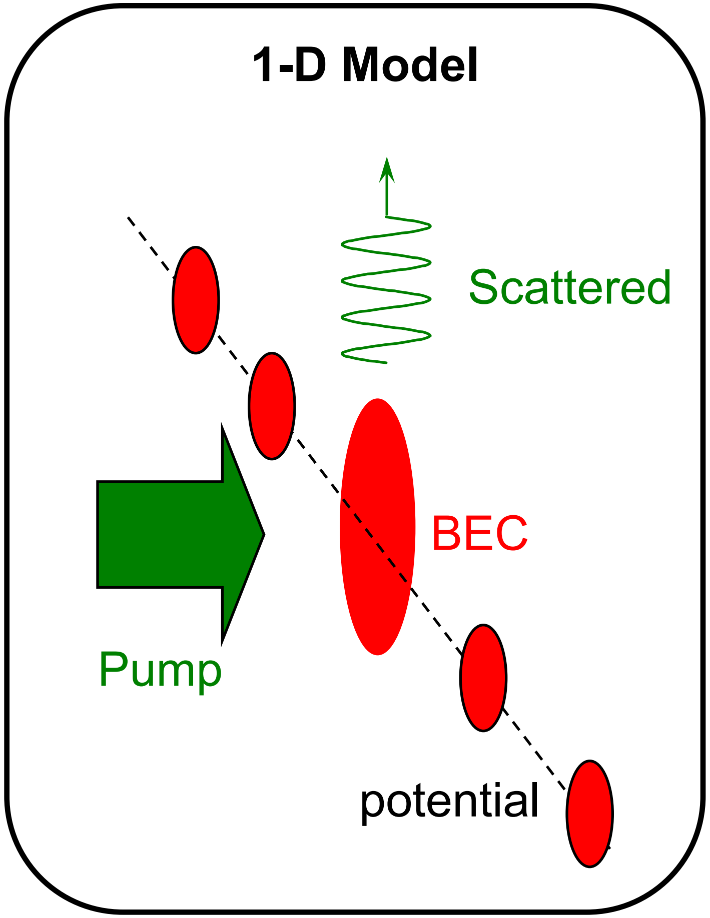
- **Evidence of sequential SR scattering**



L. Fallani et al., **PRA 71**, 033612 (2005).

MIT Experiments - Motion of atoms is two-dimensional



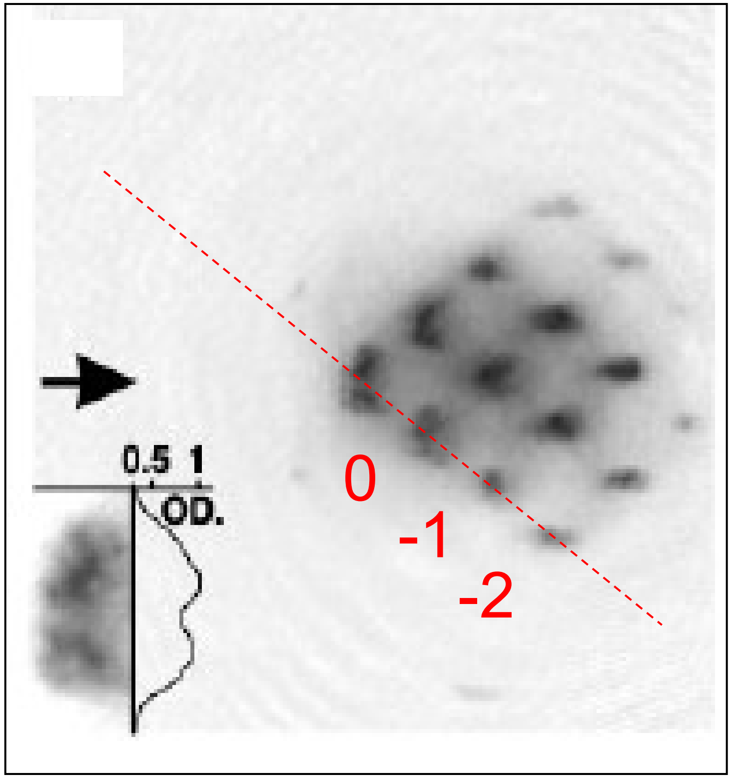


From Inouye et al., Science **285,** 571 (1999).

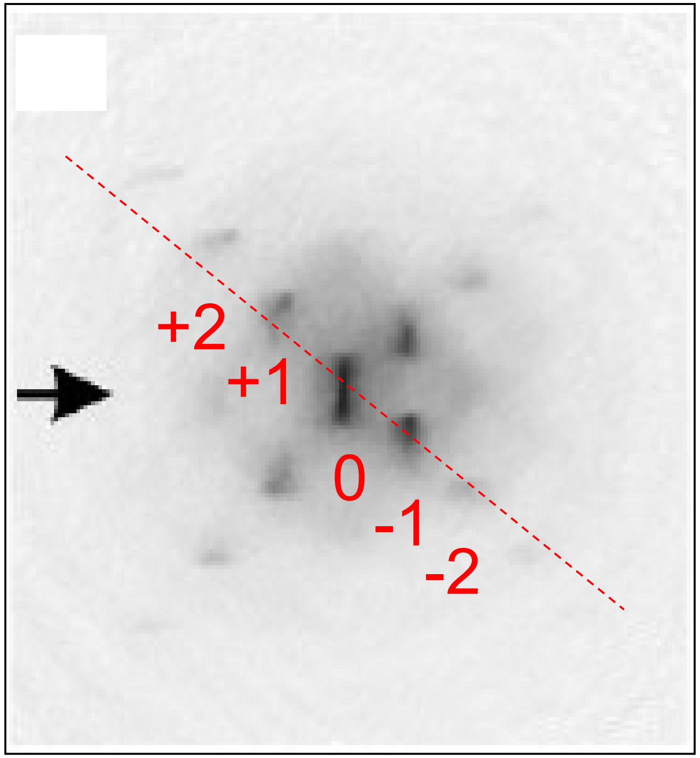
D. Schneble et. al, Science **300**, 475 (2003).

Different behaviour observed, depending on value of atom-field detuning (value of ρ)

"Superradiant Rayleigh Scattering"



"Kapiza-Dirac Scattering"



Large atom-field detuning (small ρ – quantum regime) - observe n<0 only

Small atom-field detuning (large ρ – classical regime) - observe n<0 and n>0

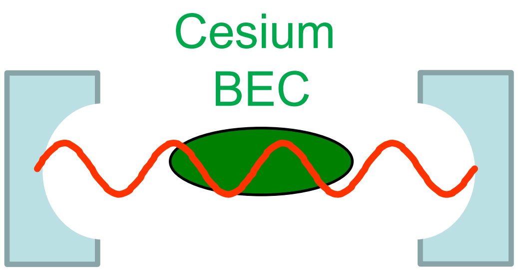
3.4 Quantum CARL – Cross-disciplinary features

Versatile testing ground for models - allows control/study of e.g.

- stochastic and chaotic dynamics
- quantum chaos
- analogues of quantum plasmas
- condensed matter systems

Current project (EPSRC) - **Modelling Condensed Matter Systems with Quantum Gases in Optical Cavities**

Collaboration between UCL (experiment), Strathclyde & Leeds (theory)



BECs in optical lattices have been used as analogues of condensed matter systems to study e.g. superfluid/insulator translation .

Cavity-BEC system has a dynamic potential and involves both short-range and long-range interactions

- new phase transitions?

- others e.g. analogue of gravitational scattering using quadrupole radiation/transitions?

Conclusions

Collective scattering of light from cold and ultracold gases are of interest for both :

Optical applications :

- New optical nonlinearities

Cross-disciplinary interest:

- Versatile testing ground for models of various coupled systems- allows control/study of e.g.
    - noise / stochastic dynamics
    - transition from regular dynamics to chaos
    - coupling range (global $\rightarrow$local)
    - quantum mechanical effects

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