UNIVERSITY OF CAMBRIDGE

BSM Search Techniques

(for the Large Hadron Collider)

Based on "A review of Mass Measurement Techniques proposed for the Large Hadron Collider", Barr and Lester, arXiv:1004.2732

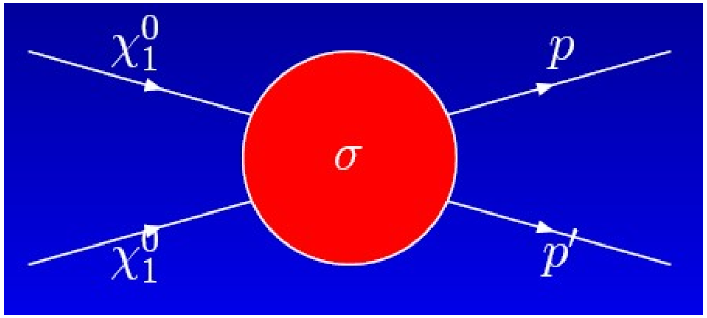
NExT-PhD Abingdon 2011

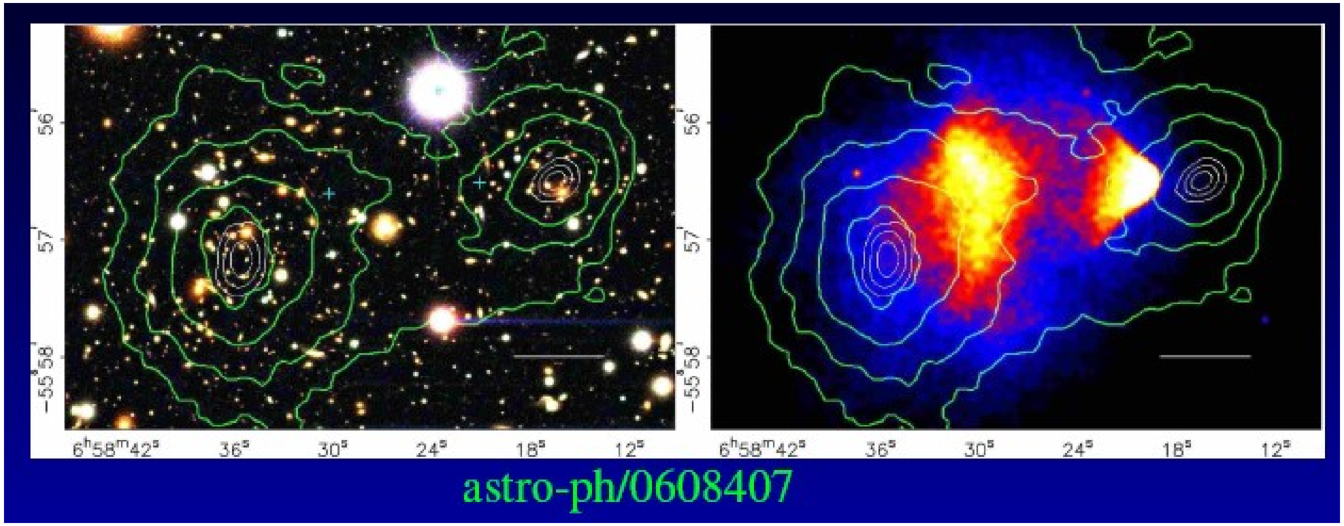
Christopher Lester
University of Cambridge

Recall ∃ awkward problems …

Aim was to fix some of these problems with the Standard Model

- Fine-tuning / "hierarchy problem" (technical) – Why are particles light?
- Does not explain Dark Matter
- No gauge coupling unification

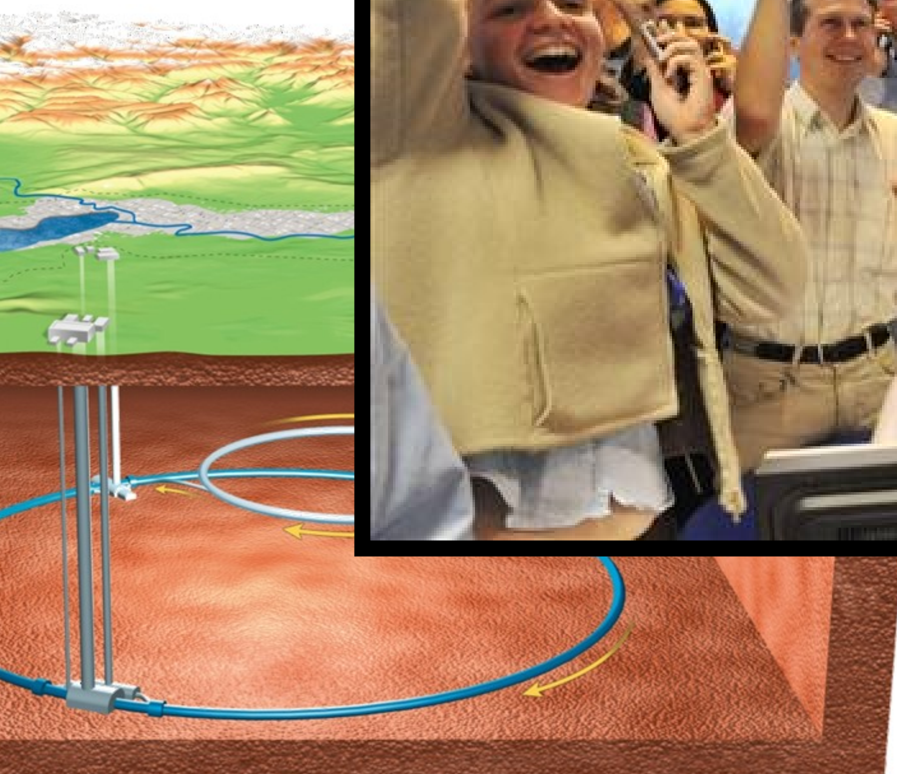




What are common features of “solutions” to these problems?

- Big increase in particle content
- Longish decay chains
- Missing massive particles
- Large jet/lepton/photon multiplicity

At some point, 5000 people will shout:



“We’ve found a …
[long pause]
… SOMETHING!”

A large collider of hadrons …
… not a collider of large hadrons

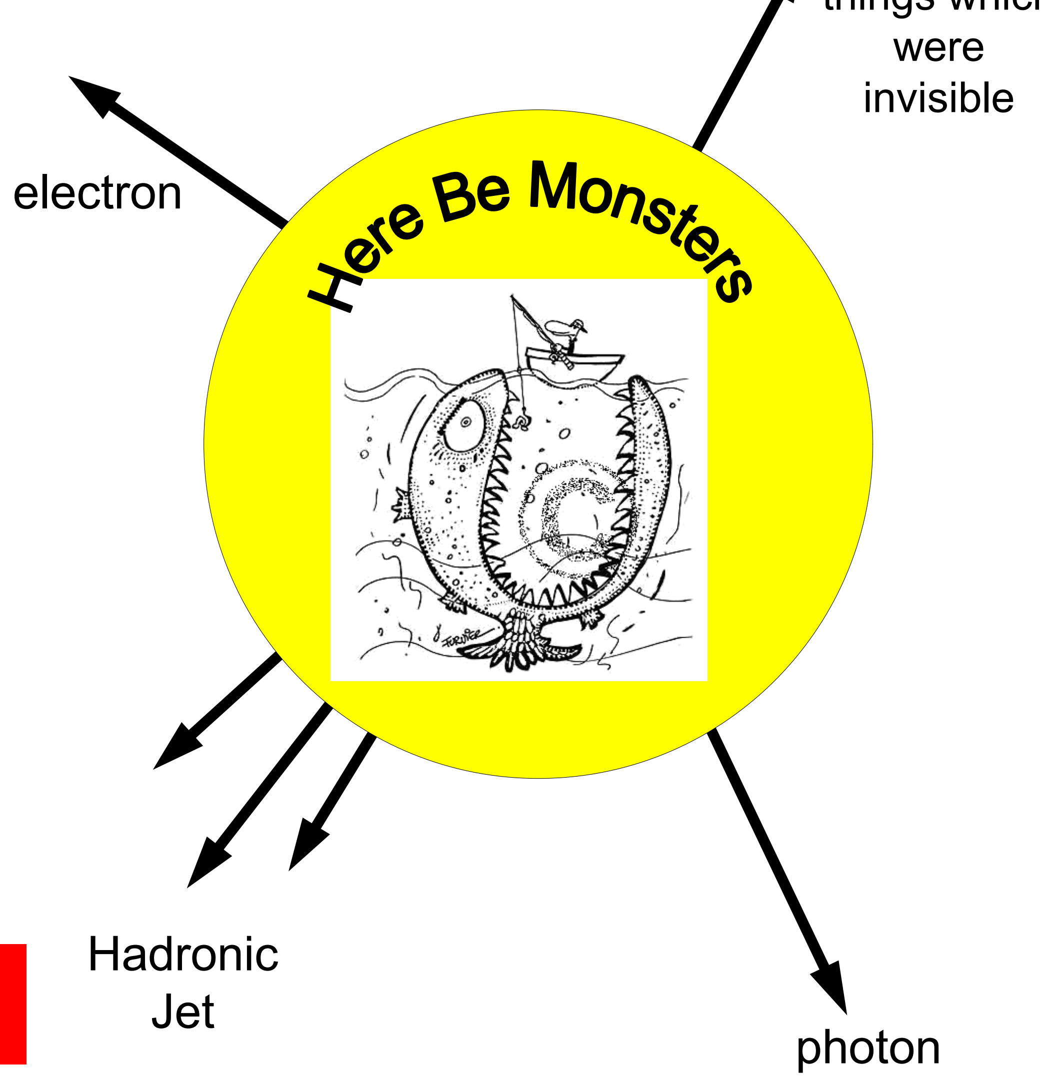
How hard is it to identify what was found?

Want to emphasise what is visible at the LHC

- Distinguish the following from each other
  - Hadronic Jets,
    - B-jets (sometimes)
  - Electrons, Positrons, Muons, Anti-Muons
    - Tau leptons (sometimes)
  - Photons

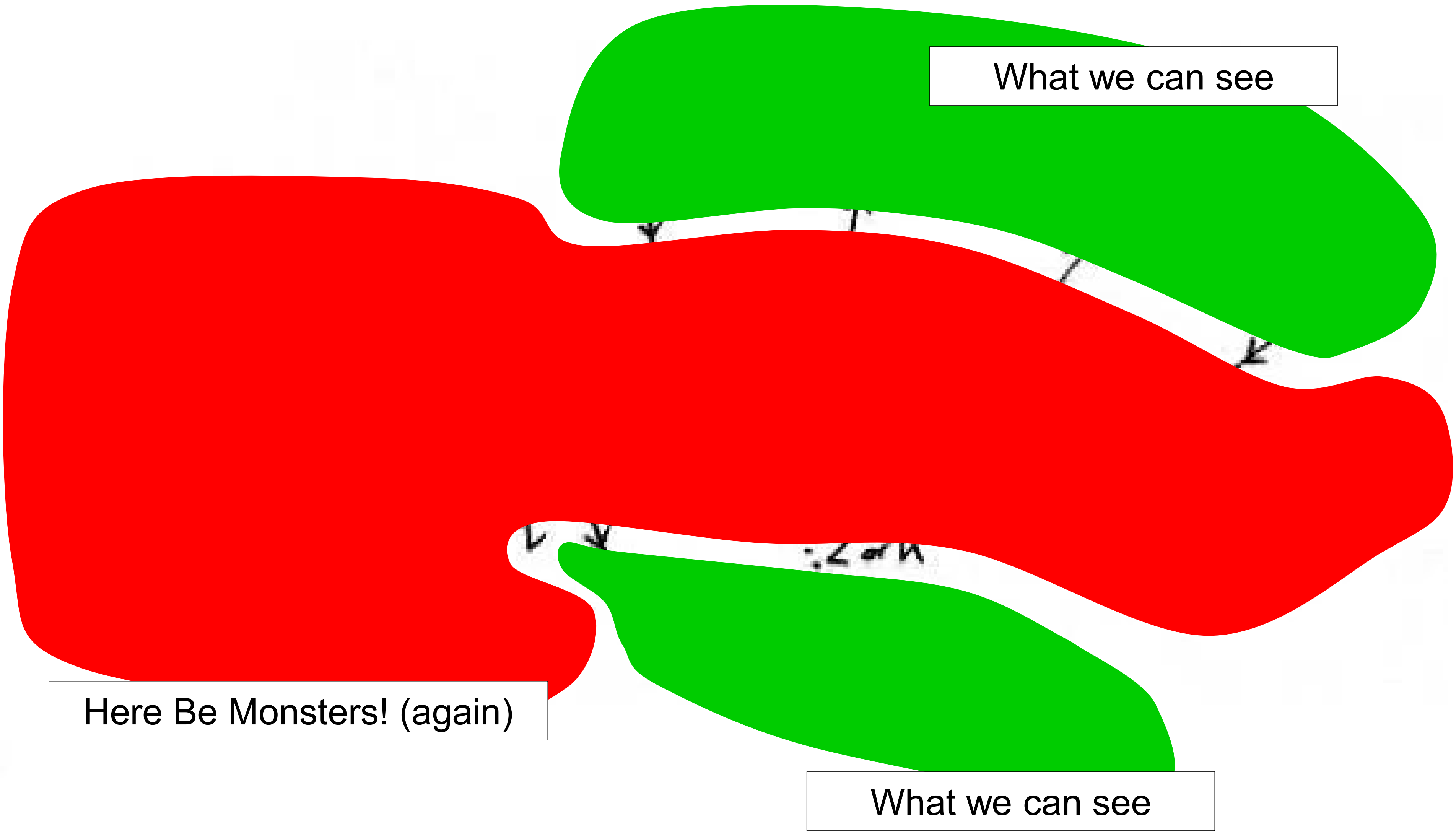
- Measure Directions and Momenta of the above.

- Infer total transverse momentum of invisible particles. (eg neutrinos)



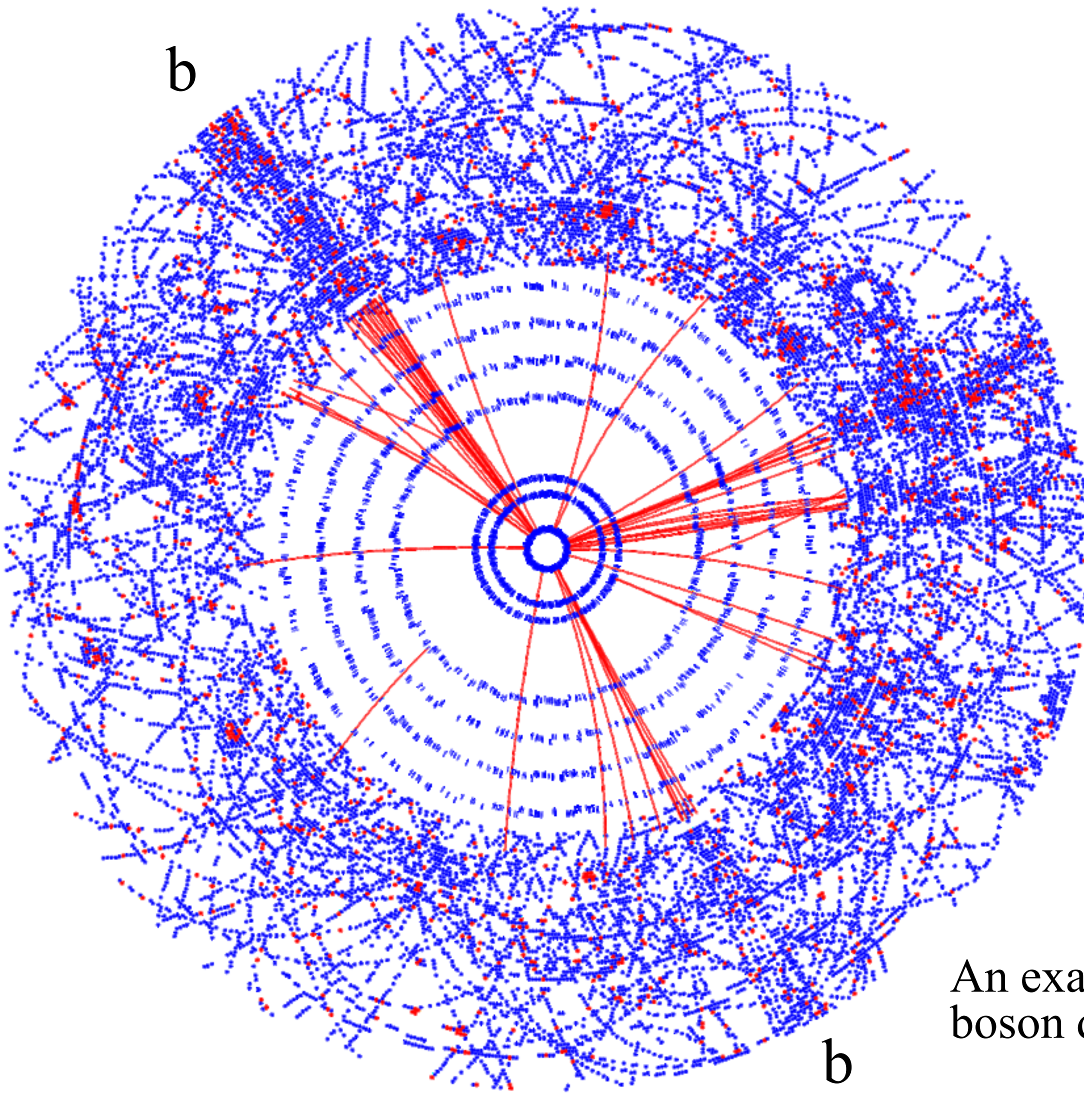
What do we NOT measure?

What might events look like?



This is the high energy physics of the 21st Century!

What events really look like scares me!



An example of an event where a higgs boson decayed to a pair of b-quarks/

Supersymmetry as Lingua Franca

Some possibilities:

- Supersymmetry
  - Minimal
  - Non-minimal
  - R-parity violating or conserving
- Extra Dimensional Models
  - Large (SM trapped on brane)
  - Universal (SM everywhere)
  - With/without small **black holes**
- "Littlest" Higgs ?
- ….

We will look mainly at supersymmetry (SUSY)

upersymmetry! CAUTION!

- It may exist
- It may not
- First look for deviations from Standard Model!

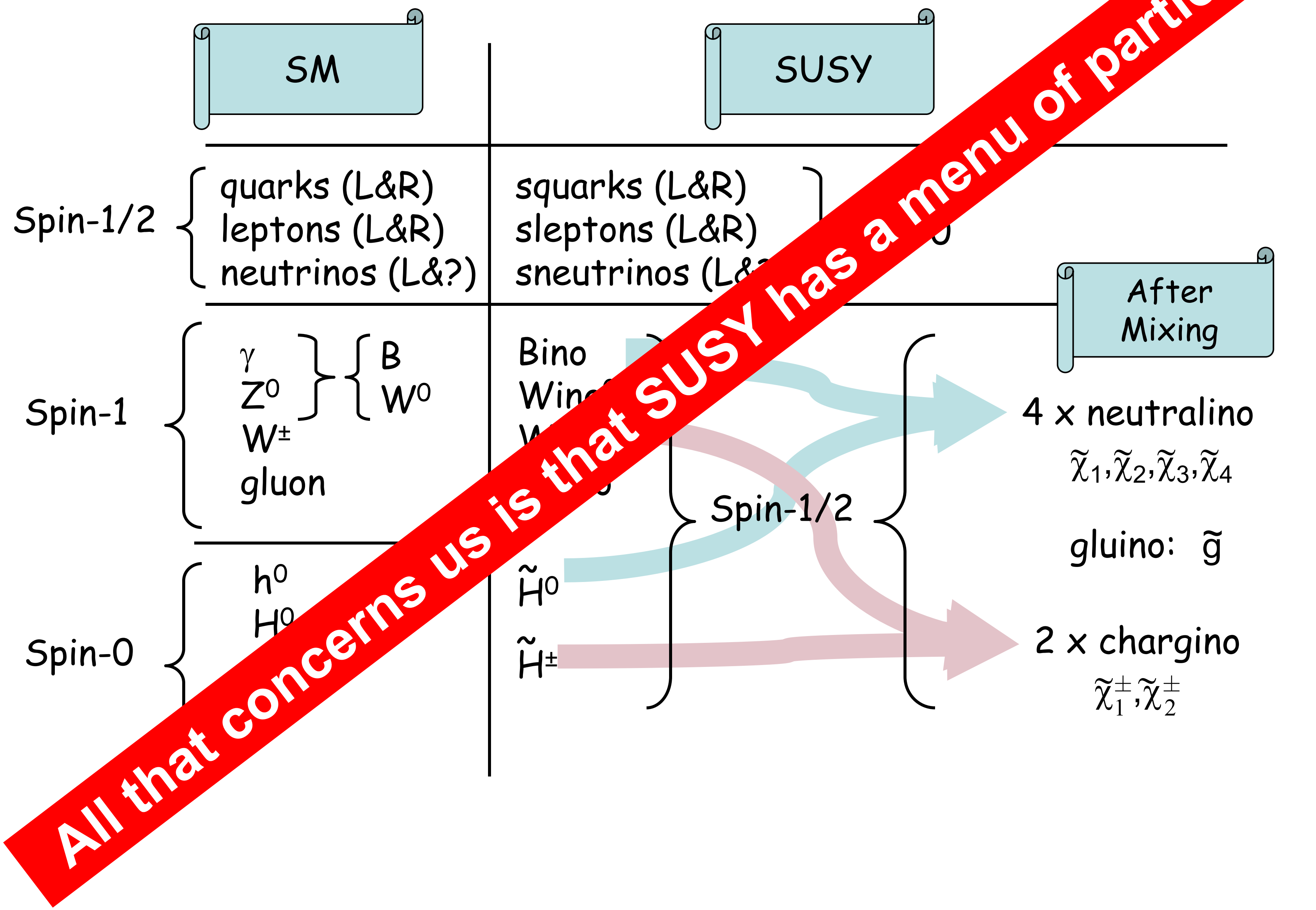
Experiment must lead theory.

Gamble:

IF DEVIATIONS ARE SEEN:

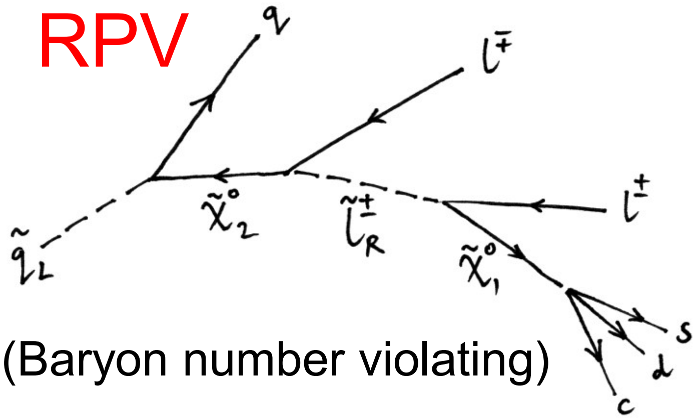
- Old techniques won't work
- New physics not simple
- Can new techniques in SUSY but can apply them elsewhere.

SUSY particle content



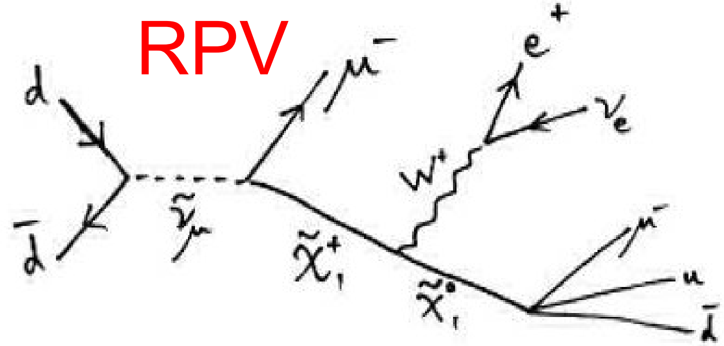
Even in SUSY many possibilities

RPV

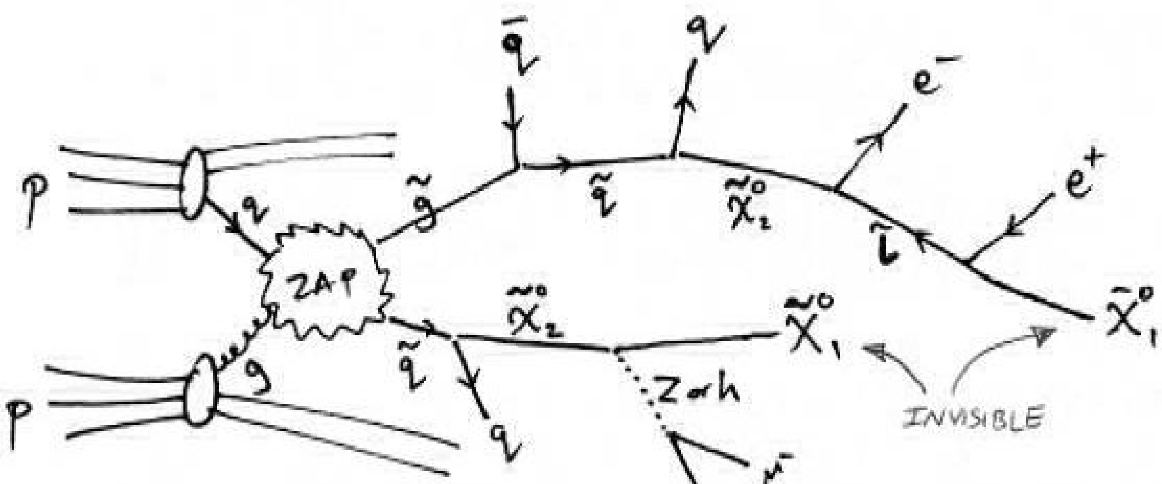


(Baryon number violating)

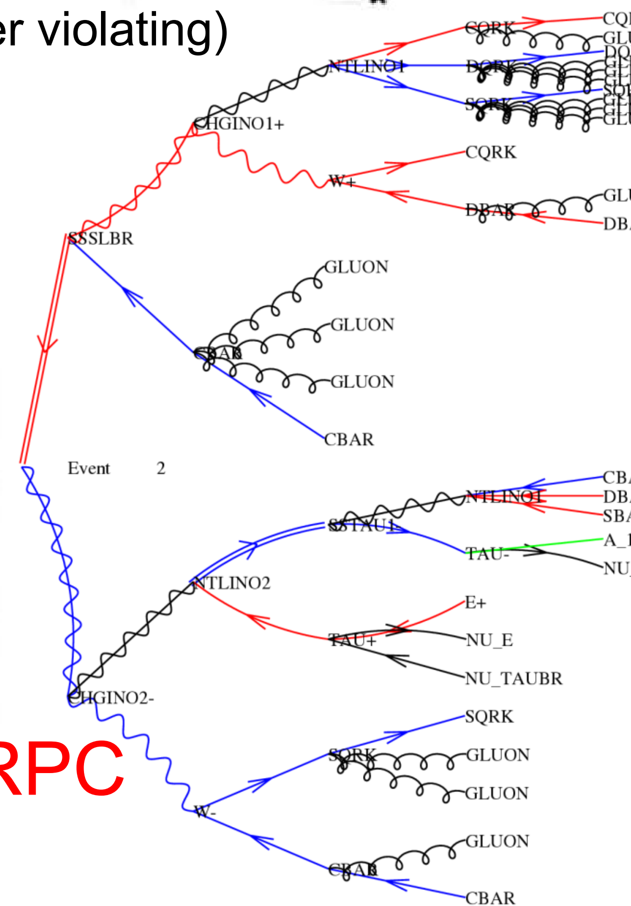
RPV



(Lepton number violating)



RPC



RPC

Do we care about masses?

- Common **Parameter** in the Lagrangian
- Expedites **discovery** – optimal **selection**
- **Interpretation**

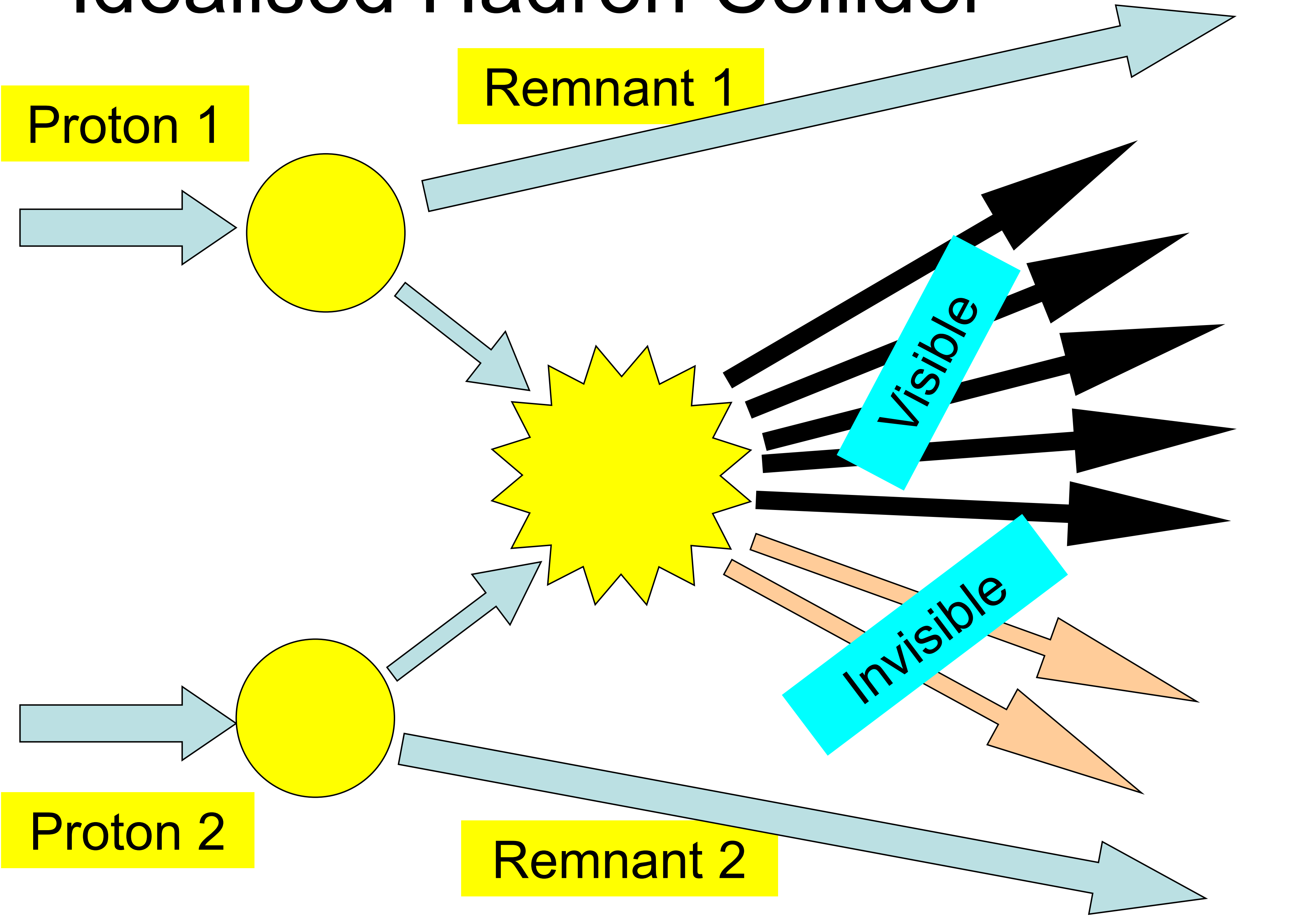
  (SUSY breaking mechanism,
  Geometry of Extra Dimensions)
- **Prediction** of new things

  Mass of W,Z ➔ indirect top quark mass "measurement" “mass measurement methods”

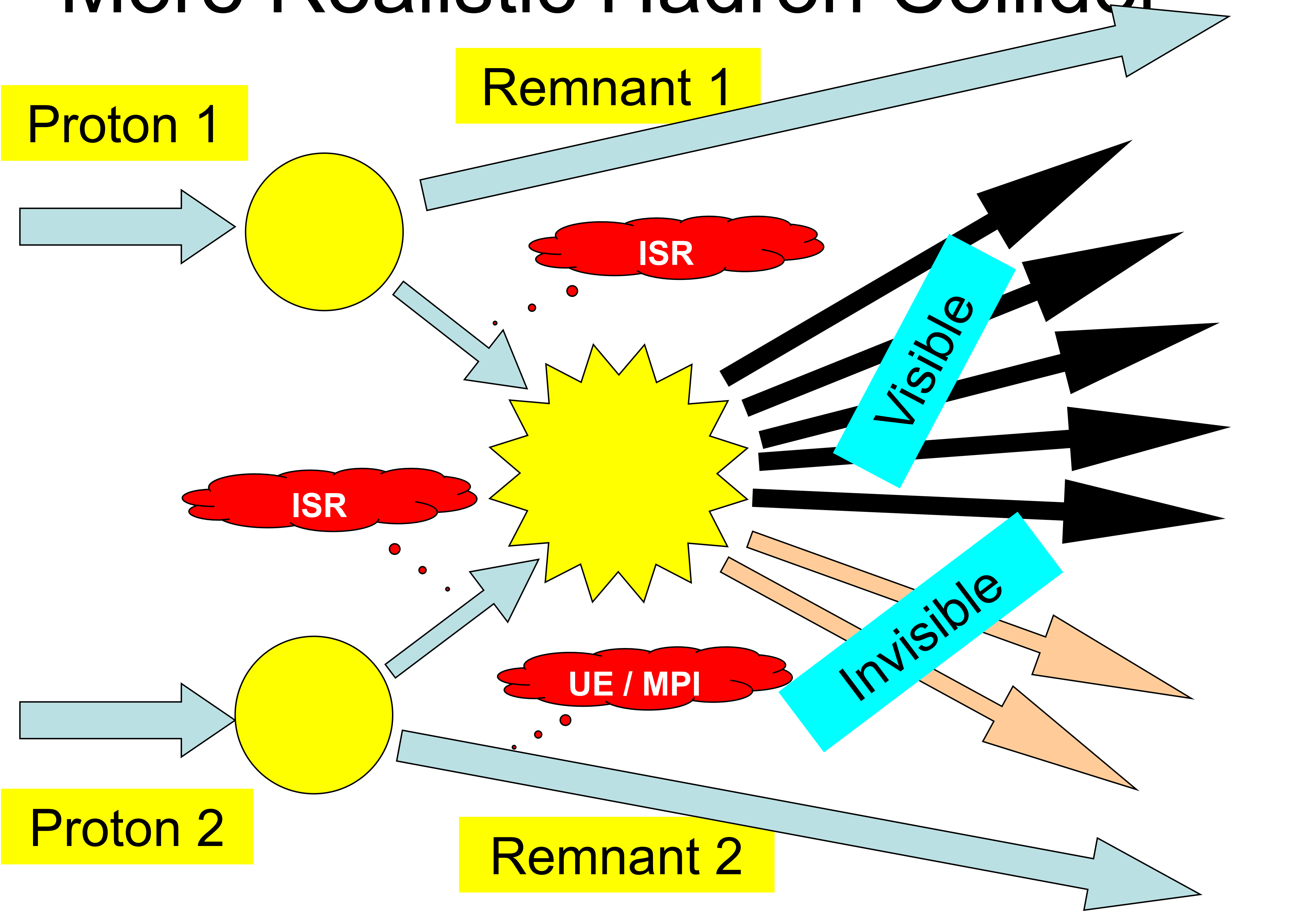
… short for …

“parameter estimation and discovery techniques”

Idealised Hadron Collider



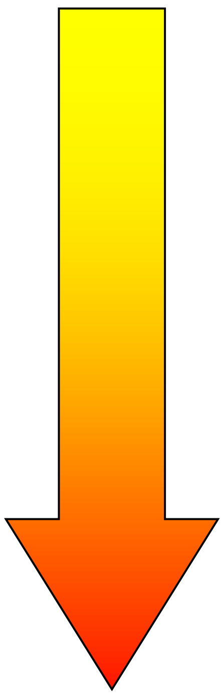
More Realistic Hadron Collider



Types of Technique

Few

assumptions



Many

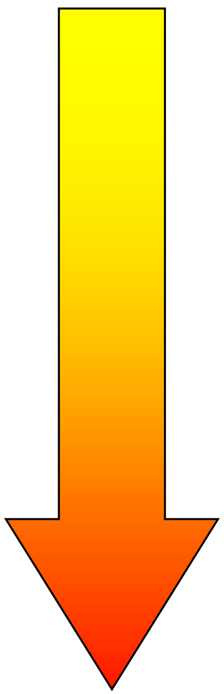
assumptions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Types of Technique

Vague

conclusions



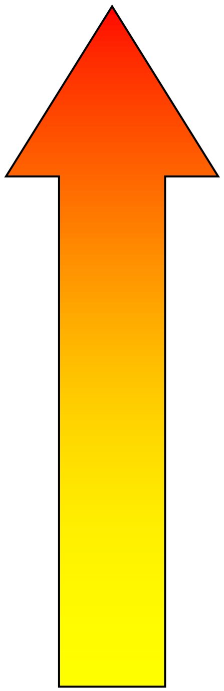
Specific

conclusions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Types of Technique

Robust



Fragile

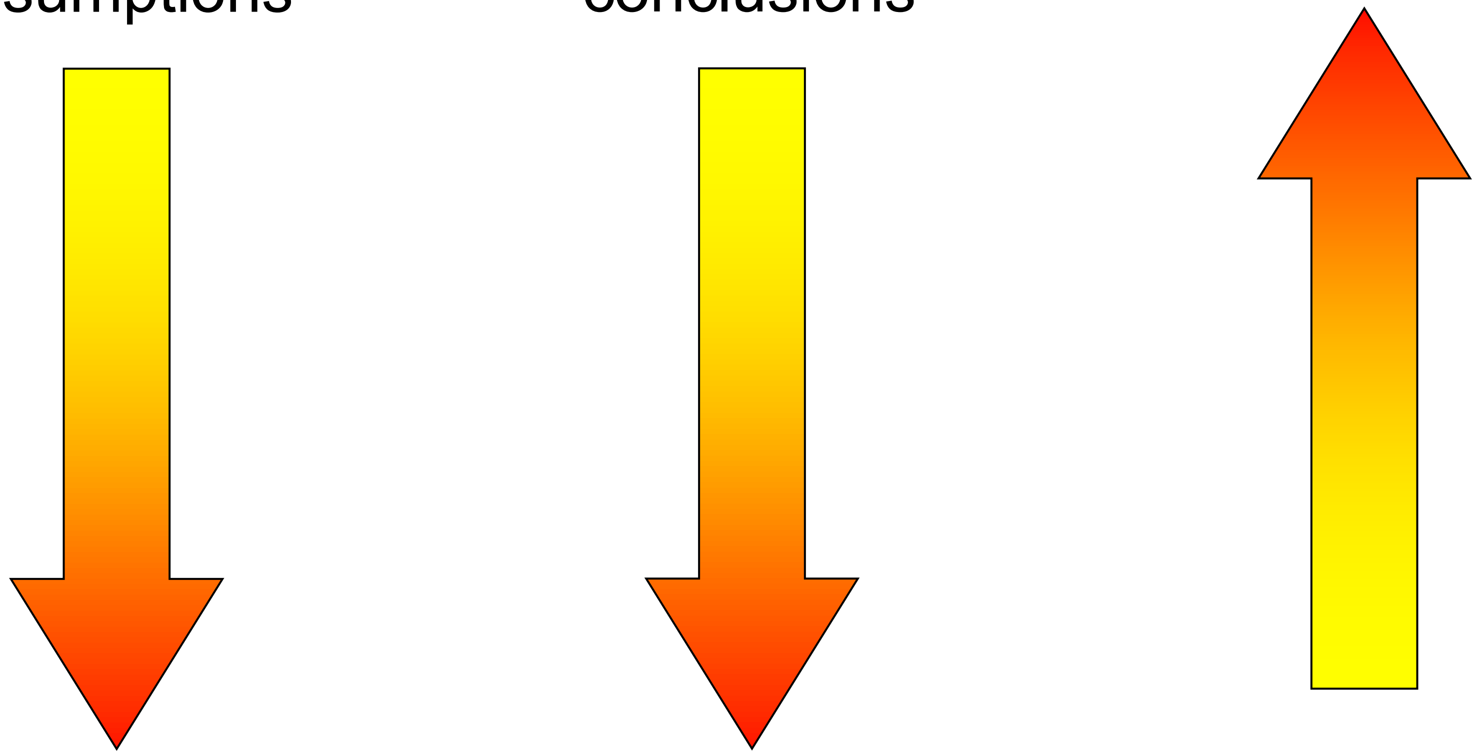
- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Interpretation : the balance of benefits

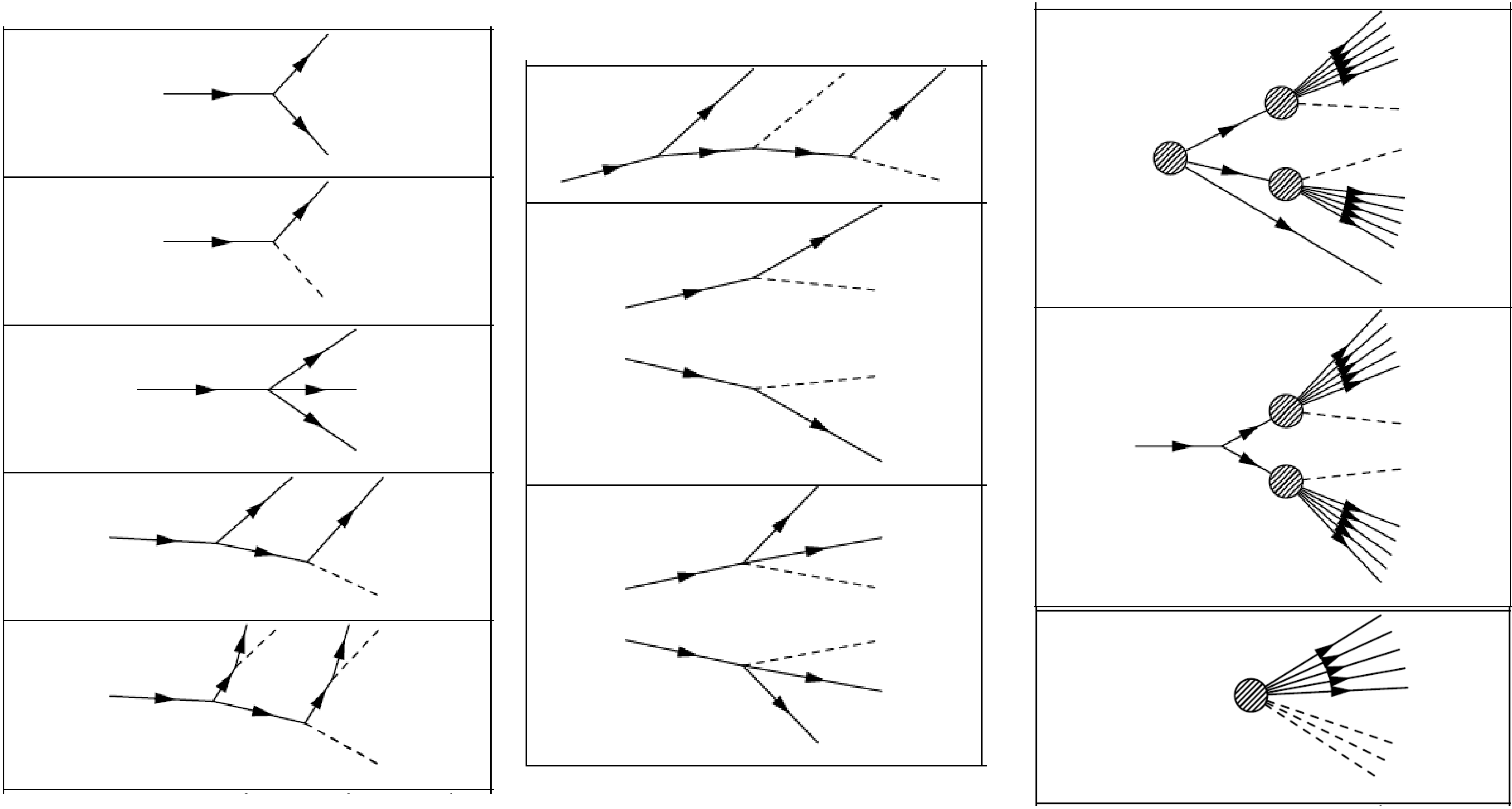
Few assumptions → **Many** assumptions

Vague conclusions → **Specific** conclusions

Fragile → **Robust**

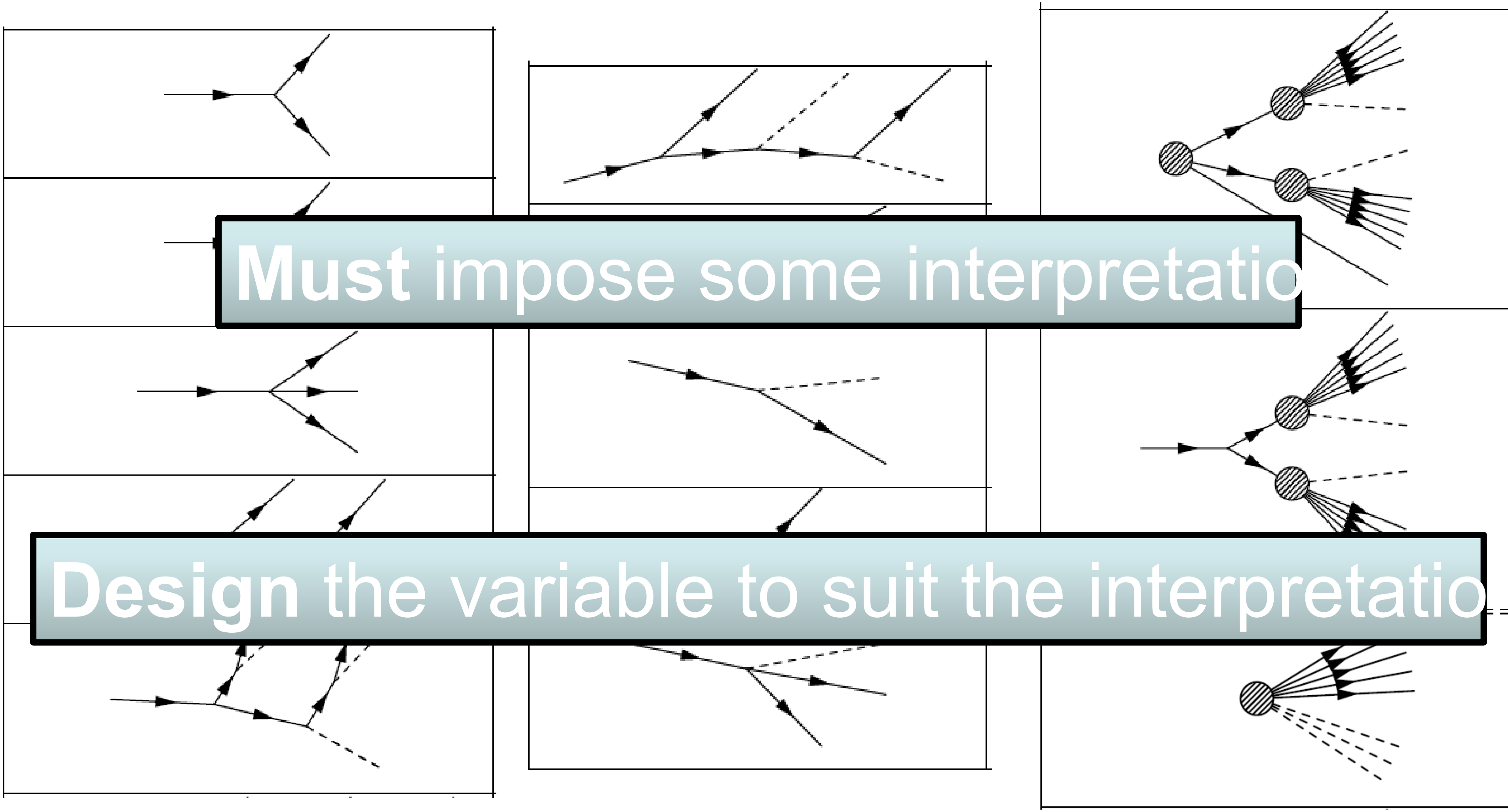


Topology / hypothesis



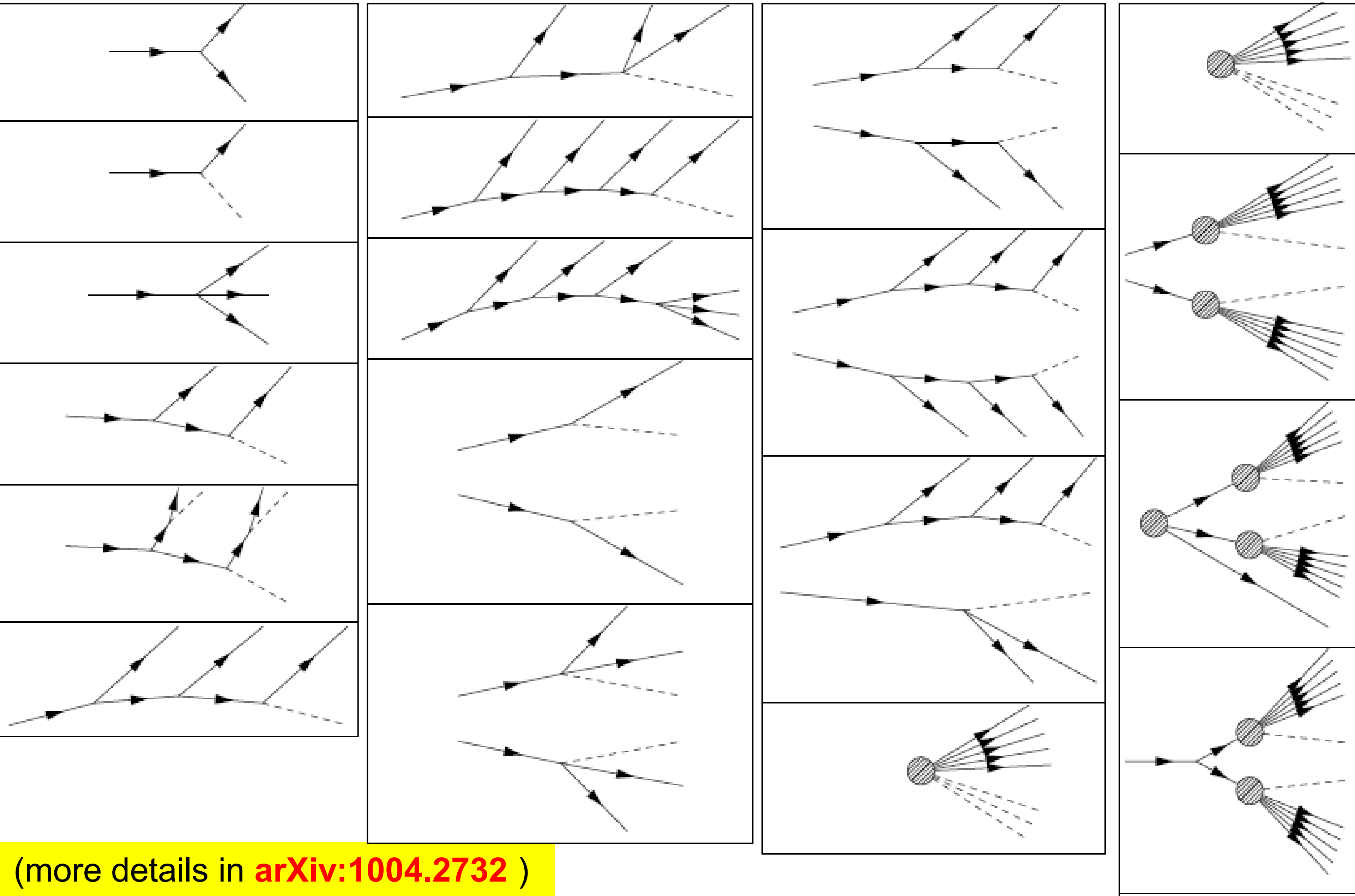
Full index in arXiv:1004.2732

Topology / hypothesis



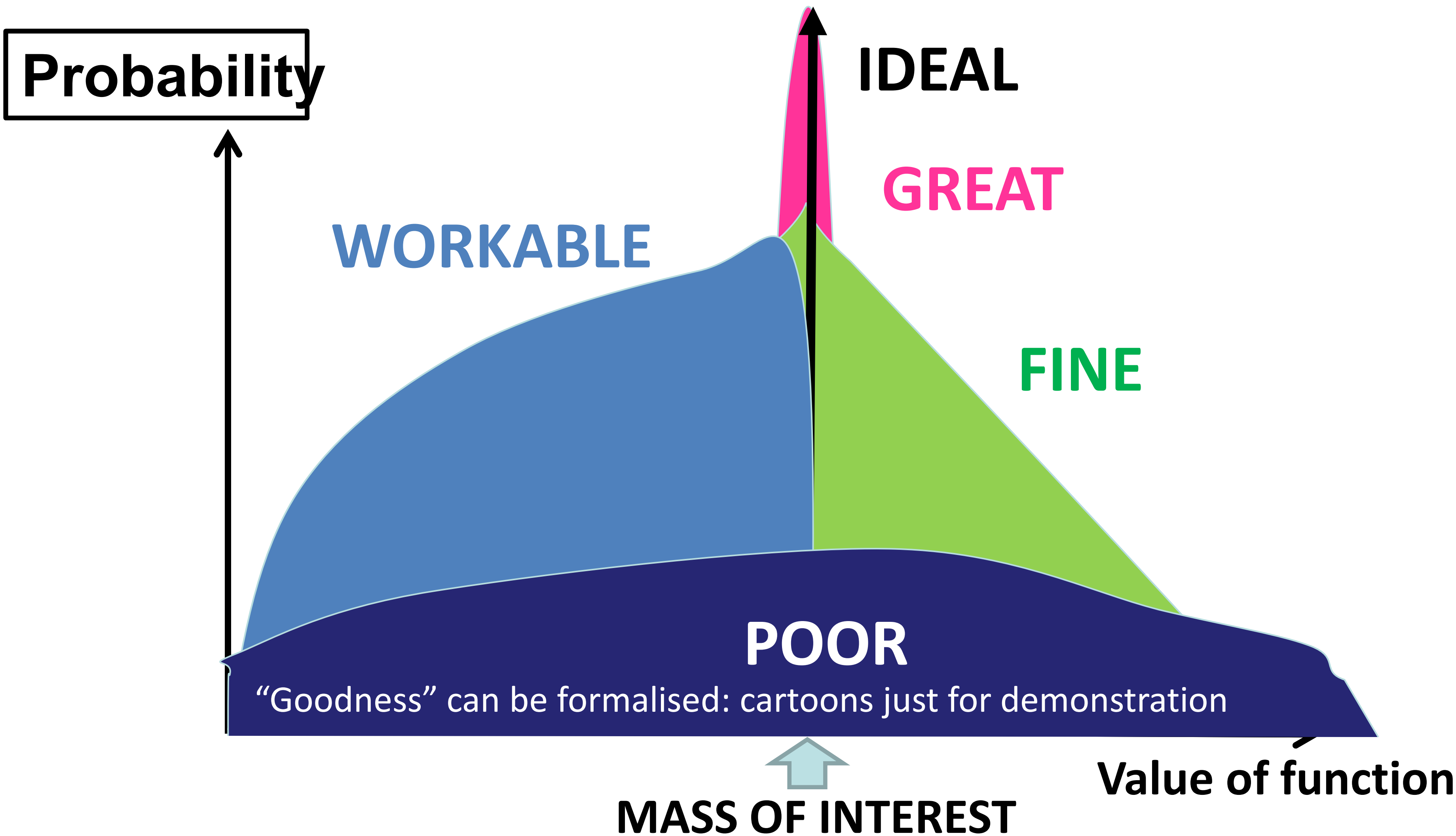
Full index in arXiv:1004.2732

Lectures are roughly ordered from **simple** to **complicated** …



(more details in **arXiv:1004.2732**)

Good vs poor variables



[much of the talk based on material in]

arXiv:1004.2732

A Review of the Mass Measurement Techniques proposed for the Large Hadron Collider

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We review the methods which have been proposed for measuring masses of new particles at the Large Hadron Collider paying particular attention to the kinematical techniques suitable for extracting mass information when invisible particles are expected.

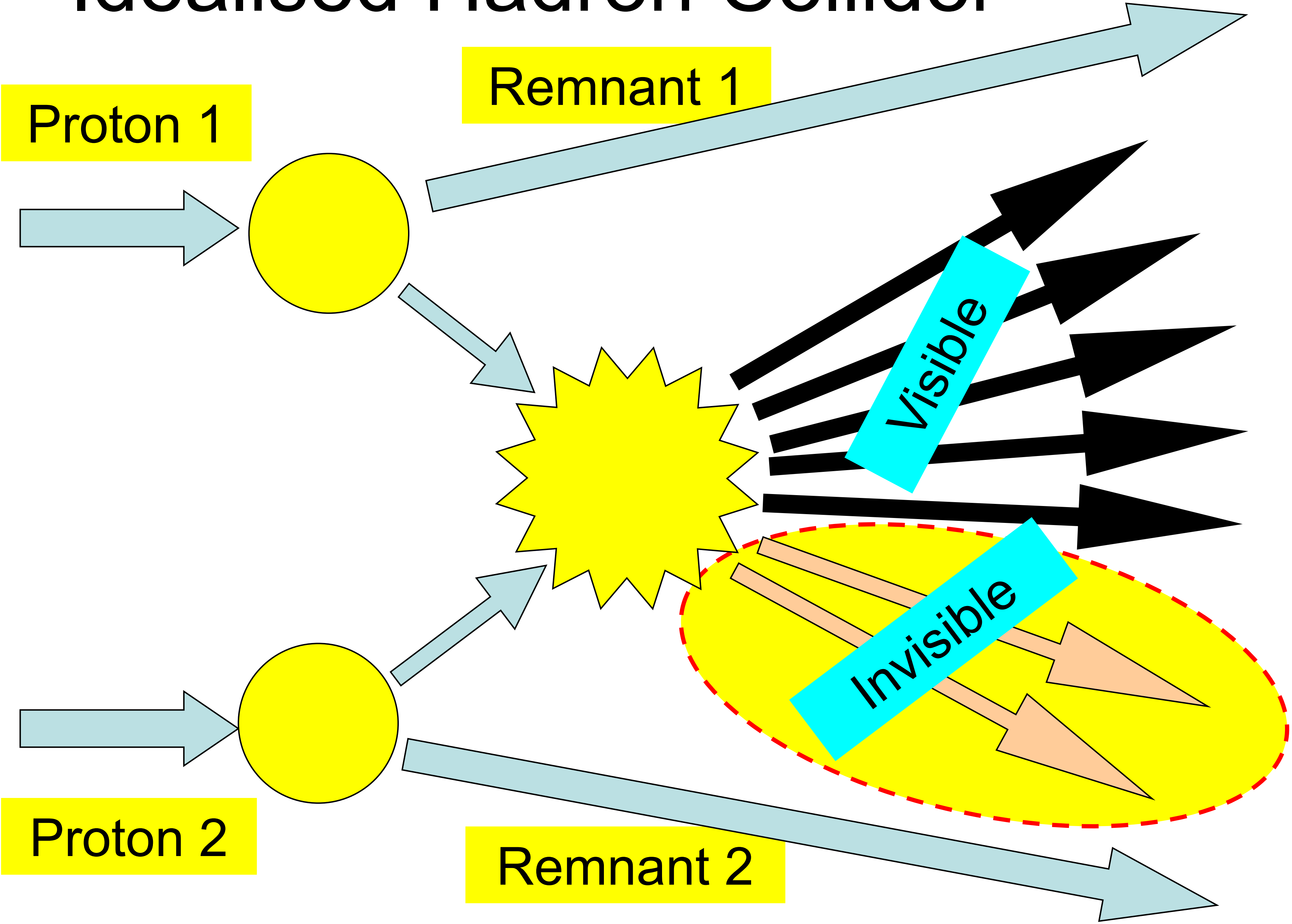
Scope and disclaimers

– will not spend much time on fully visible final states as standard mass reconstruction techniques apply
– will only consider new particles of unknown mass decaying to invisible particles of unknown mass (and other visible particles)
– selection bias – more emphasis on things I've worked with
  • Transverse masses, MT2, kinks, kinematic methods.
  • (Not Matrix Element / likelihood methods / loops)
– not shameless promotion – focus on faults!

Few assumptions,
Vague Conclusions.

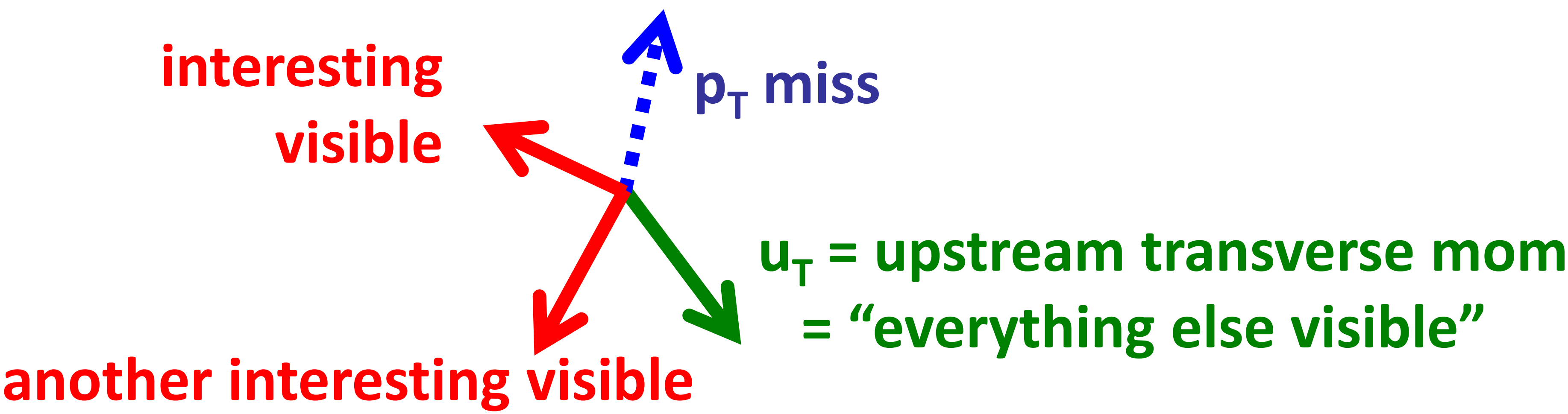
Anything with sensitivity
to mass scales.

Idealised Hadron Collider

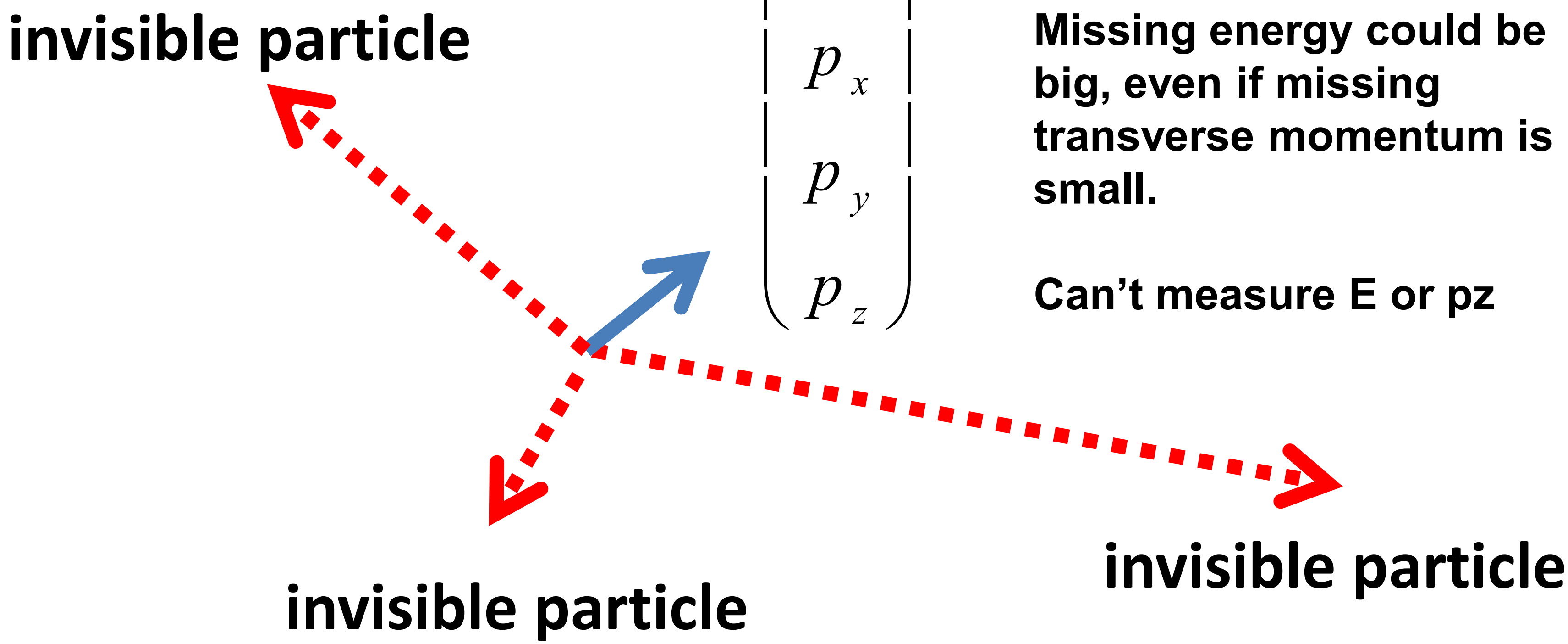


Missing transverse momentum

$$\vec{\mathbf{p}}_T^{\,miss} = -\sum_i \vec{\mathbf{p}}_T^{\,i^{th}\,visible}$$



Events have missing energy too, and it's not missing momentum



$$\begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

Total 4-momentum of invisbiles.

Missing energy could be big, even if missing transverse momentum is small.

Can't measure E or pz

Rant about missing transverse momentum

- eTmiss – aaargh
- MET – AAAARGH
- missing energy – AAAAAARRRGH

- Blame LEP?
- Calorimeter apologists?

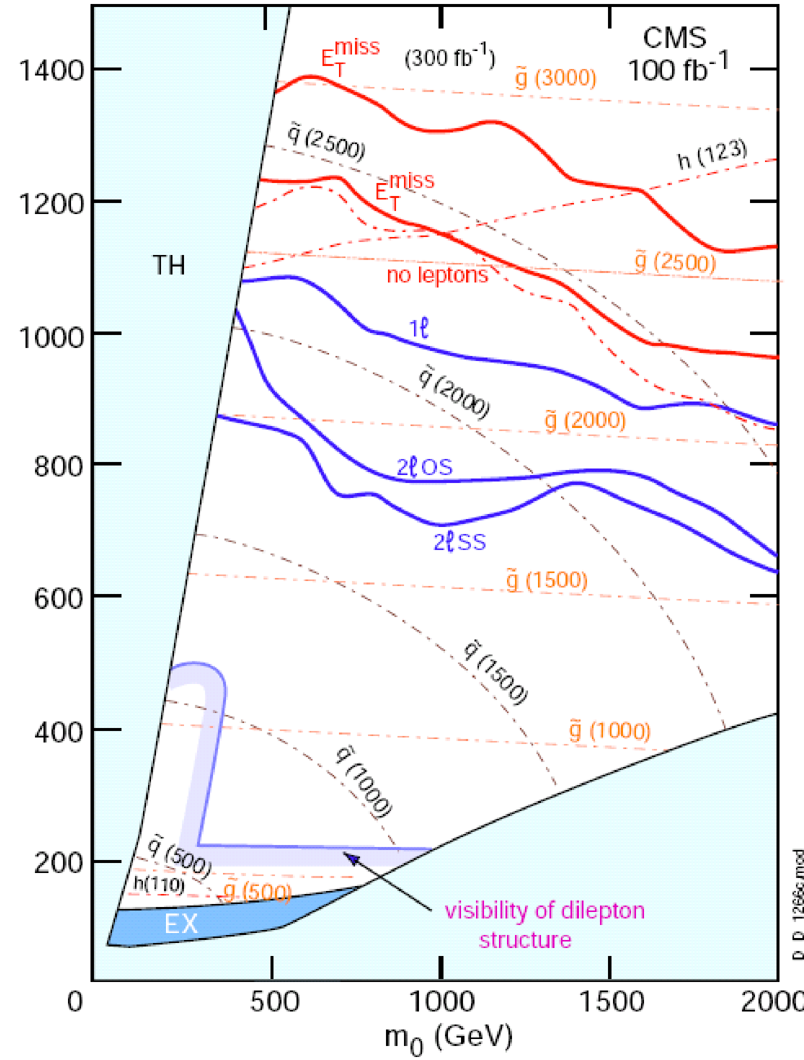
- alphaT

Main EASY signatures are:

- Lots of missing pt
- Lots of leptons
- Lots of jets

Just Count Events!

- $\not{E}_T$ ⇐ Dominant signature
- $\not{E}_T$ with lepton veto
- One lepton
- Two leptons Same Sign (SS)
- Two leptons Opposite Sign (OS)



Simply count events containing the above

Perhaps

simple = best ?

-- The End --

Am in great danger of appearing to sweep a great number of LHC analyses under here!

Not my intention at all!

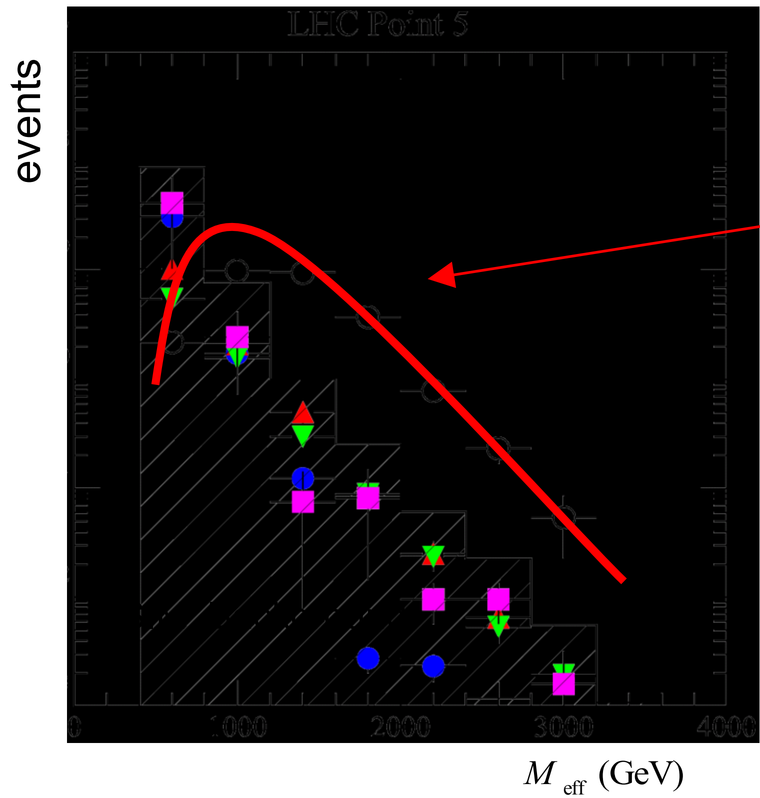
Can attempt to spot susy by counting “strange” events …

… but can we say anything concrete about a mass scale?

Next example still low-tech ….

Effective mass

What you histogram: $$M_{\text{eff}} = \mathbf{p}_{\mathrm{T}}^{\text{missing}} + \sum_i \left| \mathbf{p}_{\mathrm{T}}^{\text{jet}_i} \right|$$



You look for position of this peak and call it MeffPeak

Call it Meff and Mest too (just to confuse people!)

What might Meff peak position correlate with?

Define SUSY scale:

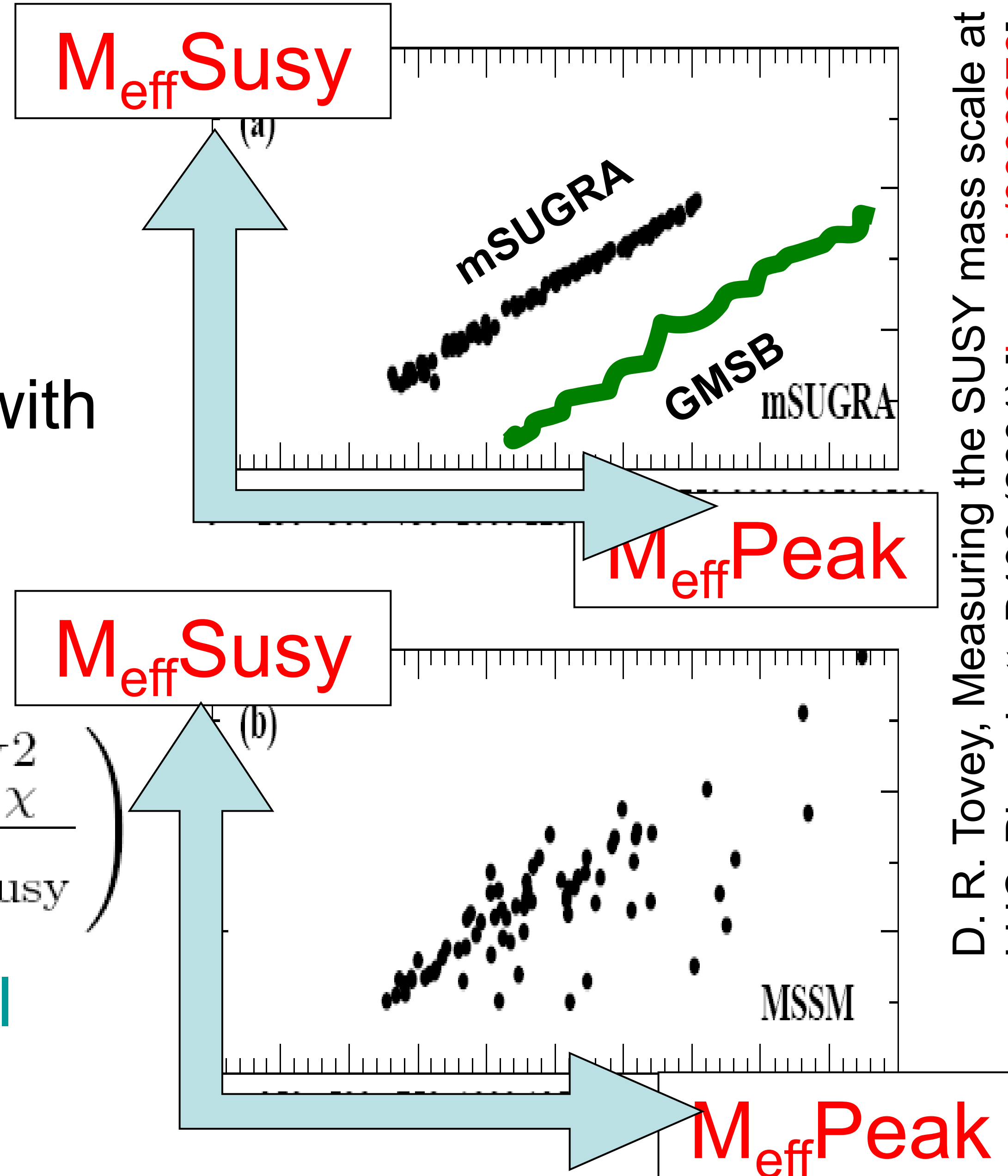
$$M_{\rm susy}^{\rm eff} = \left(M_{\rm susy} - \frac{M_\chi^2}{M_{\rm susy}} \right), \text{ with } M_{\sf SUSY} \equiv \frac{\Sigma_i\, M_i \sigma_i}{\Sigma_i\, \sigma_i}$$

$M_{effPeak}$ / M_{est} example

Observable M_{eff}Peak sometimes correlates with property of model M_{eff} defined by

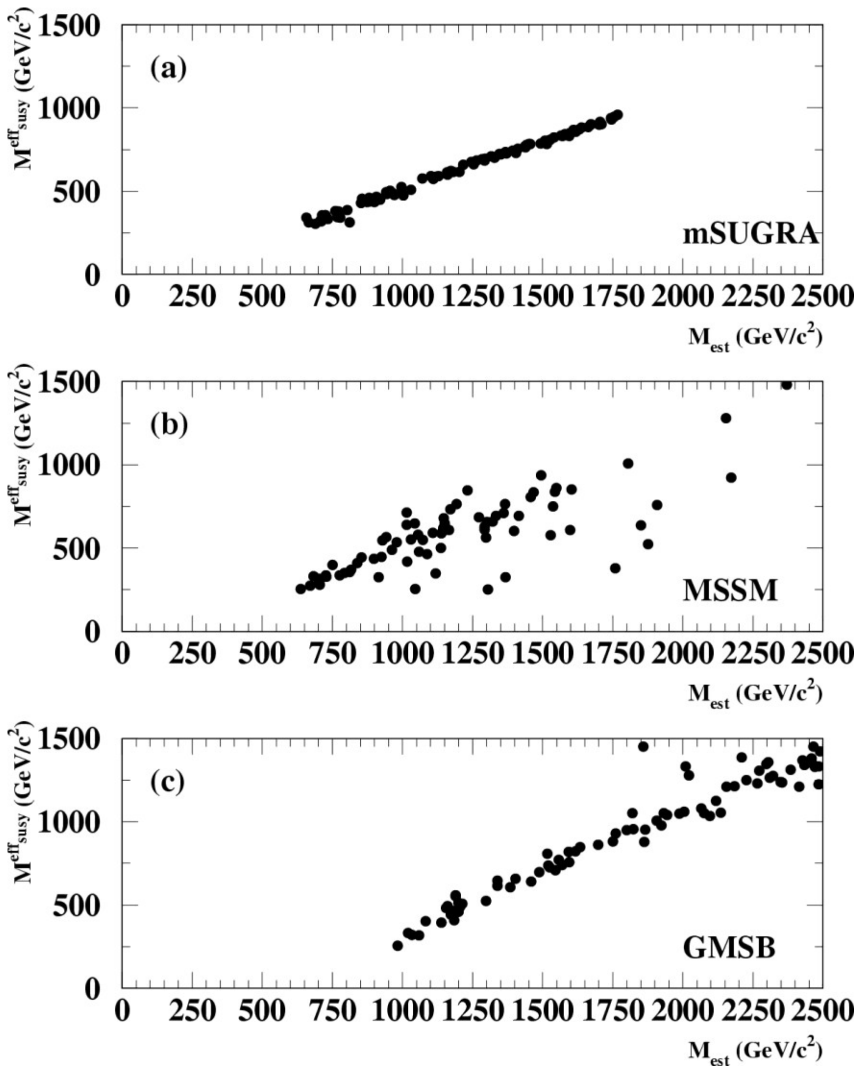
$$M_{\rm susy}^{\rm eff} = \left(M_{\rm susy} - \frac{M_\chi^2}{M_{\rm susy}} \right)$$

but correlation is model dependent



D. R. Tovey, Measuring the SUSY mass scale at the LHC, Phys. Lett. B498 (2001) [hep-ph/0006276]

Correlations between MeffPeak position and MeffSusy



(Tovey)

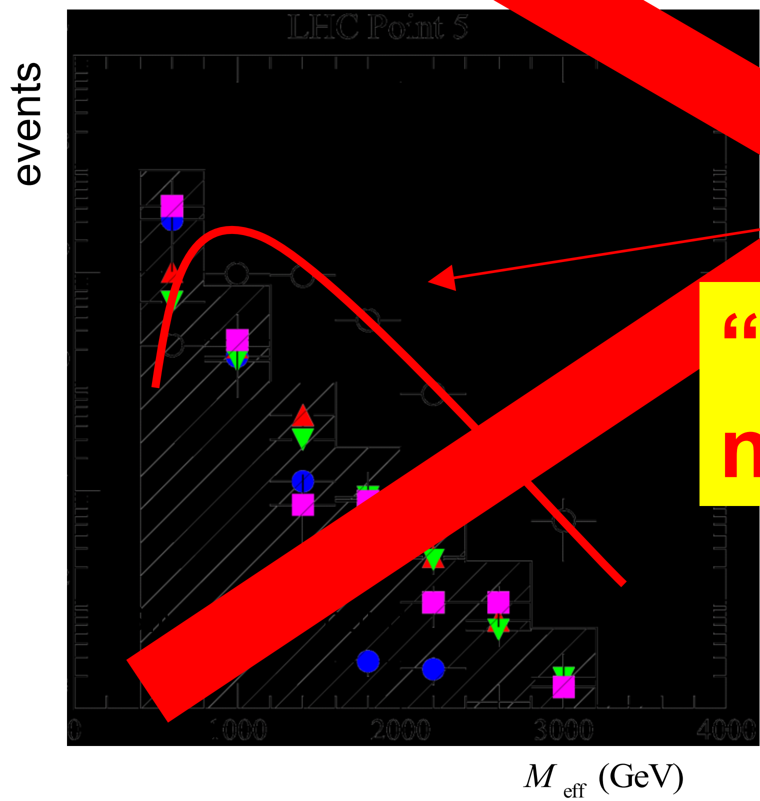
M_Hotpants ..

- Can encourage tendency to

- Create your variable, then see what might be able to measure. Oops.

Effective mass

What you histogram: $M_{\mathrm{eff}} = \mathbf{p}_{\mathrm{T}}^{\mathrm{missing}} + \sum_{i} |\mathbf{p}_{\mathrm{T}}^{i}|$



You look for position of this peak and call

"It is neither a mass, nor effective" - KM

Call it Meff too (just to confuse people!)

Meff is not alone …

Murky underworld of hidden relatives known variously as HT … same thing … sometimes

$$H_T = E_{T(2)} + E_{T(3)} + E_{T(4)} + |\not{p}_T|$$

$$E_T = E \sin\theta$$

See arXiv:1105.2977 for why sinTheta brings on nightmares.

(There are **no standard definitions** of H_T or Meff. Authors differ in how many jets are used, whether PT miss should be added etc.)

All have *some* sensitivity to the overall mass scales involved, but interpretation requires a model and more assumptions.

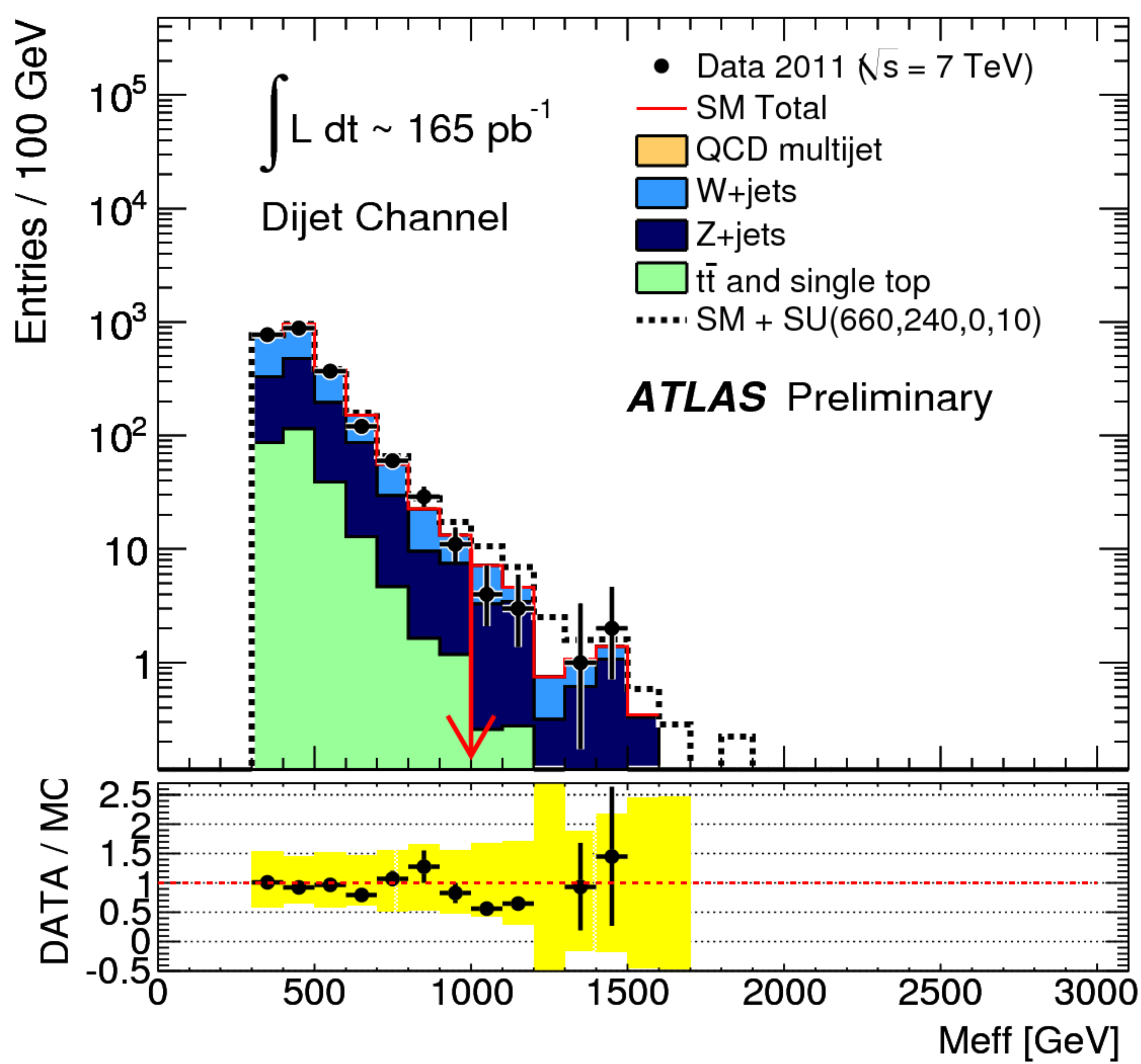
Why are we adding transverse momenta?

- Why not multiply?
  (or add logs)?

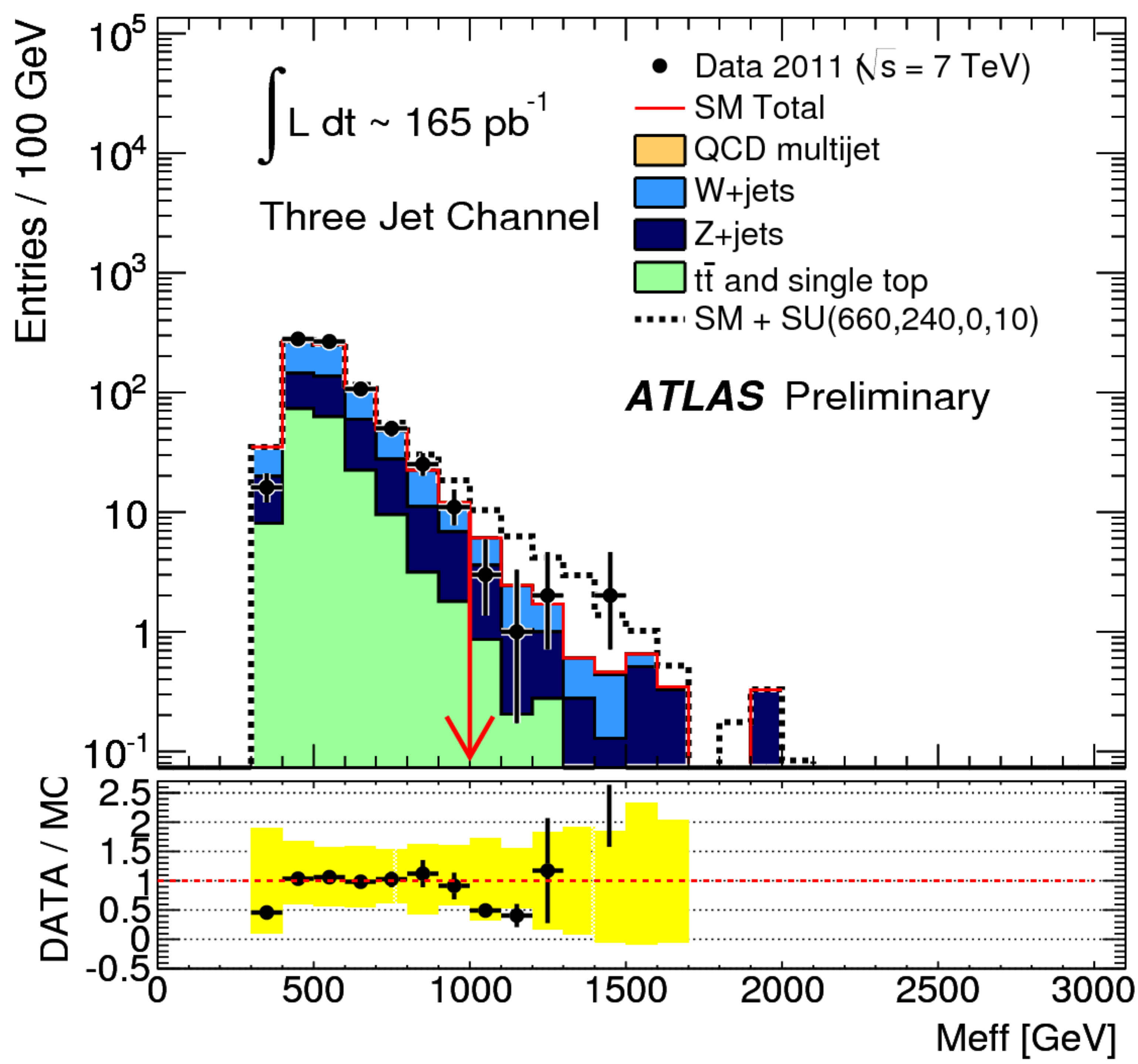
$$M_{happy} = \left(\prod_{i=1}^{n} \mathbf{p}_T^i \right)^{\frac{1}{n}}$$

- Serious proposal to use $\mathrm{Meff}^2-(u_T)^2$ in arXiv:1105.2977
- Why are the signs the same? Why equal weights? Silly?
- How many years would it take ATLAS/CMS to discover the invariant mass for Z -> a b ?

$$\mathrm{M}^2 = \left(\sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2} \right)^2$$
$$- \left(a_\mathrm{x} + b_\mathrm{x}\right)^2 - \left(a_\mathrm{y} + b_\mathrm{y}\right)^2 - \left(a_\mathrm{z} + b_\mathrm{z}\right)^2$$



Latest ATLAS 0-lepton, jets, missing transverse momentum data.

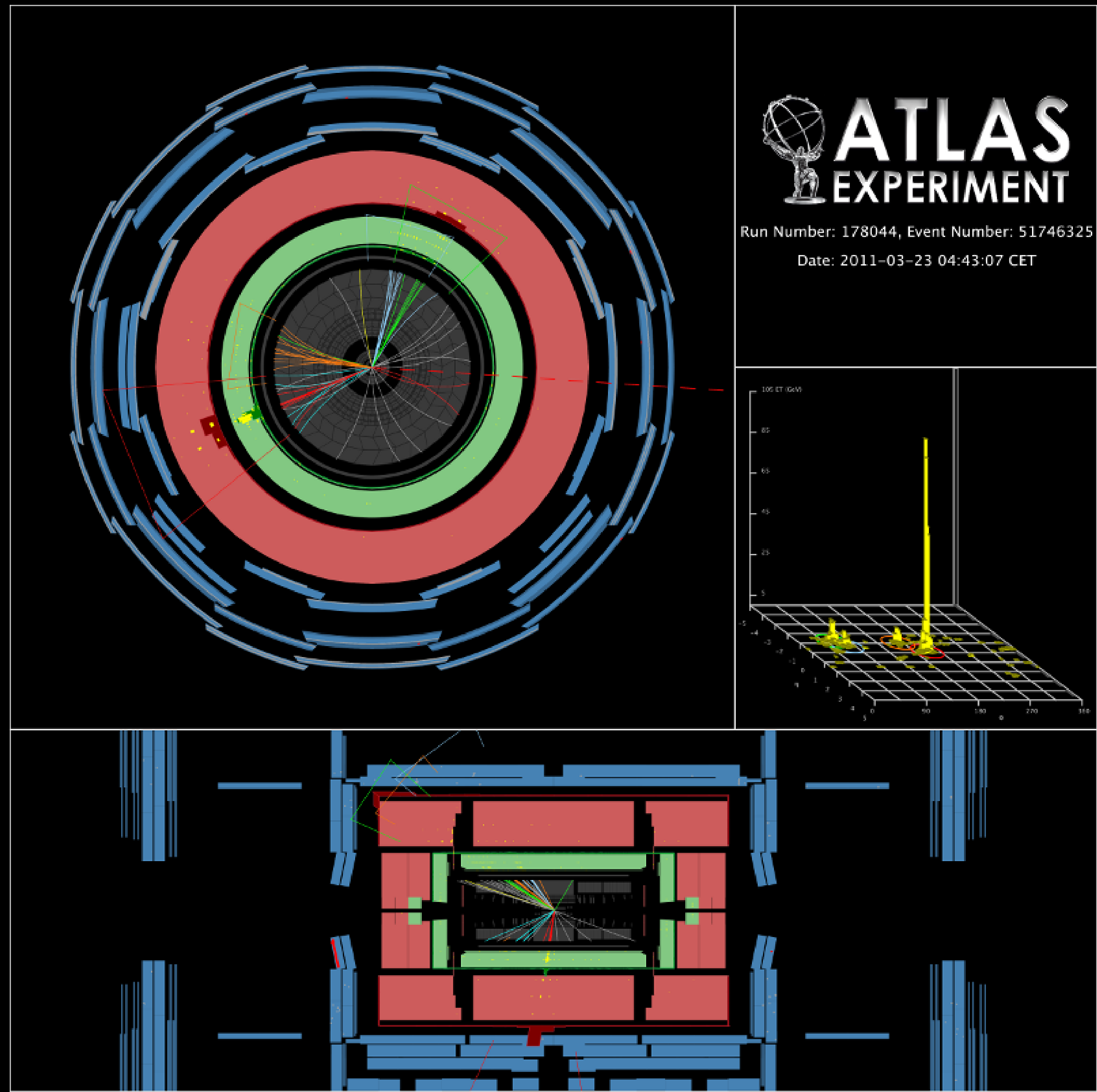


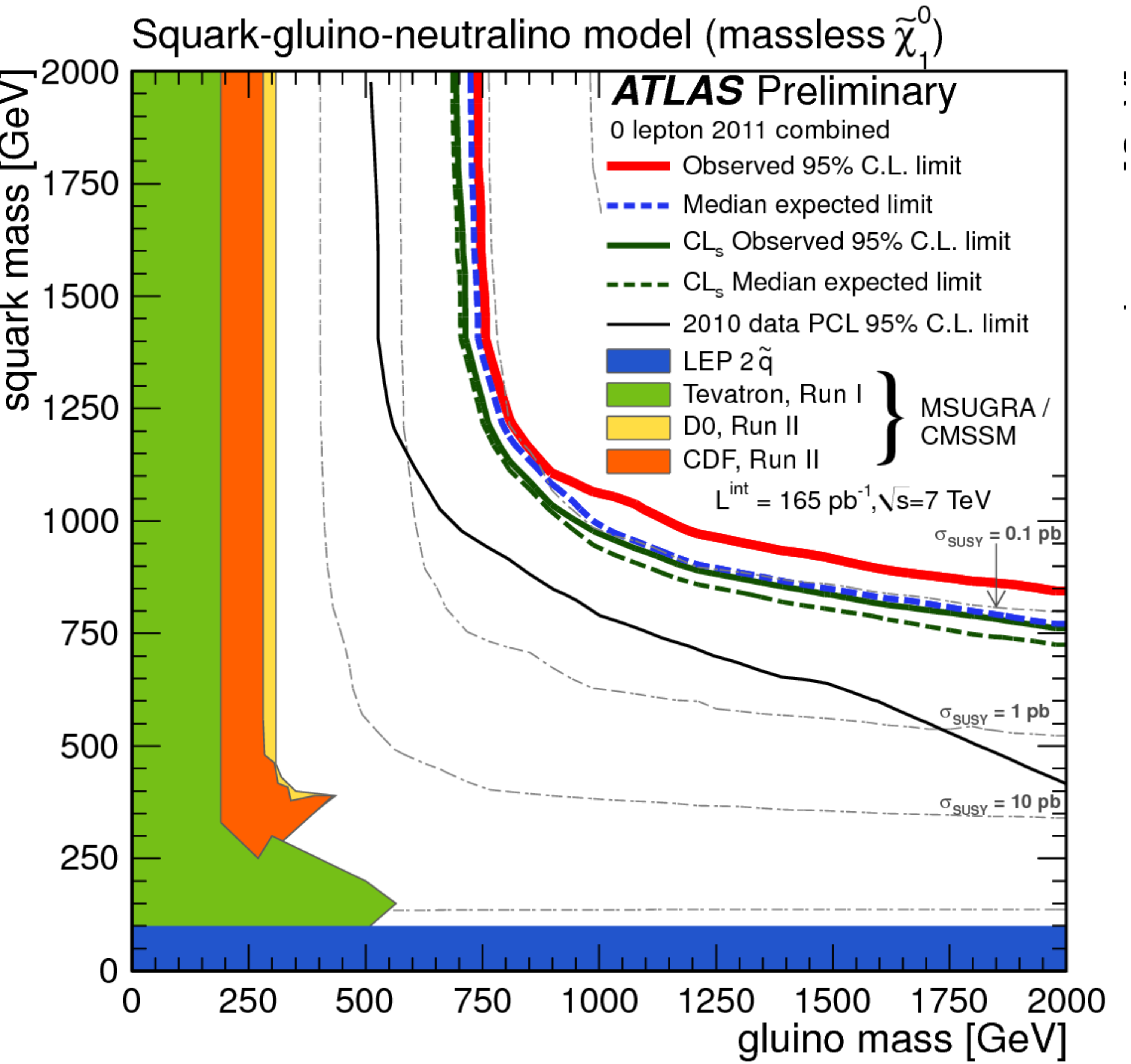
Latest ATLAS 0-lepton, jets, missing transverse momentum data.

Highest Meff event so far ….

The highest Meff in any (supposedly “clean”) ATLAS event is 1548 GeV

- calculated from four jets with pts:
  - 636 GeV
  - 189 GeV
  - 96 GeV
  - 81 GeV
- 547 GeV of missing transverse momentum.





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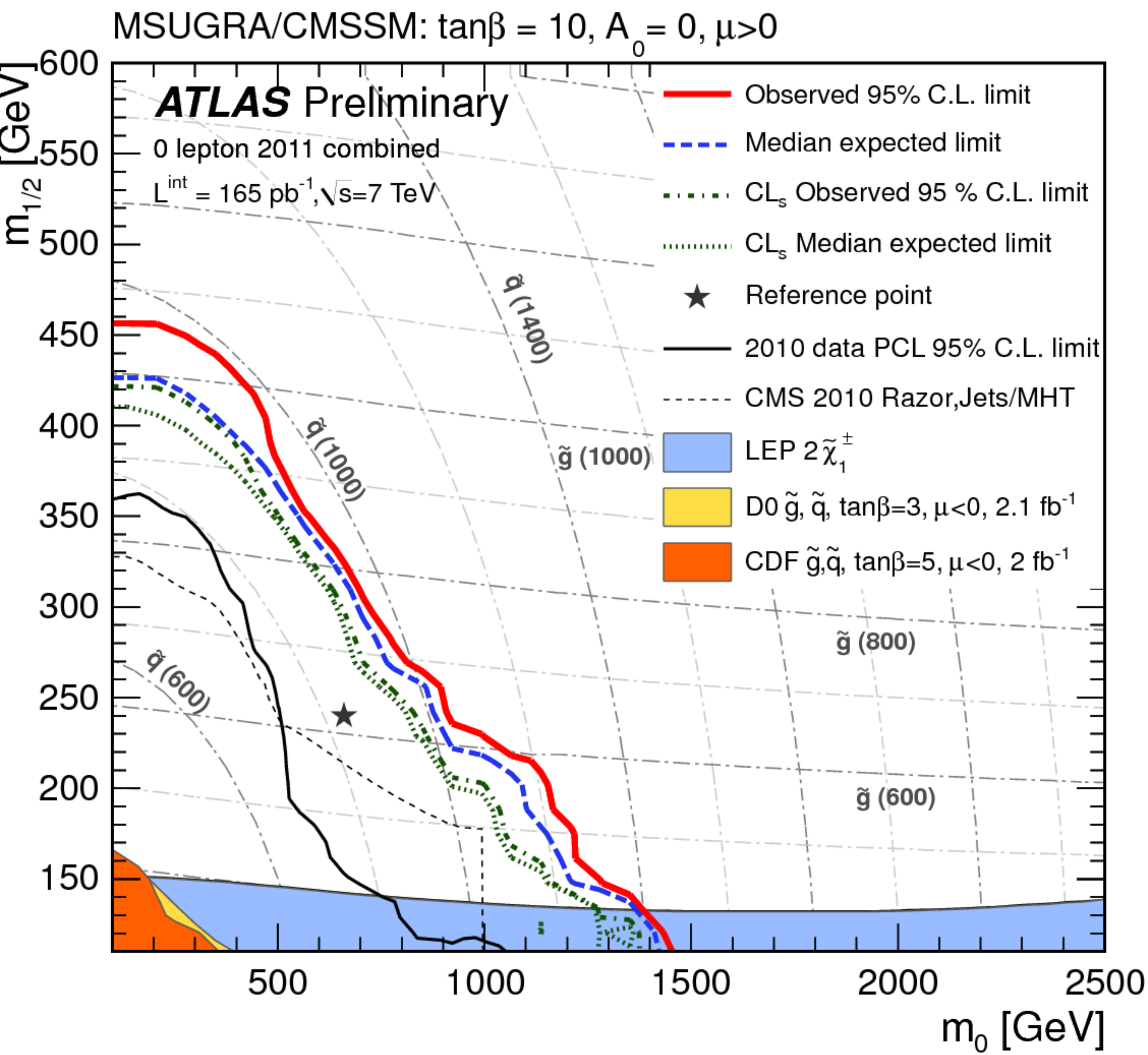


Latest ATLAS 0-lepton, jets, missing transverse momentum data.

Don’t confuse simplicity with complexity … can layer add many layers of interpretation

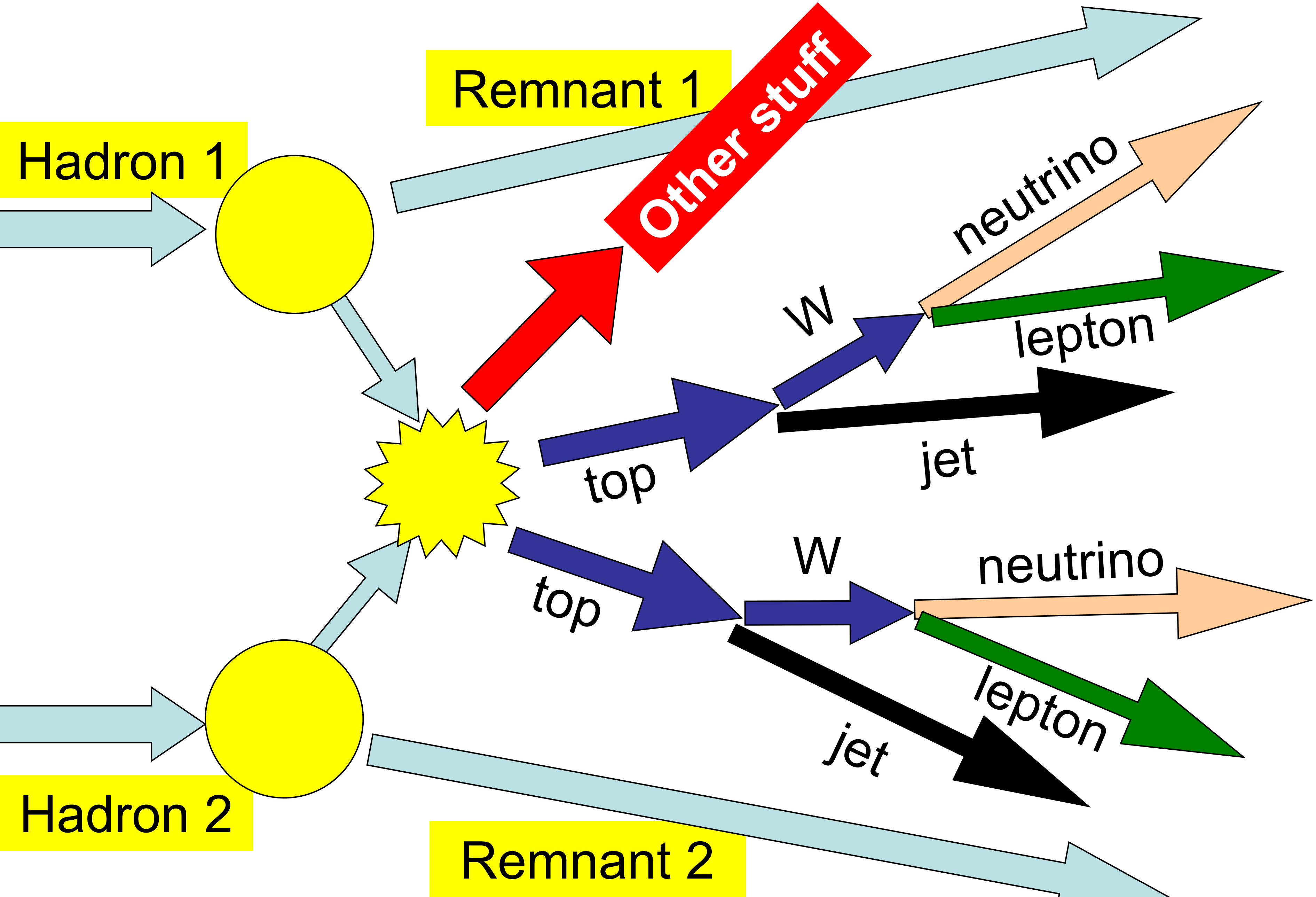
Measure top quark mass from mean lepton PT only!

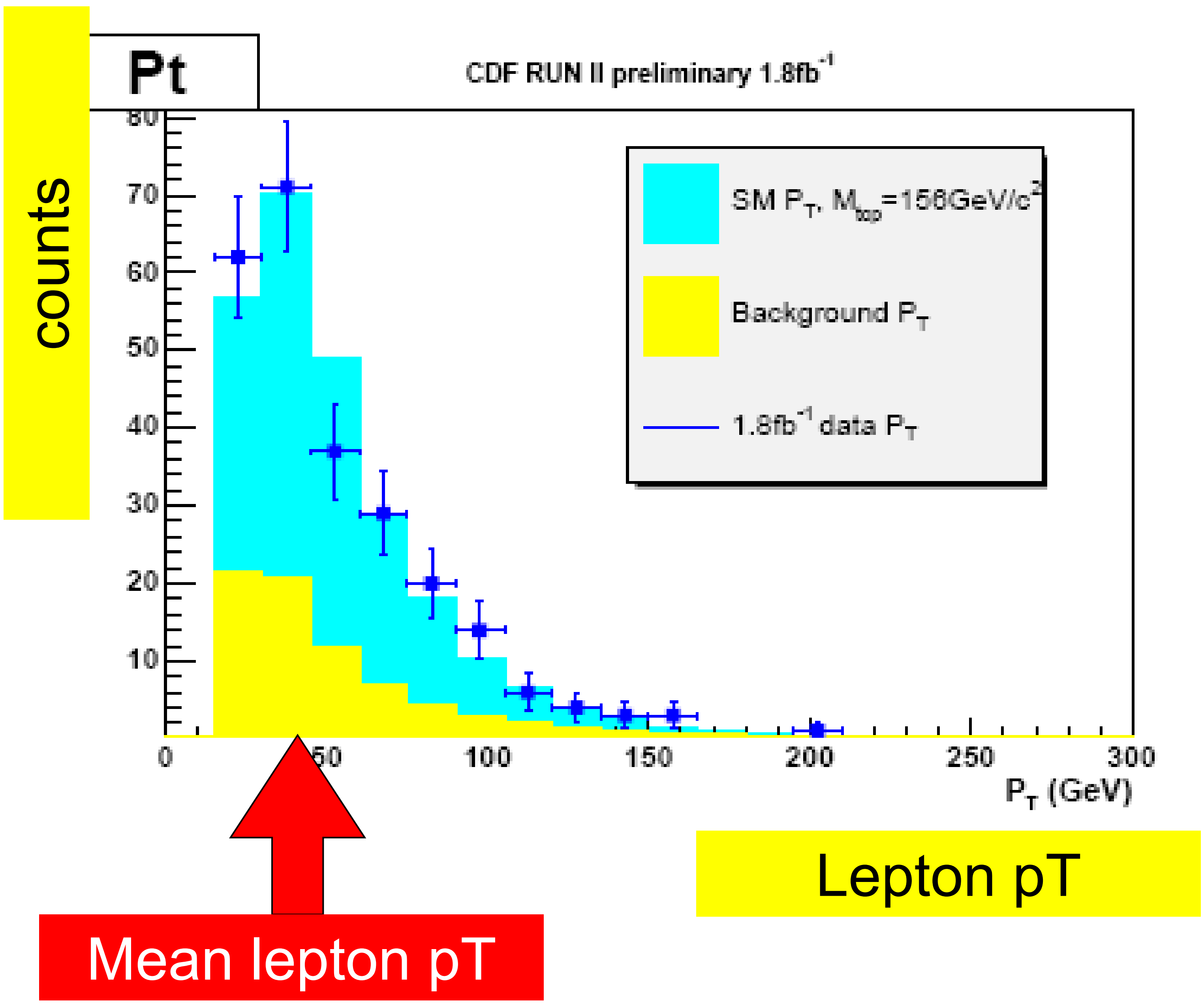
CDF note 8959

Measurement of the top quark mass from the lepton transverse momentum in the $t\bar{t} \rightarrow$ dilepton channel at the Tevatron

A new measurement of the top quark mass at 1.8 fb^{-1} integrated luminosity, using leptons' P_T in the dilepton channel is presented. A top quark mass of m_{top}=156$\pm$20$_{(stat)}$$\pm$4.6$_{(syst)}$GeV/$c^2$ is obtained with the Likelihood method and of 149$\pm$21$_{(stat)}$$\pm$5(syst)GeV/$c^2$ is obtained with the Straight Line method.

Top quark production tevatron - dileptonic

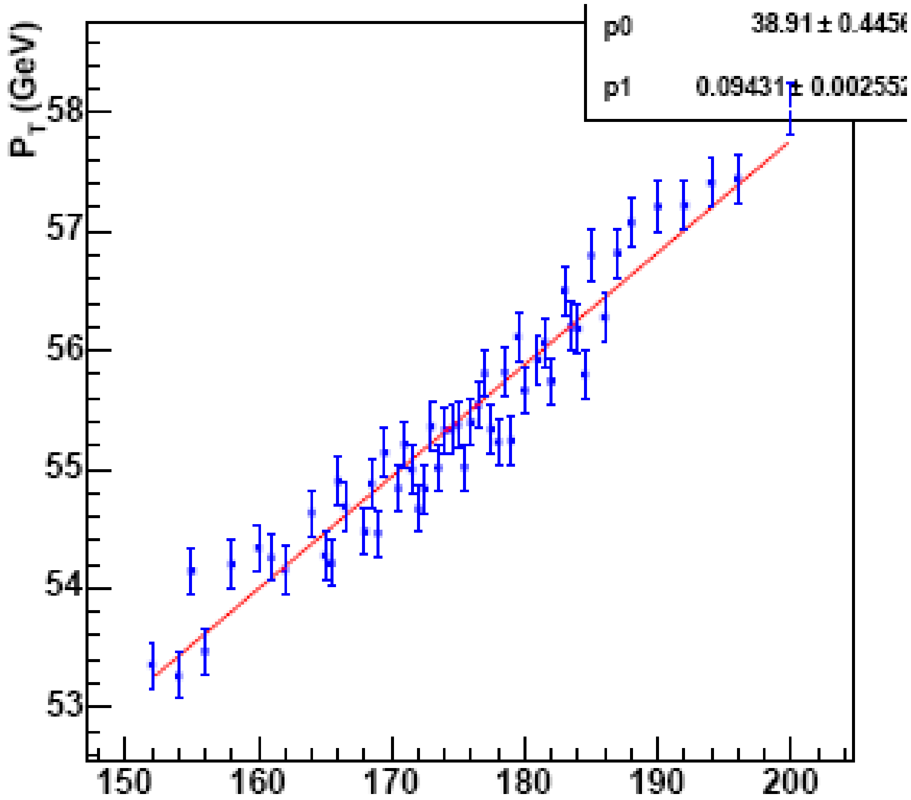




counts
Pt
CDF RUN II preliminary 1.8fb^{-1}
SM P$_T$, M$_{top}$=156GeV/c^2
Background P$_T$
1.8fb^{-1} data P$_T$
80
70
60
50
40
30
20
10
0
50
100
150
200
250
300
P$_T$ (GeV)
Lepton pT
Mean lepton pT

Frightening y-axis!

Mean lepton pT



Simulated top quark mass

Result $m_{top} = 156 \pm 20_{(stat)} \pm 4.6_{(syst)}\,\mathrm{GeV}$

Moral

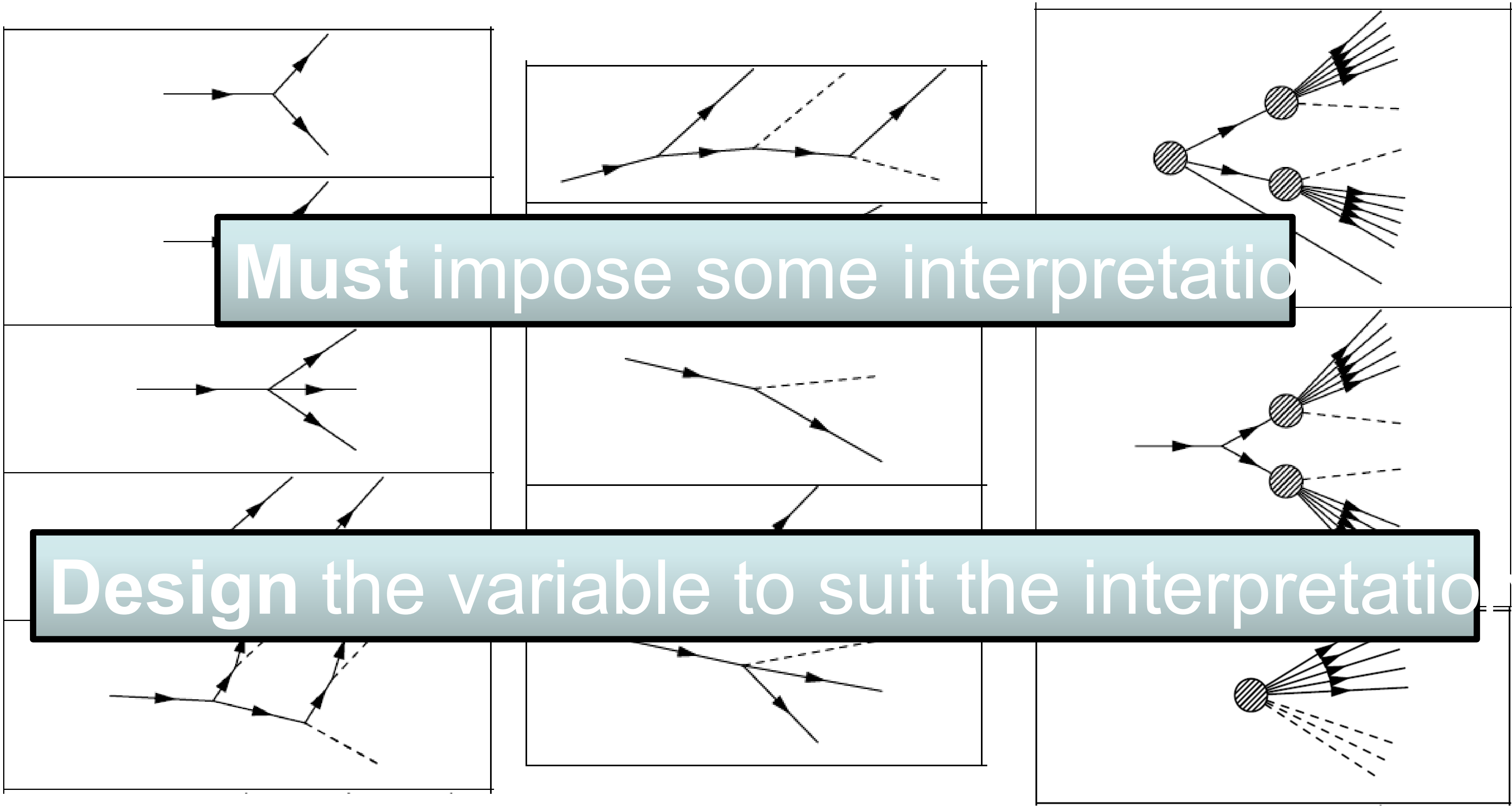
- You can monte-carlo anything.
  - example h->tau tau

- But do you trust it? Is it the best you can do?

More assumptions
Less Vague Conclusions

non-hotpants

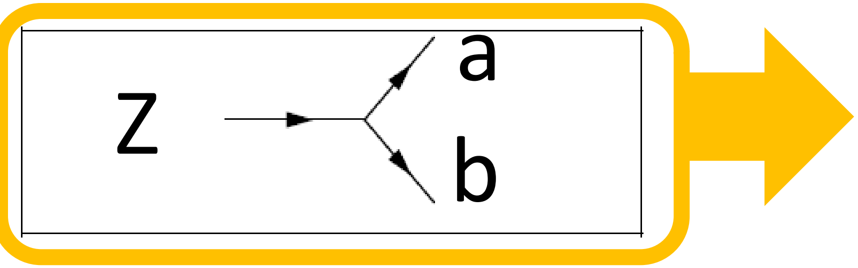
Topology / hypothesis

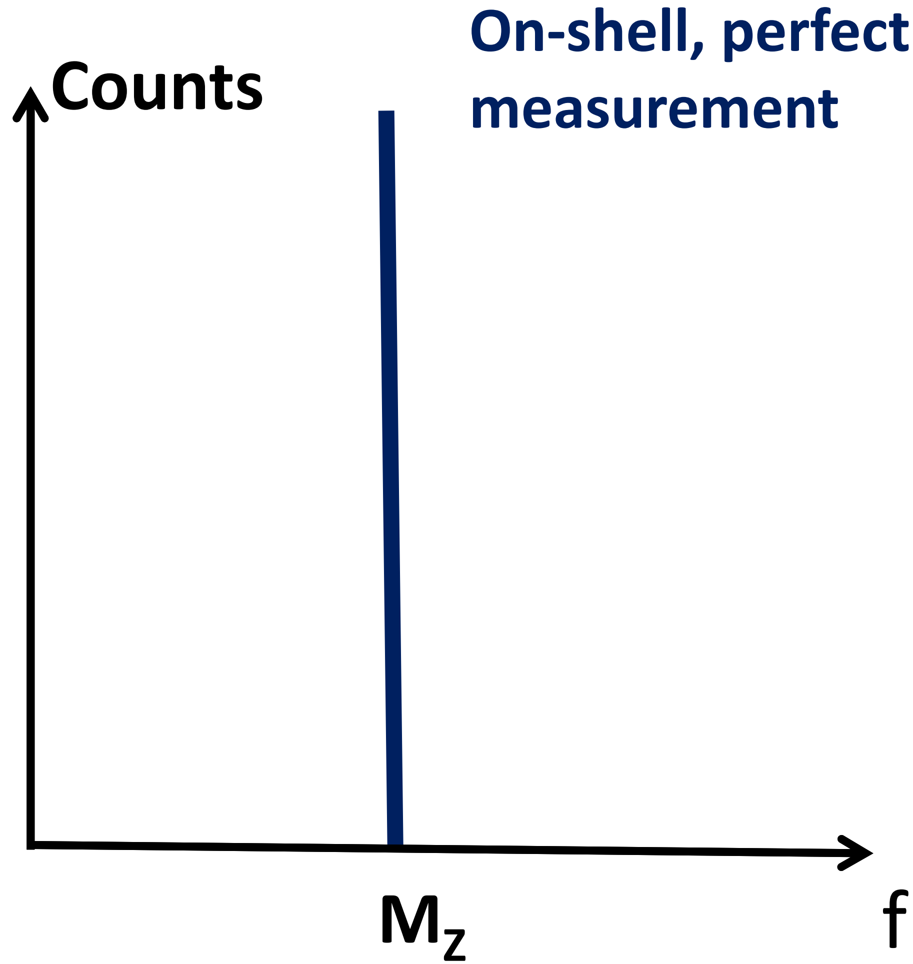


Full index in arXiv:1004.2732

All **visible**

$Z^0 \rightarrow e^+ e^-$

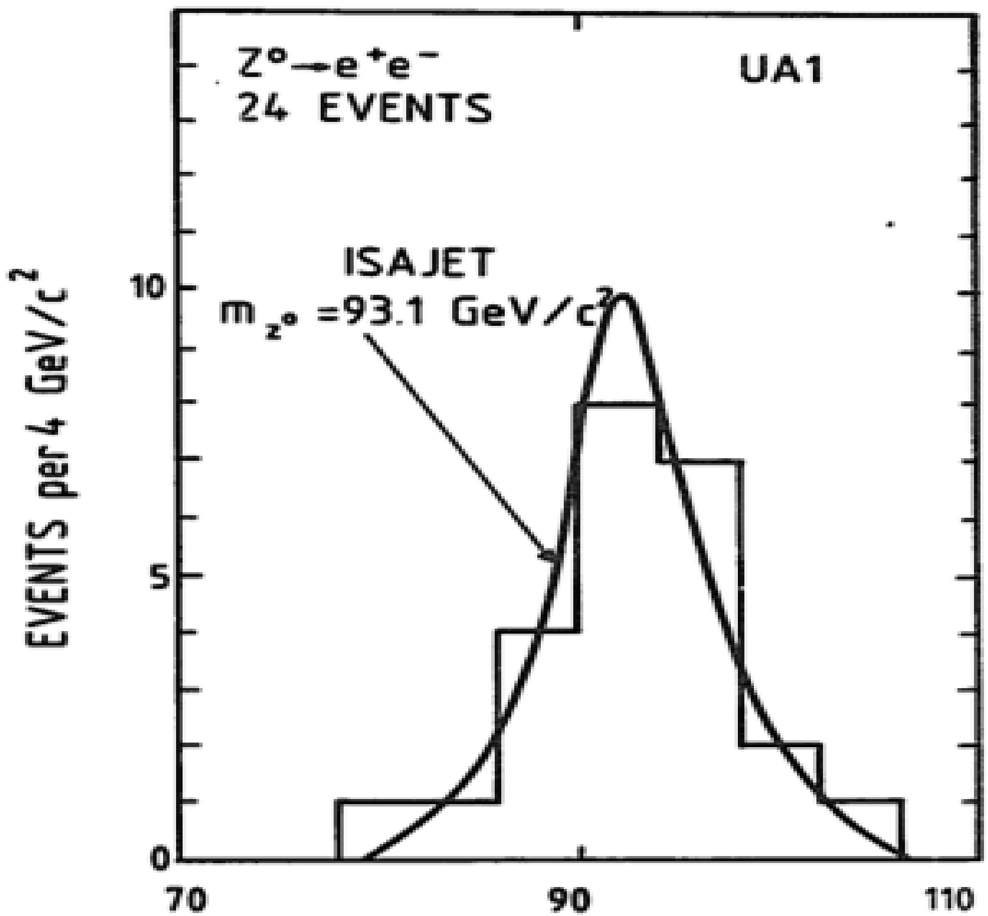




$f^2 = Z^\mu Z_\mu = (a+b)^\mu (a+b)_\mu$

$$M^2 = \left(\sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2}\right)^2 - (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2$$

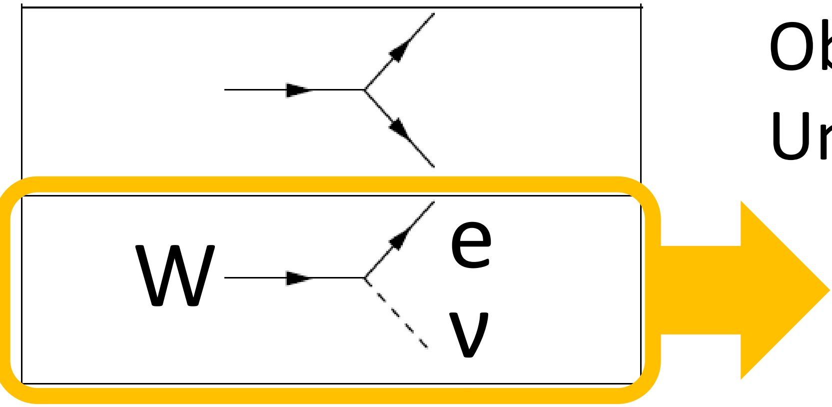
SPS – the Z boson Mass



Finite width
Detector resolution

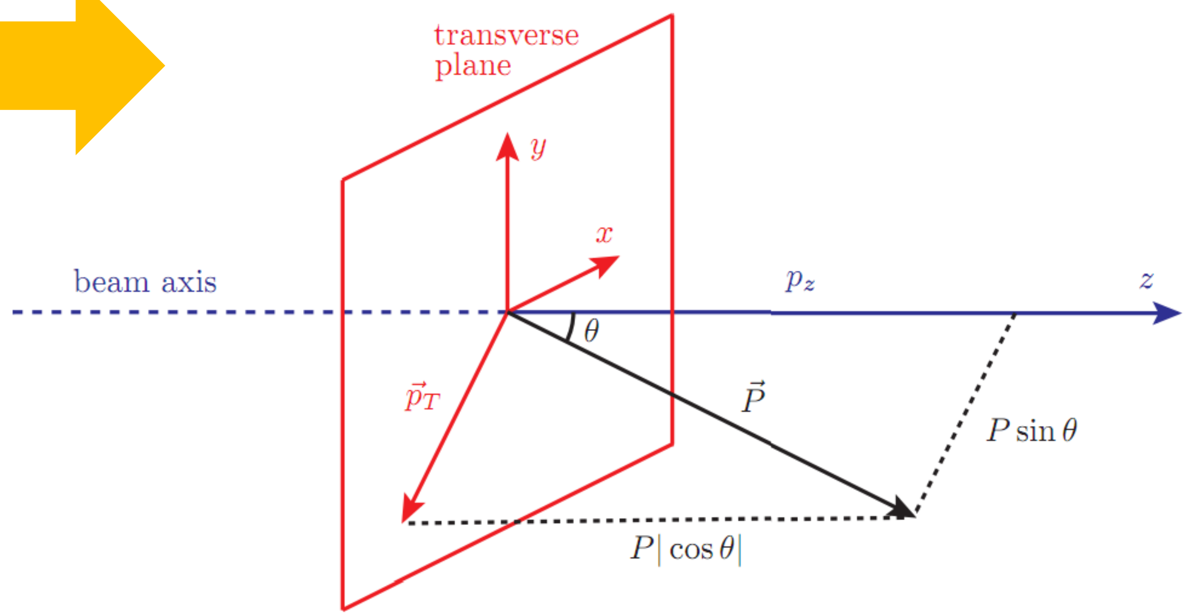
Broaden peak

Dealing with incomplete information



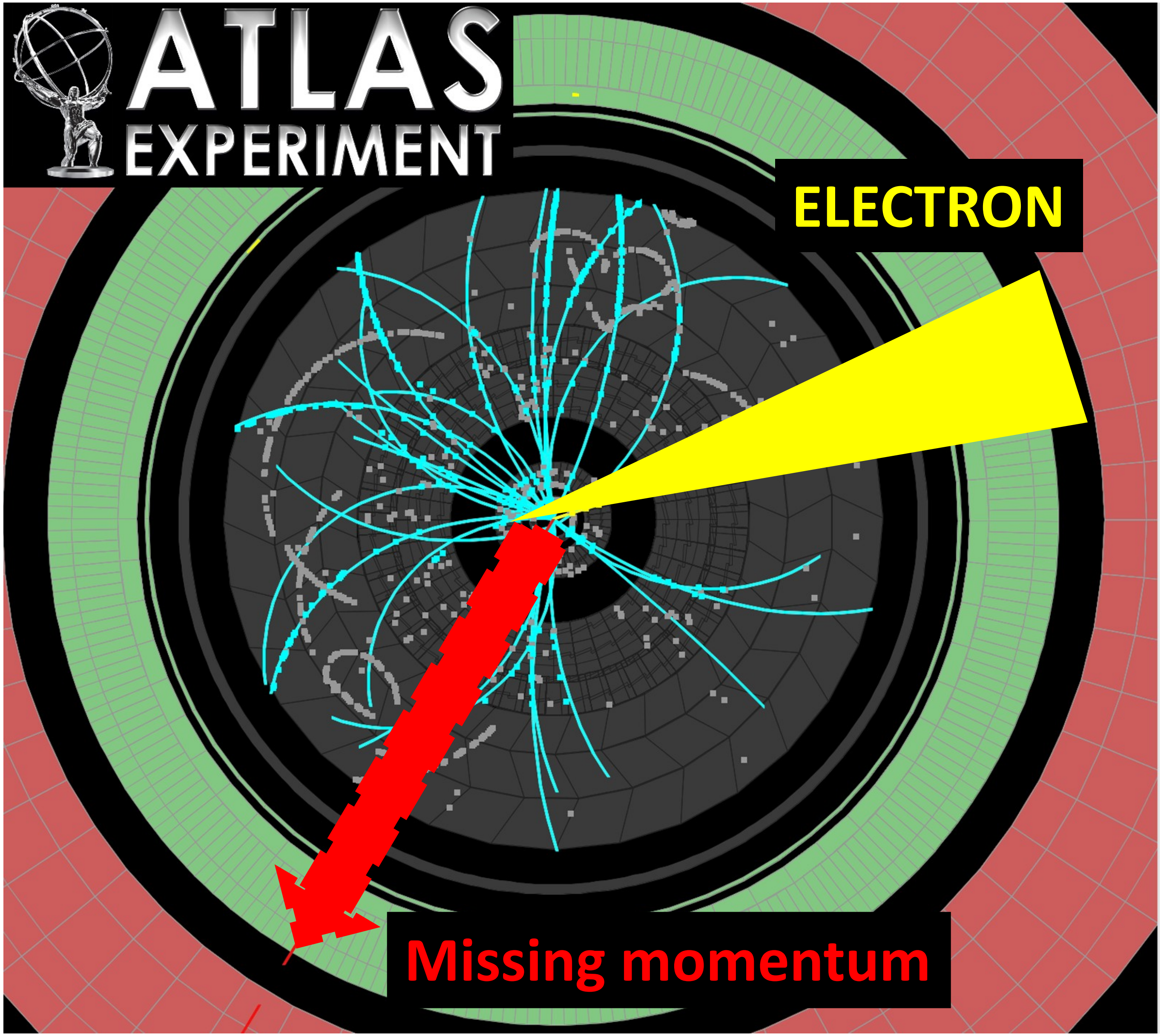
Observe: P_e (four components)

Unobserved: P_ν (does not interact)



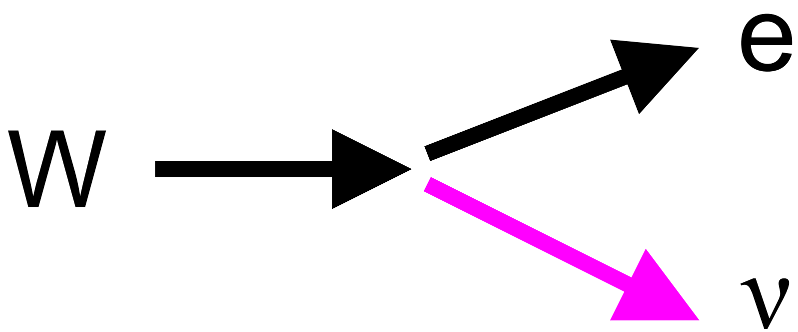
Cannot reconstruct $(P_\nu + P_e)^2$

Unobserved, but not unconstrained…


ATLAS
EXPERIMENT
ELECTRON
Missing momentum

Historical solution: (full!) W transverse mass

$$m_T^2 = m_e^2 + m_\nu^2 + 2(e_e e_\nu - \mathbf{p_T}_e \cdot \mathbf{p_T}_\nu)$$



$$e_e = \sqrt{m_e^2 + p_{Te}^2}$$

$$e_\nu = \sqrt{m_\nu^2 + p_{T\nu}^2}$$

!! NOT THIS !!

$$m_T = \sqrt{2\left|\vec{P}_{Te}\right|\left|\vec{P}_{T\nu}\right|(1 - \cos\vartheta)}$$

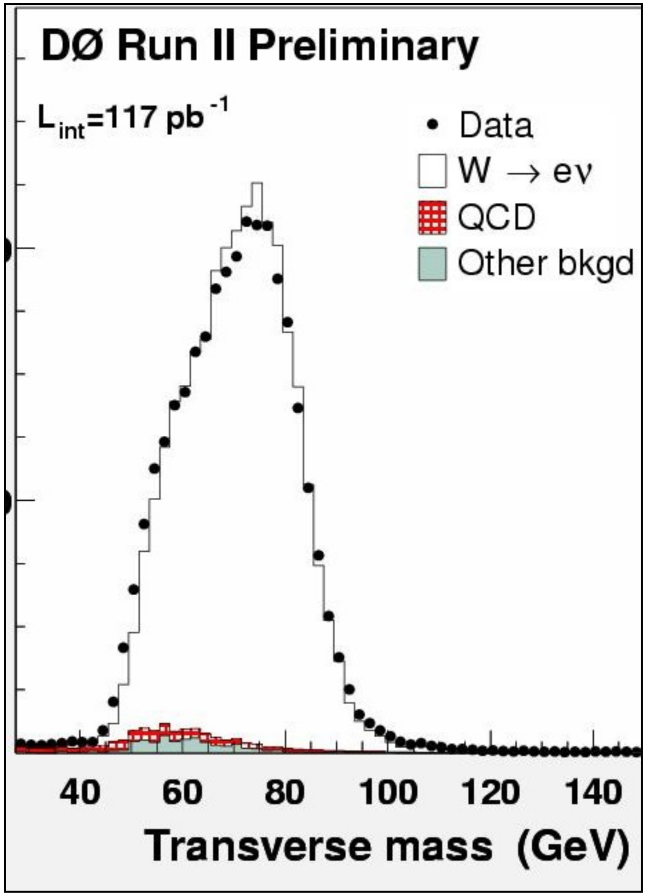
!! This is **NOT** the transverse mass !!

W transverse mass: nice properties

- In every event $m_T < m_W$ if the W is on shell

- There are events in which m_T can saturate the bound on m_W.

motivate m_T in W discovery and mass measurements.



But where did these properties come from?

Re-examine invariant mass: M→a b

$$M^2 = \left(\sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2}\right)^2$$

$$- (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2$$

$$= (E_a + E_b)^2 - (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2$$

$$= m_a^2 + m_b^2 + 2\left(E_a E_b - a_x b_x - a_y b_y - a_z b_z\right)$$

$$= m_a^2 + m_b^2 + 2\left(e_a e_b \cosh(\Delta\eta) - a_x b_x - a_y b_y\right)$$

where

$$e_a = \sqrt{m_a^2 + a_x^2 + a_y^2}$$

$$e_b = \sqrt{m_b^2 + a_b^2 + a_b^2}$$

and

$$\eta_a = \tfrac{1}{2}\ln\left((E_a + a_z)/(E_a - a_z)\right)$$

$$\eta_b = \tfrac{1}{2}\ln\left((E_b + b_z)/(E_b - b_z)\right)$$

$$\Delta\eta = \eta_a - \eta_b$$

Comparing invariant and transverse masses:

$$M^2 = m_a^2 + m_b^2 + 2\left(e_a e_b \cosh(\Delta\eta) - a_x b_x - a_y b_y\right)$$

$$M_T^2 = m_a^2 + m_b^2 + 2\left(e_a e_b \qquad\qquad - a_x b_x - a_y b_y\right)$$

Since $\cosh(\Delta\eta) \geq 1$ have $M_T \leq M$

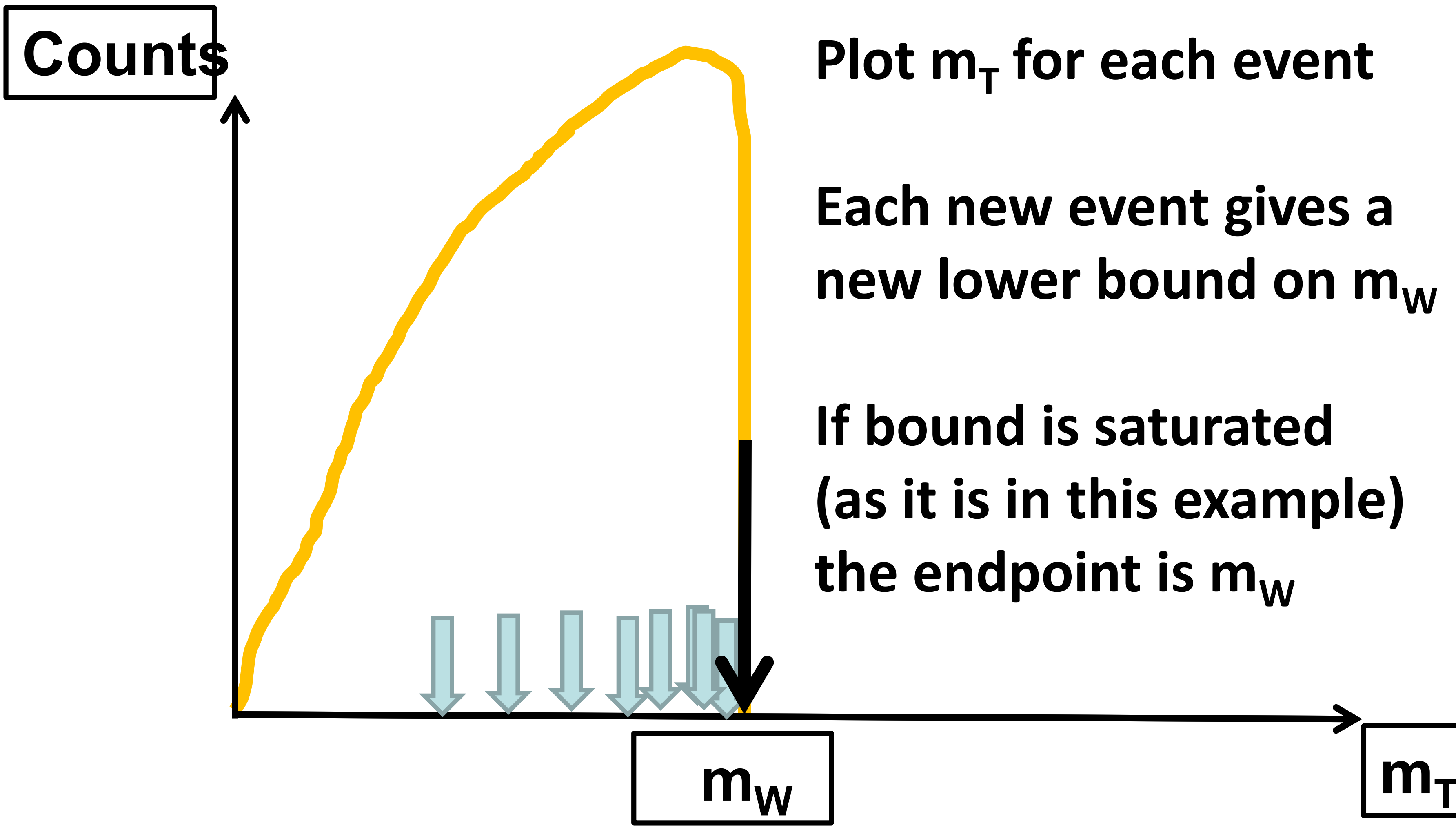
with equality when $\Delta\eta = 0$.

(Not same as throwing away z information!)

But have bound, and bound can be saturated.

Note that at this point we are assuming we know m_b.

W boson mass measurement



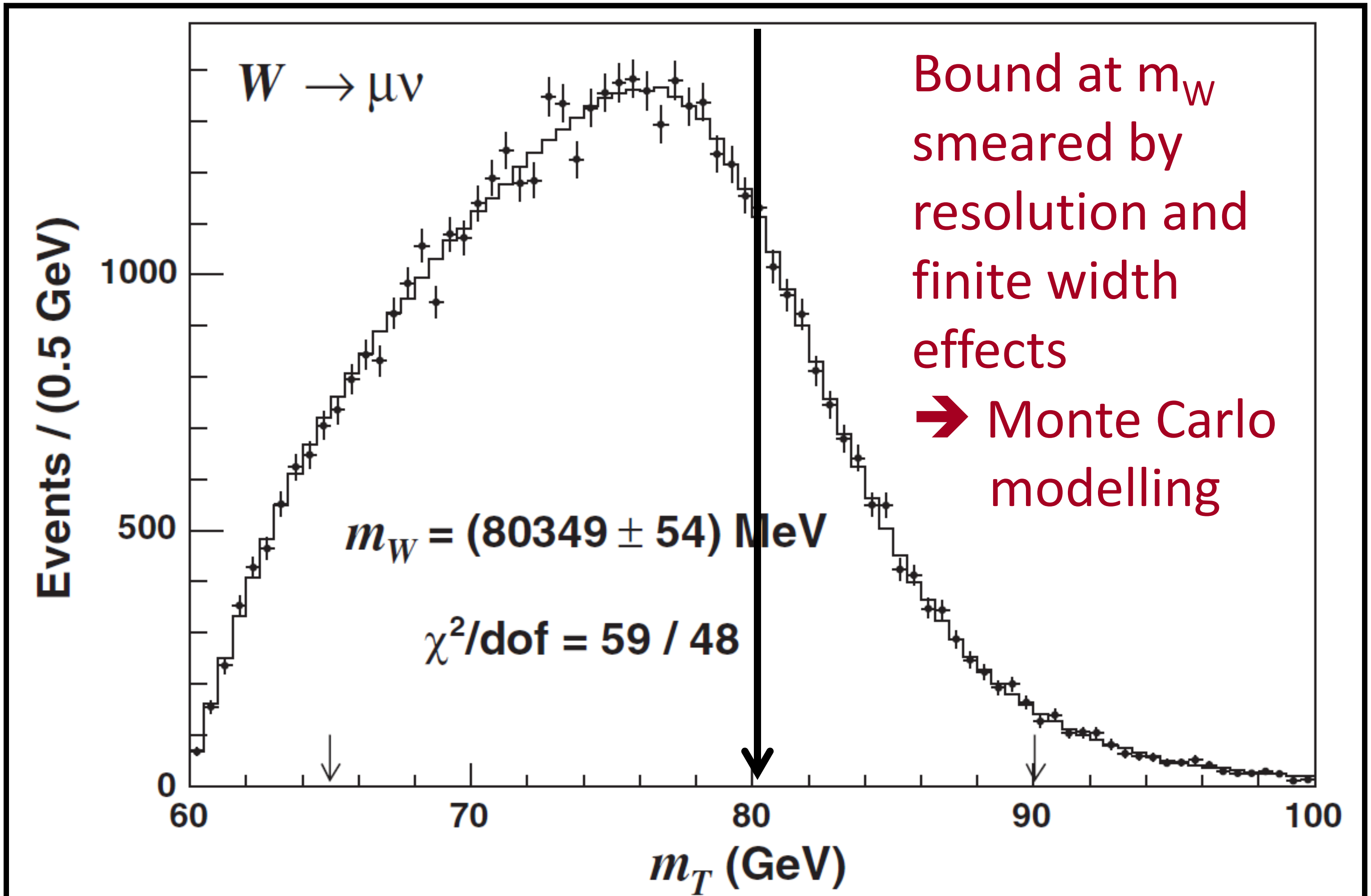
Plot m_T for each event

Each new event gives a new lower bound on m_W

If bound is saturated (as it is in this example) the endpoint is m_W

Phys.Rev.D. **77**, 112001 (2008)

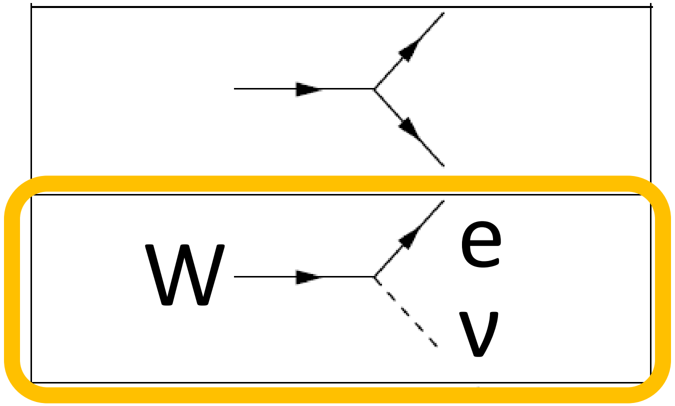
In the data….



Bound at m_W smeared by resolution and finite width effects

➔ Monte Carlo modelling

Alternative way of approaching the problem



Set out **INTENDING** to construct best lower **bound** on $(P_e+P_\nu)^2$ **given the constraints**

Constraints in this instance:

$0 = (P_\nu)^2$ ***[massless neutrino]***

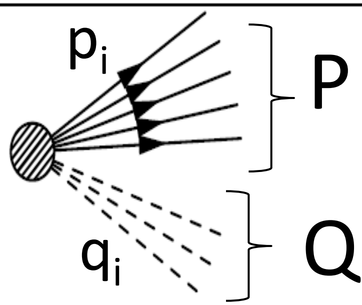
$0 = \Sigma \mathbf{p}_T = \mathbf{u}_T + \mathbf{p}_T(e) + \mathbf{p}_T(\nu)$

[momentum conservation in transverse plane]

Suggests general prescription…

(1) Propose a decay **topology**
(2) Write down your the **Lorentz Invariant** of choice
(3) Write down the **constraints**
(4) **Calculate** the bound (algebraically/numerically/mix)

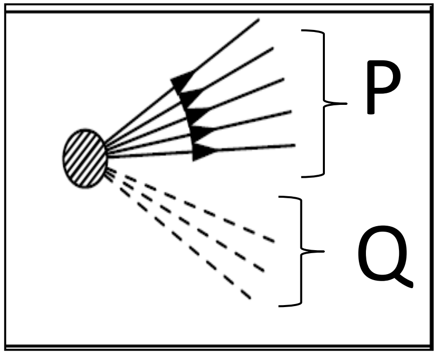
(1)



(2) $$\mathcal{M}_a \equiv \sqrt{g_{\mu\nu}\,(\mathbf{P}_a + \mathbf{Q}_a)^\mu\,(\mathbf{P}_a + \mathbf{Q}_a)^\nu}$$

(3) $$\sum_{i=1}^{N_{\mathcal{I}}} \vec{q}_{iT} = \vec{\not{p}}_T \equiv -\vec{u}_T - \sum_{i=1}^{N_{\mathcal{V}}} \vec{p}_{iT}$$

Single parent … multiple daughters



many visibles

many invisibles

$$M_{1\mathrm{T}}^{2} = \left(\sqrt{M_P^2 + \vec{\mathbf{p}}_T^{\,2}} + \sqrt{M_{\text{slash}}^2 + \vec{\mathbf{q}}_{T\text{miss}}^{\,2}}\right)^2 - u_T^2$$

$$M_{\text{slash}} = \sum_i \tilde{M}_i$$

Bound depends on *GUESS* masses of *all* invisible daughters

Most conservative: **set to zero**

[more later]

Almost exactly same as transverse mass – one small generalization

$$= \left(\sqrt{M_P^2 + \vec{\mathbf{p}}_T^{\,2}} + \sqrt{M_{\text{slash}}^2 + \vec{\mathbf{q}}_{T\text{miss}}^{\,2}}\right)^2 - u_T^2$$

$$= \left(\sqrt{M_P^2 + \vec{\mathbf{p}}_T^{\,2}} + \sqrt{M_{\text{Q}}^2 + \vec{\mathbf{q}}_{T\text{miss}}^{\,2}}\right)^2 - u_T^2$$

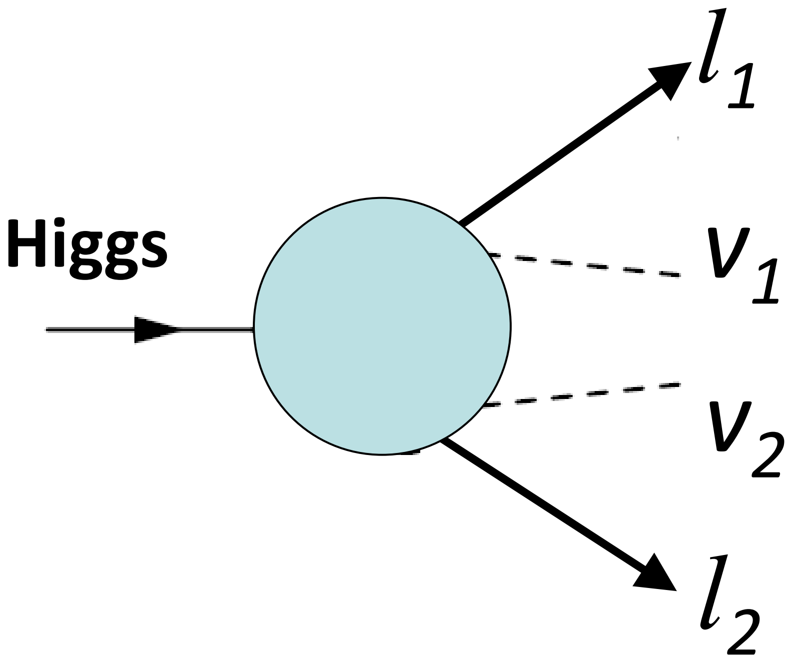
The “invisible mass” has become a parameter …. rather than the actual visible mass.

We will come back to this many times.

Suggests we should think about non-physical parameters a bit more ….

Applications of M_{1T}?

Higgs →WW* → lνlν

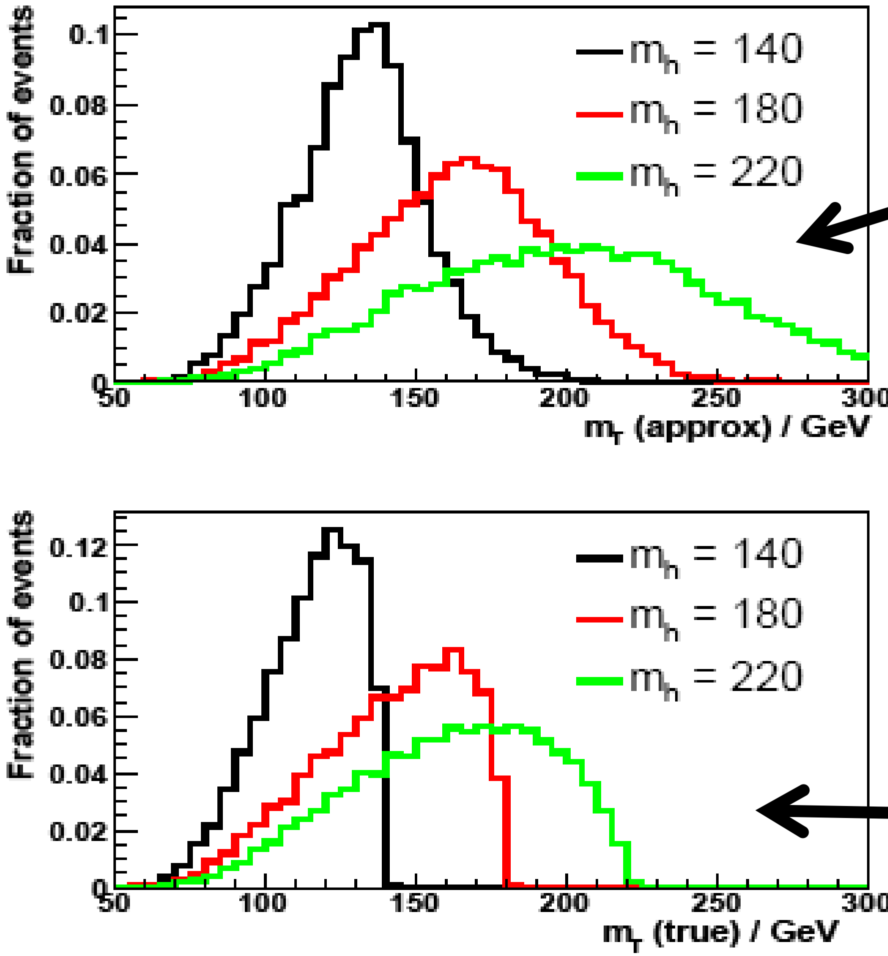


$$Q_1^\mu Q_{1\mu} = 0,$$
$$Q_2^\mu Q_{2\mu} = 0,$$

$$\vec{q}_{1\mathrm{T}} + \vec{q}_{2\mathrm{T}} = \vec{\not{p}}_{\mathrm{T}}.$$

Written up in http://arxiv.org/abs/1106.2322

Higgs →WW* → lvlv



Previous variable (not a bound)

Why are endpoints often more robust than shapes?

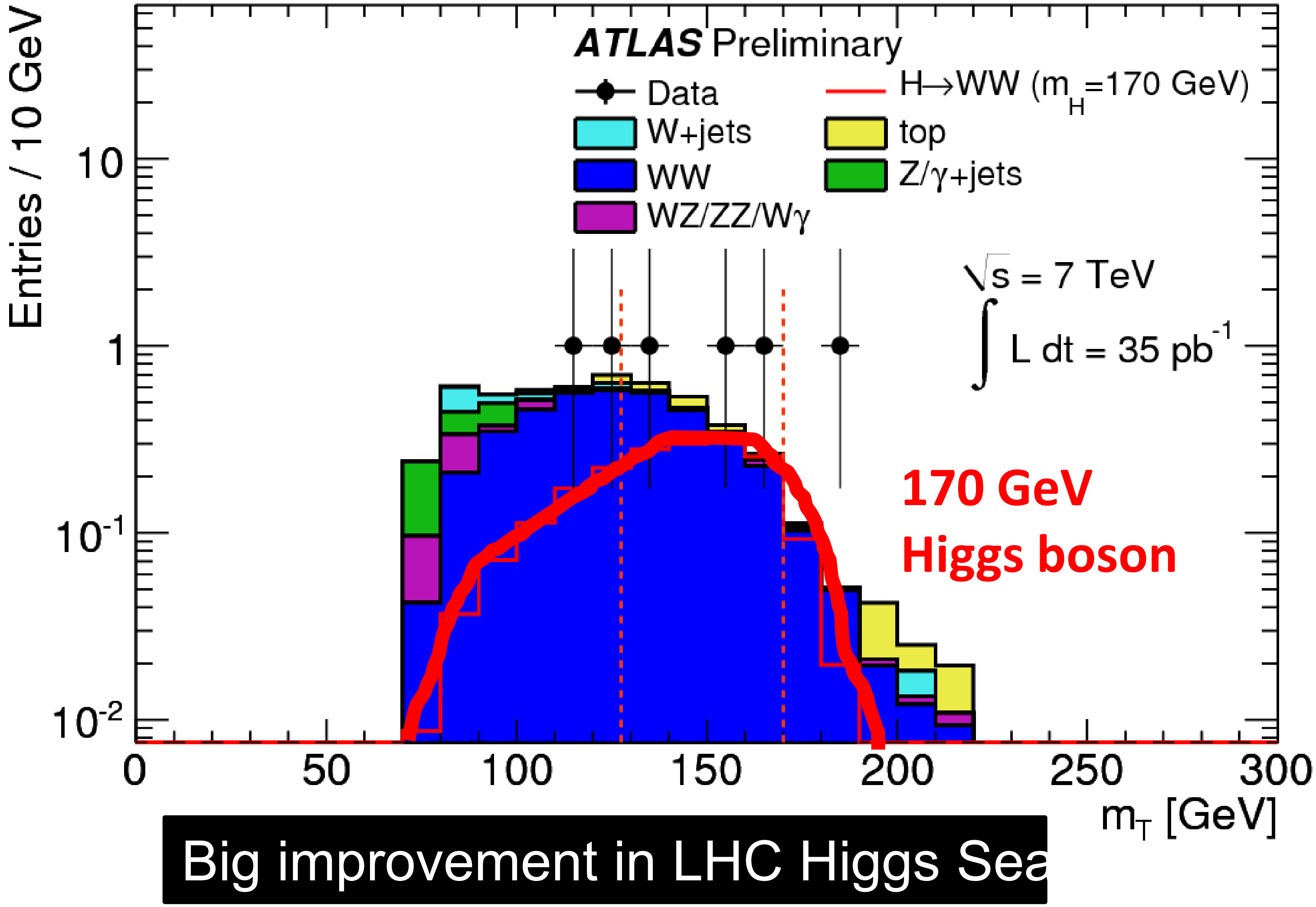
Proper bound var MTTrue = M1T

FIG. 1: Signal-only distributions of $m_T^{\rm approx}$ (top) and $m_T^{\rm true}$ (bottom) for various values of m_h (in GeV). No cuts on $\Delta\phi_{\ell\ell}^{\rm max}$ and $p_{T\,WW}^{\rm min}$ have been applied.

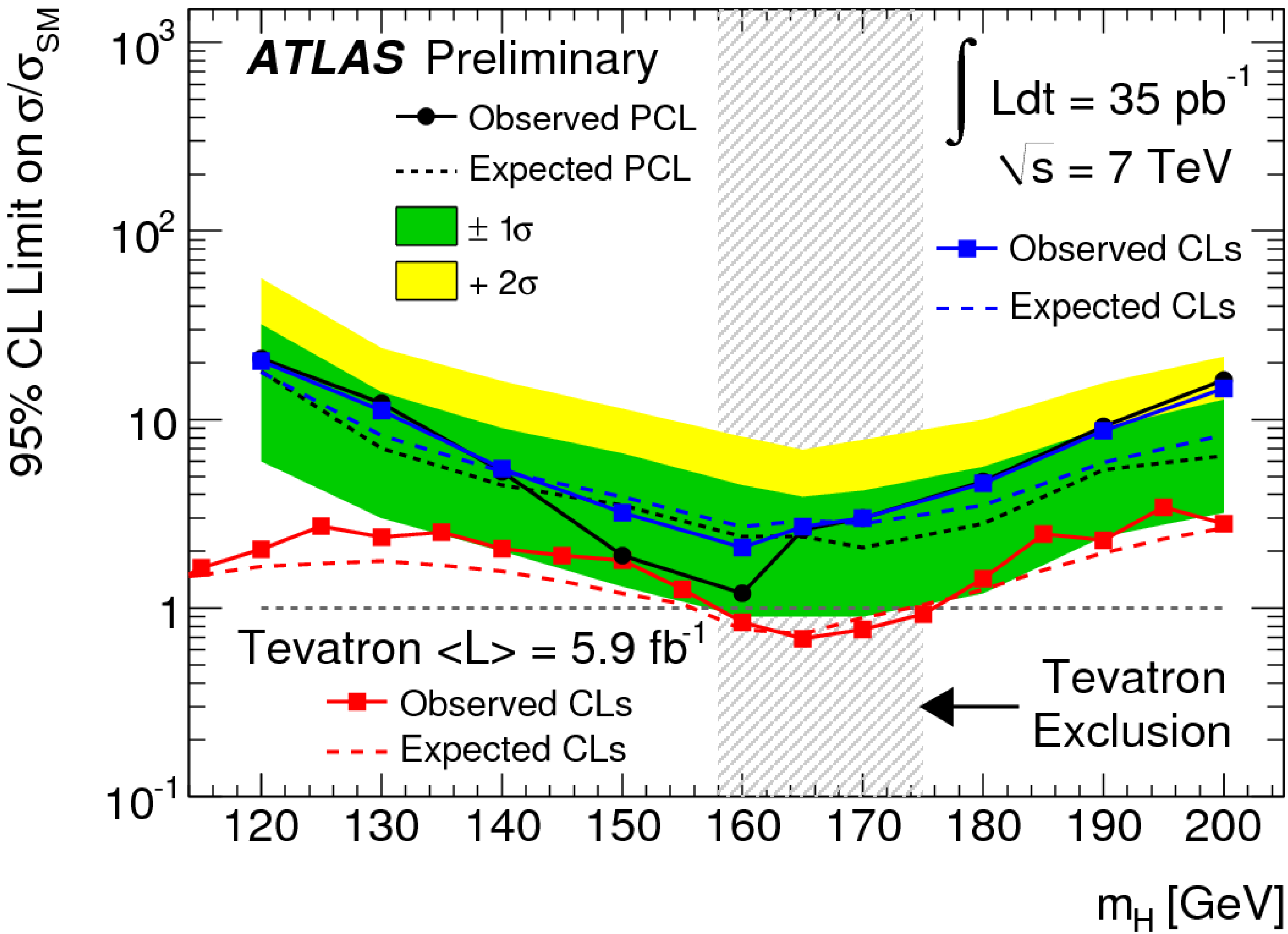
Written up in http://arxiv.org/abs/1106.2322

ATLAS-CONF-2011-005

Against the 2010 LHC data…

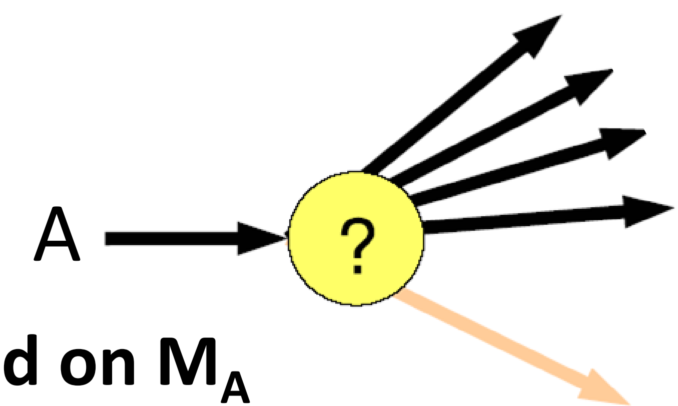


ATLAS 35/pb: H → WW → lνlν



change of topic – moving closer to BSM

What if we don't know the masses of the invisible particle(s)?

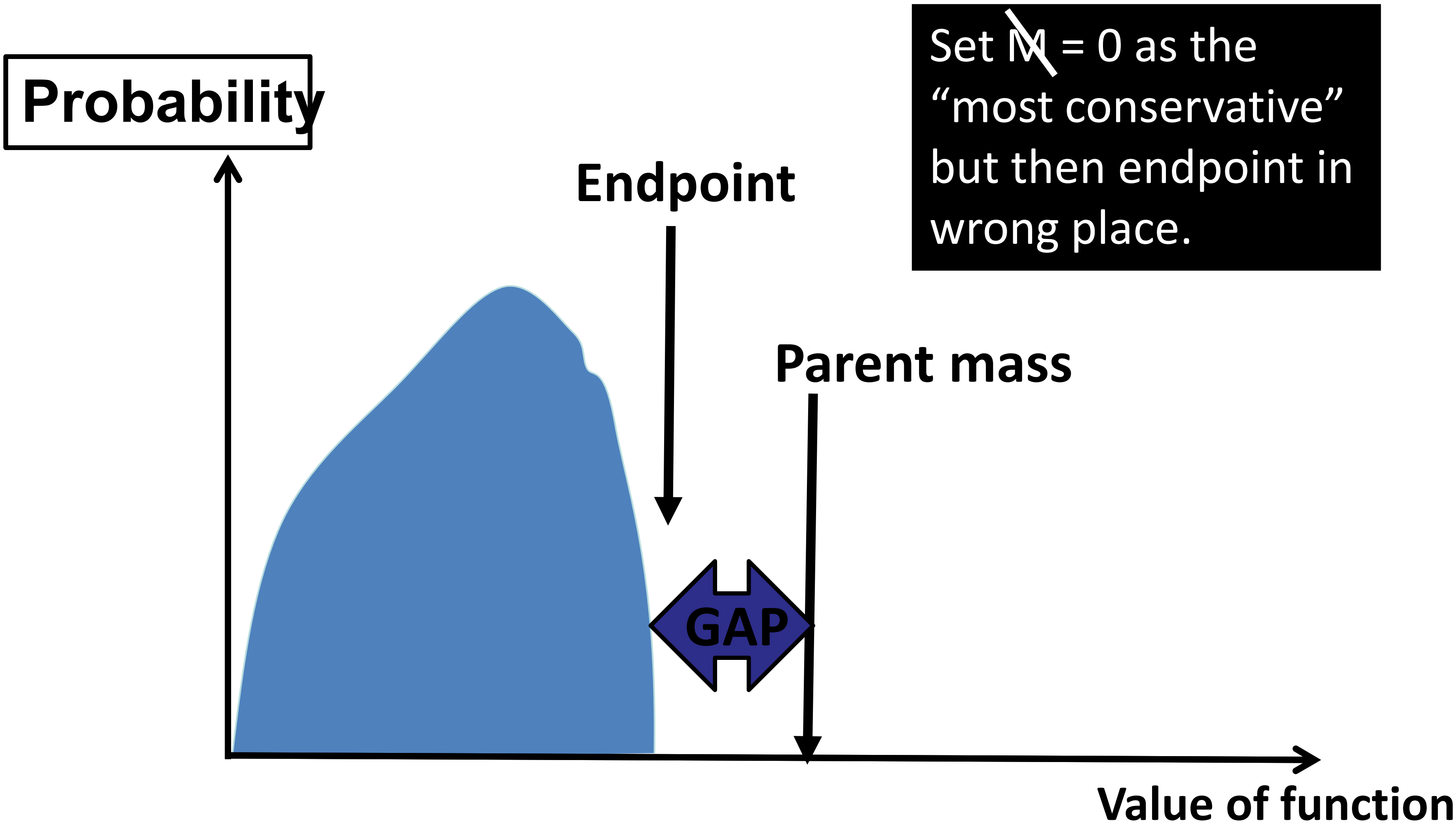


WANT bound on M_A

BUT M_B unknown…

Can we construct a maximal lower bound on M_A that depends on a hypothesis for M_B ?

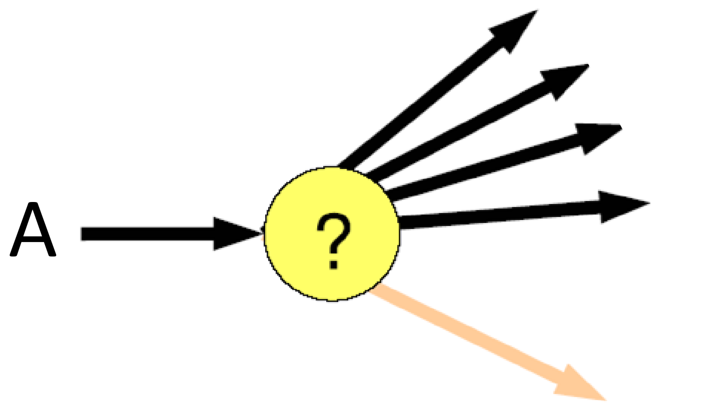
Hmm ….
"wrong M_B" not what M_T was designed for.



Let’s go back to the (full) transverse mass again for a closer look!

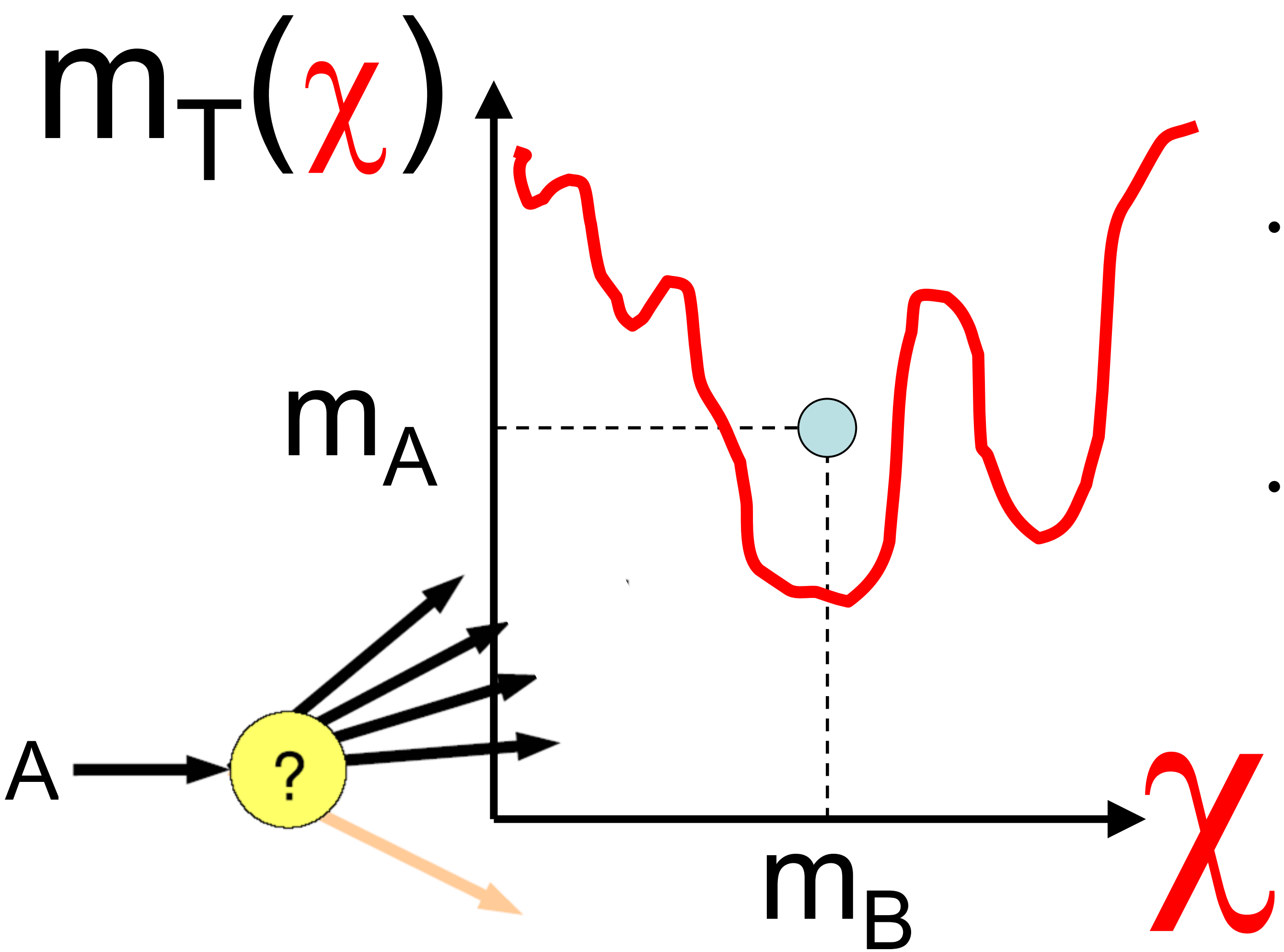
In next few slides:

χ = Guess (i.e. hypothesis) for mass of the invisible daughter



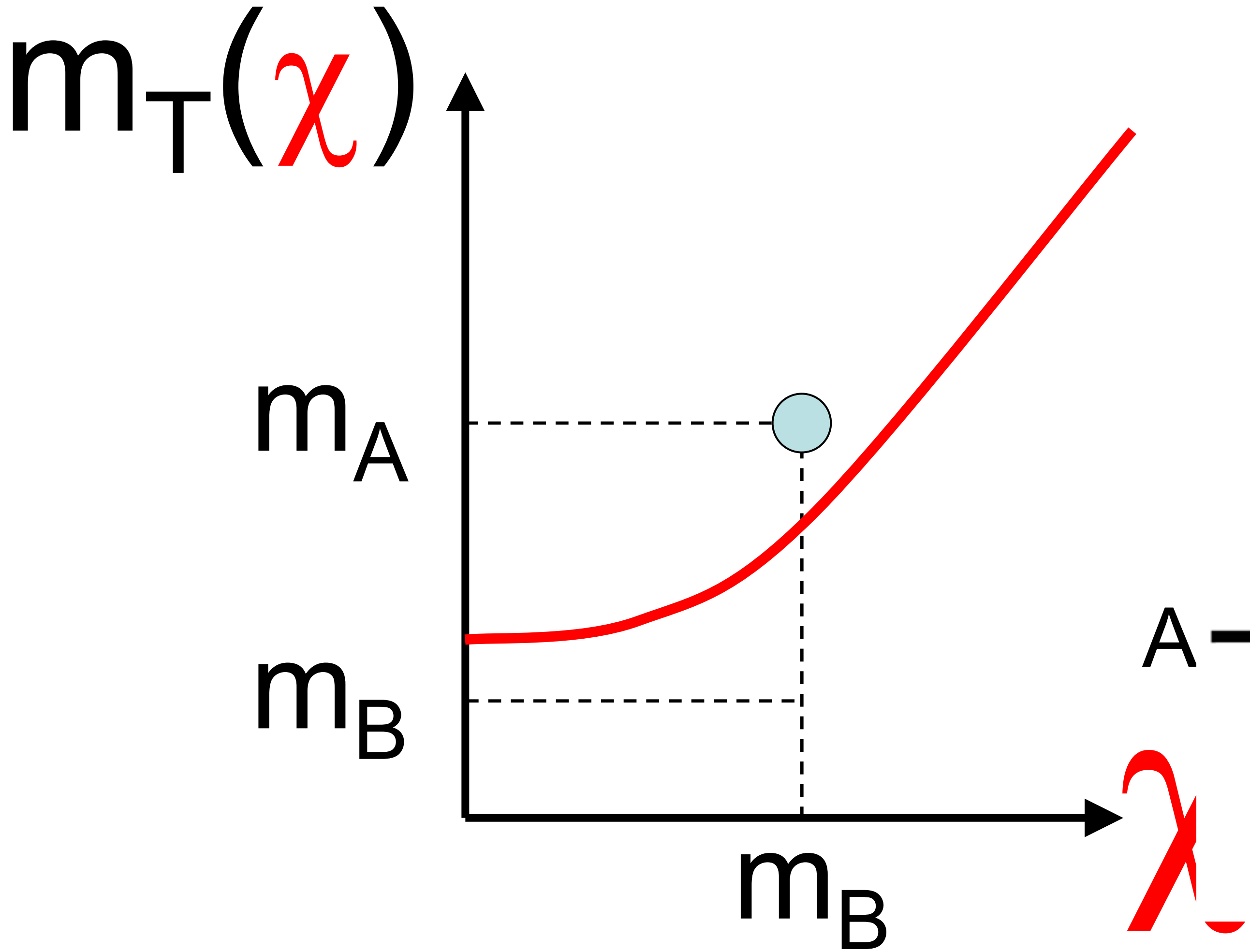
In other words, we will use χ in all the places we previously used M_B.

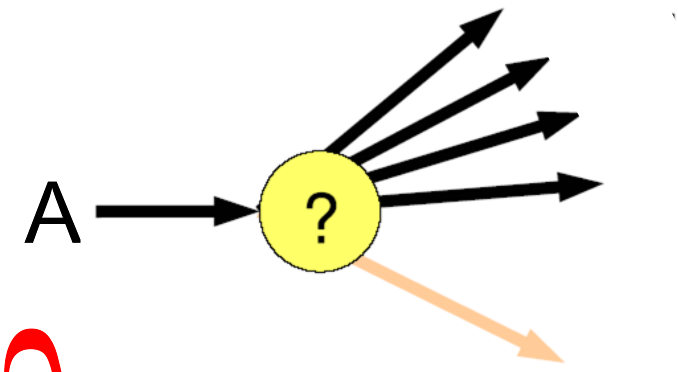
Schematically, all we have guaranteed so far is the picture below:



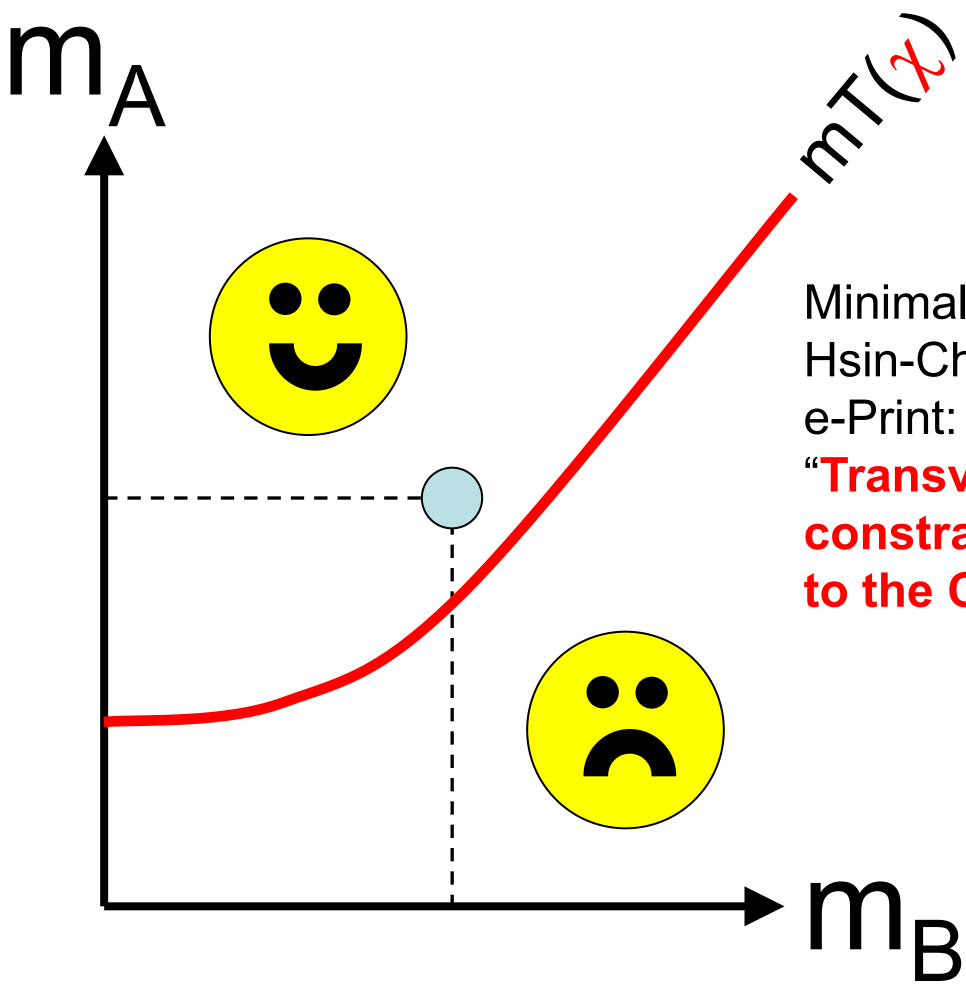
- Since "χ" can now be "wrong", some of the properties of the transverse mass can "break":
- $m_T(\chi)$ max is no longer invariant under transverse boosts! (except when $\chi=m_B$)
- $m_T(\chi)<m_A$ may no longer hold! (however we always retain: $m_T(m_B) < m_A$)

Actual dependence on invisible mass guess χ more like this:



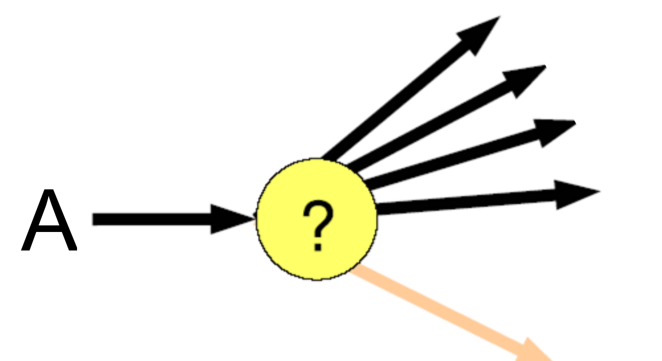


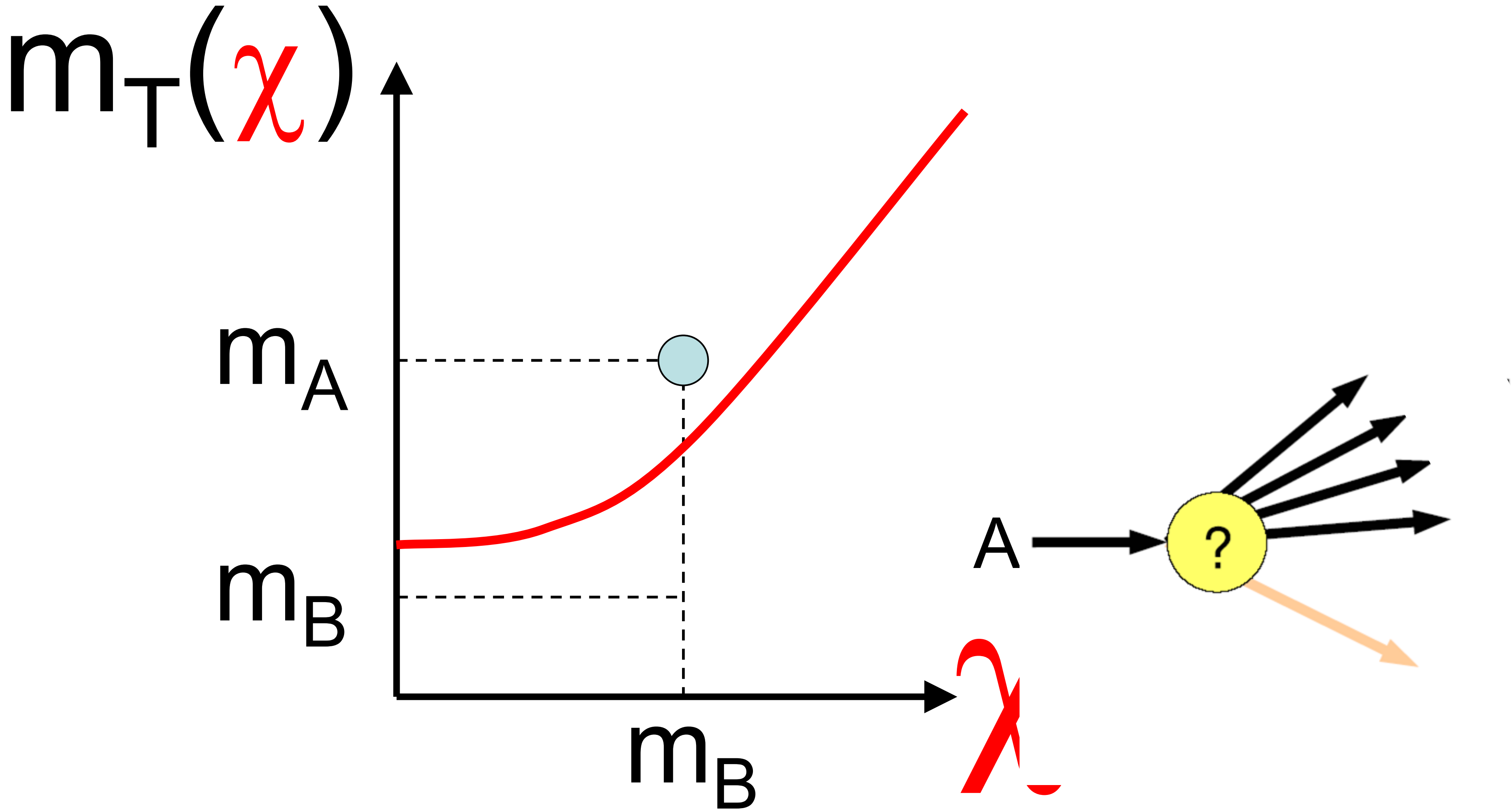
In fact, we get this very nice result:



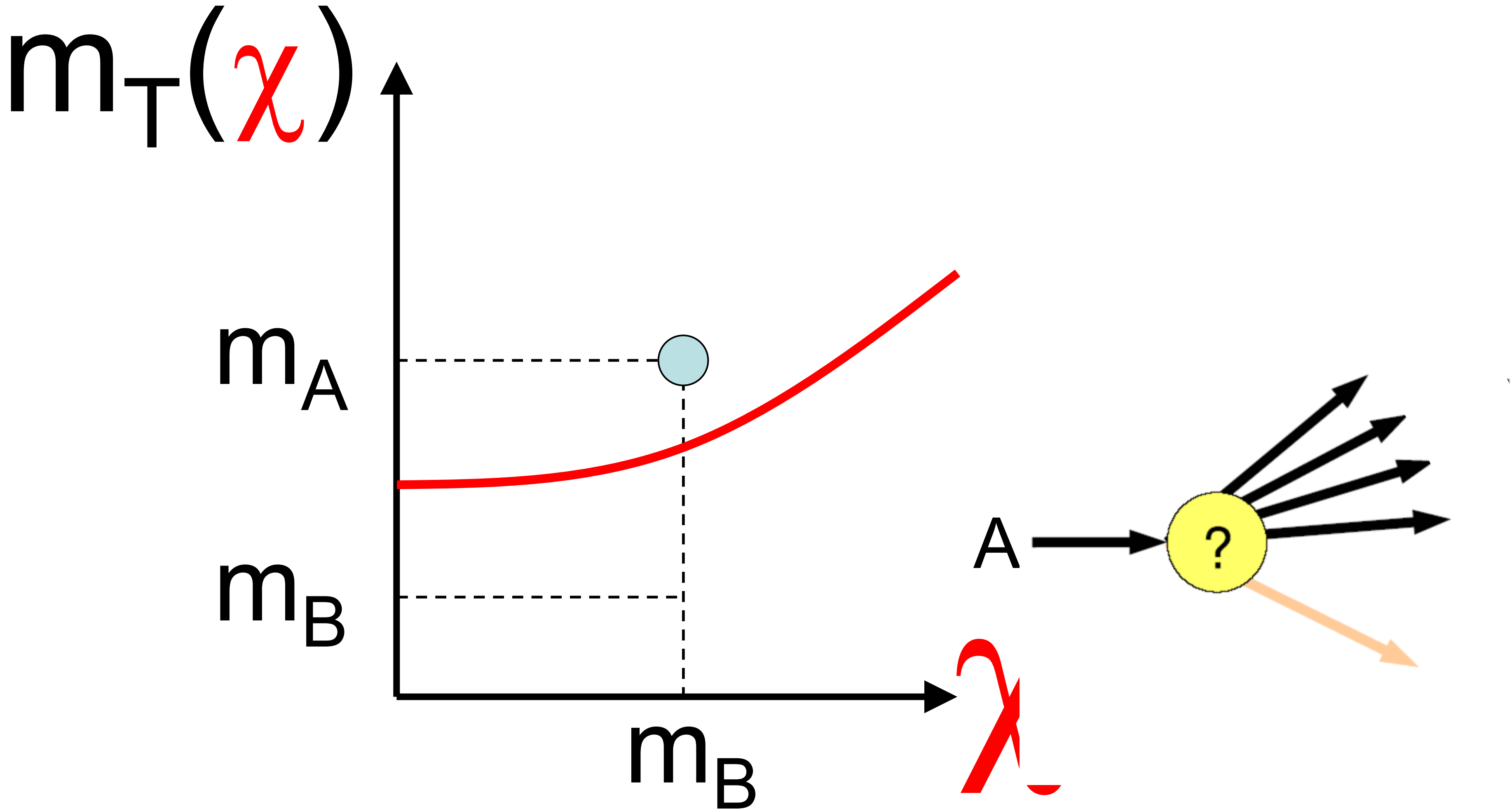
The "full" transverse mass curve is the boundary of the region of (mother,daughter) masses consistent with the observed event!

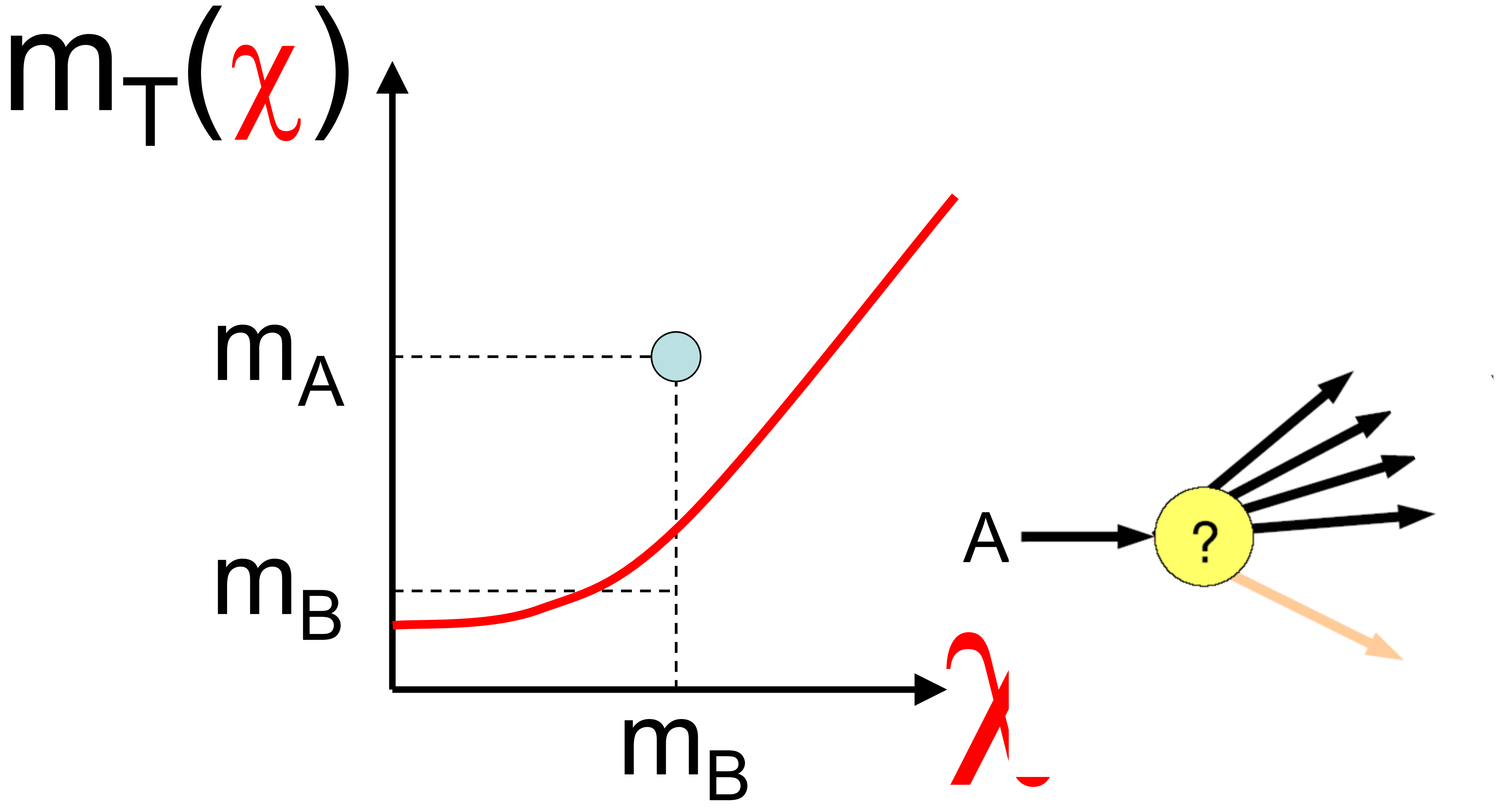
Minimal Kinematic Constraints and m(T2), Hsin-Chia Cheng and Zhenyu Han (UCD) e-Print: arXiv:0810.5178 [hep-ph] and "**Transverse masses and kinematic constraints, from the Boundary to the Crease**" **arXiv:0908.3779**



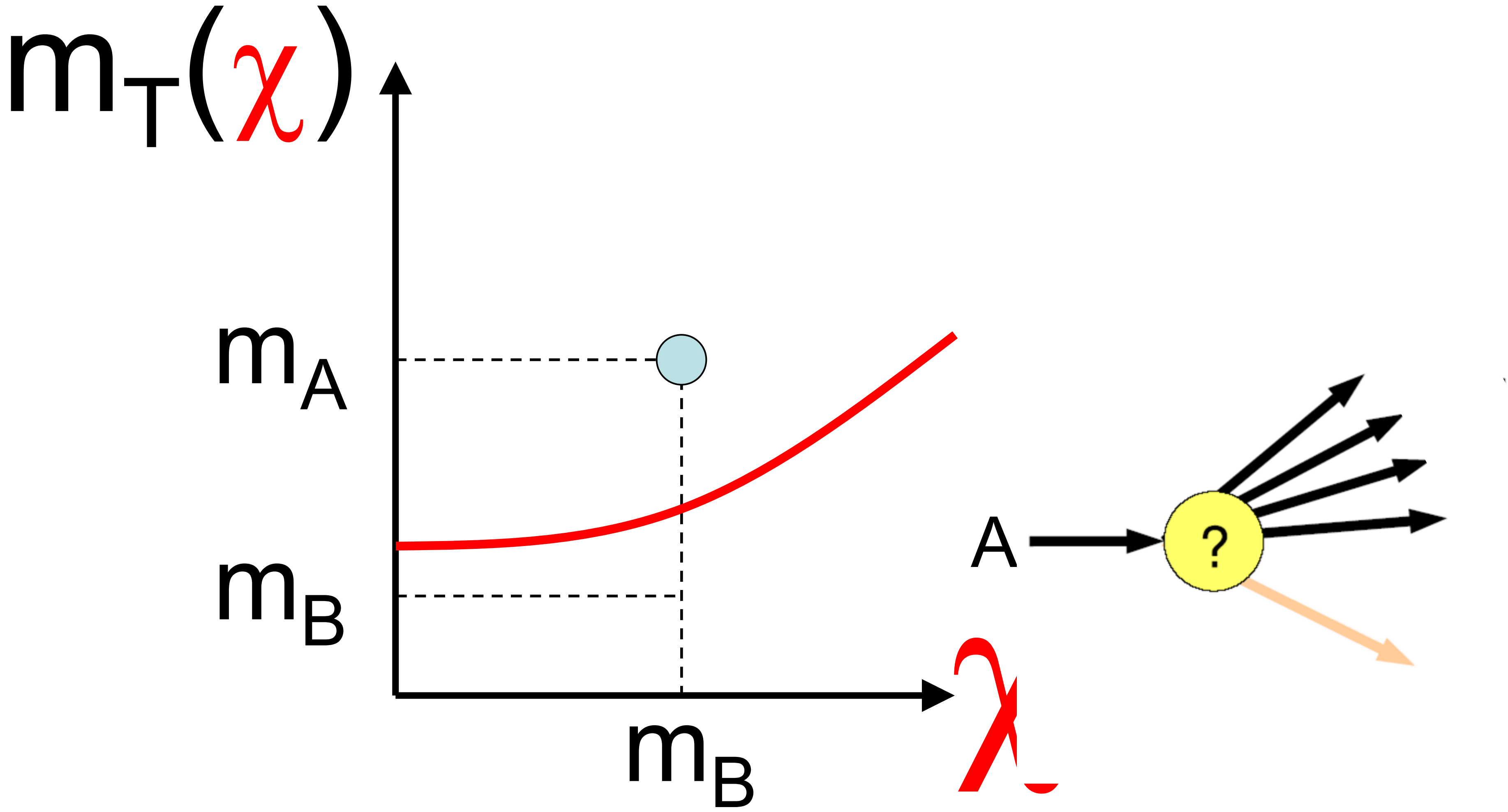

$m_T(\chi)$
m_A
m_B
m_B
χ
A
?

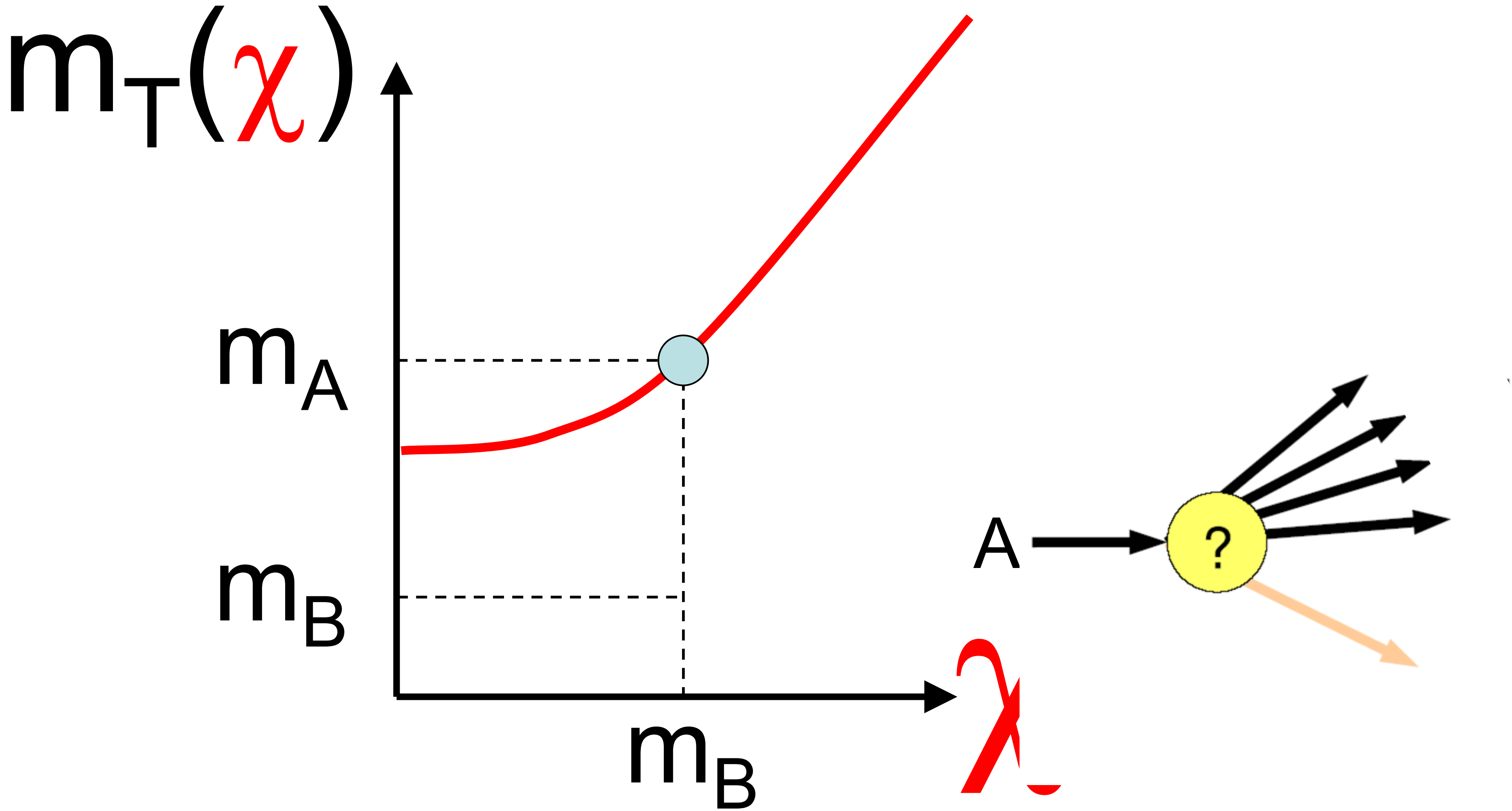
Event 2 of 8



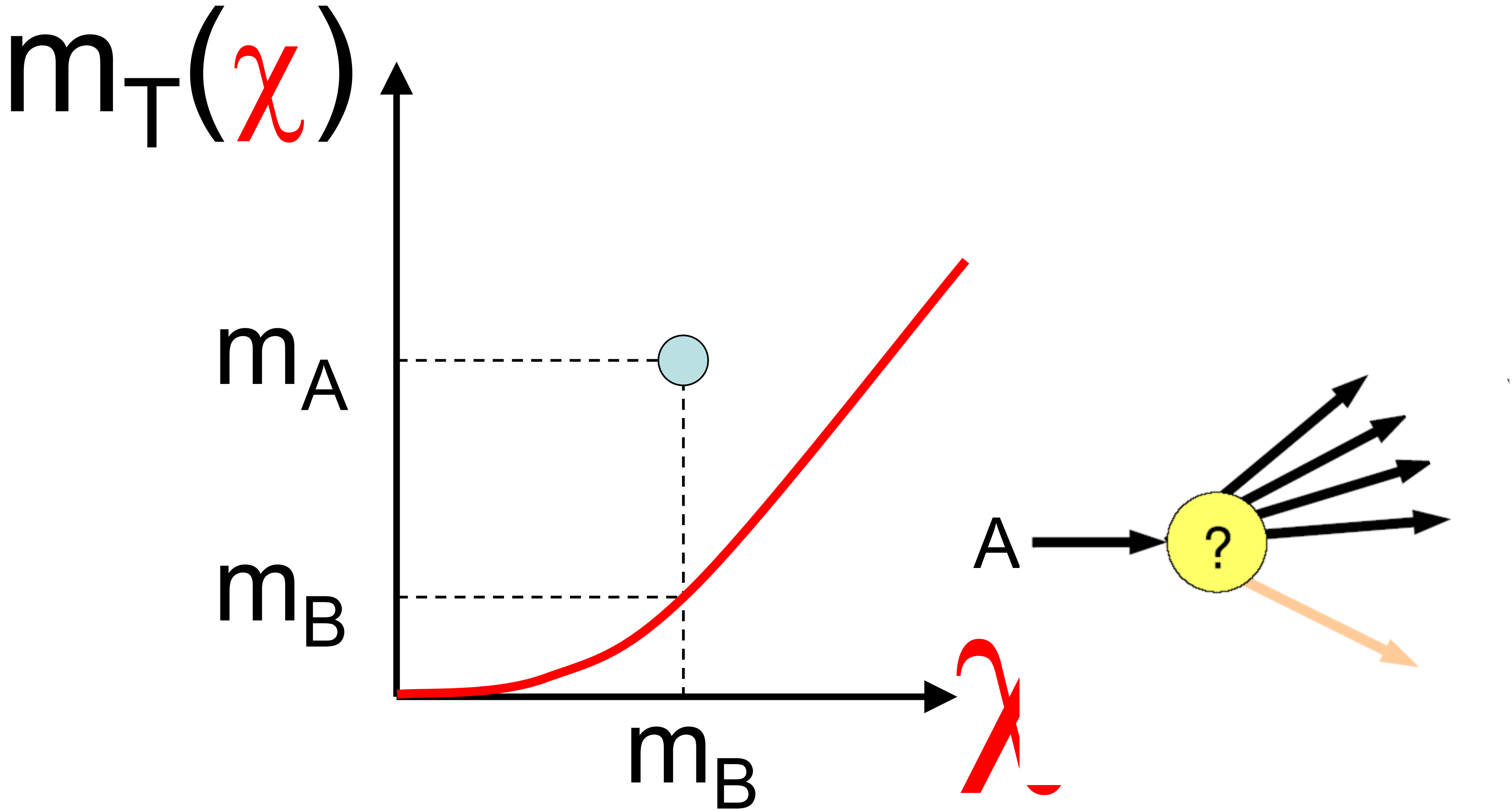


mT(χ)
mA
mB
mB
χ
A
?

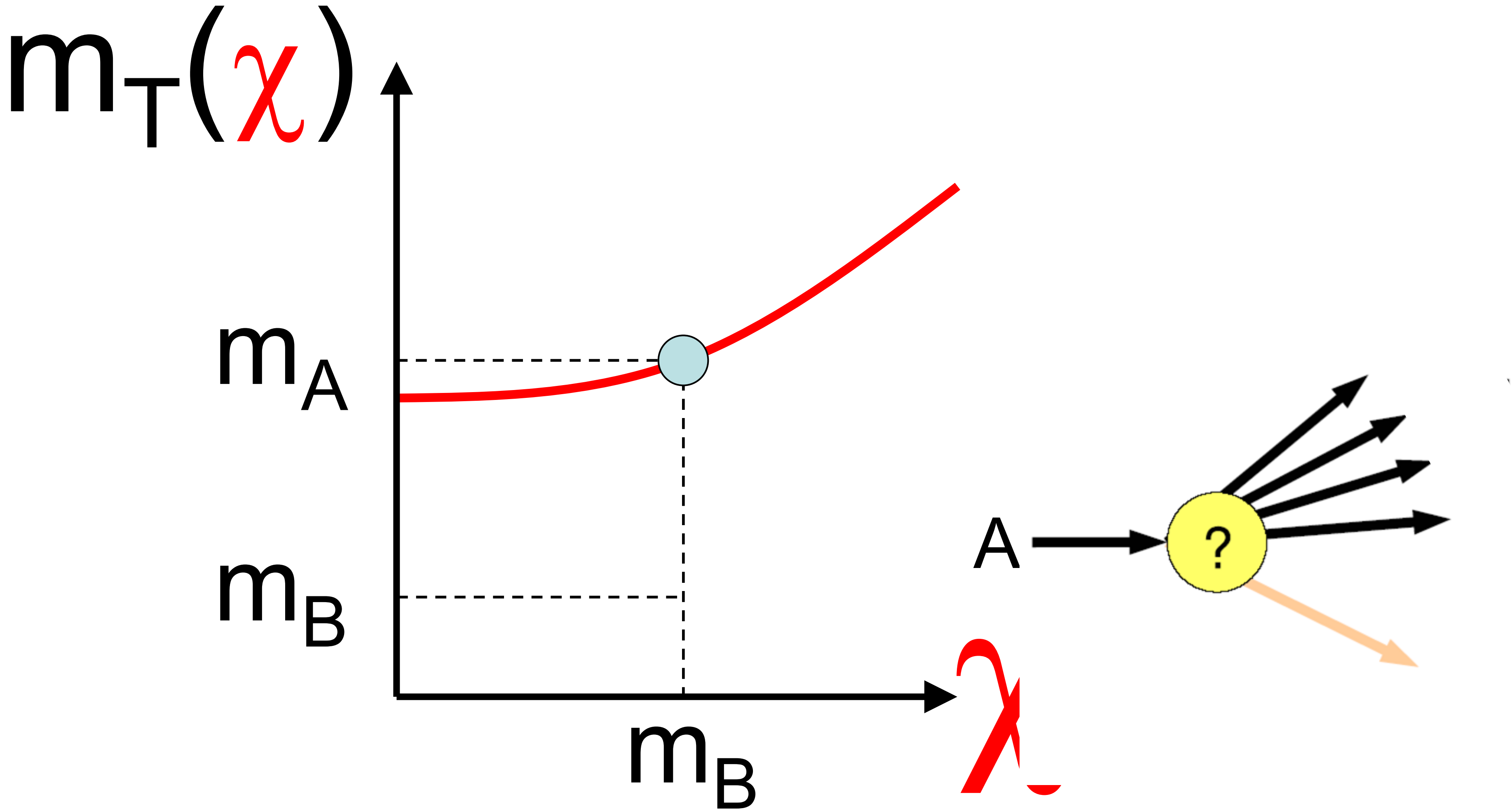

$m_T(\chi)$
m_A
m_B
m_B
χ
A
?


$m_T(\chi)$
m_A
m_B
m_B
χ
A
?

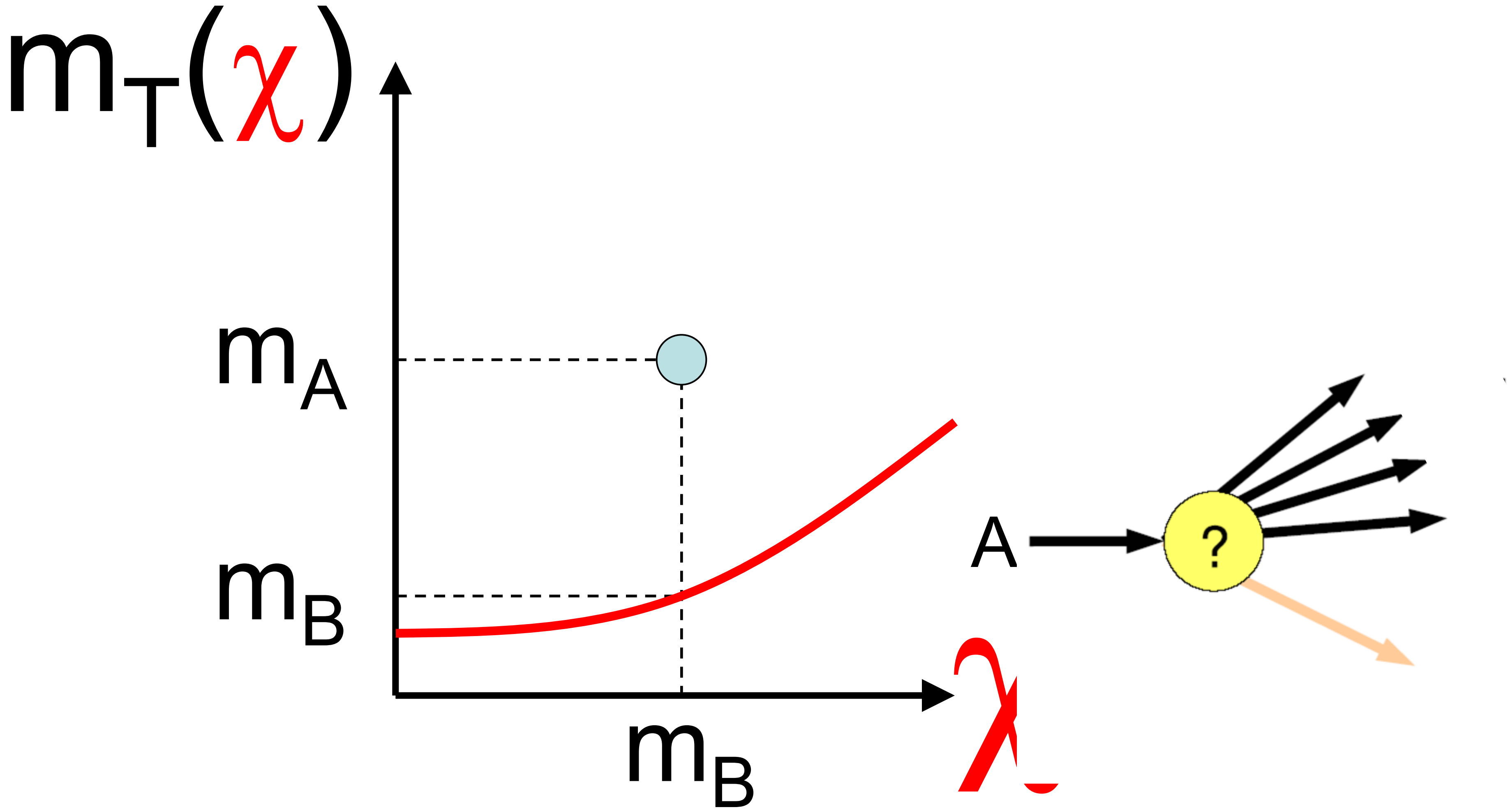
Event 6 of 8



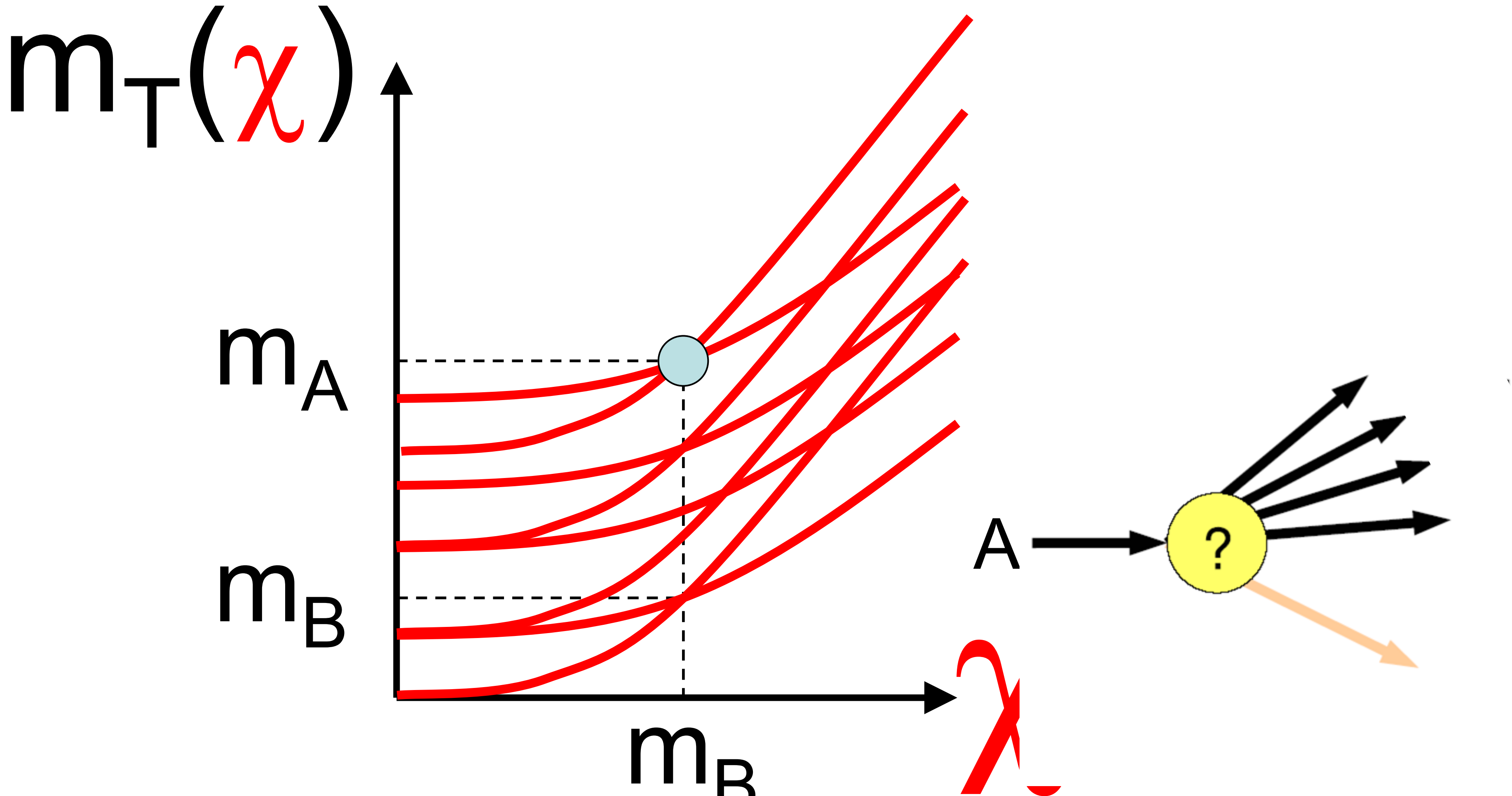
Event 7 of 8



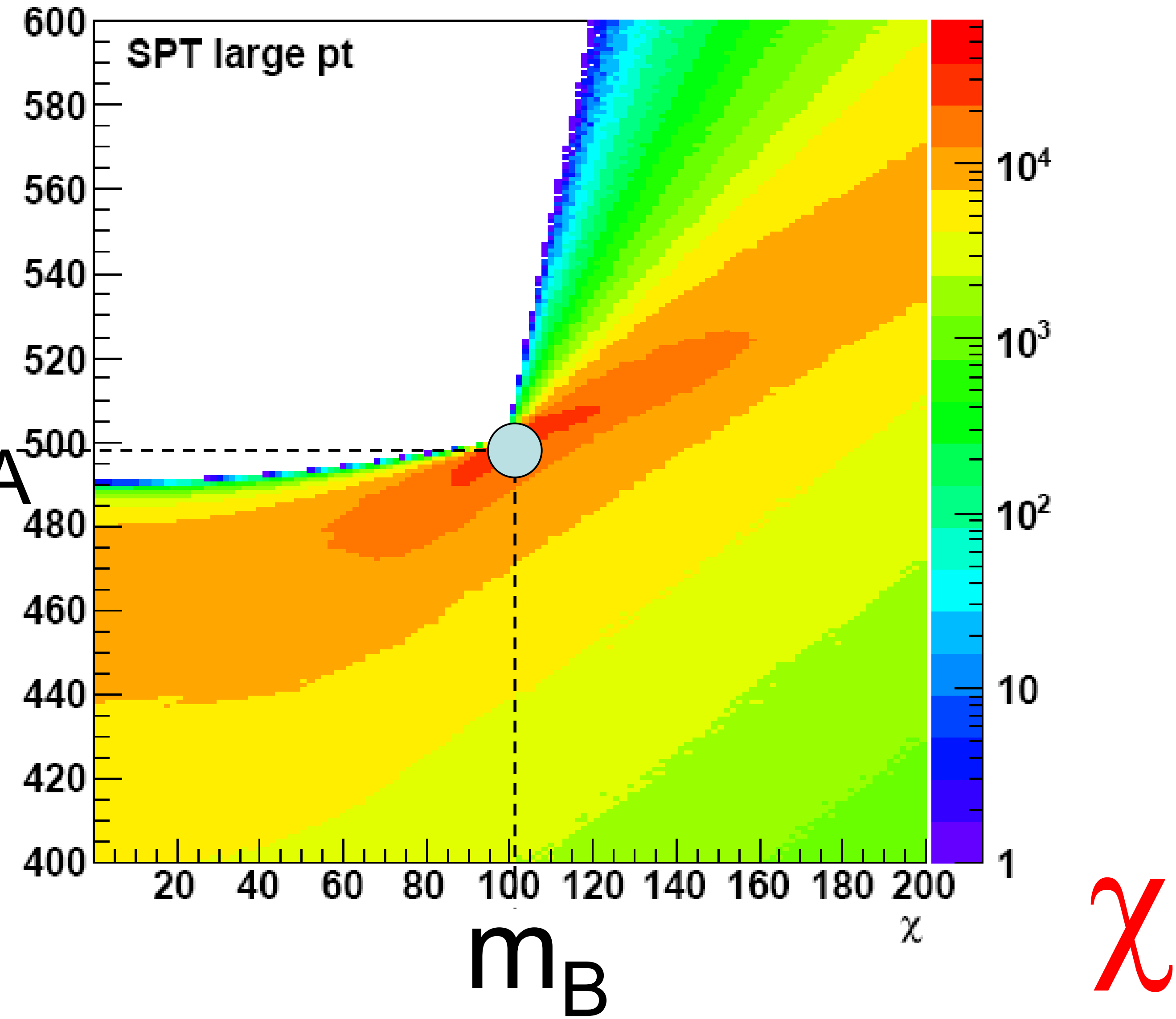
Event 8 of 8



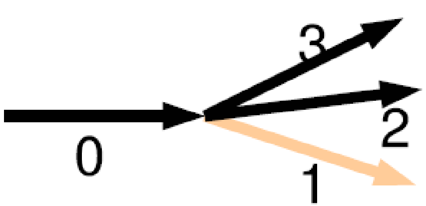
Overlay all 8 events



Overlay many events



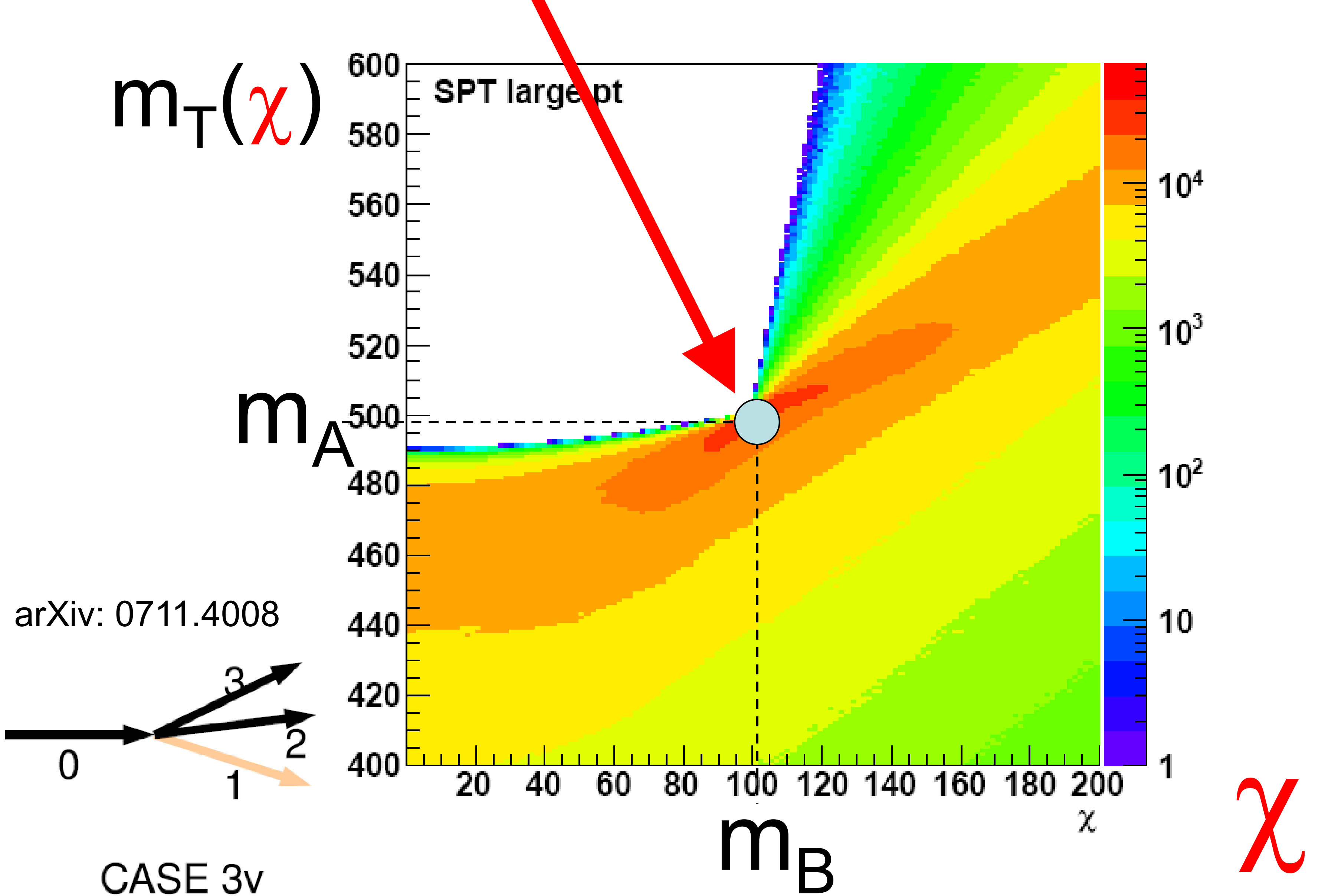
arXiv: 0711.4008



CASE 3v

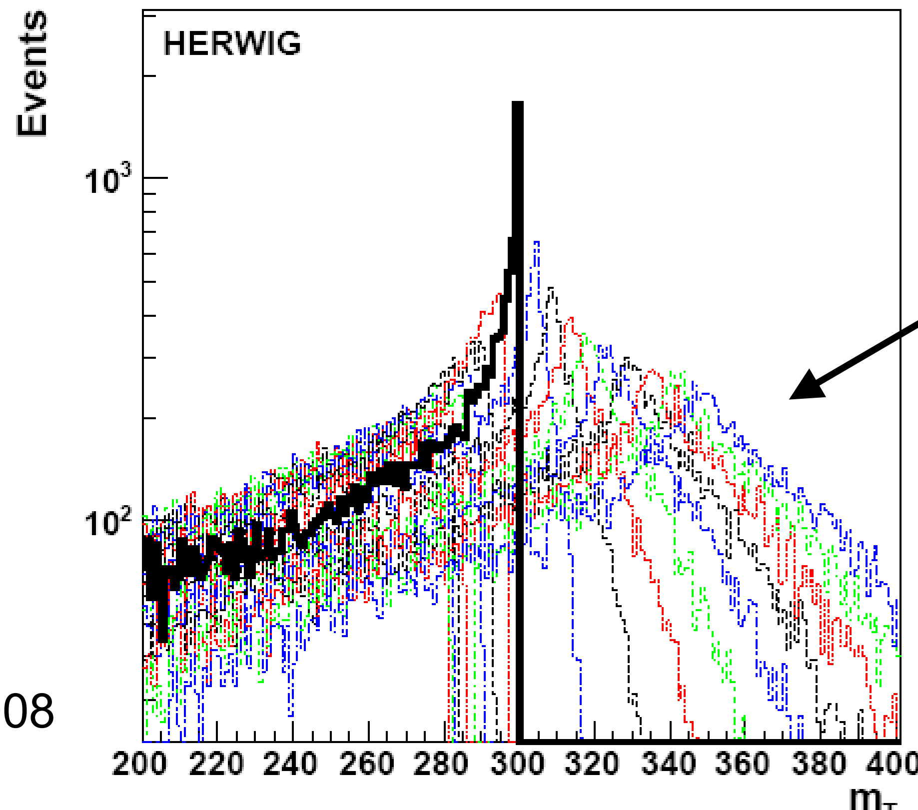
Weighing Wimps with Kinks at Colliders arXiv: 0711.4008

Here is a transverse mass “KINK”



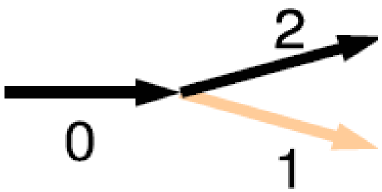
Weighing Wimps with Kinks at Colliders arXiv: 0711.4008

Alternatively, look at M_T distributions for a variety of values of chi.



Each curve has a different value of chi

arXiv: 0711.4008



CASE 2

m_T

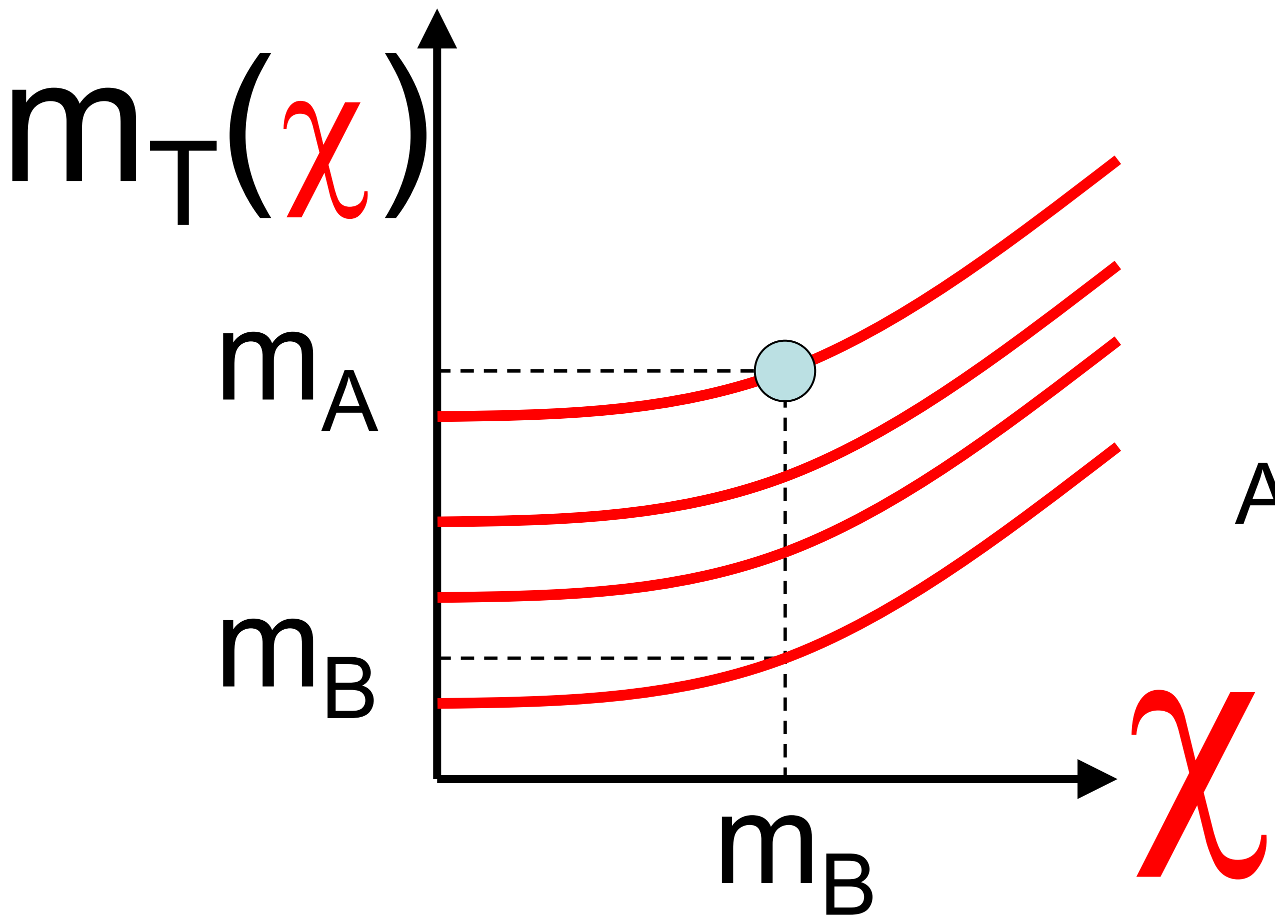
Where is the kink now?

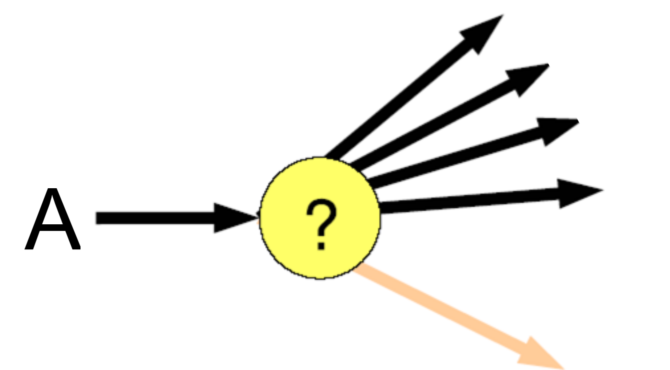
What causes the kink?

- Two entirely independent things can cause the kink:
  - (1) Variability in the "visible mass"
  - (2) Recoil of the "interesting things" against Upstream Transverse Momentum
- Which is the dominant cause depends on the particular situation … let us look at each separately:

Kink cause 1: Variability in visible mass

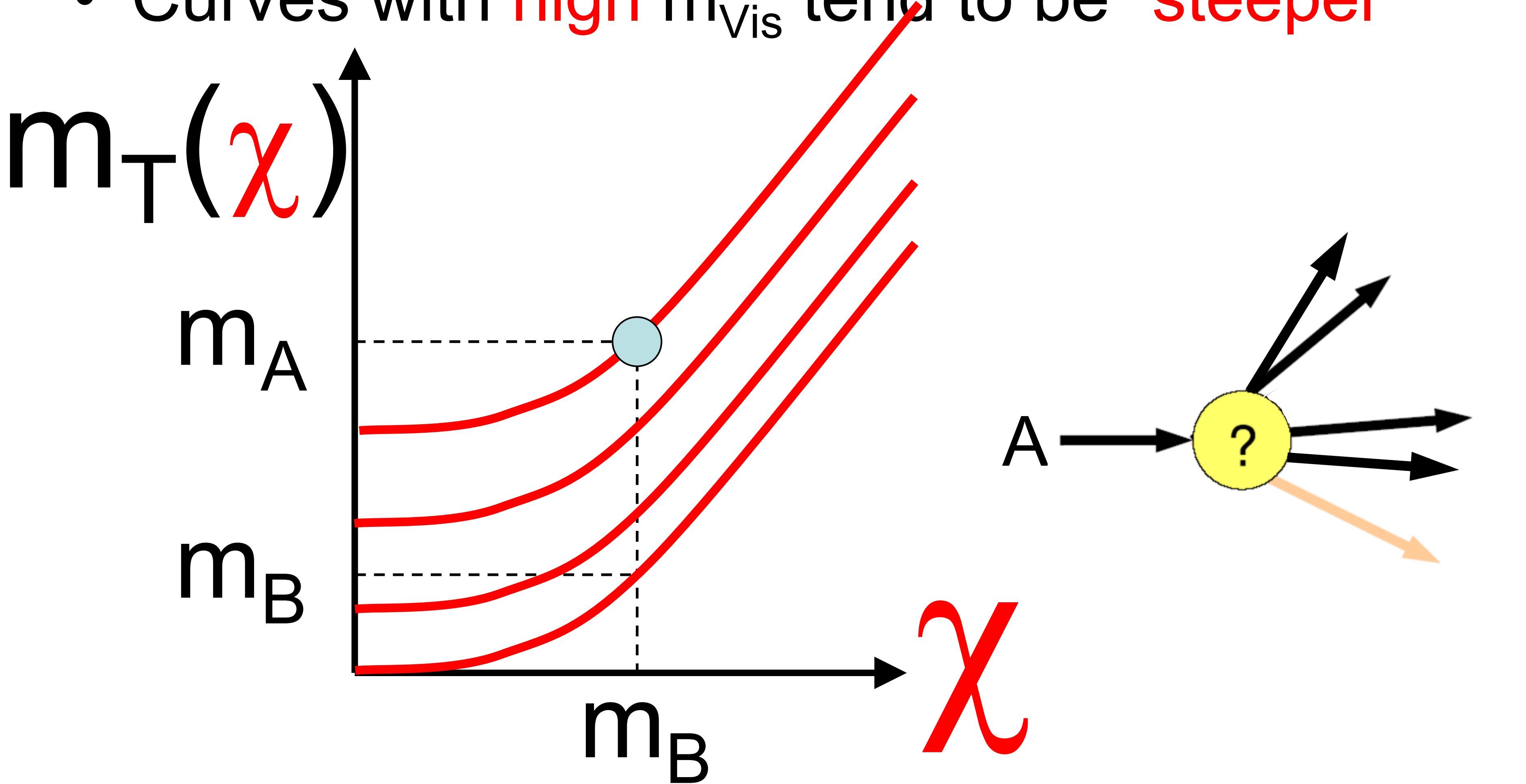
- m_{Vis} can change from event to event
- Gradient of $m_T(\chi)$ curve depends on m_{Vis}
- Curves with low m_{Vis} tend to be "flatter"

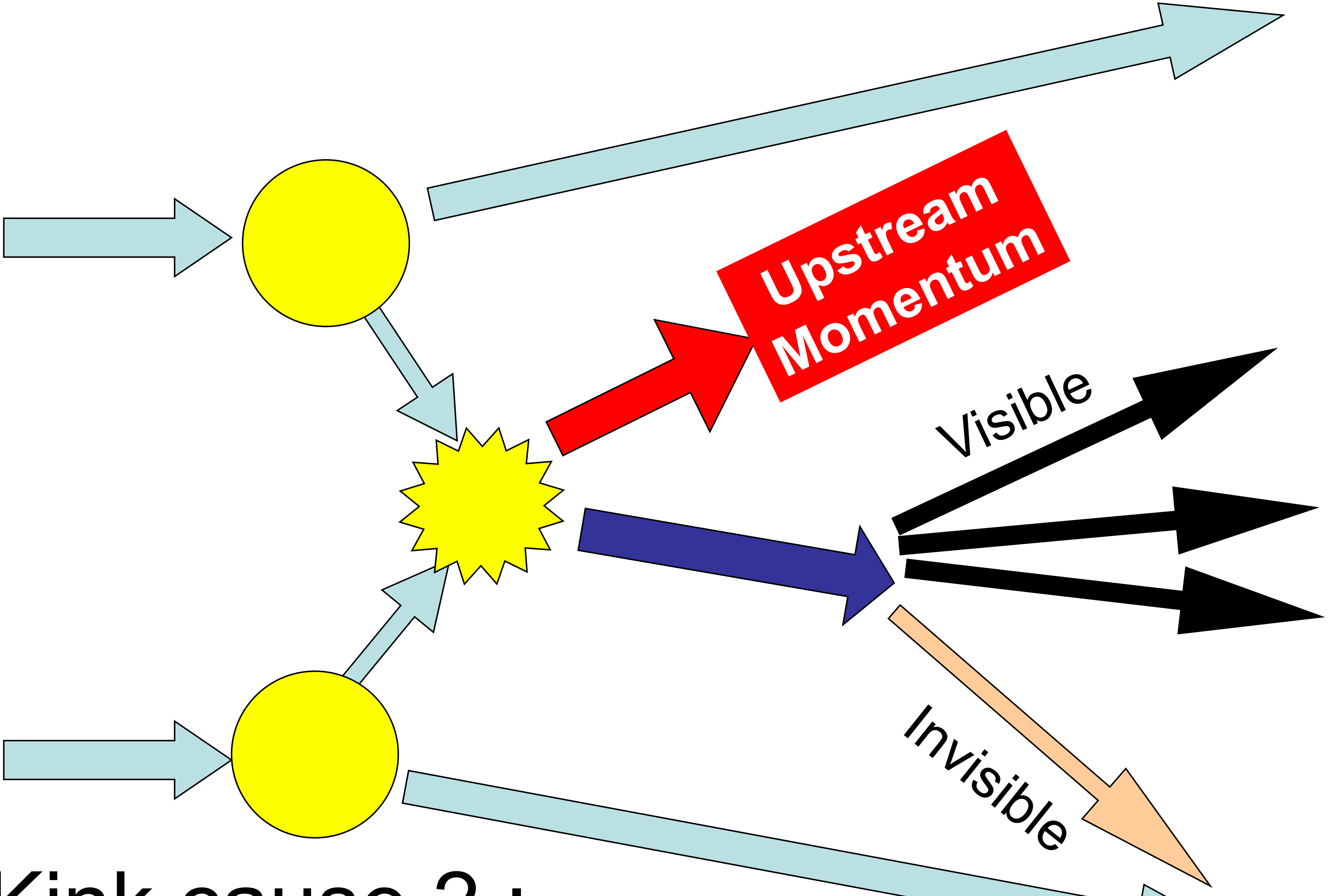




Kink cause 1: Variability in visible mass

- m_{Vis} can change from event to event
- Gradient of $m_T(\chi)$ curve depends on m_{Vis}
- Curves with high m_{Vis} tend to be "steeper"

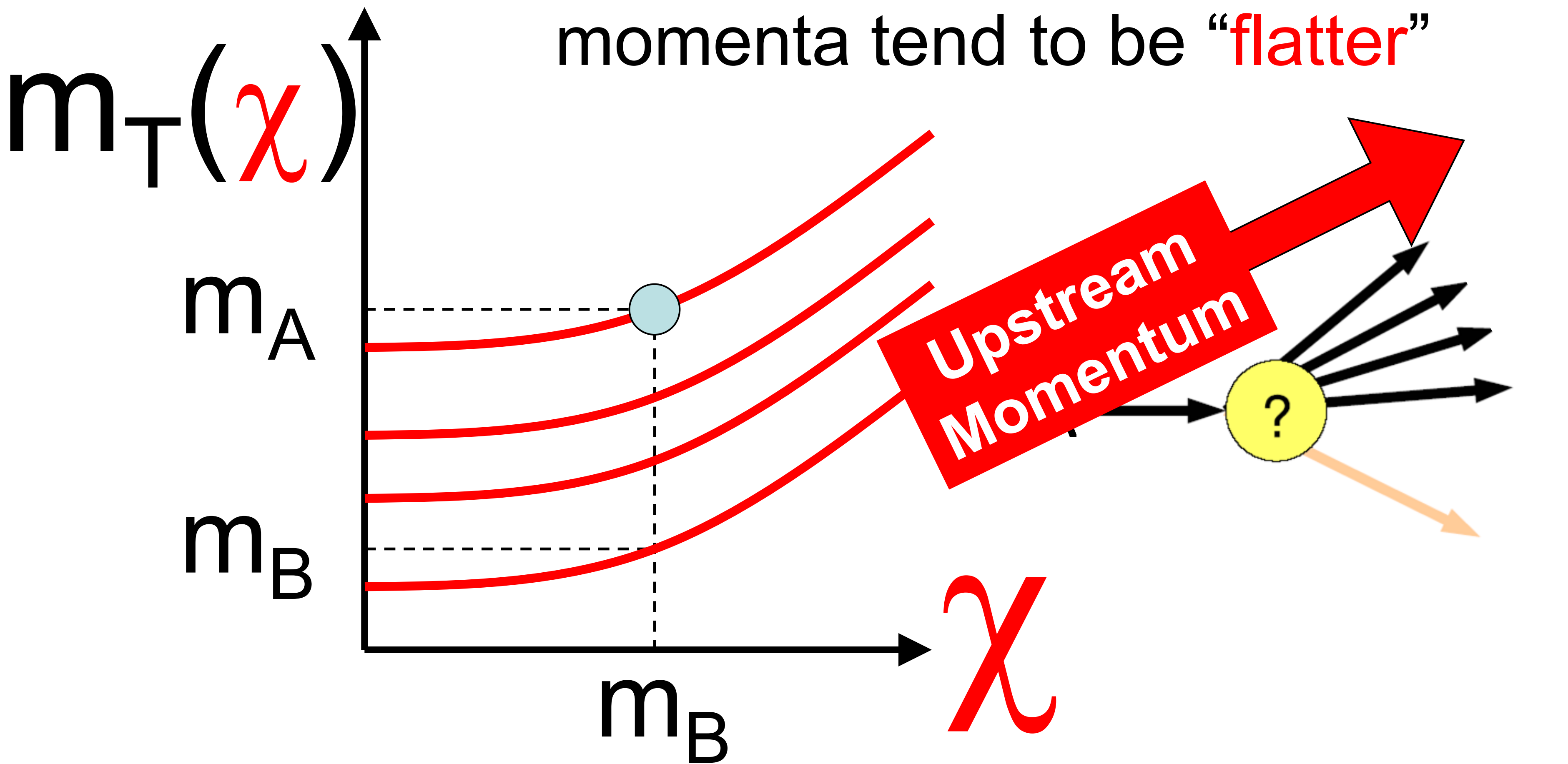




Kink cause 2 :
Recoil against Upstream Momentum

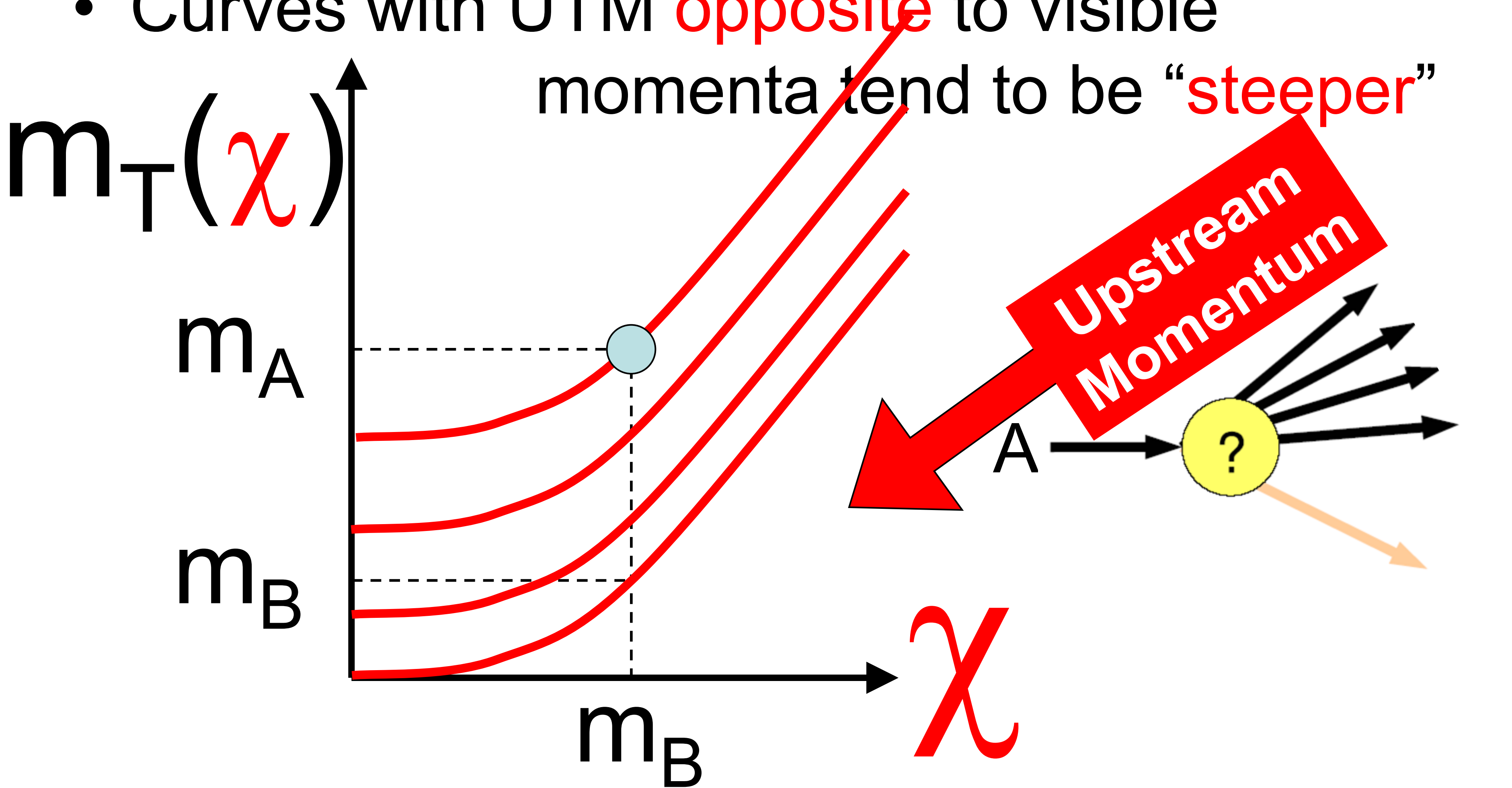
Kink cause 2: Recoil against UTM

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM parallel to visible . momenta tend to be "flatter"



Kink cause 2: Recoil against UTM

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM opposite to visible momenta tend to be "steeper"



Health warning!

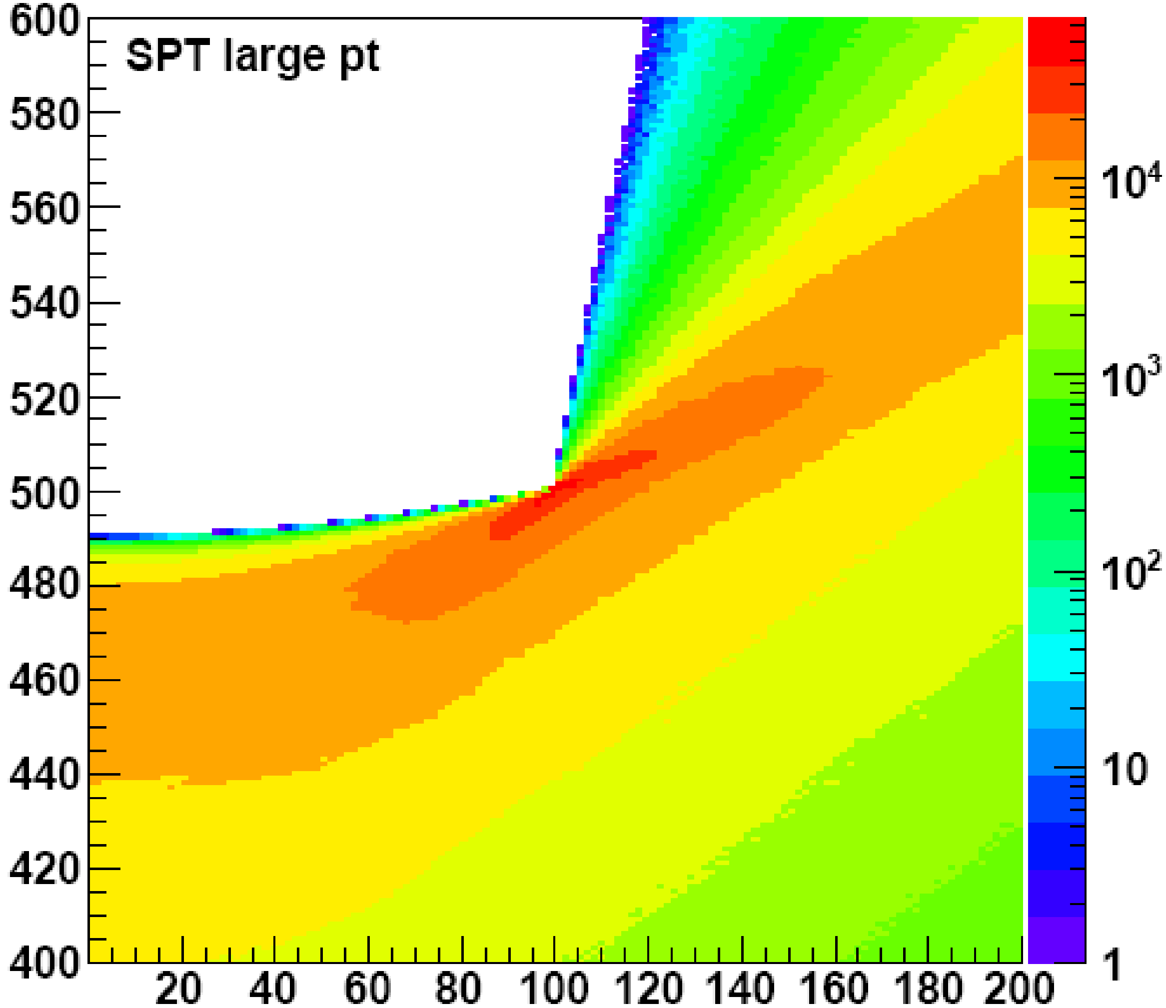
(for those of you interested in LHC dark matter constraints)

Rather worryingly, M_T kinks are at present the only known kinematic methods which (at least in principle) allow determination of the mass of the invisible particle in short chains at hadron colliders!

[We will see a dynamical method that works for single three+ body decays shortly. Likelihood methods can determine masses in pair decays too, though at cost of model dependence and CPU. See Alwall.]

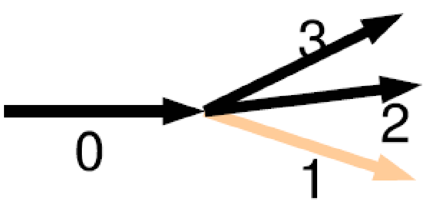
That last statement should worry you!

$m_T(\chi)$



χ

arXiv: 0711.4008

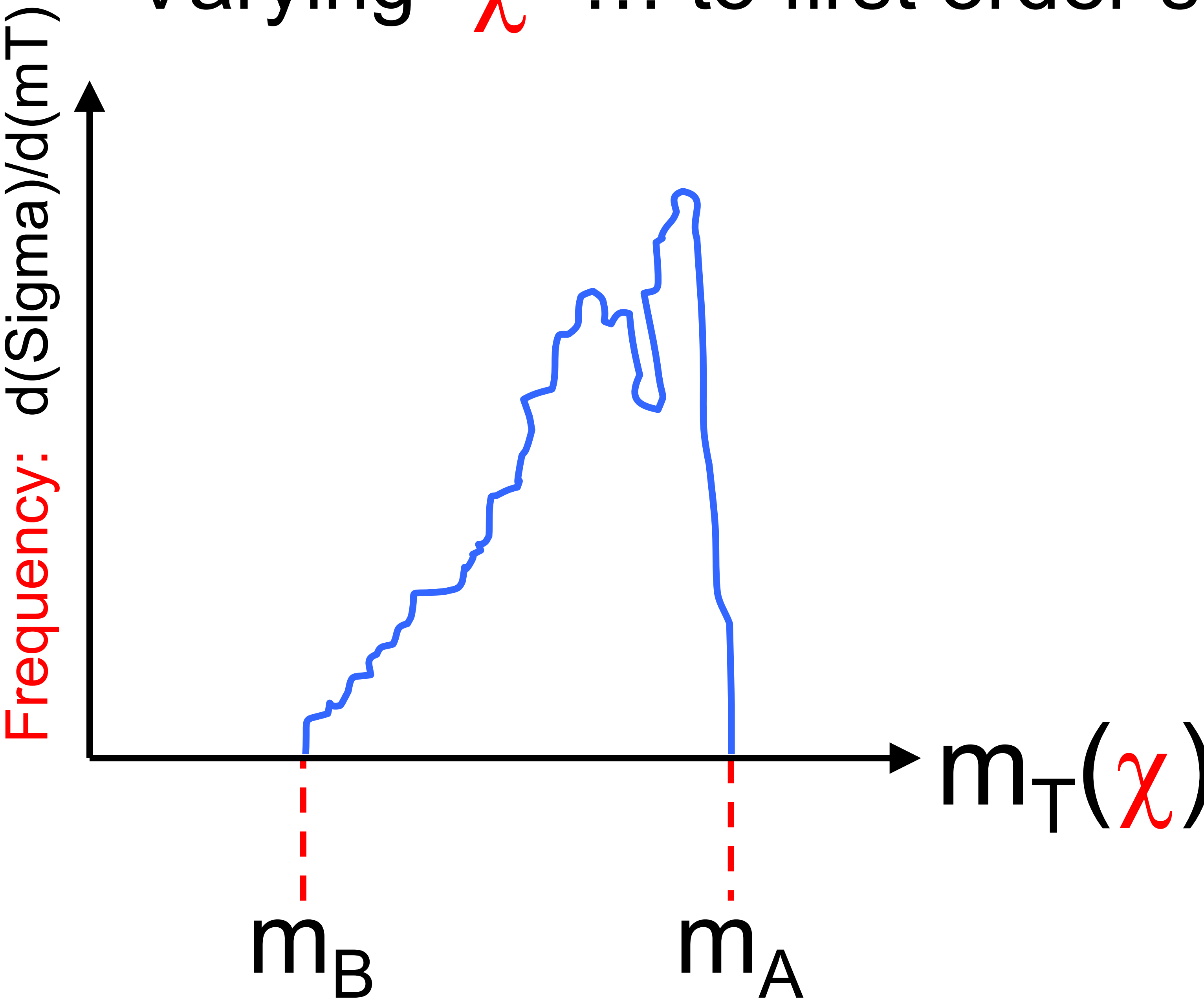


CASE 3v

Weighing Wimps with Kinks at Colliders arXiv: 0711.4008

Spot the kink

Varying "χ" … to first order see:



Take home messages for MT

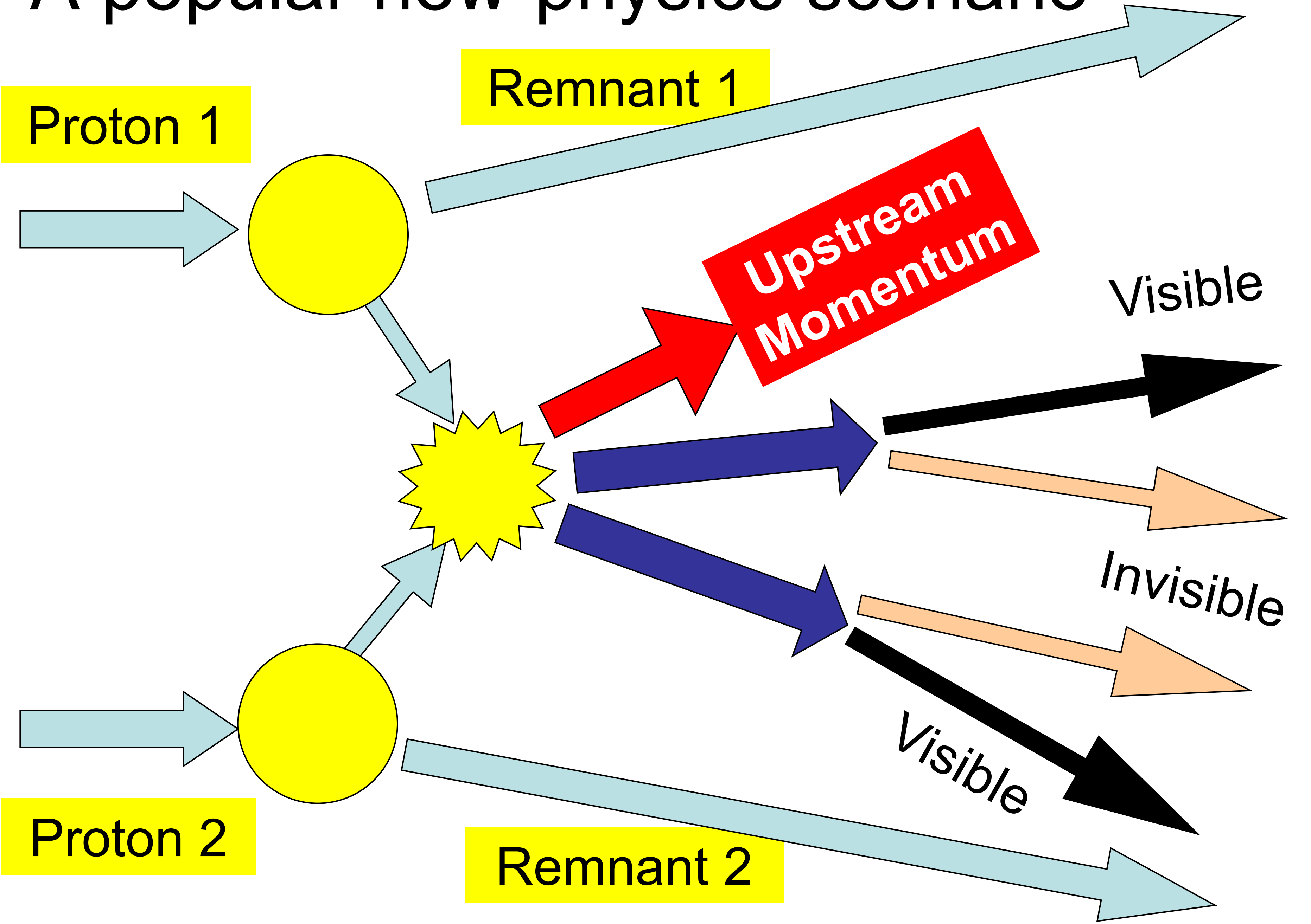
- **EASY to get MASS DIFFERENCE**
- We have two **independent kinematical** opportunities to measure **invisible daughter mass** in single particle decays:
  - "Upstream boost induced" MT kink
    - from ISR alone, useless, from real UTM, possible
  - "Variable visible mass induced" MT kink
    - impossible in 2-body decay, otherwise possible
  - HARD to set absolute mass scale

- We used pT-miss information – so only works with one invisible (so far …)

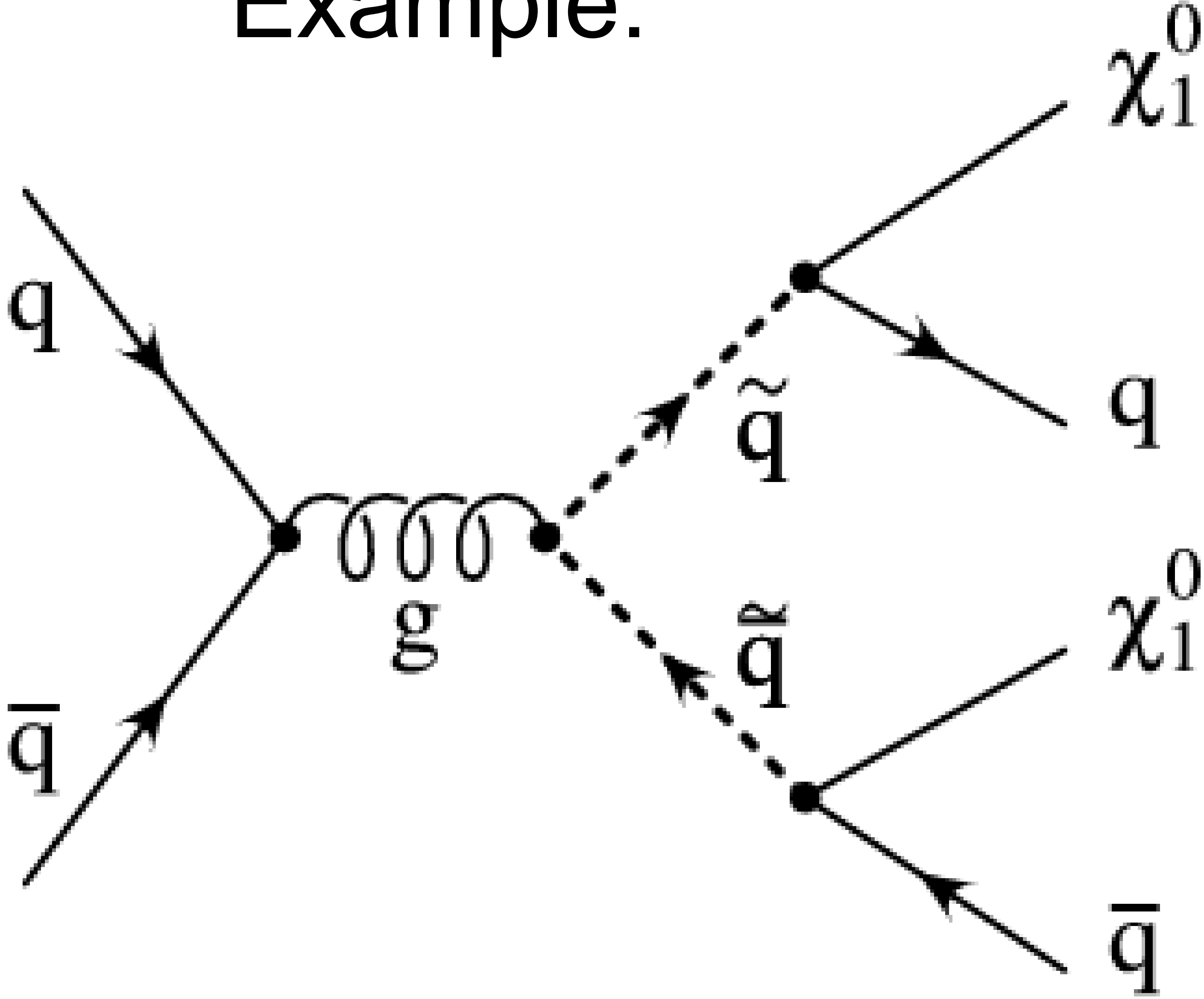
Change of topic:

How do we measure masses when there is Pair Production ?

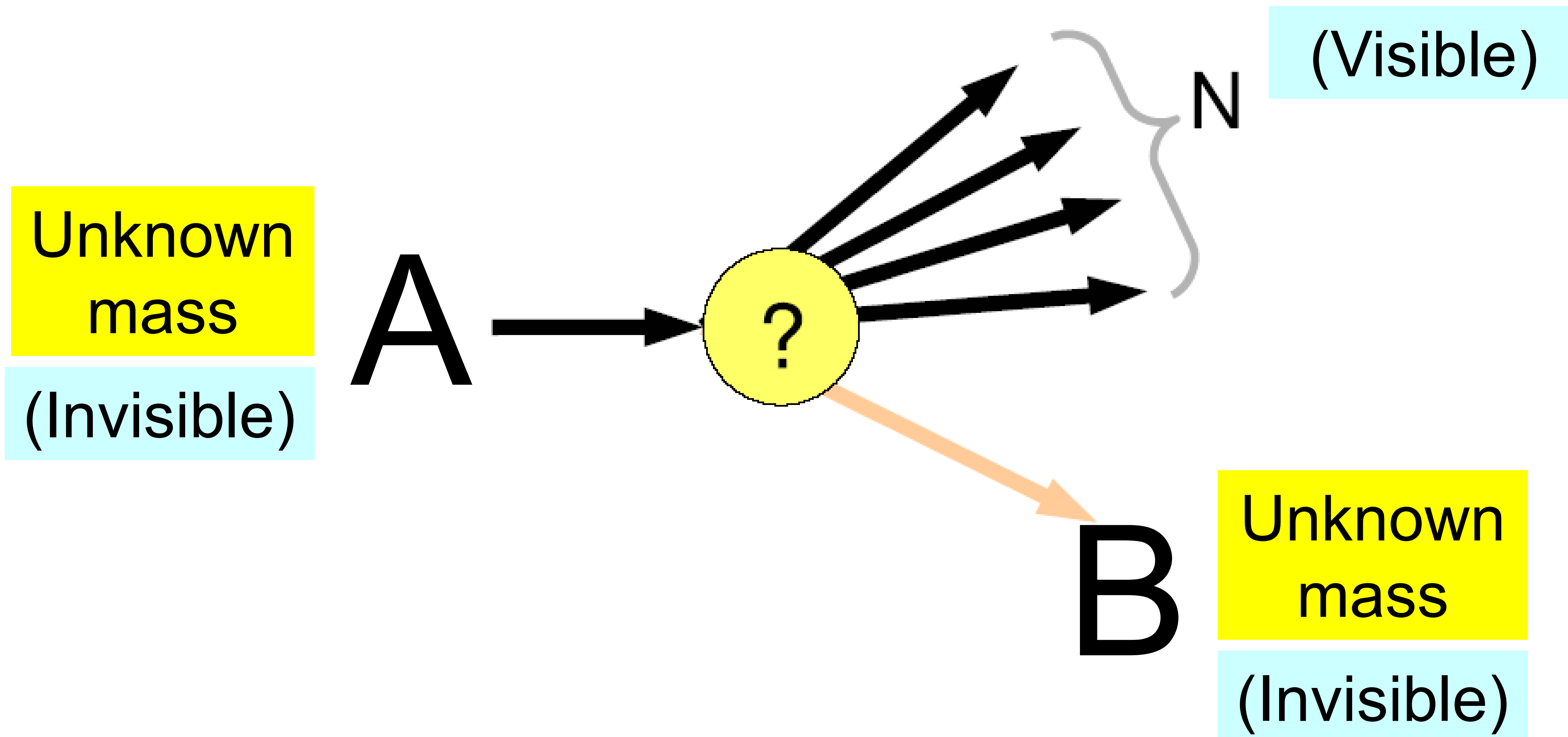
A popular new-physics scenario



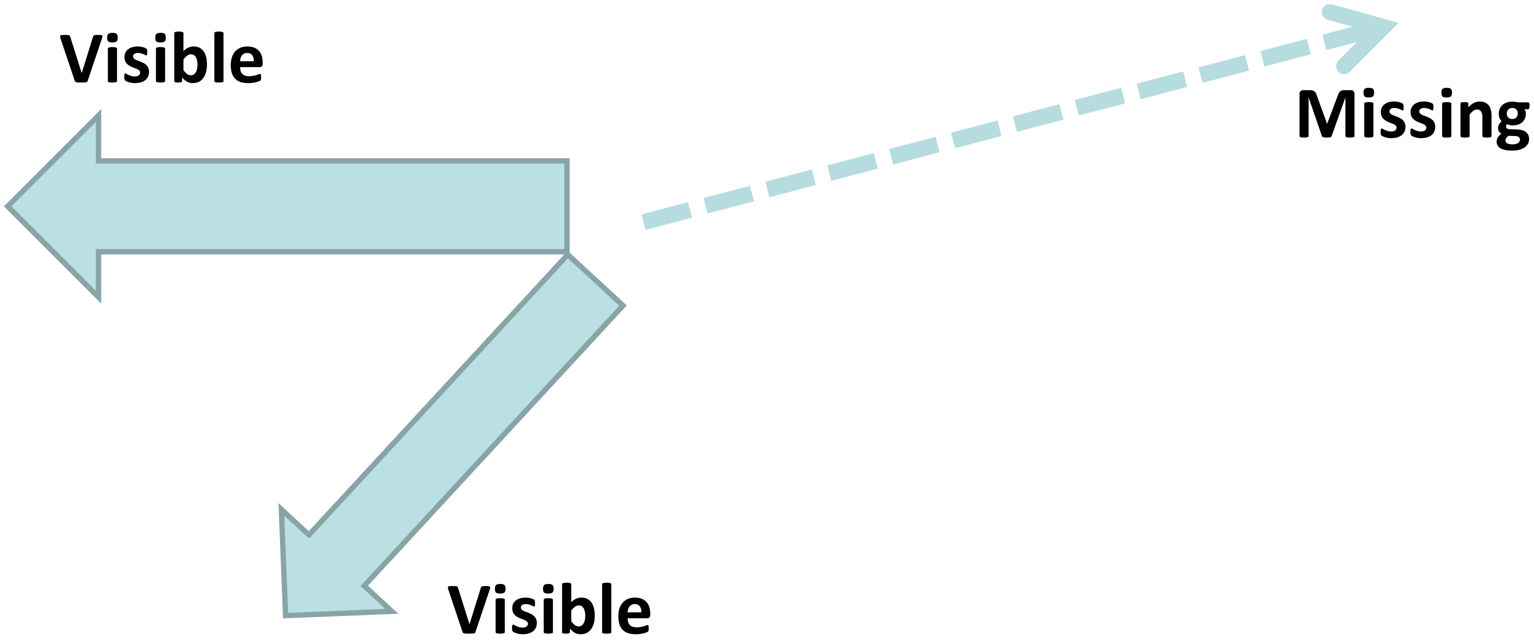
Example:

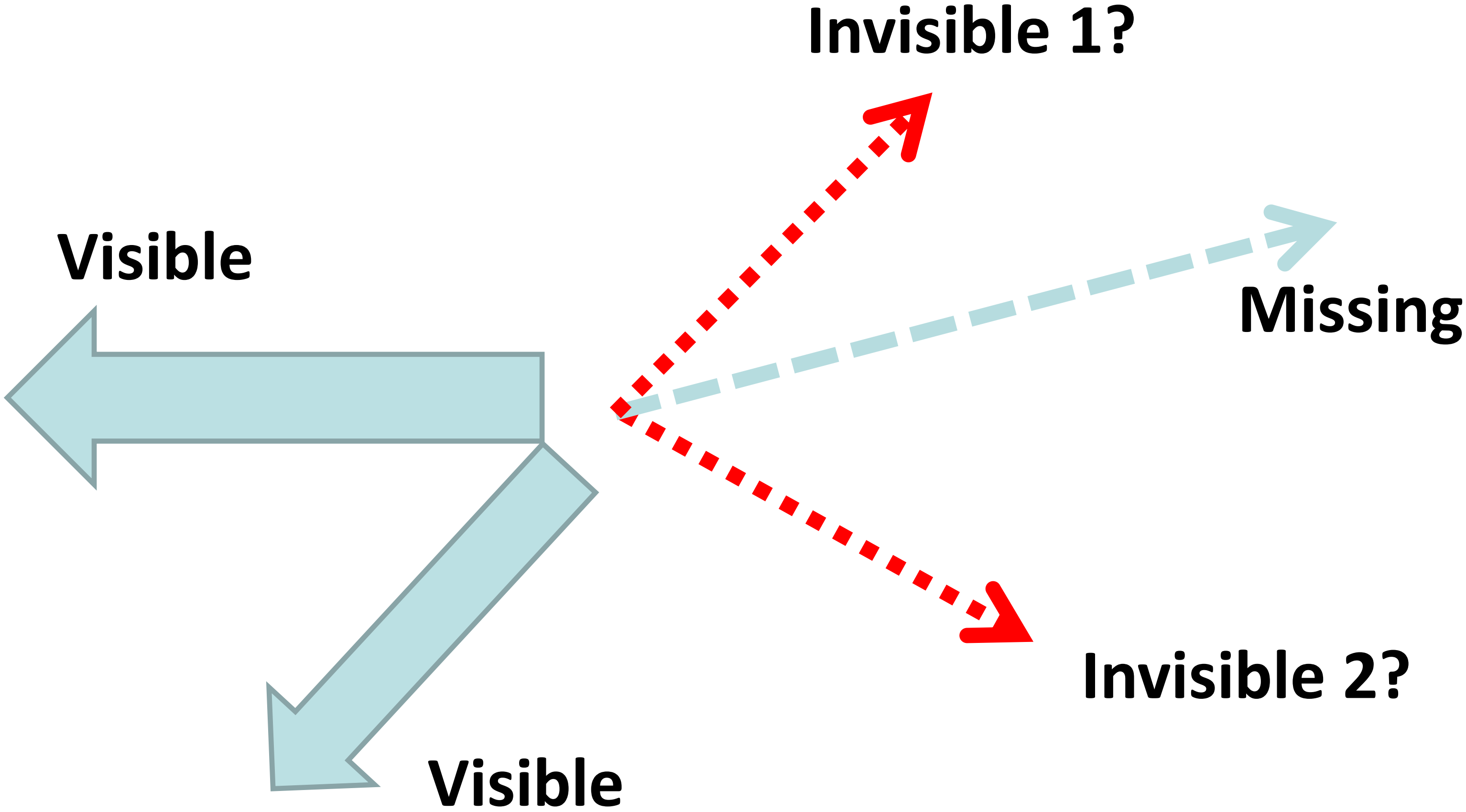


We have two copies of this:

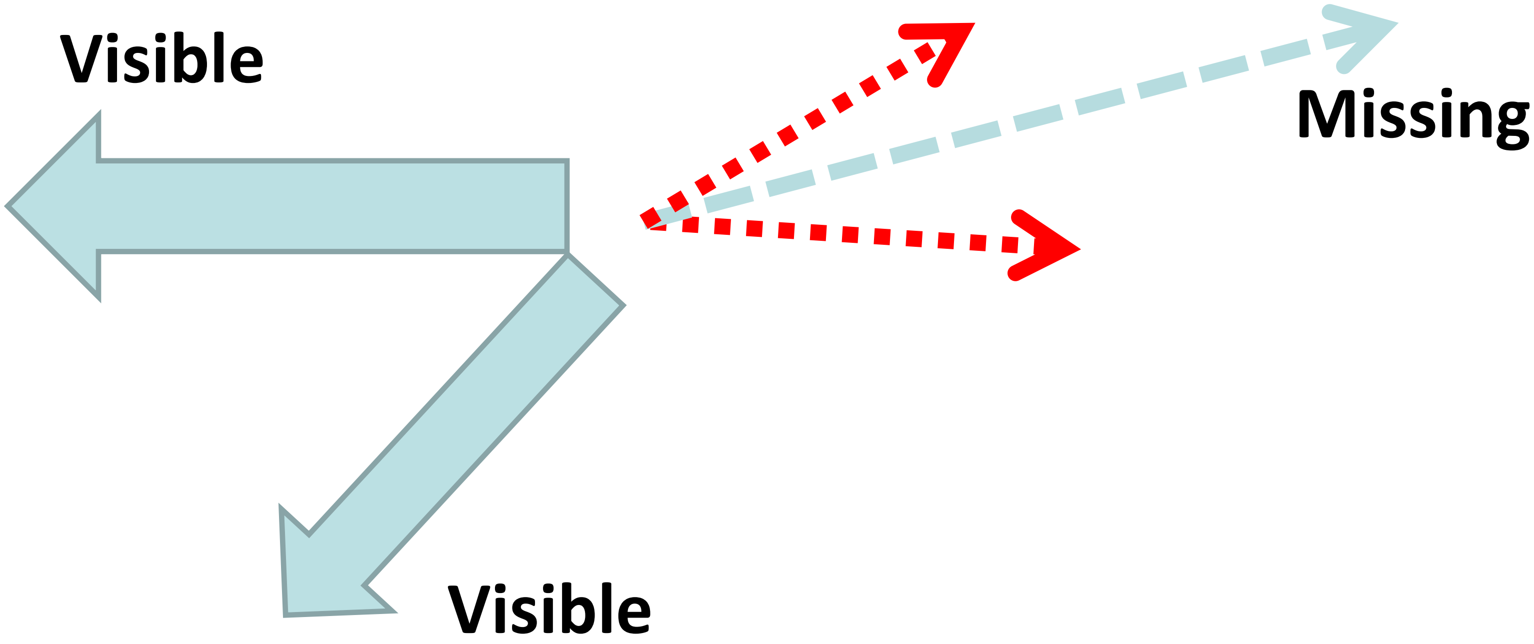


But don't know p_T of B this time! ☹

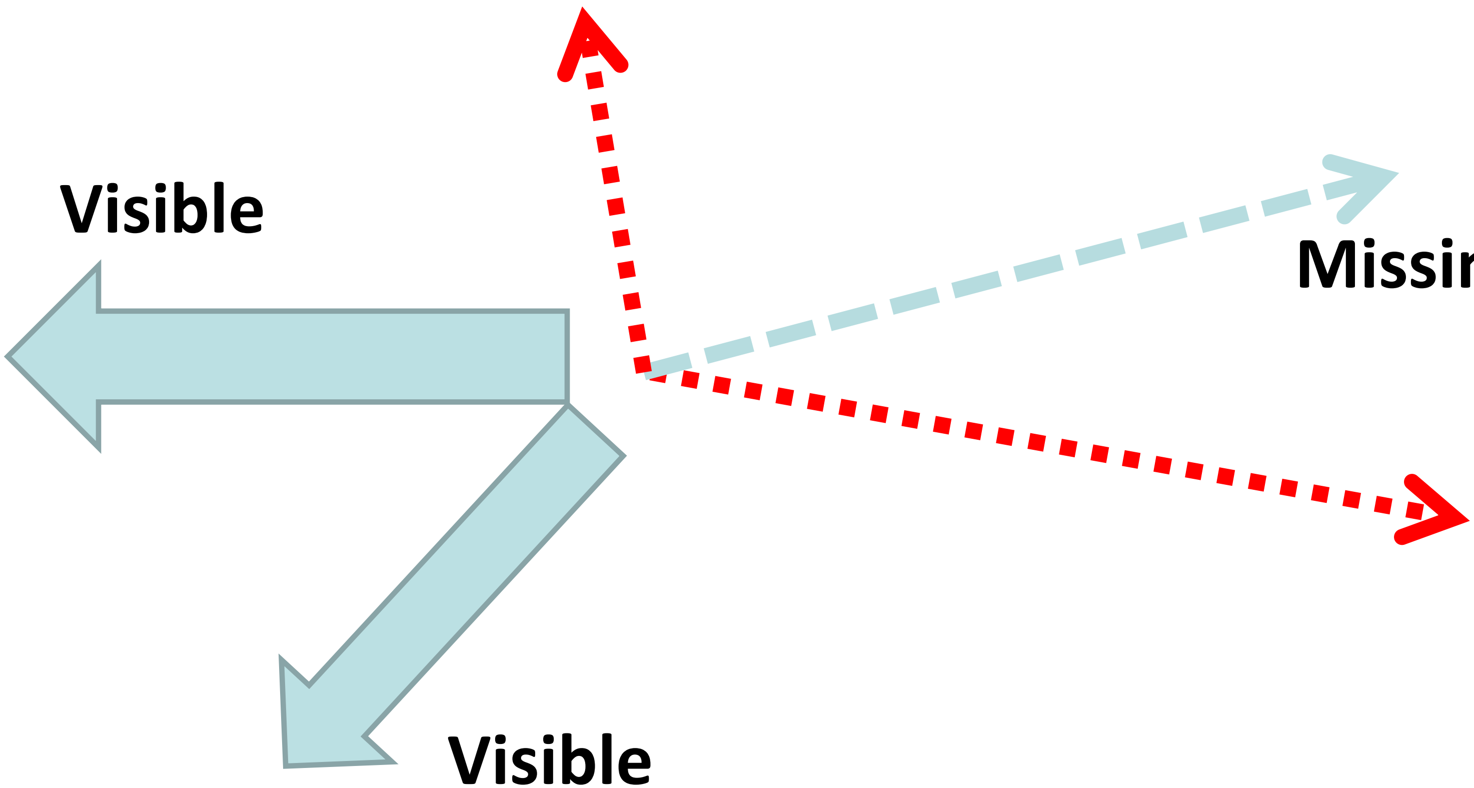

Visible
Missing
Visible



a possible “splitting”

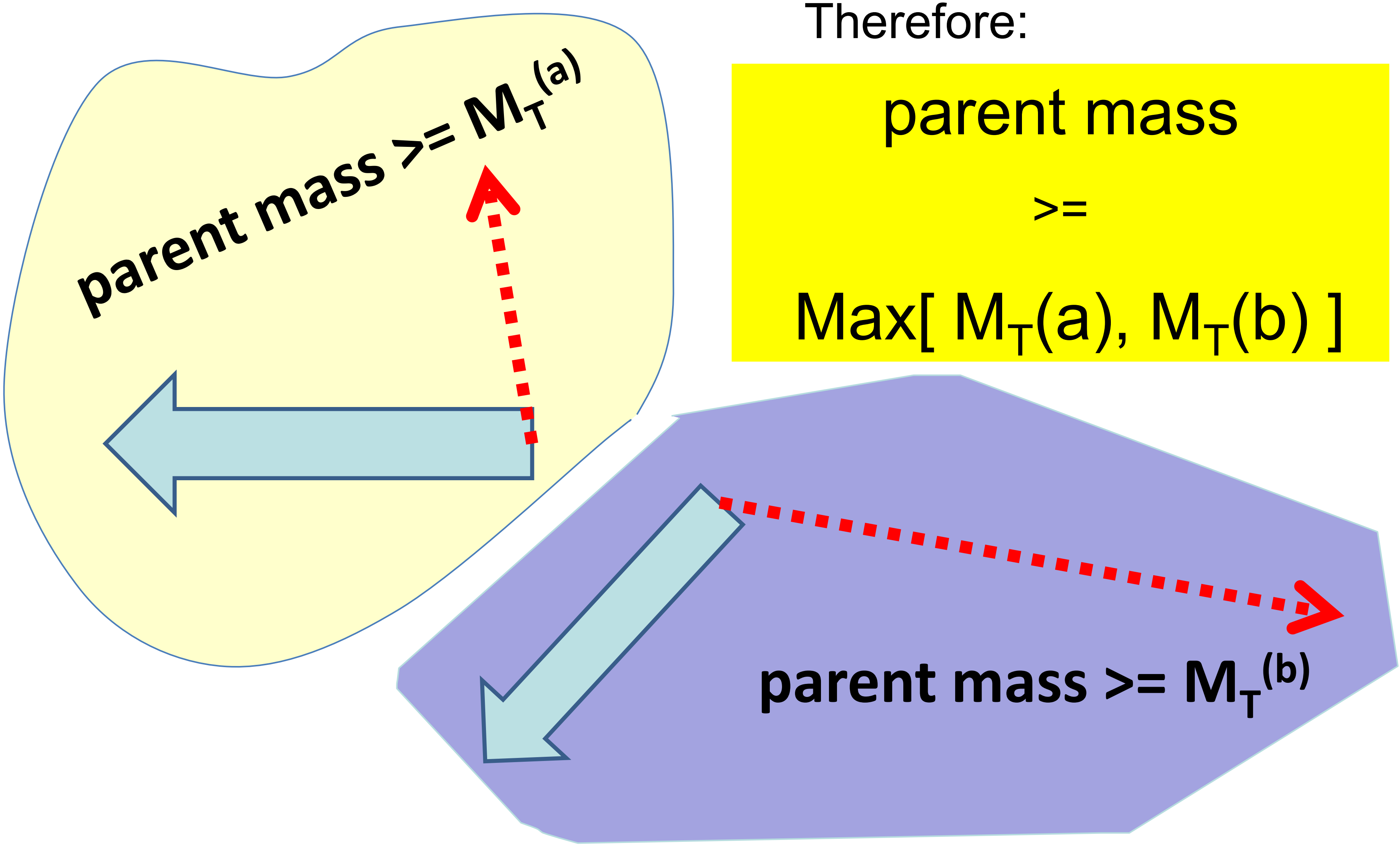


another possible “splitting”



another possible “splitting”

If **this** splitting is “correct”:



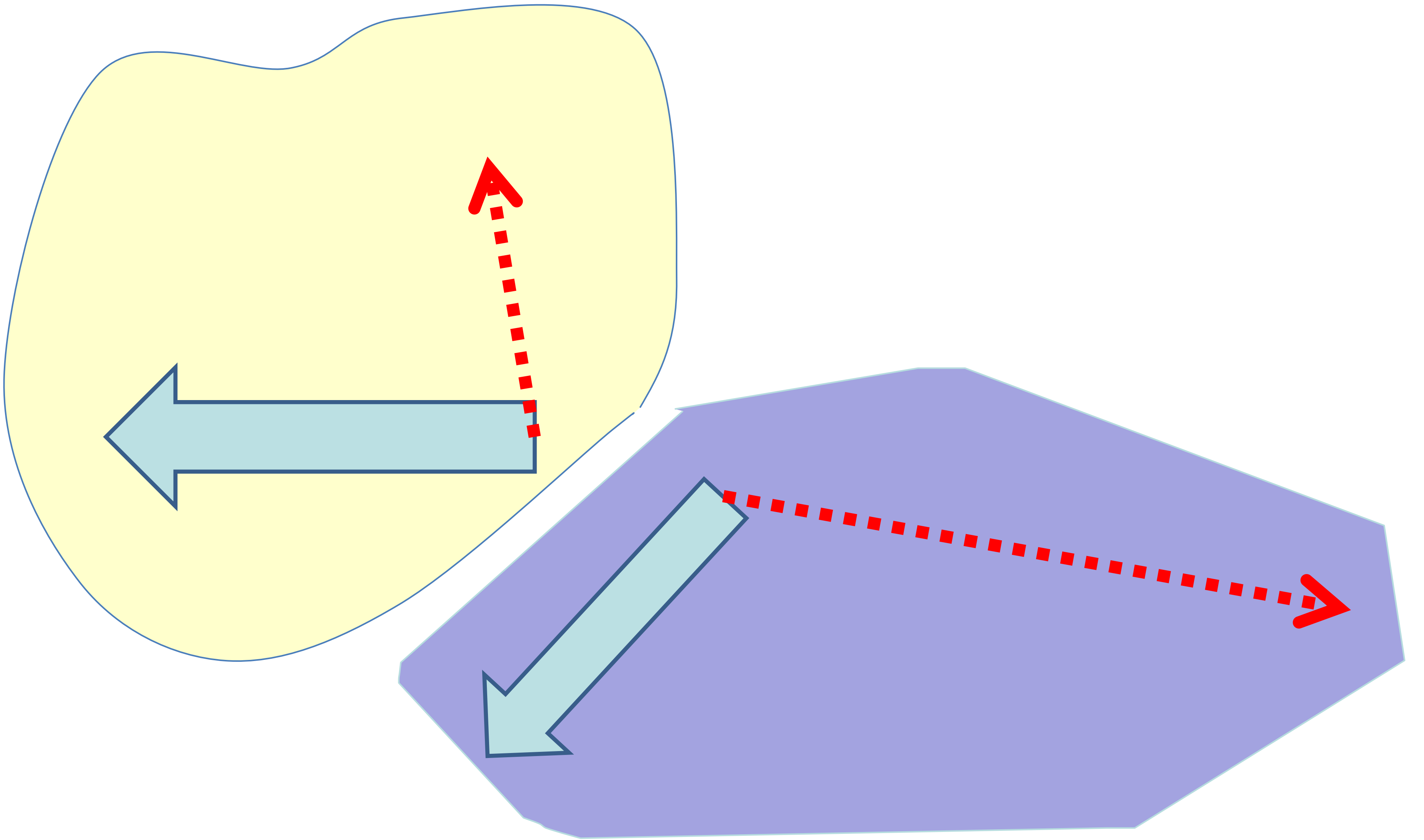
Therefore:

parent mass

>=

Max[M_T(a), M_T(b)]

But this splitting might be wrong!



But can say that:

$$\text{parent mass} \geq \underset{\substack{\text{over all splittings} \\ \text{of ptmiss}}}{\text{Min}} \left\{ \text{Max}[\, M_T(a), M_T(b) \,] \right\}$$

Lester and Summers (hep-ph/9906349)

This is m_{T2} the “Stransverse Mass”

$$m_{T2}(v_1, v_2, \not{p}_T, m_i^{(1)}, m_i^{(2)}) \equiv \min_{\sum \mathbf{q}_T = \not{\mathbf{p}}_T} \left\{ \max\left(m_T^{(1)}, m_T^{(2)} \right) \right\}$$

The most conservative partition consistent with the constraint

Take the better of the two lower bounds

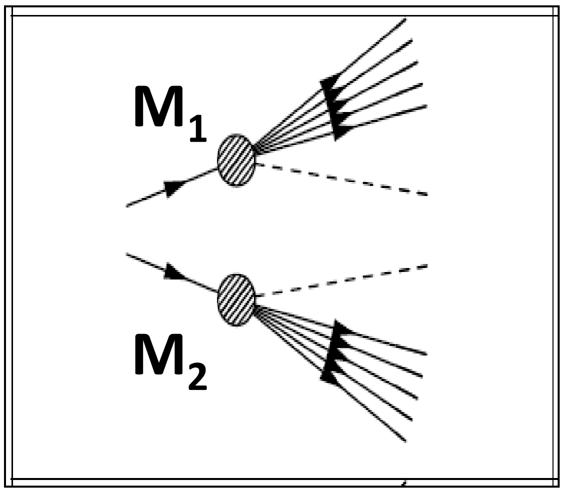
It is the <u>generalisation of transverse mass to pair production</u>.
Clear how to generalise it to any other types of production.

[Received six comments about “mis-spelling” of transverse in ATLAS editorial board!]

Note MT2 def is part of the four-step procedure:

[(1) select topology, (2) parent mass, (3) constraints, (4) find maximal lower bound]

described earlier.



Note, other approaches: MCT, Rogan, etc.

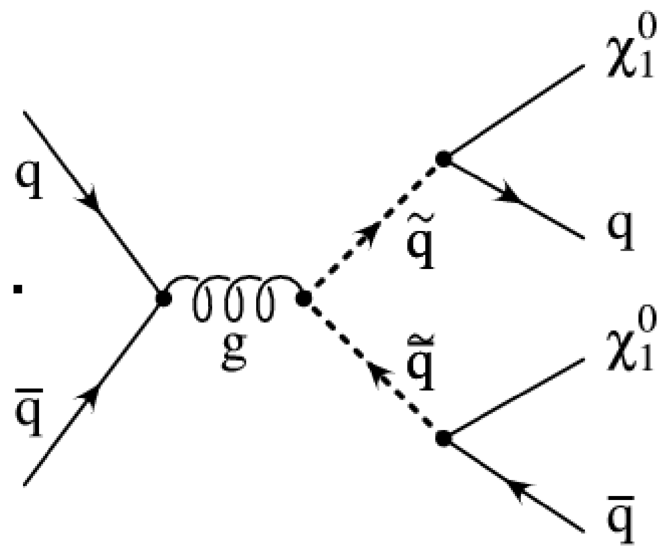
CONSTRAINTS

$$M_1 = M_2 \quad + \quad \sum_{i=1}^{N_{\mathcal{I}}} \vec{q}_{iT} = \not{\vec{p}}_T \equiv -\vec{u}_T - \sum_{i=1}^{N_{\mathcal{V}}} \vec{p}_{iT}$$

Momentum conservation in transverse plane

In other words:

- If your event is signal …



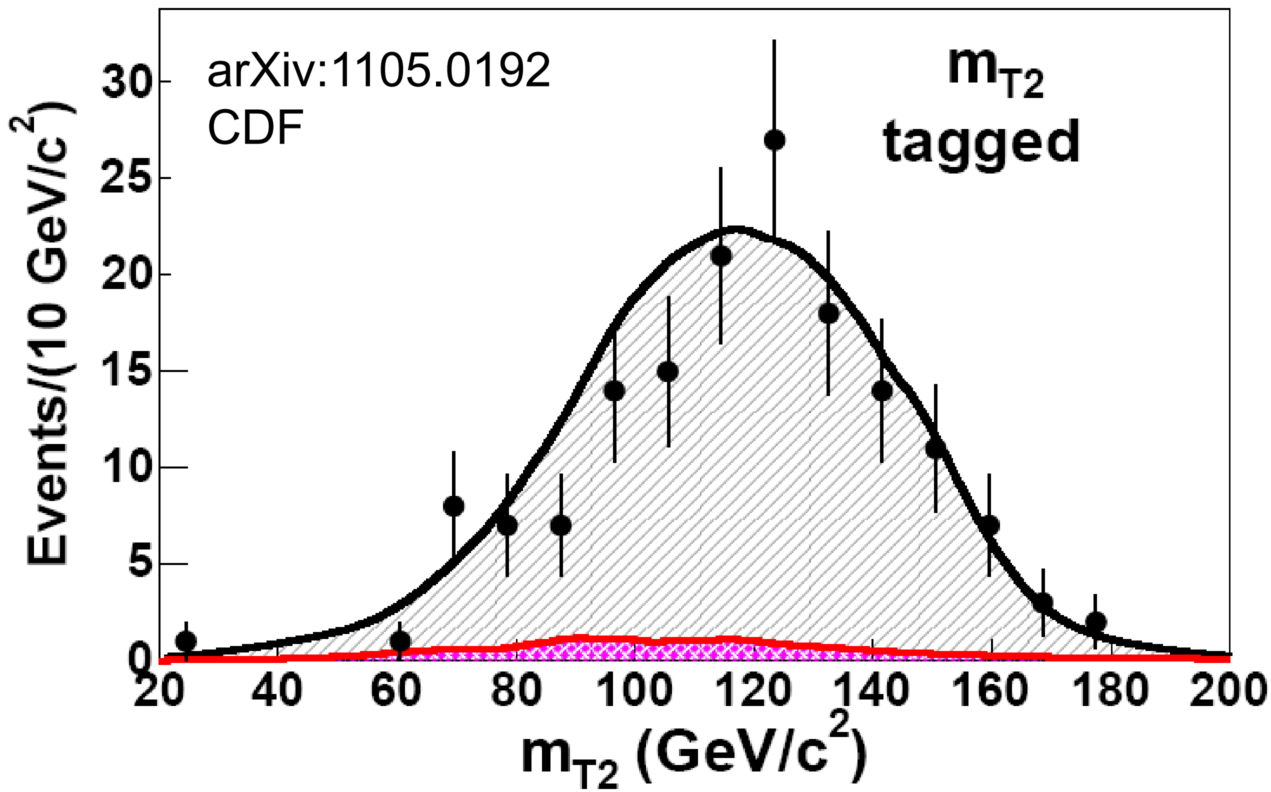
and if MT2 is “350 GeV” …

then the squark mass is >= 350 GeV.

Indeed, can show MT2 is, by construction, the best possible lower bound on the squark mass.

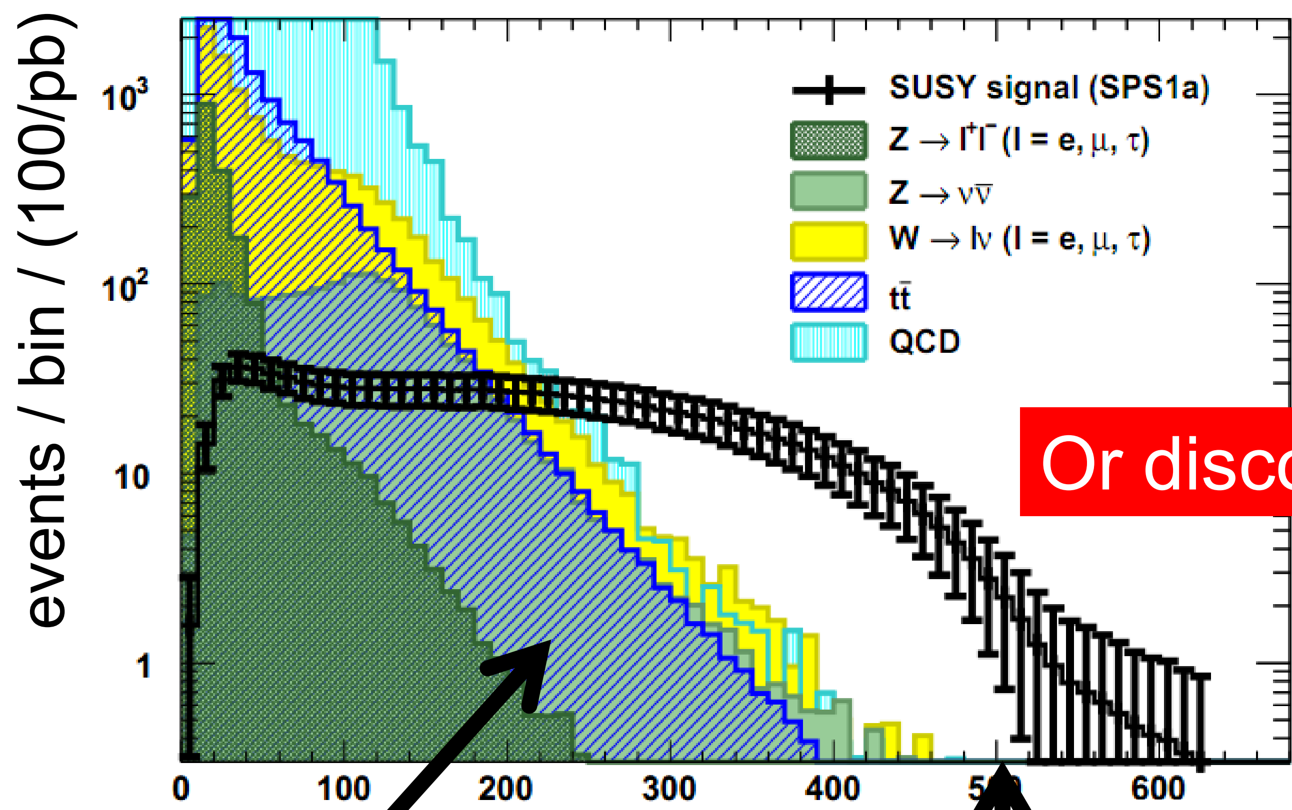
MT2 example in real data …..

- "Top Quark Mass Measurement using mT2 in the Dilepton Channel at CDF" (arXiv:0911.2956 and arXiv:1105.0192) reports that they "achieve **the single most precise measurement of m_{top} in [the dilepton] channel to date**". Also under study by ATLAS.



Top-quark physics is an important testing ground for mT2 methods, both at the LHC and at the Tevatron. If it can't work there, its not going to work elsewhere.

Example MT2 distribution …
… ?weighing? 500 GeV squarks



Or discovering?

MT2

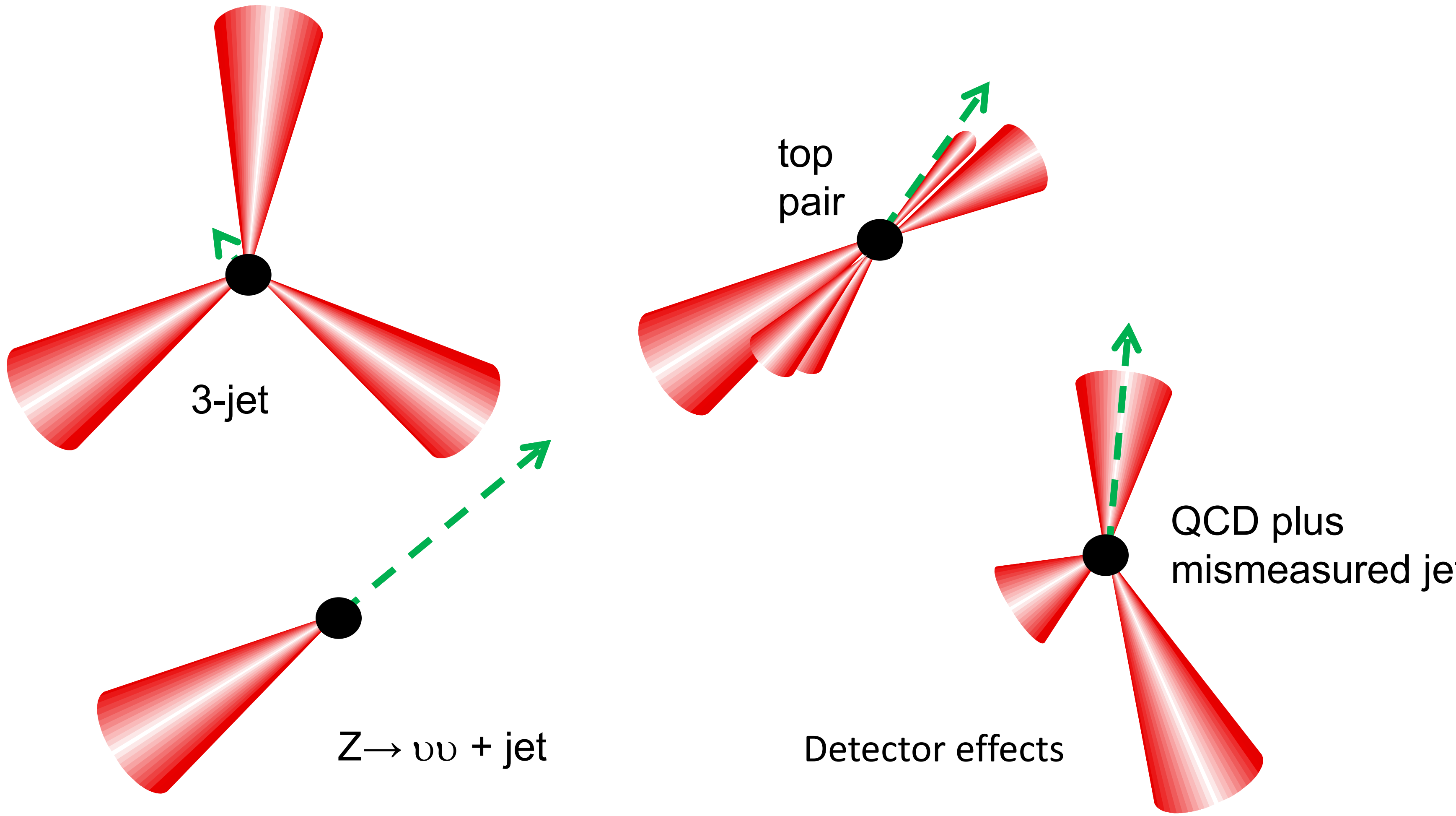
SM particles at low m_{T2}

Squark mass

arXiv:0907.2713

arXiv:0907.2713

… works because MT2 for all BGs is provably low … due to small QCD mass scale



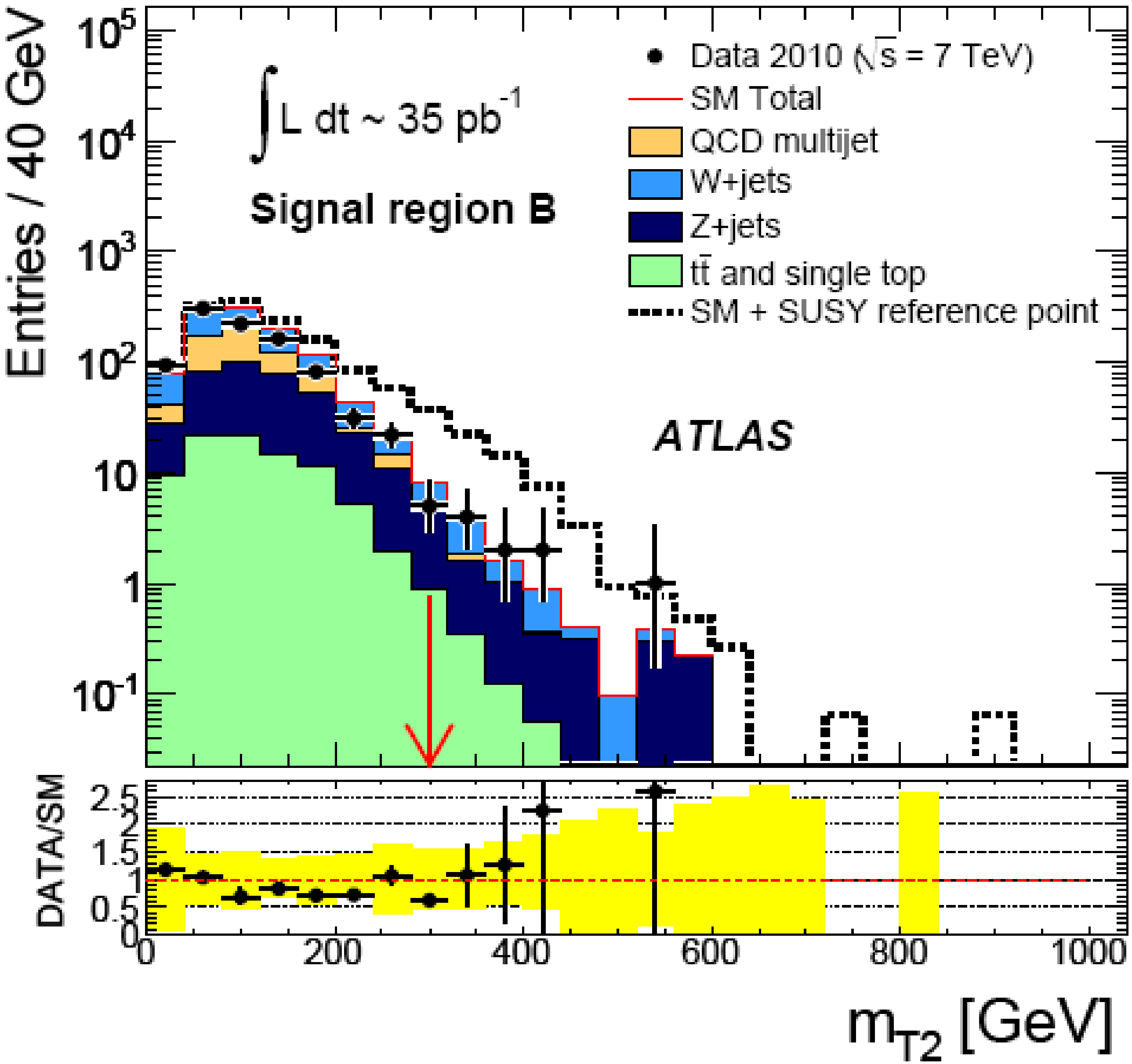
All these have m_{T2} either < m_{top} or → $m_{<}$

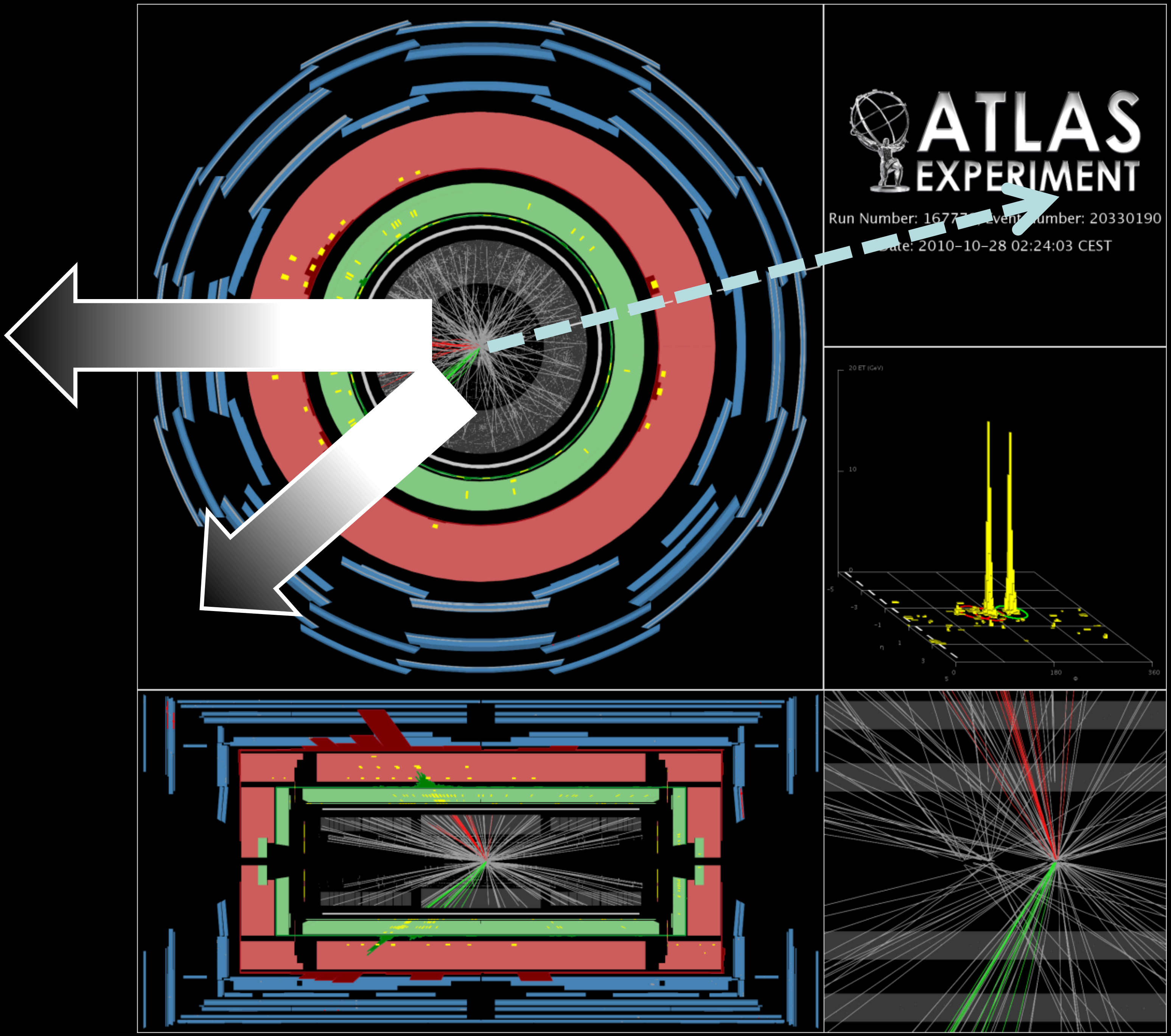
arXiv:0907.2713

Process	$m_{T2}(v_1, v_2, \not{p}_T, 0, 0)$	Comments
QCD di-jet $\to$ hadrons	$= \max m_j$ by Lemmas 1,4	
QCD multi jets $\to$ hadrons	$= \max m_j$ by Lemma 4	
$t\bar{t}$ production	$= \max m_j$ by Lemma 4	fully hadronic decays
	$\leq m_t$ by Lemmas 1,7	any leptonic decays
Single top / tW	$= \max m_j$ by Lemma 4	fully hadronic decays
	$\leq m_t$ by Lemmas 2,7	any leptonic decays
Multi jets: "fake" $\not{p}_T$	$= \max m_j$ by Lemma 5	single mismeasured jet[a]
	$= \max m_j$ by Lemma [illegible]	two mismeasured jets[a]
Multi jets: "real" $\not{p}_T$	$= \max m_j$ by Lemma 5	single jet with leptonic b decay[a]
	$= \max m_j$ by Lemma 6	two jets with leptonic b decays[a]
$Z \to \nu\bar{\nu}$	$= 0$ by Lemma 3	
$Z\,j \to \nu\bar{\nu}\,j$	$= m_j$ by Lemma 3	one ISR jet[a]
$W \to \ell\nu$ [b]	$= m_\ell$ by Lemma 3	
$W\,j \to \ell\nu\,j$ [b]	$\leq m_W$ by Lemma 2	one ISR jet[a]
$WW \to \ell\nu\ell\nu$ [b]	$\leq m_W$ by Lemma 1	
$ZZ \to \nu\bar{\nu}\nu\bar{\nu}$	$= 0$ by Lemma 3	also $= m_j$ for one ISR jet[a]
$LQ\,\overline{LQ} \to q\nu\bar{q}\bar{\nu}$	$\leq m_{LQ}$	} i.e. can take large values
$\tilde{q}\,\bar{\tilde{q}} \to q\tilde{\chi}_1^0\,\bar{q}\tilde{\chi}_1^0$	$\leq m_{\tilde{q}}$	
$q_1, \bar{q}_1 \to q\gamma_1, \bar{q}\gamma_1$	$\leq m_{q1}$	

So good for low multiplicity pair production signal discovery – disleptons?

Putting it to work for discovery




ATLAS
EXPERIMENT
Run Number: 1677
umber: 20330190
ate: 2010-10-28 02:24:03 CEST

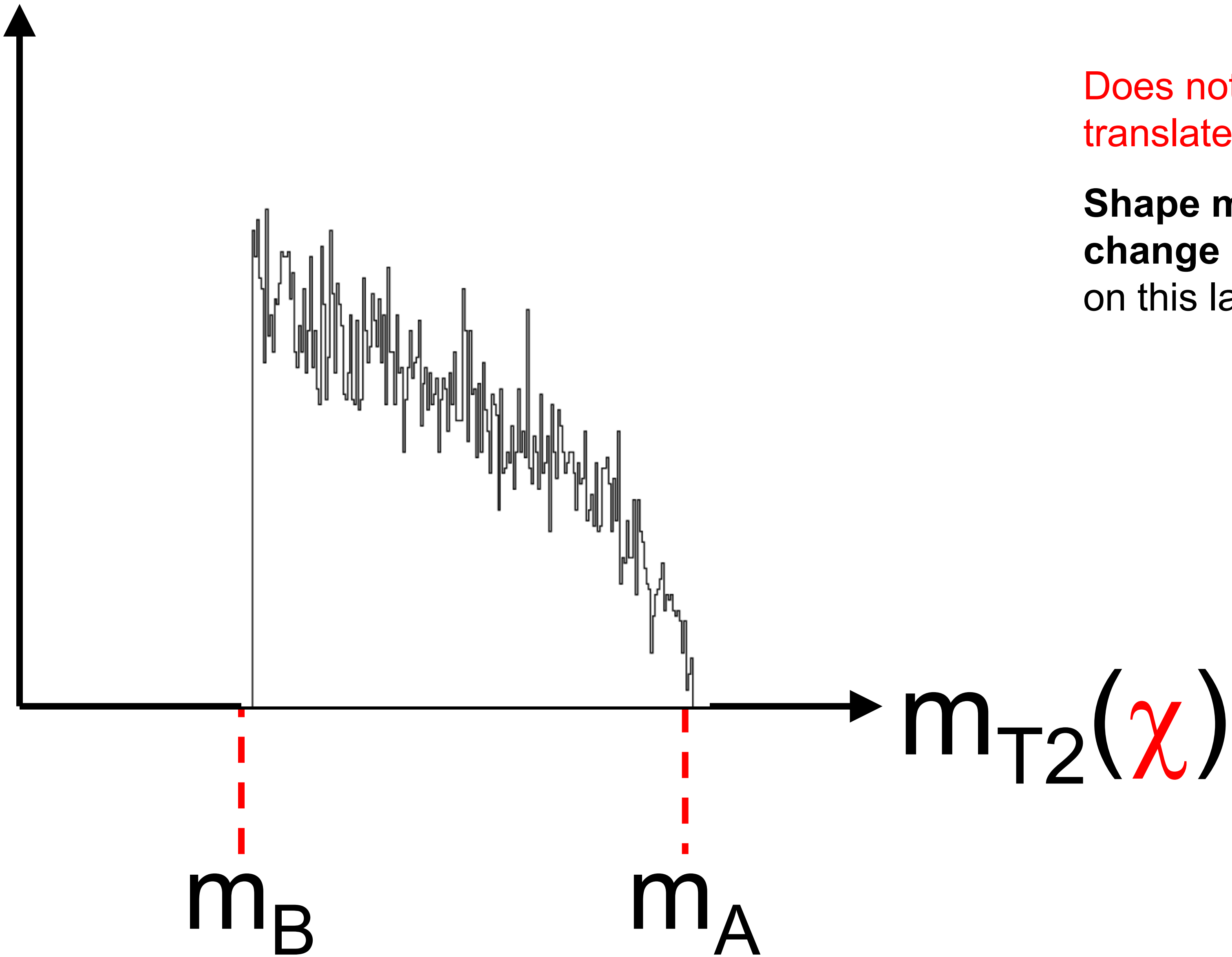
Health warning!

But note: high multiplicity environment already proving to be a challenge for mT2 (post 35/pb) and di-squark search in most recent data is being conducted with Meff. Problem is diagnosing the di-jet system.

Have dodged question of mass of invisible daughters.

What if we don’t know their masses?

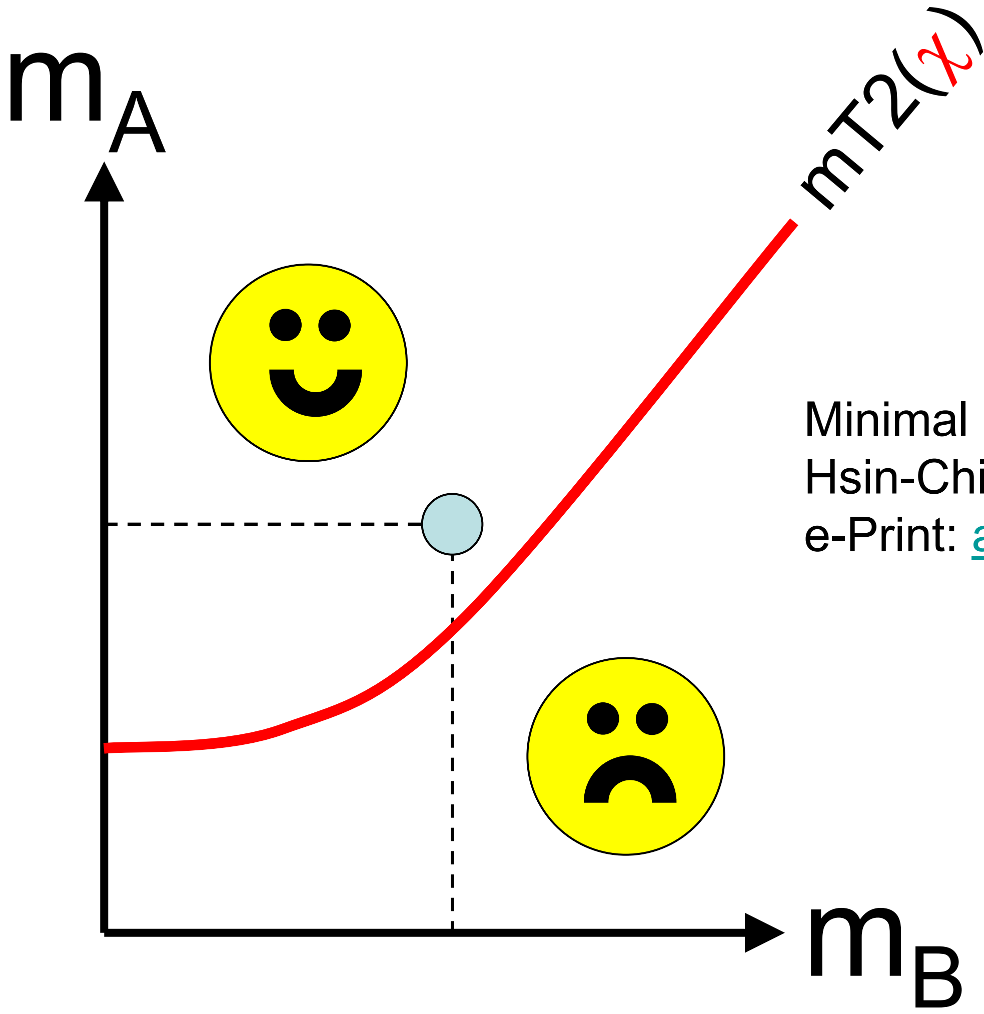
Varying "χ" … to first order



Does not just translate …

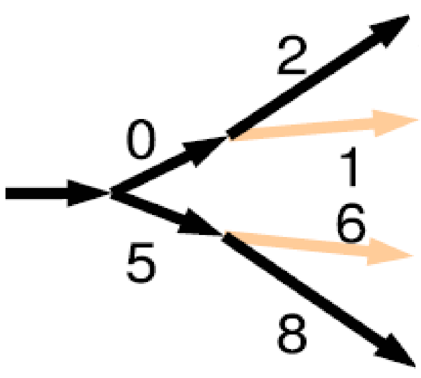
Shape may also change … more on this later.

MT2 inherits mass-space boundary from MT



The MT2(chi) curve is the **boundary** of the region of (mother, daughter) mass-space consistent with the observed event!

Minimal Kinematic Constraints and m(T2), Hsin-Chia Cheng and Zhenyu Han (UCD)
e-Print: arXiv:0810.5178 [hep-ph]



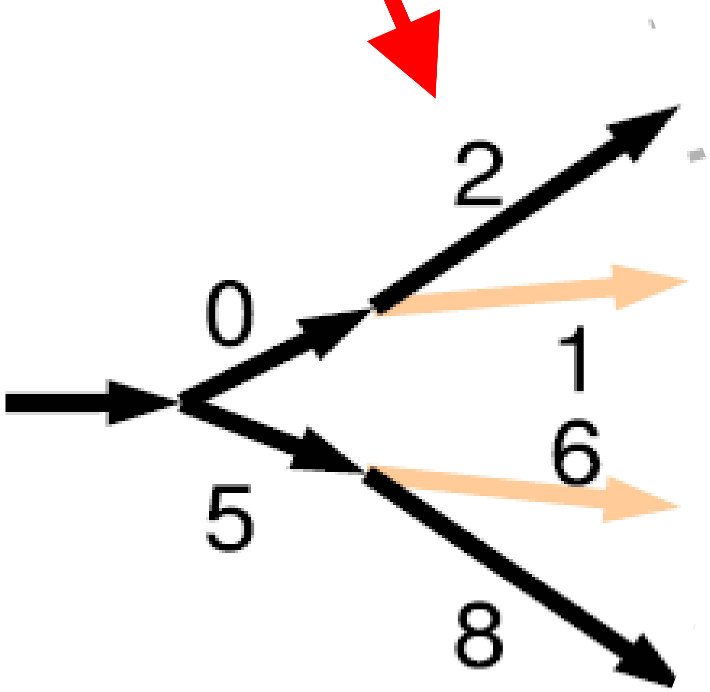
MT2 is defined in terms of MT

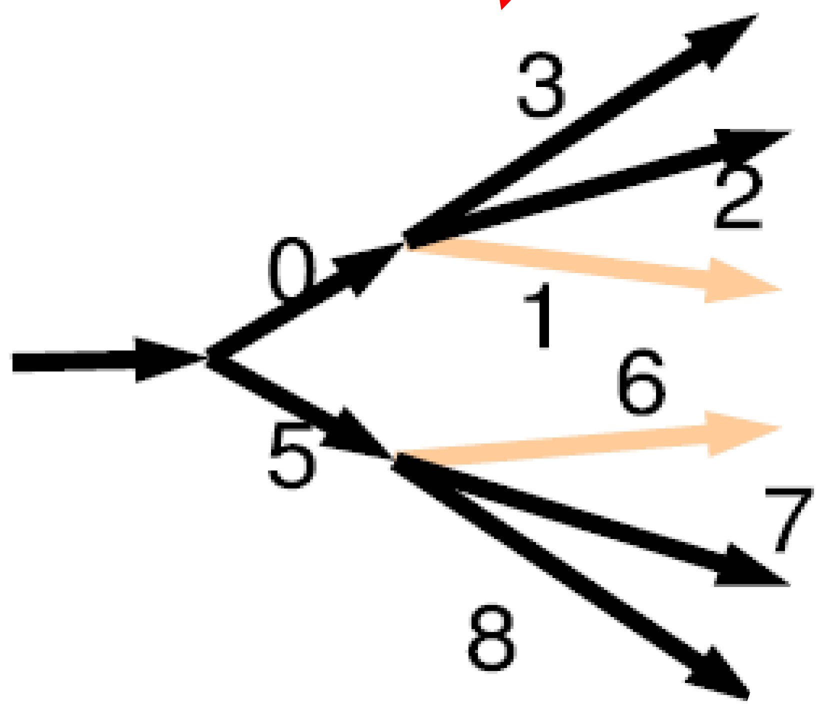
- Consequently, MT2 inherits the "kink structure" of MT and can (in principle) be used to:
  - EASILY measure the parent-daughter mass difference,
  - might PERHAPS measure the absolute mass scale using utm boosts kinks or variable visible mass kinks (HARD)

Are MT2 kinks observable ?

Expect KINK only from UTM Recoil (perhaps only from ISR!)

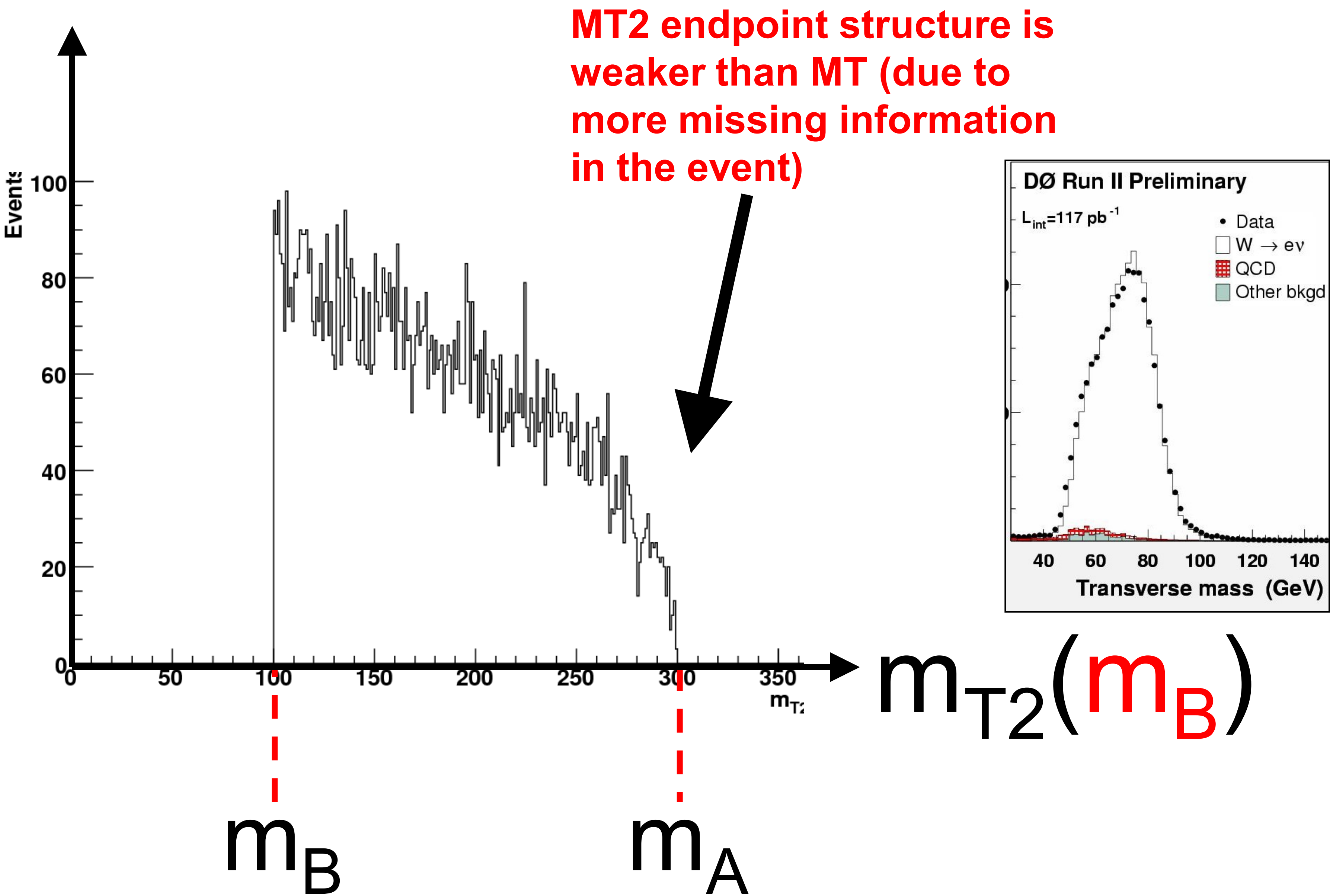
Expect stronger KINK due to both UTM recoil, AND variability in the visible masses.





arXiv: 0711.4008

Perhaps: MT2's endpoint structure is weaker than MT's.



Caveat Mensor!

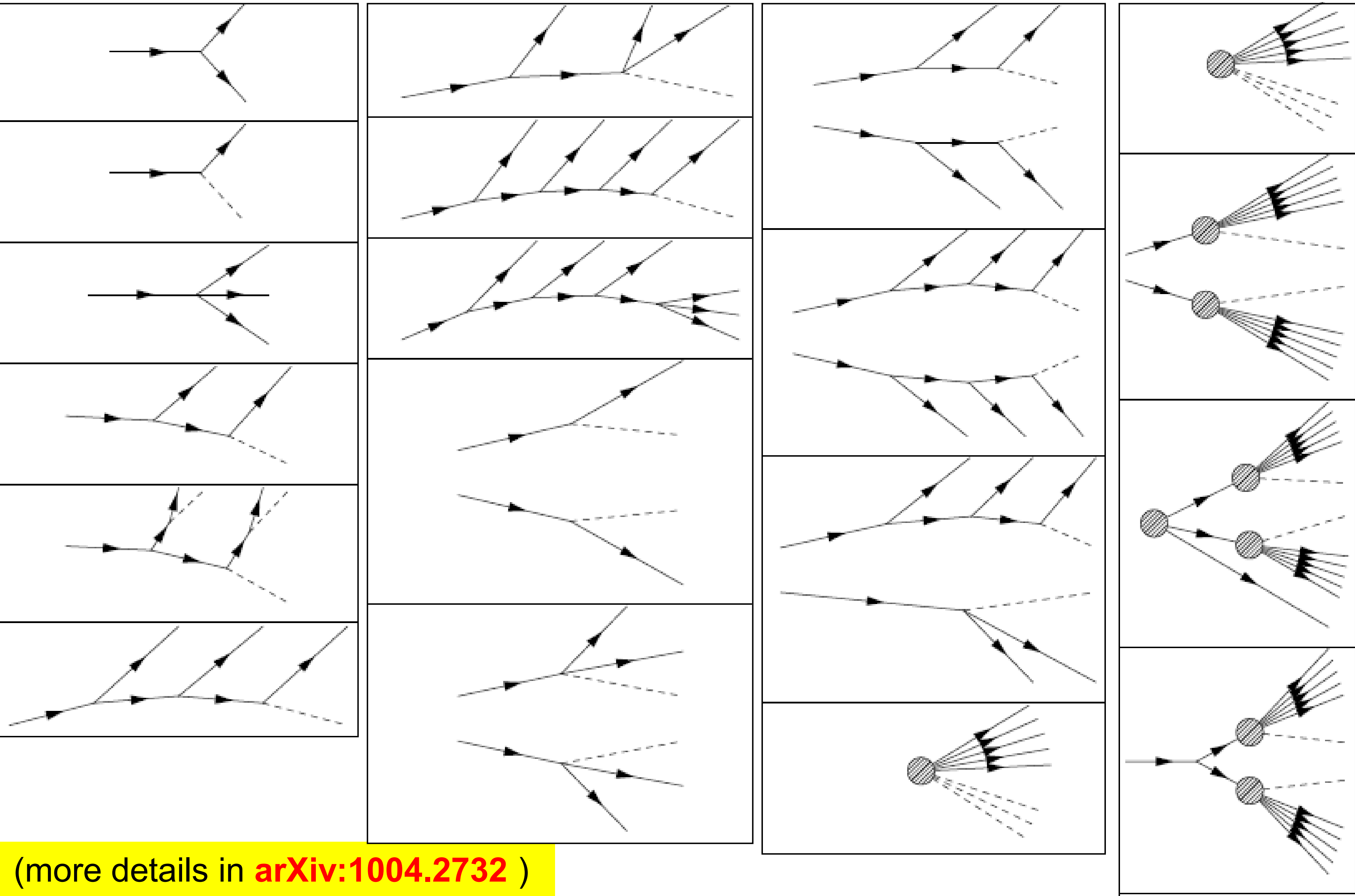
(for those of you interested in LHC dark matter constraints)

Disappointingly, M_{T2} kinks, are the only known kinematic methods which (at least in principle) allow determination of the mass of the invisible daughters of pair produced particles in short chains.

[We will see a dynamical method that works for three+ body decays shortly. Likelihood methods can determine masses in pair decays too, though at cost of model dependence and CPU. See Alwall.]

change of topic!

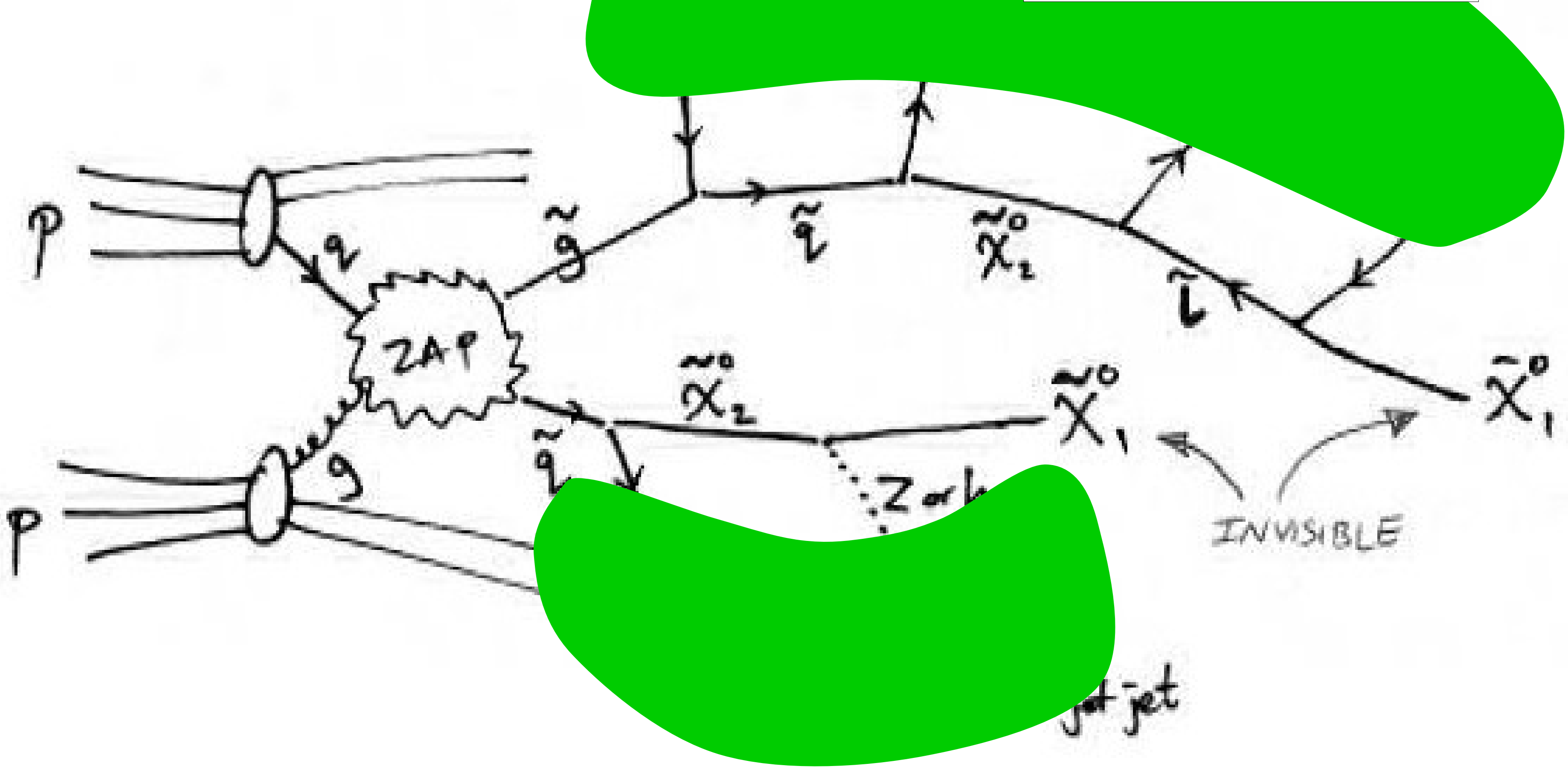
Not all proposed new-physics chains are short!



(more details in **arXiv:1004.2732**)

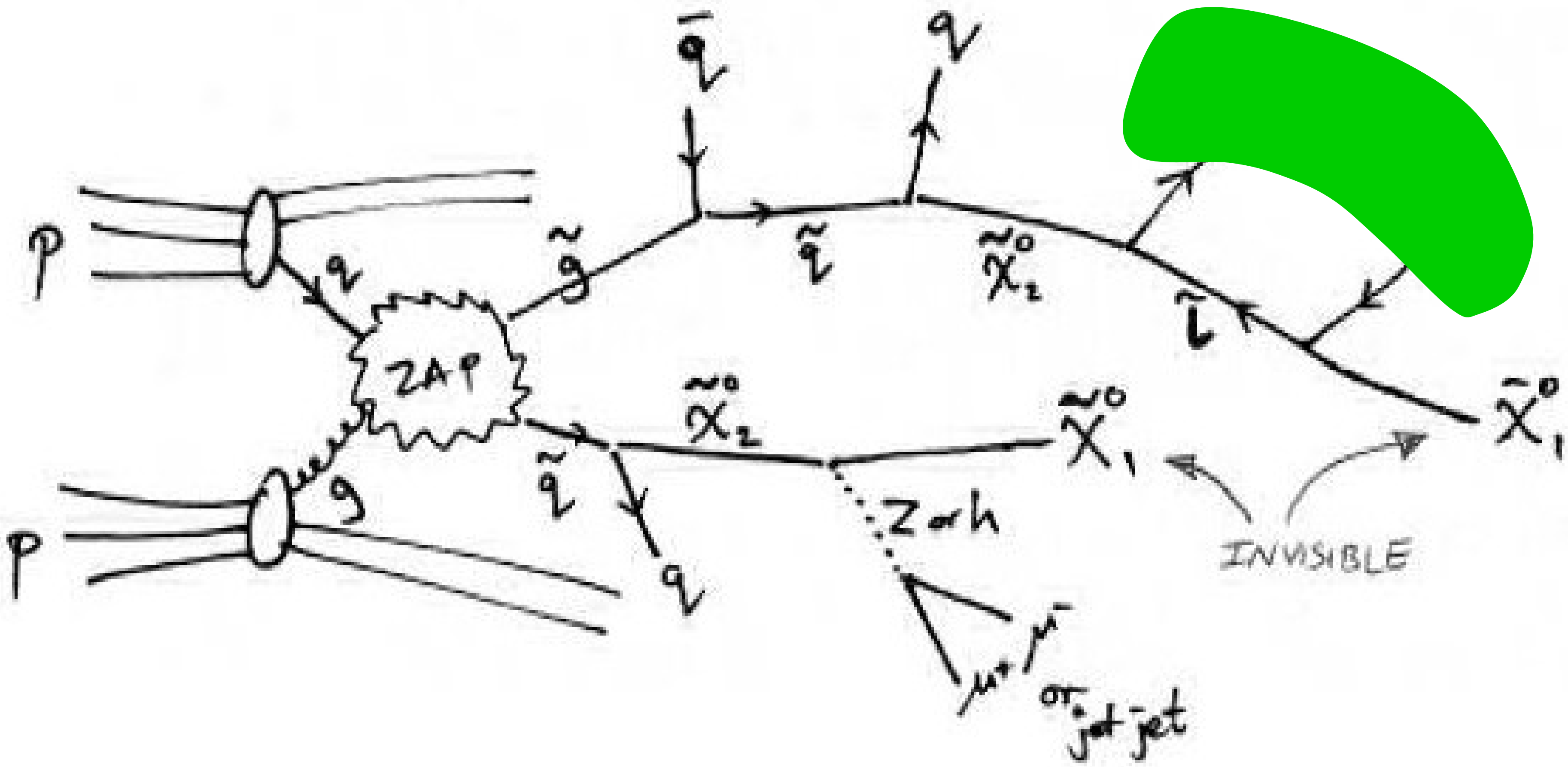
If chains a longer use “edges” or “Kinematic endpoints”

Plot distributions of the invariant masses of what you can see



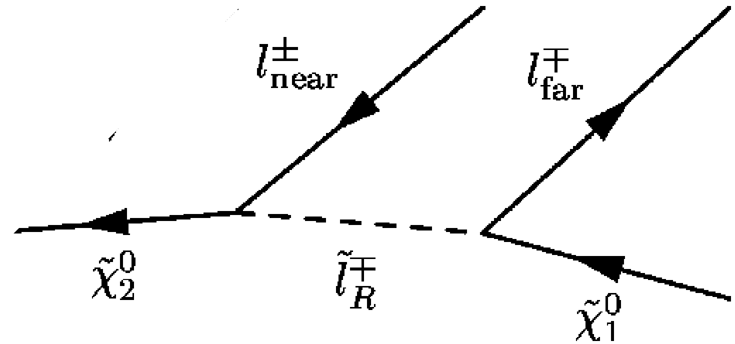
What is a kinematic endpoint?

- Consider M_{LL}

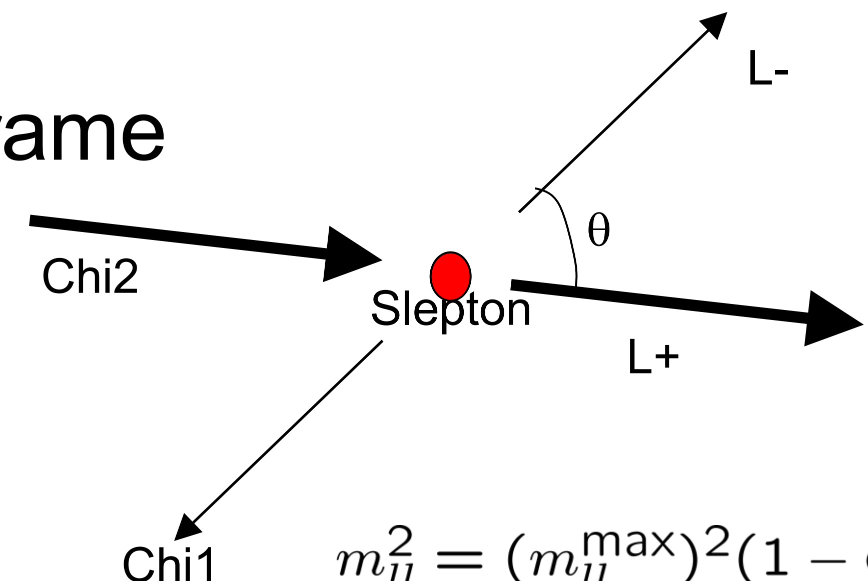


What is a kinematic endpoint?

- Zoom in on di-leptons to calculate m_{LL}

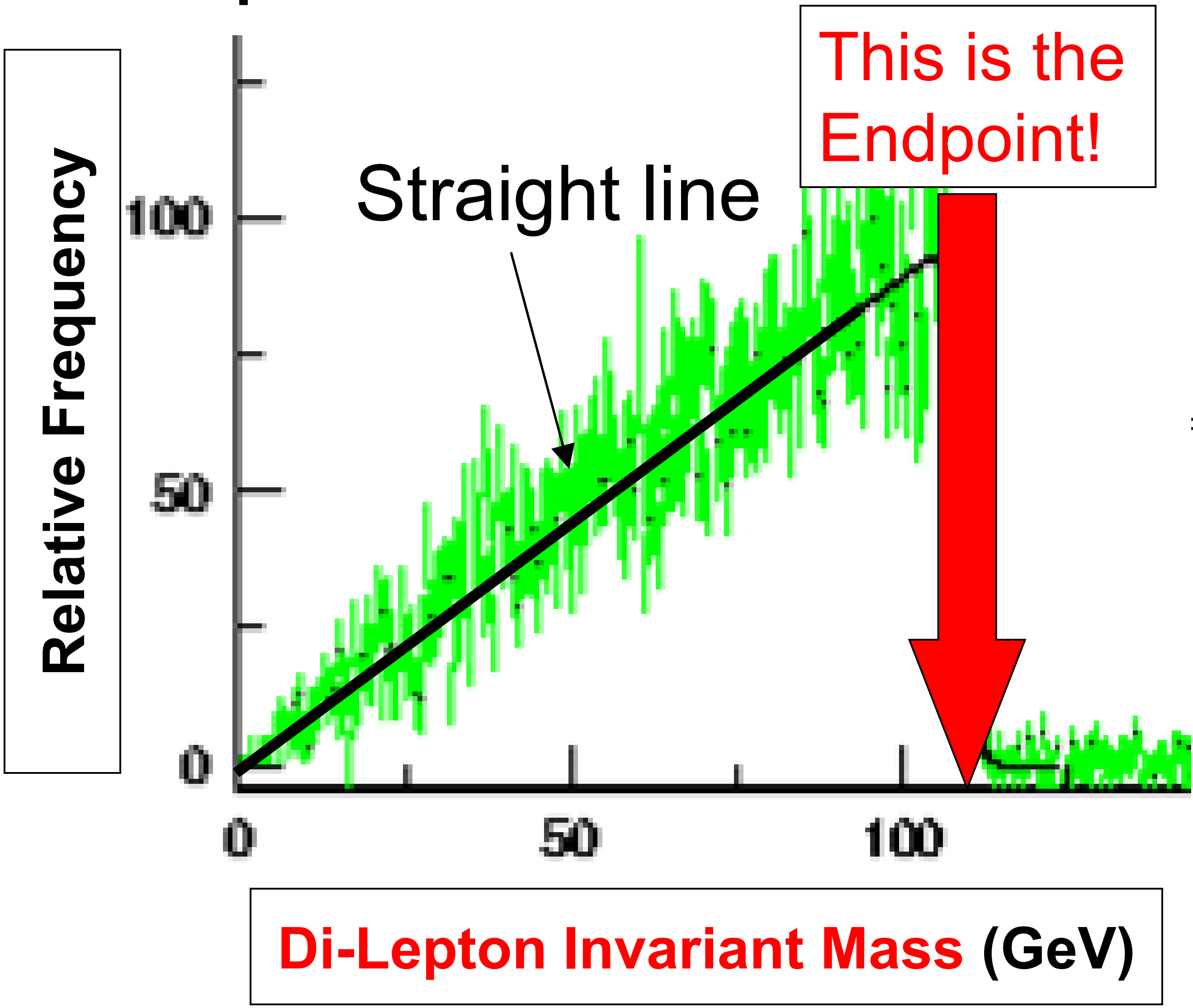


- In slepton rest-frame



$$m_{ll}^2 = (m_{ll}^{\rm max})^2(1 - \cos\theta)/2$$

Dilepton invariant mass distribution

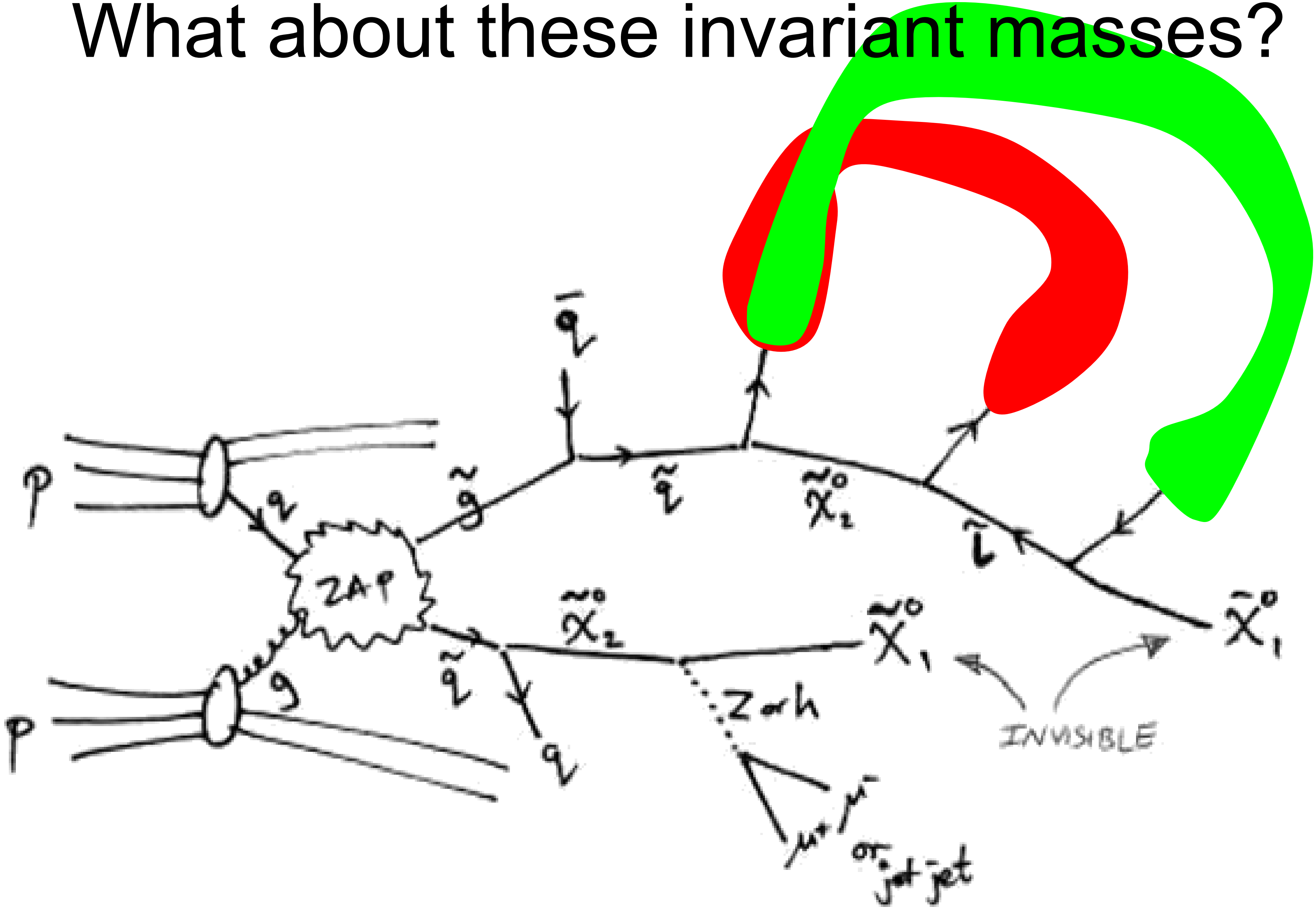


$$= \frac{\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}}$$

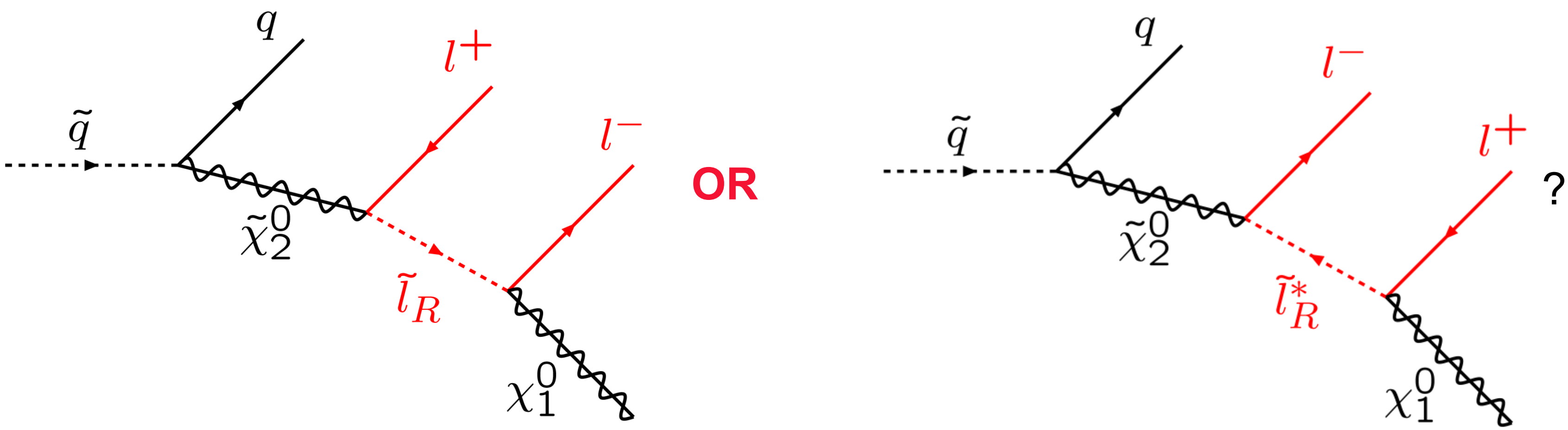
Note key difference to bounding vars

- With the bounding vars you place a bound on a property/parameter/invariant of the hypothesis or model by construction.
- With the kinematic edges and enpoints, you look for a kinematic strucure in a distribution, and use it to constrain one or more parameters of the hypothesis or model.

What about these invariant masses?



Some extra difficulties – may not know order particles were emitted



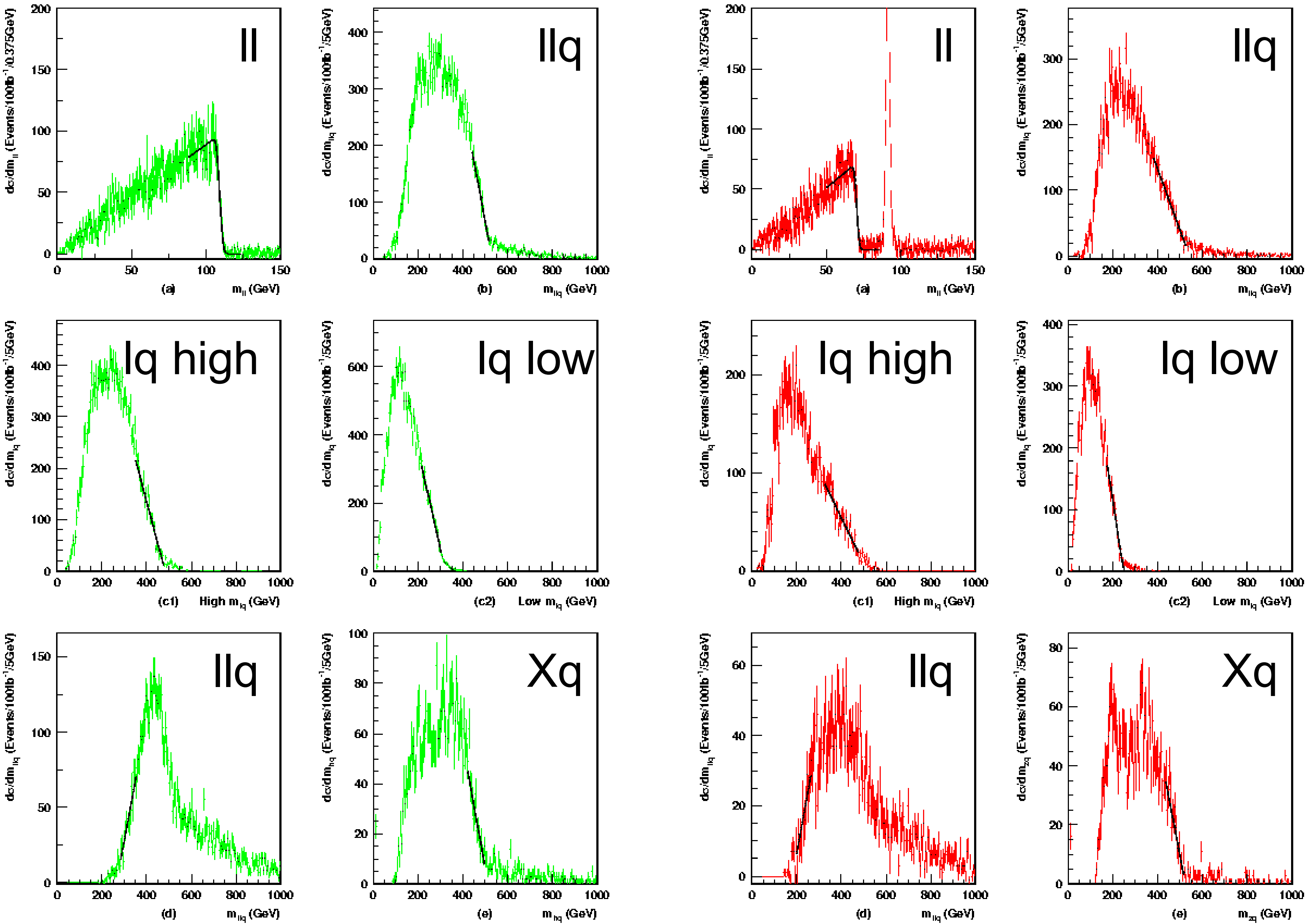
Therefore need to define order-blind variables such as

$$m_{ql}^{high} = \max[m_{ql^+}, m_{ql^-}]$$
$$m_{ql}^{low} = \min[m_{ql^+}, m_{ql^-}]$$

$$m^2_{j\ell(s)}(\alpha) \equiv \left(m^{2\alpha}_{j\ell_n} + m^{2\alpha}_{j\ell_f}\right)^{\frac{1}{\alpha}} \qquad m^2_{j\ell(d)}(\alpha) \equiv \left|m^{2\alpha}_{j\ell_n} - m^{2\alpha}_{j\ell_f}\right|^{\frac{1}{\alpha}}$$

There are many other possibilities for resolving problems due to position ambiguity. Compare hep-ph/0007009 and **hep-ph/0510356** with arXiv:0906.2417

Measure Kinematic Edge Positions



Determine how edge positions depend on sparticle masses

Related edge	Kinematic endpoint
l^+l^- edge	$(m_{ll}^{\max})^2 = (\tilde{\xi}-\tilde{l})(\tilde{l}-\tilde{\chi})/\tilde{l}$
l^+l^-q edge	$(m_{llq}^{\max})^2 = \begin{cases} \max\left[\frac{(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi})}{\tilde{\xi}}, \frac{(\tilde{q}-\tilde{l})(\tilde{l}-\tilde{\chi})}{\tilde{l}}, \frac{(\tilde{q}\tilde{l}-\tilde{\xi}\tilde{\chi})(\tilde{\xi}-\tilde{l})}{\tilde{\xi}\tilde{l}}\right] \\ \text{except for the special case in which } \tilde{l}^2 < \tilde{q}\tilde{\chi} < \tilde{\xi}^2 \text{ and} \\ \tilde{\xi}^2\tilde{\chi} < \tilde{q}\tilde{l}^2 \text{ where one must use } (m_{\tilde{q}} - m_{\tilde{\chi}_1^0})^2. \end{cases}$
Xq edge	$(m_{Xq}^{\max})^2 = X + (\tilde{q}-\tilde{\xi})\left[\tilde{\xi}+X-\tilde{\chi}+\sqrt{(\tilde{\xi}-X-\tilde{\chi})^2-4X\tilde{\chi}}\right]/(2\tilde{\xi})$
l^+l^-q threshold	$(m_{llq}^{\min})^2 = \begin{cases}[\ 2\tilde{l}(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi}) + (\tilde{q}+\tilde{\xi})(\tilde{\xi}-\tilde{l})(\tilde{l}-\tilde{\chi}) \\ \ -(\tilde{q}-\tilde{\xi})\sqrt{(\tilde{\xi}+\tilde{l})^2(\tilde{l}+\tilde{\chi})^2-16\tilde{\xi}\tilde{l}^2\tilde{\chi}}\ \ ]/(4\tilde{l}\tilde{\xi})\end{cases}$
$l^\pm_{\text{near}}q$ edge	$(m_{l_{\text{near}}q}^{\max})^2 = (\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{l})/\tilde{\xi}$
$l^\pm_{\text{far}}q$ edge	$(m_{l_{\text{far}}q}^{\max})^2 = (\tilde{q}-\tilde{\xi})(\tilde{l}-\tilde{\chi})/\tilde{l}$
$l^\pm q$ high-edge	$(m_{lq(\text{high})}^{\max})^2 = \max\left[(m_{l_{\text{near}}q}^{\max})^2, (m_{l_{\text{far}}q}^{\max})^2\right]$
$l^\pm q$ low-edge	$(m_{lq(\text{low})}^{\max})^2 = \min\left[(m_{l_{\text{near}}q}^{\max})^2, (\tilde{q}-\tilde{\xi})(\tilde{l}-\tilde{\chi})/(2\tilde{l}-\tilde{\chi})\right]$
M_{T2} edge	$\Delta M = m_{\tilde{l}} - m_{\tilde{\chi}_1^0}$

Table 4: The absolute kinematic endpoints of invariant mass quantities formed from decay chains of the types mentioned in the text for known particle masses. The following shorthand notation has been used: $\tilde{\chi} = m^2_{\tilde{\chi}_1^0}, \tilde{l} = m^2_{\tilde{l}_R}, \tilde{\xi} = m^2_{\tilde{\chi}_2^0}, \tilde{q} = m^2_{\tilde{q}}$ and X is m_h^2 or m_Z^2 depending on which particle participates in the "branched" decay.

hep-ph/0007009

updated version at arXiv:1004.2732

So now we have:

Large set of measurements

Endpoint	S5 Fit	S5 Fit error
l^+l^- edge	109.10	0.13
l^+l^-q edge	532.1	3.2
$l^\pm q$ high-edge	483.5	1.8
$l^\pm q$ low-edge	321.5	2.3
l^+l^-q threshold	266.0	6.4
Xq edge	514.1	6.6
ΔM (M_{T2} edge)	——	——

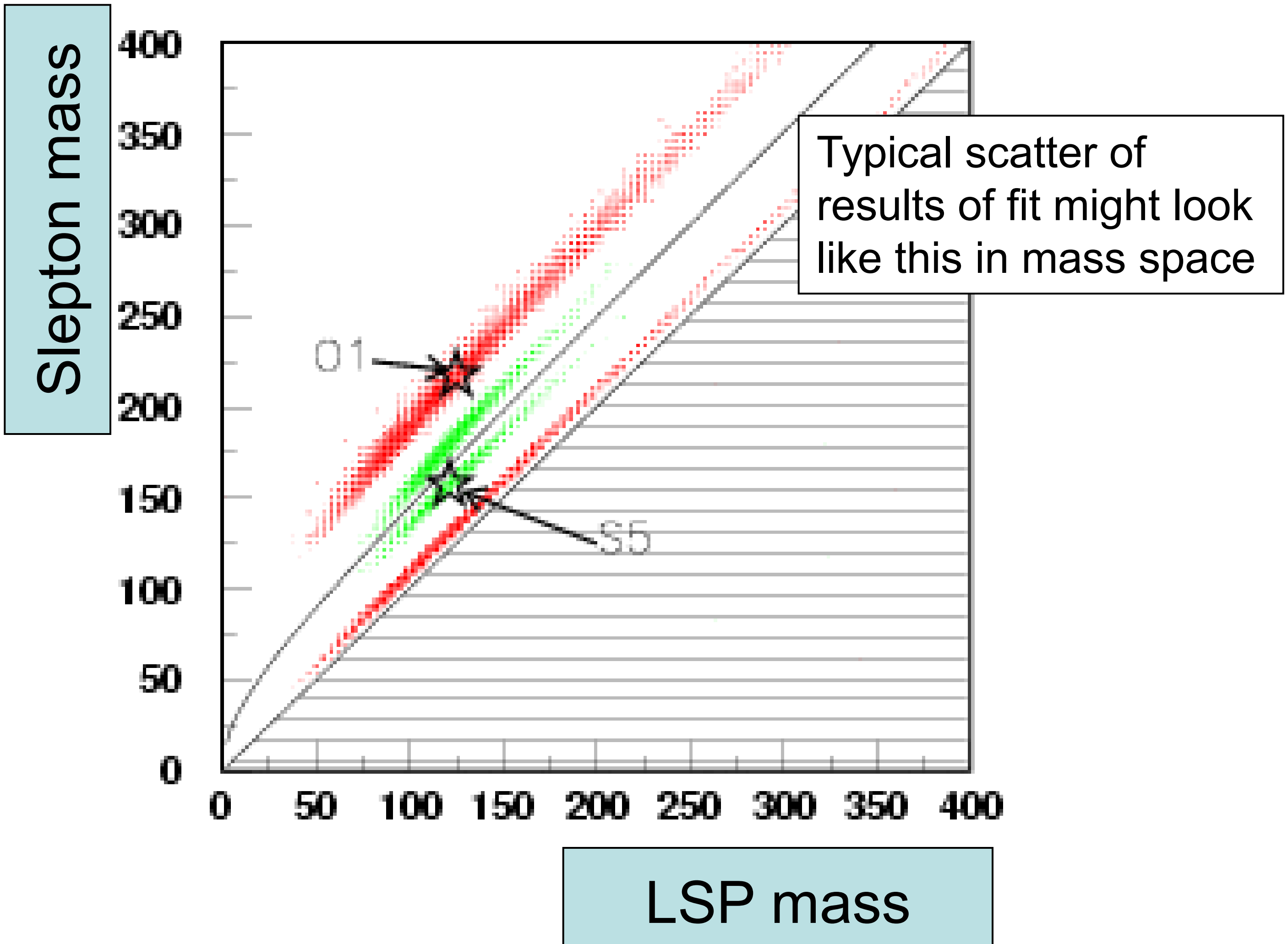
+

Theoretical expressions for edge positions in terms of masses

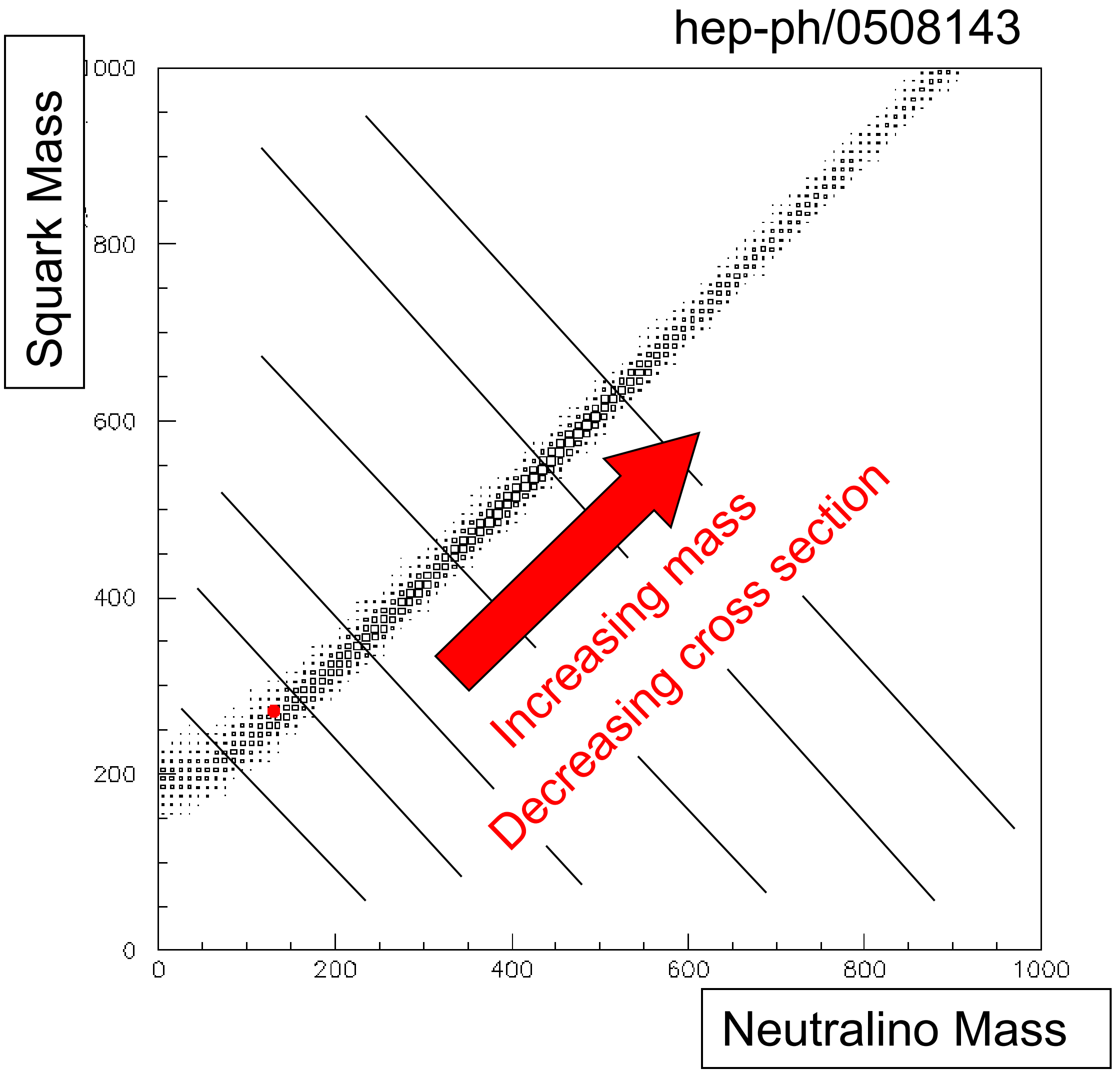
Related edge	Kinematic endpoint
l^+l^- edge	$(m_{ll}^{\max})^2 = (\tilde{\xi} - \tilde{l})(\tilde{l} - \tilde{\chi})/\tilde{l}$
l^+l^-q edge	$(m_{llq}^{\max})^2 = \begin{cases} \max\left[\frac{(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi})}{\tilde{\xi}}, \frac{(\tilde{q}-\tilde{l})(\tilde{l}-\tilde{\chi})}{\tilde{l}}, \frac{(\tilde{q}\tilde{l}-\tilde{\xi}\tilde{\chi})(\tilde{\xi}-\tilde{l})}{\tilde{\xi}\tilde{l}}\right] \\ \text{except for the special case in which } \tilde{l}^2 < \tilde{q}\tilde{\chi} < \tilde{\xi}^2 \text{ and} \\ \tilde{\xi}^2\tilde{\chi} < \tilde{q}\tilde{l}^2 \text{ where one must use } (m_{\tilde{q}} - m_{\tilde{\chi}_1^0})^2. \end{cases}$
Xq edge	$(m_{Xq}^{\max})^2 = X + (\tilde{q} - \tilde{\xi})\left[\tilde{\xi} + X - \tilde{\chi} + \sqrt{(\tilde{\xi} - X - \tilde{\chi})^2 - 4X\tilde{\chi}}\right]/(2\tilde{\xi})$
l^+l^-q threshold	$(m_{llq}^{\min})^2 = \begin{cases} [\ 2\tilde{l}(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi}) + (\tilde{q}+\tilde{\xi})(\tilde{\xi}-\tilde{l})(\tilde{l}-\tilde{\chi}) \\ \ -(\tilde{q}-\tilde{\xi})\sqrt{(\tilde{\xi}+\tilde{l})^2(\tilde{l}+\tilde{\chi})^2 - 16\tilde{\xi}\tilde{l}^2\tilde{\chi}}\ ]/(4\tilde{l}\tilde{\xi}) \end{cases}$
$l^\pm_{\rm near}q$ edge	$(m_{l_{\rm near}q}^{\max})^2 = (\tilde{q} - \tilde{\xi})(\tilde{\xi} - \tilde{l})/\tilde{\xi}$
$l^\pm_{\rm far}q$ edge	$(m_{l_{\rm far}q}^{\max})^2 = (\tilde{q} - \tilde{\xi})(\tilde{l} - \tilde{\chi})/\tilde{l}$
$l^\pm q$ high-edge	$(m_{lq({\rm high})}^{\max})^2 = \max\left[(m_{l_{\rm near}q}^{\max})^2, (m_{l_{\rm far}q}^{\max})^2\right]$
$l^\pm q$ low-edge	$(m_{lq({\rm low})}^{\max})^2 = \min\left[(m_{l_{\rm near}q}^{\max})^2, (\tilde{q} - \tilde{\xi})(\tilde{l} - \tilde{\chi})/(2\tilde{l} - \tilde{\chi})\right]$
M_{T2} edge	$\Delta M = m_{\tilde{l}} - m_{\tilde{\chi}_1^0}$

Fit all edge position for masses!
…mainly constrain mass differences

Recommend "fit" not "solve"



Cross section information is orthogonal to mass differences



How applicable are these long chain techniques ?

For the chain [redacted]
we need:

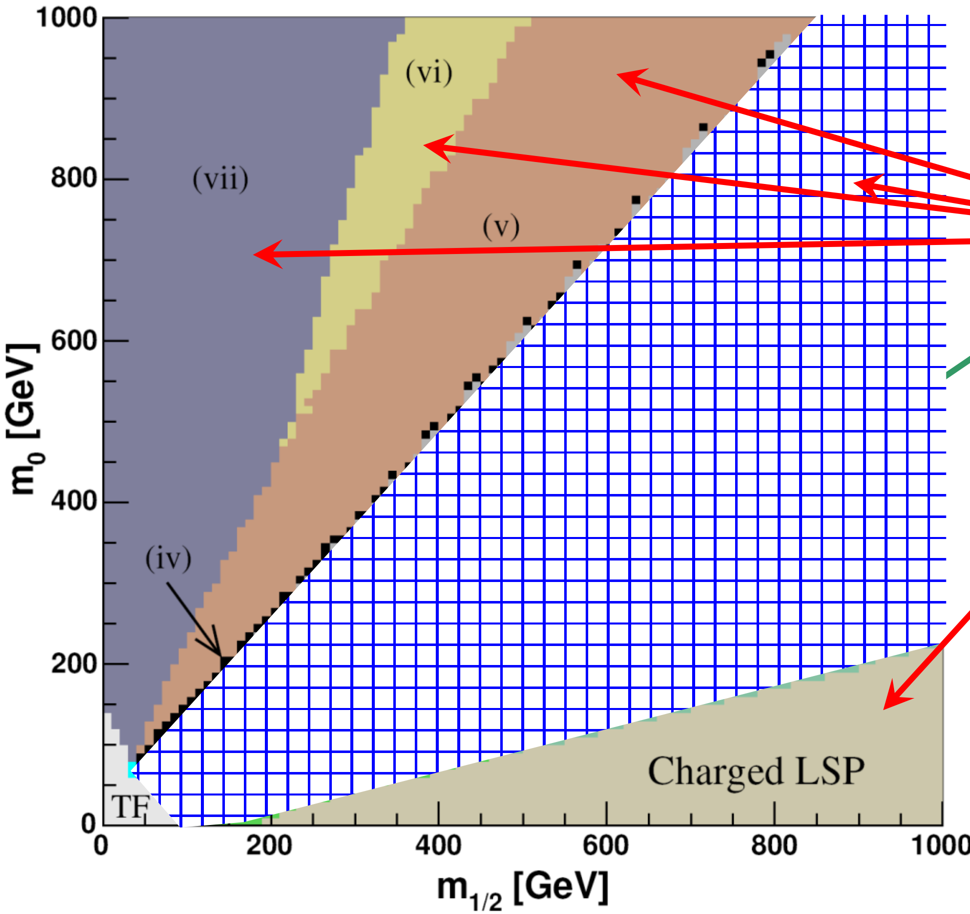
- $m_{\tilde{\chi}_2^0} > m_{\tilde{l}_R} > m_{\tilde{\chi}_1^0}$
- $m_{\tilde{g}} > m_{\tilde{q}}$

This is possible over a wide range of parameter space.

If this chain is not open, the method is still valid, but we need to look at other decay chains.

Example mSUGRA inspired scenario: $-A_0 = m_0, \tan\beta = 10, \mu > 0$

[See Allanach et al, Eur.Phys.J.**C**25 (2002) 113, hep-ph/0202233]



[redacted]

but others are possible.

The hatched area is amenable to this method in some form.

$$\left\{ \begin{array}{l} m_{\tilde{g}} > m_{\tilde{q}} \\ m_{\tilde{l}_R} > m_{\tilde{\chi}_2^0} > m_{\tilde{\chi}_1^0} \end{array} \right.$$

Its pretty hard to do $\frac{1}{10}\sigma_\alpha$ anything with this!

lighter green is where

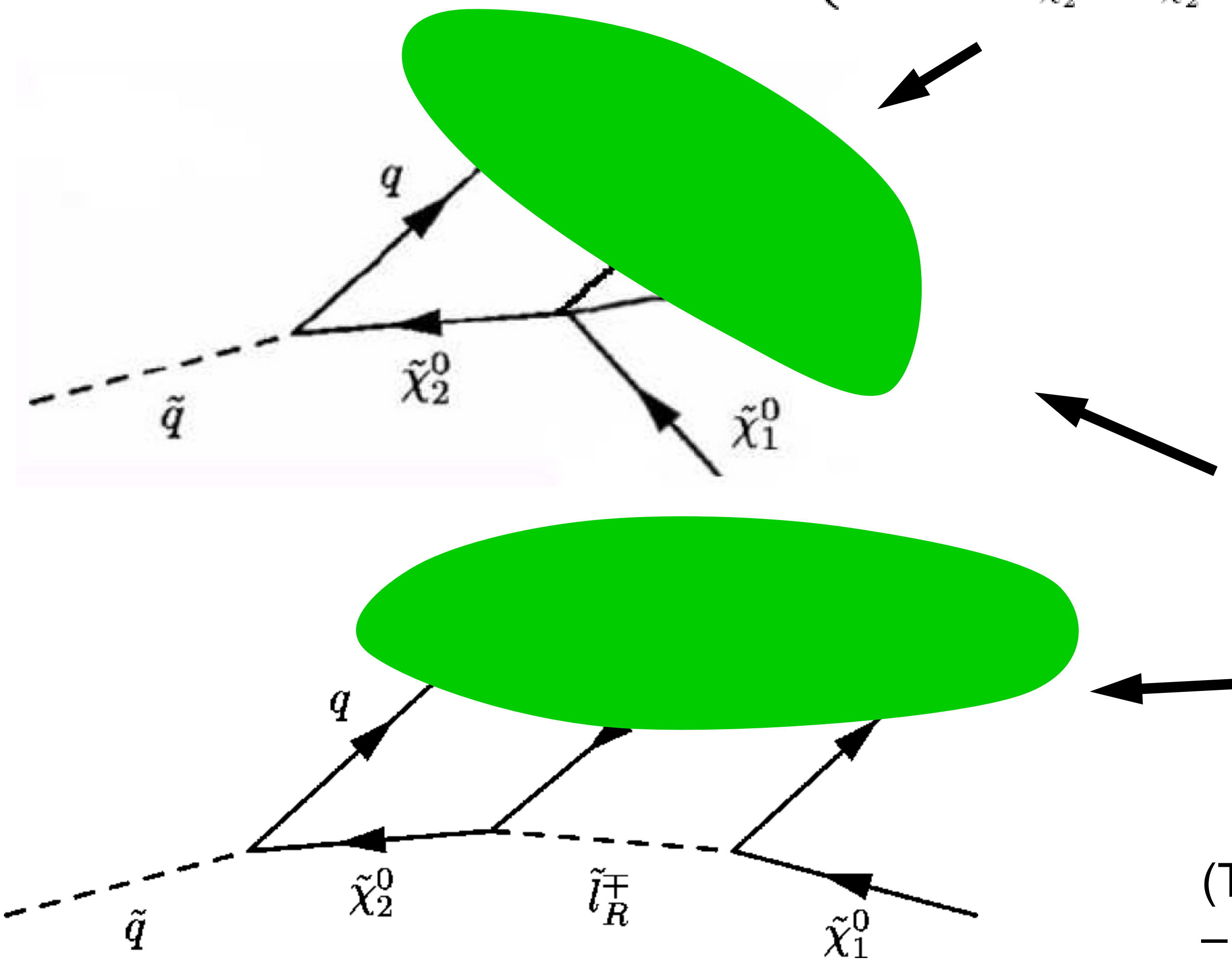
This area doesn't change much for other mSUGRA inspired scenarios.

Figure from hep-ph/0410303

Other ambiguities

$$(m_{llq}^2)^{\max} = \begin{cases} (m_{\tilde{q}} - m_{\tilde{\chi}_1^0})^2 \text{ if } m_{\tilde{\chi}_2^0}^2 > m_{\tilde{q}} m_{\tilde{\chi}_1^0} \\ (m_{\tilde{q}}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)/m_{\tilde{\chi}_2^0}^2 \text{ otherwise.} \end{cases}$$

hep-ph/0609298



Both look the same to the detector

(Though shape differs – see later)

Endpoints are not always linearly independent

e.g. if $m_{\tilde{q}_L} > m^2_{\tilde{\chi}^0_2}/m_{\tilde{\chi}^0_1}$ and $m^2_{\tilde{\chi}^0_1} + m^2_{\tilde{\chi}^0_2} > 2m_{\tilde{\chi}^0_1}m_{\tilde{\chi}^0_2} > 2m^2_{\tilde{q}_L}$

then the endpoints are

$$\left(m_{ll}^{\max}\right)^2 = (m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R})(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1})/m^2_{\tilde{l}_R}$$

$$\left(m_{qll}^{\max}\right)^2 = (m^2_{\tilde{q}_L} - m^2_{\tilde{l}_R})(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1})/m^2_{\tilde{l}_R}$$

$$\left(m_{ql_n}^{\max}\right)^2 = (m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2})(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R})/m^2_{\tilde{\chi}^0_2}$$

$$\left(m_{ql_f}^{\max}\right)^2 = (m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2})(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1})/m^2_{\tilde{l}_R}$$

$$\Rightarrow \quad \left(m_{qll}^{\max}\right)^2 = \left(m_{ll}^{\max}\right)^2 + \left(m_{ql_f}^{\max}\right)^2$$

Four endpoints not always sufficient to find the masses

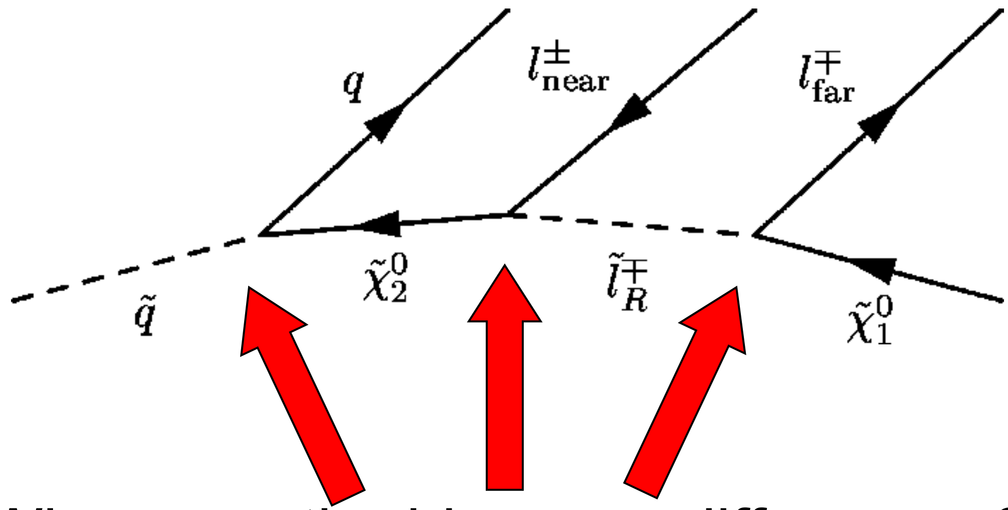
angle between leptons in slepton rest frame

- Introduce new distribution $m_{qll\ \theta>\pi/2}$ identical to m_{qll} except require $\theta>\pi/2$

It is the minimum of this distribution which is interesting

Slide from David Miller

Different parts of model space behave differently: m_{QLL}^{max}

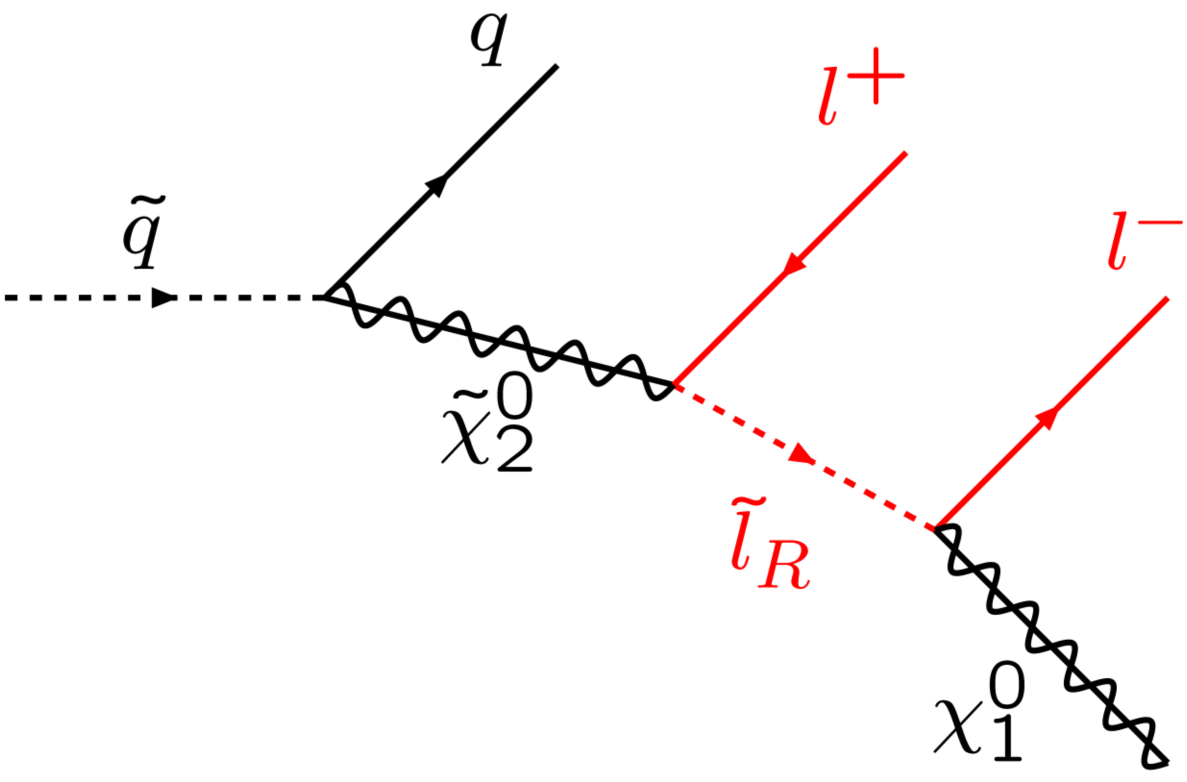


Where are the big mass differences?

hep-ph/0007009

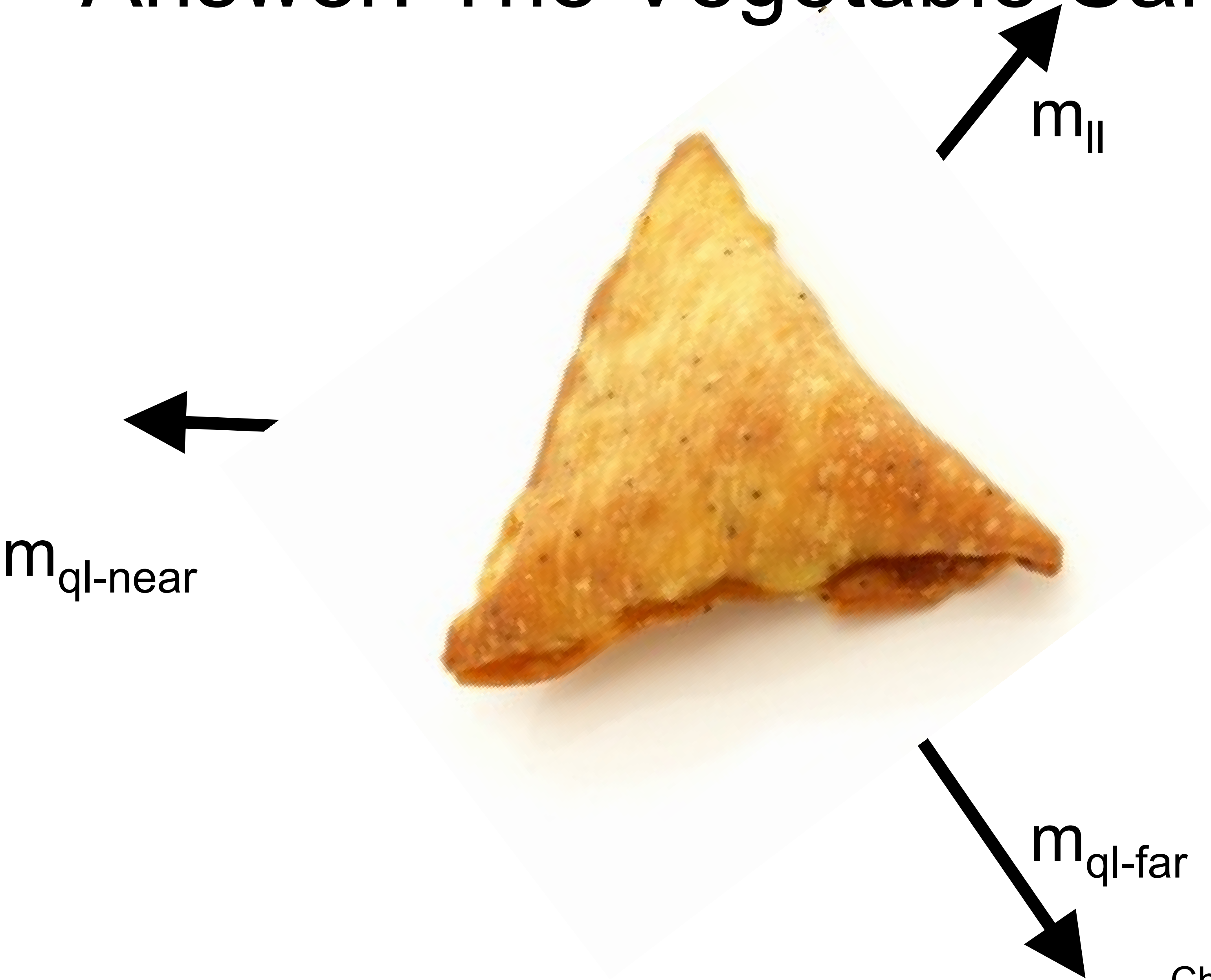
$$(m_{llq}^{\max})^2 = \begin{cases} \max\left[\frac{(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi})}{\tilde{\xi}}, \frac{(\tilde{q}-\tilde{l})(\tilde{l}-\tilde{\chi})}{\tilde{l}}, \frac{(\tilde{q}\tilde{l}-\tilde{\xi}\tilde{\chi})(\tilde{\xi}-\tilde{l})}{\tilde{\xi}\tilde{l}}\right] \\ \text{except for the special case in which } \tilde{l}^2 < \tilde{q}\tilde{\chi} < \tilde{\xi}^2 \text{ and} \\ \tilde{\xi}^2\tilde{\chi} < \tilde{q}\tilde{l}^2 \text{ where one must use } (m_{\tilde{q}} - m_{\tilde{\chi}^0_1})^2. \end{cases}$$

Which parts of $(m^2_{qlnear}, m^2_{qlfar}, m^2_{ll})$-space are populated by these events:



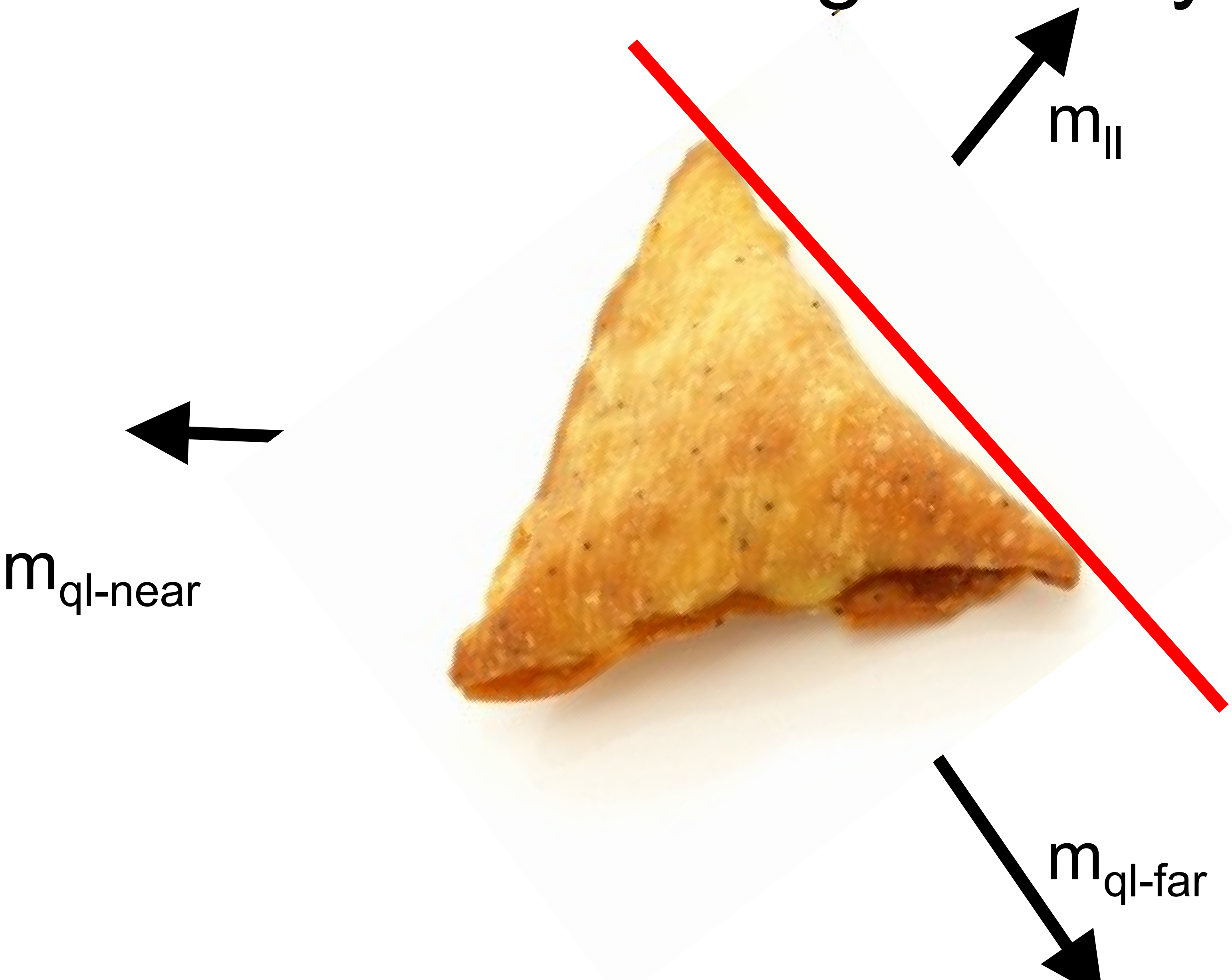
arXiv:0902.2331

Answer: The Vegetable Samosa



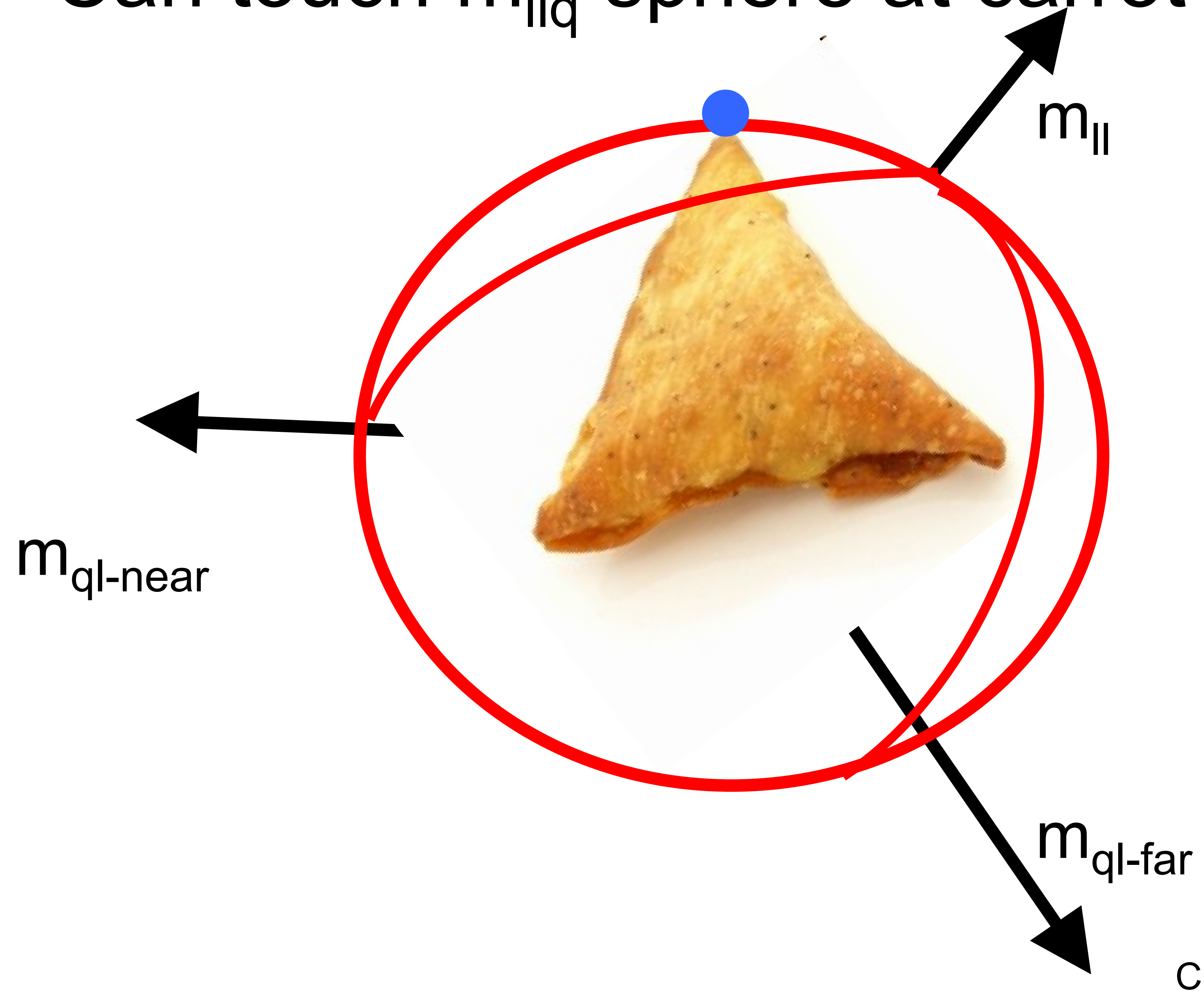
Christopher Lester

Can see ll edge clearly.



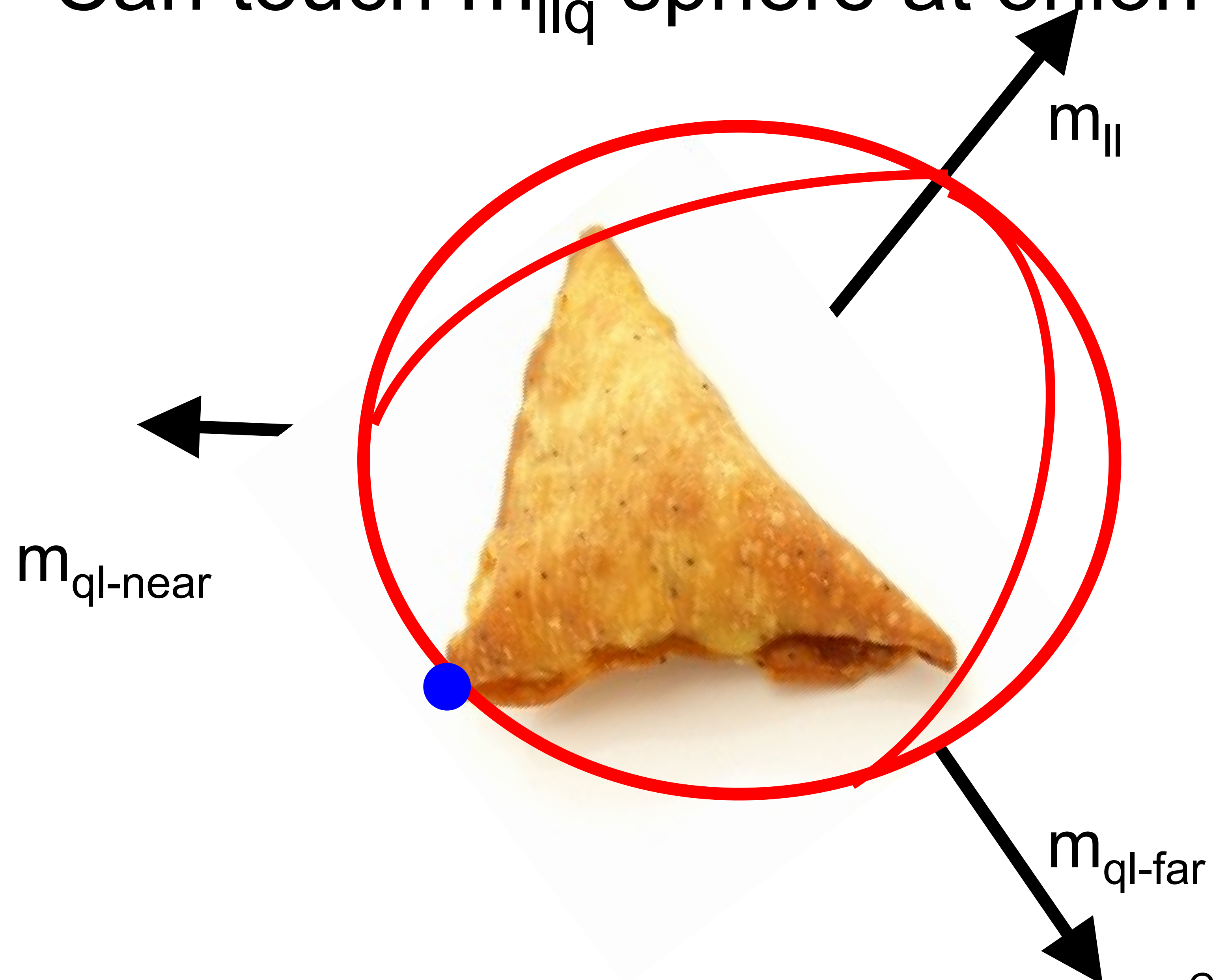
Christopher Lester

Can touch m_{llq} sphere at carrot corner



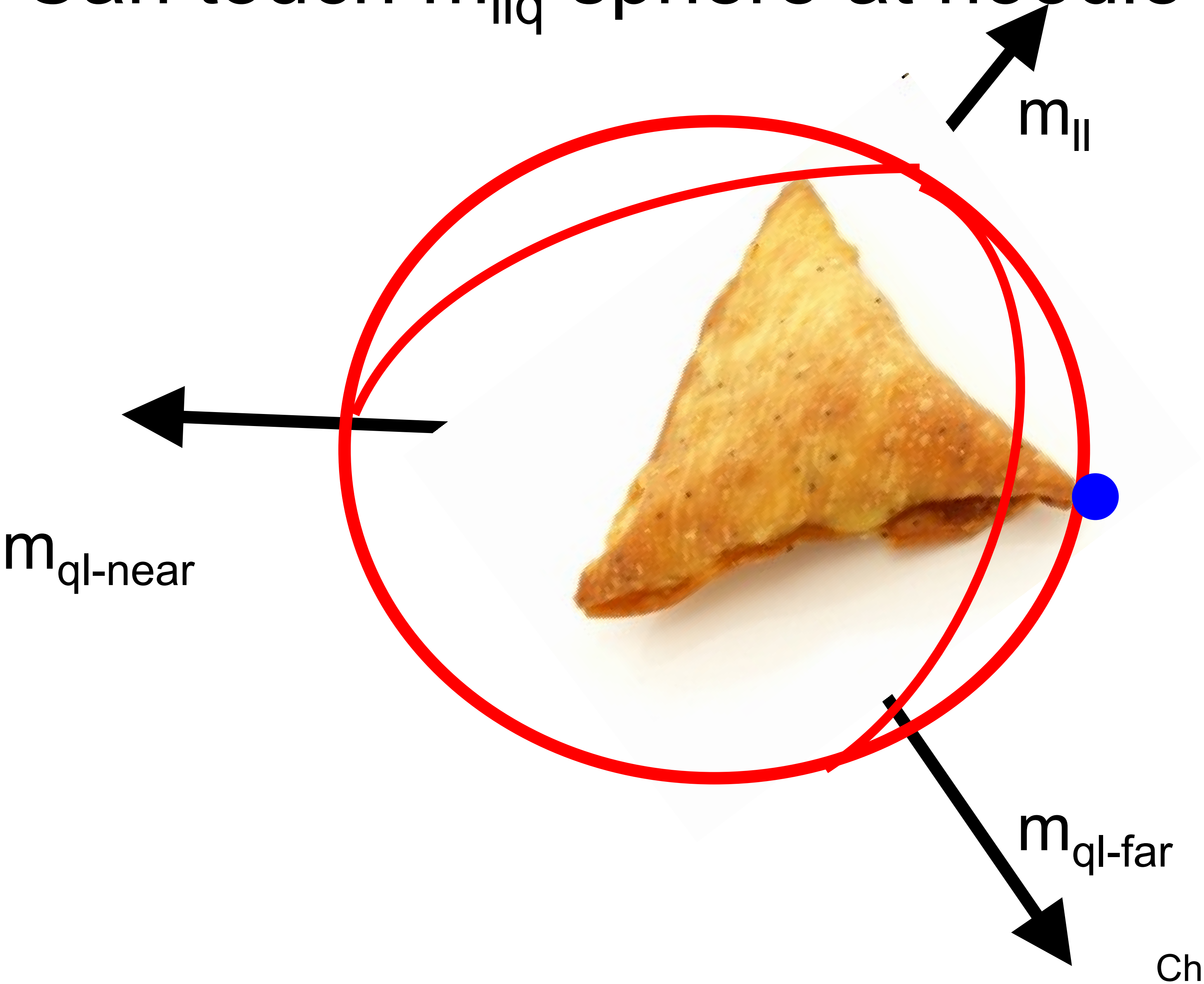
Christopher Lester

Can touch m_{llq} sphere at onion corner



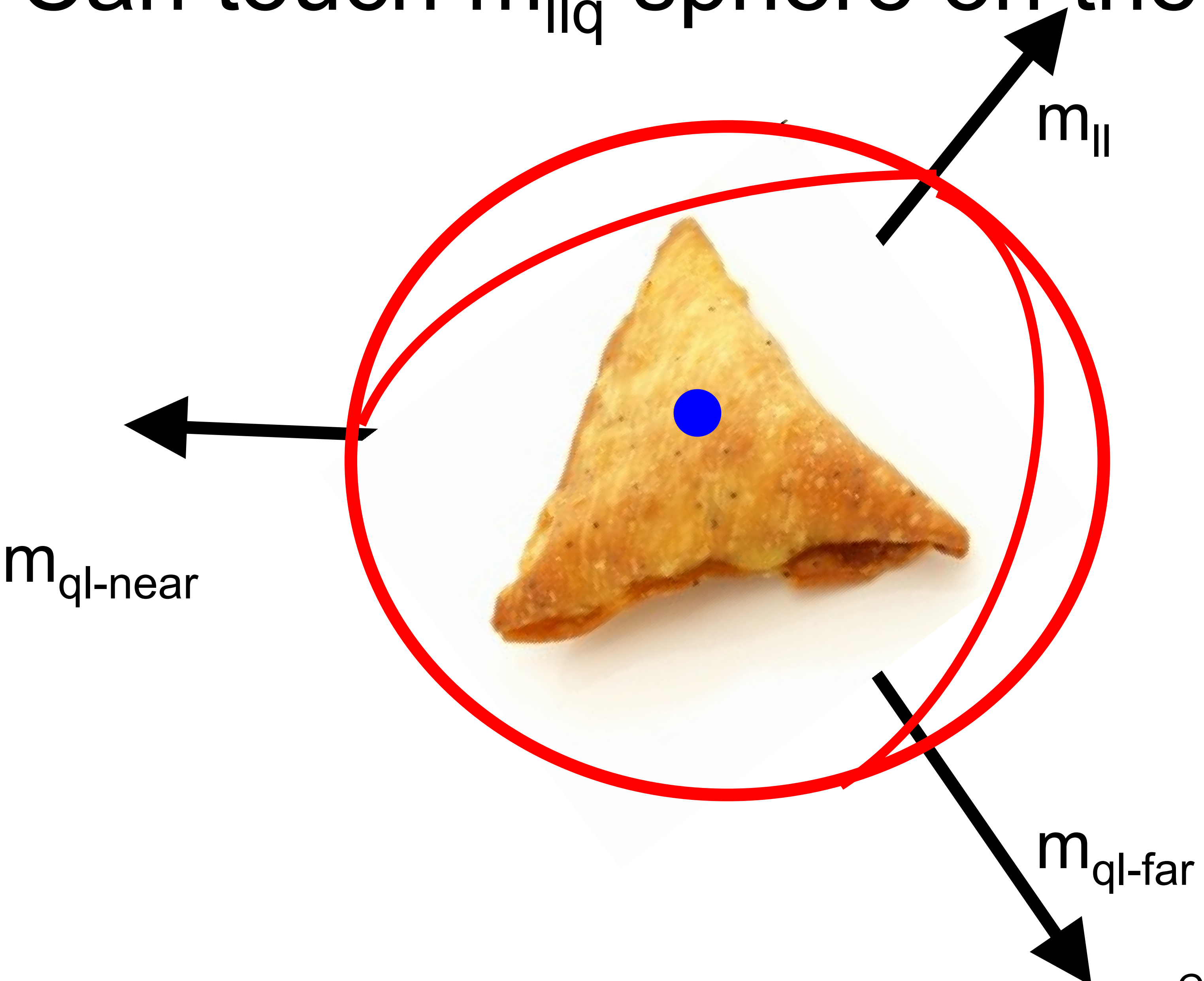
Christopher Lester

Can touch m_{llq} sphere at noodle corner



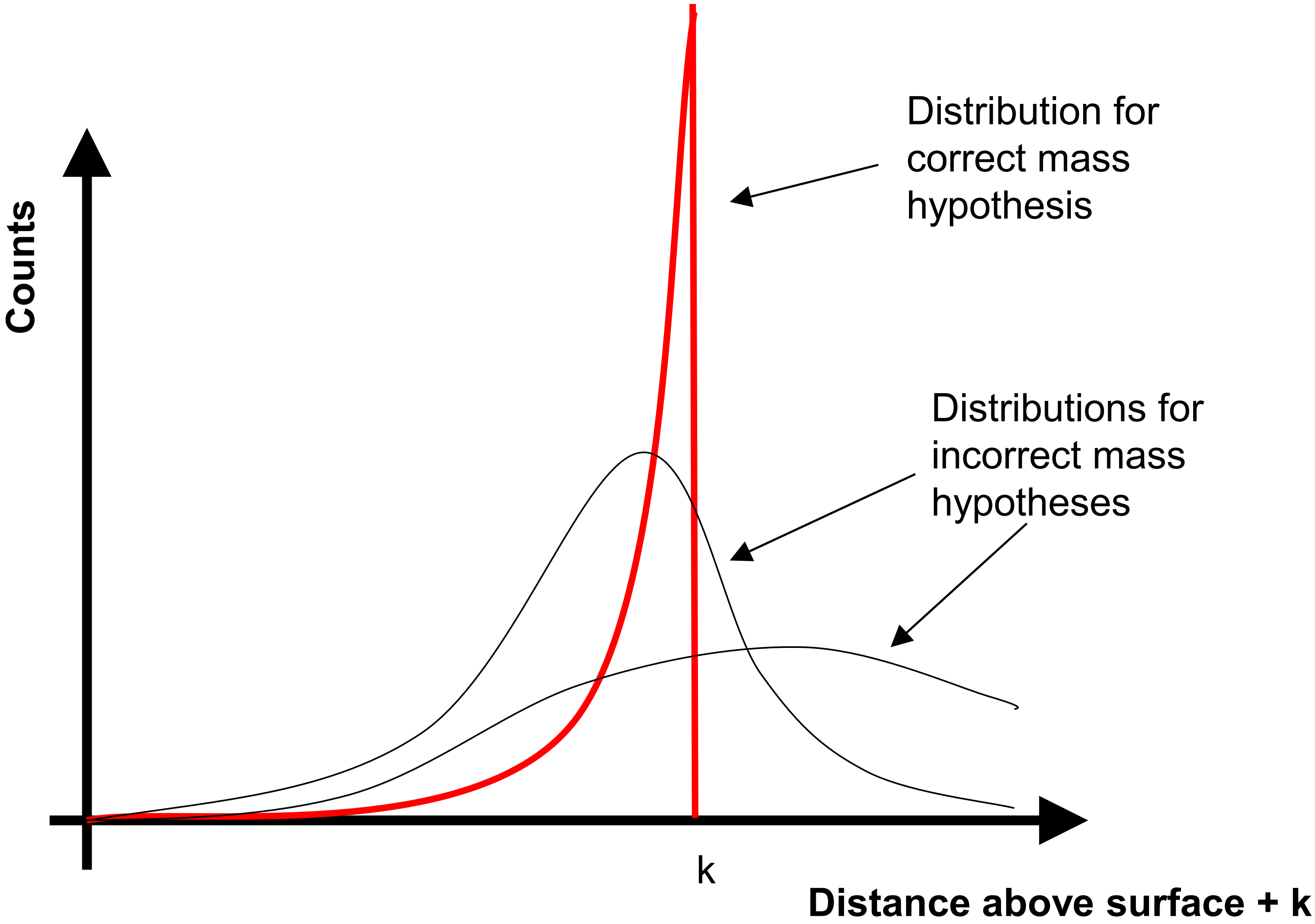
Christopher Lester

Can touch m_{llq} sphere on the "front"



Christopher Lester

So, in principle, find masses by looking for highest contrast edge.



The "shadow" (projection) of the samosa is useful for origami too

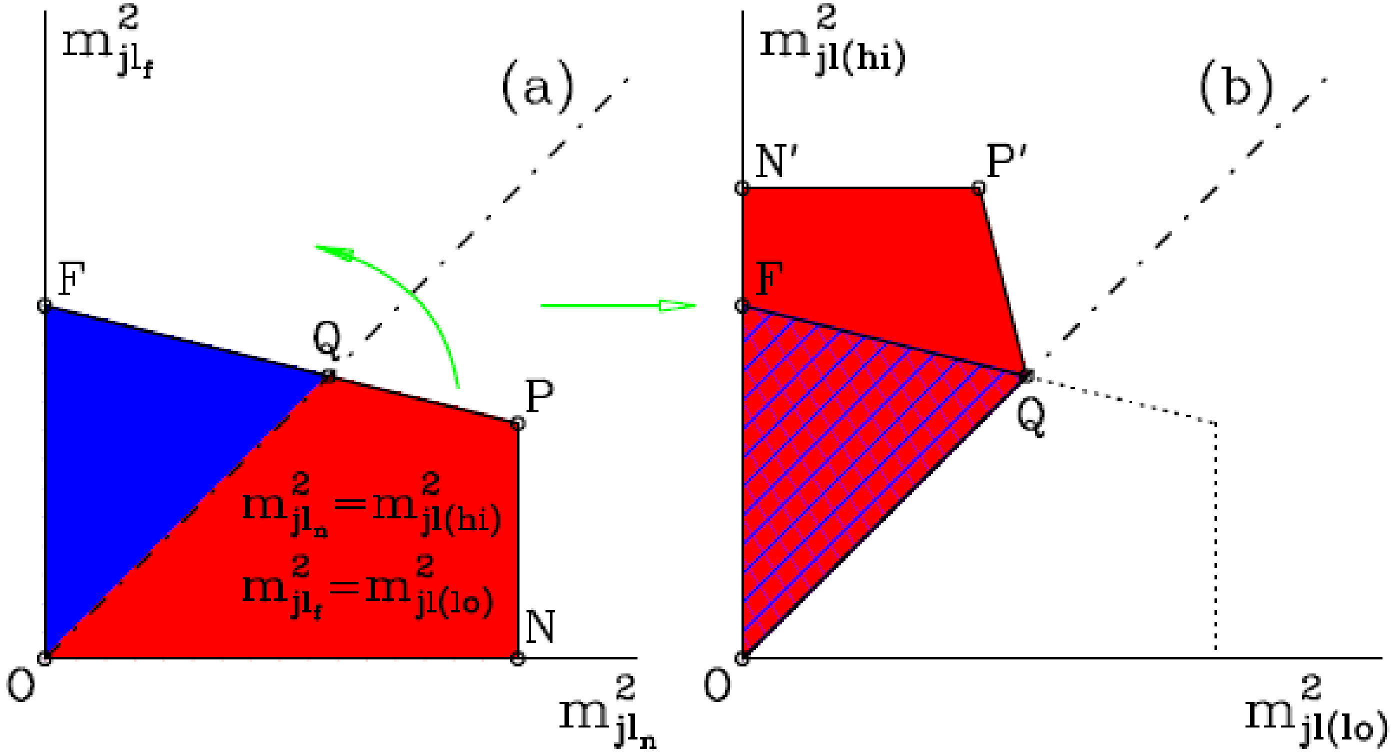
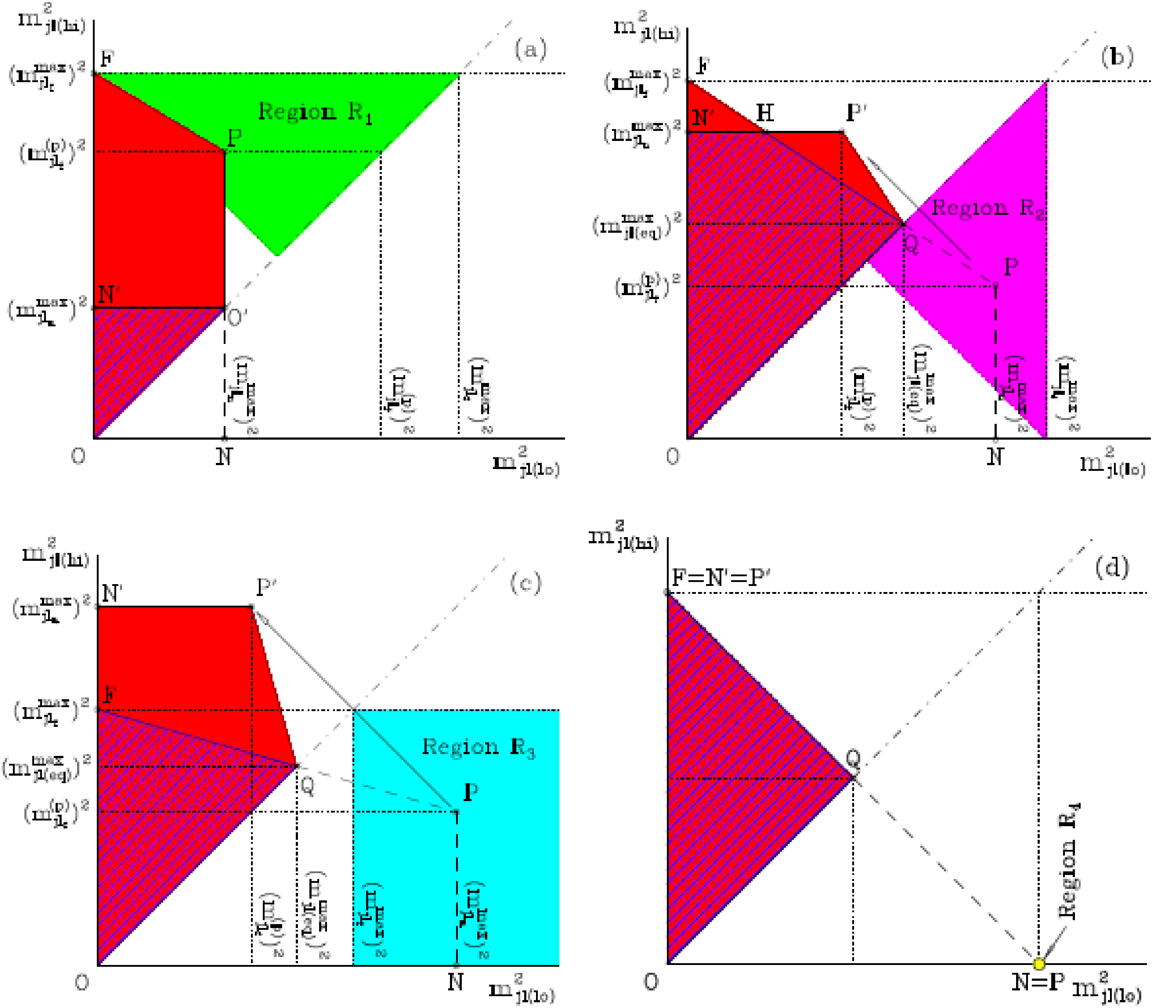


Figure 7: Obtaining the shape of the $m^2_{jl(lo)}$ versus $m^2_{jl(hi)}$ bivariate distribution by folding the $m^2_{jl_n}$ versus $m^2_{jl_f}$ distribution across the line $m^2_{jl_n} = m^2_{jl_f}$. This particular example applies to region $\mathcal{R}_3$. For the other three regions, refer to Figs. 8(a), 8(b) and 8(d).

arXiv:0903.4371


(a)
F
P
N'
O'
O
N
Region R1
(b)
F
N'
H
P'
Q
P
O
N
Region R2
(c)
N'
P'
F
Q
P
O
N
Region R3
(d)
F=N'=P'
Q
O
N=P
Region R4

Formalising an old idea … kinematic boundaries, creases, edges, cusps etc

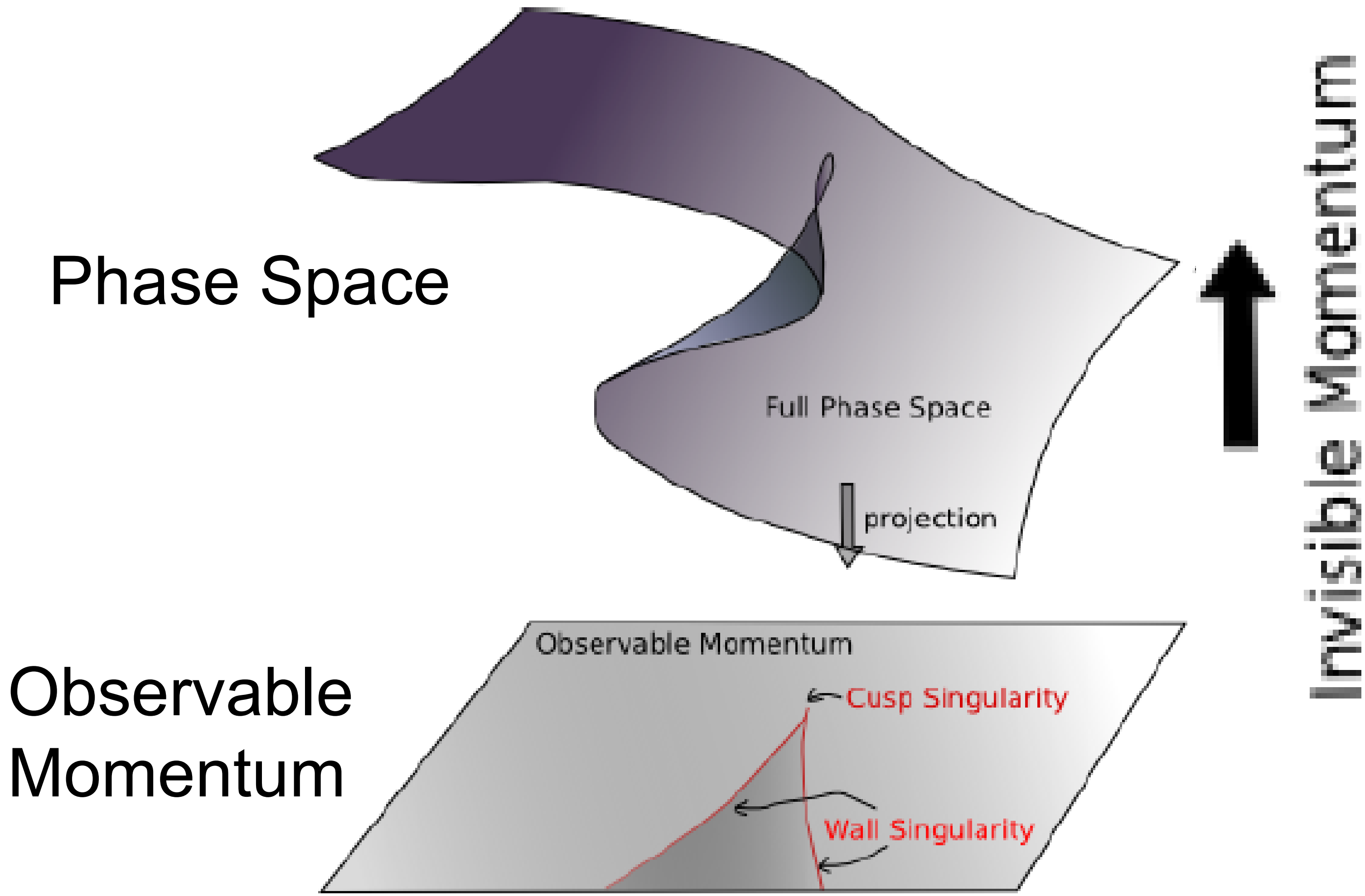
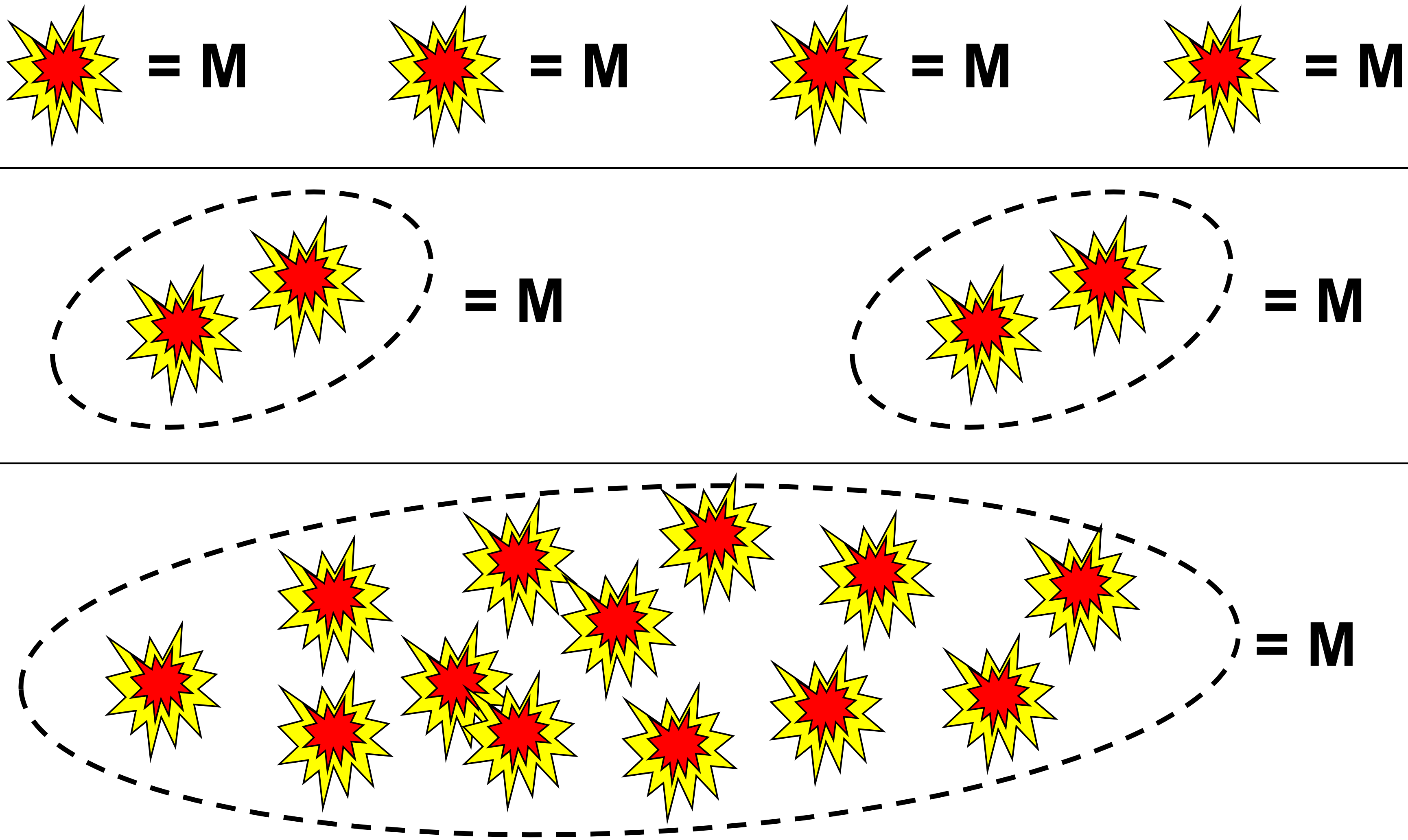


FIG. 1: A schematic diagram describing the relation between the full phase space and the projected observable phase space.

I.W.Kim: "Algebraic singularity method of mass measurement with missing energy" arXiv:0910.1149v2 [hep-ph]

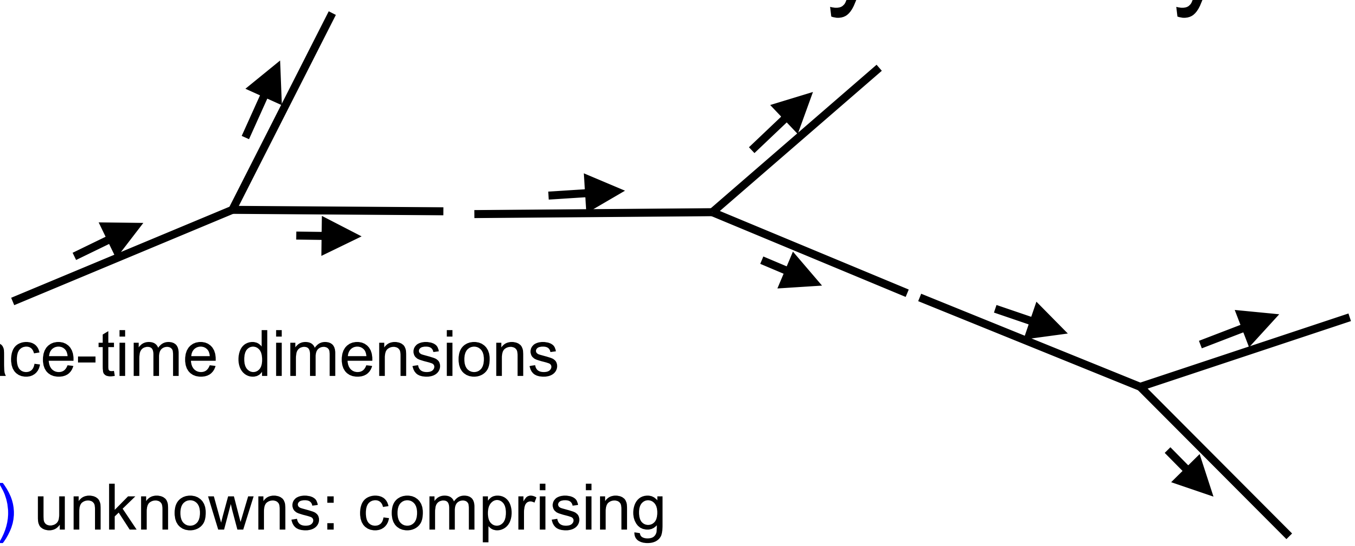
Adding even more assumptions …

Let's consider what happens when we allow ourselves to look at more than one event ….



See sections X and IX of hep-ph/0402295

N successive 2-body decays



- In D space-time dimensions
- D+(N+1) unknowns: comprising
  - D unknown momentum-components for final "missing particle"
  - (N+1) unknown backbone-particle masses
- N+1 constraints:
  - Invariant masses of the backbone-*momenta* must match the "unknown" masses
- UNKNOWNS - CONSTRAINTS = D > 0
  - Cannot solve for unknowns! ☹

Why not look at K events?

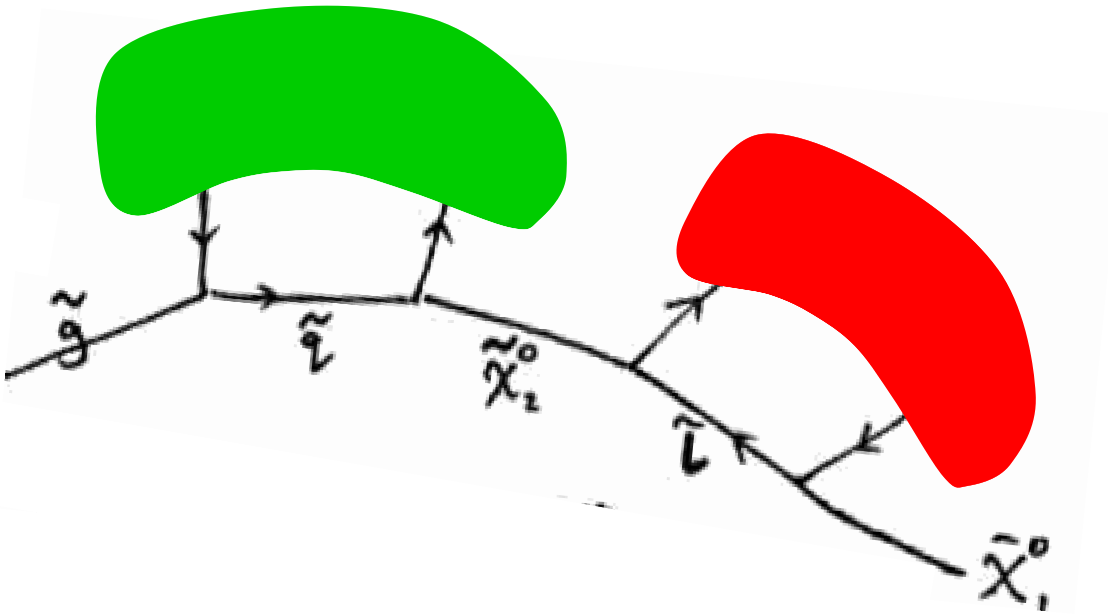
- K events, each (N successive 2-body decays)

- KD+(N+1) **unknowns**: comprising
  - KD unknown momentum-components for final "missing particle"
  - (N+1) unknown backbone-particle masses
- K(N+1) **constraints**:
  - Invariant masses of the backbone-*momenta* must match the "unknown" massses

- UNKNOWNS - CONSTRAINTS = $K(D-(N+1))+(N+1)$
- System solvable for $K \geq \dfrac{N+1}{N+1-D}$ provided $N+1>D$ i.e. $N\geq 4.$

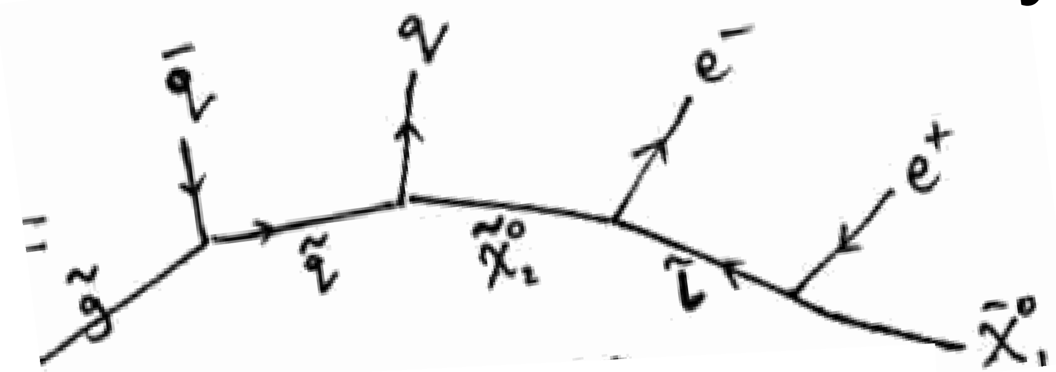
Ambiguities

- Which jet is which?
- Which lepton is which?



- So will need more events than the last calculation suggests ~ x4 ?

“Mass relation” method: summary



- Can:
  - reconstruct complete decay kinematics
  - Measure all sparticle masses
- provided that:
  - Chain has N≥4 successive two-body decays
  - One simultaneously examines at least

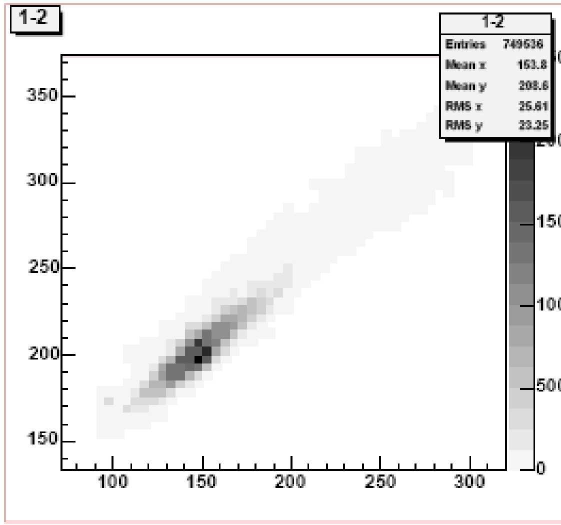
$$\frac{N+1}{N+1-D} = \frac{N+1}{N-3}$$

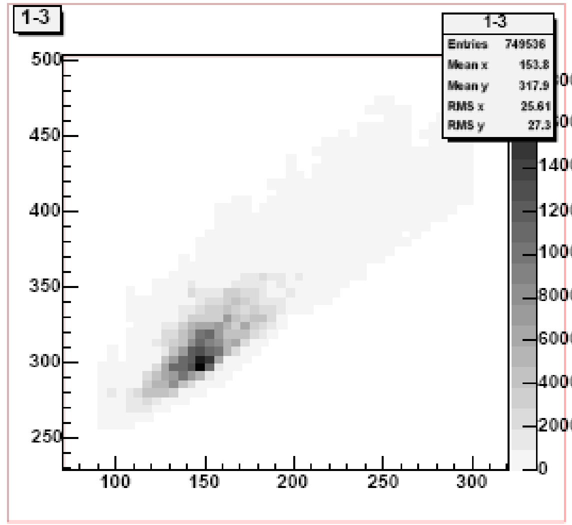
events sharing the same sparticles.

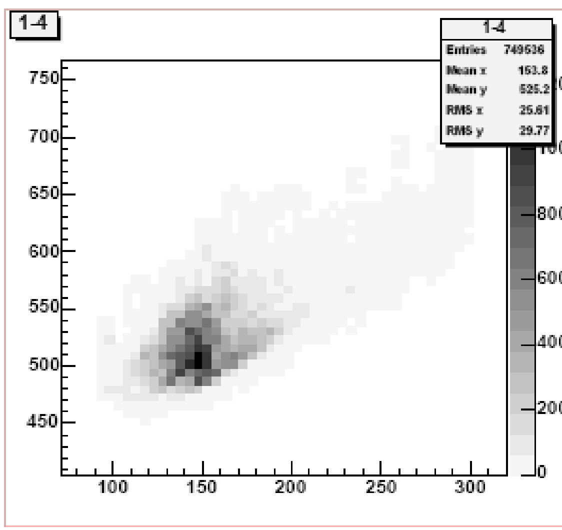
See sections X and IX of hep-ph/0402295

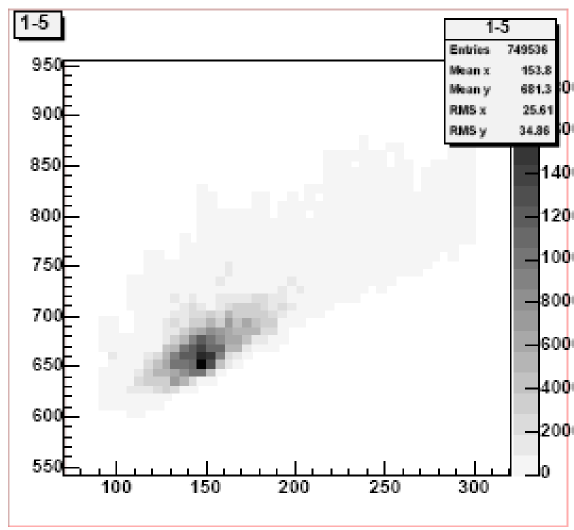
Some example reconstructed masses

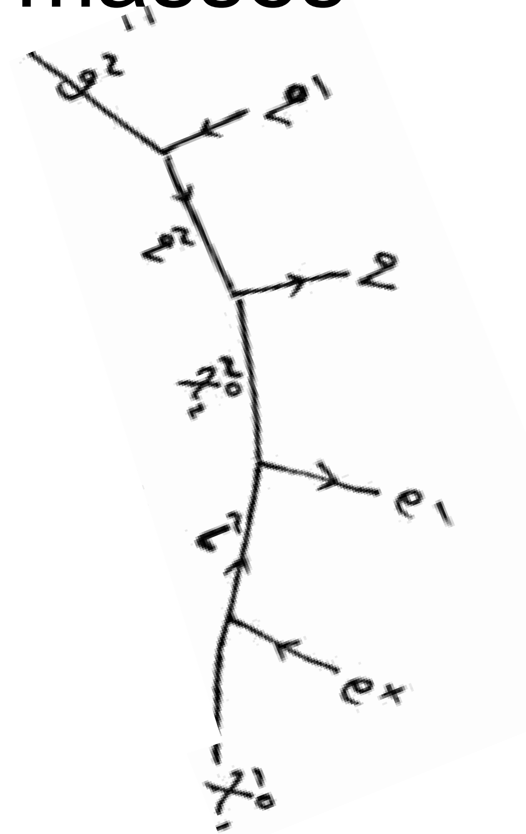
(100 events, toy MC)











Caveats:

Though see Miller hep-ph/0501033

Nobody has shown that this will work for real data.
Sample purity. Bias.
Heavily model dependent?

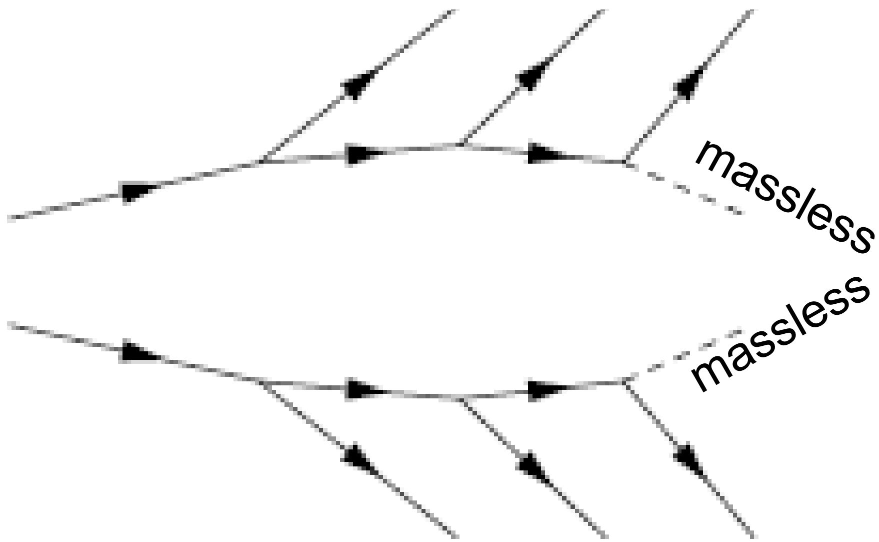
Dependence on reconstruction resolution.

N=4 two-body decays

- Fewer than 5 events
  - Under constrained, cannot solve
- 5 events
  - Can solve in principle (ignoring ambiguities)
  - Can treat events as "ideal"
- More than 5 events
  - Over constrained. Potential for inconsistency.
  - Reconstructed events will not "make sense" until resolutions are taken into account.

Another sort of “just”-constrained event

– get constraint from other “side”

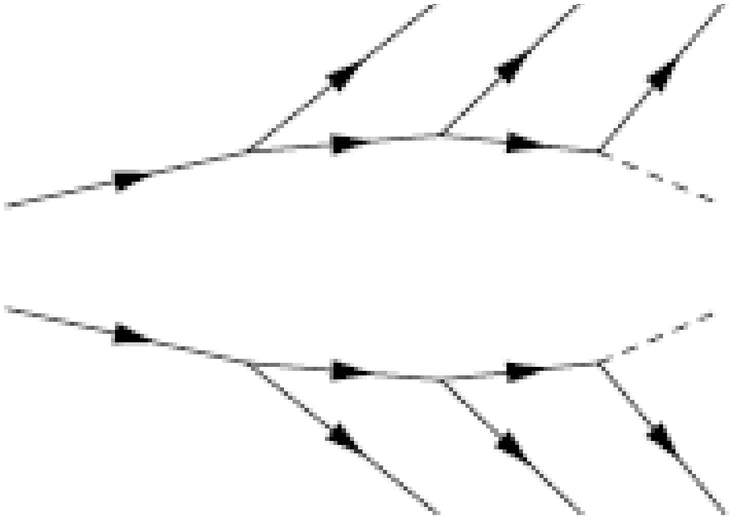


Left: case considered in hep-ph/9812233

- Even if there are invisible decay products, events can often be fully reconstructed if decay chains are long enough.
- (mass-shell constraints must be >= unknown momenta)
- Since we can use ptmiss constraint, chains can be shorter than N=4 now.

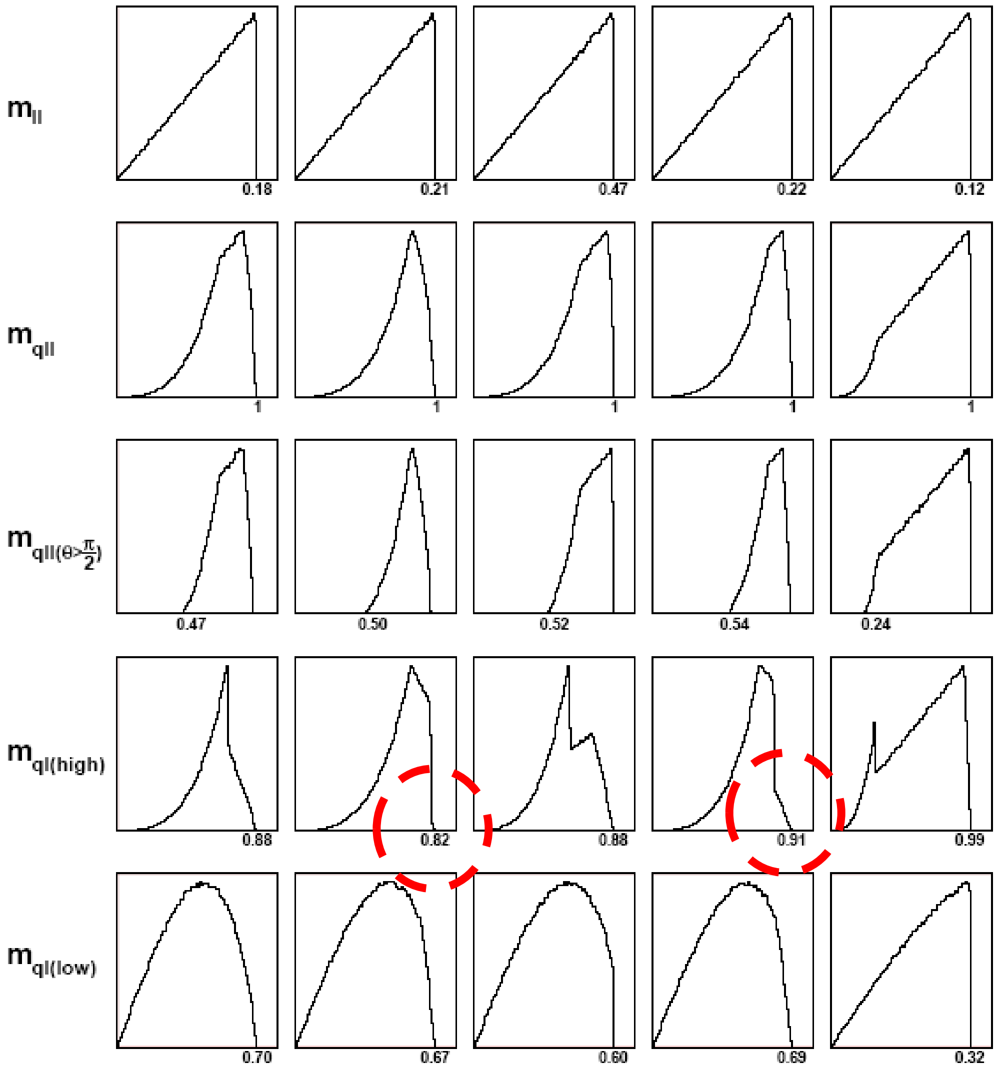
Or do both at once – pairs of double events!

- Pairs of events of the form:



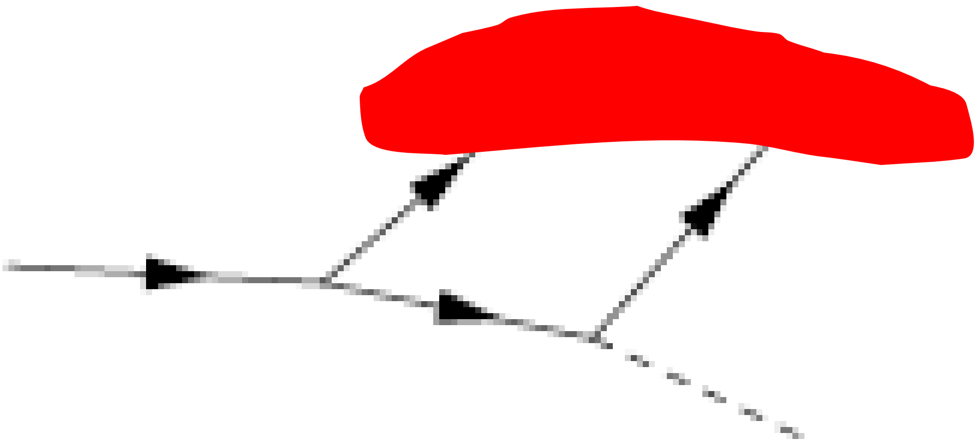
are exactly constrained.
(arXiv:0905.1344)

What about shapes of distributions?

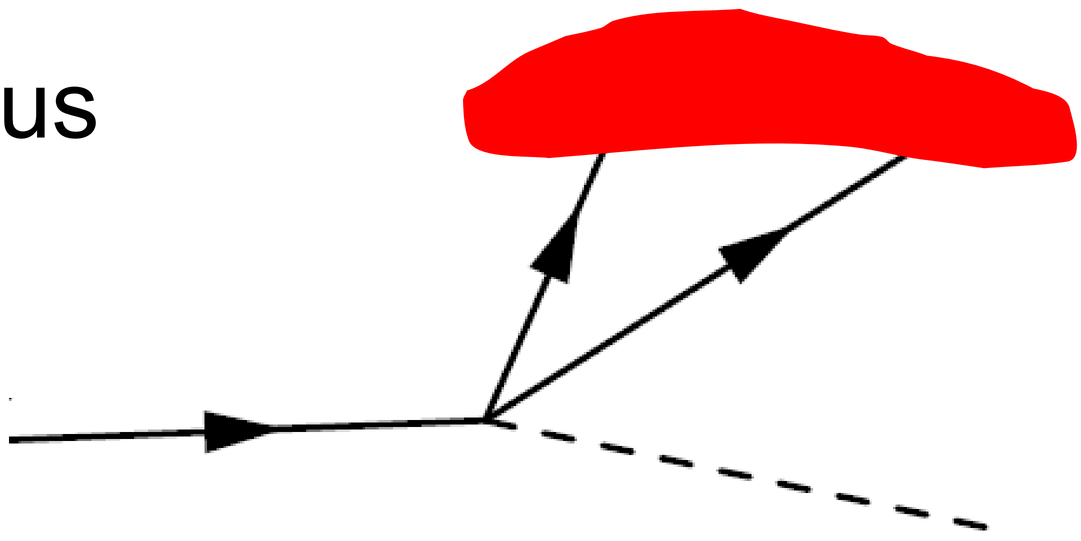


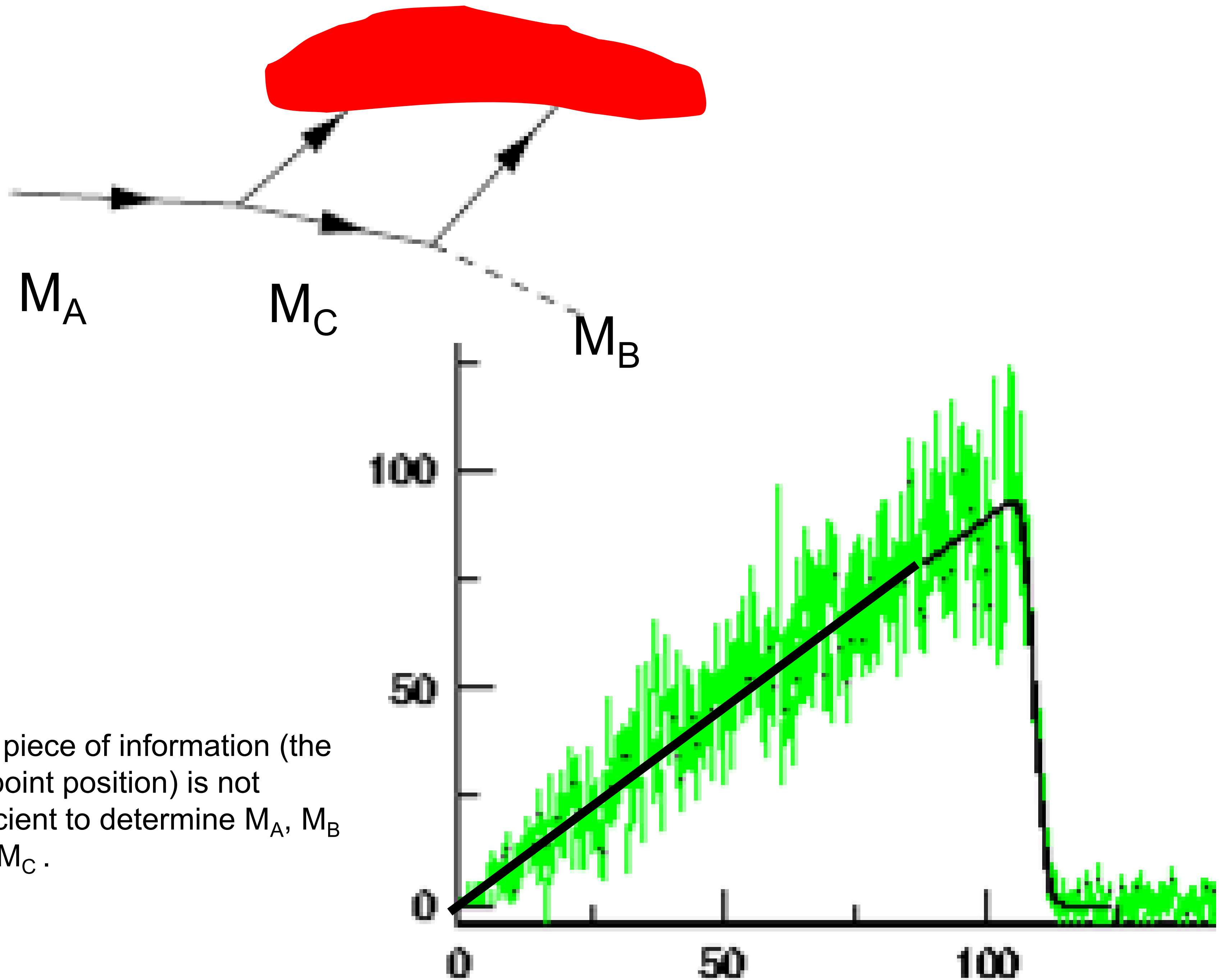
Gjelsten, Miller, Osland: hep-ph/0410303

Compare shapes of invariant mass distributions for the highlighted pairs of visible massless momenta:

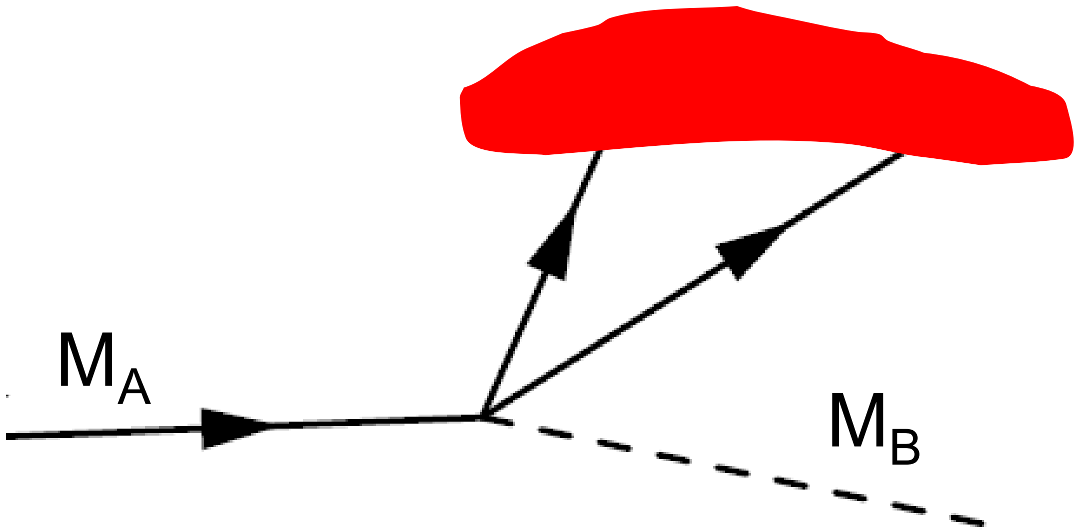


versus

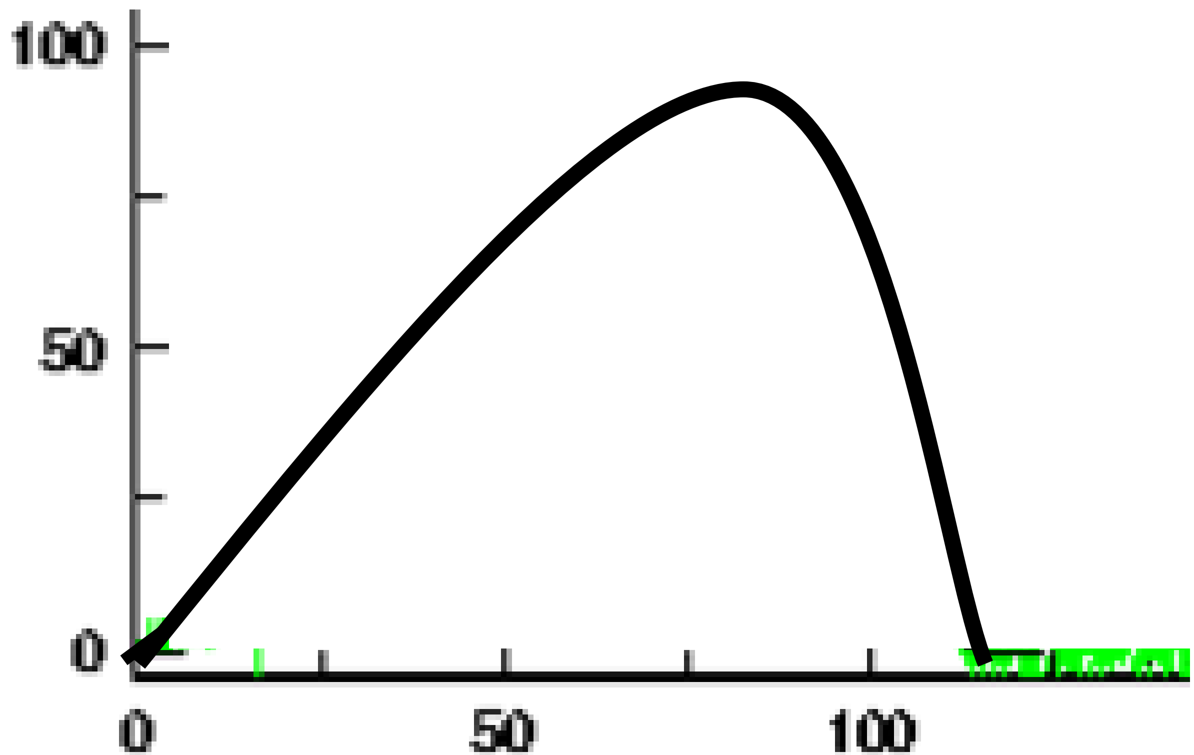




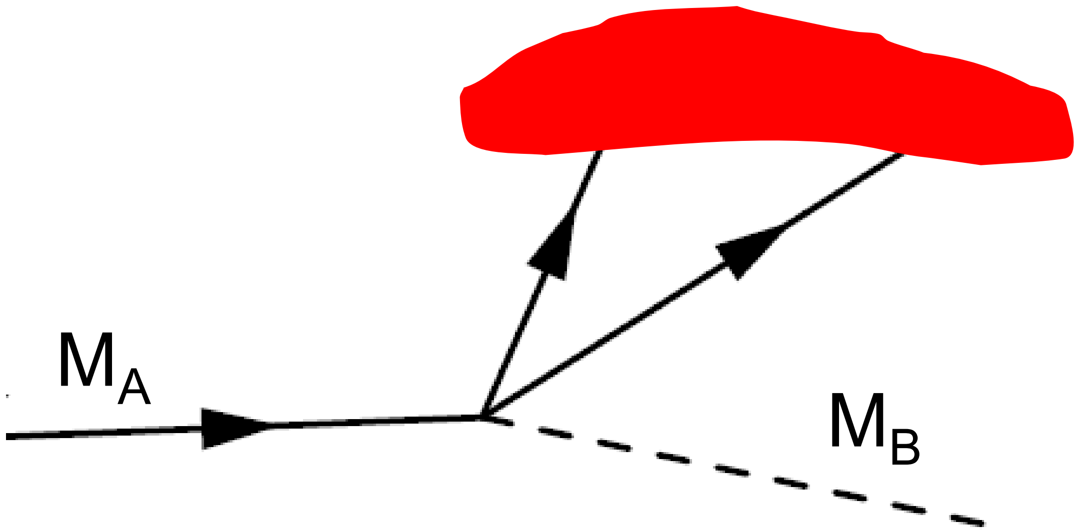
One piece of information (the endpoint position) is not sufficient to determine M_A, M_B and M_C .

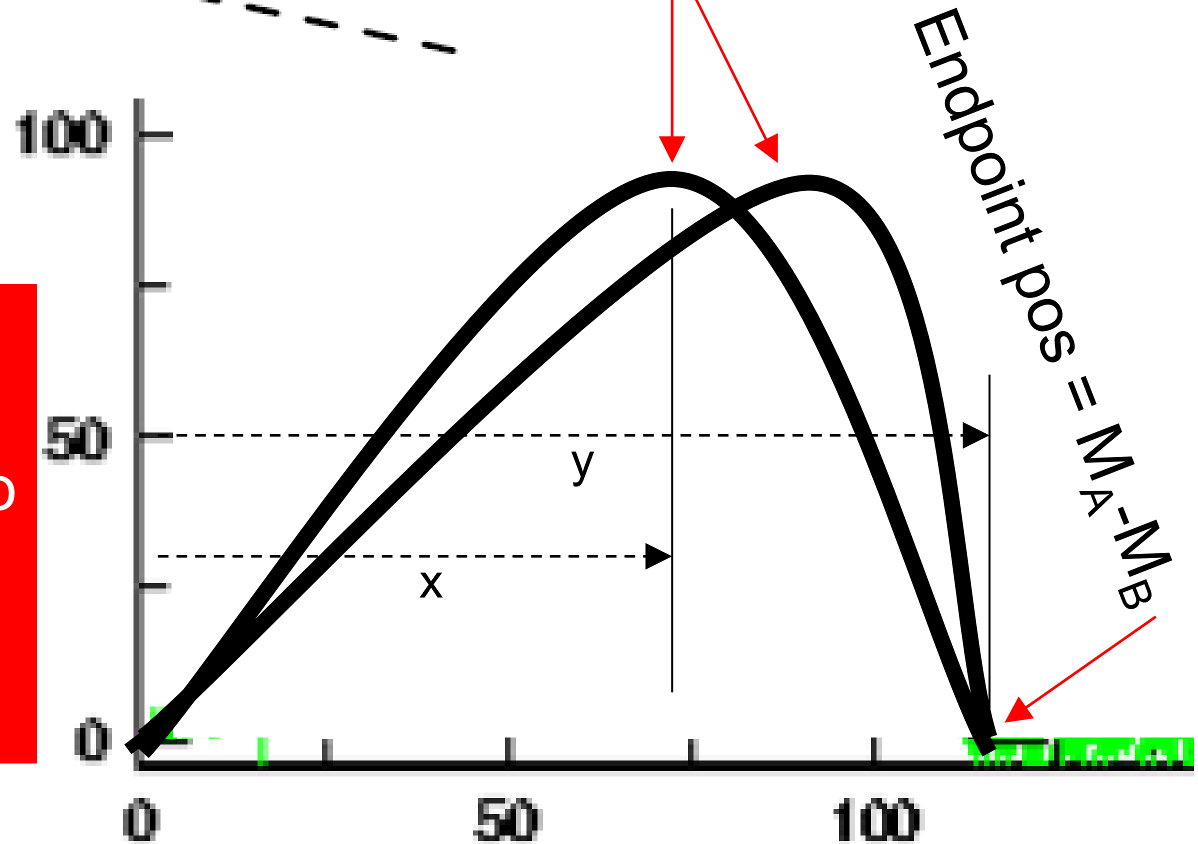


M_A
M_B



100
50
0
0
50
100





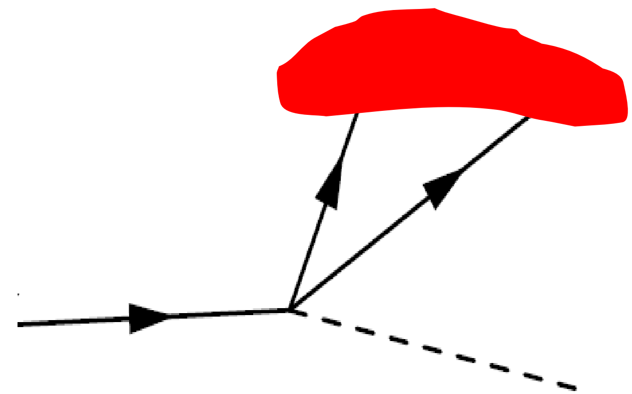
Do we have enough information from shape alone to find M_A and M_B in this three body decay, then?

Yes and no ..

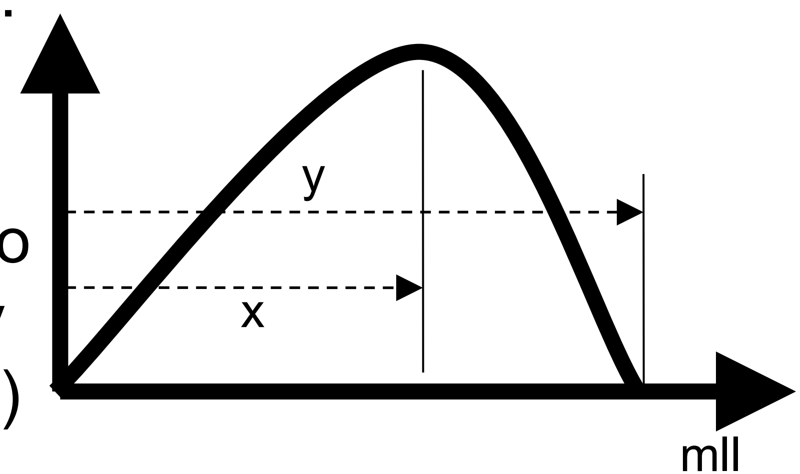
- Putting aside experimental fears concerning efficiency and acceptance corrections …
- … huge errors in the fit, and very poor sensitivity to absolute mass scale. See next exercises.
- This is why endpoints, edges and resonances are good, but shapes less so

Exercises

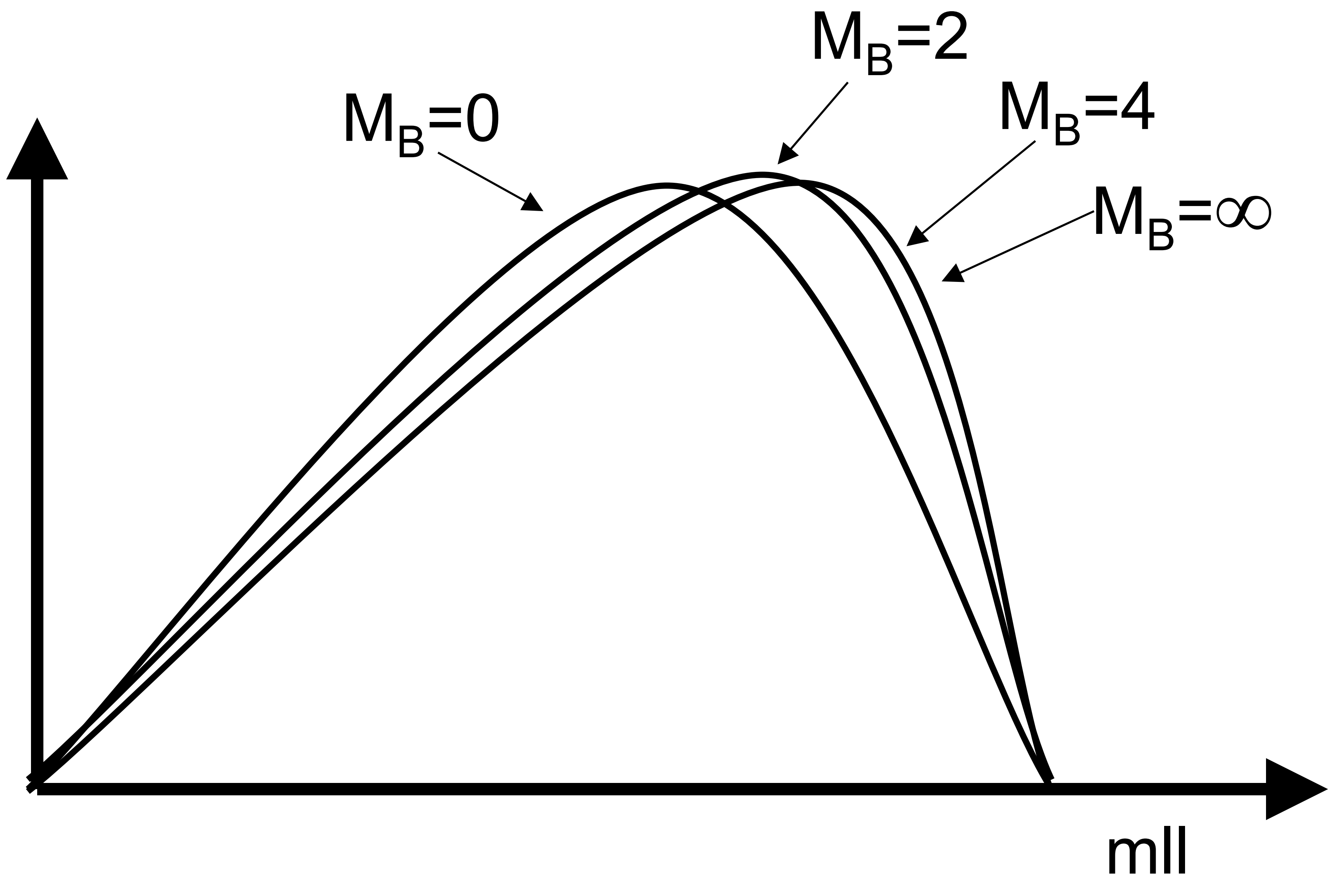
- (12) Determine the shape of the phase space distribution dσ/d(mll) (up to an arbitrary normalizing constant) for the three-body decay shown below. Assume massless visibles, and arbitrary masses for the parent and invisible.



- (13) Prove that r=x/y must lie in the range $1/\sqrt{3} \le r \le 1/\sqrt{2}$. (Note this means r can only move by ±0.06 … not far!)
- (14) Estimate how many events (approximately) would be needed to distinguish two r values differing by 0.012 (i.e. ~1/10th of allowed range)

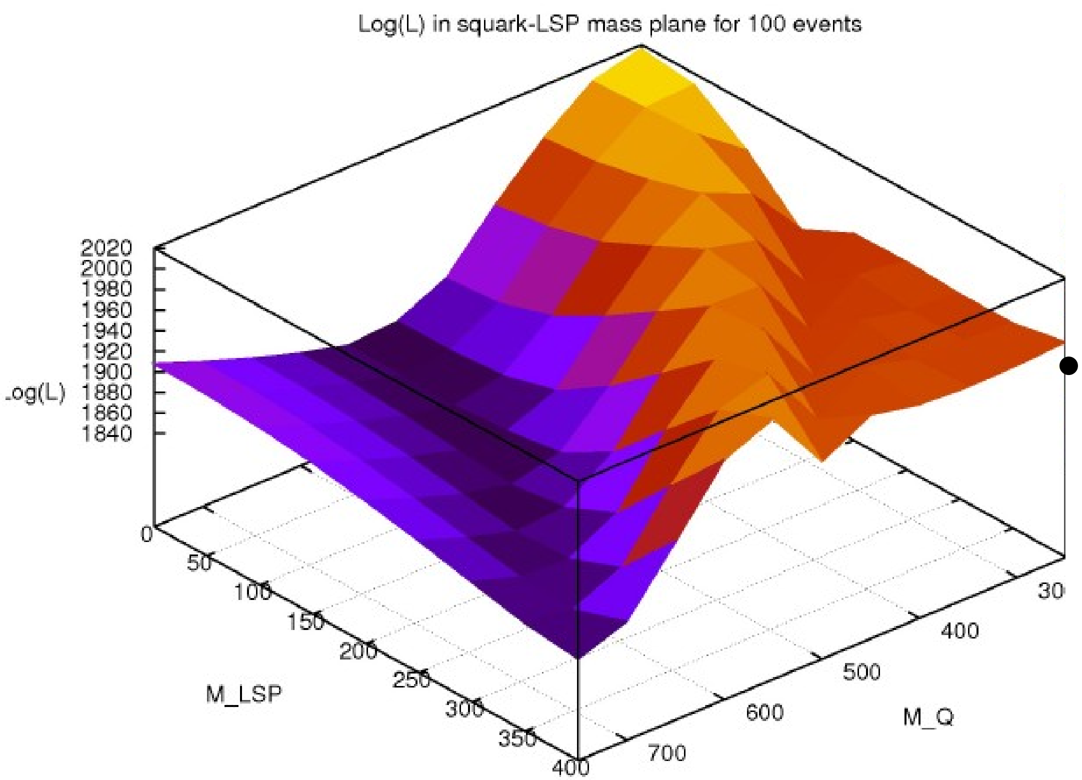


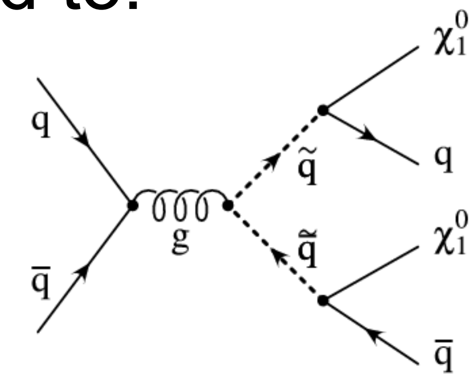
At fixed M_A-M_B you should find



The most detailed "shape" of all is the complete likelihood of the data

- Alwall et.al. (arXiv:0910.2522, arXiv:1010.2263) applied matrix element method to:

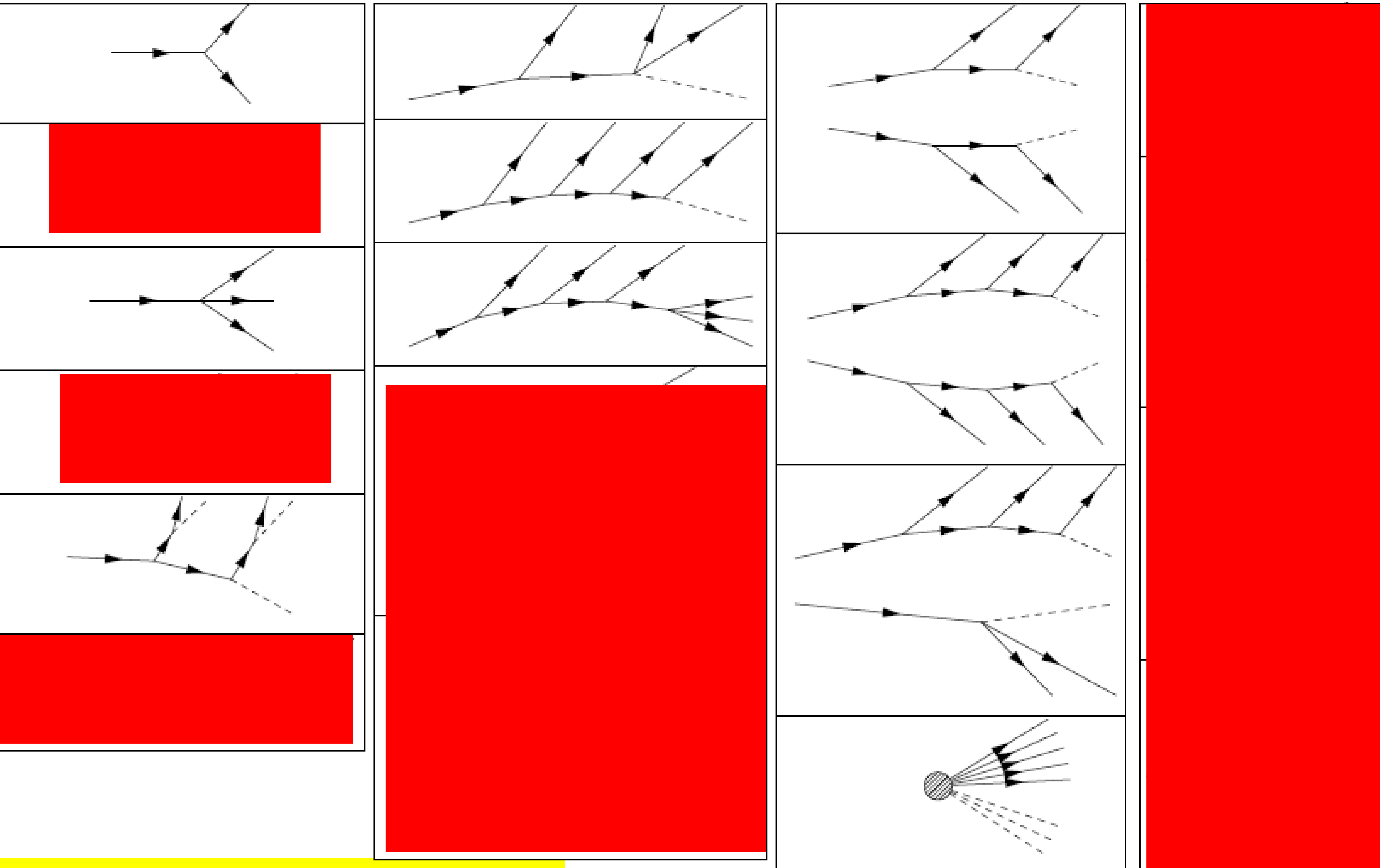




- For ~ 100 events get valley in likelihood surface with same shape as boundary of MT2 distribution

That’s probably enough on mass measurement techniques!

Have only begun to scrape the surface.



(more details in **arXiv:1004.2732**)

Not time to talk about many things

- Parallel and perpendicular MT2 and MCT
- Subsystem MT2 and MCT methods
- Solution counting methods (eg arXiv:0707.0030)
- Hybrid Variables
- Phase space boundaries (arXiv:0903.4371)
- Cusps and Singularity Variables (Ian-Woo Kim)
- Why wrong solutions are often near right ones (arXiv:1103.3438)
- Razors
- and many more!

I have only scratched the surface of the variables that have been discussed. Even the review of mass measurement methods arXiv:1004.2732 makes only a small dent in 70+ pages. However it provides at least an index …

Take home messages

- Lots of approaches to kinematic mass measurement
  - some very general, some very specific.
  - very little of the "detailed stuff" is tested in anger. Experimentalists not universally convinced of utility!
  - very often BGs present serious impediment.
  - theorists and experimenters should pay close attention to zone of applicability
- BUT
  - Finding sensible variables buys more than just mass measurements - e.g. signal sensitivity

Extras if time …

Notes:

- At TASI 2010: 75 mins per lecture:
- Lec1: 1-73 (73 slides)
- Lec2: 74-183 (110 slides)
- Lec3: 184-224 (41 slides) on masses
  - then segue into spins for another 40

Other MT2 related variables (1/3)

- MCT ("Contralinear-Transverse Mass") (arXiv:0802.2879)
  - Is equivalent to MT2 in the special case that there is no missing momentum (and that the visible particles are massless).
  - Proposes an interesting multi-stage method for measuring additional masses
  - Can be calculated fast enough to use in ATLAS trigger.

Other MT2 related variables (2/3)

- MTGEN ("MT for GENeral number of final state particles") (arXiv:0708.1028)
  - Used when
    - each "side" of the event decays to MANY visible particles (and one invisible particle) and
    - it is not possible to determine which decay product is from which side … all possibilities are tried
- Inclusive or Hemispheric MT2 (Nojirir + Shimizu) (arXiv:0802.2412)
  - Similar to MTGEN but based on an assignment of decay product to sides via hemisphere algorithm.
  - Guaranteed to be >= MTGEN

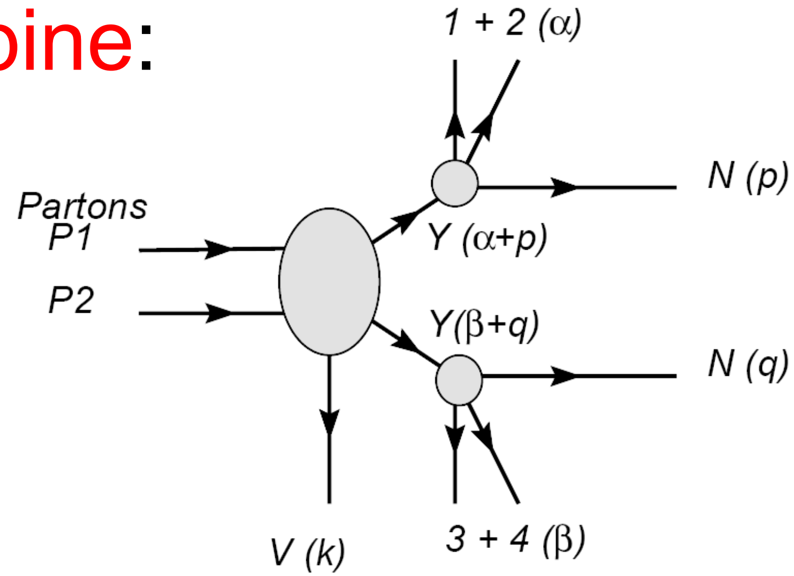
Other MT2 related variables (3/3)

- M2C ("MT2 Constrained") arXiv:0712.0943 (wait for v3 ... there are some problems with the v1 and v2 drafts)
- M2CUB ("MT2 Constrained Upper Bound") arXiv:0806.3224

- There is a sense in which these two variables are really two sides of the same coin.
  - if we could re-write history we might name them more symmetrically
  - I will call them m_{Small} and m_{Big} in this talk.

m_{Small} and m_{Big}

- Basic idea is to combine:
  - MT2



- with
  - a di-lepton invariant mass endpoint measurement (or similar) providing:

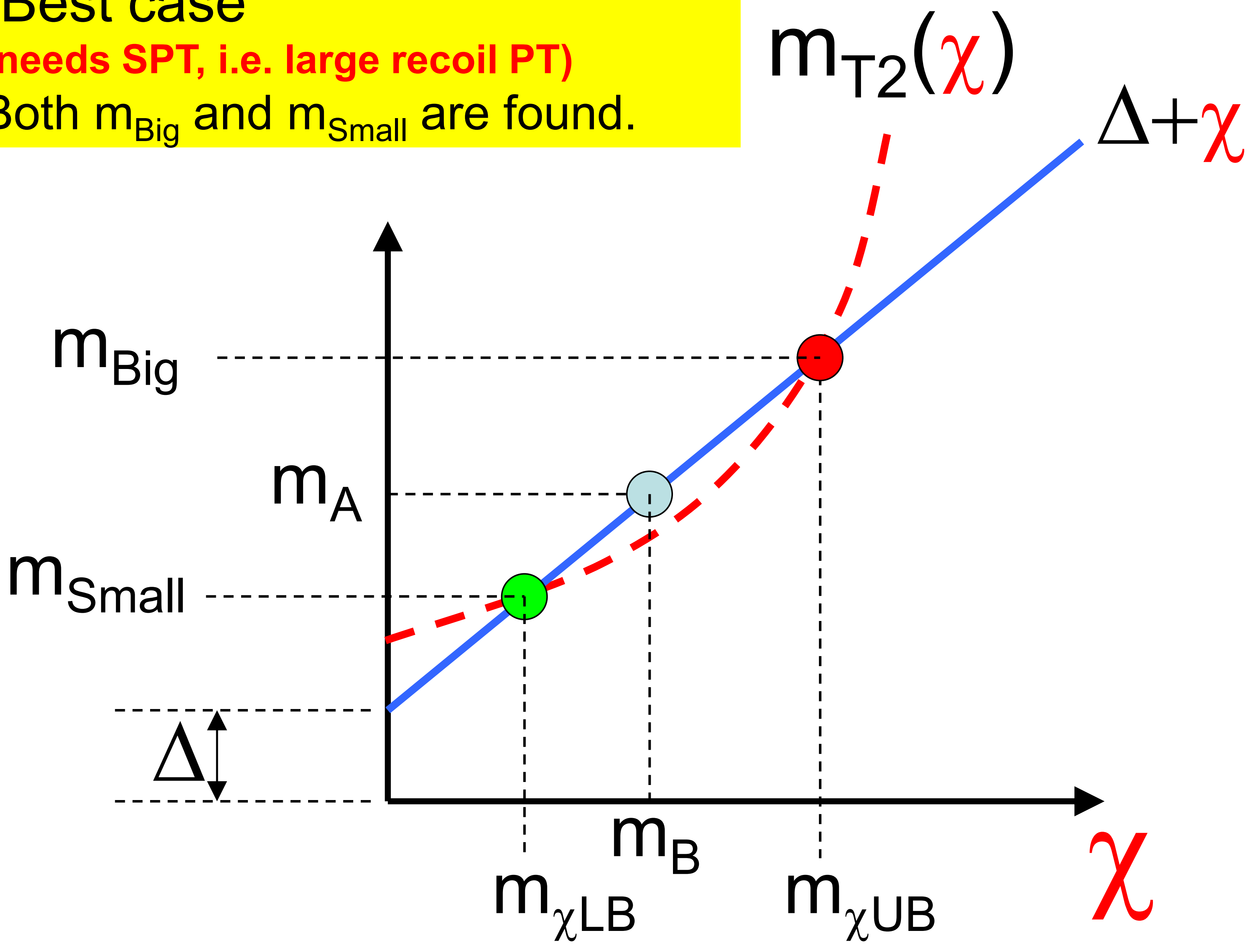
$$\Delta = M_A - M_B$$

(or M_Y-M_N in the notation of their figure above)

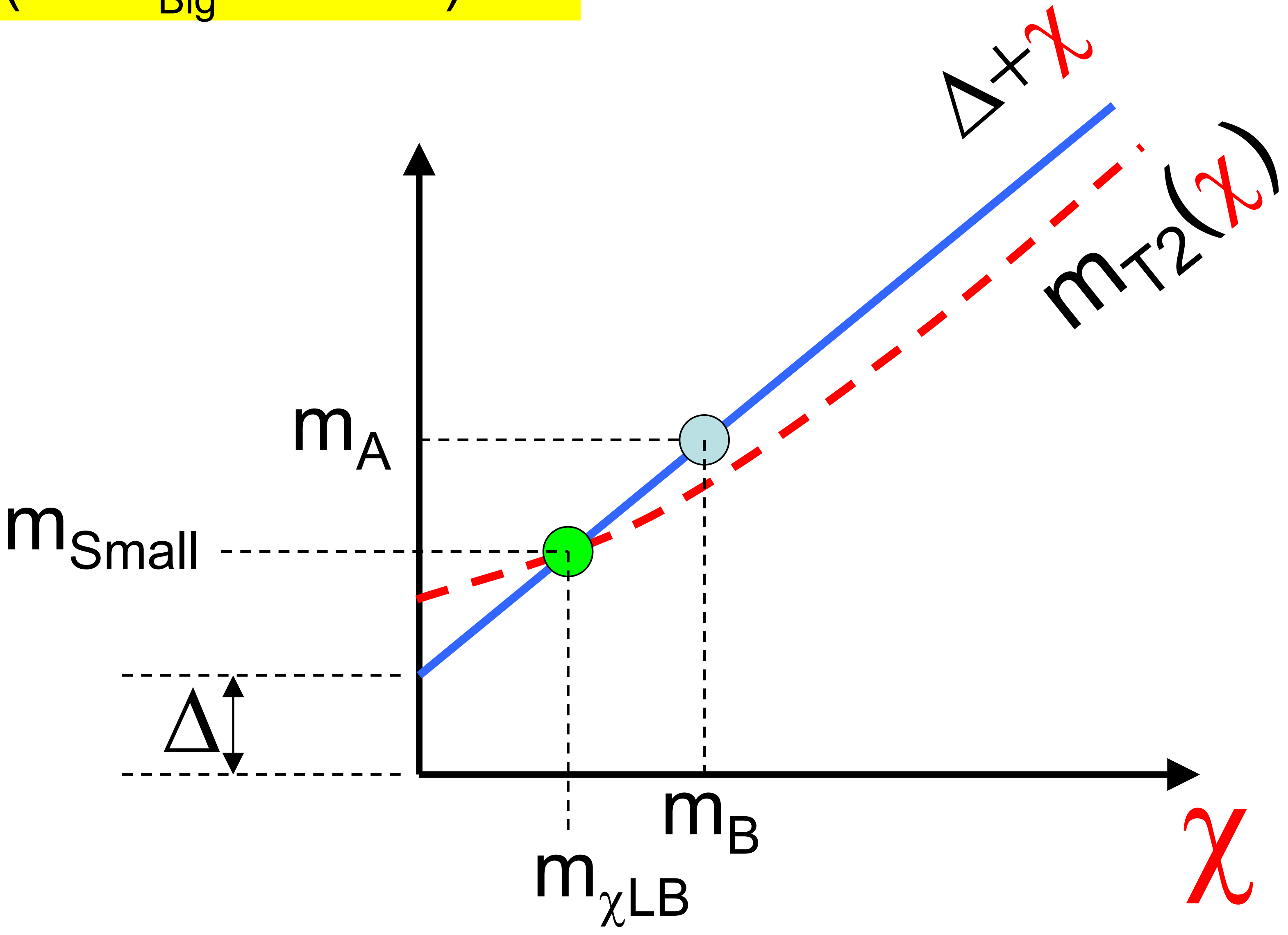
"Best case"

(needs SPT, i.e. large recoil PT)

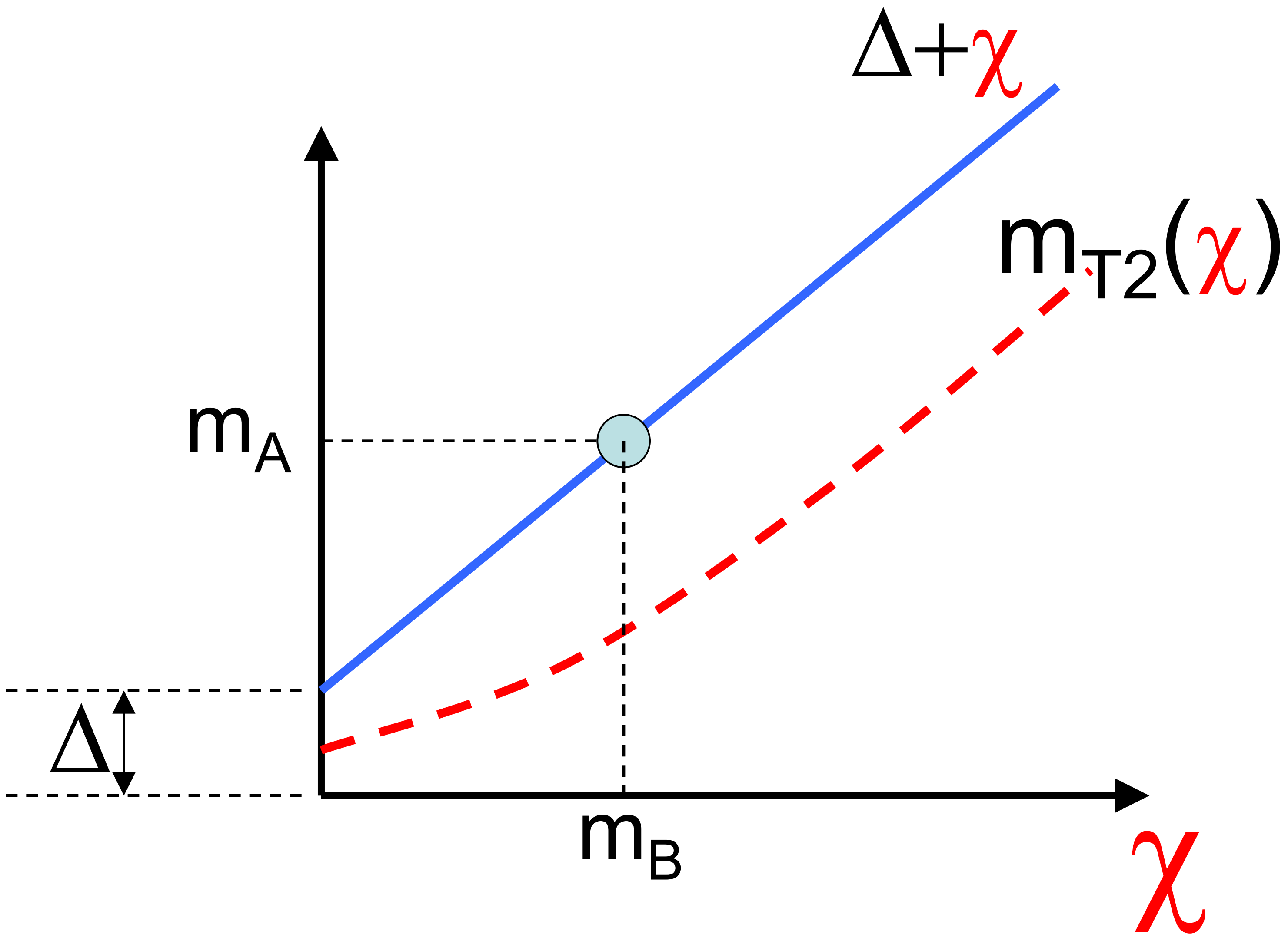
Both m_{Big} and m_{Small} are found.



"Typical ZPT case"
(no m_{Big} is found)

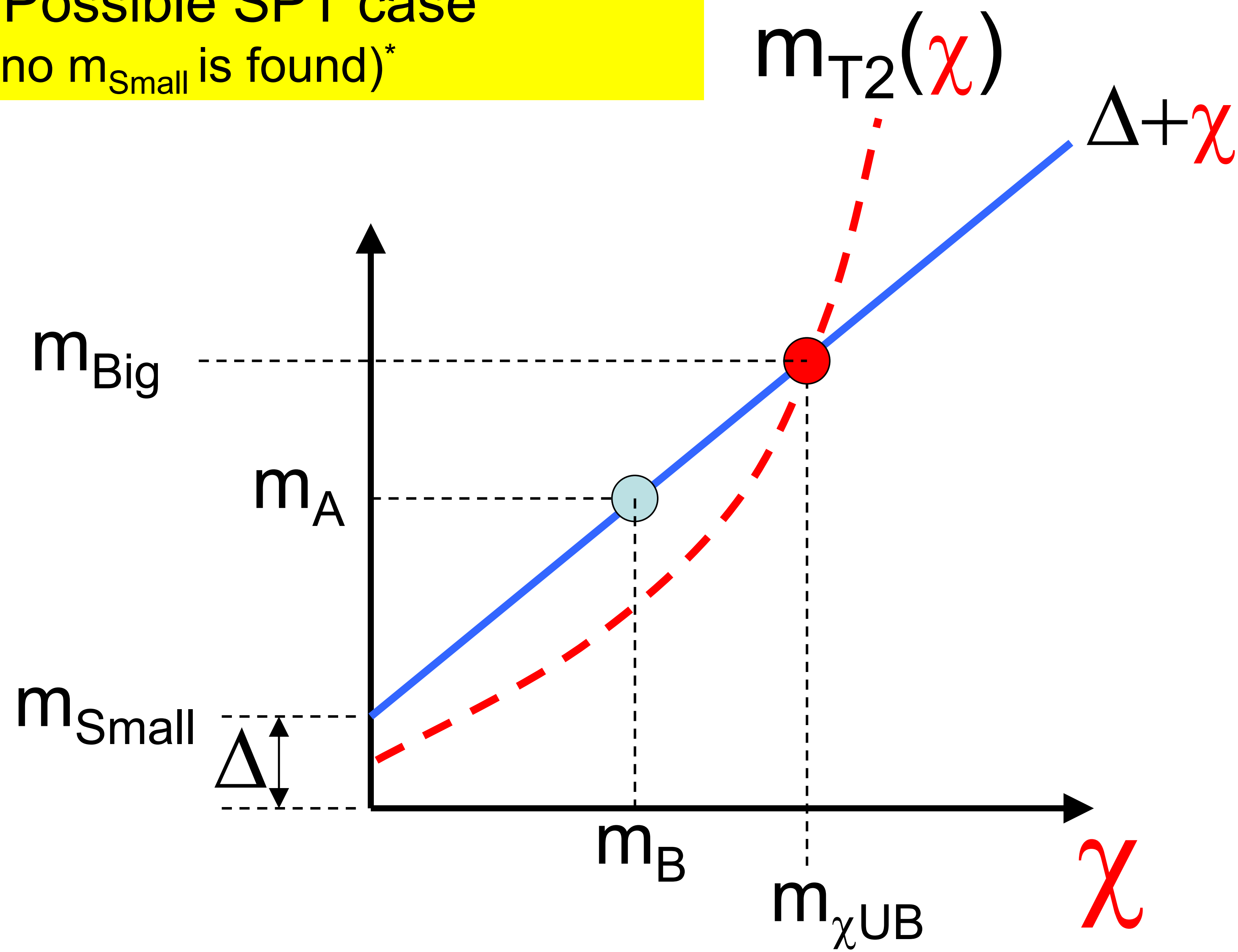


“Possible ZPT case”
(neither m_{Big} nor m_{Small} is found)*



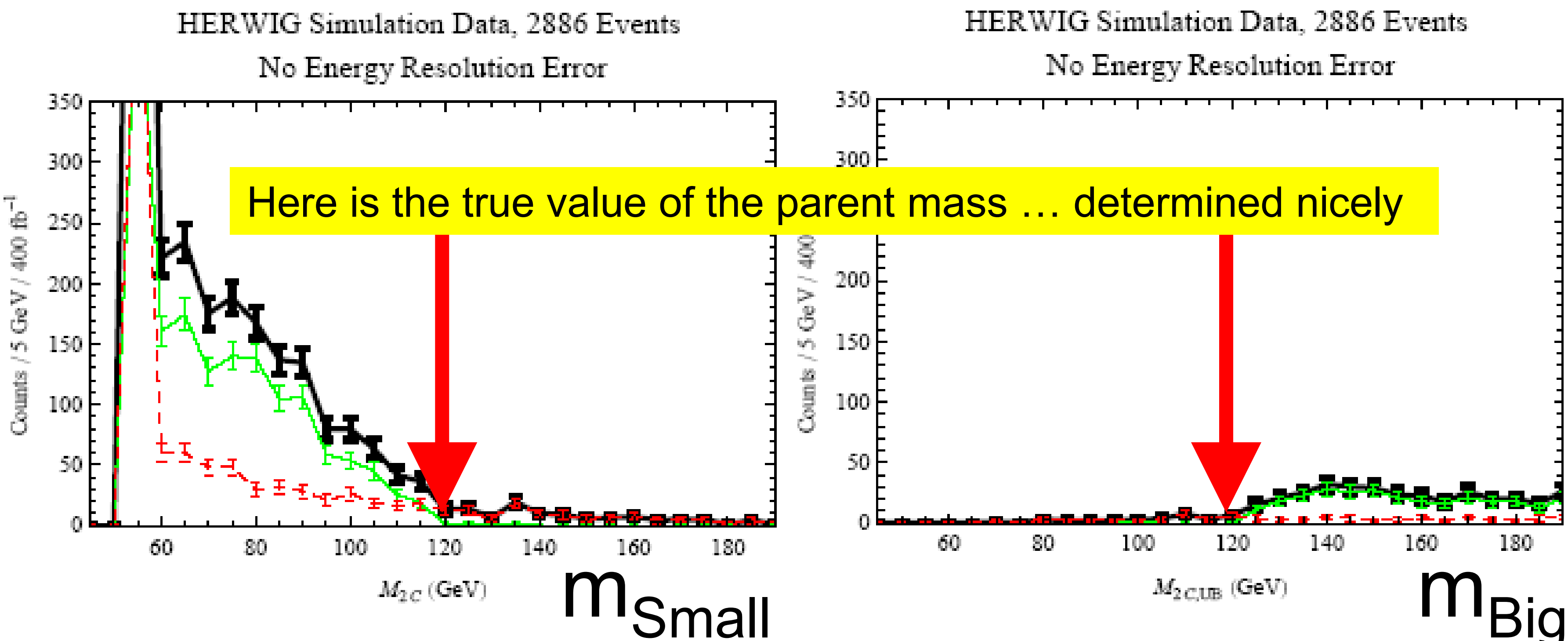
* Except for conventional definition of m_{Small} to be Δ in this case.

“Possible SPT case”
(no m_{Small} is found)*



* Except for conventional definition of m_{Small} to be Δ in this case.

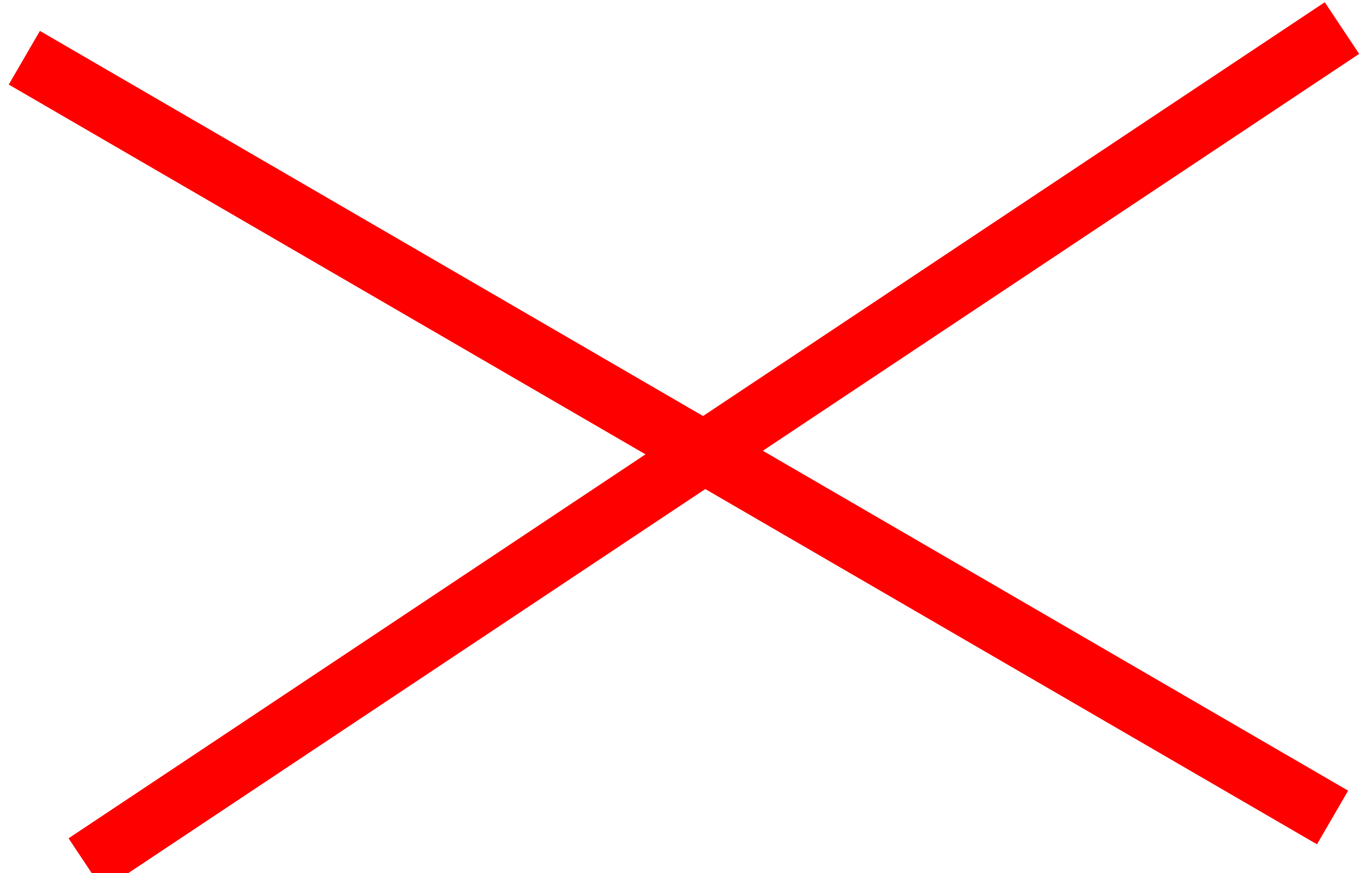
What m_{Small} and m_{Big} look like,
and how they determine the parent mass



arXiv:0806.3224

Outcome:

- m_{Big} provides the first potentially-useful event-by-event upper bound for m_A
  - (and a corresponding event-by-event upper bound for m_B called $m_{\chi UB}$)
- m_{Small} provides a new kind of event-by-event lower bound for m_A which incorporates consistency information with the dilepton edge
- m_{Big} is always reliant on SPT (large recoil of interesting system against "up-stream momentum") – cannot ignore recoil here!



LHC Specific problems

- Hadron Collider – z-boost of COM unknown
- Pile up, multiple interactions
- Production of many new particles at once?

- Multiple massive stable invisible particles?

What sort of parameter spaces?

- High dimensional
- At the very least, 8 dims
- More like ~100 dims

- No really compelling reasons to believe in any particular simple model

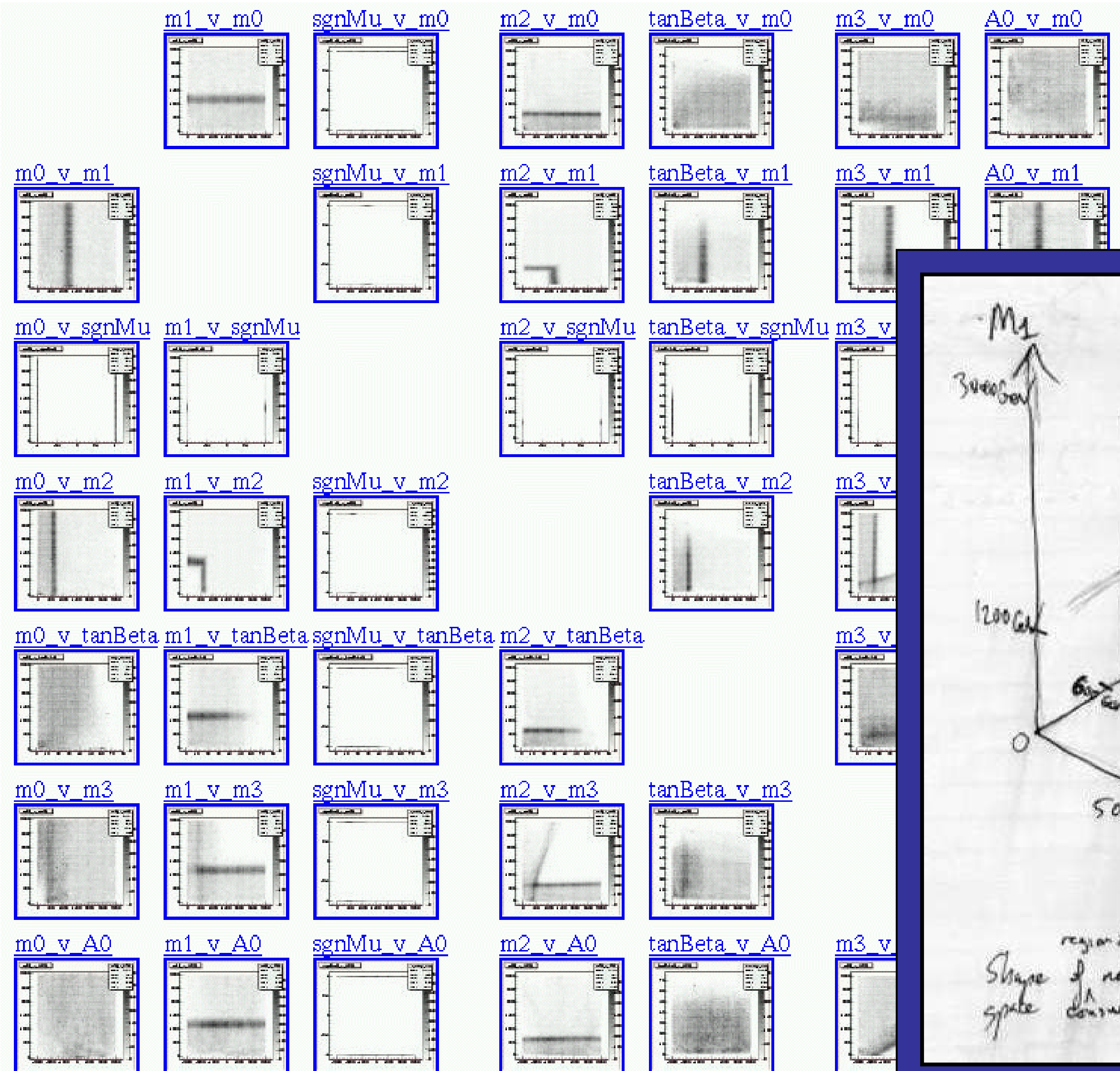
SUSY params:

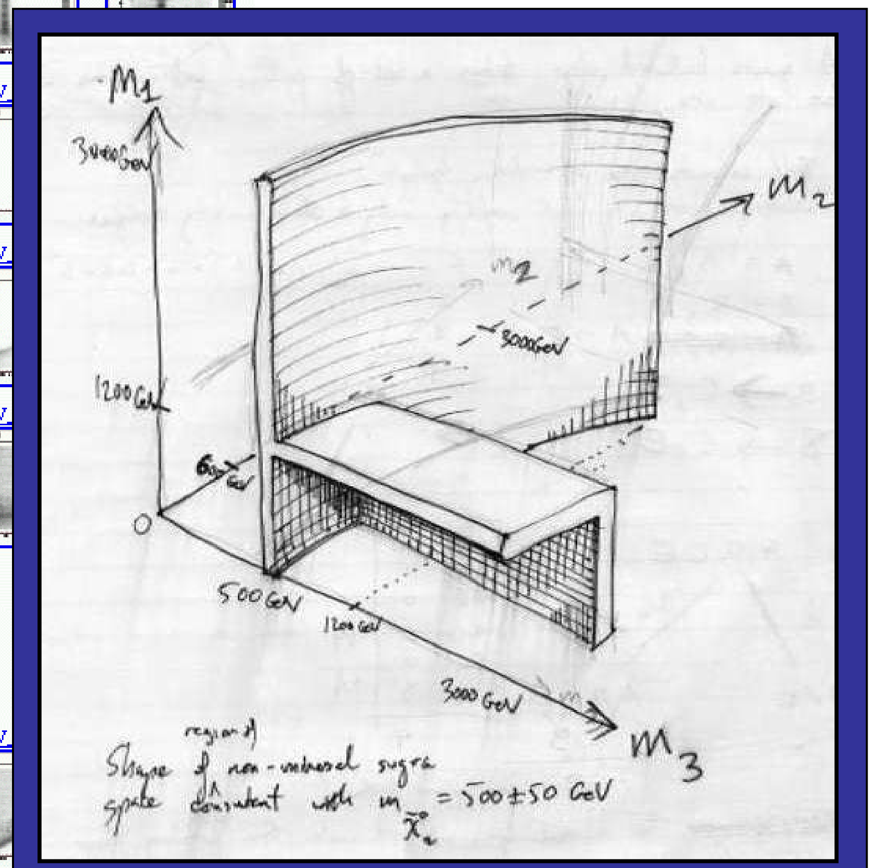
- m_0
- $M_{1/2}$
- A_0
- Tan beta
- Sgn μ

SM params:

- m_b
- m_t
- $\alpha_s(M_z)$

Unusual parameter spaces!





Shape of typical set is often something quite horrible.

Contrast with UA1/UA2

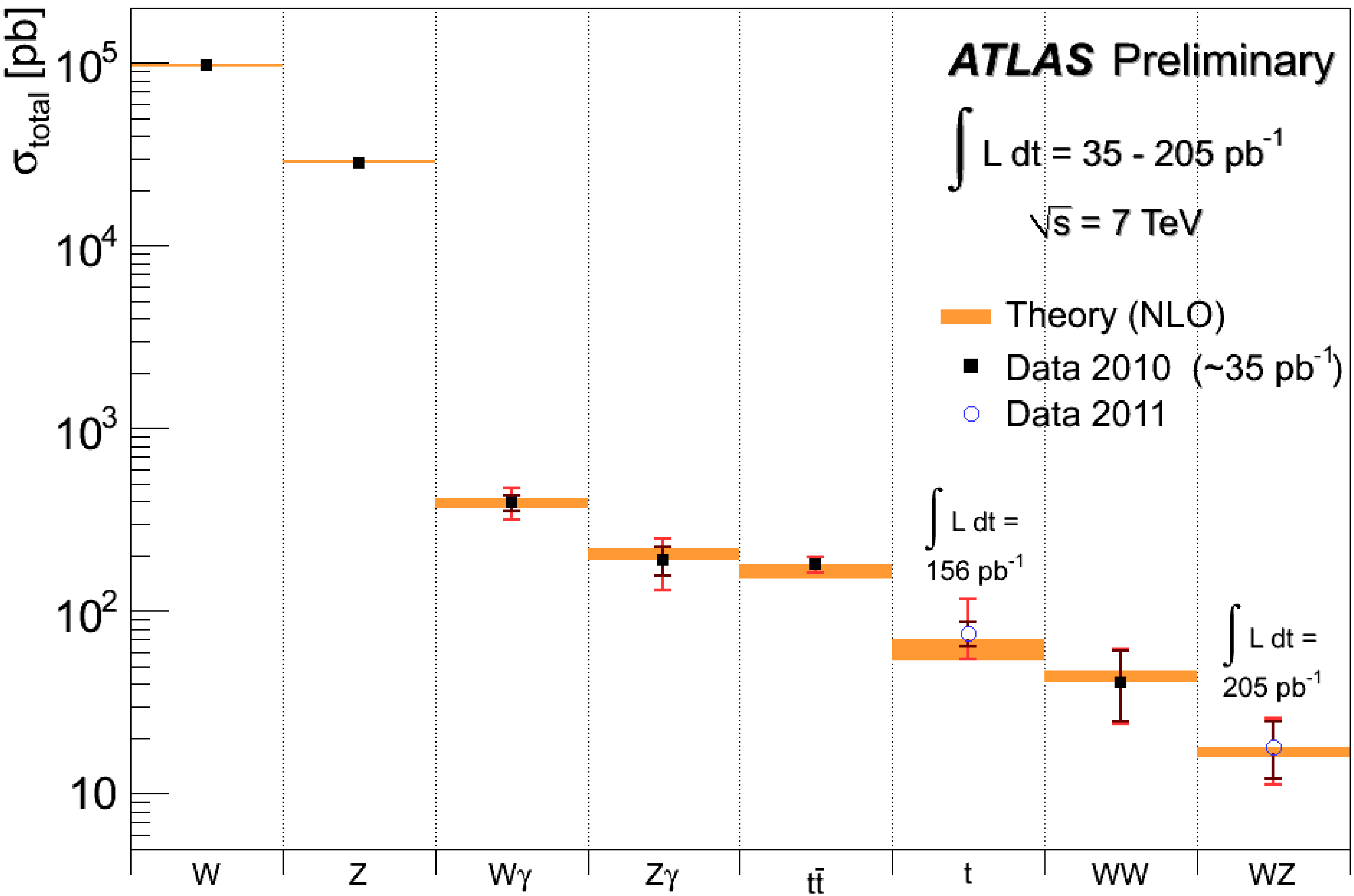
- Glashow Wienberg Salam: Phys Rev Lett 19, 1264 (1967)
  - Predictions in terms of (then) unknown θ_W:
  - M_Z > 75 GeV/c^2, $M_W > 35$ GeV/c^2
- By 1982 θ_W much constrained, giving:
  - $M_Z \approx 92 \pm 2$ GeV/c^2, $M_W \approx 82 \pm 2$ GeV/c^2
- CERN able to build UA1+UA2 (~1980) knowing the above.
- In 1983 UA1+UA2 observe W and Z at expected masses:
  - $M_Z \approx 95 \pm 3$ GeV/c^2, $M_W \approx 81 \pm 5$ GeV/c^2

A personal view of some of the recent ATLAS results

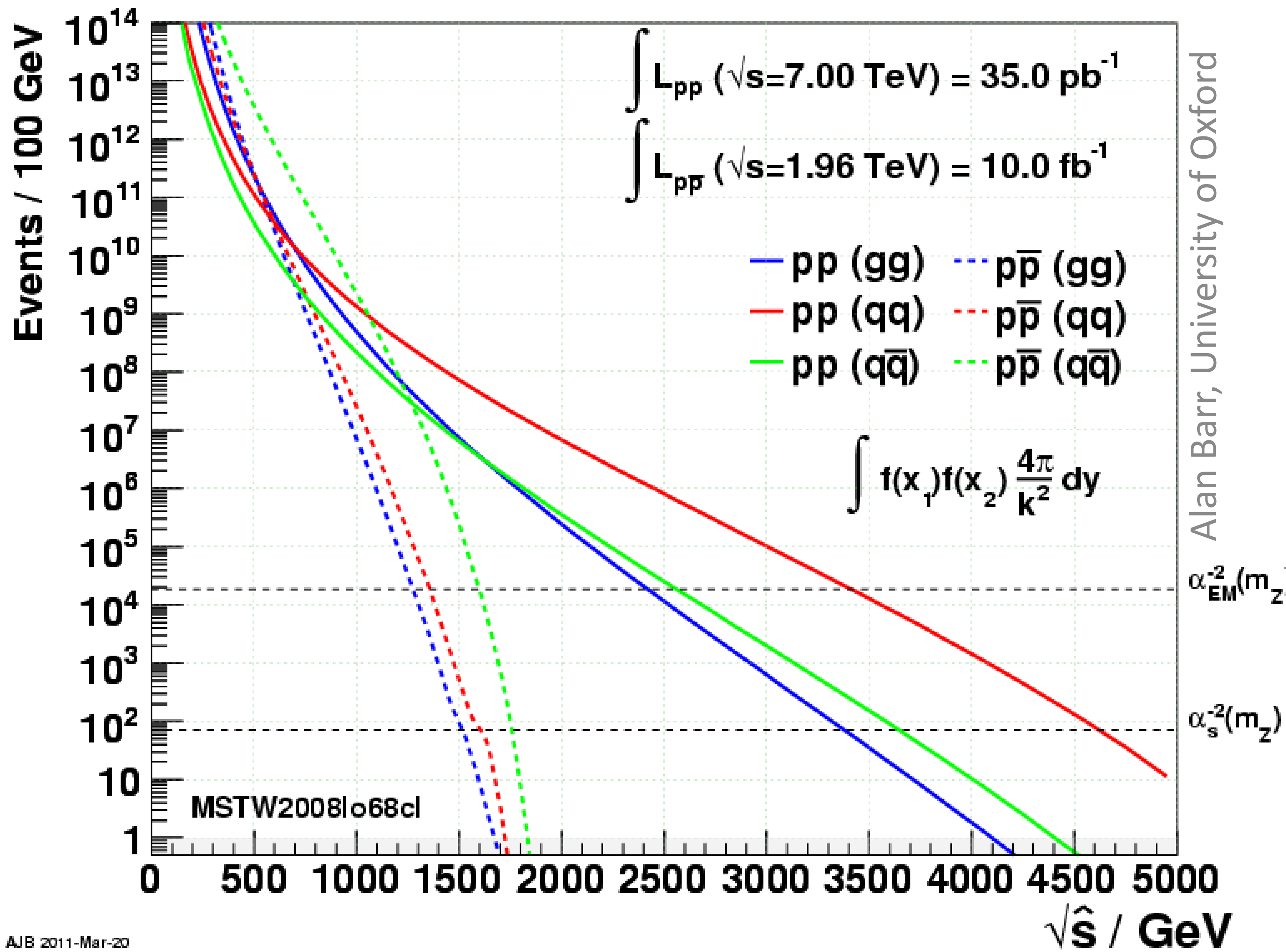
(unashamed focus on new physics searches)

Christopher Lester

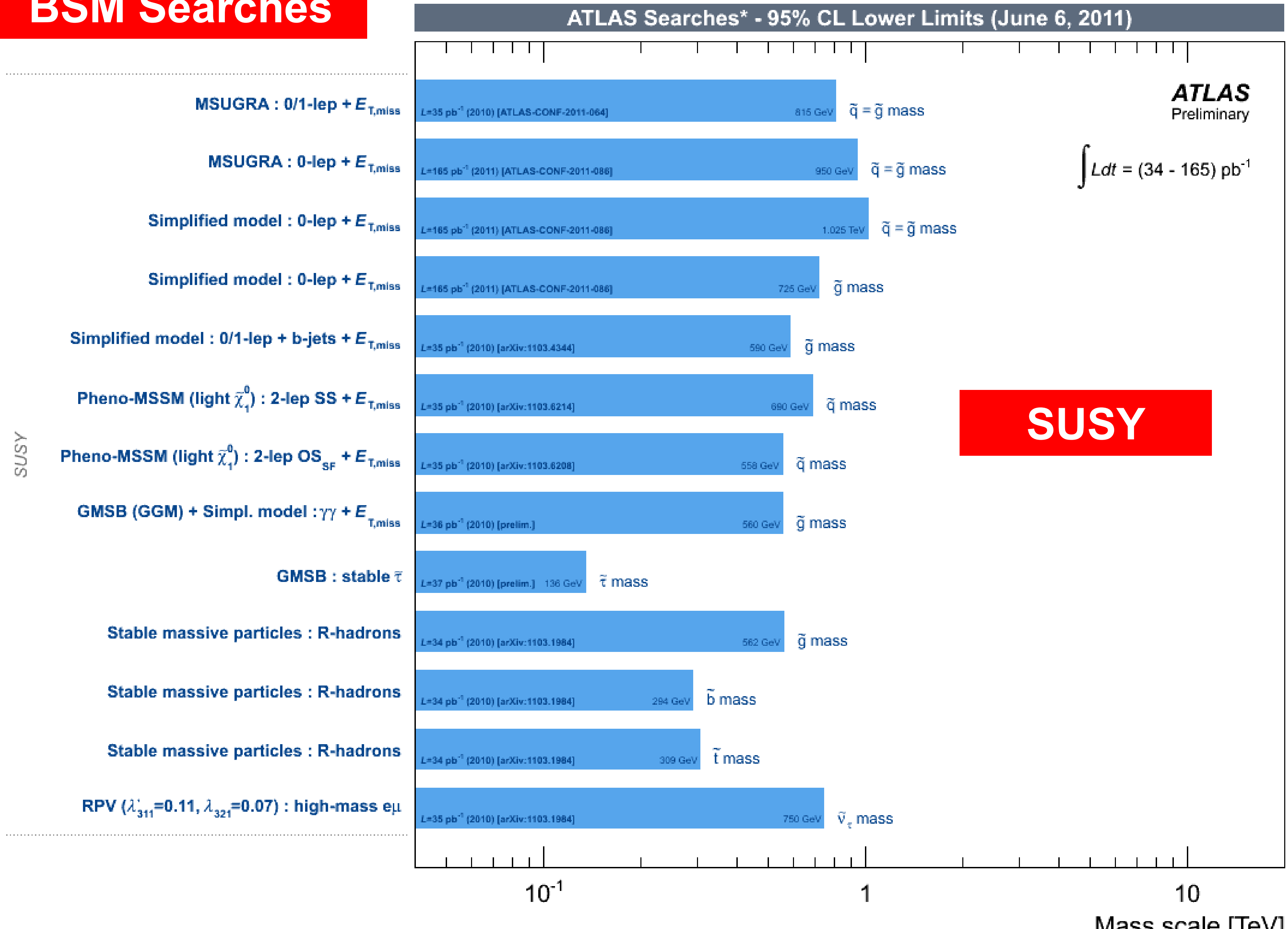
Inclusive weak boson and top quark cross section measurements by ATLAS



Parton-parton luminosity

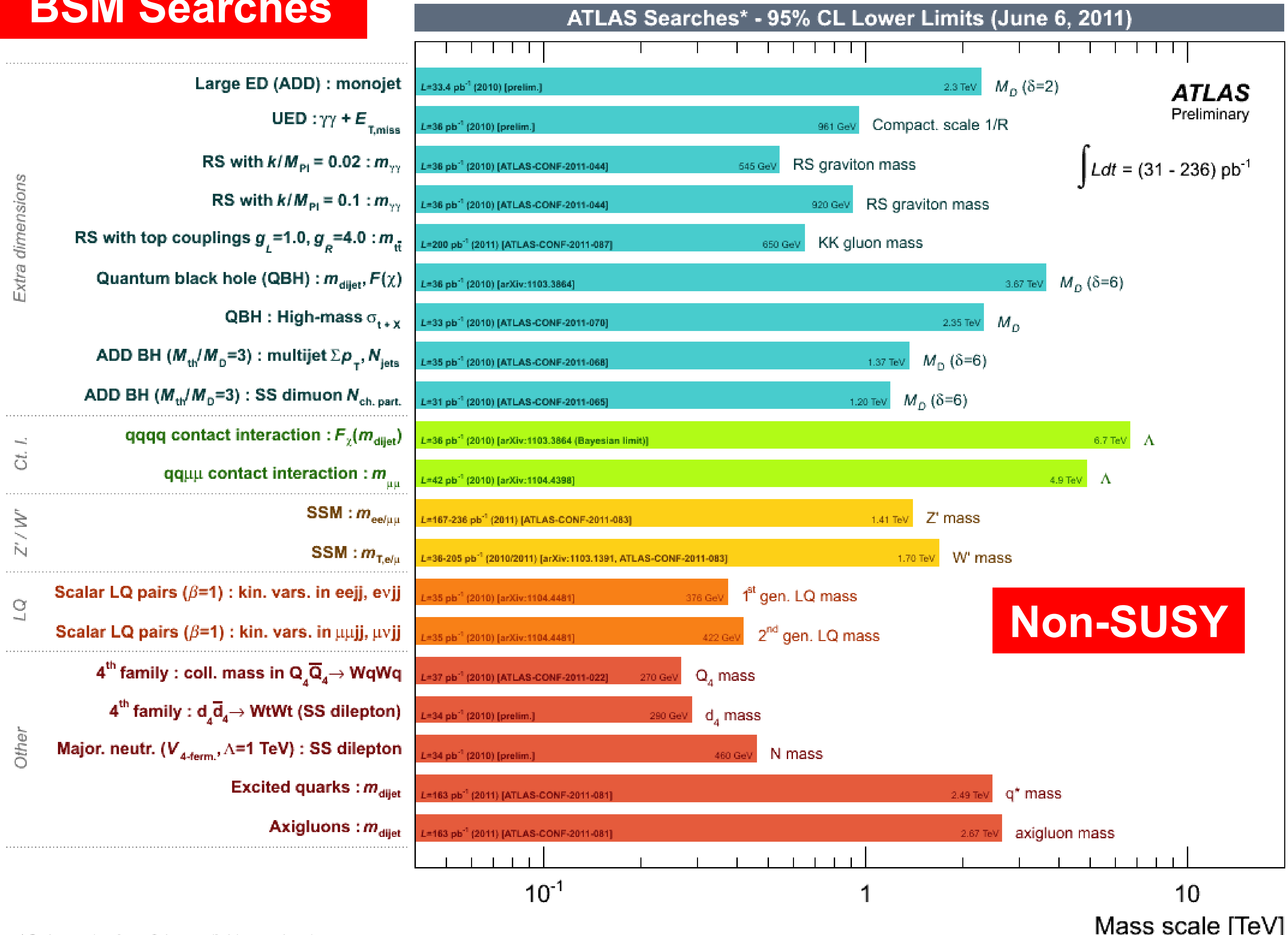


BSM Searches



*Only a selection of the available results shown

BSM Searches

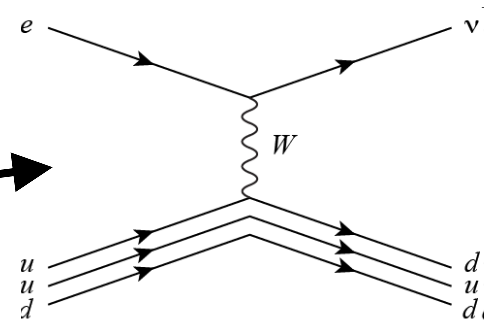


*Only a selection of the available results shown

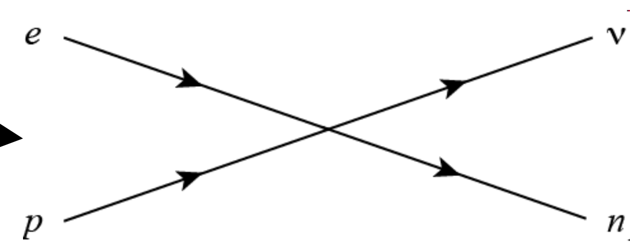
Contact interactions

Fermi theory example:

At low energies, this



looked like this:



… but now know that G_F is order one coupling suppressed by powers of W mass.

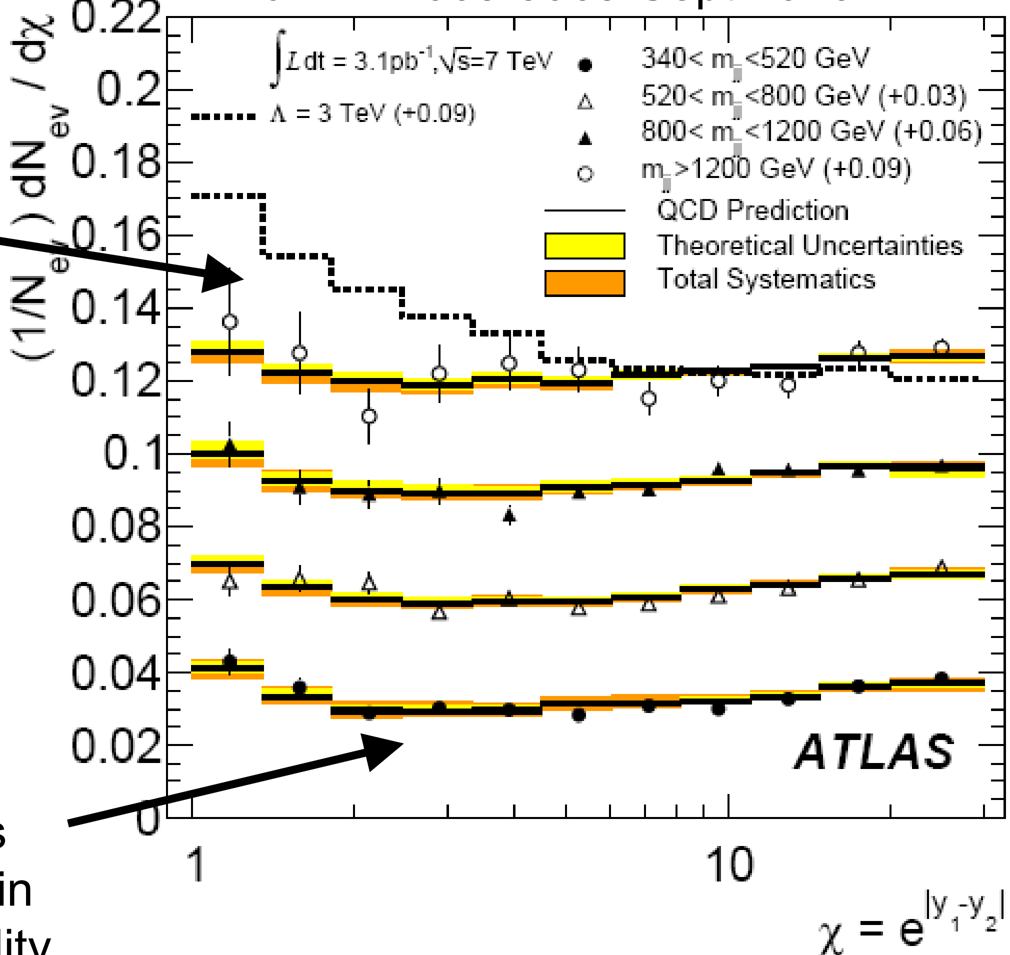
$$\frac{G_{\mathrm{F}}}{(\hbar c)^3} = \frac{\sqrt{2}}{8}\frac{g^2}{m_{\mathrm{W}}^2}$$

Can do the same sort of thing for “four quark vertex” to constrain new mass scale.

3.4 TeV contact interactions were excluded by 3/pb of data (95% CL)

qqqq contact interactions peak at small jet-jet rapidity differences

arXiv:1009.5069 Sept 2010



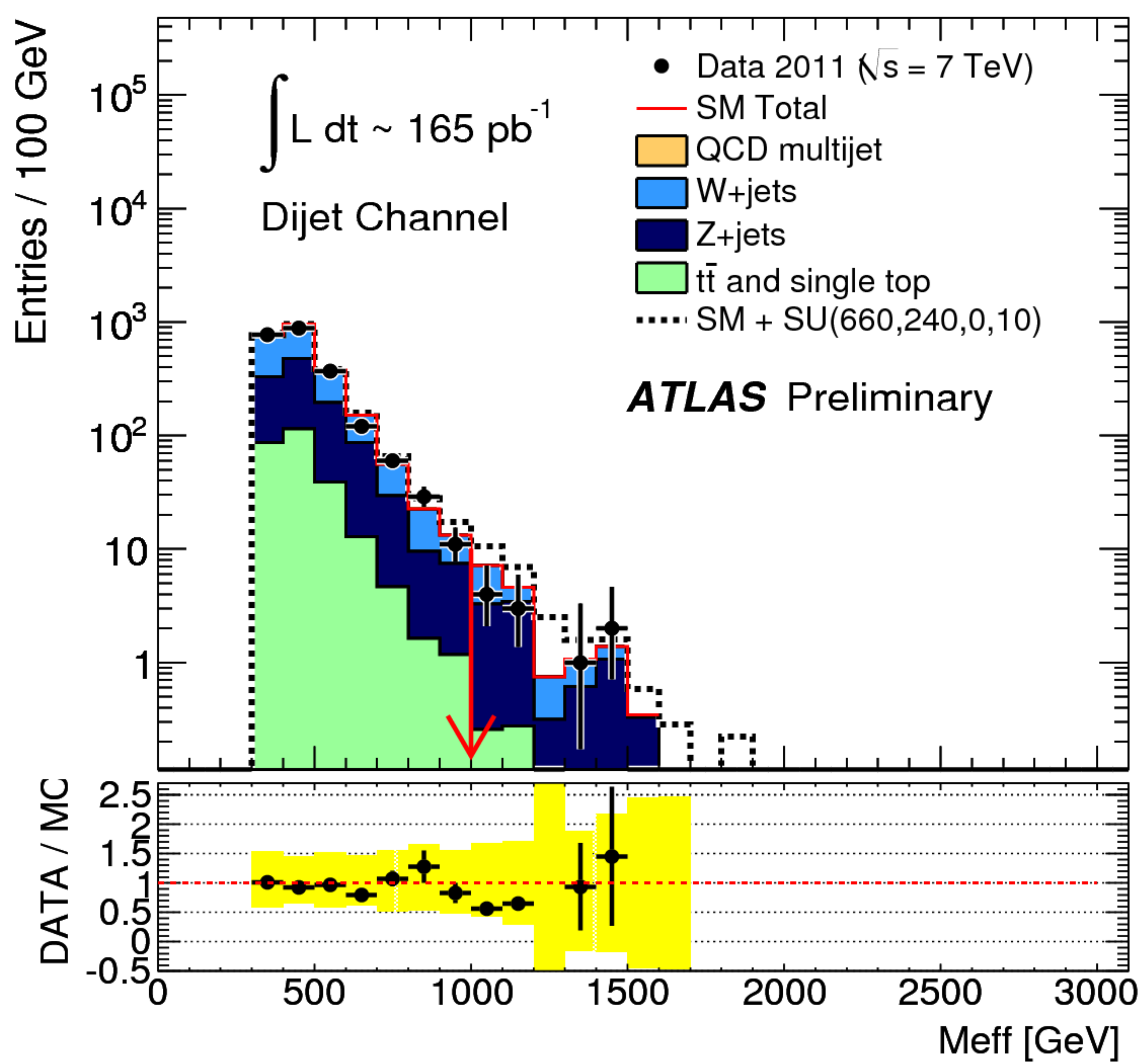
SM QCD is mostly flat in jet-jet rapidity difference

y_1 and y_2 are rapidities of the two jets.

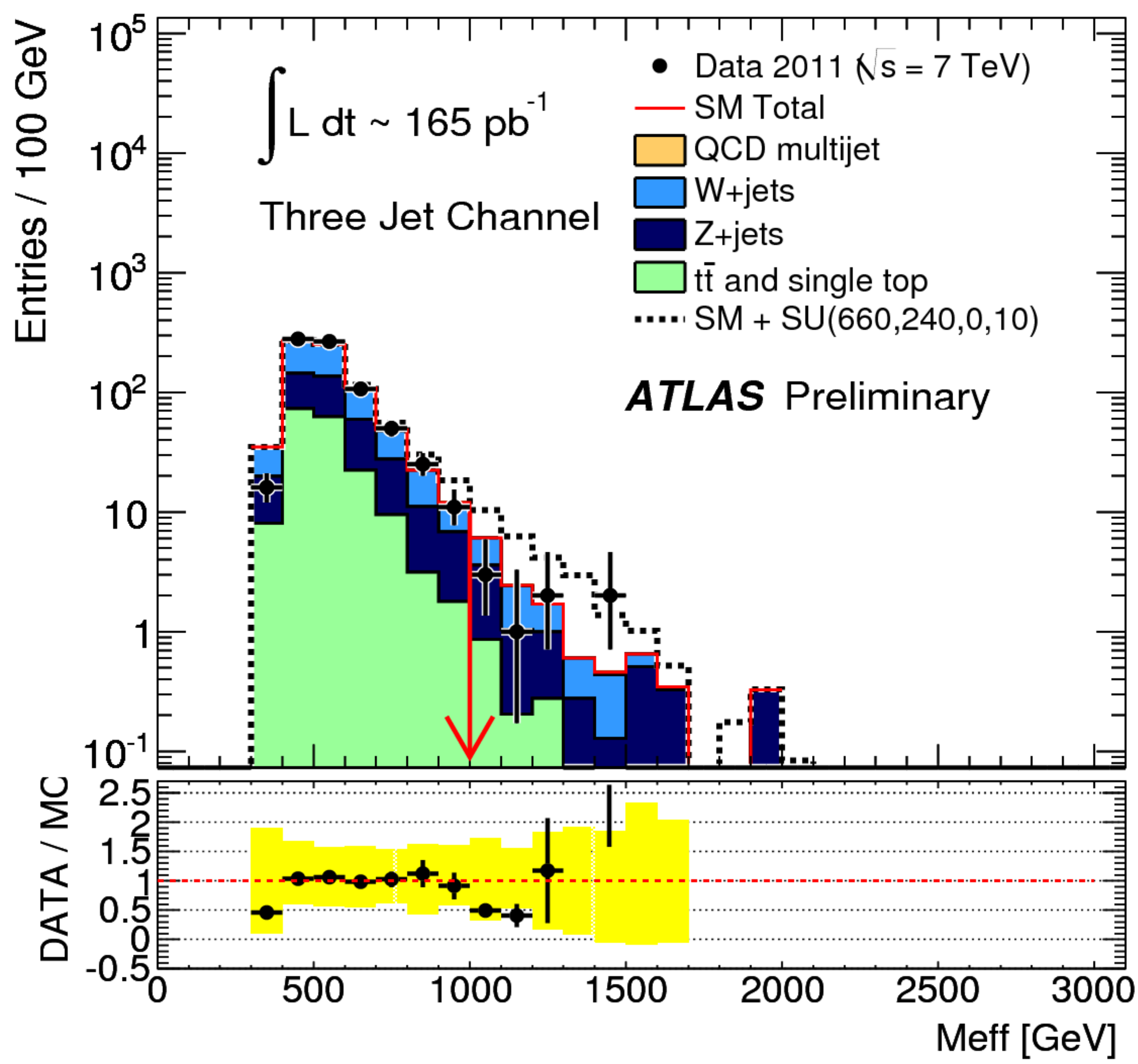
qqqq limit increases to 9.5 TeV with 36/pb
(Mar 2011: arXiv:1103.3864)

and

qqμμ limit is at 4.5 TeV with 42/pb
(April 2011: arXiv:1104.4398)



Latest ATLAS 0-lepton, jets, missing transverse momentum data.

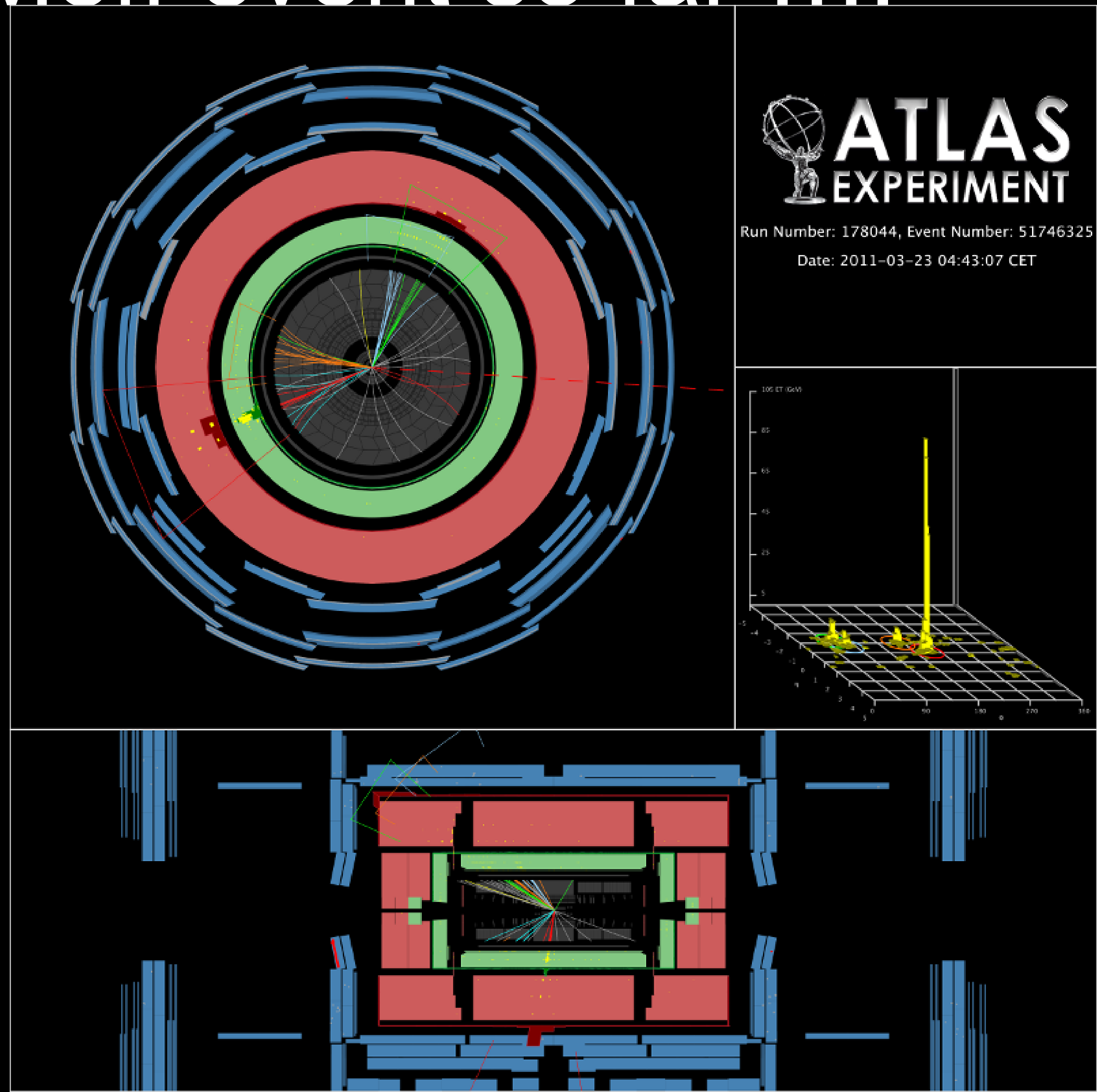


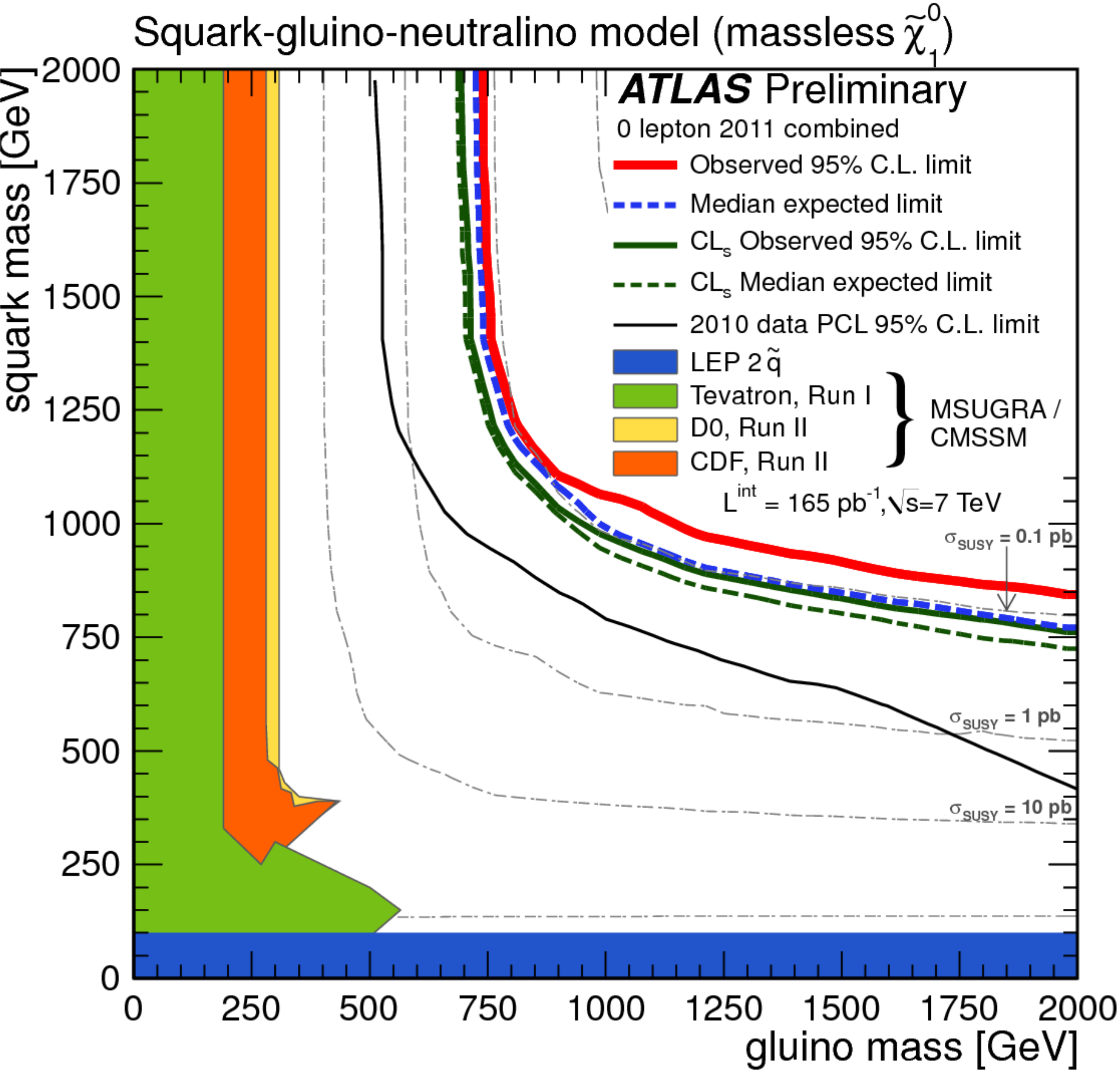
Latest ATLAS 0-lepton, jets, missing transverse momentum data.

Highest Meff event so far ....

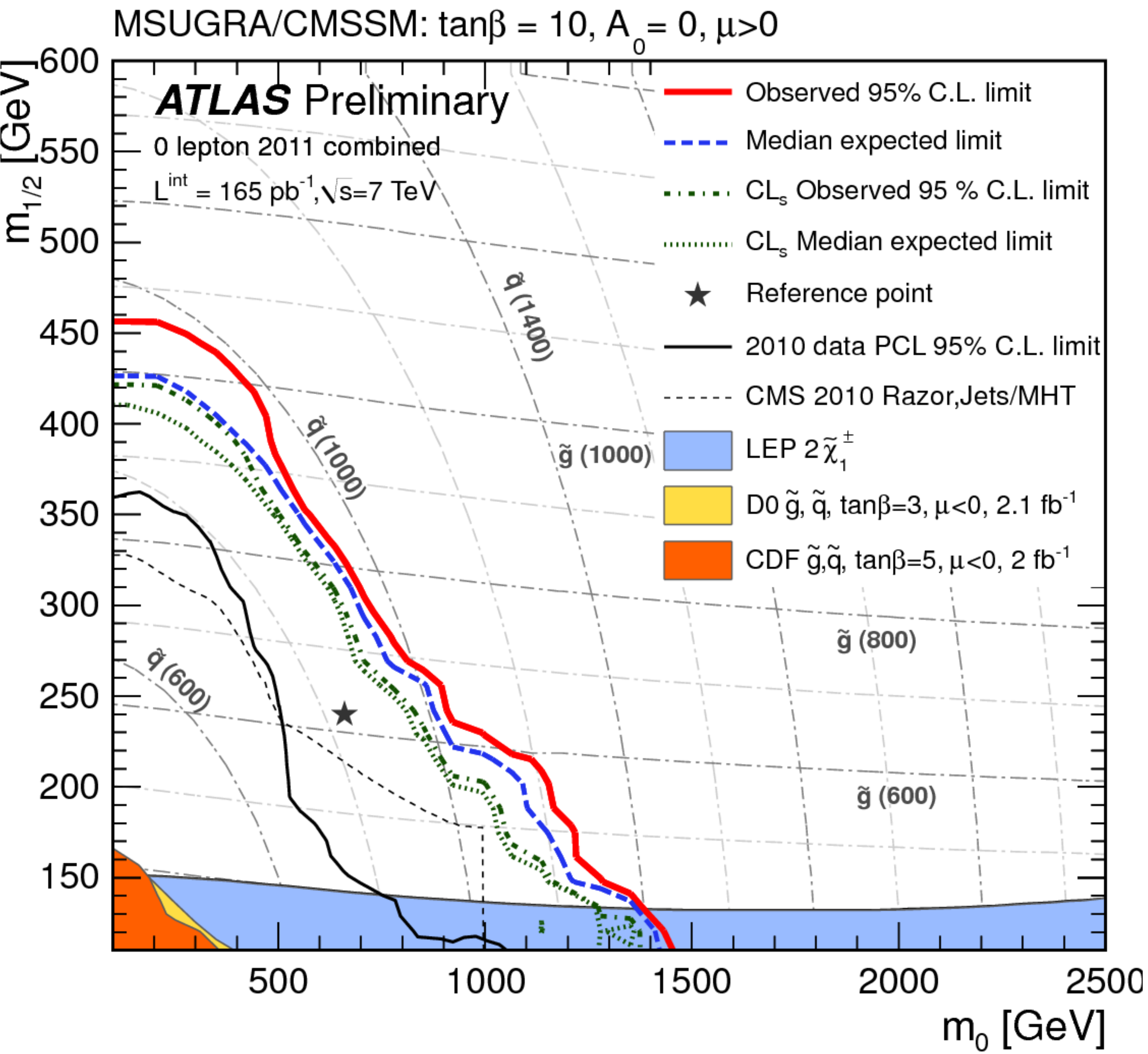
The highest Meff in any (supposedly "clean") ATLAS event is 1548 GeV

- calculated from four jets with pts:
  - 636 GeV
  - 189 GeV
  - 96 GeV
  - 81 GeV
- 547 GeV of missing transverse momentum.





Latest ATLAS 0-lepton, jets, missing transverse momentum data.

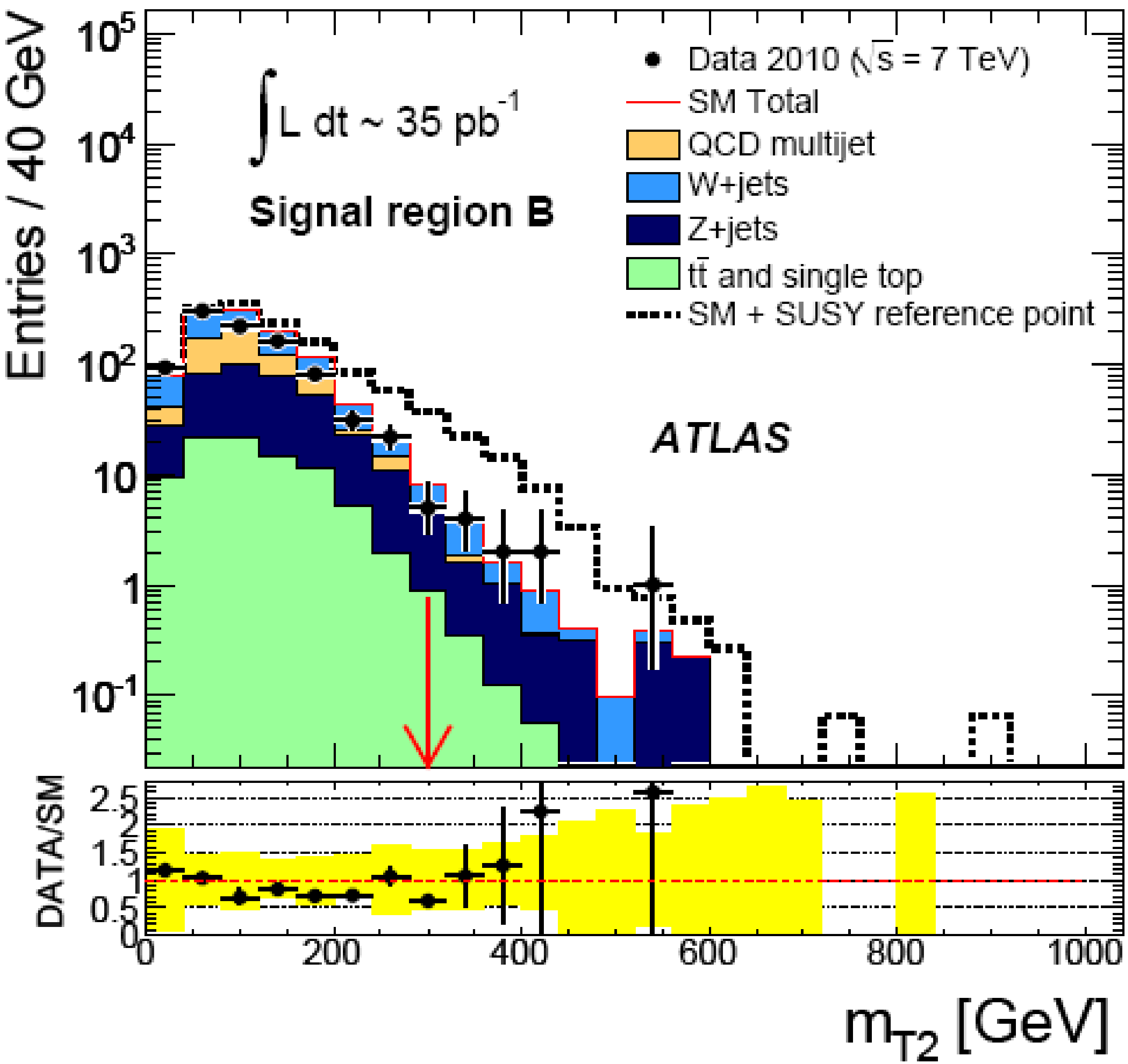


Latest ATLAS 0-lepton, jets, missing transverse momentum data.

Less well tested areas

- neutralino mass close to squark or gluino mass
- signatures with not many jets

So far MT2 only competitive at 35/pb

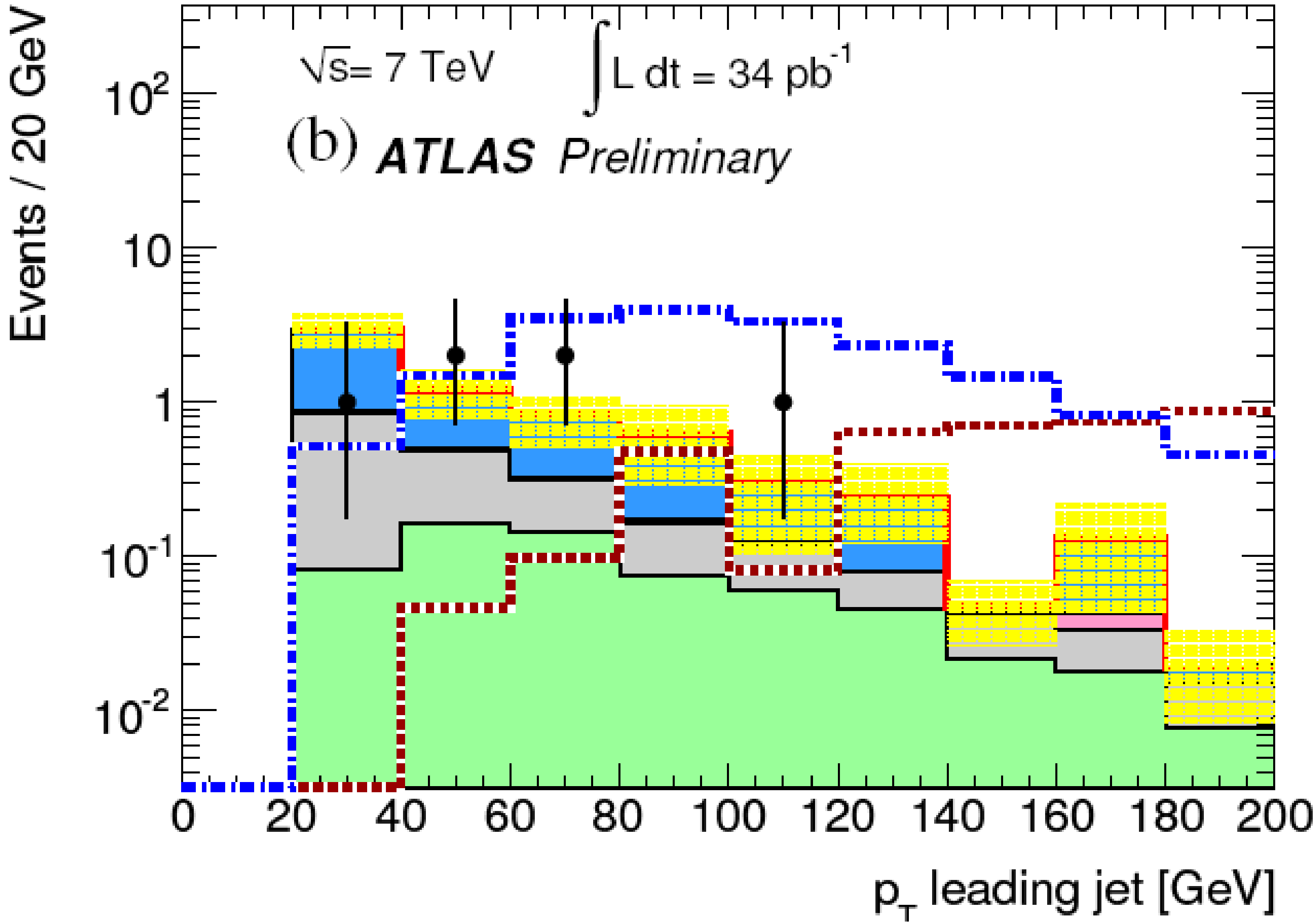


Multi-leptons (with jets and MPT)

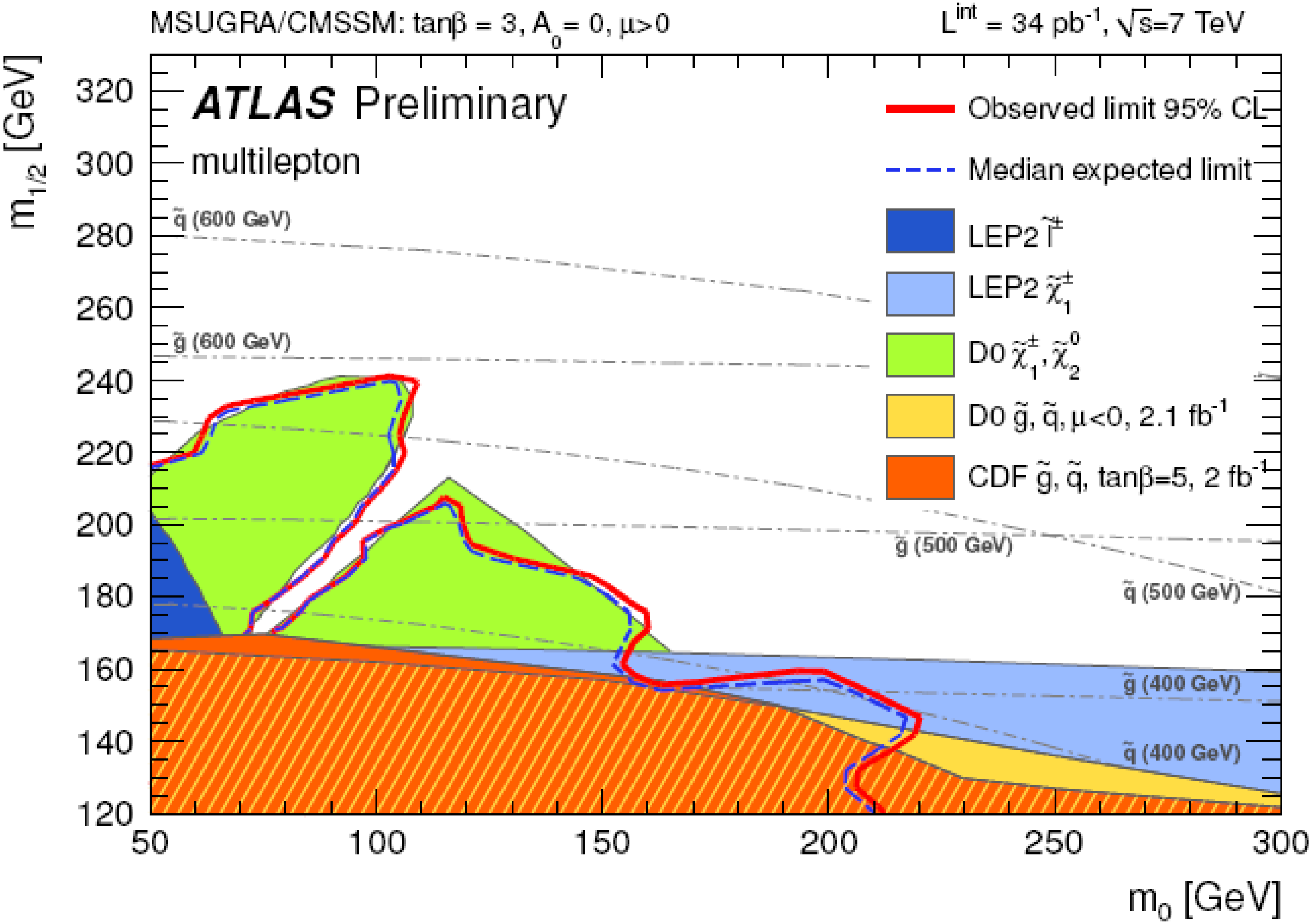
- Require 3 leptons (any flavour and charge combos)
  - 20 GeV electron/muon
  - 20 GeV electron/muon
  - 20 GeV electron / 10 GeV muon
- Require 2 jets > 50 GeV and MPT > 50 GeV to suppress ttbar and Z+jets
  - so not sensitive to direct multilepton production.

http://cdsweb.cern.ch/record/1338568/files/ATLAS-CONF-2011-039.pdf

Multi-leptons

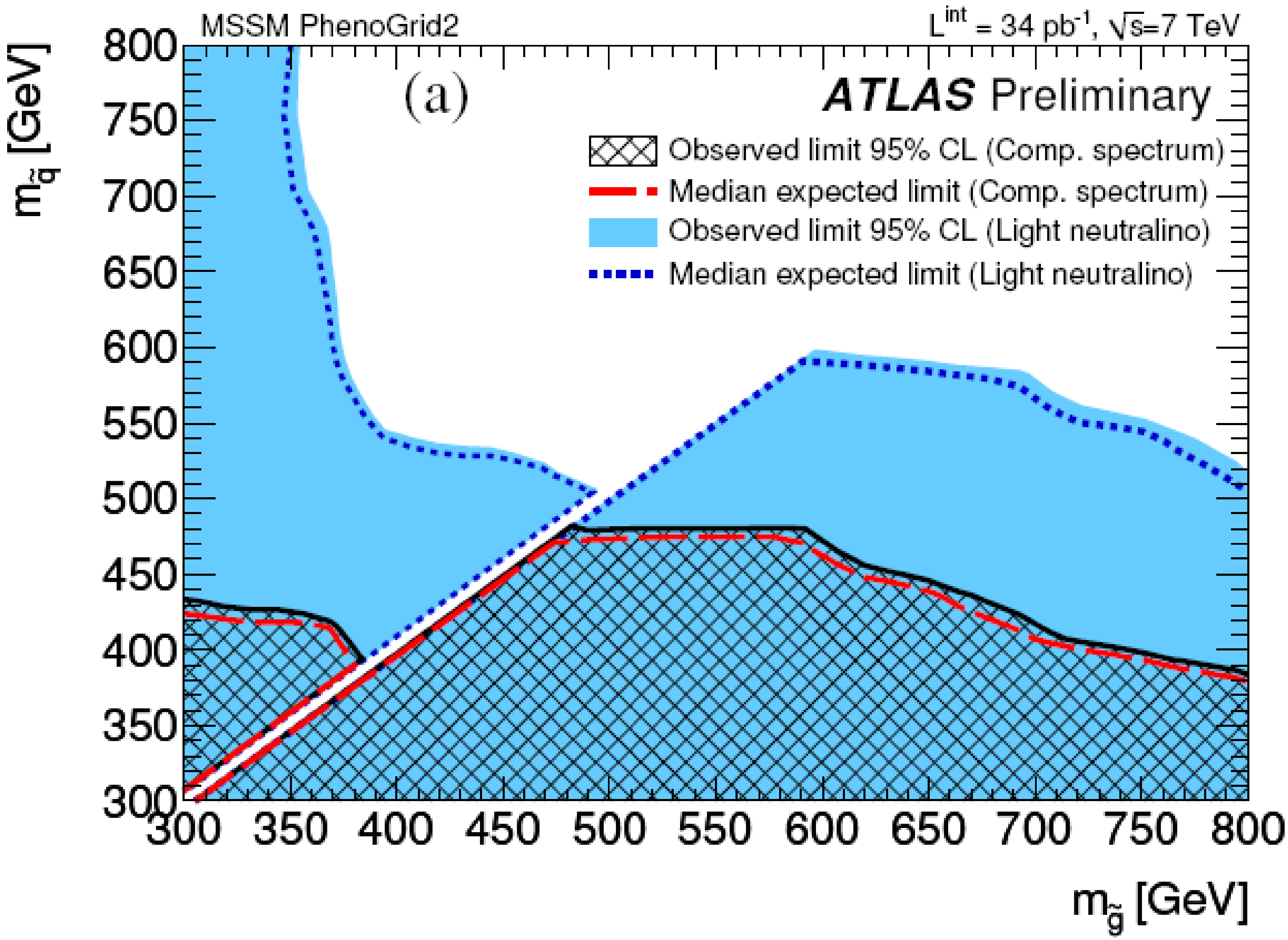


Multi-leptons



http://cdsweb.cern.ch/record/1338568/files/ATLAS-CONF-2011-039.pdf

Multi-leptons



- Your favourite multi-lepton producing model is probably not ruled yet, unless you know it makes lots of jets too …

Excesses in e+e- or mu+mu- over e+mu- and e-mu+ ?

- Can we focus on flavour-conserving BSM signals, and reduce sensitivity to BG modelling?

$$\mathcal{S} = \frac{N(e^\pm e^\mp)}{\beta(1-(1-\tau_e)^2)} - \frac{N(e^\pm \mu^\mp)}{1-(1-\tau_e)(1-\tau_\mu)} + \frac{\beta N(\mu^\pm \mu^\mp)}{(1-(1-\tau_\mu)^2)}$$

http://arxiv.org/abs/1103.6208

Aim is that analysis doesn't really need to know these numbers very well:

	$e^\pm e^\mp$	$e^\pm \mu^\mp$	$\mu^\pm \mu^\mp$
Data	4	13	13
Z/γ^*+jets	0.40±0.46	0.36±0.20	0.91±0.67
Dibosons	0.30±0.11	0.36±0.10	0.61±0.10
$t\bar{t}$	2.50±1.02	6.61±2.68	4.71±1.91
Single top	0.13±0.09	0.76±0.25	0.67±0.33
Fakes	0.31±0.21	-0.15±0.08	0.01±0.01
Total SM	3.64±1.24	8.08±2.78	6.91±2.20

2lepton flavour subtraction :

If the assumption is made that the branching fractions for $e^{\pm}e^{\mp}$ and $\mu^{\pm}\mu^{\mp}$ final states in new physics events are identical, and the branching fraction for $e^{\pm}\mu^{\mp}$ final states is zero, [illegible] Al-

i.e. lepton flavour conserving limit of order 10 events for order 35 events/pb indicates limit for cross section for lepton flavour conserving production is about ~0.3 pb

http://arxiv.org/abs/1103.6208

fairly model independent

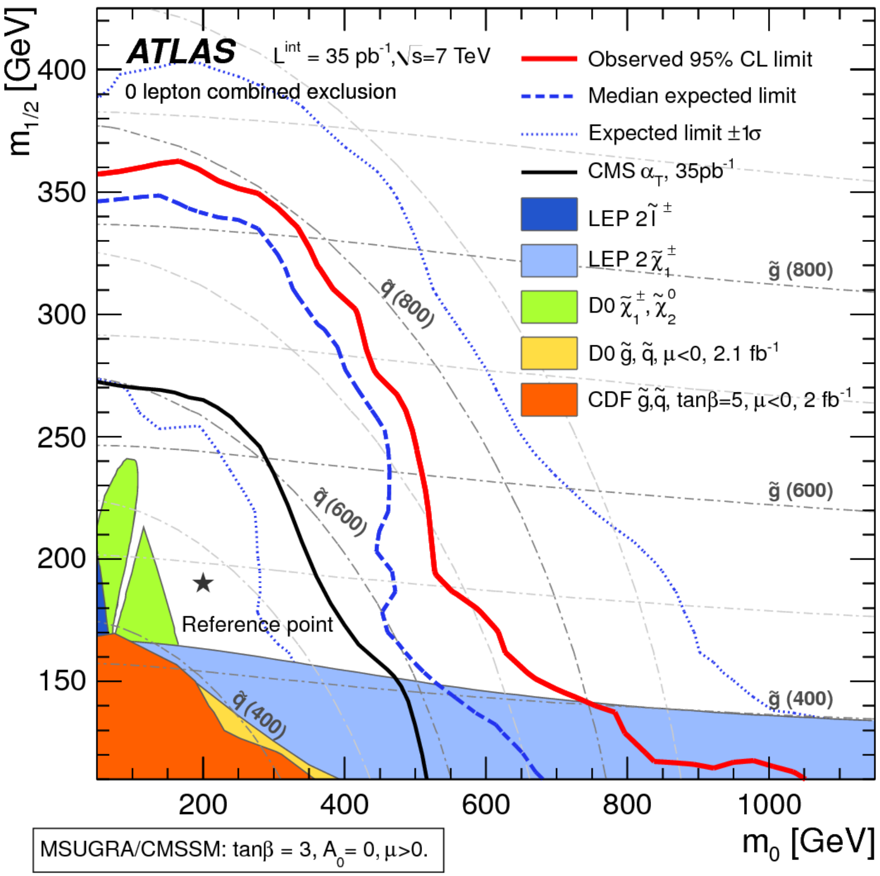
competitive with results from the jets+mpt analysis:

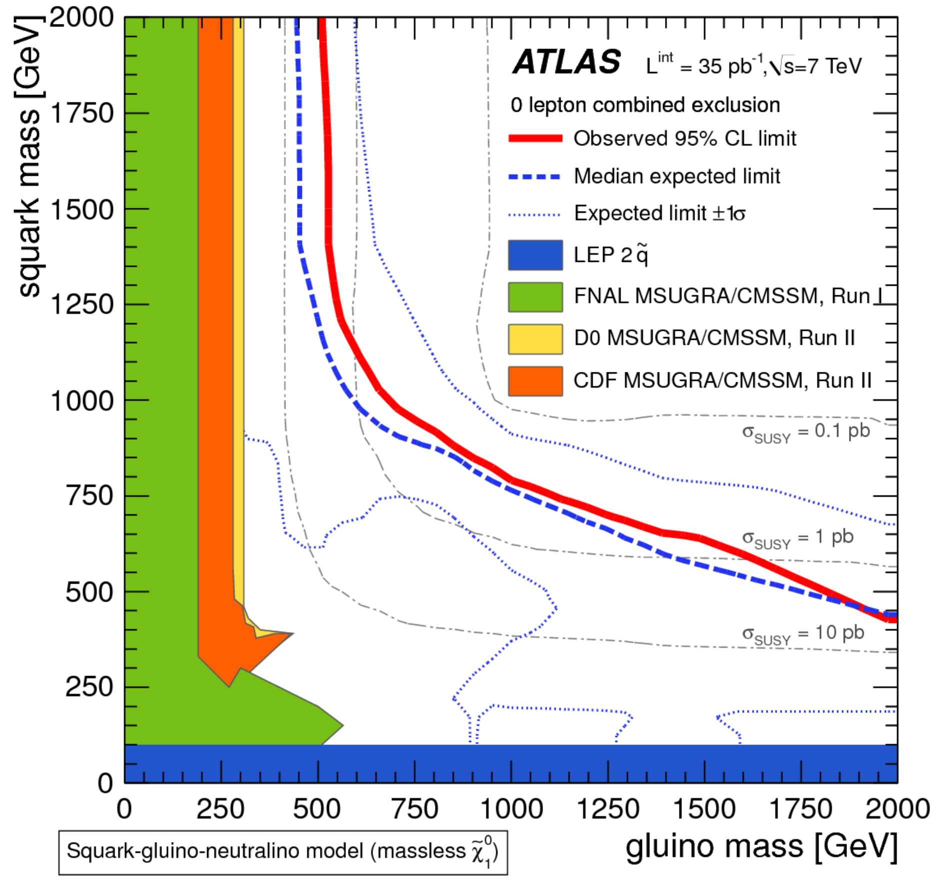
Exclude non-SM effective cross sections (σ x BR x Acc x Eff):

A: 1.3 pb B: 0.35 pb C: 1.1 pb D: 0.11 pb

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/susy-0lepton_01/

.. competitive with 35/pb limits on strong BSM production



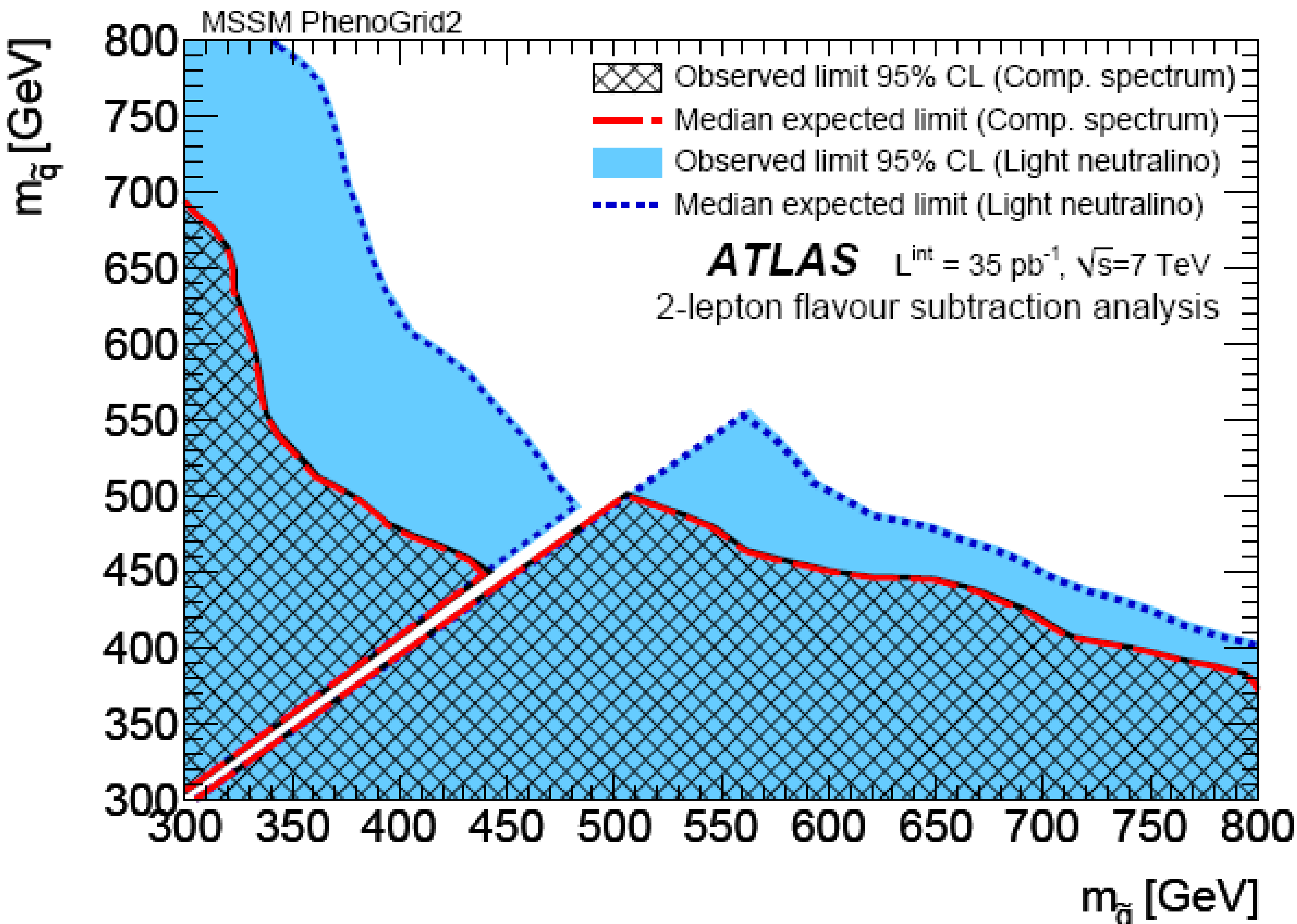


Exclude non-SM effective cross sections (σ x BR x Acc x Eff):

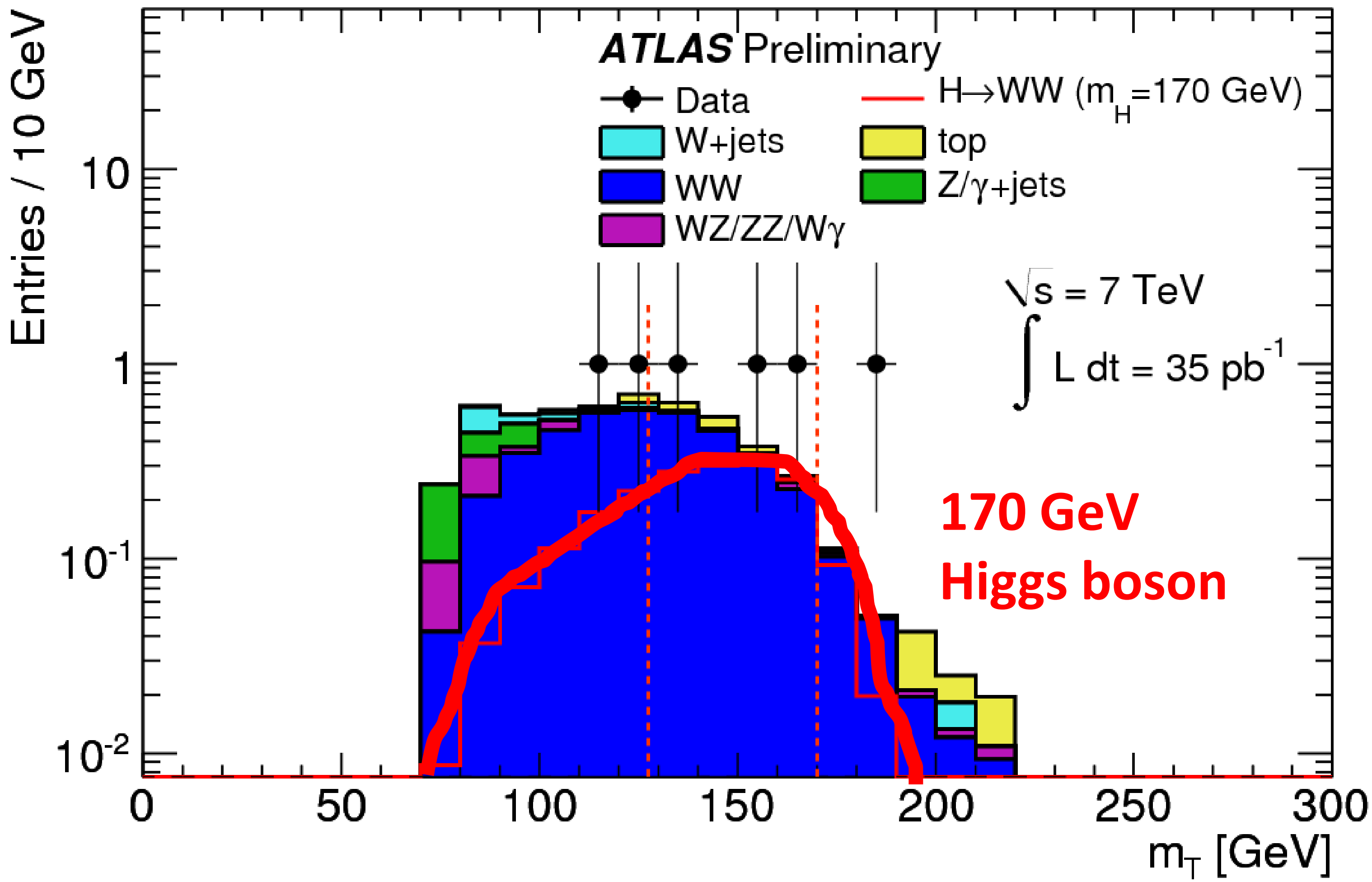
A: 1.3 pb   B: 0.35 pb   C: 1.1 pb   D: 0.11 pb

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/susy-0lepton_01/

This presentation arguably less helpful:



Against the 2010 LHC data…



ATLAS-CONF-2011-005

ATLAS 35/pb: H → WW →