QUANTUM GRAVITATIONAL EFFECTS ON GRAND UNIFIED THEORY

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OUTLINE

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Motivation and Goal

- The strong and electroweak force unify in 10^{16} GeV
- The quantum field can be described under a simple representation. Eg SU(5), SO(10).
- GUT provide a large number of fields and particles. (easily to reach 1000) $N \equiv N_0 + N_{1/2} - 4N_1$
- Accordingly, the scale of quantum gravity effect will be reduced by $\mu_* = M_{\mathrm{Pl}}/\eta$

with $\eta = \sqrt{1 + \frac{N}{12\pi}}$

- Thus QG effect has lower scale and provide more significant influence than we thought on GUT scale. The GUT modes have to take this modification into account.
- Reviewing previous work, the lowest order of QG effect is $\frac{c}{\hat{\mu}_*}\mathrm{Tr}\left(G_{\mu\nu}G^{\mu\nu}H\right)$

$\hat{\mu}_* = \mu_*/\sqrt{8\pi} = \hat{M}_{\mathrm{Pl}}/\eta$ with $\hat{M}_{\mathrm{Pl}} = 2.43 \times 10^{18}\,\mathrm{GeV}$

$$-\frac{1}{4}(1+\epsilon_1)\,F_{\mu\nu}F^{\mu\nu}_{\mathrm{U(1)}} - \frac{1}{2}(1+\epsilon_2)\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}_{\mathrm{SU(2)}}\right)$$
$$-\frac{1}{2}(1+\epsilon_3)\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}_{\mathrm{SU(3)}}\right)$$

with coupling constants $g_i \to (1+\epsilon_i)^{-1/2} g_i$

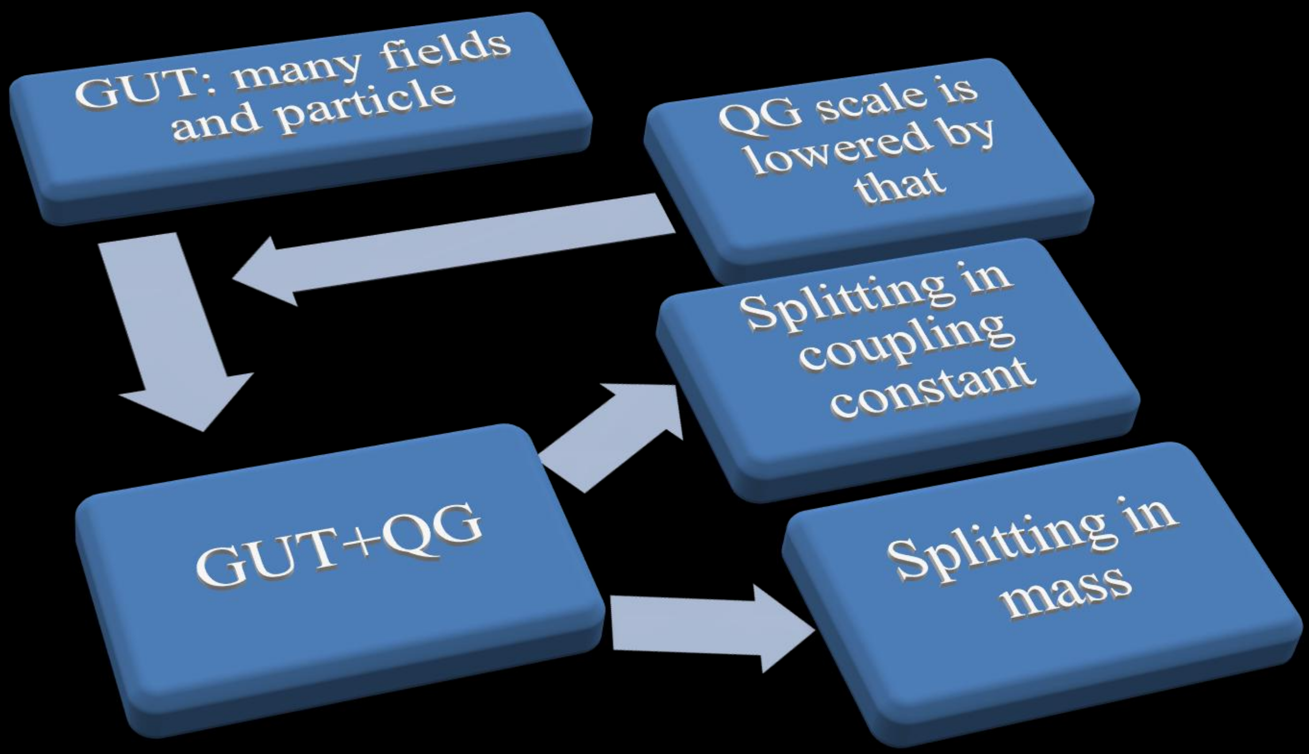
$$\epsilon_1 = \frac{\epsilon_2}{3} = -\frac{\epsilon_3}{2} = \frac{\sqrt{2}}{5\sqrt{\pi}}\,\frac{c\eta}{\sqrt{\alpha_G}}\,\frac{M_X}{\hat{M}_{\mathrm{Pl}}}\,.$$

- That example indicates the unification coupling constant is split.

$$\alpha_G = (1+\epsilon_1)\,\alpha_1(M_X) = (1+\epsilon_2)\,\alpha_2(M_X) = (1+\epsilon_3)\,\alpha_3(M_X)\,.$$

- The analogue results is expected to be attended for the splitting of unification mass.

Brief logic map



Fermion Mass in SU(5) Model

- The fermion masses are produced by the Yukawa coupling term.
- With the Lagragian:

$$\mathcal{L} = \{G_d \bar{f^c}_{jR} \Psi^j_{kL} H^k(5) + G_u \varepsilon_{jklmn} \bar{\Psi^c}^{jk}_L \Psi^{lm}_L H^n(5)\} + h.c.$$

$$= \frac{-2G_d M_w}{\sqrt{2} g_2}(\bar{d}d + \bar{e}e) \quad - \frac{2G_u M_w}{\sqrt{2} g_2} 8[\bar{u}u]$$

- From the coefficient , the masses of down quark, up quark and electron are read.

QG effect: Dimension 5 operator

- With the operator proposed by Ellis and Gaillard

$$\begin{aligned}
O_5 = &\frac{a}{\bar{\mu}_*}\{\phi_{mn}\bar{f}^{mk}H^l_k\Psi^n_l\} \\
&+\frac{b}{\bar{\mu}_*}\{\phi_{mn}H^{mk}\bar{f}^l{}_k\Psi^n_l\} \\
&+\frac{c}{\bar{\mu}_*}\varepsilon^{mnpql}\{\Psi_{mn}\Psi_{pq}H_k\phi^k_l\} \\
&+\frac{d}{\bar{\mu}_*}\varepsilon^{mnpkl}\{\Psi_{mn}\Psi_{pq}H_k\phi^q_l\}
\end{aligned}$$

- Implementing the vacuum expectation value (vev) of two Higgs fields which break the unified symmetry and electroweak symmetry respectively.

$$\langle 0 \mid H(5) \mid 0 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{2M_w/g_2}{\sqrt{2}} \end{pmatrix}$$

$$\langle 0 \mid \phi(24) \mid 0 \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} (2\sqrt{2}/5g_1)M_x$$

- That results in the mass term from the quantum gravitational effect.

$$\mathcal{O}_5 = 4M_w M_X/5g_1g_2\,[(\frac{a+b}{\bar{\mu}_*})(2d\bar{d}) + (\frac{a-b}{\bar{\mu}_*})\frac{9}{2}(e^-e^+)]$$
$$+4M_w M_X/5g_1g_2\,[(\frac{c'-d}{\bar{\mu}_*})(2d\bar{d}) + (\frac{c-d}{\bar{\mu}_*})\frac{9}{2}(e^-e^+)]$$
$$+\frac{c}{\bar{\mu}_*}4M_w M_X/5g_1g_2[-\frac{3}{2}(-2\bar{u}u)]$$

Results

Putting all of them together and redefining factor

$$\mathcal{L} + O_5 = -m_d(1+\varepsilon_1+\varepsilon_2)(\bar{d}d) - m_e(1+\frac{9}{2}\varepsilon_1 - \frac{9}{2}\varepsilon_2)(\bar{e}e) - m_u(1+\varepsilon_3)(u\bar{u})$$

$$m_d = \frac{2M_w}{\sqrt{2}g_2} G_d$$

$$m_e = \frac{2M_w}{\sqrt{2}g_2} G_d$$

$$m_u = \frac{2M_w}{\sqrt{2}g_2} 8G_u$$

$$\varepsilon_1 = \frac{-\sqrt{2}g_2}{2M_w G_d}(4M_w M_X/5g^2)\frac{a}{\bar{\mu}_\star}$$

$$\varepsilon_2 = \frac{-\sqrt{2}g_2}{2M_w G_d}(4M_w M_X/5g^2)\frac{b}{\bar{\mu}_\star}$$

$$\varepsilon_3 = \frac{-\sqrt{2}g_2}{2M_w G_d}(4M_w M_X/5g_1 g_2)\frac{3}{8}\frac{c}{\bar{\mu}_\star}$$

- The QG effects are revealed in the modification factors (ε) which split the unification of masses.
- These correction will be more significant and close to order 1 because of the fact that $\bar{\mu}_*$ in the denominator is suppressed by the number of fields; thus increases the modification term.
- The fermion masses are modified in the way that each of them shift inparalleled with some factor in common.

$$m_d \rightarrow m_d(1+\varepsilon_1+\varepsilon_2)$$

$$m_u \rightarrow m_u(1+\varepsilon_3)$$

$$m_e \rightarrow m_e(1+\frac{9}{2}\varepsilon_1-\frac{9}{2}\varepsilon_2)$$

Conclusion

- The gravitational effect play an important role to the splitting of unification mass.
- The large number of the unification group we consider, the more significant effect form QG will apply.
- These effects influence every unification model not only to SU(5). To be precise in other model with larger number of field react to this effect more drastically. Thus some unified may become not unified and vice versa.