Warped Spaces in more than Five Dimensions.

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Introduction

- The problem with extra dimensional scenarios is many free parameters. How should we determine the geometry, topology and dimensionality of extra dimensions?
- Top down approaches, notably string theory, give rise to many possibilities.
- Here I will consider a bottom up approach motivated by the gauge hierarchy problem.
- Attempt to address the question

  > *If one assumes that the gauge hierarchy problem is resolved, via the warping of an extra dimension, then what are the phenomenologically viable spaces?*

- Restrict the study to spaces that are warped with respect to a single direction and models in which the SM gauge fields propagate in the extra dimension.

Resolving the Hierarchy Problem in Extra Dimensions

To resolve hierarchy problem would like $M_{\rm fund} \sim m_{\rm fund}^{\rm Higgs}$. Consider a $4+1+\delta$ dimensional space;

$$ds^2 = a^2(r)\eta_{\mu\nu}dx^\mu dx^\nu - b^2(r)dr^2 - c^2(r)d\Omega_\delta^2$$

where $d\Omega_\delta^2 = \gamma_{ij}d\phi^i d\phi^j$ and $i,j = 1\ldots\delta$. In the 4D effective theory, parameters modified by two effects:

Volume Effects

Fields propagating in the bulk (e.g. gravity) have couplings scaled by volume of extra dimensions. (AADD Model)(Arkani-Hamed, Dimopoulos, Dvali '98)(Antoniadis, Arkani-Hamed, Dimopoulos, Dvali '98).

$$M_{\rm P}^2 \sim \int d^{\delta+1}x \; a^2 b c^\delta \sqrt{\gamma}\; M_{\rm Fund}^{\delta+3}.$$

Warping

Fields localised in the space have masses suppressed by gravitational redshifting. (RS Model)(Randall, Sundrum '99)

$$m_{\rm 4D}^2 = a^2(r_{\rm ir})m_{\rm fund}^2$$

If $M_{\rm P} \sim M_{\rm fund}$ then a warp factor $a(r_{\rm ir})^{-1} \equiv \Omega \sim 10^{15}$ would offer potential resolution to hierarchy problem.

Resolving the Hierarchy Problem in Extra Dimensions

Hence two aspects to resolving gauge hierarchy problem.

- Can the space be stabilised, without fine tuning, such that small effective Higgs mass (and hence small EW scale) is generated? (Goldberger & Wise '99)
- Can the model satisfy constraints from existing observables without resorting to a large KK mass (M_{KK})?
- If $M_{\mathrm{KK}} \gg$ EW scale then firstly Higgs Mass potentially quadratically sensitive to M_{KK} and secondly curvature and size of extra dimensions is typically no longer at the Planck scale.

Restrict study to models in which EW symmetry is broken by 'SM like' Higgs.

With the SM gauge fields free to propagated in the bulk, we focus on the consistency with electroweak observables.

Gauge Field KK Decomposition

Here carry out KK decomposition before EW symmetry breaking.

$$A_\mu = \sum_n A_\mu^{(n)}(x^\mu) f_n(r) \Theta_n(\phi_1, \dots, \phi_\delta) \quad \text{s.t.} \quad \int d^{1+\delta}x \; bc^\delta \sqrt{\gamma} f_n f_m \Theta_n \Theta_m = \delta_{nm},$$

where

$$f_n'' + \frac{(a^2 b^{-1} c^\delta)'}{(a^2 b^{-1} c^\delta)} f_n' - \frac{b^2}{c^2} \alpha_n f_n + \frac{b^2}{a^2} m_n^2 f_n = 0 \quad \text{and} \quad -\frac{1}{\sqrt{\gamma}} \partial_{\phi_i} (\sqrt{\gamma} \gamma^{ij} \partial_{\phi_j} \Theta_n) = \alpha_n \Theta_n.$$

Now $n = (n_1, \dots, n_{1+\delta})$, i.e. in more than five dimensions many additional KK modes. Since $\alpha_n \geqslant 0$ first KK mode always corresponds to $\alpha_n = 0$.

The relative coupling of the KK Gauge fields to the Higgs

$$F_n \equiv \frac{f_n(r_{\text{ir}})}{f_0(r_{\text{ir}})} \quad \text{or} \quad F_n \equiv \frac{f_n(r_{\text{ir}}) \Theta_n(\phi_{\text{ir}})}{f_0(r_{\text{ir}}) \Theta_0(\phi_{\text{ir}})}$$

The relative coupling of the KK Gauge fields to the Fermion Zero modes

$$F_\psi^{(n)} \equiv \frac{f_\psi^{(0,n,0)}}{f_\psi^{(0,0,0)}} \quad \text{where} \quad f_\psi^{(l,n,m)} \equiv \int d^{\delta+1}x \; ba^3 c^\delta \sqrt{\gamma} \, f_L^{(l)} \Theta_L^{(l)} \, f_n \Theta_n \, f_L^{(m)} \Theta_L^{(m)}.$$

Location of the Higgs

Corrections to EW observables occur at tree level because Higgs mixes KK modes resulting in corrections to W/Z masses and couplings. Location of Higgs important.

If Higgs is localised to codimension one brane

$$S=\int d^Dx\sqrt{-G}\left(\mathcal{L}_{\rm SM}+\frac{\delta(r-r_{\rm ir})}{b}\left[|D_\mu\Phi|^2-V(\Phi)\right]\right),$$

then, due to orthogonality of KK profiles (or equivalently conservation of higher dimensional momentum), only $n=(n_1,0,\dots,0)$ modes will mix with W/Z zero mode.

$$|D_\mu\Phi|^2\supset\sum_{n,m}\frac{g^2v^2}{4}f_nf_mW_\mu^{+(n)}W^{-(m)\mu}+\frac{(g^2+g'^2)v^2}{8}f_nf_mZ_\mu^{(n)}Z^{\mu(m)}$$

If Higgs localised to a three brane

$$S=\int d^Dx\sqrt{-G}\left(\mathcal{L}_{\rm SM}\right)+\int d^4x\sqrt{-g_{\rm ir}}\left[|D_\mu\Phi|^2-V(\Phi)\right]$$

Conversely with Higgs on 3 brane, all KK modes will mix,

$$|D_\mu\Phi|^2\supset\sum_{n,m}\frac{g^2v^2}{4}f_n\Theta_nf_m\Theta_mW_\mu^{+(n)}W^{-(m)\mu}+\frac{(g^2+g'^2)v^2}{8}f_n\Theta_nf_m\Theta_mZ_\mu^{(n)}Z^{\mu(m)}.$$

Electroweak Observables

Electroweak corrections often parameterised in terms of corrections to W/Z propagator, the S and T parameters. (Peskin & Takeuchi '92). Tree level contributions from KK gauge fields given by

$$S \approx -\frac{4M_Z^2 c^2 s^2}{\alpha} \sum_n \frac{2F_n F_\psi^{(n)}}{m_n^2 + M_z^2(F_n^2 - 1)} + \mathcal{O}(m_n^{-4})$$

$$T \approx \frac{1}{\alpha} \sum_n \frac{M_w^2(2F_n F_\psi^{(n)} - F_n^2)}{m_n^2 + M_w^2(F_n^2 - 1)} - \frac{M_z^2(2F_n F_\psi^{(n)} - F_n^2)}{m_n^2 + M_z^2(F_n^2 - 1)} + \mathcal{O}(m_n^{-4}).$$

In RS model one can include a fermion bulk mass term and localise the fermions towards UV brane reducing $F_\psi^{(n)}$ and hence the contribution to S parameter.

One can also replace 'SM' bulk $SU(2) \times U(1)$ gauge symmetry with a custodial $SU(2)_R \times SU(2)_L \times U(1)$ gauge symmetry. This gives a partial suppression of the T parameter. (Agashe, Delgado, May, Sundrum '03)

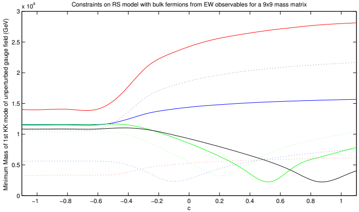


Figure: Constraints on RS model from S_Z^2 (Red line), M_W (blue line), Γ_Z (green line), Γ_{had} (black line), R_e (blue dots), Γ_{inv} (red dots), $\Gamma_{l^+l^-}$ (green dots) and A_e (black dots). While on the x axis is 5D bulk Dirac Mass.

Alternatively look at individual observables. Typically tightest constraint comes from weak mixing angle.

$$s_Z^2 \approx s_p^2 \left(1 - \frac{c_p^2}{c_p^2 - s_p^2} \sum_{n=1} \left[\frac{m_z^2 F_n^2}{m_n^2} - \frac{m_w^2 \left(F_n - F_\psi^{(n)} \right)^2}{m_n^2} \right] + \mathcal{O}(m_n^{-4}) \right)$$

where $s_p^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2} G_f \hat{M}_Z^2}}\right)$. Or with custodial symmetry

$$s_Z^2 \approx s_p^2 \left(1 + \frac{c_p^2}{c_p^2 - s_p^2} \sum_{n=1} \left[\frac{m_w^2 F_\psi^{(n)\,2}}{m_n^2} - \frac{2 m_w^2 F_n F_\psi^{(n)}}{m_n^2} + s'^{\,2} m_w^2 \left(\frac{\tilde{F}_n^2}{\tilde{m}_n^2} - \frac{F_n^2}{m_n^2} \right) + \mathcal{O}(m_n^{-4}) \right] \right).$$

The size of EW constraints is quadratically sensitive to F_n (or linearly with a custodial symmetry).

F_n in Five Dimensions

From EOM of KK gauge field with NBC's it can be shown that

$$\Omega^2 = \frac{b^2(r_{\rm uv}) f_n(r_{\rm uv}) f_n''(r_{\rm ir})}{b^2(r_{\rm ir}) f_n(r_{\rm ir}) f_n''(r_{\rm uv})}, \quad f_n''(r_{\rm ir}) + m_n^2 \Omega^2 f_n(r_{\rm ir}) = 0 \quad \text{and} \quad f_n''(r_{\rm uv}) + m_n^2 f_n(r_{\rm uv}) = 0$$

Hence with a large warp factor the KK gauge fields profiles will always be very peaked towards IR.

Further still when one can include a fermion bulk mass term, M, the fermion zero mode profile can be localised towards UV

$$f_L^{(0)}(r) = \frac{\exp\left[\int_c^r b(\tilde{r}) M d\tilde{r}\right]}{\sqrt{\int_{r_{\rm uv}}^{r_{\rm uv}} \frac{b}{a} \exp\left[\int_c^r 2b(\tilde{r}) M d\tilde{r}\right]}},$$

with $F_n = \frac{\sqrt{\int b\, dr}\, f_n(r_{\rm ir})}{\sqrt{\int b f_n^2\, dr}}$ this implies that

$$\boxed{F_\psi^{(n)} \leqslant F_n \quad \text{and} \quad F_n > 1.}$$

In practice $F_n \gtrsim 6-7$ with $\Omega = 10^{15}$ (RS model $F_n \approx 8.3$). Hence spaces with a large warp factor will always have sizeable EW constraints. In agreement with (Delgado, Falkowski '07)

This is difficult to generalise to more than five dimensions.

The Toy Model

Here in order to investigate $D > 5$ dimensions consider, arguably, simplest bottom up extension of RS model. (Kogan, Mouslopoulos, Papazoglou, Ross '01)

$$S = \int d^{5+\delta}x\sqrt{G}\left[\Lambda - \frac{1}{2}M^{3+\delta}\mathcal{R} + \mathcal{L}_{\mathrm{bulk}}\right].$$

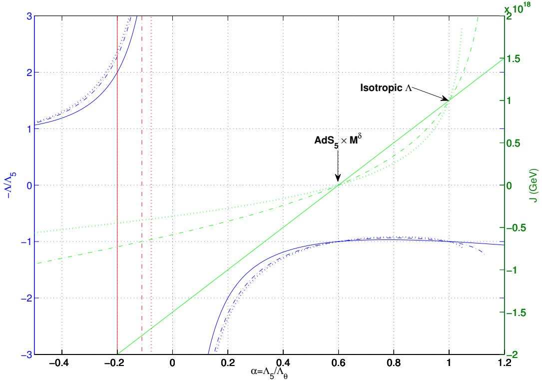
with an anisotropic bulk cosmological constant

$$\Lambda = \begin{pmatrix} \Lambda\eta_{\mu\nu} & & & & \\ & \Lambda_5 & & & \\ & & \Lambda_\theta & & \\ & & & \ddots & \\ & & & & \Lambda_\theta \end{pmatrix} \quad \text{and define} \quad \alpha \equiv \frac{\Lambda_5}{\Lambda_\theta}.$$

Then solve the Einstein equations with ansatz warped w.r.t single direction. Admits solutions of the form

$$\boxed{ds^2 = e^{-2kr}\eta_{\mu\nu}dx^\mu dx^\nu - dr^2 - \sum_{i=1}^{\delta} e^{-2Jr}d\theta_i^2}$$

Solutions of Einstein Equations.

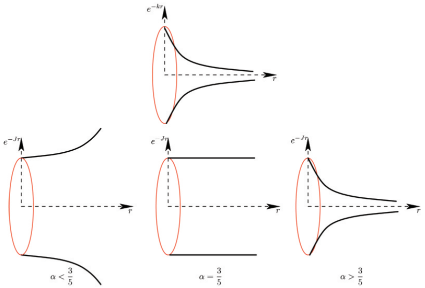


The solutions to the Einstein equations for 6D (solid lines), 8D(dash-dot lines) and 10D (dot-dot lines). J is plotted in green and $-\frac{\Lambda}{\Lambda_5}$ in blue. The red lines correspond to $2k + \delta J = 0$. Here $\Omega \equiv e^{kR} = 10^{15}$ and $M_{KK} \equiv \frac{k}{\Omega} = 1$ TeV. $AdS_5 \times M^{\delta}$ studied in (Davoudiasl, Hewett, Rizzo '02).

$$ds^2 = e^{-2kr}\eta_{\mu\nu}dx^{\mu}dx^{\nu} - dr^2 - \sum_{i=1}^{\delta} e^{-2Jr}d\theta_i^2$$

Solutions of Einstein Equations (cont.).

Three classes of solution:

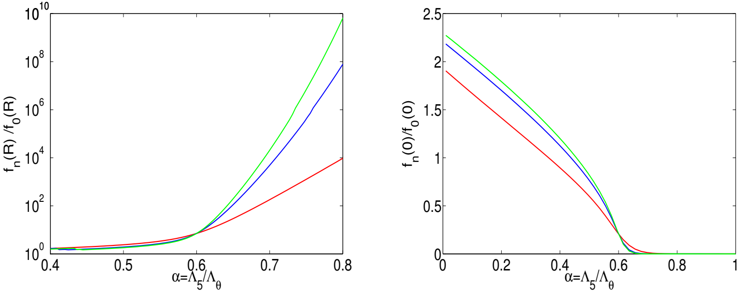


Effective Planck mass:

$$M_P^2 = M_{\rm Fund}^{3+\delta} \frac{(1 - e^{-(2k+\delta J)R})(R_\theta)^\delta}{(2k + \delta J)}.$$

Hence if $2k + \delta J > 0$ and $k \sim J \sim R_\theta^{-1} \sim M_{\rm Fund} \sim M_P$ then $\Omega \sim 10^{15}$ would resolve hierarchy problem.

F_n in Spaces with Anisotropic Bulk Cosmological Constant



(a) F_n

(b) Coupling to fields on UV brane (F_ψ)

Figure: The relative coupling of the $\alpha_n = 0$ KK gauge modes in 6D (red), 8D (blue) and 10D (green). Here $\Omega \equiv e^{kR} = 10^{15}$ and $M_{\rm KK} \equiv \frac{k}{\Omega} = 1$TeV.

Find, analogous to AADD model, volume of internal space scales KK couplings differently from flat zero mode.

Electroweak constraints

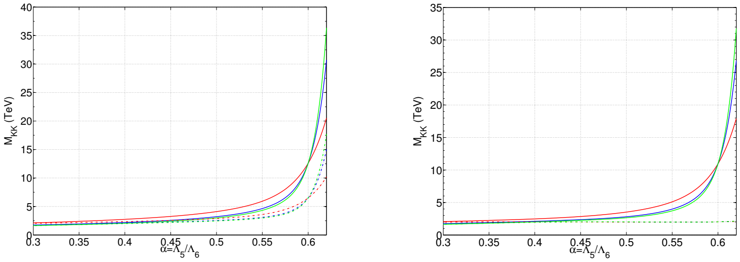


Figure: Lower bound on $M_{KK} = \frac{k}{\Omega}$ from s_Z^2 constraint for two models, bulk $SU(2) \times U(1)$ gauge symmetry (left) and bulk $SU(2)_R \times SU(2)_L \times U(1)$ custodial symmetry (right). With the fermions localised on the IR brane (solid lines), UV brane (dashed lines).

- If one assumes Higgs is localised to a codimension one brane and for simplicity fermions are localised on 3 branes in either the IR or UV of the space. Then tree level contribution to EW observables from KK gauge fields suppressed for $\alpha < 0.6$.
- Constraints for $\alpha > 0.6$ not plotted since KK modes strongly coupled so tree level analysis not valid.
- Overall lower bound corresponding to $F_n^2,\ F_n F_\psi \sim 1$ is $M_{KK} \gtrsim 2 - 2.5$ TeV

KK decomposition of Gauge fields

Assuming δ extra dimensions are toroidal i.e $\partial_i^2\Theta_n=-\frac{l_i^2}{R_\theta^2}\Theta_n$, the gauge profiles given by:

$$f_n''-(2k+\delta J)f_n'-\sum_{i=1}^{\delta}e^{2Jr}\frac{l_i^2}{R_\theta^2}f_n+e^{2kr}m_n^2f_n=0.$$

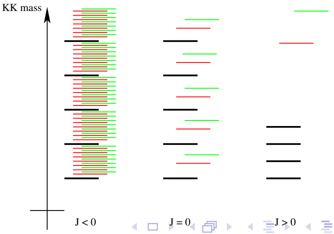
substituting $x_n^2=\frac{e^{2kr}m_n^2-e^{2Jr}\sum_{i=1}^{\delta}\frac{l_i^2}{R_\theta^2}}{\gamma^2}$ such that $x_n'=\gamma x_n$ then;

$$f_n(x)=Nx_n^{\frac{1}{2}\frac{2k+\delta J}{\gamma}}\left(\mathbf{J}_{-\frac{1}{2}\frac{2k+\delta J}{\gamma}}(x_n)+\beta\mathbf{Y}_{-\frac{1}{2}\frac{2k+\delta J}{\gamma}}(x_n)\right)$$

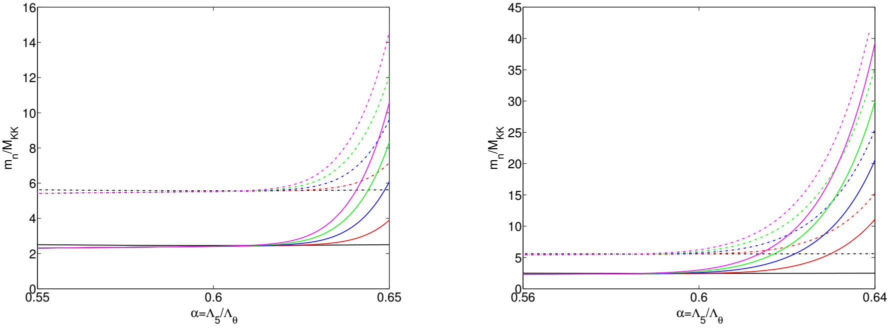
$$\boxed{m_n\sim X_n\frac{\sqrt{\gamma^2+\frac{e^{2JR}}{R_\theta^2}\sum_i^{\delta}l_i^2}}{e^{kR}}}$$

where

$X_n\sim\mathcal{O}(1)$.



Gauge Fields KK Masses



The first 5 KK masses in 6D with $R_\theta = R$ (left) and $R_\theta = 0.1R$ (right).
$\Omega \equiv e^{kR} = 10^{15}$ and $M_{KK} \equiv \frac{k}{\Omega} = 1\text{TeV}$.

If internal space is shrinking towards IR then bulk gauge fields becomes strongly coupled to IR localised Higgs.
If internal space is growing towards IR then gauge propagator has a high density of poles.

Potential Dual Theories?

- Higher dimensional AdS spaces conjectured to have dual field theory via AdS/CFT.
- With IR localised Higgs, RS model conjectured to be dual to a theory closely related to technicolour.
- Original AdS/CFT was dualism between solution of type IIb string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and $\mathcal{N} = 4$ super-Yang Mills theory $SU(N_C)$.
- Radius of S^5 determines number of colours (Aharony, Gubser, Maldacena, Ooguri & Oz '00)

$$\int_{S^5} F_5 = N_C.$$

  where F_5 is the Ramond Ramond field strength tensor.
- An interesting example of a solution in which the N_C was reduced using the Seiberg duality is the Klebanov-Strassler solution (Klebanov & Strassler '00).
- The result is a deformed conifold without a conical singularity.

The Klebanov-Strassler Solution

Klebanov-Strassler solution has complicated form but internal space approximately $S_3 \times S_2$ with radii shrinking towards IR.

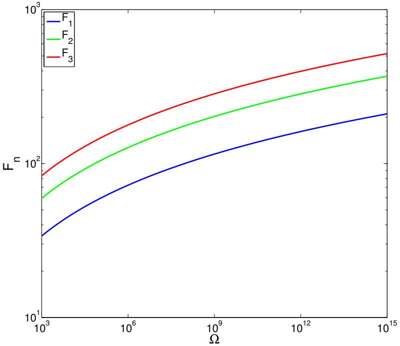


Figure: Couplings of first three '$\alpha_n = 0$' gauge modes to a field localised in the IR.

If we again allow gauge fields to propagate in the bulk then again they become strongly coupled. With $\Omega = 10^{15}$ then $F_1 \approx 210$.

Conclusions

- These results assume SM gauge fields propagate in the bulk also assume a large warp factor.
- Scale of EW corrections determined at tree level by relative gauge couplings.
- Very difficult to find space in which KK gauge fields have weaker coupling than zero mode ($F_n < 1$).
- With internal space growing towards IR, potentially can reduce contributions to EW observables regardless of location of fermions or custodial symmetry.
- However uncertainties in phenomenological implications of high density of KK modes.
- Any deviation from $AdS_5 \times M^\delta$ gives rise to significant change in phenomenology from that of RS model.