General gauge mediation in higher dimensions

Moritz McGarrie

NEXT Workshop, July 2011

Based on

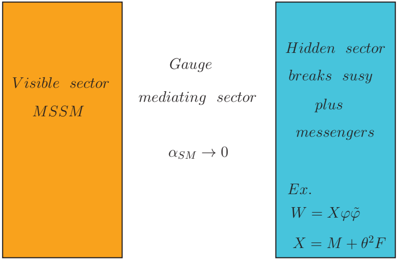
arXiv:1004.3305 M.M., Rodolfo Russo
arXiv:1009.0012 M.M.
arXiv:1009.4696 M.M., Daniel Thompson
arXiv:1101.5158 M.M.

Queen Mary University of London

Outline

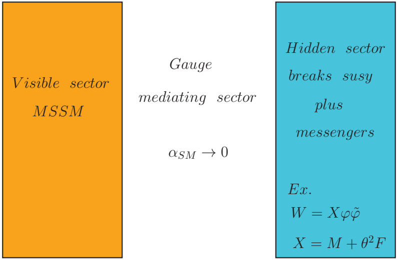
1. Review GGM
2. Extend GGM to 5D on an interval $R^{1,3} \times S^1/\mathbb{Z}_2$
3. Motivate duality: the two site model
   as vector meson dominance
4. Extend to a slice of AdS space
5. Conclude

General gauge mediation (Meade, Seiberg, Shih)



- We require hidden-visible sector decoupling as $\alpha_{SM} \longrightarrow 0$
- Work perturbatively in α_{SM}
- Characteristic scales M and F
- Let's use $W = X\varphi\tilde{\varphi}$ as our benchmark model

General gauge mediation (Meade, Seiberg, Shih)



- We require hidden-visible sector decoupling as $\alpha_{SM} \longrightarrow 0$
- Work perturbatively in α_{SM}
- Characteristic scales M and F
- Let's use $W = X\varphi\tilde{\varphi}$ as our benchmark model

Motivations:

extract soft masses, explore strong coupling, apply dualities, model dependent from mode independent features? ,....

General gauge mediation

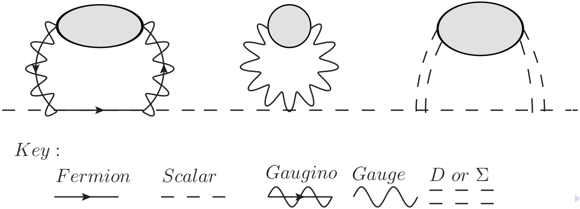
$$V_{WZ} = \theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D$$

$$S_{int} = 2g_{SM}\int d^4x \int d^4\theta \mathcal{J} V = g_{SM}\int d^4x (JD - \lambda j - \bar{\lambda}\bar{j} - j^\mu A_\mu)$$

- The effective Lagrangian at order g^2 leads to gaugino masses.

$$\delta\mathcal{L}_{eff} = \quad -g^2\tilde{C}_{1/2}(0)\lambda\sigma^\mu\partial_\mu\bar{\lambda} - \frac{g^2}{4}\tilde{C}_1(0)F_{\mu\nu}F^{\mu\nu} + \frac{g^2}{2}\tilde{C}_0(0)D^2$$
$$-\frac{g^2}{2}(M\tilde{B}_{1/2}(0)\lambda\lambda + c.c.) + \ldots$$

- One loop effects lead to sfermion masses at order g^4
- $\tilde{C}_s$ are Fourier transforms of the space-time current correlators.
- sfermion mass diagrams:

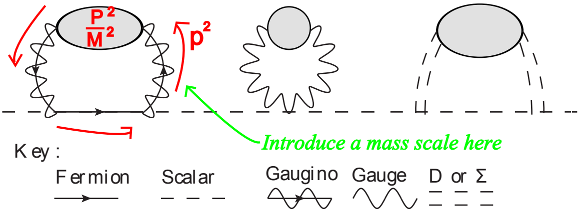


General gauge mediation (5D)

- For a given hidden sector we get a supertraced combination of current correlators

$$[3\tilde{C}_1(p^2/M^2) - 4\tilde{C}_{1/2}(p^2/M^2) + \tilde{C}_0(p^2/M^2)] = \Omega(p^2/M^2)$$

- Even for a perturbative hidden sector this is still a function that must be expanded
- Expanding in $\frac{M^2}{p^2} \to 0$ leads to "Gauge mediation" $+O(1/p^2)$
- Expanding in $\frac{p^2}{M^2} \to 0$ leads to "Gaugino mediation" $+O(p^2)$
- We should suppress the momenta in the outer loop of these diagrams to obtain "Gaugino mediation"
- To suppress the loop momenta p^2 we need to introduce a mass scale in the game.



Add an extra dimension

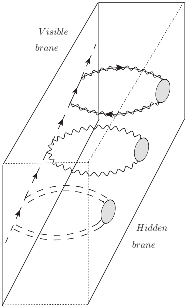
- If we want...
  - suppressed scalar soft masses
  - a geometric interpretation visible-hidden sector decoupling $\alpha_{SM} \longrightarrow 0$
  - Analogues of Vector Meson Dominance of QCD
- ...then it is convenient to add an extra dimension
- So let's look at three typical examples of adding an extra dimension:
  - Flat $S^1/\mathbb{Z}_2$
  - Two site model
  - Warped $S^1/\mathbb{Z}_2$

General gauge mediation in 5d

5d $\mathcal{N}=1$ Super Yang Mills

$$S_{5D}^{SYM}=\int d^5x\,\mathrm{Tr}\left[-\frac{1}{2}(F_{MN})^2-(D_M\Sigma)^2-i\bar{\lambda}_i\gamma^M D_M\lambda^i+(X^a)^2+g_5\,\bar{\lambda}_i[\Sigma,\lambda^i]\right]$$

$$SU(2)_R \quad X^a \;,\; a=1,2,3. \quad \lambda^i=\begin{pmatrix}\lambda^i_{L\alpha}\\ \bar{\lambda}^{i\dot{\alpha}}_R\end{pmatrix}, \quad i=1,2. \;\text{ with }\; \lambda^i=\epsilon^{ij}C\bar{\lambda}_j^T$$



The fixed points are $\delta(x_5)$ and $\delta(x_5-\ell)$

Compactify on $R^{1,3}\times S^1/\mathbb{Z}_2$ reduces to 4d $\mathcal{N}=1$ SYM with

$$(+\;parity)\quad V=-\,\theta\sigma^\mu\bar{\theta}A_\mu+i\bar{\theta}^2\theta\lambda_L-i\theta^2\bar{\theta}\bar{\lambda}_L+\frac{1}{2}\bar{\theta}^2\theta^2 D$$

$$(-\;parity)\quad \Phi=\frac{1}{\sqrt{2}}(\Sigma+iA_5)+\sqrt{2}\theta(-i\sqrt{2}\lambda_R)+\theta^2 F$$

where the identifications between 5d and 4d fields are

$$D=(X^3-D_5\Sigma)\quad F=(X^1+iX^2).$$

Bulk propagator

- Example of Bulk propagator from the fermions with kinetic terms

$$\sum_n \bar{\lambda}_L \sigma^\mu \partial_\mu \lambda_L + \sum_n \bar{\lambda}_R \sigma^\mu \partial_\mu \lambda_R$$

- using $\lambda(x,y)_L = \lambda(x)_L \sum_n \frac{1}{\sqrt{\ell}} Cos(\frac{n\pi y}{\ell})$

$$\langle \bar{\lambda}(x,y)\lambda(0,y')\rangle = \frac{1}{\ell}\sum_{n,m}\frac{\delta_{nm}}{p^2} Cos(\frac{n\pi y}{\ell}) Cos(\frac{m\pi y'}{\ell})$$

- A geometric sum of mass insertions from $\sum_n \lambda_L \partial_5 \lambda_R + \sum_n \bar{\lambda}_L \partial_5 \bar{\lambda}_R$ gives

$$\langle \bar{\lambda}(x,y)\lambda(0,y')\rangle = \frac{1}{\ell}\sum_{n}\frac{Cos(\frac{n\pi y}{\ell}) Cos(\frac{n\pi y'}{\ell})}{p^2 + (\frac{n\pi}{\ell})^2}$$

- From "brane to brane" gives

$$\langle \bar{\lambda}(x,0)\lambda(0,\ell)\rangle = \frac{1}{\ell}\sum_{n}\frac{(-1)^n}{p^2 + (\frac{n\pi}{\ell})^2}$$

sfermion mass formulas

- For $\delta(x_5)$ like fixed points translation invariance is broken $\rightarrow$ The currrents couple to all kk modes.

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4p}{(2\pi)^4} p^2 \sum_n \frac{(-1)^n}{p^2 + (\frac{n\pi}{\ell})^2} \sum_{\hat{n}} \frac{(-1)^{\hat{n}}}{p^2 + (\frac{\hat{n}\pi}{\ell})^2} \Omega(p^2/M^2)$$

- Matsubara summation of full kk tower

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \left(\frac{p\ell}{Sinh(p\ell)}\right)^2 \Omega(p^2/M^2)$$

- only zero modes. 4d limit

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \Omega(p^2/M^2)$$

- zero mode and first mode $m_1^2 = (\frac{\pi}{\ell})^2$ vector meson dominated

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \left(\frac{m_1^2}{p^2 + m_1^2}\right)^2 \Omega(p^2/M^2)$$

Essentially a form factor $f(p^2)$ times the 4d GGM answer.

Energy Scales

- Let's use an example of massless mode and $+1$ kk mode

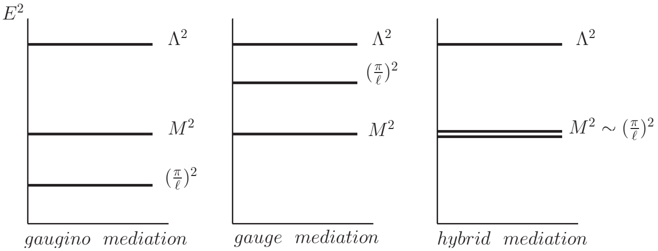


Figure: Relative mass scales that determine the sfermion mass

- If we introduce 1 kk mode mass scale $m_1 = (\frac{\pi}{\ell})$ (or vev of a Higgs)
- We find different regimes for the scalar soft masses
- We cannot reach hybrid mediation using a Taylor expansion in p^2

Full kk model compared to minimal model

- So how do the form factor behave?

- Matsubara summation

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \left(\frac{p\ell}{Sinh(p\ell)}\right)^2 \Omega\left(\frac{p^2}{M^2}\right)$$

- zero mode and first mode $m_1^2 = (\frac{\pi}{\ell})^2$

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \left(\frac{m_1^2}{p^2 + m_1^2}\right)^2 \Omega\left(\frac{p^2}{M^2}\right)$$

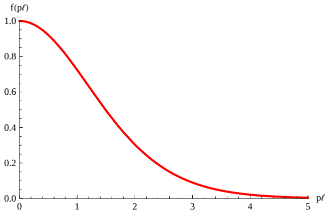


Figure: A plot of $f(p\ell) = (p\ell)^2 / \sinh^2(p\ell)$

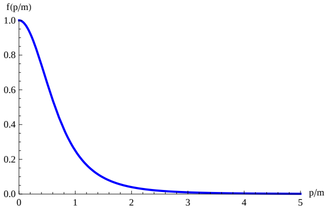


Figure: A plot of $f(\frac{p}{m_1}) = \left(1/(\frac{p^2}{m_1^2} + 1)\right)^2$

- Keypoint: The simpler model of 1kk mode captures the same essential physics as the full summation

Deconstruction of general gauge mediation

- The key point: This model is essentially the same as the truncated kk model of the 5d case.

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \left(\frac{m_1^2}{p^2 + m_1^2}\right)^2 \Omega(p^2/M^2)$$

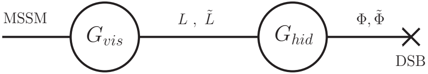


Figure: Two site lattice model

- $W = X\Phi\bar{\Phi} + K(L\bar{L} - v^2)$
- kk eigenstates $\tilde{A}_\mu^0, \tilde{A}_\mu^1$
- masses $m_0^2 = 0, m_1^2 = 2v\sqrt{g_1^2 + g_2^2}$
- diagonalise lattice eigenstates to mass eigenstates
- $m_k^2 = 8g^2v^2 \sin^2(\frac{k\pi}{N}) \quad k = 0, 1, ..., N-1$

- Bosonic sector is vector meson dominance model
- Can be realised from Seiberg duality dynamically (1008.2215)
- Suggests extensions to AdS

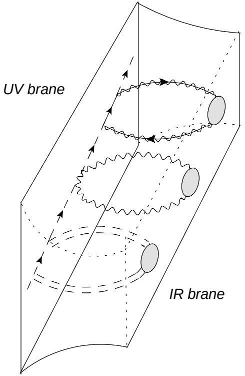
Warped general gauge mediation

This setup allows for any susy breaking sector to be located on the IR brane

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$V = -\theta\sigma^a\bar{\theta}\delta^\mu_a A_\mu + i\bar{\theta}^2\theta e^{-\frac{3}{2}k|y|}\lambda - i\theta^2\bar{\theta}e^{-\frac{3}{2}k|y|}\bar{\lambda} + \frac{1}{2}\bar{\theta}^2\theta^2 e^{-2\sigma}D$$

$$V = \sum_n \frac{1}{\sqrt{2\ell}}V_n(x)f_n^2(y)$$



$$S_{int} = 2g_5 \int d^5 x e^{-2k|y|}\int d^4\theta \mathcal{J} V\delta(y-\ell)$$

- An off-shell supersymmetric action using "theta-warping" $\tilde{\theta} = e^{-\frac{k|y|}{2}}\theta$, $e^a_\mu(x,y) = e^{-\sigma}\delta^a_\mu$
- The mass scale we introduce is k
- $m_n \sim (n - \frac{1}{4})\pi k e^{-k\ell}$
- Mass scales are naturally hierarchically small eg $\hat{F} = e^{-2k\ell}F$, $\hat{M} = e^{-k\ell}M$

Warped general gauge mediation

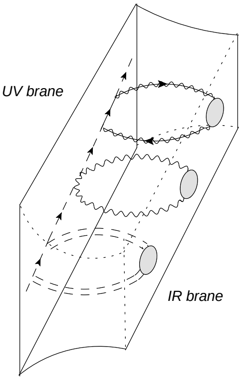
- propagator

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4p}{(2\pi)^4} \tilde{G}(0,\ell)\tilde{G}(0,\ell)p^2\Omega(p^2/M^2)$$

$$\tilde{G}(0,\ell) = \frac{1}{2\ell}\sum_n \frac{f_n^{(2)}(y)f_n^{(2)}(y')}{p^2+m_n^2}$$

- eigenmasses $m_n \sim n\pi k e^{-k\ell}$

warped eigenfunctions $f_n^2(y)$.

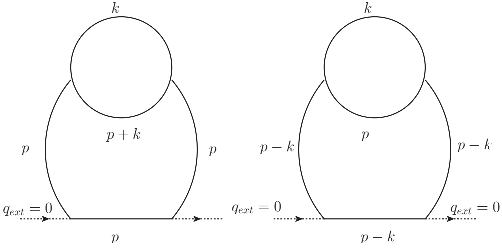


- 4d limit
- only zero modes mediate the message
- $m_\lambda \sim (\frac{\alpha}{4\pi})\frac{\hat{F}}{\hat{M}}$
- $m_{\tilde{f}}^2 \sim (\frac{\alpha}{4\pi})^2|\frac{\hat{F}}{\hat{M}}|^2$

- 5d limit
- all modes contribute
- $m_\lambda \sim (\frac{\alpha}{4\pi})\frac{\hat{F}}{\hat{M}}$
- $m_{\tilde{f}}^2 \sim (\frac{\alpha}{4\pi})^2 e^{-k\ell} f(k,\ell,M)|\frac{\hat{F}}{\hat{M}}|^2$

Accessing hybrid mediation

Labelling momenta in two loop diagrams



- The first case is typical for GGM. The second case mixes mass scales of the inner loop with the outer loop.
- Can be solved exactly for the case of 1kk mode below the UV cutoff e.g. minimal gaugino mediation
- (As long as one specifies a perturbative hidden sector eg $W = X\varphi\tilde{\varphi}$)

$$m_{\tilde{f}}^2 = (\frac{\alpha}{4\pi})^2 (\frac{F}{M})^2 S(x,y) \quad , \; x = \frac{F}{M^2} \quad , \quad y = \frac{m_1}{M}$$

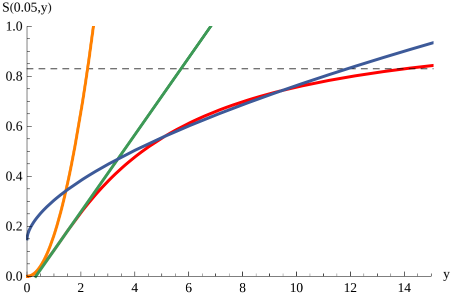
(This formula $S(x,y)$ is due to R.Auzzi & A.Giveon arXiv:1011.1664)

“The gaugino mediation variations”

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \left(\frac{m_1^2}{p^2 + m_1^2}\right)^2 \Omega(p^2/M^2)$$

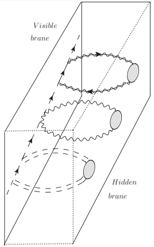
$$m_{\tilde{f}}^2 = \left(\frac{\alpha}{4\pi}\right)^2 \left(\frac{F}{M}\right)^2 S(x, y) \ , \quad x = \frac{F}{M^2} \quad , \quad y = \frac{m_1}{M}$$

- 4d limit (Dashed)
  $S = constant$
- Gaugino mediation limit/5d limit
  (Orange) $S \simeq y^2 \simeq \frac{1}{(M\ell)^2}$
- Hybrid regimes
  (Green) $S \simeq y \simeq \frac{1}{(M\ell)}$
- or (Blue)
  $S \simeq y^{1/n} \simeq \frac{1}{(M\ell)^{1/n}}$



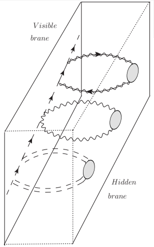
- Key message: We can have various scalar mass suppressions not just the gaugino mediated limit.

The End!



- summary messages
  - To achieve suppressed scalar soft masses we should introduce a mass scale $\simeq$ extra dimension.
  - These models don't have to lead to "gaugino mediation"
  - plenty of open/speculative questions....can we determine the UV theory from vector mesons? How much of AdS/QCD can be applied to susy breaking? Do more exotic things like black holes in AdS tell us something interesting about susy breaking or the hidden sector?

The End!



- summary messages
  - To achieve suppressed scalar soft masses we should introduce a mass scale $\simeq$ extra dimension.
  - These models don't have to lead to "gaugino mediation"
  - plenty of open/speculative questions....can we determine the UV theory from vector mesons? How much of AdS/QCD can be applied to susy breaking? Do more exotic things like black holes in AdS tell us something interesting about susy breaking or the hidden sector?

Thanks for listening

Appendices follow

Matsubara summation

- "brane to brane" propagator:

$$S = \sum_n h(k_5) = \frac{1}{2\ell} \sum_n (-1)^n \frac{1}{k^2 + (k_5)^2}$$

- We would like to remove the sum on k_5 so we can carry out an integration on only the k^2 momenta.
- Replace the sum with a complex auxiliary function $g(ik_5)$, that has poles at the sum values:

$$g(z) = \frac{\beta}{e^{(\beta z)} - 1} \quad , \quad \beta = 2\ell$$

- We apply the residue theorem

$$\oint \frac{dk_5}{2\pi} g(z) h(z) = \oint \frac{dk_5}{2\pi} \frac{1}{2\ell} \frac{2\ell}{e^{i2k_5\ell} - 1} \frac{e^{ik_5\ell}}{k^2 + (k_5)^2} = \sum \mathrm{Res}[g(z)h(iz)]|_{z=ik_5}$$

- choose a contour that only captures the poles at $k_5 = \pm ik$

$$S = \frac{1}{k \mathrm{Sinh}\, k\ell}$$

BACK

Higgs mechanism

- kk masses are generated by a super Higgs mechanism with $\partial_5 = vev$ (Ex. $U(1)$):

$$-\frac{1}{4}F_{MN}F^{MN} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}F_{\mu 5}F^{\mu 5}$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_\mu A_5 \partial^\mu A^5 - 2\partial_\mu A_5 \partial^5 A^\mu + \partial_5 A_\mu \partial^5 A^\mu)$$

Generating the KK tower

- The kinetic terms for the scalar linking fields $D_\mu = \partial_\mu + igA_{i\mu} - igA_{i+1\mu}$

$$\sum_i (D_\mu Q_i)^\dagger D^\mu Q_i = \sum_i (D_\mu Q_i)^\dagger D^\mu Q_i$$

- Expand around the vev (put in by hand)

$$Q_{i\alpha}^{\beta} = v\delta_{\alpha}^{\beta} + \phi_{i\alpha}^{\beta}$$

- This generates

$$\mathcal{L} \supset g^2 v^2 \sum_{i=0}^{N-1} (A_i^{a\mu} - A_{i+1}^{a\mu})^2 \quad \text{or} \quad \frac{1}{2} A_{i\mu}^a \mathcal{M}_{ijab}^2 A_j^{b\mu}$$

- Diagonalising this mass matrix gives

$$\tilde{A}_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} e^{i(\frac{2\pi jk}{N})} A_j$$

- with $m_k^2 = 8g^2 v^2 \sin^2(\frac{k\pi}{N}) \qquad k = 0, 1, ..., N-1$

Lattice propagator

- Gauge field mixed space propagator from lattice site k to q:

$$\langle p^2; k, q\rangle = \langle V(x)_k V(0)_q\rangle = \frac{\langle V_k V_q\rangle}{p^2}$$

- insert a closure relation $\mathbb{I} = \sum_j |\tilde{V}_j\rangle\langle\tilde{V}_j|$ with eigenmasses $m_j^2 = 8g^2 v^2 \sin^2(\frac{j\pi}{N})$
- then use $\langle\tilde{V}_j|V_q\rangle = \frac{1}{\sqrt{N}} e^{i(\frac{2\pi jk}{N})}$ to obtain

$$\langle p^2; k, q\rangle = \frac{1}{p^2}\sum_j \langle V_k|\tilde{V}_j\rangle\langle\tilde{V}_j|V_q\rangle = \frac{1}{N}\sum_j e^{-i(\frac{2\pi jk}{N})} e^{i(\frac{2\pi jq}{N})}\frac{1}{p^2}$$

- then a geometric sum of mass insertions gives

$$\langle p^2; k, q\rangle = \frac{1}{N}\sum_j e^{-i(\frac{2\pi jk}{N})} e^{i(\frac{2\pi jq}{N})}\frac{1}{p^2 + m_j^2}$$

BACK

Warped eigenfunctions

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

- The vector superfield equation of motion is
$$[e^{2\sigma}\eta^{\mu\nu}\partial_\mu\partial_\nu + e^{2\sigma}\partial_5(e^{-2\sigma}\partial_5)]V(x,y) = 0$$

- $V = \sum_n \frac{1}{\sqrt{2\ell}} V_n(x) f_n^{(2)}(y)$
$$f_n^{(s)}(y) = \frac{e^{s\sigma/2}}{N_n}\left[J_1(\frac{m_n e^\sigma}{k}) + b(m_n)Y_1(\frac{m_n e^\sigma}{k})\right] \quad , \quad N_n \simeq \frac{1}{\sqrt{m_n e^{-k\ell}\pi\ell}}$$

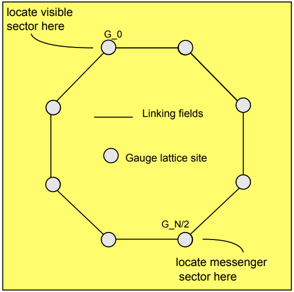
- Orthonormality
$$\frac{1}{2\ell}\int_{-\ell}^{\ell} e^{(2-s)\sigma} f_n^{(s)}(y) f_m^{(s)}(y) dy = \delta_{nm}$$

- Expanding $f_n^{(2)}(y)$ for large masses give
$$f_n^{(s)}(0) f_n^{(s)}(\ell) \simeq 4(k\ell)(-1)^n e^{-k\ell/2}$$

BACK

Deconstruction of general gauge mediation

- Each lattice site is a standard model parent gauge group $SU(5)$
- Lattice sites are linked together using bifundamental chiral superfields



	$SU(5)_0$	$SU(5)_1$	$\cdots$	$SU(5)_{N-2}$	$SU(5)_{N-1}$
Q_0	$\square$	$\overline{\square}$	$\cdots$	1	1
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Q_{N-1}	1	1	$\cdots$	$\square$	$\overline{\square}$
$\bar{\mathbf{5}}_{1,2,3}$	$\overline{\square}$	1	$\cdots$	1	1
$\mathbf{10}_{1,2,3}$	$\substack{\square\\\square}$	1	$\cdots$	1	1
H_d	$\square$	1	$\cdots$	1	1
H_u	$\square$	1	$\cdots$	1	1

Hidden sector

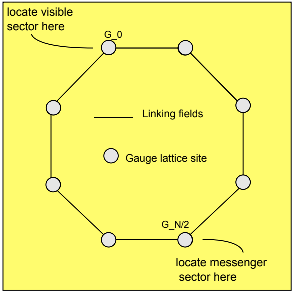
	$SU(5)_{\frac{N}{2}}$
X	1
φ	$\square$
$\tilde{\varphi}$	$\overline{\square}$

- $W = X\varphi\tilde{\varphi}$

Deconstruction of general gauge mediation

Key features:

$$m_{\tilde{f}}^2 = -g^4 \int \frac{d^4 p}{(2\pi)^4} p^2 \langle p^2; 0, \frac{N}{2} \rangle \langle p^2; 0, \frac{N}{2} \rangle \Omega(p^2/M^2)$$



- N lattice sites, spacing $a = \frac{1}{\sqrt{2}gv}$. $\ell = Na$
- diagonalise lattice eigenstates to mass eigenstates
- $m_k^2 = 8g^2 v^2 \sin^2(\frac{k\pi}{N}) \qquad k = 0, 1, ..., N-1$

$$\tilde{V}_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} e^{i(\frac{2\pi jk}{N})} V_j$$

- mixed space propagator

$$\langle p^2; k, q \rangle = \frac{1}{N} \sum_{j=0}^{N-1} e^{-i(\frac{2\pi jk}{N})} e^{i(\frac{2\pi jq}{N})} \frac{1}{p^2 + m_j^2}$$